# Tire Force Estimation Utilizing Wheel Torque Measurements and Validation in Simulations and Experiments

***Anton Albinsson (*Corresponding author*), *Fredrik Bruzelius, **Mats Jonasson,*Bengt Jacobson**

***Chalmers Technical University**
412 96 Gothenburg Sweden
Phone: +46 (0)31 772 10 00
E-mail: anton.albinsson@chalmers.se,
fredrik.bruzelius@chalmers.se,
bengt.jacobson@chalmers.se

****Volvo Cars**
405 31Gothenburg Sweden
Phone: +46 (0)31 59 00 00
E-mail: mats.jonasson@volvocars.com

This study investigates a new tire force estimator based on the recursive least square (RLS) method. Tire force estimation with known driving wheel torque is studied and compared to the case with torque estimation from the internal combustion engine. This is motivated by a future scenario with electric propulsion, which reasonably gives improved wheel torque estimations. Sensitivity to vehicle parameters and challenges with individual lateral tire force estimation are also investigated. The results, experimental and simulation data, show good performance and potential for tire force estimation using the RLS method.

Topics / Modeling and simulations, State estimation

## 1. INTRODUCTION

Active safety systems are becoming more advanced in order to meet higher requirements from both the market and legislations. With more accurate and new information about the vehicle states, the current and future active safety systems can be improved. The tire forces are important vehicle states that can be used for vehicle parameter estimation and tire identification online.

For electric vehicles new opportunities exists for estimation and for improving active safety systems. The main advantage with electric motors, in an estimation context, is that the delivered torque to the wheels is more straightforward to estimate compared to the torque from an Internal Combustion Engine (ICE). Additionally, electric propulsion is typically also used for braking and can be wheel individual, compared to traditional propulsion. Electric motors can then be considered as extra sensors that can be utilized. This study investigates how a better estimation of the applied wheel torque can improve both lateral and longitudinal tire force estimation. In [1,2] both lateral and longitudinal tire forces are estimated but no reliable estimate of the wheel torque is available and hence no individual longitudinal tire force estimation is attempted. [3,4] utilizes the wheel dynamics and the driving wheel torque to estimate the longitudinal tire forces but the validation is done using simulations only. The present study highlights problems associated with tire force estimation, such as uncertain parameters and individual tire force estimation. Individual lateral tire force estimation has been discussed in [1,2,4]. These estimations works well under special conditions constrained by the assumptions made, e.g. uniform friction, low level of lateral acceleration. [5] use a large number of vehicle parameters which is not possible to estimate online and the estimator is prone to be very vehicle specific. A more thorough discussion about the parameters affecting the individual tire force estimation is presented here. The estimator is based on a relatively simple Recursive Least Square (RLS) method compared to the Kalman filter methods used in [1-5]. One of the main reasons for estimating tire forces is to combine the data with a slip estimator in order to identify vehicle and tire parameters. The tire force estimates are to be considered as inputs to the tire model. Including a tire model in the tire force estimator would thus not add any new information to the system unless the tires are identified offline. No tire model has been used to estimate the tire forces in this study which makes the estimator robust to variations in tire parameters and road surfaces. Validation of the estimator was done with simulations using IPG Carmaker and data from real experiments.

## 2. ESTIMATOR STRUCTURE

The input to the estimator is the lateral and longitudinal body acceleration, yaw rate, wheel speeds on all four wheels, steering wheel angle and the propulsion torque on the front wheels. The test data used in the study is from tests performed using a front wheel drive vehicle equipped with an internal combustion engine and not an electrical motor. The test vehicle was fitted with torque wheels for all tests. As a result of the vehicle not being equipped with electric motors, the torque measurements from the torque



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wheels on the propelling front axle were used as inputs to the estimator instead of the electric motor torque. This assumes that the electric motor torque can be estimated with high accuracy. As a reference the ICE torque estimation available on the CAN-Bus was used and an even torque distribution between the left and right tire was assumed to represent a vehicle equipped with an ICE only. It is also assumed that the vehicle is equipped with a GPS for estimation of rolling radius.

The Estimator is divided into two main functions, vertical tire force estimation and planar tire force estimation, see Fig. 1 for an overview of the estimator structure. The main focus though is to estimate the axle lateral forces, rear axle longitudinal forces and individual front longitudinal tire forces. All estimations are based on the RLS method with exponential forgetting factors, see for example [6]. Without the forgetting factors the measurements at all sample times would be weighted equally and the estimates would eventually converge to a static value. Additionally the forgetting factors act as a filter as the current estimate is based on the previous measurements as well. The forgetting factors can be considered tuning parameters and their values are a tradeoff between noise and convergence speed, or in this case tracking performance. The implemented RLS method also offers the opportunity to include weighting factors for the different measurements. In addition no assumptions about the state dynamics are necessary when the RLS method is used. The tire forces are dependent on driver inputs that are hard to model, such as steering angle, motor torque etc. It is not possible to describe the tire force dynamics without incorporating a full model that would introduce substantial uncertainties.

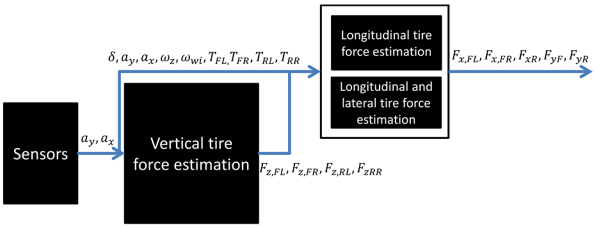


Fig. 1, Tire force estimator structure

The planar tire force estimator is designed to work for both pure longitudinal forces as well as for combined longitudinal and lateral forces. In order to keep the lateral tire force estimation from drifting during low levels of excitation the planar tire force estimator was split into two modes, one to handle straight driving and one to handle combined forces. Hence, lateral tire forces are not estimated if the steering wheel angle has been close to zero for a longer period of time. The lateral forces are assumed to be negligible when this condition is true. In order to avoid discontinuities in the output from the two estimators, both estimators will run simultaneously for a short period of time before the output is switched. An additional condition, for switching between the longitudinal and the combined tire force estimator, is introduced on the measured lateral acceleration to estimate lateral tire forces on banked roads.

### 2.1 Models used for tire-force estimation

The basic vehicle model used in the estimator is a planar two-track model combined with a steady-state load transfer model, see Fig. 2.

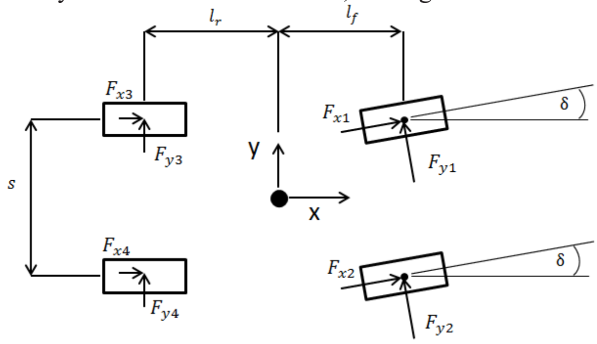

Fig. 2 Two-track model

The lateral load transfer equation (eq. 1) includes steady state suspension effects. $\Delta F_{zyi}$ is the change in vertical load on the left and right side of the axle (i=f,r), $c_{\varphi i}$ is the roll stiffness on the axle, $h_i$ is the roll center height for the axle, $l_i$ is the longitudinal distance from the CoG to the axle, $h'$ is the CoG height above the roll axis and $s_i$ is the axle track width. For longitudinal load transfer the suspension effects are neglected as the distribution between the left and right tire should ideally be equal on a smooth surface (eq 2).

$$\Delta F_{zyi} = \frac{1}{s}\left(\frac{c_{\varphi i}}{c_{\varphi 1}+c_{\varphi 2}-mgh'}h' + \frac{L-l_i}{l}h_i\right)ma_y \tag{1}$$

$$\Delta F_{zxi} = \pm\frac{ma_x h}{2L} \tag{2}$$

Vertical tire forces are approximated with the vehicle weight, longitudinal CoG position and estimates of the longitudinal and lateral load transfer, eq. 3. The vertical tire forces are used as an input to the planar tire force estimator for approximation of the rolling resistance of the individual front tires and to approximate the difference $F_{y1} - F_{y2}$ in the yaw moment equation, eq 7.

$$F_{zi} = \frac{mg(L-l_i)}{2L} \pm \Delta F_{zyi} \pm \Delta F_{zxi} \tag{3}$$

The planar tire force estimator is divided into two parts to utilize the fact that the wheel torque can be determined with high accuracy. The individual longitudinal tire forces on the front wheels are estimated in the first part solving eq. 4 for $F_{xi}$ where $I_{wi}$ is the wheel inertia, $R_{ei}$ is the radius of the wheel and e is the normal force offset at the ground. The torque measured by the torque wheel is used as the electric motor torque $T_{EMi}$ and $F_{z_i}$ is the estimated vertical tire force. The wheel angular acceleration is obtained by a first order discrete differentiation of the wheel angular speed signal.

$$I_{wi}\dot{\omega}_{wi} = T_{EMi} - F_{xi}R_{ei} - F_{z_i} * e \tag{4}$$

The output from the estimator based on the wheel dynamics, $F_{x1}$ and $F_{x2}$ are used as inputs to the vector based RLS estimator describing the two-track model, see eq. 5,6,7. The lateral and longitudinal accelerations are also inputs to the system and



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straightforward to use as they are measured directly. The yaw acceleration is obtained from the yaw rate sensor by a first order discrete differentiation. The equations presented below are only valid, in the sense that they are used in the estimator, as long as the friction brakes are not applied. If the friction brakes are applied, the wheel torque equations must be incorporated in the two-track model and the brake torque distribution should be known.

$$ma_x = F_{x1}\cos(\delta) + F_{x2}\cos(\delta) + F_{x3} + F_{x4} - F_{y1}\sin(\delta) - F_{y2}\sin(\delta) - F_{drag} \quad (5)$$

$$ma_y = \left(F_{y1} + F_{y2}\right)\cos(\delta) + F_{y3} + F_{y4} + (F_{x1} + F_{x2})\sin(\delta) \quad (6)$$

$$I_z\dot{\omega}_z = \left(F_{y1} + F_{y2}\right)\cos(\delta)\, l_f + \left(F_{y1} - F_{y2}\right)\sin(\delta)\frac{s}{2} - \left(F_{y3} + F_{y4}\right)\left(L - l_f\right) + (F_{x4} - F_{x3})\left(\frac{s}{2}\right) + (F_{x2} - F_{x1})\left(\frac{s}{2}\right)\cos(\delta) + (F_{x1} + F_{x2})\sin(\delta)\, l_f \quad (7)$$

Although the two-track model described above contains terms with the individual tire forces in the longitudinal and lateral direction, equation 5 and 6 only contains the sum per axle which makes it impossible to estimate individual lateral tire forces. The yaw moment equation (eq. 7) includes terms where the left and right side are distinguishable for longitudinal forces, rear and front, and the front lateral tire forces but the number of unknowns is still more than the number of equations available. Due to the vehicles front wheel drive configuration, the rear longitudinal tire forces are small during cruising and acceleration and their contribution to the yaw dynamics can therefore be neglected. If the vehicle is fitted with active yaw control that utilizes the friction brakes, the individual rear longitudinal tire forces need to be estimated when the system is active. Depending on the application, the estimator may be turned off at these maneuvers or an estimation of the brake torque based on the brake pressure may be used. The accuracy of the friction brake torque estimation is normally low with a standard sensor set and should be avoided if possible.

The difference between the left and right lateral tire force in eq. 7 is estimated based on the vertical load distribution according to eq. 8. The equation is based on the assumption that the left and right wheel utilizes the same amount of friction. It was found that the difference between the left and right tire lateral force is overestimated using this equation. However, it was also found that the lateral axle force estimation was improved when this assumption is used in eq. 7, even with large errors in the vertical tire force estimation.

$$F_{yi} = F_{y,axle}\frac{F_{zi}}{F_{z,axle}}\ (i = L, R) \quad (8)$$

Due to the limited possibility to distinguish left and right tire forces, the output from the planar tire force estimator is limited to $F_{yF}$, $F_{yR}$, $F_{xR}$ (with $F_{x1}, F_{x2}$ already estimated from eq 4.) where:

$$F_{yF} = F_{y1} + F_{y2} \quad (9)$$

$$F_{yR} = F_{y3} + F_{y4} \quad (10)$$

$$F_{xR} = F_{x3} + F_{x4} \quad (11)$$

The measurement vector for the RLS method in the case of combined lateral and longitudinal forces is then:

$$Y = [Y_1\ Y_2\ Y_3]^T \quad (12)$$

Where

$$Y_1 = ma_x - (F_{x1} - F_{x2})\cos(\delta) + F_{drag} \quad (13)$$

$$Y_2 = ma_y - (F_{x1} + F_{x2})\sin(\delta) \quad (14)$$

$$Y_3 = I_z\dot{\omega}_z - (F_{x1} + F_{x2})\sin(\delta)\, l_f - (F_{x2} - F_{x1})\left(\frac{s}{2}\right)\cos(\delta) \quad (15)$$

$$F_{drag} = \frac{1}{2}C_{dA}v_x^2 \quad (16)$$

The wheel speed signals were used to approximate the longitudinal vehicle velocity in eq 16. This approximation works well as long as at least one wheel has a low longitudinal slip. Considering that the longitudinal velocity is only used for the estimation of the drag force acting on the vehicle, this approximation is accurate enough. On the other hand if the tire force estimator would be combined with a slip angle estimator, a more accurate estimation of $v_x$ is required for the slip angle estimation.

When the steering wheel angle has been close to zero for a few seconds and only the longitudinal dynamics are of interest, the measurement vector is reduced to $Y_1$ and the estimated lateral forces are set to zero. As mentioned previously an additional condition on the lateral acceleration is added to estimate the lateral forces on banked roads where significant lateral tire forces could be present without any steering wheel angle.

Another interesting property of the system can be observed from eq. 13 and 5. The rear longitudinal force $F_{xR}$ is only dependent on this measurement and the terms in the equations must be correct in order to have an accurate estimation. $F_{yF}$ is also included in eq. 5 but the estimation of $F_{yF}$ is more robust since it is also based on the measurement of the lateral acceleration. $F_{xR}$ can be viewed as a sink for the errors in the rest of the assumption made in eq. 13 and 5. This is normally not a problem as $F_{xR}$ is small for a front wheel drive vehicle during normal driving and hence not very interesting to estimate. $F_{xR}$ will however be large during braking and for braking maneuvers it is therefore important to design the estimator in a way which allows the error to be collected in the estimation of the brake torque instead. If for instance a model that estimates the brake torques based on the brake pressure were to be used, the error could end up in the front lateral tire force estimation instead. The torque wheels cannot distinguish between the friction brake torque and the propulsion torque from the engine. This means that estimator designed to handle braking maneuvers, with unknown friction brake torque, could not be evaluated with the available test data. No test cases where the vehicle utilizes the friction brakes were therefore investigated in this study.



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## 2.2 Vehicle parameters

All vehicle parameters used in the estimator are assumed to be known. Some of these parameters can be estimated utilizing the information from the sensors. For example, the effective rolling radius of the tires can for instance be estimated using the GPS and wheel velocities. Nonetheless, a number of parameters cannot be identified online and it is for this reason important to investigate how errors in the vehicle parameters propagate to the tire force estimation. All vehicle parameters used in the estimator can be seen in Table 1.

Table 1, Vehicle Parameters

| Notation | Description |
|---|---|
| m | Mass of the vehicle |
| $I_z$ | Yaw inertia |
| $l_f$ | CoG distance from front axle |
| $L$ | Wheelbase |
| s | Track width |
| $C_{dA}$ | Drag coefficient |
| $I_w$ | Wheel inertia |
| $R_e$ | Effective rolling radius |
| e | Offset for vertical tire force |
| $c_{\varphi i}$ | Roll stiffness i=front/rear |
| $h_i$ | Roll center height i=front/rear |
| h | CoG height |
| $h'$ | Height of CoG above Roll centre axis |
| nst | Steering ratio (Steering wheel to wheel) |

## 3. SIMULATION RESULTS

Simulations were performed in IPG Carmaker using the standard demo car BMW M5 with modified parameters to fit the test vehicle. An advantage with individual wheel torque estimation is the known yaw moment contribution from the longitudinal tire forces. Fig. 3 shows the estimation of the front axle lateral force at a steady state maneuver with a lateral acceleration of around 0.8g. It is assumed that the torque distribution is biased with 70% of the propelling torque distributed to the front right wheel and 30% to the front left wheel. The results illustrate the importance of individual torque wheel estimation when either torque vectoring or a locking differential is used. If only information from the accelerometers were to be used it would not be possible to distinguish the left and right front longitudinal tire forces. An accurate estimation of the individual wheel torque is important for vehicles with either of these configurations but especially for vehicles which utilizes torque vectoring.

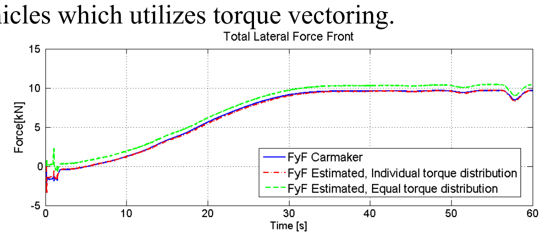


Fig. 3, Lateral force estimation with torque vectoring, Steady state circle driving (Radius=100m), high-μ

Even though the individual longitudinal tire forces on the front axle can be known with good accuracy from the driving torque, individual lateral tire force estimation is still challenging when using standard sensors. [1,2] assumes that the lateral tire force distribution between the left and right tire is proportional to the vertical tire force distribution according to eq. 8. Fig. 4 shows the individual lateral tire force estimation based on this relation for a lane change maneuver where the vertical tire force estimation is assumed to be perfect. It was found that distributing the lateral force based on the vertical tire force overestimates the difference in lateral tire force between the left and the right wheel.

There are several reasons for this. The distribution is based on the assumption that the left and right tire utilizes the same amount of friction, which is not true in general. The nonlinearity of the cornering stiffness as a function of vertical load has not been taken into consideration and the effects of combined slip are neglected, see [7]. Another contributing factor which is neglected is the suspension compliance. The outer wheel generates a higher lateral tire force compared to the inner wheel in a corner due to load transfer and the resulting compliance steer will hence be greater on this wheel. This leads to a difference between the left and right tire steer angles and consequently the slip angles. The effect of lateral force steer is vehicle dependent but was significant for the vehicle used in this study. The individual front steering angles are not measured and it is not possible to estimate them easily without substantial modelling assumptions

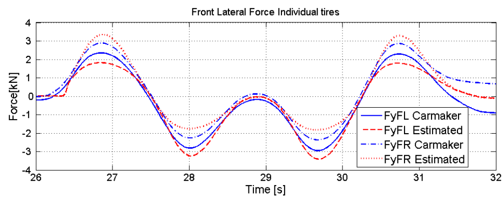


Fig. 4, Individual lateral tire forces front, Lane change maneuver

## 4. EXPERIMENTAL RESULTS

The experimental data was collected using a normal sized passenger car equipped with torque wheels. The test vehicle used is front wheel drive with an internal combustion engine. Signals from the vehicles CAN-Bus was recorded and used as an input to the tire force estimator. A small delay is present between the measured forces and the estimations as a result of the input data to the estimator being collected via the CAN-Bus. The torque measurements from the torque wheels, when used as an input to the estimator, are delayed by 70ms to match the data from the CAN-Bus.

The torque wheel measurements were also used as a reference for the tire force estimator. The raw data from the torque wheels have a large offset which needs to be removed. The raw data was calibrated at manoeuvres where the tire forces are approximately known. For instance the lateral tire forces were set to



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zero when the vehicle was driving in a straight line. The longitudinal forces and the effective wheel torques were set to be equal to the approximated rolling resistance when the ICE was not producing any torque. Due to these offsets the torque wheel measurements should not be considered to be 100% accurate since a small offset will remain. Regardless, torque wheels are a good reference for approximate values and general behaviour of the tire forces.

The front longitudinal tire forces are estimated from the wheel torque utilizing the wheel dynamics. There are a number of parameters in this equation (eq 4.) which could be wrong. An error in the rolling resistance force would for instance give an equally large error in the total front longitudinal force estimation. This error is small compared to the driving torque except when cruising at low velocities or when the vehicle is free rolling. The effective rolling radius on the other hand describes the gain between the tire force and the wheel torque and the error will be proportional to the amplitude of the applied torque.

As shown in Fig. 5 the longitudinal force estimation based on the torque from the ICE management system is accurate during slow events. On the other hand during faster events where the torque changes rapidly, the propulsion torque is overestimated. At 19-23 seconds an aggressive lateral manoeuvre is executed which affects the comparison. As long as the ICE is operating close to steady state conditions the torque estimation is quite good. Normally though, it is more interesting to estimate the tire forces in transient manoeuvres when the system is properly excited. As the accuracy of the longitudinal tire force estimation is directly dependent on the accuracy of wheel torque estimation, accurate wheel torque estimation is vital.

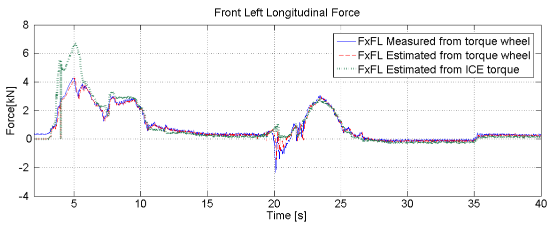


Fig. 5, Estimation of front individual longitudinal tire forces, Straight line acceleration, High-μ

The results from an aggressive lateral maneuver (same test run as Fig. 5) where a damped sinus wave is given as a steering input with the first turn to the left can be seen in Fig. 6. The lateral axle force estimation works well if all vehicle parameters are known. However, when values for the mass, longitudinal CoG position and yaw inertia of the same vehicle with 200 kg less load on the rear axle were used, a large error was found in the rear lateral tire force estimation. This could be expected considering that the mass parameter will change the total lateral force that is required to balance the measured acceleration. The CoG position and the yaw inertia affect the distribution between the front and rear lateral tire forces. A normal passenger car can have a total load variation of around 300kg or around 15% which is significant. This extra load could be in the form of passengers or cargo and it is not obvious of how an increased weight affects the longitudinal CoG position and the yaw inertia. For this reason the mass, yaw inertia and the longitudinal CoG position should be estimated separately.

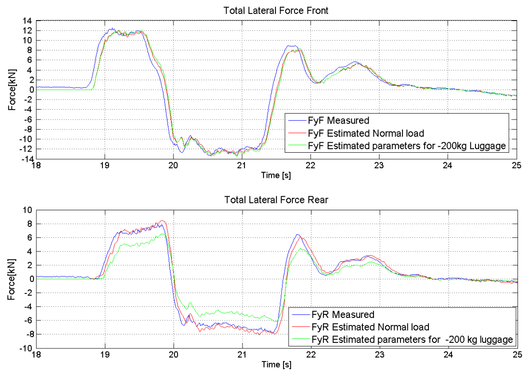


Fig. 6, Lateral tire force estimation, Aggressive lateral maneuver, High-mu, ESC-intervention

The yaw inertia, the mass and the longitudinal CoG position is only a few of the parameters which affect the tire force estimation. The estimated vertical tire forces are also an input to the planar tire force estimator. In order to study the planar tire force estimation accuracy with errors in the vertical tire force estimation, the center of gravity height was doubled. As seen from Fig. 7, a large error in the center of gravity height parameter does not affect the lateral axle force estimation significantly. Similar results were found for the estimated rear axle lateral force and for the case when the total roll stiffness was doubled. The errors in the parameters affecting the estimated load transfer in the test cases described are so large that the lateral axle force estimation can be considered to be robust to significant errors in the vertical tire force estimation.

As expected the accuracy of the vertical tire force estimation is reduced with large errors in the parameters in the load transfer equation, see Fig. 8. Furthermore, Fig. 8 illustrates the modelling error when a steady state load transfer model is used. Compare the measured vertical tire force with the estimated value for a CoG height of 0.5m. It is clear that the transient load transfer, which is mainly influenced by the dampers and the bushings, is ignored. However, adding roll damping to the load transfer equations would introduce more uncertainties that may decrease the accuracy of the estimation. The dampers will furthermore generate different forces on the left and right side due to different characteristics in bump and rebound. A model that includes the vertical motion of the wheel and body would then be required in order to model the dampers influence on individual vertical tire forces. Due to the increased complexity of the model with roll damping the steady state load transfer model was chosen as the best option even though it does not manage to capture the transient load transfer.


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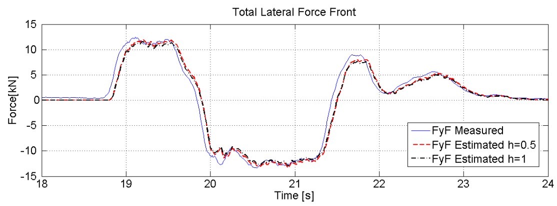

Fig. 7, Lateral front axle force estimation with wrong CoG height, Aggressive lateral maneuver

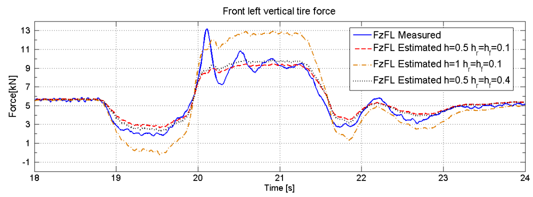

Fig. 8, Example of vertical tire force estimation with different vehicle parameters, Aggressive lateral maneuver

Another contribution to the difference between the lateral force estimation with different roll parameters is the estimated roll angle. When the vehicle body has a roll angle relative to the wheels, the accelerometer will not measure the acceleration in the same coordinate system as the wheels, since a component of the earth's gravity will be measured as well. The lateral acceleration measurement was thus corrected with the estimated steady-state roll angle. The equation used to estimate the roll angle is based on the same assumption as the lateral load transfer equation (eq. 1). On the other hand when the roll angle effect has been compensated for, no correction due to road bank angle has to be made since the acceleration measured in the wheels coordinate system needs to be balanced by tire forces in the same coordinate system. In other words if a lateral acceleration is measured in the wheel coordinate system, lateral tire forces must be present as well. Fig. 9 shows an example where the vehicle is travelling on a banked road and the lateral tire forces are estimated. The driver is steering to the left but the vehicle is turning to the right due to the large bank angle of the road. The estimator still manages to estimate the lateral axle forces without any estimation of the road bank angle. The same reasoning can be used for longitudinal tire forces but with angles around the vehicles y-axis instead.

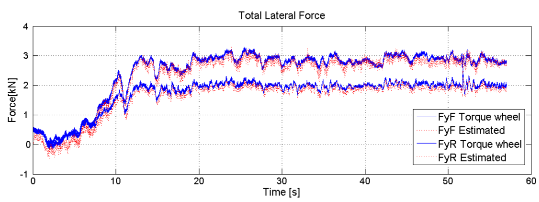

Fig. 9, Total axle lateral force in banked corner

## 5. CONCLUSIONS

A tire force estimator that utilizes individual wheel torque measurements based on a straight forward RLS method was developed. The estimator performs well, given known vehicle parameters, for estimation of front individual longitudinal tire forces, axle lateral forces and the rear axle longitudinal force. It is also robust to variations in tires and road surfaces as no tire model was used. The torque estimation from the engine management system can be used to estimate front individual wheel torques if the engine is operating close to steady state and the torque distribution between the left and right tire is known.

The tire force estimator is sensitive to errors in some of the vehicle parameters which can vary significantly between two different journeys. Large errors were also found for the individual lateral tire force estimation. Hence, a more accurate model for the distribution of lateral tire forces is needed but will introduce new uncertainties. The axle lateral force estimation on the other hand is robust to large errors in the vertical tire force estimation and to large road bank angles.

Future work will include vehicle and tire parameter estimation. The estimator will be extended to include braking maneuvers with both friction brakes and electric motors. The estimated forces are intended to be used in vehicle control functions, such as ESC and ABS. The functions will be used to evaluate performance and the requirements of the estimator.

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