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ON THE EVALUATION OF MATERIAL FORCES IN FRACTURE MECHANICS

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Summary. The sensitivity of the potential energy in inhomogeneous continua with respect to a so called "material motion" is studied. In particular, the energetic changes induced by the virtual extension of a crack is of interest. By varying the potential energy with respect to a change in the material domain measured from a fixed/reference configuration, a thermodynamically consistent crack-driving force is obtained, in a general geometric and material nonlinear setting. The mesh-sensitivity of the material force is examined for a single-cracked specimen under varying loading cases and material properties.

1 INTRODUCTION

The research on configurational, or material, forces dates back to the work of Eshelby[1] in the 1950's. Substantial contributions in this field were given by Maugin[2] and Gurtin[3]. The theoretical formulation and the numerical framework of material forces in fracture mechanics for hyperelastic material and geometrically nonlinear setting have been presented by Steinmann[4 5]. The consideration of energy dissipation from inelastic material response together with the energy changes induced by configurational changes have been described in Runesson et al.[6].

Starting from the expression for the potential energy in the material domain Ω_X, the variation of the respective field due to the virtual evolution of a crack is accounted for, according to Tillberg et al.[7]. For this purpose, an absolute/fixed configuration Ω_ξ is introduced (Fig. 1), which remains fixed during material motion as well as the spatial motion due to loading, whereby the general geometric and material non-linear setting of the problem is emphasized. In the absence of material volume forces or surface tractions, the potential energy Π equals the integrated volume-specific free energy ψ, which is a function of the deformation gradient $\boldsymbol{F}$ and the state variables in $\underline{\boldsymbol{k}}$, as shown in (1).

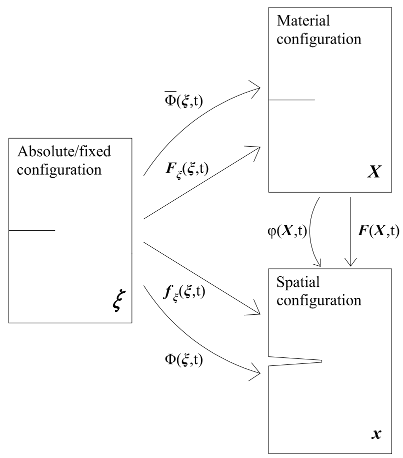


Figure 1: Material, spatial and absolute configurations considered, allowing for evolving topologies via $\overline{\Phi}(\xi, t)$.

$$\Pi_{int}(\boldsymbol{X}) = \int_{\Omega_X} \psi_X(\boldsymbol{F}, \underline{\boldsymbol{k}}) d\Omega_X \tag{1}$$

Upon introducing the configurational changes in a variational formulation, the directional derivative of (1) with respect to an arbitrary change in the Lagrangian domain $\delta\boldsymbol{X}$ is determined via the transformation of the integration domain to the absolute domain Ω_ξ.

2 CRACK-DRIVING FORCE

The mathematical consideration of the material forces presented briefly in Section 1 results in the definition of a thermodynamically consistent crack-driving force $\boldsymbol{G}$, which, as shown in Runesson et al.[6], can be split in two parts: the configurational $\boldsymbol{G}^{\text{CONF}}$ and the dissipative $\boldsymbol{G}^{\text{MAT}}$. Introducing the parametrization of $\delta\boldsymbol{X}$ as $\delta\boldsymbol{X} = \overline{W}\delta\boldsymbol{a}$, where $\overline{W}$ is a weight function and $\delta\boldsymbol{a}$ the virtual extension of the crack, we obtain

$$\boldsymbol{G} = \boldsymbol{G}^{\text{CONF}} + \boldsymbol{G}^{\text{MAT}} \tag{2}$$

where the configurational force can be expressed as

$$\boldsymbol{G}^{\text{CONF}} = \int_{\Omega_X} \left[-\boldsymbol{\Sigma} \cdot \left(\boldsymbol{\nabla}_X \overline{W}\right)\right] d\Omega_X \tag{3}$$

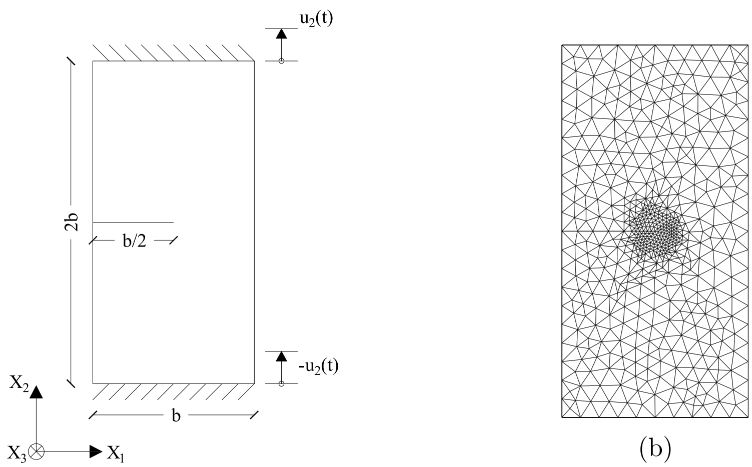
while the dissipative force takes the form

$$\boldsymbol{G}^{\text{MAT}} = \int_{\Omega_X} \underline{\boldsymbol{K}} \star [\underline{\boldsymbol{k}} \otimes \boldsymbol{\nabla}_X] \cdot \overline{W} d\Omega_X \tag{4}$$

Note that in (3), the Eshelby stress tensor, $\boldsymbol{\Sigma} = \psi \boldsymbol{I} - \boldsymbol{F}^T \partial\psi/\partial\boldsymbol{F}$, has been introduced. In (4), the "dissipative" stresses term $\underline{\boldsymbol{K}} = -\partial\psi/\partial\underline{\boldsymbol{k}}$ arises, where $\underline{\boldsymbol{k}}$ accounts for the internal variables that determine the constitutive state of the material.

3 NUMERICAL EXAMPLE

A simple two-dimensional single-cracked plate is considered under mode I deformation, as shown in Fig. 2. As to the constitutive model, hyperelasto-plasticity with isotropic linear hardening in the post-yielding region is adopted. Firstly, equilibrium of the direct motion problem is sought for. Upon convergence of each load step, the field and internal variables of interest are stored. Then, as a post-processing step, the configurational and the "dissipative" part of the previously defined crack-driving force are computed, according to (3) and (4).



(a) (b)

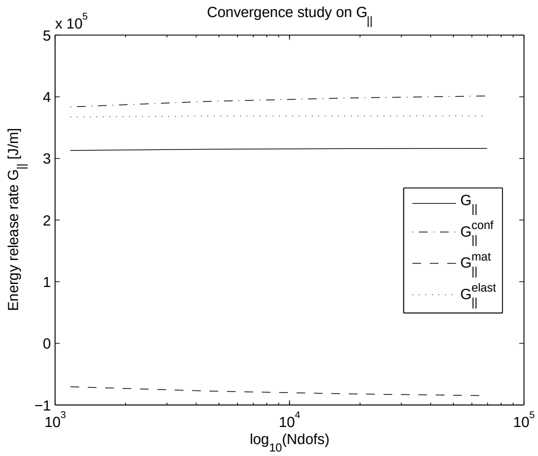
Figure 2: Edge-cracked specimen. (a) Continuous domain. (b) Discretization.

The convergence properties of the crack-driving force $\boldsymbol{G}$ and its components $\boldsymbol{G}^{\mathrm{CONF}}$ and $\boldsymbol{G}^{\mathrm{MAT}}$ are depicted in Fig. 3. The tangential and the perpendicular components of $\boldsymbol{G}$, denoted $G_{\|}$ and $G_{\perp}$, respectively, are considered separately.

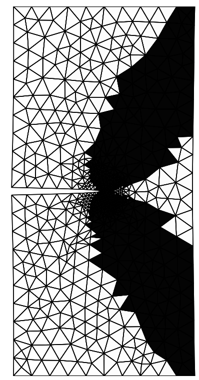
4 CONCLUSIONS

At the present, a thermodynamically consistent crack-driving force has been implemented in a two-dimensional finite element model in a geometrical and material nonlinear setting. The material constitutive model used was hyperelasto-plasticity.

From the study of a single-cracked specimen under mode I deformation (Fig. 2), it can be observed for the tangential component of the crack-driving force shown in Fig. 3a, that the model works well for hyperelastic material response. At the respective case, only ${G_{\|}}^{\mathrm{CONF}}$ is present. As regards elasto-plasticity, a lower value for $G_{\|}$ is obtained, as reported also in Tillberg et al.[7]. The divergence of the separate components comprising the total force $G_{\|}$ should be noted, due to their sensitivity with mesh refinement. The divergent subparts of $G_{\|}$ result in a total force that behaves well under mesh refinement, thus a force which can be used in simulations, as long as its constitutive parts, ${G_{\|}}^{\mathrm{CONF}}$ and ${G_{\|}}^{\mathrm{MAT}}$, are limited.



(a)



(b)

Figure 3: Convergence of crack-driving force. (a) Tangential component. (b) Plastic deformation.

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