On Cooperative Control of Automated Driving Systems from a Stability and Safety Perspective

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To my family
Abstract

Over the last few decades, congested traffic network have become a serious problem in many countries. Congestions result in time losses, increase of fuel consumption and also raise the risk of accidents. Intelligent transportation systems may contribute to mitigate such problems. Advancement together with the reduction in cost of embedded computing, on-board vehicle sensors and wireless communication paved the way for introduction of automated driving systems. Vehicle platooning is an example of an automated driving systems which can be implemented to improve the traffic situation.

To enable vehicle platooning with short inter-vehicles distances a control strategy is required that can guarantee passenger safety, comfort and stability of the platoon, so called string stability. While string stability is naturally defined in the frequency domain, stating safety and comfort requirements and vehicle limitations is more convenient as time domain specifications. Hence, fulfilling all the requirements and specifications simultaneously is not a trivial task.

This thesis deals with the development of distributed model-based control strategies for a vehicle platoon. The aim of the control strategy is to enable platooning with a short inter-vehicle distance while fulfilling string stability criterion and maintaining safety and comfort. To achieve this, two approaches are proposed, i) translating string stability criterion into time domain requirement and ii) combining frequency domain control design techniques with Model Predictive control framework into a single control problem. Particular attention is given to ease the proposed methods for real time implementations. The control design is decoupled into longitudinal and lateral motion control and the methods presented can guarantee string stability and constraint satisfaction in both motion directions. Furthermore, a safety verification method based on reachability analysis technique and invariant set theory is proposed for safety analysis of automated driving systems for a given controller. The findings in this thesis are verified through simulations and field experiments.

Keywords: Intelligent Transportation, Platooning, String Stability, Distributed Control, Reachability Analysis.
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List of publications

This thesis is based on the following five appended papers:

Paper 1


Paper 2


Paper 3

R. Kianfar, P. Falcone and J. Fredriksson, “A Distributed Model Predictive Control (MPC) Approach to Active Steering Control of String Stable Cooperative Vehicle Platoon”, Proceeding of the 7th IFAC Symposium on Advances in Automotive Control, September 2013, Tokyo, Japan.

Paper 4

List of publications

Paper 5


Other publications

In addition to the appended papers, the following papers are also written by the author of the thesis:


# Contents

Abstract  

Acknowledgements  

List of publications  

Contents

## I  Introductory chapters

### 1  Overview

1.1 Platooning state of the art  
1.2 Enable platooning  
1.2.1 Challenges in control problem  
1.3 Thesis contributions  
1.4 Thesis outline

### 2  Advanced driver assistance systems

2.1 Fully autonomous driving  
2.2 Automated driving (vehicle platooning)  
2.2.1 Longitudinal vehicle automation  
2.2.2 Lateral vehicle automation  
2.2.3 Challenges in vehicle automation

### 3  Vehicle dynamics modeling

3.1 Longitudinal dynamics  
3.2 Lateral vehicle dynamics  
3.3 Global frame

### 4  Cooperation topology and stability of vehicle platoon

4.1 Cooperation topology  
4.2 String stability

vii
Contents

4.2.1 Mathematical preliminaries .................................... 25
4.2.2 Norm measure ..................................................... 25
4.2.3 Cooperation topology ........................................... 26
4.2.4 Spacing policy .................................................... 27
4.2.5 Homogeneity vs heterogeneity ................................ 28
4.2.6 Longitudinal vs lateral direction ............................. 28
4.3 Appendix ............................................................ 29

5 Tools ........................................................................ 31
5.1 Receding horizon control ........................................... 31
5.2 Reachability analysis and invariant set theory .................. 34

6 Summary of included papers ........................................ 39

7 Concluding remarks and future research directions .......... 45
7.1 Concluding remarks .................................................. 45
7.2 Future research directions ......................................... 47

References .................................................................... 49

II Included papers

Paper 1 Design and Experimental Validation of a Cooperative
Driving System in the Grand Cooperative Driving Chal-
lenge ........................................................................ 59
1 Introduction ............................................................... 59
2 Problem statement/GCDC context .................................. 61
  2.1 Urban scenario ....................................................... 61
  2.2 Highway scenario ................................................... 62
  2.3 Evaluation criteria ................................................. 62
  2.4 Safety requirements .............................................. 64
3 Overview of the cooperative driving system architecture .... 64
  3.1 System inputs ......................................................... 64
  3.2 System outputs ...................................................... 66
  3.3 Functions overview .............................................. 66
4 Communication .......................................................... 67
5 Sensor fusion ............................................................. 69
6 Control ...................................................................... 73
  6.1 Vehicle modeling .................................................... 73
  6.2 Control problem statement and requirement satisfaction 75
  6.3 Controller design ................................................... 76
Contents

7 Results ........................................ 79
  7.1 Vehicle model identification .................. 79
  7.2 Simulation results ............................. 80
  7.3 Experimental results ......................... 85
8 Conclusions .................................. 86
9 Acknowledgements .............................. 89
References ...................................... 89

Paper 2 A Control Matching Model Predictive Control Approach to String Stable Vehicle Platooning 93
1 Introduction .................................. 93
2 Vehicle Modeling .............................. 95
3 Constraints and Time Domain Requirements .... 97
  3.1 Safety: ..................................... 97
  3.2 Performance: ................................ 97
  3.3 Acceleration requirement: .................... 97
  3.4 Actuator limitations: ......................... 98
  3.5 Desired velocity range: ...................... 98
  3.6 String stability: ............................ 98
4 Control Problem Formulation ................... 99
  4.1 String stable controller ...................... 99
  4.2 Model predictive controller ................. 100
5 Feasibility and Stability ....................... 104
  5.1 Preliminaries ................................ 104
  5.2 Persistent feasibility ......................... 105
  5.3 Asymptotic stability ......................... 106
6 Simulations and Experimental Results .......... 107
  6.1 Description of the Experimental Set-up .... 107
  6.2 Scenario 1 (vehicle following) ............. 110
  6.3 Scenario 2 (string stability) ............... 111
  6.4 Scenario 3 (constraint satisfaction) ....... 115
7 Conclusion ................................... 119
References ................................... 119

Paper 3 A Distributed Model Predictive Control (MPC) Approach to Active Steering Control of String Stable Cooperative Vehicle Platoon 125
1 Introduction .................................. 125
2 Vehicle Modeling .............................. 127
  2.1 Inter-vehicle dynamics ....................... 127
  2.2 Actuator dynamic ............................ 128
  2.3 String stability ............................. 129

ix
Paper 4 Combined Longitudinal and Lateral Control Design for String Stable Vehicle Platooning within a Designated Lane

1 Introduction .............................................. 143
2 Modeling ................................................... 144
  2.1 Longitudinal vehicle dynamics ......................... 146
  2.2 Lateral vehicle dynamics .............................. 147
3 Design Requirements and Limitations ...................... 147
  3.1 Frequency domain requirements (longitudinal) .......... 147
  3.2 Time domain requirements and limitations (longitudi-
                nal) ........................................... 148
  3.3 Time domain requirements and limitations (lateral) . 149
4 Control Design ............................................ 149
  4.1 String stable longitudinal control ..................... 150
  4.2 Corrective constraint satisfier ........................ 151
  4.3 Lateral Control ....................................... 152
5 Results .................................................... 154
  5.1 Longitudinal tracking performance and string stability 154
  5.2 Longitudinal constraint satisfaction ..................... 156
  5.3 Lateral control performance ........................... 158
6 Concluding Remarks ...................................... 158
References .................................................. 158

Paper 5 Safety Verification of Automated Driving Systems

1 Introduction .............................................. 163
2 Background and preliminaries on Reachability analysis and
   invariant set theory ..................................... 165
3 ACC design example ...................................... 167
  3.1 Modeling ............................................. 168
  3.2 Control design ....................................... 169
4 Safety and performance requirements ........................ 171
5 Safety verification of automated driving systems (ACC) ...... 173
  5.1 Minimum achievable safe inter-vehicle distance ....... 174
  5.2 Persistent constraint satisfaction (Maximal Admissi-
                  ble Safe set) .............................. 175
Contents

5.3 Robustness analysis .................................. 175
5.4 Time delay ........................................... 177
6 Results and discussions ................................. 178
  6.1 Simulation results .................................. 178
  6.2 Robustness analysis ................................. 183
  6.3 Experimental results ................................. 183
  6.4 Vehicle model identification ......................... 185
  6.5 Emergency braking scenario ......................... 186
7 Conclusion .............................................. 188
8 Acknowledgements ....................................... 191
Preliminaries and background ............................ 191
References ................................................ 193
Part I

Introductory chapters
Chapter 1

Overview

The increase of number of vehicles in daily traffic as well as the transportation growth lead to traffic flow problems, like congestions. Over the last few decades congested traffic network have become a serious problem for the society. Congestions result in time losses, increase the air pollution by increasing the CO\textsubscript{2} level, increase the fuel consumption and increase the possibility of fatal accidents. According to the European Union Road Federation 2010, the tonne-kilometer on the EU-27 road network has grown by 45.5\% over the period 1995-2008, at a rate of 2.9\% per year. Similarly, the passenger-kilometer has grown by 21.4\%, at a rate of 1.5\% per year. In 2008, this growth led to shares of 72.5\% and 72.4\% of the total inland EU-27 transportation of goods and passengers, respectively, to take place on the road network. In a separate report from the European Union Commission it is stated that 40000 people die in road accidents every year. This report also states that road accidents are the main cause of death for people under the age of 45 in Europe. Considering congestions problem and the problems associated to that, motivates questioning whether the existing road network has, at the current growth rate, the capacity to meet the future demands for safe road transportation of both goods and passengers. The non-stopping demand for more transportation, requires developing more road and road infrastructure. However, in mega-cities which are mainly subjected to congestions problem, developing the road network is not a feasible solution anymore. Fortunately, thanks to advances in vehicular, communication and information technologies traffic congestion can be alleviated, by enabling cooperation among vehicles to better exploit the usage of existing roads capacity.

Intelligent Transportation Systems (ITS) in a wide sense refers to advances in the infrastructure unit, roadside units and also intelligent vehicle, see Figure 1.1. Automated driving is an example of intelligent transportation system, which consists of hardware and software mechanisms enabling
Chapter 1. Overview

Figure 1.1: Intelligent transportation system (ITS), courtesy of the U.S. Department of Transportation

automated driving of multiple vehicles (vehicle platoons).

1.1 Platooning state of the art

Driver’s reaction is subjected to error and time delay, i.e. it takes some time until the driver reacts to changes in the environment. In addition even for a driver with full attention is not trivial to find the optimal maneuver in terms of safety and fuel consumption. Driver’s reactions usually is made based on her limited perception of the environment, e.g. eye sight and sounds. Consequently delay in the reaction and limitation in the information that a driver has access to result in a non-optimal decision made by the driver. This can have a great impact on the traffic flow, which can result in collisions or other undesired phenomena in traffic. Automated driving or in particular vehicle platooning can instead, help to mitigate congestions problem and increase passengers’ safety. In a vehicle platoon, a chain of vehicles follow each other in an automated way. The first vehicle in the platoon is called the leader and the rest of vehicles are called followers. In the simplest case, every vehicle in the platoon measures its position with respect to its preceding vehicle using on-board sensors e.g. radar, lidar or camera and maintain a safe distance to its preceding vehicle by controlling its velocity. While in a more advanced version vehicles are also equipped with wireless communication which allows sharing critical information regarding the dynamics vehicles states or/and the road ahead. An example of platooning in a real scenario can be seen in Figure 1.2.
1.1. Platooning state of the art

The idea of platooning can be traced back to the eighties when the California Partner for Advanced Transportation System (PATH) was established to develop and investigate the impact of vehicle-highway cooperation and communication systems [1]. Since then, this idea has been further studied and developed by many researchers, see e.g. [2–5]. Vehicle platooning has also been studied through a few real world experiments. One of the first real world experiment was carried out at PATH California where a longitudinal vehicle platoon of four vehicles was tested in 1994. Later in 1998 a vehicle platooning with eight fully automated vehicle was tested. In similar project entitled KONVOI, longitudinal and lateral controller developed and tried on a platoon of five trucks [6]. Advances in vehicular technology and communication systems led to further study the benefit of vehicle platooning. SARTRE, a European Commission Co-Funded established to further study the feasibility of implementing vehicle platooning in public motorways [7]. In 2011 Grand Cooperative Driving Challenge (GCDC) teams from industry and academia competed in both urban and highway driving scenario [8]. Further details on the history of vehicle platooning can be found in [9]. The main advantages of platooning are:

- **increased traffic throughput**: automated driving or platooning can reduce the inter-vehicle distance between vehicles which results in a better usage of road capacity. The result of a recent study, [10], shows that the highway capacity can be increased up to 43% if all the vehicles in the highway enable platooning using on-board sensors (camera and radar). The same study shows that this figure can boost up to 273% providing that the vehicles also use wireless communications.

- **reduced fuel consumption**: by reducing the inter-vehicle distance between the vehicle the aerodynamic drag is also decreased. Reduction
in the aerodynamic drag result in reduction of the fuel consumption. The result of a study, [11, 12], show that the fuel consumption can be reduced up to 7% for a vehicle platoon with only two trucks. In addition to decreasing the inter-vehicle distance, platooning can also contribute to reduction in fuel consumption by avoiding unnecessary acceleration and deceleration.

- **reduced air pollution**: apparently reducing the fuel consumption leads to reduction of air pollutant as well.

- **increased safety**: human reaction is naturally subjected to delay and the delay depends on the cognitive status of the driver. Statistics shows that human error is the main source of accident in almost 90% of the car accident. Hence, platooning can also mitigate accidents by enabling a safe inter-vehicle distance between the vehicles.

- **increased comfort**: driving a car can be as unpleasant as it is sometimes enjoyable. Every driver waste a significant amount of time while driving in a congested road. Platooning provides the opportunity to the driver to be relaxed or even let the driver spend the time on more desirable task, e.g. reading news, surfing.

### 1.2 Enable platooning

To drive with a short inter-vehicle distance and without jeopardizing the safety demands a delicate engineering design. To accomplish platooning, interactions between three modules, i.e. communication, sensing and control, are required. Thanks to advances in wireless communication, vehicles can send/receive information to/from other vehicles, i.e. vehicle-to-vehicle (V2V) communication and also the infrastructure, i.e. vehicle-to-infrastructure (V2I) communication. The communication messages can be transmitted in real-time, fail-safe and reliable way based on the communication standard protocol, IEEE.11p. Wireless communication can be used to enhance the sensing module. In other words, wireless communication can provide additional useful information to the control modules which goes beyond the information that can be measured using on-board sensors. The sensing module consists of on-board sensors and positioning devices such as GPS and compass. On-board sensors, e.g. radar, camera and lidar which can provide information about the adjacent vehicles. With the use of advanced GPS technology, position of all the vehicles in the platoon is available to the rest of platoon with a centimetre position accuracy. The information delivered by the communication module (V2V or V2I) and the
measured information from the sensing module are fused to obtain a good estimate of desired quantities. Then, the fused information are sent to the control module which is responsible for decision making. The control unit is responsible for maintaining a desired safe distance to the preceding vehicle. The control action is sent to the vehicle actuators which are i) powertrain and brake in case of longitudinal control and ii) steering wheel in case of lateral control. An overall system architecture of an automated vehicle in a platoon is depicted in Figure 1.3.

### 1.2.1 Challenges in control problem

Driving in a close distance put a high demand on the controller. The controller should be able to guarantee safety and comfort of the passengers while respecting the limitations in the actuators. Furthermore when vehicles moves as a chain (vehicle platoon) not only the stability of individual vehicle should be considered in the control design but also the stability of the whole platoon together plays a key role on the overall performance of the platoon. The platoon stability is referred as string stability and defined as the capability of a platoon in attenuating disturbances in position error, velocity error and acceleration as the disturbances propagate towards the tail of the platoon [13–16]. The concept of string stability will be discussed further in detail in Chapter 4. String instability can introduce shock waves in traffic flow which may result in collisions. However, string stability cannot solely guarantee the safety, comfort and performance. For example in a heterogeneous vehicle platoon (platoon with non-identical vehicles or/and control structure), suppression of a desired signals, e.g. acceleration does not guarantee the suppression in spacing error. Hence, as the inter-vehicle distance reduces safety becomes a critical issue to consider. In this thesis two approaches are proposed to guarantee safety of string stable controllers. In Papers 1-4, a constraint optimal control framework is used to explicitly account for safety, comfort and other desired specifications directly in the design stage. Considering different sources of uncertainty in an automated driving system, e.g. delay in communication, measurement noise and uncertainty in the actuator model, the need for a verification method becomes vital. In paper 5, a posteriori model-based verification method based on set theory is proposed to verify if a string stable controller guarantee safety, comfort and other specifications.
1.3 Thesis contributions

The aim of this thesis is i) to develop a distributed/decentralized control strategy which can enforce string stability of a vehicle platoon in longitudinal and lateral direction and simultaneously accounts for different constraints arising from specifications and limitations in the control problem. The main challenge lies in the fact that combining frequency specification (string stability) and time domain specification (safety, actuator limitations, etc.) into a single control problem is not trivial, ii) to propose a safety verification method which can be used to safety verification of a given automated/cooperative controller in presence of model uncertainty, measurement noise and delays. The main contributions of the thesis are as follows,

- presenting results on the development of controllers both in longitudinal and lateral direction to enable vehicle platooning. The novelty with control design lies in combing string stability, safety, comfort and actuator limitations into a single control design.

- three approaches to achieve string stability and constraint satisfaction are proposed, i) translation of string stability definition from frequency domain into time domain, Papers 1 and 3 ii) using a control matching technique to match the behaviour of an MPC controller to a string stable frequency domain-based designed controller, Paper 2 iii) to use an MPC-based ad-hoc controller which can correct the control
command of a desired string stable controller when the constraints are active Paper 4.

- a linear control strategy for the combined longitudinal and lateral control is proposed which enable vehicle platooning within a designated lane. By scheduling over the longitudinal velocity, the nonlinear model is divided to an LTI and LPV systems describing longitudinal and lateral dynamics, respectively, details are given in Paper 4.

- to avoid extensive simulation and expensive experiments, a mathematical framework based on the reachability analysis and set theory is proposed to safety verification of automated driving system. The method is in particular useful for the cases that controller cannot explicitly guarantee the fulfilment of specifications and requirements, e.g. safety. Such method is presented in Paper 5

- the proposed verification method is applied to study the impact of providing additional information e.g. acceleration of preceding vehicle to the controller on the inter-vehicle spacing error. The method is also extended to account for model mismatch, measurement noise and delays.

- the results presented in Papers 1, 2, 4 and 5 of this thesis are experimentally validated using prototype vehicles.

### 1.4 Thesis outline

This thesis consists of two parts. Part I, provides context and a brief background for the second part. Part II includes five papers which serve as the core for the thesis. Part I comprises seven chapters. Chapter 1 provides an introduction to platooning which helps the reader to become familiar with the concept of automated driving. In Chapter 2 an overview of advanced driver assistance systems with emphasize on autonomous and cooperative systems is given. Chapter 3 gives a brief background on vehicle modeling. In Chapter 4, important properties of a vehicle platoon, i.e. cooperation topology and string stability are introduced. A brief survey on the various definition of string stability is presented as well. Chapter 5 gives an overview of mathematical tools used in this thesis. A summary of the appended papers is in Chapter 6. At the end, Chapter 7 finalizes Part I with concluding remarks and future works.
Chapter 2

Advanced driver assistance systems

Over the last two decades advances in the vehicular technology, communication and control systems have led to introduction of several advanced functionalities by the automotive industry, e.g. Anti Blocking System (ABS), Vehicle Stability Control (VSC) and Lane Departure Warning (LDW), Figure 2.1. Such systems are usually referred as Advanced Driver Assistance Systems (ADAS) by the automotive industry. The primary objectives of ADAS are to assist the driver either when the safety is endangered or when the driver demands more comfort. However, recent research and rapid development of sensing technology and embedded computing have made it possible to have automated vehicles with a level of autonomy which goes far beyond the capability of standard ADAS systems.

Figure 2.1: (a) Lane departure warning system (b) Camera mounted on the back of mirror.
Chapter 2. Advanced driver assistance systems

2.1 Fully autonomous driving

The dream of having fully autonomous vehicles has been initiated a few decades ago. However, this dream did not come true until almost ten years ago when the first DARPA Grand Challenge (2004) was held in the Mojave Desert in the US. In that challenge several teams from leading universities and companies competed against each other. The idea was to develop fully autonomous vehicles which could travel a certain route in the desert without any interaction with humans. Even though none of the teams managed to finish the competition in that event, the first serious attempt for having fully autonomous vehicle was made. In DARPA Grand Challenge (2007, Urban Challenge) several teams competed over a course of 96 km in urban area. Vehicles were supposed to obey all traffic rules while avoiding collision with any possible obstacles. [17].

One of the successful team which participated in DARPA is the Google driverless car, [18]. In Figure 2.2 the Google car is shown. The vehicle is equipped with a laser scanner on the roof which is rotating and provides a map of the objects surrounding the vehicle. Furthermore, the vehicle is also equipped with three radars, camera, orientation sensor and etc.

However, having such fully autonomous vehicles in every day transportation systems, that can operate in all traffic situations seems a bit unrealistic at the moment. The main reason is the high cost of having so many expensive sensory systems on normal vehicles. Furthermore, safety verification of such autonomous vehicles requires tremendous amounts of work.

2.2 Automated driving (vehicle platooning)

Automated driving or in particular vehicle platooning can be considered as an intermediate step to fill the gap between manual driving and fully autonomous driving. In vehicle platooning, vehicles can operate fully or semi-autonomously in some part of the route, e.g. highways and return the control back to the driver when needed. The idea is to exploit the already available sensing module and actuators in production to have a cost efficient product.

2.2.1 Longitudinal vehicle automation

A well known example of a semi-autonomous system is adaptive cruise control (ACC), which was launched in 1995’s by the car maker Mitsubishi. ACC is an enhanced version of cruise control (CC). In an ACC system, the relative distance and velocity between two adjacent vehicles are measured.
2.2. Automated driving (vehicle platooning)

![Figure 2.2: (a) Google first prototype vehicle (b) Google autonomous car.](image)

Using a radar or lidar, Figure 2.3(a). Then, based on these measurements, the control unit maintains a safe distance between vehicles by controlling the accelerator and brake pedal. ACC has a hierarchical control architecture, meaning that, the controller consists of an upper and a lower level as depicted in Figure 2.4. The upper level controller is responsible to determine the desired acceleration while the lower level controller is responsible to provide the commanded acceleration.

![Figure 2.3: (a) A radar based ACC system (b) Lane keeping system.](image)

**Impact of ACC and CACC on traffic efficiency**

The ACC systems proved to have a positive impact not only on the safety and comfort but also on the traffic throughput as well. However, the normal ACC system suffer from the inability of enabling a string stable platoon. Furthermore, due to the rather large time gap that is usually chosen for ACC
Chapter 2. Advanced driver assistance systems

![Hierarchical structure of ACC diagram](image)

**Figure 2.4: Hierarchical structure of ACC**

systems, the effect on the traffic throughput is not significant. Hence, rapid advances in fast and reliable wireless communication, resulted in the introduction of a new system, the so-called Cooperative Adaptive Cruise Control (CACC). CACC can be seen as an add-on to ACC, which exploits wireless communication. Through wireless communication vehicles can exchange information such as maximum braking capability, intended acceleration and commanded control signal. Furthermore, wireless communication can provide information about the status and topography of the road ahead, which can have a great impact on the fuel economy of heavy duty vehicles, [12]. By providing additional information to the controller, wireless communication can contribute in maintaining the stability of the platoon as well [19]. The overall information provided by communication can be exploited to enhance controller which is responsible for decision making.

ACC and CACC may be used to enable platooning, [19, 20]. In Grand Cooperative Driving Challenge (GCDC) 2011, nine international teams competed against each other. The competition scenarios were made such that the performance of CACC systems proposed by different teams were evaluated in both urban and highway scenario, [21]. GCDC was one of the first event in which the efficiency of CACC was evaluated in a heterogeneous environment. Furthermore, over the last decade, the impact of (semi) au-
tonomous systems such as ACC and CACC on the traffic throughput and also their capability in alleviating congestion problem studied by several researchers, [22–24]. The results in [24] reveal that CACC systems can lead to drastic improvement in the traffic efficiency. The study in [24] is limited to passenger cars and is not considering any overtaking maneuver. However, it indicates that the traffic flow can increase from 2100 vehicle/hour/lane on a 100% manual highway to 2900 vehicle/hour/lane on a 20% manual, 20% ACC and 60% CACC equipped highway.

### 2.2.2 Lateral vehicle automation

Lane keeping system (LKS) is another functionality in high-end vehicles, which can be used to enable automated driving systems. An LKS automatically control the steering wheel in order to keep the vehicle within the lane markings and also makes the steering wheel turn in order to negotiate a curve. In an LKS, the lateral displacement of the vehicle w.r.t. the road center line is measured and is compensated by adjusting the steering wheel angle. To measure the lateral displacement either magnetometer sensors (look-down approach) or vision sensors (look-ahead approach) are used. The LKS based on magnetometer sensors was developed and demonstrated in 1996 by researcher at PATH, [25]. Later, car manufacturer, e.g., Nissan, Ford and Volvo started to develop LKS which are able to measure the lateral offset at a distance ahead of vehicles using vision sensors, see Figure 2.3(b). By combining the already existing ACC system with LKS, automation within a lane can be enabled. Volvo is going to launch a new system on their Volvo XC90 model called ACC+steer assist, which can be considered as a further step towards vehicle automation Figure 2.5, [26].

### 2.2.3 Challenges in vehicle automation

As stated, CACC shows a good potential to improve traffic flow by reducing the inter-vehicle distance between vehicles and also contributing in the stability of the platoon. However, we should note that reducing the inter-vehicle distance can introduce undesired effects, e.g. jeopardizing safety and string instability. Even though, most of the aforementioned studies indicate a good potential to improve traffic flow using CACC, there are still challenges such as string stability, robustness and safety verification to be addressed.

One solution to enable vehicle automation in the lateral direction is to use lane keeping systems, however, in a vehicle platoon with short inter-vehicle distance or in a snowy day, the road lane markings may not be visible by the vision sensors which can cause problems. An alternative
approach to LKS is a vehicle following approach, which each vehicle follows the predecessor vehicle rather than the lane markings. However, a poor design for such solution may result in instability of vehicle platoon in the lateral direction.

Figure 2.5: ACC+ steer assist function Volvo XC90.
Chapter 3

Vehicle dynamics modeling

In this chapter, the basics of vehicle dynamics are presented. The aim of this chapter is to provide a basic understanding of vehicle dynamics and the modeling assumptions that are used in this thesis. From a control perspective it is more desirable to work with linear models as long as they are good approximations of the real process. First, the longitudinal motion of a vehicle is modeled. Then, the model is extended to capture the lateral vehicle dynamics. Finally, fundamental limitations of the vehicle capabilities which, mainly arise from the tire friction is discussed. For further information on vehicle dynamics the reader is referred to [27, 28].

3.1 Longitudinal dynamics

The longitudinal motion of a vehicle can be modeled as a point mass using a force balance (Newton’s second law).

\[ m \ddot{v}_x = F_{xf} + F_{xr} - F_d - F_{roll} - F_g, \]  

(3.1)

where \( m \) and \( v_x \) are the mass and velocity of the vehicle, respectively. \( F_{xf} \) and \( F_{xr} \), \( F_d \), \( F_{roll} \) and \( F_g \) are the longitudinal tire forces at the front and rear tires, the aerodynamic drag, the rolling resistance and gravity forces, respectively. Figure 3.1 illustrates the forces acting on the vehicle. The longitudinal tire forces depend on 

i) the so-called slip ratio, 

ii) the normal load and 

iii) the friction coefficient between the tires and the road. The slip ratio is defined as,

\[ \kappa = - \frac{v_{xw} - \omega r_w}{v_{xw}}, \]  

(3.2)

where \( v_{xw} \), \( \omega \) and \( r_w \) are the longitudinal velocity at the axle of the wheel, rotational velocity of the wheel and effective tire radius, respectively, according to Figure 3.2. The typical characteristic of longitudinal tire force
Chapter 3. Vehicle dynamics modeling

Figure 3.1: Vehicle model

Figure 3.2: Tire modeling notation
3.1. Longitudinal dynamics

versus the slip ratio is depicted in Figure 3.3(a). As can be seen in the figure, for small slip ratio, the longitudinal tire force is proportional to the slip ratio,

\[ F_{xf} \simeq C_{\kappa f} \kappa_f, \]
\[ F_{xr} \simeq C_{\kappa r} \kappa_r, \]

where \( C_{\kappa f} \) and \( C_{\kappa r} \) are the longitudinal tire stiffness for the front and rear tire, respectively. However, for larger slip ratio the relation between longitudinal tire force and slip ratio becomes nonlinear, which requires a more sophisticated model, for further information of tire characteristic the reader is referred to [28].

![Figure 3.3: Tire forces (a) Longitudinal force as a function of longitudinal slip \( \kappa \), for different slip angles \( \alpha \) (b) Lateral force as a function of side slip angle \( \alpha \), for different slip values \( \kappa \).](image)

The aerodynamic drag force is denoted by \( F_d \) and is represented as

\[ F_d = \frac{1}{2} \rho C_d A_F (v_x + v_w)^2, \]

where \( \rho \), \( C_d \), \( A_F \) and \( v_w \) are the air mass density, drag coefficient, frontal area of the vehicle and wind velocity, respectively. The rolling resistance is denoted by \( F_{roll} \) and is proportional to the normal forces, i.e.,

\[ F_{roll} = f(F_{Nf} + F_{Nr}), \]

where \( F_{Nf} \) and \( F_{Nr} \) are the normal forces at the place of front and rear tires and \( f \) is the rolling resistance coefficient. Finally the gravitational force can be written as,

\[ F_g = mg \sin(\theta), \]
where $\theta$ is the slope of the road.

As described in Chapter 2, for control purposes, the longitudinal model of a vehicle is divided into an upper and a lower level. While the upper level model should capture the dynamics between the desired acceleration (upper level control input) and the actual acceleration that the vehicle delivers, the lower level model describes the dynamics between the acceleration and the actuator inputs (brake/throttle). The longitudinal acceleration dynamics can be described by the following nonlinear differential equation, \[29\],

\[
\dot{a}_x = f(v_x, a_x) + g(v_x)\eta,
\]

where $a_x$ is the longitudinal acceleration and $\eta$ is the control input of lower level controller. The functions $f(\cdot, \cdot)$ and $g(\cdot)$ are defined as,

\[
f(v_x, a_x) = -\frac{2K_d}{m}v_xa_x - \frac{1}{\tau(v_x)}[a_x + \frac{K_d}{m}v_x^2 + \frac{d_m}{m}]
\]

\[
g(v_x) = \frac{1}{m\tau(v_x)}
\]

where $\tau$, $K_d$ and $d_m$ are the engine time constant, aerodynamic drag coefficient and mechanical drag. As far as the low level controller is concerned, nonlinear control synthesis, e.g., feedback linearization control techniques can be used to calculate the actuator command for tracking the desired acceleration, \[30, 31\]. Utilizing a feedback linearization control law,

\[
\eta(t) = mu(t) + K_d v_x^2(t) + d_m + 2\tau(v_x)K_d v_x(t)a_x(t),
\]

in \(3.8\) gives the following system model,

\[
\dot{a}_x(t) = -\frac{1}{\tau}a_x(t) + \frac{1}{\tau}u(t),
\]

as far as the higher level controller is concerned this first order linear model is widely used in the literature. Equation \(3.11\) is a low pass filter which describes the imperfection of the lower level controller in tracking the desired acceleration. In Paper 1, 2, 3 and 5 of this thesis, the model described by \(3.11\) is used for the control synthesis and analysis.

### 3.2 Lateral vehicle dynamics

In the previous section, the longitudinal dynamics of the vehicle along its longitudinal axis is presented. In this section, the lateral dynamics of the vehicle is presented. Here, lateral dynamics refers to both the dynamics of vehicle along the axis perpendicular to the longitudinal axis of the vehicle.
3.2. Lateral vehicle dynamics

and also the yaw dynamics. First, we introduce the basic of the so-called bicycle model, which is a well accepted model to capture the lateral vehicle’s motion. A bicycle model assumes identical slip angles for the left and right wheel on each axis. However, this assumption is reasonable for negotiating curves of moderate radius at normal driving velocity. Hence, the bicycle model sometimes is referred to as the single track model as well. Applying Newton’s second law along the $y$ axis depicted in Figure 3.4,

\[ m a_y = F_{yr} + F_{yf} \]  \hspace{1cm} (3.12)

where $a_y$, $F_{yr}$ and $F_{yf}$ are the inertial acceleration of the vehicle along the $y$ axis at the central of gravity (c.o.g) and the lateral tire force of rear and front tires, respectively. The inertial acceleration $a_y$ consists of lateral acceleration $\dot{v}_y$ in the vehicle body frame and also the centripetal acceleration $v_x \dot{\psi}$. Hence, the inertial acceleration can be written as,

\[ a_y = \dot{v}_y + v_x \dot{\psi} \]  \hspace{1cm} (3.13)

where $\dot{v}_y$, $v_x$ and $\dot{\psi}$ are the acceleration in the body frame, longitudinal velocity in the body frame and the angular velocity of body frame coordinate in inertial frame. By replacing (3.13) in (3.12), the vehicle motion along

Figure 3.4: Vehicle model

the $y$ axis can be written as,

\[ m(\dot{v}_y + v_x \dot{\psi}) = F_{yf} + F_{yr} \]  \hspace{1cm} (3.14)

The lateral forces $F_{yf}$ and $F_{yr}$ are proportional to the so-called slip angle of front and rear tires, respectively. As can be seen in Figure 3.3, the slip
angle of the tire is defined as the angle between the orientation and velocity of the tire.

\[ \alpha = \arctan \left( \frac{v_{y \omega}}{v_{x \omega}} \right) \]  

(3.15)

where \( v_{y \omega} \) and \( v_{x \omega} \) are the lateral and the longitudinal velocity of the tire. Slip angle \( \alpha_f \) of the front wheel can be written as,

\[ \alpha_f = \delta - \theta_f \]  

(3.16)

where, \( \delta \) and \( \theta_f \) are the steering angle and the angle between the velocity vector of front wheel and longitudinal axis of the vehicle. Similarly, the slip angle of the rear tire can be described as,

\[ \alpha_r = -\theta_r \]  

(3.17)

where \( \theta_r \) is angle between the velocity vector of rear wheel and longitudinal axis of the vehicle. Therefore, the lateral forces of front and rear tires can be written as:

\[ F_{yf} = C_{\alpha f} (\delta - \theta_f) \]  

(3.18)

\[ F_{yr} = C_{\alpha r} (-\theta_f) \]  

(3.19)

where, \( C_{\alpha f} \) and \( C_{\alpha r} \) are the cornering stiffness of the front and rear tires, respectively. However, the proportionality relation between lateral force and slip angle only holds for small slip angles. For larger slip angles, more sophisticated models are required, [28]. Figure 3.3(b) shows the lateral force of tire versus the slip angle.

The yaw dynamics is described via the moment equation around the z axis in Figure 3.4,

\[ I_z \ddot{\psi} = F_{yf} l_f - F_{yr} l_r \]  

(3.20)

where \( I_z \), \( l_r \) and \( l_f \) are the moment of inertia around the z axis and the distances between the rear and front tires and c.o.g of the vehicle, respectively. Under the assumptions of i) small slip angles and ii) constant longitudinal velocity \( v_x \), the longitudinal and lateral vehicle dynamics can be decoupled and described by a set of linear differential equations. Harsher driving style results in larger slip angles where the tire forces enter to the nonlinear region. However, for the application considered in this thesis, i.e., vehicle platooning it is reasonable to assume that the vehicle only operates within the linear region of tires. A further study on the validity of linear model in describing the tire’s slip-force relation can be found in [32].
3.3 Global frame

The vehicle model developed in the previous section is based on a coordinate frame which is fixed to the vehicle. However, it might also be of interest to describe the vehicle’s motion with respect to the global coordinate frame as well. The vehicle’s equations of motion in the global frame are

\[
\begin{align*}
\dot{Y} &= v_x \sin(\psi) + v_y \cos(\psi) \\
\dot{X} &= v_x \cos(\psi) - v_y \sin(\psi)
\end{align*}
\]

(3.21) \hspace{1cm} (3.22)

where \(v_x\) and \(v_y\) are the longitudinal and lateral velocity of vehicle’s c.o.g. As can be seen from (3.21) and (3.22), the vehicle motion’s dynamics in the global frame is nonlinear, see Figure 3.5.

![Figure 3.5: Vehicle in inertial frame](image)
Chapter 4

Cooperation topology and stability of vehicle platoon

In this chapter, two important and interrelated concepts regarding to vehicle platooning are introduced. First, an overview of the cooperation topologies in a vehicle platoon is introduced. Different cooperation topologies may result in different choices of the control structure. Hence it can have a great impact on overall performance of a vehicle platoon. Secondly, string stability as an important concept in vehicle platooning is introduced and formally defined. Finally, we briefly discuss how the stability of a vehicle platoon can be affected by different choices of cooperation, inter-vehicle spacing policy and homogeneity vs heterogeneity.

4.1 Cooperation topology

As mentioned in Chapter 1, every vehicle in a platoon may be equipped with a wireless communication link, which can send and receive information to/from other vehicles in the platoon. In this thesis, there is no intention to give an extensive survey of different cooperation topologies, instead, the most common cooperation topologies in the literature are introduced. As can be seen from Figure 4.1, the simplest communication topology is when every vehicle in the platoon only receives information from its immediate predecessor and send information only to its immediate follower. This topology is preferable from an implementation point of view and is commonly adopted by CACC [19, 33]. The second common assumed communication topology for a platoon requires that all vehicles in a platoon receive information from the leader of the platoon in addition to their preceding vehicle. This configuration is illustrated in Figure 4.2. The advantage of this topology is that the extra information provided to the controller may result in
improved control performance [2]. This topology is particularly useful to obtain string stability for the spacing error, as is explained in the next section. However, such topology might require a more complicated control algorithm and also a wireless communication system with higher bandwidth. Another topology to consider is when every vehicle in the platoon exchanges information with both its preceding vehicle and its follower, see Figure 4.3. This type of structure is inspired by the way that a human driver drives. A human driver normally tries to adjust the speed and distance to the preceding car by constantly monitoring the preceding and the following car.

4.2 String stability

String stability is an important property, which refers to the capability of a platoon in attenuating any disturbance/error introduced by the leader or any other vehicle in the platoon. The disturbance and error can be considered, e.g., with respect to acceleration, [8], and position error between vehicles, [2], respectively. Hence, a platoon is string stable if any distur-
4.2. **String stability**

Bance/error with respect to the desired signal damps out as it propagates toward the tail of the platoon. In this section an overview of string stability w.r.t. different i) norm measures and ii) cooperation topologies, iii) inter-vehicle spacing is presented. Furthermore, string stability w.r.t. to the homogeneity vs heterogeneity and w.r.t. the direction of vehicle motion is studied. Note that the discussion presented in this chapter is limited to the string stability of linear systems, for study on string stability of nonlinear systems the reader is referred to, [34].

### 4.2.1 Mathematical preliminaries

String stability can be defined mathematically as a norm condition in the frequency domain w.r.t. different signals and norm measures. Considering \( \gamma_i, i = 1, \ldots, N \) as the desired signals to be suppressed in a platoon with \( N \) vehicles. Denote \( G_i(j\omega) \) as the transfer function between input \( \gamma_{i-1} \) to output \( \gamma_i \). The input-output relation can be described through convolution in the time domain.

\[
\gamma_i(t) = g(t) * \gamma_{i-1}(t),
\]

where \( g(t) = \mathcal{L}^{-1}(G_i(j\omega)) \) is the impulse response of the system. The peak norm (\( \mathcal{L}_\infty \)-norm) and the total energy (\( \mathcal{L}_2 \)-norm) of a scalar signal \( \gamma_i(t) \) are defined as,

\[
\| \gamma_i \|_\infty = \sup_{t \geq 0} |\gamma_i(t)|, \quad \| \gamma_i \|_2 = \left( \int_0^\infty \gamma_i(t)^2 dt \right)^{1/2}.
\]

The \( H_\infty \) of transfer function \( G_i(j\omega) \) is defined as,

\[
\| G_i(j\omega) \|_\infty \triangleq \sup_\omega |G_i(j\omega)| = \sup_{\gamma_{i-1} \neq 0 \in \mathcal{L}_2} \frac{\| \gamma_i \|_2}{\| \gamma_{i-1} \|_2}.
\]

The \( \mathcal{L}_1 \) norm of the impulse response \( g_i(t) \) is defined as,

\[
\| g_i \|_1 = \int_0^\infty |g_i(t)| dt = \sup_{\gamma_{i-1} \neq 0 \in \mathcal{L}_\infty} \frac{\| \gamma_i \|_\infty}{\| \gamma_{i-1} \|_\infty}.
\]

### 4.2.2 Norm measure

The following string stability criteria can be defined,

**Definition 1** A vehicle platoon is \( \mathcal{L}_2 \) string stable if the energy of the output signal is less than the energy of the input signal, i.e.,

\[
\| G_i(j\omega) \|_\infty \leq 1 \quad \forall i = 2, \ldots, N, \forall \omega,
\]

25
Chapter 4. Cooperation topology and stability of vehicle platoon

this gives,
\[ \| \gamma_i \|_2 \leq \| \gamma_{i-1} \|_2 \]

Definition 2 A vehicle platoon is \( L_\infty \) string stable if the maximum magnitude of the output signal is less than the maximum magnitude of the input signal, i.e.,
\[ \| g_i \|_1 \leq 1 \quad \forall i = 2, \ldots, N, \]
this gives,
\[ \| \gamma_i \|_\infty \leq \| \gamma_{i-1} \|_\infty \]

Definition 3 A vehicle platoon is string stable without overshoot in the frequency domain if (4.6) holds and the impulse response does not change sign, i.e., [35],
\[ \| g_i \|_1 \leq 1 \quad \forall i = 2, \ldots, N, \text{ and } g_i(t) \geq 0 \quad \forall t. \]

Note that the string stability condition (4.6) only guarantee that the maximum magnitude of the desired signal is attenuated. This does not say anything about the sign of the signals. Hence, even if this condition holds, in case of different signal signs, dangerous situations may occur.

Lemma 1 If the impulse response \( g_i(t) \geq 0 \), then the following holds [4],
\[ \| g_i \|_1 = \| G_i(j\omega) \|_\infty \]

Proof: From linear system theory and using the Laplace definition,
\[ | G_i(0) | \leq \| G_i(j\omega) \|_\infty \leq \| g_i \|_1, \]
\[ | G_i(0) | = \left| \int_0^\infty g_i(t) dt \right| \leq \int_0^\infty | g_i(t) | dt = \| g_i \|_1, \]
when \( g_i(t) \geq 0 \),
\[ \| g_i \|_1 = \| G_i(j\omega) \|_\infty \]

4.2.3 Cooperation topology

Depending on the type of cooperation topology and specifications, two definitions for string stability can be given, i.e., i) predecessor-follower string stability and ii) leader-followers string stability

Definition 4 (Predecessor-follower) A vehicle platoon is predecessor-follower \( L_2 \) string stable if
\[ \| \frac{\gamma_i(j\omega)}{\gamma_{i-1}(j\omega)} \|_\infty \leq 1 \quad \forall i = 2, \ldots, N, \forall \omega \]
4.2. String stability

Definition 5 (Leader-follower) A vehicle platoon is leader-followers $L_2$ string stable if

$$\left\| \frac{\gamma_i(j\omega)}{\gamma_1(j\omega)} \right\|_\infty \leq 1 \quad \forall i = 2, \ldots, N, \forall \omega \quad (4.13)$$

Note that condition (4.12) is more stringent compared to (4.13). Similarly $L_\infty$ and without overshoot string stability can be defined for the two topologies mentioned previously. To guarantee string stability, a controller should be designed such that condition (4.12) or (4.13) is satisfied. An example of $L_\infty$ string stable platoon with respect to acceleration signal is depicted in Figure 4.4. The red dashed signal is the acceleration of lead vehicle and as can be seen from the figure the acceleration signal is attenuating from the leader to the last follower (blue dashed signal).

![Figure 4.4: String stable platoon with respect to acceleration signal. The red dashed signal: acceleration of the lead vehicle and blue dashed: acceleration of the last follower.](image)

4.2.4 Spacing policy

Depending on the type of adopted spacing policy, string stability may or may not be achieved,

- **Constant spacing** A spacing policy where the vehicles in the platoon follow each other at a constant space.

- **Constant headway time** Headway time refers to the time that a vehicle can reach its preceding vehicle if travels at the current speed. A spacing policy where vehicles maintain a constant headway w.r.t. their preceding cars is called constant headway time policy.

It is known that for a constant spacing policy communication with the leader of the platoon is essential to obtain string stability, see e.g., [2], [36]
and [37]. This is a general result and holds for any linear controller due to the complementary sensitivity integral constraint. In [38] it is shown that even a bi-directional control strategy cannot overcome this problem. On the other hand for a constant headway policy string stability w.r.t. all aforementioned cooperation topologies and all different norm measure can be achieved without the need of communication with the leader, [13, 35]. In [5], the minimum required headway to achieve string stability with a unidirectional control structure is studied. Further studies on the spacing policies and its effect on the string stability and traffic flow capacity is given in [39].

4.2.5 Homogeneity vs heterogeneity

A vehicle platoon that consist of identical vehicles, controllers and cooperation topology is referred to as homogeneous vehicle platoon. On the other hand a heterogeneous platoon may consist of vehicles with different, e.g., dynamics and/or different control structure. While, extensive studies of string stability of homogeneous platoons can be found in the literature, as far as heterogeneous string stability is concerned, the literature is rather sparse. One of the early studies of heterogeneous platoon was done by [30]. In [14, 40] string stability of heterogeneous vehicle platoon under a constant spacing policy and a decentralized control structure are studied. These studies show that under a constant spacing policy, contrary to the homogeneous case, the spacing error cannot be attenuated uniformly in the platoon. Later in [41], a CACC control structure for string stable heterogeneous vehicle platoon is presented. Further studies with particular attention on the feasibility of CACC system for real time implementation can be found in [42], Paper 1 and Paper 2 in this thesis. Note that, In the homogeneous vehicle platoon string stability w.r.t. a desired signal results in string stability w.r.t. to other signals, however, this is not true for a heterogeneous vehicle platoon. This can be simply explained by looking at the transfer function between the position error and acceleration of two adjacent vehicles. For the former case the two transfer functions are equivalent while for the latter case such result does not hold in general, see Section 4.3 and [41].

4.2.6 Longitudinal vs lateral direction

While string instability in the longitudinal direction may result in shock-waves and consequently accidents, string instability in the lateral direction may result in vehicles ending in the wrong lane after, e.g., a lane change maneuver. Hence, in a vehicle-following setup with automatic steering, string stability in the lateral direction should be considered in the control design.
as well. Similar to the longitudinal case, string stability in the lateral direction can be defined as norm condition. However, string stability is defined w.r.t. the lateral offset between the vehicles, [43, 44] and Paper 3 in this thesis.

### 4.3 Appendix

In this section, as an example the spacing error dynamics and acceleration between two adjacent vehicles are derived and compared. Denote \(p_k, v_k, a_k\) and \(u_k\) as the absolute position, velocity, acceleration and control command for vehicle \(k\), respectively. Define the initial conditions as,

\[
\begin{align*}
p_{k-1}(0) &= v_{k-1}(0) = a_{k-1}(0) = v_k(0) = a_k(0) = 0, \\
p_k(0) &= -\delta.
\end{align*}
\]

The longitudinal vehicle dynamics for vehicle \(k\) can be written as,

\[
\begin{align*}
sP_k(s) &= V_k(s) + p_k(0) \tag{4.15a} \\
sV_k(s) &= A_k(s) \tag{4.15b} \\
A_k(s) &= H_k(s)U_k(s) \tag{4.15c}
\end{align*}
\]

where \(s\) is the Laplace variable and \(H_k(s) = \frac{1}{1+\tau s}\), hence

\[
\begin{align*}
P_{k-1}(s) &= \frac{H_{k-1}(s)}{s^2}U_{k-1}(s), \\
P_k(s) &= \frac{H_k(s)}{s^2}U_k(s) - \delta/s, \tag{4.16}
\end{align*}
\]

the inter-vehicle spacing error between the vehicle \(k\) and \(k-1\) is defined as,

\[
E_k(s) = P_{k-1}(s) - P_k(s) - hV_k(s) - \delta/s, \tag{4.17}
\]

manipulating the equations above gives the following

\[
E_k(s) = G_{k-1}(s)U_{k-1}(s) - (1 + hs)G_k(s)U_k(s). \tag{4.18}
\]

Considering a linear control law, e.g., a state feedback control law \(C(s) = k_p + k_ds + k_\alpha s^2\), the control command becomes \(U_k(s) = C_k(s)E_k(s)\). Plug in \(U_k(s)\) in equation (4.17) and considering an initial spacing error \(e_k(0)\) leads to

\[
E_k(s) = \frac{G_{k-1}(s)C_{k-1}(s)}{1 + (1 + hs)G_k(s)C_k(s)}E_{k-1}(s) + \frac{1}{1 + (1 + hs)G_k(s)C_k(s)}e_k(0)/s. \tag{4.19}
\]
Similarly, the transfer functions between the accelerations of two adjacent vehicles can be derived. From equation (4.19) and considering \( U_{k-1}(s) = C_{k-1}(s)E_{k-1}(s) \) and \( e_k(0) = 0 \), we have,

\[
U_k(s) = \frac{C_k(s)G_{k-1}(s)}{1 + (1 + hs)G_k(s)C_k(s)}U_{k-1}(s).
\] (4.20)

Then, from (4.15c) we have,

\[
A_k(s) = H_k(s)U_k(s) = \frac{H_k(s)C_k(s)G_{k-1}(s)}{1 + (1 + hs)G_k(s)C_k(s)}U_{k-1}(s),
\] (4.21a)

\[
\Rightarrow \frac{A_k(s)}{A_{k-1}(s)} = \frac{H_k}{H_{k-1}} \frac{C_k(s)G_{k-1}(s)}{1 + (1 + hs)G_k(s)C_k(s)}
\] (4.21b)

\[
\Rightarrow \frac{A_k(s)}{A_{k-1}(s)} = \frac{C_k(s)G_k(s)}{1 + (1 + hs)G_k(s)C_k(s)}
\] (4.21c)

Comparing (4.17) and (4.15) shows that the spacing error transfer function and acceleration transfer function between two adjacent vehicles are equivalent only if the following holds:

\[
G_{k-1}C_{k-1} = G_kC_k.
\] (4.22)

While (4.22) is true for a homogeneous vehicle platoon, this is not generally the case for a heterogeneous vehicle platoon.
Chapter 5

Tools

This chapter is dedicated to an overview of the tools and techniques which are used in this thesis, to formulate the control design problem and safety verification problem of automated vehicles.

5.1 Receding horizon control

Control of a vehicle platoon can be formulated as an optimization problem. Model predictive control (MPC) is a powerful tool to handle constraints on control inputs and states is considered as the control scheme in our automated driving system. In most cases, MPC can be formulated as a quadratic programming (QP), i.e. a quadratic objective function subject to linear constraints. Consider a platoon of $n$ vehicles, a centralized control strategy can be formulated as the following,

$$
\begin{align*}
\min_{U} & \quad x(N)^T P x(N) + \sum_{k=0}^{N-1} x(k)^T Q x(k) + u(k)^T R u(k) \\
\text{subject to} & \quad x(k+1) = A x(k) + B u(k) \\
& \quad x(k) \in \mathcal{X}, u(k) \in \mathcal{U}, \\
& \quad \mathcal{E} x(k) + \mathcal{F} u(k) \leq 0 \quad k = 1 \cdots, N
\end{align*}
$$

where,

$$
U = \begin{bmatrix} u_1^T & u_2^T & \cdots & u_n^T \end{bmatrix}^T,
$$

$$
X = \begin{bmatrix} v_1, a_1 & e_{p,2}, v_2, a_2 & \cdots & e_{p,n}, v_n, a_n \end{bmatrix}^T,
$$

are the control input (commanded acceleration) and the state vectors for the entire platoon, respectively, with $v_i, a_i, e_{p,i}, e_{v,i}$ denoting the velocity,
acceleration, spacing error and relative velocity for \( i \)th vehicle, respectively, 
\( P, Q \) and \( R \) are weighting matrices with appropriate dimensions to penalize
the final state, state and input signal, \( N \) is the prediction horizon, \( \mathcal{X}, \mathcal{U} \) are
admissible set of states and control signals, respectively.

The system dynamics of a vehicle platoon are coupled through their states.

\[
x_1(k + 1) = A_1x_1(k) + B_1u_1(k) \\
x_i(k + 1) = A_ix_i(k) + B_iu_i(k) + E_ix_{i-1}(k) \quad i = 2, \ldots, n.
\]

(5.3a) (5.3b)

However, by considering the coupling term \( E_ix_{i-1} \) as an external distur-
bance, i.e. \( \omega_i(k) = x_{i-1}(k) \), the state update equations (5.3) become decou-
pled and the centralized optimization problem (5.1) can be cast as a decen-
tralized optimization problem for each vehicle. Hence, the MPC problem
for decoupled sub-systems (vehicles) can be written as,

\[
\min_{U_i} x_i(N)^TP_ix_i(N) + \sum_{k=0}^{N-1} x_i(k)^TQ_ix_i(k) + u_i(k)^TR_iu_i(k)
\]

(5.4a)

subject to

\[
x_i(k + 1) = A_ix_i(k) + B_iu_i(k) + E_i\omega_i(k), \quad i = 2, \ldots, n.
\]

(5.4b)

\[
x_i(k) \in \mathcal{X}_i, u_i(k) \in \mathcal{U}_i, \quad k = 1, \ldots, N
\]

(5.4c)

\[
\mathcal{E}_ix_i(k) + \mathcal{F}_iu_i(k) \leq 0, \quad i = 2, \ldots, n.
\]

(5.4d)

At every time step \( k \), each vehicle solves the optimization problem (5.4)
using new measurement. First optimal move \( u_i(k) \) is applied to the plant,
then, at the next time step \( k + 1 \) the same procedure is repeated. This is
illustrated in Figure. 5.1. The weighting matrices \( Q_i \) and \( R_i \) are used to
penalize the states and control input, respectively. Penalizing the states
guarantee that the spacing error, relative velocity and acceleration become
smaller, while penalizing the control command ensure that the acceleration
and braking remains on a reasonable level. Minimizing the control command
has direct effect on the fuel consumption. Hence, the choice of \( Q_i \) and \( R_i \)
can be seen as a trade off between the performance and fuel consumption.
Solving the optimization problem online using new measurements provides
feedback effect in the controller. It should be noted that the (QP) problem
presented here, is considered as a class of convex optimization which efficient
solvers are available for, e.g. [45, 46]. For further information on the subject
the reader is referred to [47].
5.1. Receding horizon control

Figure 5.1: Model predictive control scheme
Chapter 5. Tools

5.2 Reachability analysis and invariant set theory

To increase the traffic throughput, it is desirable to reduce the inter-vehicle distance between vehicles in a platoon. Decreasing the inter-vehicle distance can also result in reduction of fuel consumption. However, reducing the inter-vehicle distance apparently can increase the risk of rear end collisions. As described earlier, every vehicle in a platoon is equipped with a sensing module, e.g. radar and camera, which measures the relative distance and velocity between the vehicles. In addition to that, each vehicle can also be equipped with wireless communication which makes it possible to receive information like acceleration of preceding vehicle. The information sensed by sensing module and received by communication are fused and sent to the control unit. Then, using the received measurement and based on a motion model, the control unit calculate a commanded acceleration which will be sent to either throttle or brake system. However, the overall performance of such complex system which requires interaction between different subsystems, largely depends on the accuracy of sensor measurement, communication delay, packet drops, model mismatch and delay in the throttle and brake actuator. Although, extensive simulations can be used to verify system reliability and performance in different situation, any analysis based on simulation may not capture all the phenomena and may require an enormous amount of time. To verify such complex system mathematical tools are required. Reachability analysis technique and invariant set theory can be used to safety verification of autonomous systems. In this section, a brief overview on the aforementioned methods is presented which can serve a background for the second part of this thesis.

A few definitions are introduced and basic results on reachability analysis, are presented. For further study regarding invariant set theory and reachability analysis, the reader is referred to [48] and [49].

**Definition 6** A polyhedron $\mathcal{P} \in \mathbb{R}^n$ is the intersection of finite number of closed halfspaces in $\mathbb{R}^n$

$$\mathcal{P} = \{x \in \mathbb{R}^n | Hx \leq h\}$$  \hspace{1cm} (5.5)

**Remark:** A closed polyhedron is called a polytope.

**Definition 7** The Minkowski sum of two polytopes $\mathcal{R}$ and $\mathcal{Q}$ is a polytope defined as,

$$\mathcal{R} \oplus \mathcal{Q} = \{x + y \in \mathbb{R}^n | x \in \mathcal{R}, y \in \mathcal{Q}\}$$  \hspace{1cm} (5.6)
5.2. Reachability analysis and invariant set theory

Definition 8 The Pontryagin difference of two polytopes \( \mathcal{R} \) and \( \mathcal{Q} \) is a polytope defined as,

\[
\mathcal{R} \ominus \mathcal{Q} = \{ x \in \mathbb{R}^n | x + q \in \mathcal{R}, \forall q \in \mathcal{Q} \} \tag{5.7}
\]

Definition 9 The convex hull of a set of points \( \{ X^i \}_{i=1}^{N_x} \) is the smallest convex set which contains \( X \).

\[
hull(X) = \{ x \in \mathbb{R}^n : x = \sum_{i=1}^{N_x} \lambda_i X^i, 0 \leq \lambda_i \leq 1, \sum_{i=1}^{N_x} \lambda_i = 1 \} \tag{5.8}
\]

Definition 10 Composition of an affine mapping \( f \) and a polyhedron \( \mathcal{P} \), with \( f \) as,

\[
f : z \in \mathbb{R}^m \mapsto Az + b, \quad A \in \mathbb{R}^{m \times m}, \quad b \in \mathbb{R}^m \tag{5.9}
\]

is defined as,

\[
f \circ \mathcal{P} = \{ y \in \mathbb{R}^m | y = Ax + b \quad \forall x \in \mathbb{R}^n, \quad Hx \leq h \} \tag{5.10}
\]

Denote by \( f_a \) the state update function of an autonomous system,

\[
x(k + 1) = f_a(x(k), \omega(k)), \tag{5.11}
\]

where \( x(k) \) and \( \omega(k) \) are the state and disturbance vector, respectively. The system (5.11) is subject to the following constraint,

\[
x \in \mathcal{X}, \quad \omega \in \mathcal{W}, \tag{5.12}
\]

where \( \mathcal{X} \) and \( \mathcal{W} \) are polytopes in \( \mathbb{R}^n \) and \( \mathbb{R}^d \), respectively.
Chapter 5. Tools

**Definition 11** For the autonomous system (5.11), we denote the robust one-step reachable set for initial states $x(0)$ contained in the set $S$ as,

$$\text{Reach}^{f_a}(S, W) = \{ x \in \mathbb{R}^n : \exists x(0) \in S, \exists \omega \in W | x = f_a(x(0), \omega) \},$$

this is the set of states, the system dynamic (5.11) can evolve to from a given set of initial states $S$ for some disturbance $\omega \in W$.

In Figure (5.3), the reachable set from the set of initial states $S$ under the dynamical system (5.11) is represented. The reachable set is represented for two time instances with blue and purple sets. The green arrows show how the initial states $x_0$ evolve into the reachable sets.

![Figure 5.3: N-step reachable set. $S$ represents the set of initial states. The blue and purple sets are the reachable set for two time instances. $X$ is the admissible set.](image)

**Definition 12** For the autonomous system (5.11), the robust Pre set is defined as the dual of one-step reachable set,

$$\text{Pre}^{f_a}(T, W) = \{ x \in \mathbb{R}^n : f_a(x(k), \omega(k)) \in T, \forall \omega \in W \},$$

this is the set of initial states which under the system dynamics (5.11) can in one-step evolve to the target set $T$ for $\forall \omega \in W$.

The backward reachable set from the target set $T$ for a couple of steps is depicted in Figure (5.4)
Figure 5.4: N-step backward reachable set. $\mathcal{T}$ represents the target set (desired set). The yellowish sets are the backward reachable set for different time instances. $\mathcal{X}$ is the admissible set.
Chapter 6

Summary of included papers

This chapter provides a brief summary of the papers that constitute the base for this thesis. Full versions of the papers are included in Part II. The papers have been reformatted to increase readability and to comply with the layout of the rest of the thesis.

Paper 1


This paper presents a simulation and experimental study of a string stable cooperative adaptive cruise controller. The first part of the paper briefly describe the implementation of a communication module based on IEEE protocol. This follows by a brief explanation about the sensor fusion module which was used to filter the data in a real time scenario. The rest of the paper is dedicated to the evaluation of proposed control strategy. The proposed control strategy is an MPC approach which can handle different constraints and specifications while stabilizing the vehicle dynamics. Constraints and specifications are resulted from safety constraints, actuator limitations and performance requirements. Finally traditional frequency domain definition of string stability is translated into time domain and is accounted for in the control design. The simulation and experimental results indicate the effectiveness of the proposed method.
Chapter 6. Summary of included papers

The author of this thesis is responsible for designing the MPC controller and collecting the experimental data related to the controller, conducting the comparison between the controllers. Furthermore, the author of the thesis is responsible for editing the overall paper.

**Paper 2**


As discussed in Chapter 4, string stability of a vehicle platoon is defined in the frequency domain and traditionally is enforced by design of controllers in the frequency domain as well. However, to enable a safe vehicle platooning, safety requirements, actuator limitations and passenger comfort should also be explicitly considered in the control design. While it is more convenient to state string stability condition in the frequency domain, the latter specifications are better fitted in time domain. Combining the specifications from frequency and time domain into a single control design is not trivial. In this paper a novel approach is proposed to combine the aforementioned specifications into a single control design. First a linear controller in the frequency domain is designed in order to guarantee the string stability of the platoon. Then, an MPC controller is designed to handle the time domain specifications (constraints). Finally, an optimization technique is used to tune the weighting matrices of an MPC controller such that its behaviour matches with the linear controller. As a result, as long as the constraints are not active the behaviour of the MPC controller is identical to the string stable controller. When a violation of constraints is predicted over the prediction horizon, the behaviour of the two controllers is not identical anymore and MPC controller makes sure that the constraints are fulfilled. Simulations and experimental results show the effectiveness of proposed method in both maintaining string stability and also fulfilling the constraints.

The author of this thesis is responsible for the main ideas, has performed the simulations, has conducted the experiments and prepared the manuscript. The work has been done under the guidance and supervision of the co-authors.

**Paper 3**

R. Kianfar, P. Falcone and J. Fredriksson, “A Distributed Model Predictive Control (MPC) Approach to Active Steering Con-
A distributed receding horizon control strategy to active steering control of vehicle platoon is presented in this paper. In particular, automation of vehicle platoon in the lateral direction is considered in this work. The main idea in this paper is to develop a control strategy which can  

1) guarantee the stability of individual vehicles  
2) keep the vehicle within the linear region of tire’s operation  
3) enforce the stability of entire platoon, i.e., lateral string stability.  

To accomplish this, the control problem is formulated as an optimization problem (MPC-based approach) and is solved locally by each vehicle. It is assumed that every vehicle in the platoon can receive information from its preceding vehicle and send information to its follower. The information is sent via communication links between vehicles. To achieve the first objective a linear MPC controller is implemented by each vehicle such that the stability of individual vehicle is enforced. To keep the vehicle within the linear region of the tires, constraints on the lateral slip angle are imposed and embedded into the control problem formulation. Finally, to obtain string stability, it is assumed that each vehicle broadcasts an intention over a future prediction horizon. The intention is the optimal open loop trajectory calculated by the local MPC controllers over a finite time prediction horizon. Furthermore, unlike to Paper 2, lateral string stability is formulated as a time domain constraint, which is accounted for in the control design. The simulation results show the effectiveness of proposed approach.

The author of this thesis is responsible for the main ideas, has performed the simulations and prepared the manuscript. The work has been done under the guidance and supervision of the co-authors.

**Paper 4**


In Paper 1 and 2 string stability and constraints satisfaction for a vehicle platoon in the longitudinal direction are studied and two novel approaches for enforcing string stability and constraints are proposed. In Paper 3, under
Chapter 6. Summary of included papers

the assumption of constant longitudinal velocity, the longitudinal and lateral dynamics of the vehicles are decoupled. Hence, a distributed MPC approach is proposed to enforce string stability in the lateral direction. In this paper a combined longitudinal and lateral control problem of a vehicle platoon is considered. Contrary to Paper 3 where the longitudinal vehicle velocity is assumed to be constant, an LPV-based MPC is proposed to account for the variations in the longitudinal velocity. Furthermore, by combining linear frequency and time domain control strategies technique, a novel control strategy is proposed to guarantee string stability (a frequency domain specification) and constraints satisfaction (time domain specifications) simultaneously. The proposed method is validated through simulations and experiments with two prototype vehicles.

The author of this thesis is responsible for the main ideas, has performed the simulations, has conducted the experiments and prepared the manuscript. The work has been done under the guidance and supervision of the co-authors.

Paper 5


In many practical cases the safety of given controllers should be verified a posteriori. Hence, in this paper a method based on reachability analysis technique and invariant set theory is presented to safety verification of automated driving systems. The main idea is to develop a mathematical framework to safety verification of automated driving systems to avoid extensive simulations and expensive experiments. The evaluation is carried out for two different linear controllers and the results are compared. As an example, the method is applied to study the minimum required safe distance between two adjacent vehicles in a vehicle platoon. The method is further extended to account for model uncertainty, delay and possible measurement noise. The proposed method can be used to calculate the maximal admissible safe set. In words, a set which vehicle safety is guaranteed for all the future time. Such set can be used as a reference for guaranteeing the safety of automated driving systems, e.g., ACC or CACC. Effectiveness of the proposed approach is validated via simulations and partially through field experiment as well.

The author of this thesis is responsible for the main ideas, has performed the simulations, has conducted the experiments and prepared the
manuscript. The work has been done under the guidance and supervision of the co-authors.
Chapter 7

Concluding remarks and future research directions

In this chapter a summary of the thesis is given and some directions for future research are mentioned.

7.1 Concluding remarks

This thesis investigates how automated/cooperative driving or in particular vehicle platooning can be enabled to mitigate congestions problem and raise the vehicles safety. Alleviating congestions and other problem associated to that have a clear benefit in traffic flow, fuel economy and air pollution.

The focus of this thesis is to i) develop control algorithm which can enable vehicle automation in a safe and efficient way ii) present a mathematical framework which can be used for safety and performance verification of automated driving systems.

In Paper 1 and 3, methods are presented to develop model based control strategies enabling vehicle platooning in the longitudinal and the lateral directions, respectively. The main challenge is to account for constraints and string stability simultaneously in the control design. The proposed control strategy can formulate these limitations and specifications into a constrained optimal control problem which is solved in a receding horizon. In Papers 1 and 3, time-domain definitions for string stability both in the longitudinal and the lateral directions are introduced and translated into inequality constraints. Hence, decentralized MPC controllers are developed which can simultaneously guarantee safety and performance while enforcing string stability. The proposed method requires communication between two adjacent vehicles to enforce string stability. Each vehicle calculates and sends an intention of its movement for finite steps ahead to its follower. A
Chapter 7. Concluding remarks and future research directions

A combination of old and future information (intention) is used as a constraint to enforce string stability.

The main advantage of the proposed method in Papers 1 and 3 is their ability in translating the string stability criterion from frequency domain into time domain constraints and enforcing that together with other time-domain requirements. However, this approach requires intensive communication between vehicles which can be challenging from a practical point of view. Hence, in Paper 2, an alternative approach based on control matching technique is proposed. In particular, a two step design procedure is proposed which can combine string stability and constraint satisfaction into a single MPC-based control design problem. The main advantage of this method compared to the one proposed in Papers 1 and 3 is that this approach does not require an intention of the preceding vehicles, thus less communication bandwidth is required. Proposed strategy is effectively implemented on a platoon of three vehicles.

The combined longitudinal and lateral control of vehicle platoon is addressed in Paper 4. It is known that a combined longitudinal and lateral vehicle dynamics result in a nonlinear system which is not favourable from a control design perspective. It is very common in the literature that by considering the longitudinal vehicle velocity as constant, the nonlinear dynamics can be decoupled into linear longitudinal and lateral dynamics. However, this simplistic assumption can have negative impact on the vehicle stability. In Paper 4, by considering the longitudinal velocity as a measured scheduling parameter, the nonlinear dynamics decouple into an LTI and an LPV model. Then, by combining frequency domain control techniques, e.g. $H_{\infty}$-design and MPC-based control design, a control strategy is proposed which accounts for string stability, safety and speed variations. Simulation and experimental results show that string stability is guaranteed as long as the constraints are not active and if the controller predicts constraints violation over the prediction horizon, a corrective control action is applied to keep the vehicle within the constraint.

Unlike the methods proposed in this thesis, many of the automated driving controllers used in practice are incapable of guaranteeing specifications like safety a priori. Hence, in Paper 5, reachability analysis technique and invariant set theory are used to safety verification of automated driving system in presence of uncertainty, delay and model mismatch as a posteriori. As an example the proposed method can be applied to study the minimum required safe inter-vehicle distance between vehicles. The reachability analysis can be used to calculate a set that can guarantee safety over a finite or even infinite horizon. Finally, it should be emphasized that most of the techniques presented in this thesis are experimentally validated.
7.2 Future research directions

Further development towards industrialization of vehicle platooning is one way to extend the findings in this thesis. To achieve this the control techniques proposed in this thesis can be extended to guarantee string stability in presence of model uncertainty, imperfect measurements and unreliable communication. Such idea can be pursued by extending Paper 1, 2, 3, 4. In particular, in Paper 2 and 4 robust string stability can be achieved using linear robust control theory to guarantee string stability in presence of model mismatch and imperfect communication. Another approach to achieve robust string stability is to extend Paper 1 and 3 by introducing uncertainty in the MPC frame work. Extending the approach to account for such purpose can be done by adjusting the prediction model for accommodating model uncertainty, imperfect measurements and packet losses. Apparently, such cases can be handled by exploiting techniques from robust/stochastic MPC theory. In this case, the computational burden increases since instead of a QP-problem, e.g., a min-max optimization problem should be solved over the prediction horizon of the MPC controller which may not be suitable for real time implementations.

Along the same goal, i.e., industrialization of techniques proposed in this thesis, further verification and validation is required. Although, the longitudinal control techniques proposed in this thesis are tested experimentally, verification of the lateral controller proposed in Paper 3 and 4 through real time implementations are good extensions.

Inter-vehicle distance plays an important role in efficiency of vehicle platoon in terms of fuel consumption and road capacity. In Paper 5, methods are proposed to calculate the minimum achievable safe distance. Depending on the quality of the communication channel between vehicles and as well as the quality of the measurements, inter-vehicle distances can be increased or decreased. Obviously, the quality of the communication channel can vary during a trip and from vehicle to vehicle. An extension to the work presented in Paper 5 is to estimate a level of confidence for the communication channel and the measured data in real time. This information can be used to calculate the minimum achievable safe distance in real time. This requires modification of algorithm presented in Paper 5 to suit for real time implementation. Another direction to extend the scope of Paper 5 is to extend the model so it can better capture the engine and brake dynamics. Hybrid modeling could be an alternative, which apparently results in verification of hybrid systems.
References


References


References


References


