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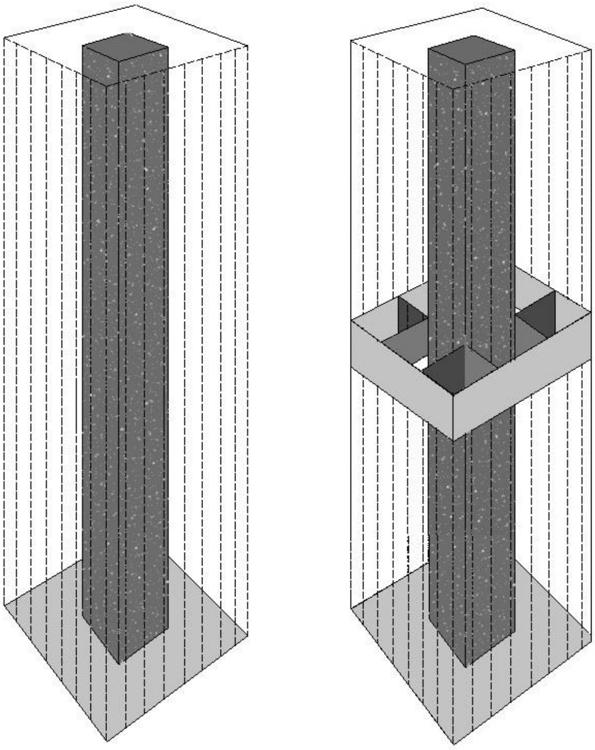

Sizing Optimisation of Structural Systems of Tall Buildings

Preliminary design in serviceability limit state

Master of Science Thesis in the Master's Programme Structural Engineering and Building Technology

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Department of Civil and Environmental Engineering
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CHALMERS UNIVERSITY OF TECHNOLOGY

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Cover:
Structural systems of tall buildings; shear wall-braced structure and core structure with outrigger.

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ABSTRACT

This project concerns sizing optimisation of structural systems for tall buildings in preliminary design and the serviceability limit state. Sizing optimisation is an important part of preliminary design in order to achieve a structure with minimal costs and carbon footprint.

Based on a case study on two types of structural systems for office buildings of four different heights with area 36x36 m^2 were optimised and analysed. The aim was to develop recommendations for choosing, checking and optimising structural systems in preliminary design and to investigate which variables that are the most decisive in sizing optimisation.

The optimisation procedure was based on the Lagrange Multiplier technique, where the objective was to minimise the needed volume of material with respect to the constraint of a maximum allowed lateral deflection.

Results show that a core structure with one outrigger is more efficient than a shear wall-braced structure, as the core structure with outrigger needs a smaller total volume of material. The case study of a 245 m tall building showed that for a core structure with one outrigger the needed total volume of material was 14 % less than the total needed volume of material for a corresponding shear wall-braced structure.

Concerning the core structure with one outrigger, the results show that there is no obvious location along the height of the structure where the outrigger performs optimally. For the case of a trapezoidal wind load applied on the structure, the results have shown that the optimal location of one outrigger along the height of the building lays at a level corresponding to 50 % to 75 % of the height.

Results also show that the maximum required thickness of the core walls was reduced by 23 % up to 50 % for the four different heights when one outrigger was provided to the shear wall braced structure.

Furthermore, when optimising according to the Lagrange Multiplier technique and with only one constraint of the maximum allowed lateral deflection, results show that the acceleration increases when the volume of material is reduced.

Key words: Tall buildings, sizing optimisation, Lagrange, outrigger, shear wall, structural system, volume of material, acceleration, wind load, dynamic, preliminary design.

Dimensionsoptimering av stomsystem för höga byggnader
Preliminär dimensionering i bruksgränstillstånd
Examensarbete inom Structural Engineering and Building Technology
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SAMMANFATTNING

Detta projekt avser dimensionsoptimering av stomsystem för höga byggnader i preliminär dimensionering och i bruksgränstillstånd. Dimensionsoptimering är en viktig del i det preliminära skedet av ett byggprojekt för att erhålla ett effektivt stomsystem med minimala kostnader och miljöpåverkan.

Med utgångspunkt från en fallstudie har två typer av stomsystem för kontorsbyggnader med fyra olika höjder, med en yta 36x36 m^2, optimerats och analyserats. Målet var att utarbeta rekommendationer för val, kontroll och optimering av stomsystem i preliminär dimensionering och att undersöka vilka variabler som är avgörande för dimensionsoptimering.

Optimeringsprocessen baserades på Lagrange Multiplikator metod där målet var att minimera erforderlig materialvolym med avseende på ett villkor som är för maximal tillåten topputböjning.

Resultaten visar att ett stomsystem med kärna och en utriggare är effektivare än ett stomsystem som enbart består av en kärna eftersom stomsystemet med kärna och utriggare kräver en mindre total materialvolym. I fallstudien med en 245 m hög byggnad visade det sig att ett stomsystem med kärna och utriggare behövde 14 % mindre total materialvolym än ett stomsystem med enbart en kärna.

När det gäller stomsystem med kärna och en utriggare visar resultaten att det inte finns en uppenbar position längs höjden av stommen där effekten från utriggaren blir optimal. Detta beror till stor del på förhållandet mellan styvheterna hos kärnan, utriggaren och fasadpelarna och formen på den pålagda vindlasten. För fallet med en trapetsformad vindlast visar resultaten att den optimala placeringen av utriggaren längs byggnadens höjd är på en nivå motsvarande 50 % till 75 % av höjden på byggnaden.

Resultaten visar även att den maximala väggtjockleken på kärnan reducerades mellan 23 % och 50 % för de fyra olika studerade höjderna när en utriggare tillfördes stomsystem med enbart en kärna.

Resultaten från optimeringen med Lagrange Multiplikator metod med hänsyn till endast ett villkor, som är en maximal tillåten topputböjning, visar att accelerationen ökar när materialvolymen reduceras.

Nyckelord: Höga byggnader, dimensionsoptimering, Lagrange, utriggare, skjuvvägg, stomsystem, materialvolym, acceleration, vindlast, dynamik, preliminär dimensionering.

Contents

Preface

This master's thesis concerns sizing optimisation of tall building structures in preliminary design in the serviceability limit state.

The project has been carried out in collaboration with the structural engineering firm VBK. We wish to express our sincere gratitude to our supervisor Erik Samuelsson, structural engineer at VBK, for his professional knowledge and help during this project. Thanks to the help and feedback from Erik the value of the project has significantly increased.

A person who has helped us solve important issues in this project is the building aerodynamic expert Kamal Handa. We thank him for his help and patience, and for the instructive times and all the laughter we shared.

We are very grateful for the support from Bill Baker, structural engineer at SOM in Chicago. Even though he is a busy person he took the time give us important feedback which solved many of our issues. We appreciate it a lot.

We would like to direct our gratitude towards our examiner Professor Björn Engström for his valuable advice and his patience during this thesis project. We have had the pleasure of having Björn as our teacher in several courses during our master's programme at Chalmers University.

For many joyful memories during the project we would like to thank our opponents Marcus Eriksson and Elias Fritzson.

Finally, we would like to express special thanks to our families, who always have supported us through life and during our studies.

Göteborg, June 2014

Nawar Merza & Ashna Zangana

Notations

$u_{\max}$	Maximum tip deflection at the top of the building
$u_{\max}{}^{*}$	Maximum tip deflection at the top of the building including 2nd order effect
H	Height of the building
B	Width of the building
h	Storey height
$G_{k,i}$	Permanent self-weight of the structure
P_k	Pre-stressing force
$Q_{k,1}$	Main variable load
$\gamma_{Q,1}$	Partial safety factor for the main variable load
$Q_{k,i}$	Secondary variable load
$\gamma_{Q,i}$	Partial safety factor for the secondary variable load
$\psi_{0,i}$	Reduction factor for secondary variable load
$q_{d,sls}$	Design load in serviceability limit state
M_i	Bending moment at storey i
EI_i	Bending stiffness of member i
EA_c	Axial stiffness of a column
E	Modulus of elasticity
I_i	Moment of inertia of member i
w	Wind load on the structure
v_m	Mean wind velocity
v_b	Reference wind velocity
c_d	Effect of vibrations of the structure due to turbulence
c_s	Effect on wind actions from the non-simultaneous occurrence of peak wind pressures on the surface
q_p	Wind velocity pressure
d	Lever arm between each side of the building
b	Length of an outrigger
F_i	Equivalent lateral force at storey i
$\overline{f}_i$	Axial force due to unit virtual load in bar i

P_b	Global critical buckling load of the structure
P	Global Axial load on the structure
s	Factor of stability according to Vianello's method
AF	Amplification factor for second order analysis
f_n	Natural frequency of the building
g	Gravitational acceleration
$\ddot{X}_{\max}$	Acceleration of the building
m	Mass of the building
m_0	Equivalent mass of the building
δ_s	Structural damping
δ_a	Aerodynamic damping
$f(A)$	Objective function as a function of the member's area A
$g(A)$	Constraint function as a function of the member's area A
α	Shape correction factor
i	Radius of gyration
λ	Lagrange Multiplier
$L(A,\lambda)$	Lagrangian function
L_i	Length of member i

1 INTRODUCTION

1.1 Historical perspective

According to Smith and Coull (1991) tall buildings became a more attractive solution as more advanced materials, such as steel, were introduced and also when the elevator was invented. Nowadays tall buildings could be constructed without the need of having very thick walls at the bottom floors and also the top floors became as attractive as the bottom floors since the elevator made it easy to reach the upper floors. Tall buildings were requested in large cities where the population was growing together with a need of accommodation.

The first tall building supported by a metal frame was an 11-storey tall in Chicago and the year was 1883. Eight years later The Masonic Temple in Chicago with its 20 stories was constructed with braced façade frames. That was when it was realised that wind load has to be considered in design of high rise structures.

The development of using reinforced concrete for tall buildings was slow and by the time the Empire State Building was constructed, with a height of 102 stories, the tallest concrete building was only 23 stories. During and after the 1970's, several structural systems for tall buildings were developed due to economical and efficiency issues.

1.2 Background and problem description

Tall buildings, as for bridges and other large structures, require huge amount of material, energy, planning and economy to get built. In every tall building project it is important for structural engineering companies to be able to present attractive offers, when competing with other companies, to acquire a certain design project. It is of interest to present an efficient structure in order to give a proposal that is attractive to the contractors as well as the owners. An efficient structure does not only provide minimum material usage and economic solution, but also it minimizes the carbon footprint which is a major factor for a structural engineer to consider when designing large structures. This is of major concern in the world today in general, namely that too large emissions are taking place that damages the nature of our planet. This is why the problem description of this project concerns minimising volume of material so that the carbon footprint is minimised.

This project was developed in corporation with the structural engineering company VBK Konsulterande Ingenjörer AB. VBK has an interest of increasing their knowledge about tall building structures and to be in the front edge within the subject of tall buildings in Sweden.

According to DLS Dynamics (2010) the taller a building is the more inefficient it becomes. Simultaneously as the height of building increases, the construction costs increases. This can be seen in Figure 1.1.

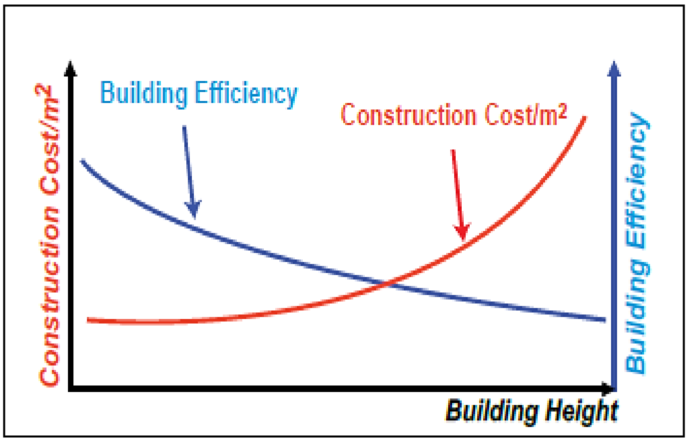

Figure 1.1: Relation between building height and its efficiency and construction cost respectively [DLS Dynamics (2010)].

The issue of sustainable environmental development is complicated and is not solved only by minimising the volume of material of buildings. Nevertheless, it is still a contribution towards sustainable environmental development which a structural engineer should see as an important responsibility.

According to Jayachandran (2009) an optimised structural system of a tall building in preliminary design minimizes erection costs and results in better fabrication and construction. The cost of the structural system, amongst other, depends on the weight of the structure, which in turn depends on the initial design. Furthermore, a structure that is optimised in preliminary design also leads to an improvement of the foundation design. Therefore a size optimised structure leads to lower construction costs and increased structural efficiency meaning that the gap between the lines in Figure 1.1, for buildings of greater heights, becomes smaller.

According to Mehta and Meryman (2009) 50 years ago the consumption of concrete in the world was approximately 1 ton per capita. Today the magnitude is approximately 3 tonnes per capita. Particularly concerning tall buildings, the total volume of material and the structural efficiency are important factors with regard to economy and sustainability. It is therefore important in an early stage of a construction project, where preliminary design is taking place, to be able to make a good estimation of material usage and structural layout. Key parameters that affect the material usage and structural performance of a building, that can help a structural engineer to create an optimised preliminary design, are therefore of interest to map.

1.3 Aim

The purpose of this project was to optimise the volume of material, with regard to the maximum lateral deflection, of two structural systems of tall buildings and to examine the behaviour of the structures.

An objective of this project was to examine the influence of different parameters on the total volume of material required for a structure with a constrained maximum allowed lateral deflection as well as to examine the structural behaviour of two structural systems of tall buildings.

A further objective was to provide recommendations concerning factors that are decisive in the process of preliminary design of tall buildings.

1.4 Chosen methodology

Literature studies were to be carried out in order to obtain knowledge about tall buildings and their structural behaviour and performance. The literature study should also include studying optimisation methods and existing requirements in Eurocodes.

The literature studies, along with assumed cases of tall buildings of different heights, should be the basis for choices of promising structural systems. These structural systems were to be further studied in the project.

Another way to sort out suitable structural systems, for the case specific conditions in this project, could have been by performing brief structural analysis of the different structures through which an additional comparison could have been made. Nevertheless, as some structural systems are known to be efficient within certain limits, it would be easy to cancel out the inappropriate alternatives.

The different assumed cases were to be optimised with regard to maximum allowed lateral deflection. The arising acceleration of the optimised structures was calculated and evaluated. The optimisation method to be chosen is presented and explained in Chapter 7.

Parametric studies were to be carried out in order to capture the behaviour of the chosen structural systems. The results from the parametric studies are presented in Chapter 10.

All calculations were performed in Matlab since this program provides fast iterations and easy handling of parameters. The program also handles vectors and matrices in a way making it easy to test and verify the program.

The results of the calculations and parametric studies should be summarised in figures showing the needed volume of material for different heights of a specific structural layout of a tall building. In order to compile recommendations for structural engineers in preliminary design of a tall building, factors that are decisive for the structural behaviour and the needed volume of material should be highlighted. These factors should be analysed so that it is clear how they affect the structural behaviour and the required volume of material. In parallel to this it is important to emphasise what factors that may not have much of an effect on the structural behaviour and required volume of material.

As a final point, an approach for preliminary design of tall buildings based on the whole project should be suggested and further studies should also be presented.

1.5 Limitations

The structural systems that were chosen were studied with respect to limited cases which are presented in Chapters 8 and 9. The optimisation of the chosen structures, within the frame of the chosen cases, is carried out only with respect to the constraint of maximum lateral deflection.

Shear deformations are neglected as the shear contribution to the total lateral displacement of tall buildings is relatively small in comparison to the contribution from flexure.

There are several kinds of serviceability criteria such as limited crack width, deflection and stress. Nevertheless, in this project, the criteria of main interest were maximum lateral deflection and acceleration at the top of the building, due to the fact that focus was laid on preliminary design. Thus, no detailed sizing and analysis of individual members were carried out.

Torsional effects are not considered in this project.

1.6 Structure of the report

The report consists of four main parts:

1. The first part, Chapters 2-3, presents criteria that need to be considered in design of tall buildings and facts about different types of structural systems for tall buildings. Advantages and disadvantages of the different structural systems are highlighted in this part.
2. The second part, Chapters 4-7, concerns methods of analysis of the structures presented in the first part. More precisely this part contains methods to estimate lateral deflection by different methods, suitable for different types of structures, and a method is presented for how stability of tall buildings with varying stiffness can be handled. Finally the second part of the report presents how dynamic analysis and the structural optimisation is applied and handled in this project.
3. The third part, Chapters 8-9, presents assumed conditions for the case studies in this project and choices are made for promising structural systems. Furthermore, the calculation procedure is presented and explained in this part.
4. The fourth and final part, Chapter 10-12, is where the results are presented and analysed. The analysis of the results is also connected to the literature study which is presented in the previous parts of the report. Furthermore, some conclusions are drawn concerning the whole project and the results and general recommendations for preliminary design of tall buildings are suggested. In the end of the report suggestions for further studies are presented.

2 Design criteria in general

Requirements that can be decisive for what type of structural system that is chosen for a building are among others architectural requirements. The architectural requirements depend on whether the building will serve as a residential building or if it is intended for commercial use. Nevertheless when a very tall building is designed, then structural requirements become the most significant design criteria [Smith and Coull (1991)].

Since a building will serve a function, it is of importance to design structural systems for buildings in order to allow properly functioning service systems, such as ventilation, water supply and waste disposal. The main criteria considered in this project are of structural kind and the essential ones are that the structural system can resist loads as well as having sufficient lateral stiffness [Smith and Coull (1991)]. This means that structural systems of buildings, and their structural elements should be designed to resist loads that are likely to act on the structure, during construction and service, also in order to fulfil criteria for serviceability [Eurocode 0, CEN (2010)].

The intended service life, which is chosen and considered in design of a structure, depends on what purpose the structure should fulfil. According to Eurocode 1, CEN (2010), there are different categories for different purposes. For structural parts in buildings a suggestion for the intended service life is 50 years. For structural parts in "monumental buildings" and bridges the suggestion is 100 years as intended service life.

The two fundamental limit states, which are used to verify the structural performance, are ultimate and serviceability limit states. These limit states are described in Sections 2.2 and 2.3 respectively.

2.1 Actions on a structure

Structural systems of tall buildings must have capacity to carry loads that can be either static or dynamic. Actions on a structure can be classified as following:

- Vertical loads
    - Permanent loads (dead load)
        - Dead load of structural members
        - Dead load from non-structural members and fixed equipment and installations
    - Imposed loads (live load)
        - Load from occupants, furniture, equipment and storage
        - Snow load
- Lateral loads
    - Wind
- Other effects
    - Geometric imperfections
    - Earthquake
    - Accidental actions

The imposed load depends on the type of activity taking place on a certain floor, such as parking areas, hospital, office or residential areas. The magnitude of the imposed load is specified for different situations in Eurocode 1, CEN (2010), and in the National Annex in Sweden, EKS 9 (2013). In this project the magnitude of the live loads, i.e. imposed loads for an office building is 2.5 kN/m^2 [EKS 9 (2013)].

For tall structures subjected to lateral actions reasonable simplifications can be justified. The lateral actions, such as wind and earthquake, can be treated as equivalent static loads reduced to a series of horizontal loads applied at each storey of a tall building. This simplification is sufficient to give a good estimation of the internal forces developed in the structure.

The wind load varies along the height and it depends on the surrounding environment. More about how wind load is determined is described in Section 9.4.

Geometric imperfections, known as unintended inclination of vertical members, must be considered when designing and analysing structures [Taranath (2011)]. The moment that arises in the foundation of a building, due to lateral loading, is increased due to geometric imperfections. This is why geometric imperfections can be handled as a lateral load applied on each storey.

2.2 Ultimate limit state

The verification in the ultimate limit state concerns the safety of humans as well as the safety of the structure itself. When equilibrium is lost, meaning that additional load cannot be resisted by the structural system, the ultimate limit state is reached. This is when deformations become out of control, when a mechanism has developed or the material itself breaks.

In the ultimate limit state it is important not only to take into consideration the worst loads that the structure can be subjected to, but also the second order effects must be considered. It is also important to identify critical elements in structural systems and design these in order to prevent progressive collapse in case of accidental actions.

Generally, a fundamental equilibrium check of buildings must be made with respect to the stability of the building against overturning. In Figure 2.1 it is illustrated that lateral actions give rise to a moment M_w around bottom edge of a building and this so called overturning moment must be smaller, giving a low probability of failure, than the resisting moment M_G that arises due to the vertical load of the building [Smith and Coull (1991)]. Furthermore, stability check second order effects must be included.

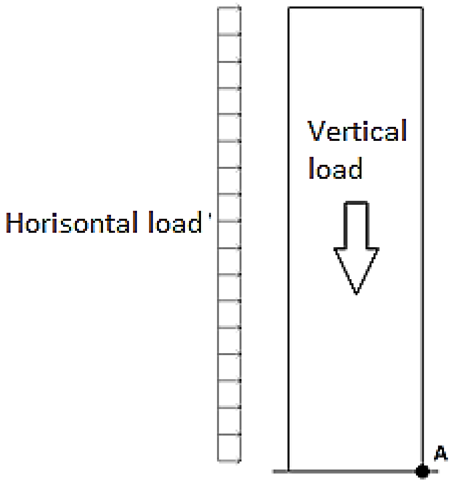

Figure 2.1: The stability against overturning of a building. The moment that arises at point A due to horizontal load w must be smaller than the moment at A due to vertical load.

2.2.1 Overturning moment

According to Eurocode 0, CEN (2010), the following equation (2.1) must be satisfied to ensure safety against overturning

$$E_{d,dst} \leq E_{d,stb} \qquad (2.1)$$

where

$E_{d,dst}$ = design value of overturning load effects

$E_{d,stb}$ = design value for stabilising load effects.

2.2.2 Load carrying capacity

The following equation (2.2) must be satisfied to ensure that the capacity of a structure or structural component is sufficient

$$E_d \leq R_d \qquad (2.2)$$

where

E_d = design value of load effect,

R_d = design value of corresponding load carrying capacity.

Checking the capacity of a structure also includes global and local stability check. Local instability can lead to progressive collapse, if the structure is not redundant.

2.3 Serviceability limit state

According to Eurocode 0 the serviceability limit state design is performed in order to ensure adequate function of structural systems in normal service, the comfort of humans and acceptable appearance of a structure. Therefore it must be verified that

$$E_d \leq C_d \tag{2.3}$$

where

C_d = design value of a specific serviceability criterion,

E_d = design value of load effects that is determined for the relevant load combination with respect to the specific serviceability criterion.

There are several kinds of serviceability criteria such as limited crack width, deflection and stress. Nevertheless, in this project, the criteria of main interest were maximum lateral deflection and acceleration at the top of the building, due to the fact that focus was laid on preliminary design. Thus, no detailed sizing and analysis of individual members were carried out.

2.3.1 Lateral deflection

A way of measuring the stiffness of tall buildings is by the so called *drift index*. The drift index is the maximum lateral deflection at the top of the building divided by the total height of the same. There is no specified limit for an acceptable drift index. An example of a generally used drift index limit for tall buildings is shown in criterion 2.4.

$$\frac{u_{\max}}{H} \leq \frac{1}{500} \tag{2.4}$$

where

$u_{\max}$ = maximum tip deflection at the top of the building.

H = height of the building.

Usually it is included in the design process to choose a suitable limit for the drift index. In different countries, and depending on what service the building will provide, the drift index can vary considerably. According to Smith and Coull (1991) the drift index can vary from 0.001 to 0.005. Even though this might differ in different situations, they all have the following in common; lower values are used for residential buildings and hotels in comparison to commercial buildings, such as office buildings, where higher values are accepted.

Important to notice, as an example, is that if the limit for lateral deflection is fulfilled, it does not necessarily mean that dynamic comfort criteria are fulfilled [Smith and Coull (1991)].

2.3.2 Acceleration

Buildings that have too large acceleration can become uninviting, and hence inhabitable, even though they can carry the loads that arise due to oscillating wind action [Smith and Coull (1991)]. It is therefore important to perform a dynamic analysis of structures in cases where dynamic loads could cause large accelerations [CEN (2010)].

It is generally known that acceleration is the most important parameter in order to evaluate the human response to vibrations with respect to comfort. Still there are no exact limits for comfort criteria [Smith and Coull (1991)].

The International Organisation for Standardisation (ISO) provides guidelines for evaluation of human response to low-frequency horizontal motion. The guidelines are similar to the ones provided by Smith and Coull (1991) and can be seen in Figure 2.2.

Perception of motion is subjective, meaning that different persons might perceive the same magnitude of motion differently.

Different limits of acceleration are justified for buildings with different activities. For calmer and quieter activities, such as office and residential buildings, the threshold for maximum allowed acceleration is relatively low. For buildings intended for other types of activities, for example where physical activity is common, the threshold is not as strict.

Acceleration (m/sec^2)	Effect
<0.05	Humans cannot perceive motion
0.05–0.10	Sensitive people can perceive motion; hanging objects may move slightly.
0.1–0.25	Majority of people will perceive motion; level of motion may affect desk work; long-term exposure may produce motion sickness
0.25–0.4	Desk work becomes difficult or almost impossible; ambulation still possible
0.4–0.5	People strongly perceive motion; difficult to walk naturally; standing people may lose balance.
0.5–0.6	Most people cannot tolerate motion and are unable to walk naturally
0.6–0.7	People cannot walk or tolerate motion.
>0.85	Objects begin to fall and people may be injured

Figure 2.2: Levels of acceleration corresponding to the degree perception of humans. [Smith and Coull (1991)]

2.4 Fire safety

To consider fire in design is specifically important for tall buildings, as the safety regulations are stricter for tall buildings in comparison to lower buildings. This is due to the fact that if a fire occurs in a tall building, it might lead to more severe consequences than for a low-rise building. Furthermore, the evacuation is more complicated as external evacuation is limited.

There are many factors that affect the structural system when considering fire safety. One major aspect is the requirement to make it possible to evacuate occupants in case of fire. For example, this can be achieved by elevators in protected structural cores.

In case of fire the structural materials in tall buildings will eventually be exposed to high temperatures. This affects the bearing capacity of the structural system, as the modulus of elasticity more or less decreases with increasing temperature depending on the type of material.

When a fire starts in a building, the building must be able to withstand its loads for a certain time before it collapses [CEN (2010)]. The reason is that the occupants must have time to evacuate the building and the fire-fighters must have a chance to execute their mission before the structure collapses. If a structural system is made in steel, it is extra sensitive to increased temperatures, but the collapse can still be delayed by measures such as an isolating spray paint layer and gypsum boards that cover the steel.

A way of enhancing the fire-safety of tall buildings is by preventing the fire from extending from one area to another in the building. For instance, the fire should not be able to spread through the ventilation system from one room to another or from one floor to another and the fire should not easily be able to spread through openings in the façade from one floor to another.

For more thorough information about what to consider in design with regard to fire, reference is made to Eurocode 1 where methods for fire analysis are provided and to Samuelsson and Svensson (2007) where additional information is summarised.

2.5 Foundation

Differential settlements, arising from different compressive forces at different areas in the foundation, might lead to redistribution of sectional forces in any type of structural system. Furthermore, deformation induced stresses could arise in statically indeterminate structural systems, for instance non-braced structures with moment resisting connections. Another problem that might arise in cases where the foundation is not infinitely stiff is that a rotational settlement could occur. This would lead to a an increased maximum lateral deflection at the top of the building and also increase the second order effects that could lead to instability [Smith and Coull (1991)]. These are reasons why differential and rotational settlements should be prevented by a proper design of tall buildings.

Uplift is another issue that the structural system, specially the foundation, should be designed for. Uplift could occur when overturning moment due to wind load becomes too large [Smith and Coull (1991)].

2.6 Effects of imposed deformation

Imposed deformations that arise in structural systems are either stress dependent or stress independent strains. When differential strains arise non-structural members are forced to carry load which is not acceptable. An example of strain that might affect the structural behaviour of a structural system is axial shortening of members subjected to compression. This occurs in columns and load bearing walls and it is important to take into account and compensate for this axial shortening in detailed design.

The two following sections describe different type of strains, which need to be considered in detailed design. Nevertheless, these effects were neglected in this project as the project focuses on preliminary design.

2.6.1 Stress dependent strain

Stress dependent strains are elastic strain and creep strain. Elastic strain depends on the magnitude of the stress and modulus of elasticity of the material. Creep develops over time and the total creep deformation depends on the loading historey.

Furthermore problems might arise if the strengths of connected structural members are not developed simultaneously. This could for example be the case if precast and cast in-situ structural members are connected to each other and different creep strains arise in the different members.

2.6.2 Stress independent strain

Thermal strain develops due to temperature changes regardless of what stress the material is subjected to. All type of materials need to deform, more or less, due to temperature changes. An example of how a whole structure can be affected by temperature change is when a tall building is subjected to sunlight on one façade, while the opposite façade remains in shadow. This can lead to a differential thermal deformation between the two opposite façades resulting in deflection of the whole structure [Smith and Coull (1991)].

Another stress independent strain is shrinkage. This is an issue that must be considered mainly for concrete structures, but also for timber structures. Steel structures do not shrink independently of stress.

2.7 Acoustics and indoor climate

Acoustics is an important issue, as it affects the comfort of the occupants. Different structural systems react differently to acoustic vibrations. A structural system can be adjusted, e.g. by inserting insulation where it is needed, in order to ensure comfortable environment. Nevertheless, it is preferable to consider acoustics already in the early design of a structural system to prevent many late adjustments of this kind, as they can be costly and less efficient.

To obtain a structurally efficient structural system, by minimising the amount of material used, is a way to try to reach a sustainable design. Simultaneously, it is of importance to keep in mind that other issues might affect whether a design is sustainable or not in long term. The building's ability to store heat is such an issue. Having a structure with low thermal storage capacity usually results in a larger need of heating and cooling systems, whereas a structure with high thermal storage capacity regulates its indoor climate on its own to some extent. This topic as well as acoustics was not investigated in this project.

3 Structural systems of tall buildings

From a structural engineering point of view, the choice of the structural system of a tall building would ideally involve only the selection and arrangement of the structural members to ensure safety and functionality. However, reality is not that simple as there are many other parameters, such as architectural, fire and acoustic issues, that play an important role on the choice of a suitable structural system.

It is however important to note that the taller and more slender a building, the more important the structural factors become.

There are many different types of structural systems of tall buildings that have been developed and yet engineers are trying to come up with new efficient systems. According to Smith and Coull (1991), a rough estimation of how economically feasible each structural system is for different heights is listed as following

- Non-braced frames with moment resisting connections: 1-25 stories
- Truss-braced structures: 1- the very tallest
- Shear wall-braced structures: 1-35 stories
- Core structures with outriggers: 40- the very tallest
- Tube structures: 40- the very tallest
- Interacting structural systems: 1- the very tallest

The type of structural system that is suitable for a certain ranges of number of stories depends also on the slenderness of the structure and other issues such as foundation conditions, architectural constraints and fire safety. These issues and other factors that need to be considered when a structural system for a tall building is to be chosen are described and discussed in Chapter 8.

Tall buildings have two types of lateral deformations, which depend on the slenderness of the structure, namely shear deformation and flexural deformation. This is visualised in Figure 3.1.

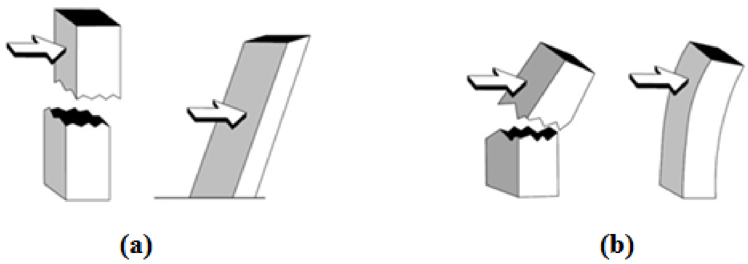

Figure 3.1: The deformed shape of a cantilever, which a tall building could resemble, arises due to two types of deformation; (a) shear; (b) flexure.

It is important for a structural engineer to be aware of these two phenomena when performing analysis of the lateral behaviour of a structure in order to estimate a realistic lateral displacement of the structure.

The more slender a tall building is, the larger percentage of the total lateral displacement arises due to flexure. Vice versa; the more non-slender a structure is, the larger percentage of the total lateral displacement arises due to shear. This is clearly shown in Figure 3.2, [Neuenhofer (2006)]. For a structure where the height is at least five times the widest width of the structure, the shear contribution to the total lateral displacement is negligible. However, this phenomenon does not arise for non-braced frame structures. Their behaviour is explained in Section 3.1.

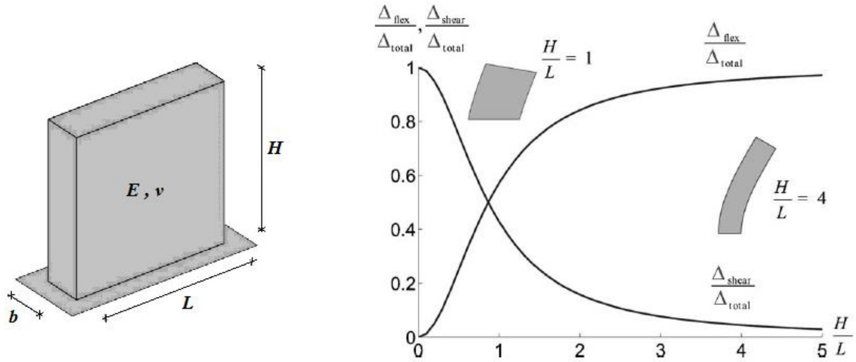

Figure 3.2: Total lateral displacement of buildings, with different height-to-width ratios, consisting of shear- and flexural contributions. [Neuenhofer (2006)]

3.1 Non-braced frame structures

Non-braced frame structures consist of columns and beams rigidly joined together in moment-resisting connections as shown in Figure 3.3. In this project, a non-braced frame structure is referring to a frame with no bracings but with moment resisting connections.

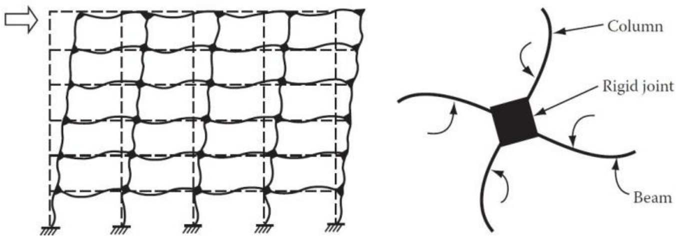

Figure 3.3: Non-braced frame subjected to horisontal load at the top. [Taranath (2011)]

As presented earlier in Chapter 3 tall buildings have two types of lateral deformation modes which depend on the slenderness of the structure. In general the more slender a building is, the more it behaves in flexure. However, this does not hold for structures with moment resisting connections, such as non-braced frame structures. These structures behave predominantly in a shear mode. A structure of this type within the range of 20 to 30 stories will approximately have a total lateral displacement consisting of 80-90 % shear racking, while flexure only contributes to 10-20 % [Chen and Lui (2005)]. It is therefore important for a structural engineer to consider shear deformation, as well as flexural deformation, specifically in non-braced frames.

3.1.1 Sway due to vertical load

For a structural engineer to be able to handle and solve problems, it is necessary to make some simplifications in modelling. For example, when designing and analysing a structure the case can be simplified to be symmetric, even though in reality this will not always be the case. The consequences of simplifications are especially important to have in mind concerning non-braced frame structures, because there is a phenomenon called side-sway which depends on whether a load is symmetric or not.

According to Schodek (1997), side-sway occurs when the structure or the load is asymmetric and the structure reacts so that equilibrium is fulfilled. The effect is that the structure translates laterally when loaded asymmetrically; see Figure 3.4a. As can be seen in Figure 3.4c, the moment at the right corner will be larger than the moment at the left corner of an ideal non-braced frame structure where the joint is not allowed to rotate. As the restraint at the base of the column depends on the moment at its upper joint, it will lead to a larger restraint at the base of the right column and a smaller restraint at the base of the left column. When checking the horizontal equilibrium of the frame it can be seen that it will not be fulfilled due to the unequal restraints. Equilibrium must be fulfilled and the only way for the frame to achieve equilibrium is by translating to the left. In this way the moment will be slightly enlarged at the left corner since its angle is forced to be smaller, while it is a reduced moment at the right corner which is opened up to a larger angle. This will also result in an increased restraint at the base of the left column and a decreased restraint at the base of the right column. To sum up, in order to fulfil equilibrium the frame sways to the side so that the corner moments will be equal.

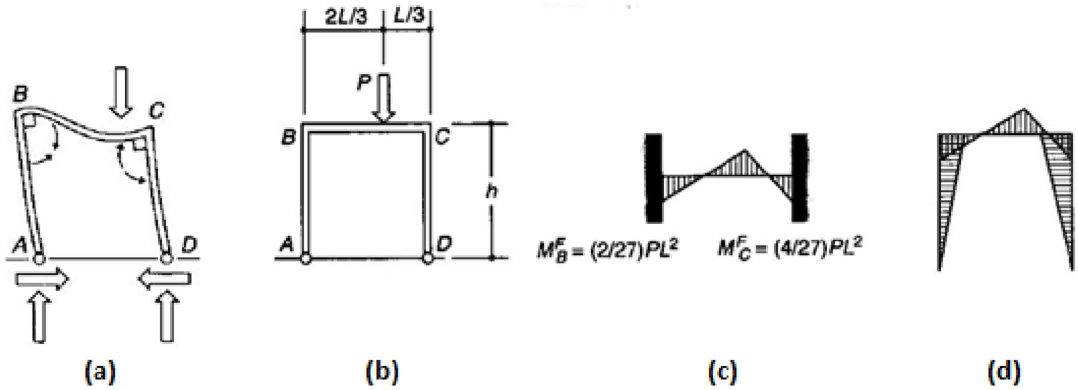

Figure 3.4: Influence of asymmetric loading; (a) Reactions due to external load; (b) Asymmetric load case on frame; (c) Magnitude of moment in the beam; (d) Moment distribution. [Schodek (1997)]

Vertical load such as self-weight and live load is resisted by frame action with load paths from the beams to the columns and further to the foundation. In general, the weight of a non-braced frame structure is increasing linearly with the number of stories. Consequently, the weights and sizes of the columns can be assumed to be proportional to the axial forces accumulated [Sasson (1995)].

Wind load is the dominant lateral load on tall buildings. Non-braced frames with moment resisting connections resist lateral load by the interaction between the columns, beams and their connections. Hence, the magnitude of the lateral displacement at the top of a non-braced frame depends on the flexural stiffness of these members [Smith and Coull (1991)].

From a structural engineering point of view the main advantage with this type of structural system is that the rigid connections between beams and columns reduce the positive bending moment for the beams as well as they reduce the effective length of the columns. This generally results in smaller member sizes compared to simply supported members. From an architectural point of view the non-braced frame structure provides open arrangement of structural members, allowing freedom for interior and exterior planning such as e.g. location of doors and windows [Smith and Coull (1991)].

According to Smith and Coull (1991), the economical height limit of non-braced frame structures is approximately 25 stories, but, in case of taller structures non-braced frame structure can be very costly in order to keep the lateral deflection of the structure at an acceptable level. This is due to the moment resisting connections, which are complicated to design. If concrete is used as material, longer construction time as well as expensive formwork is required. If steel is used, it requires expensive fire proofing system as well as expensive moment-resisting detailing and maintenance are required as well [Taranath (2011)].

The fact that smaller sizes of members are needed in non-braced frame structures compared to simply supported members is therefore only true up to the limit height above mentioned. The reason is because the magnitude of the bending, shear and axial forces depends on the building height. According to Schodek (1997) and as can be seen in Figure 3.5, when these forces become too large, the beams and columns will also have to increase in proportion. This could overturn the positive feature of non-braced frame structures with regard to space utilisation. Additionally, too large members will most likely lead to loss of space for stories, if the height of the structure is limited.

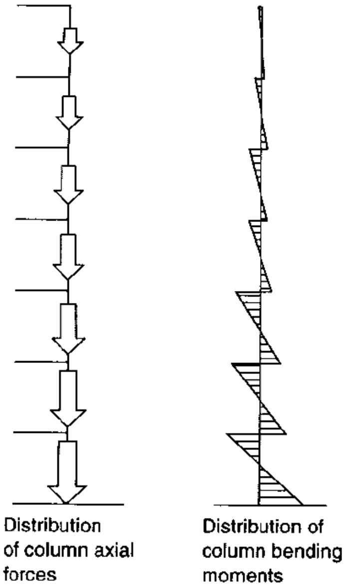

Figure 3.5: Non-braced frame axial forces and bending moments with height. [Schodek (1997)]

A structure with moment resisting connections is more sensitive to differential settlements compared to a system of simply supported members. When for example a differential settlement arises, stresses will arise in a structure with moment resisting connections. A simply supported structure is not affected in the same way as a non-braced frame structure due to translation of one of its supports, in relation to the other supports. However, a non-braced frame structure would be affected as its upper part can be seen as a beam with partially fixed edges. If one fixed edge is translated vertically in relation to the other edge, then deformation induced stresses will appear in the different parts of the frame [Schodek (1997)].

Furthermore, the foundation for a non-braced frame structure needs to take care of an additional lateral force, which puts greater demands on the foundation than if it would have been a simply supported frame. This phenomenon is a horizontal restraint as shown in Figure 3.6.

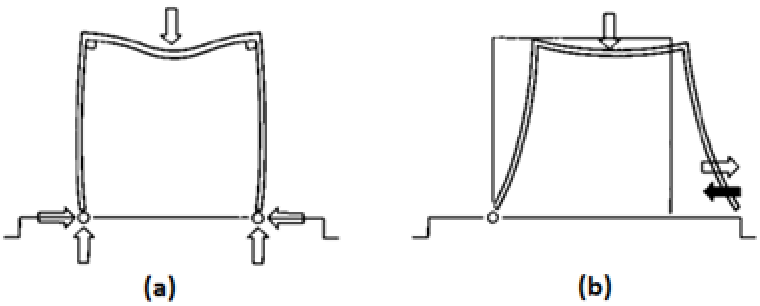

Figure 3.6: (a) Horisontal restraint at column base of non-braced frame structure; (b) realise of horizontal restraint. [Schodek (1997)]

3.2 Truss-braced structures

A truss-braced structure is essentially a beam-column structural system with truss units that resist lateral loads by means of diagonal and beam structural members, see Figure 3.7. Centric bracing trusses are configured so that the horizontal bars, diagonal bars and the chords meet in one point which can be seen in Figure 3.8. Eccentric bracing trusses do not meet in one point as can be seen in Figure 3.11. In the coming paragraphs two different types of truss-braced structures, centric and eccentric truss-braced structures, are presented and their behaviours are explained.

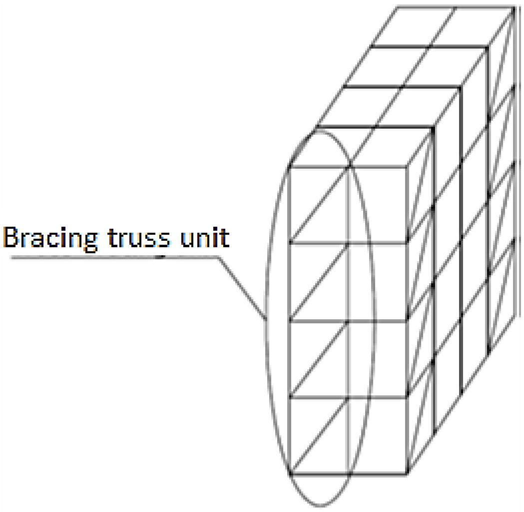

Figure 3.7: Truss-braced structure.

3.2.1 Centric bracing truss

The behaviour of truss-braced structures when subjected to lateral loads is similar to that of an I-beam. While the column-beam system carries the gravity load, the bracing truss units carry the lateral load. To clarify how the bracing units work, the diagonal bars, simply called diagonals, and horizontal bars act as webs transferring "shear" and the two chords act as flanges transferring "moment" by axial tension and compression in each chord. The different members that trusses consist of are defined in Figure 12. The axial shortening and lengthening of the chords tend to cause a flexural deformation of the structure, whilst the diagonals and girders that transfers lateral loads tend to cause deformation of diagonals in a shear-racking shape. Therefore, when calculating the lateral deflection of a braced frame, the engineer must take both these effects into account. Otherwise, the final lateral deflection might be underestimated. Both effects and how the different members in a truss unit are transferring load are illustrated in Figure 3.9.

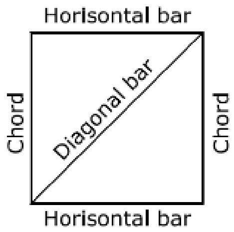

Figure 3.8: Definition of members in a truss.

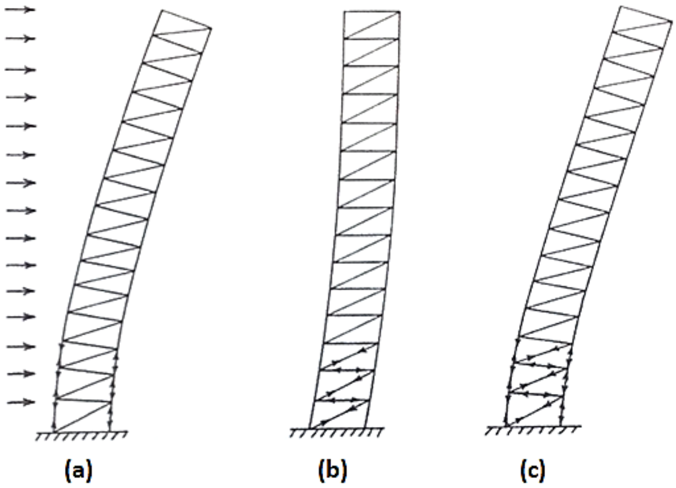

Figure 3.9: Different structural behaviour of a bracing truss unit subjected to lateral load; (a) Flexural deformation; (b) Shear deformation; (c) Combined flexural and shear deformation. [Smith and Coull (1991)]

What characterises a truss-braced structure is that the lateral deflection is controlled by the axial stiffness of diagonals, horizontal bar and chord members. Furthermore, the deflection also depends on the width of the truss bracing.

The determination of member forces in a bracing truss is based on the same theory as for a truss in general, meaning that no bending action is introduced in the joint and, hence, the load is resisted by axial responses only. In reality this is not fully true, as the joints always transfer a small magnitude of moments. However, in order for the "real" structure to function as the theory implies, meaning that only axial work is performed, the centre axis of each structural member must meet in the centre of the adjacent joint. By this, the transmission of forces will be predominantly in axial action, and therefore, this simplification works well. Figure 3.10 shows some of many different layouts which are possible for centric bracing units.

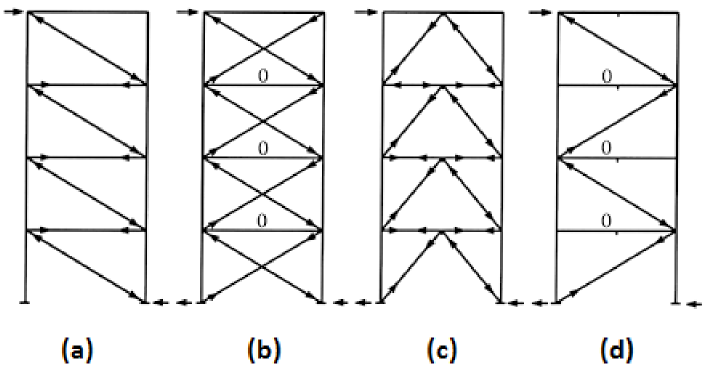

Figure 3.10: Centric bracing units; (a) Single-diagonal bracing; (b) X-bracing; (c) Inverted V-bracing; (d) Single-diagonal alternate direction bracing. [Taranath (2011)]

Steel is the most common material used for bracing units due to the fact that it has high strength both in tension and in compression and it requires relatively small dimensions in comparison to, for example, concrete. When a braced structure is subjected to lateral loading parts of the bracing unit will work in tension, which is why steel is a suitable material for the bracing. This is to be compared to concrete which does not work as well in tension. Concrete bracings could be used, but then it has to be as an x-bracing, see Figure 3.10b. For this type the bracing will only work in compression and each diagonal has to be able to take the shear that arises due to lateral loading, meaning that only one of the diagonals is actively working.

Generally use of diagonal bracing is an effective way of resisting lateral load, as it requires relatively little material and it is suitable for low-rise buildings as well as for very tall buildings. Furthermore, the centric bracing truss liberates the girders and columns from resisting lateral load that is subjected to the building and therefore these members only need to be designed with respect to their own self weight and permanent and imposed loads. This means that the girders will be of the same size no matter what height, or what storey, they will be at.

What characterise the high efficiency of truss bracings is that load is carried by axial response of the members only. The structural efficiency can be optimised, especially when the load carrying system that resists lateral load is located at the perimeter of the building, which is the case for core structures with outriggers. More about core structures with outriggers is presented in Section 3.5.

The reason why it is the most efficient way to carry load is due to the fact that the whole cross section of the members is equally activated when the truss is resisting an applied load. This is to be compared to a member working in flexure, where the stress distribution is not uniform across its cross section and, hence, the material is not utilised in the same efficient way as an axially loaded member is.

Placing bracing truss units in the façade can limit or interfere with architectural features. These features can be e.g. doors and windows, which have to be considered. However, bracings can also be placed in dividing walls, so that they are not visible [Coull and Smith (1991)]. For residential buildings it may not be suitable to have bracing elements in the façades. On the other hand it can be a part of the architectural expression in the façade in other types of buildings. Examples of where bracing can be used in the facade to enhance function and aesthetics is in exhibition halls, train stations, large office buildings etc.

3.2.2 Eccentric bracing truss

In eccentric braced frames, the lateral load is not only transferred axially through diagonal members but also through the so called “links” in shear and bending. A link is a segment e that is a part of the floor members, see Figure 3.11. Eccentric braced frames are mainly for seismic design, where extremely large lateral forces occur in the structure. Since some of the load action is resisted through in bending and shear in the links, the axial stress in the diagonals is reduced. Hence, buckling of the diagonals and horizontal bars is delayed [Popov (1986)].

According to Smith and Coull (1991), due to the plastic behaviour of the short links resulting in high ductility, the eccentric bracing truss provides large plastic deformation capacity and can sustain many cycles of earthquake loading. However the eccentric bracings are less stiff and less efficient than centric bracing truss since shear and moment are introduced in addition to axial load in eccentric bracing trusses.

Eccentric braced frames will not be further considered, as seismic design is not within the scope of this project.

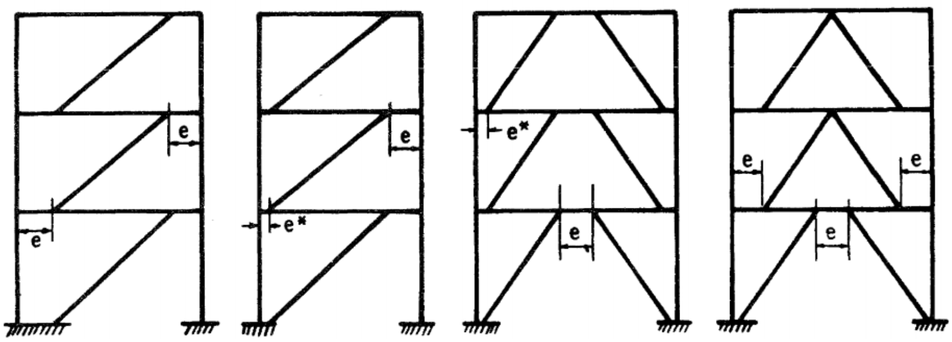

Figure 3.11: Different configurations of eccentric bracing units. [Popov et. Al (1986)]

3.3 Shear wall-braced structures

Shear wall-braced structures are common structural systems for e.g. residential buildings but can be used to a wide range of other types of buildings. The entire structure is laterally braced by either individual concrete shear walls, combined shear wall units or a combination of both as shown in Figure 3.12. Combined shear wall units consist of several shear walls, stretching in different directions in plane, that are rigidly joined together in order to form one bracing unit.

These individual bracing members and combined units usually continue down to the base to which they are rigidly connected. The main task of these bracings is to transfer lateral loads down to the foundation mainly by flexural action, due to their slenderness, but they can also carry gravity loads.

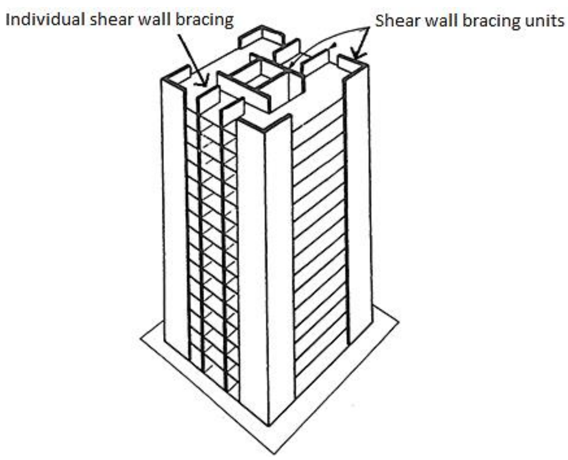

Figure 3.12: A tall building braced by individual shear walls and combined shear wall units. [Smith and Coull (1991)]

It is important to be aware of that the structure shown in Figure 3.12 is a simplification as columns are not shown. In reality shear wall-braced structures may also include columns that help to carry gravity loads, particularly for shear wall-braced structures with parking garage in the bottom part of the building where open space is required.

Individual shear wall members and combined units they usually have a high height-to-width ratio and therefore have a predominant flexural behaviour. The economical height of shear wall-braced structures is approximately up to 35 stories due to their in-plane stiffness [Coull and Smith (1991)]. However, a combination of shear with e.g. non-braced frame structure with moment-resisting connections can increase the efficiency of the entire structure, and thus, increase the economical height of the building up to approximately 50 stories.

From an architectural point of view shear wall-braced structures are suitable for buildings in need of partitions, i.e. to enclose spaces such as apartments in a residential building. A further advantage is that shear wall-braced buildings perform well with regard to sound and fire insulation between different parts of the building and therefore it is a well suited structural system for e.g. residential buildings and hotels. [Coull and Smith (1991)]

From a structural engineering point of view shear walls have very high in-plane stiffness and are therefore suitable for tall buildings. These individual shear wall bracing members can be arranged in a way to form one functioning structural unit. See Figure 3.13 where two types of bracing units are provided for the structure; the L-shaped units positioned in the corners of the building and the double-I-beam shaped elevator core unit in the middle of the building. Examples of several types of shear wall units are shown in Figure 3.13.

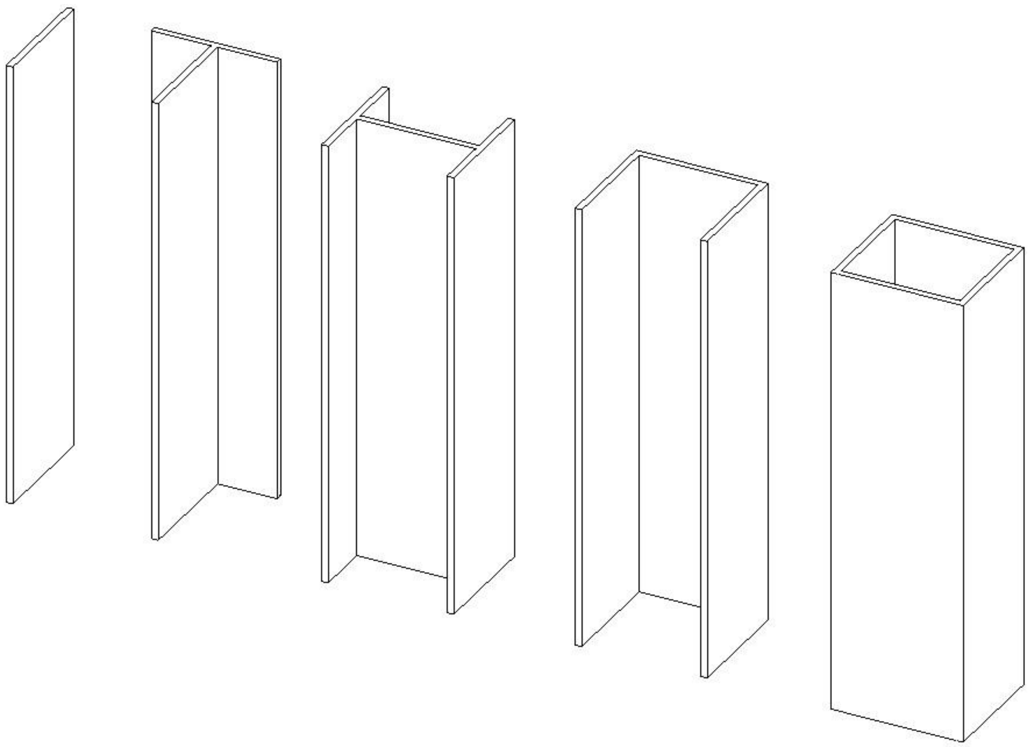

Figure 3.13: Examples of individual shear walls and combined shear wall units. From left to right; Individual shear wall, T- unit, I-unit, C-unit, Core unit. Observe that these usually contain openings which are not shown in the figures.

Consequently, depending on the configuration of shear walls the engineer can develop bracing units with very high flexural and torsional stiffness. As an example, the quadratic core unit shown in Figure 3.13 has higher torsional stiffness than the I-shaped unit. This is obvious due to Steiner's theorem. It is important to be aware of the fact that shear walls must be rigidly attached to each other in order to function as one combined unit.

Shear walls have shown to perform well in earthquakes as a result of their ductility [Coull and Smith (1991)].

Since shear wall-braced structures usually are made of concrete, the limited tensile strength is known to be the main problem with increasing height of the building. This is due to the fact that the flexural tensile stresses increase with the height as the bending moment caused by lateral load is a function of the height. Knowing this, it is an advantage to arrange shear walls in the structural system so that they carry gravity loads in order to minimise the flexural tensile stresses in the wall [Coull and Smith (1991)]. It is therefore of importance to know that the arrangement of the shear walls in a building plays an important role for the efficiency of the structural system. Shear walls that are not used to carry gravity load might be subjected to high tensile stresses. This could lead to that the amount of reinforcement needed significantly increases resulting in higher costs. In extreme cases there might be a need of prestressing, which also increases costs of bracing members.

3.4 Tube structures

A tubular structure is basically a giant tubular beam cantilevering out of the ground. The basic idea of all tubular structures is to place as much as possible of the load-carrying structural members in the exterior façades of the building. Consequently, the stiffness of the structure is maximised by engaging the maximised lever arm of the entire building to resist the lateral loads. More specifically, the entire structure can be considered as a beam with closely spaced columns in the exterior façades of the structure acting as "flanges" that resist the overturning moment due to lateral loads. These "flanges" help the core to resist lateral loads as well as transferring gravity loads down to the foundation. Principally, when the wind hits the building, two facades act as flanges resisting the overturning moment, while the other two façades parallel to the direction of the wind act as "webs" resisting shear. This is illustrated in Figure 3.14.

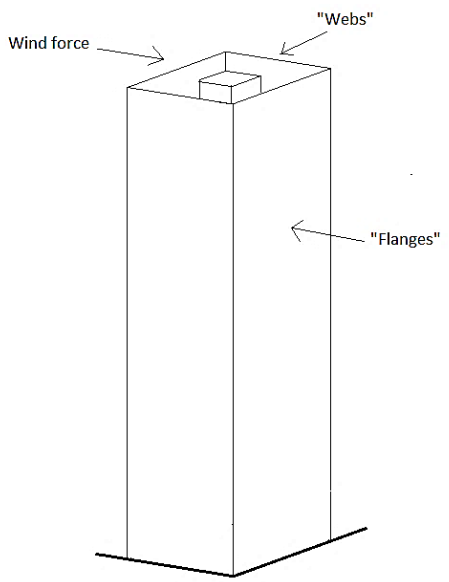

Figure 3.14: Principle of structural systems of tube structures.

There are three types of tube structural systems which are shown in Figure 3.15. The following sections explain the basic behaviour of the structural systems respectively.

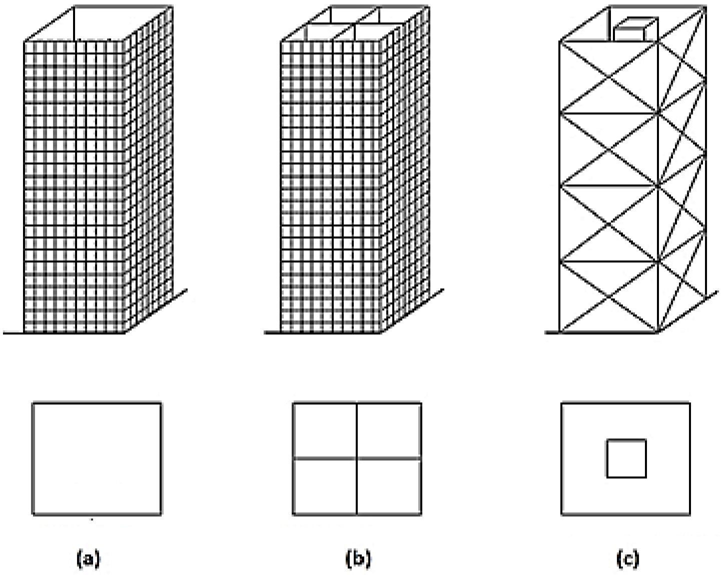

Figure 3.15: Tube structural systems of type (a) framed tube; (b) bundled tube; (c) trussed tube.

3.4.1 Framed tube structures

The framed tube system is historically the first tubular structural system developed for very tall buildings. The structural system consists of façades acting as a three dimensional non-braced frame structure with moment resisting connections. The façade consists of closely-spaced columns rigidly jointed with beams. Each façade wall of the building is rigidly connected to the other in order to form one three dimensional tubular unit that resists both lateral and gravity loads [Smith and Coull (1991)]. An example of tall buildings that had framed tube as structural system was the old World Trade Center, which can be seen in Figure 3.16. It consisted of two buildings situated at Manhattan, New York, and they founded on solid bedrock. In case of a non-braced frame structure with moment resisting connections it is preferable to have solid bedrock beneath, as the system is sensitive to differential settlements in the foundation. This issue of sensitivity is more thoroughly described in Section 3.1.

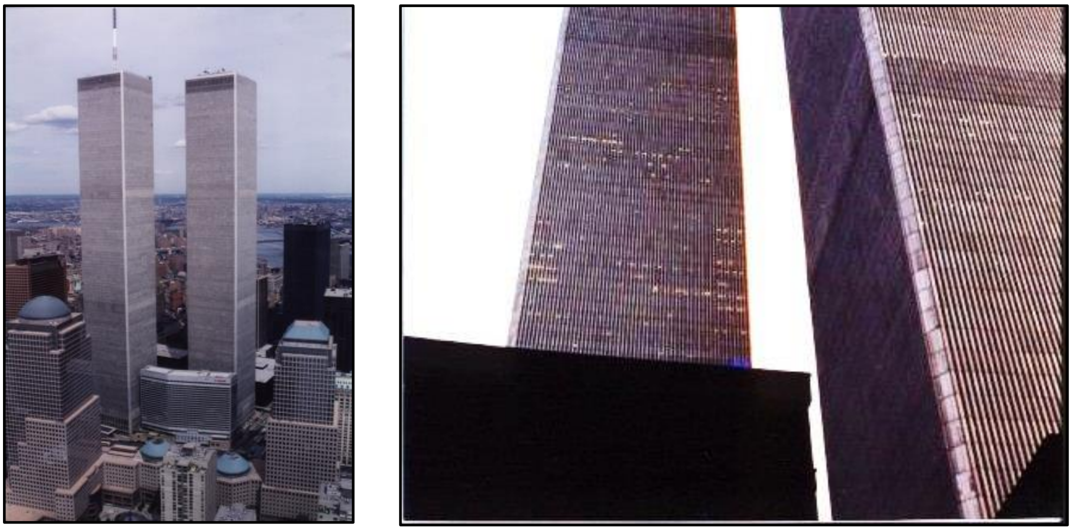

Figure 3.16: The old World Trade Center with a framed tube as its structural system. A consequence of the structural system is the small openings in the façade [Janberg (1998) and (2000)].

The framed tube system, compared to bundled and trussed tubes, is not such an efficient structure. This is due to the fact that when the lateral load acts on the façades, the frame deforms in a shear mode resulting in a reduction of the effective stiffness of the structural system. How a non-braced structure with moment resisting connections behaves when loaded is described in Section 3.1.

A further phenomenon which reduces the efficiency of a framed tube is known as *shear lag*. Shear lag can occur in the "flanges" of the structure where axial forces due to lateral load arise. The flanges suffer from shear lag effect due to the fact that the columns closest to the corners of the façade attract more load than the columns intermediate position in the façade. Consequently, the intermediate columns lag behind those closest to the corners and therefore are therefore less utilised. The effect of shear lag is shown in Figure 3.17.

The only way to minimise shear lag in a framed tube is to place the columns in the façade as close to each other as possible. In his way a more homogenous flange is obtained. Consequently, the closely spaced columns minimise the sizes of the windows in the façade and can therefore be seen as an architectural restraint, see Figure 3.16. Furthermore, the moment-resisting connections for such a tall building increase the cost significantly in comparison with e.g. the trussed tube system which is described in Section 3.4.3.

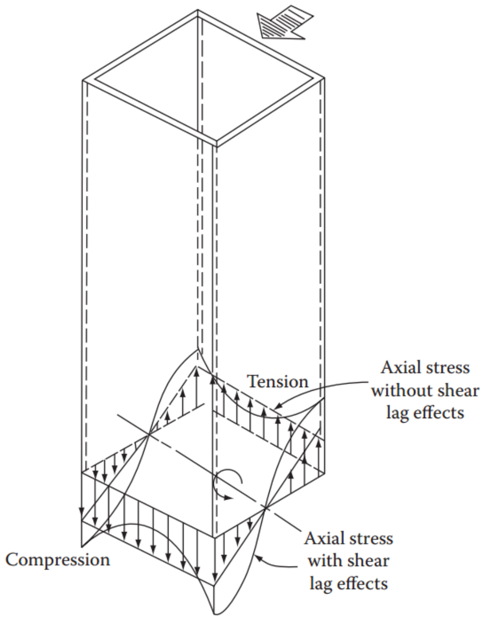

Figure 3.17: A tubular structure subjected to lateral load. The shear lag effect tends to cause a non-uniform stress distribution, resulting in a less efficient structure. [Taranath (2011)]

3.4.2 Bundled tube structures

A bundled tube structure consists of several tubes connected together in order to form a three dimensional tubular unit that has high torsional, flexural and shear rigidity. A bundled tube structure is shown in Figure 3.15b. Note that the figure shows an example of a bundled tube structure consisting of four tubes, but a bundled tube structure could also consist of other numbers of tubes. Furthermore, the individual tubes can be of different heights. As an example, Figure 3.18 shows a 442 m tall building named “Willis Tower” located in Chicago, USA. The Willis Tower consists of nine interacting tubes with different heights. These tubes act together as one three dimensional giant fixed-end cantilever. The developer of this structural system, Fazlur Rahman Khan, wanted to create a system for tall buildings that behaves dominantly in flexure. A tall structure with dominating flexural behaviour is more efficient than if shear behaviour is significant, as more of the lateral load is carried in flexure. Since a single tube unit behaves predominantly in flexure, Khan figured out that putting several cores together would give an effective tall structure mainly responding in flexure. Nevertheless, some shear deformation always arises.

A bundled tube structural system is more efficient than a corresponding framed tube. The reason can be clarified by referring to the Willis Tower as an example. Its structural system can be viewed as a giant beam cross-section with four “webs” across the entire width of the section, see Figure 3.19. When the structure is subjected to lateral loads causing it to bend, the high in-plane stiffnesses of the floor diaphragms constrain the interior “webs” to deform equally with the external “webs” in the façades. The shear forces are resisted by each “web” in proportion to its lateral stiffness.

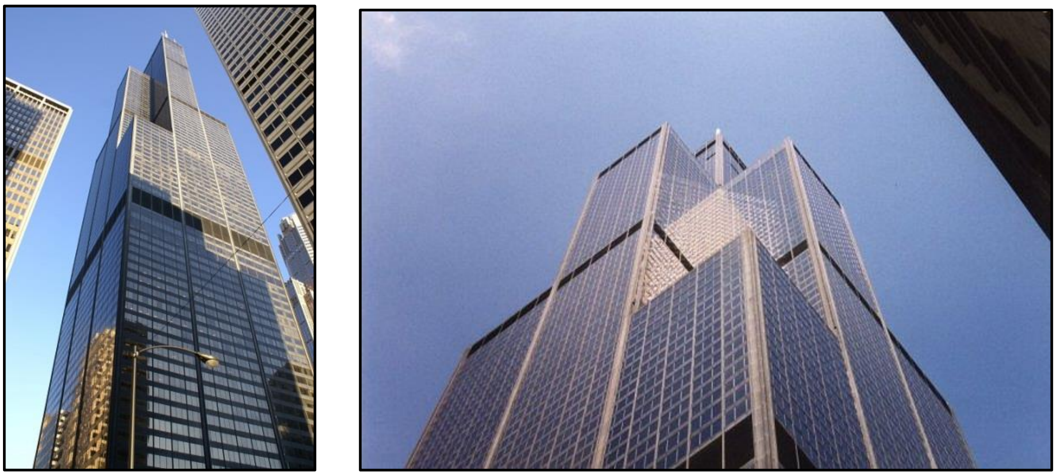

Figure 3.18: The Willis Tower with a bundled tube structural system [Janberg (2008)].

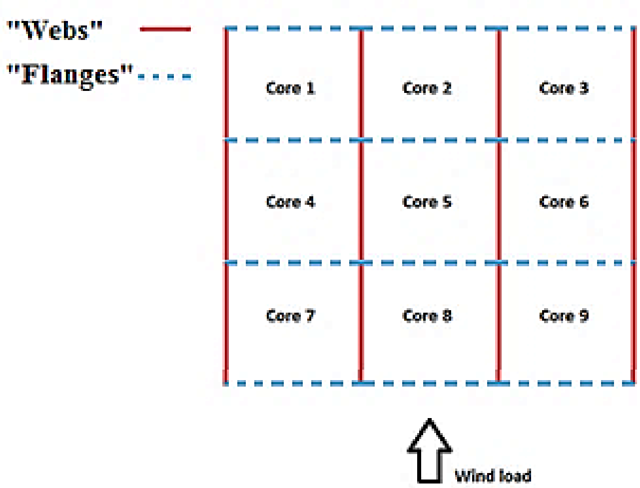

Figure 3.19: A plan view of Willis Tower, consisting of bundled tubes, subjected to wind load. The structural system is highly rigid with regard to torsion, flexure and shear.

Most important with regard to the shear lag effect, the end columns of the interior web frames will be mobilised directly by the web frames and therefore more stressed. The result is a more uniform axial stress distribution along the flange and a significant reduction of the shear lag effect. Thus, the bundled tube structural system is more efficient than the framed tube. [Taranath (2011)]

A further advantage with the bundled tube structural system is the fact that the sizes of the windows can be larger thanks to a more uniform axial stress distribution. Therefore, the bundled tube provides larger window sizes.

Despite the clear advantage and the higher structural efficiency of a bundled tube compared to a framed tube, the bundled tube structural system is not as efficient as the trussed tube structural system, which is presented in the following section.

3.4.3 Trussed tube structures

From a structural point of view, the most efficient tubular structural system is the trussed tube. This structural system could also be beneficial in an architectural point of view, if the diagonals are used as an aesthetic expression. The façades of the building consist of trusses that are connected to each other to function as one three dimensional truss unit that is highly rigid with regard to torsion, flexure and shear. Figure 3.20 shows an example of a 459 m tall building named John Hancock Tower located in Chicago, USA. This building was the tallest building in the world in 1968 with its trussed tube structural system.

Figure 3.20: John Hancock Tower in Chicago, USA. Its structural system consists of a trussed tube [Janberg (2008)].

What makes this structural system more efficient than both framed and bundled tube systems is the fact that the lateral load is resisted mainly by the four façades acting as one three dimensional truss. The diagonals are connected to the columns and eliminate the effects of shear lag in both the “flanges” and the “webs”. The result is that the structure, when subjected to lateral loads, behaves like a truss-braced frame resisting loads by axial action, which is the most efficient way of resisting loads. The behaviour of a truss-braced structure is described in Section 3.2.

A further advantage with a trussed tube structural system is how the diagonals help the columns to carry the gravity loads as well as minimising the shear lag effect. Differences in the amount of gravity load on the different columns are evened out by the diagonals. This is achieved by redistributing axial load from the more stressed columns to the ones that are less stressed. In other words, the diagonals help to even out the load between the columns.

A consequence of the efficiency of this structural system is that the columns can be placed with a larger distance from each other providing larger window openings than e.g. what a framed tube would allow in the façades.

A consequence of the diagonals in the façade is that they might disturb, as they block several windows throughout the height of the building, see Figure 3.20. On the other hand, the diagonals may also serve well for architectural purposes, as for the case in John Hancock Tower [Taranath (2011)].

A disadvantage of this structural system is that there are large steel elements that are relatively exposed. These are more sensitive to fire than e.g. concrete elements. Nevertheless the steel elements can be protected by special insulating paint that expands when heated and they should be designed so that they can withstand fire for a certain required amount of time. A further disadvantage is the fact that steel is prone to corrosion and therefore requires a lot of maintenance.

3.5 Core structures with outriggers

A core structure with outriggers is known as a very efficient structural solution for tall and slender structures that ranges from 40 stories to very tall buildings. Figure 3.21 shows a structure with centric core and two outriggers. The core can be placed either centric or offset as can be seen in Figure 3.22. In this project, only core structure with one outrigger with a centric core was further investigated.

This type of structural system is suitable for slender buildings. Consequently its overall flexural behaviour becomes the major contributor to the total lateral deflection. The main task for the outriggers is to involve the façades columns and by that increase the stiffness of the structure with regard to lateral load. This increase is gained due to the fact that the effective depth of the effective cross-section is increased. The outrigger forces the columns in the façades to be tensioned at the windward and compressed at the leeward side. Furthermore, when the structure is subjected to lateral loads, the outriggers prevent bending deformation of the core, resulting in reduced bending moment in the core as well as reduced lateral deflection. [Coull and Smith (1991)]

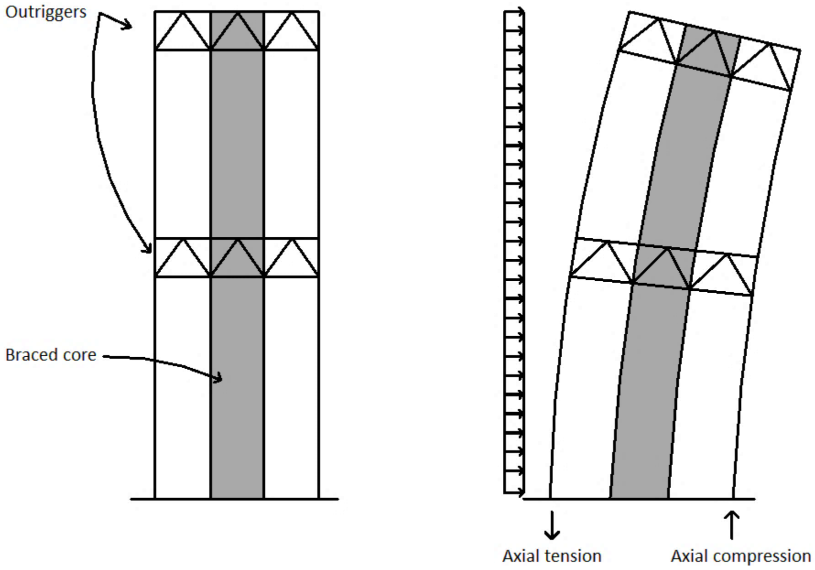

Figure 3.21: Core structure with centric core and two outriggers subjected to lateral load. The outriggers impose tension and compression on the windward and leeward side respectively.

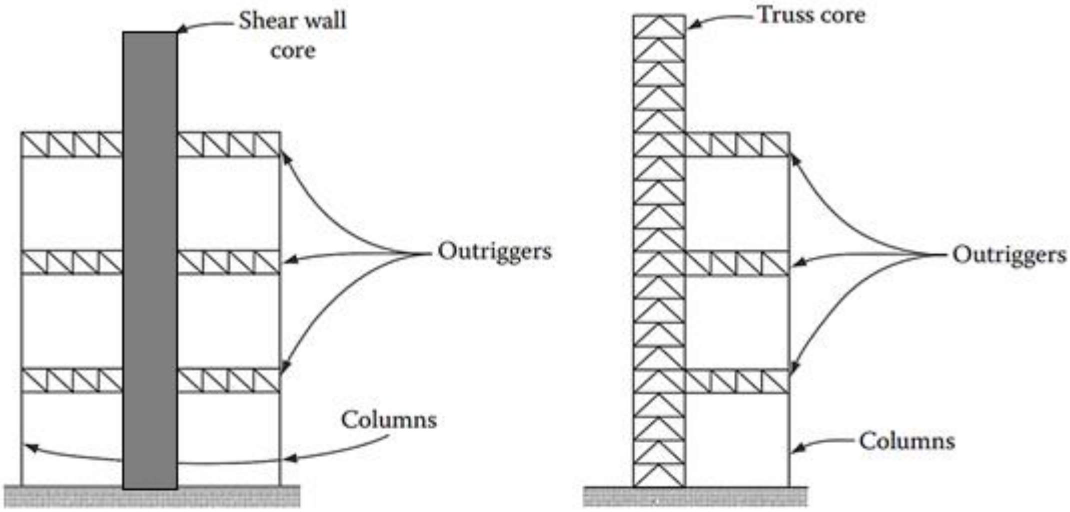

Figure 3.22: Outrigger braced core structure with (a) centric core; (b) offset core. [Adopted from Taranath (2011)]

From an architectural point of view the core structure with outrigger and centric core provides a column-free plan layout between the central core and the exterior columns, which makes this type of structural system well suitable for e.g. office buildings.

From a structural engineering point of view core structures with outriggers are highly efficient systems for tall and super tall buildings. This is due to the fact that the structural system involves the axial stiffness of the exterior columns to help resist the overturning moment. A further advantage of this structural system is that it equalises the difference in axial strain between the columns in the building, e.g. axial shortening due to very high axial loads. Core structure with outrigger structural system also gives the designer the possibility of deciding the optimum location of the outriggers depending on what is needed to be achieved. For example, if the tip deflection is the main problem in the design, the designer can place the outrigger in the higher regions of the structure in order to minimise the tip deflection. On the other hand, if the design problem is due to very large base moment at the bottom of the structure, then the outriggers can be placed in the lower regions of the structure in order to minimise the bending moment.

Core structures with outriggers are efficient and cost-effective solutions with regard to acceleration as these systems significantly minimises the lateral drift of the building, which is important to ensure occupant comfort. [Taranath (2011)]

The stories where the outriggers are placed can be used for several purposes. Two examples could be installation systems but also as so called refuge floors. Refuge floors can be used for evacuation in case of fire or other types of emergencies. According to Robertson, structural engineer of Shanghai World Financial Center, the occupants can be evacuated to these stories where they are protected in case of fire, if it is not possible to safely escape the building through the elevator shafts. This could be due to an event of smoke and fire in the staircases or in the elevator shafts. In this way the fire-fighters gain some time to rescue the occupants who are isolated in safe stories and not locked in areas filled with smoke and fire.

Despite the clear advantage gained by adding outriggers to the structure there is however an efficiency limit for the number of outriggers that can be added in a structure, above which additional outriggers will not increase the stiffness of the structure significantly. According to Smith and Coull (1991) a very tall building that features this structural system may include up to four outriggers. However, there is generally no need of adding more than four outriggers in a super tall building.

The main disadvantage of core structure with outrigger, with respect to the structural behaviour, is the fact that the outriggers only increase the flexural stiffness of the structure, but do not increase the shear resistance. The shear is resisted only by the core.

A further disadvantage of this structural system is that a number of rentable stories are lost to provide space for the outriggers. However, the floors where the outriggers are placed can be used for other activities or for installations such as ventilation, electricity etc. The space could also be used as refuge floor for evacuation as previously mentioned. [Taranath (2011)]

3.6 Interacting structural systems

There are many possible ways that different structural systems can be combined in order to increase the structural efficiency as well as the economical height of a structure. Since the behaviour of interacting systems is complex and this project focuses on the pure structural systems and their preliminary design, interacting systems were not further studied. For more information about how interacting systems work the reader is referred to Taranath (2011) and Smith and Coull (1991).

4 Estimation of lateral deflection

It is of importance to limit the lateral deflection as it may affect for instance the function of non-structural components such as doors and elevators. Too large lateral deflections, meaning that the drift index is larger than the maximum allowed, can lead to excessive cracking, loss of stiffness and redistribution of forces to non-structural elements [Smith and Coull (1991)]. Furthermore, an excessive deflection may lead to discomfort for occupants of a building. It is because of serviceability criteria that it is of great importance to limit the lateral deflection.

In the following sections it is described how the lateral deflection can be estimated for different types of structural systems.

4.1 Truss idealised structures

To estimate the lateral deflection of a truss idealised structures there is a method called Virtual Work. The virtual work method is an analytical method based on the concept of conservation of energy where the calculation procedure easily can be systemised by tabulation. This method serves well for estimation of lateral deflection of structures that are idealised as trusses. [Smith and Coull (1991)]

Assuming that all structural members in a truss-braced structure only transfers load axially, the maximum lateral deflection of the structure can be determined according to the following equation (4.1)

$$u_{\max} = \sum \overline{f}_i \cdot \left(\frac{FL}{EA}\right)_i \qquad (4.1)$$

where

$\overline{f}_i$ = the axial force in member i due to the "virtual" unit load of magnitude 1

$\left(\frac{FL}{EA}\right)_i$ = the deformation of member i with its specific axial stiffness EA.

In equation (4.1) the axial force F in member i is calculated with regard to the "real" lateral load on the structure, e.g. wind load. Furthermore, the axial force $\overline{f}_i$ in member i is calculated with regard to the "virtual" load of magnitude 1 applied at storey i where the lateral drift is searched. This is illustrated in Figure 4.1 showing the principle of virtual work. The figure shows the "real" and the "virtually" loaded structure respectively. The following calculation procedure explains the virtual work method:

- Use the "real" load on the structure to determine all member forces F by using the method of joints.
- Apply the virtual unit load of magnitude 1 at the joint and in the direction where the deflection is sought. In the example shown in Figure 4.1, the horizontal deflection at joint f is desired.

- Determine all member forces due to virtual unit load 1 in order to obtain $\bar{f}_i$ for all members. Having L, E and A known, determine the maximum lateral deflection u_{max} by summing up the contributions from all members. This is perfomed by using equation (4.1).

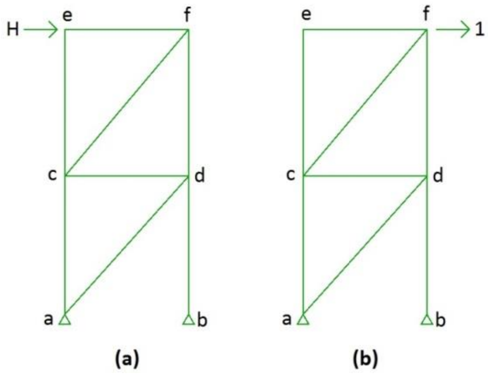

Figure 4.1: A truss-braced structure (a) "real" structure subjected to horisontal load H; (b) "virtual" structure subjected to a unit load 1 at joint f where the lateral deflection is sought.

4.2 Structures behaving mainly in flexure

The top lateral deflection of structures that behave predominantly in flexure, such as shear wall-braced structures, can be estimated analytically by integrating the curvature along the whole structure. For a cantilever, such as a tall building, the lateral deflection at any point x_1 along the cantilever is estimated as following [Engström, (2011)]:

$$u(x_1) = \int_0^{x_1} \frac{M(x_1)}{E \cdot I(x_1)} \cdot (x_1 - x) \cdot dx \qquad (4.2)$$

By setting $x_1 = H$, e.i. the height of the structure, the top lateral deflection of the structure is obtained.

It is important to be aware of the fact that only the flexural deformation accounted for, while the shear deformations are ignored. Since this project concerns relatively tall structural systems with predominant flexural behaviour, this way of estimating the lateral deflection serves well in the preliminary design process. Nevertheless it is important to emphasise that in tall building design shear deformation does always occur, but, depending on the type of structural system and the slenderness of the structure, shear deformation is usually of a much smaller magnitude than flexural deformation.

4.3 Special methods for non-braced frame structures

For non-slender non-braced frame structures shear racking is the major cause for the lateral deflection. The reason is that the girders and columns bend. A further reason is that non-braced frame structures have many large "holes", making the structure to act as a beam with a lot of holes in it. The shear racking of a non-slender non-braced frame structure depends on two effects; rotation of the joints due to double bending of the beam and rotation of the joints due to double bending of the columns.

At each storey, when the drift due to beam flexure is estimated, it is assumed that the columns are flexurally rigid. In a similar way the drift due to column flexure is estimated under the assumption of flexurally rigid beams. Hence, each storey drift is calculated and summarised to the total accumulated drift at the top of the building [Smith and Coull (1991)].

The total drift u_i at storey i is taken to be

$$u_i = u_{ig} + u_{ic} + u_{if} \tag{4.3}$$

where

u_{ig} is the storey drift due to girder flexure

u_{ic} is the storey drift due to column flexure

u_{if} is the storey drift due to global bending.

The drifts in a storey due to beam and column flexure are illustrated in Figure 4.2a and 4.2b.

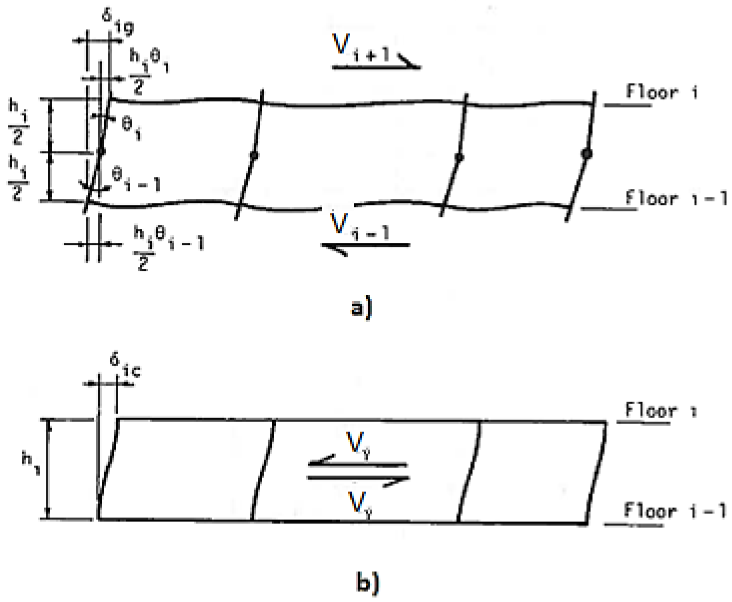

Figure 4.2: Storey drift due to (a) beam flexure; (b) column flexure. [Smith and Coull (1991)]

Including the expression for each term in equation (4.3), the lateral deflection formula takes the following form

$$u_i = \frac{V_i \cdot h_i^2}{12 \cdot E \cdot \sum \left(\frac{I_g}{L} \right)_i} + \frac{V_i \cdot h_i^2}{12 \cdot E \cdot \sum \left(\frac{I_c}{L} \right)_i} + h_i \cdot A_0^{\ i} \tag{4.4}$$

where

V_i is the shear force at storey i.

h_i is the height of storey i.

E is the modulus of elasticity of the girder/column.

I_g and I_c are the moment of inertia of the girder and column respectively.

L_i is the sum of the lengths of the bays of a storey (width of a storey).

$A_0^{\ i}$ is the area of the curvature diagram seen in Figure 4.3 from the base of the building up to the mid-height of the storey i.

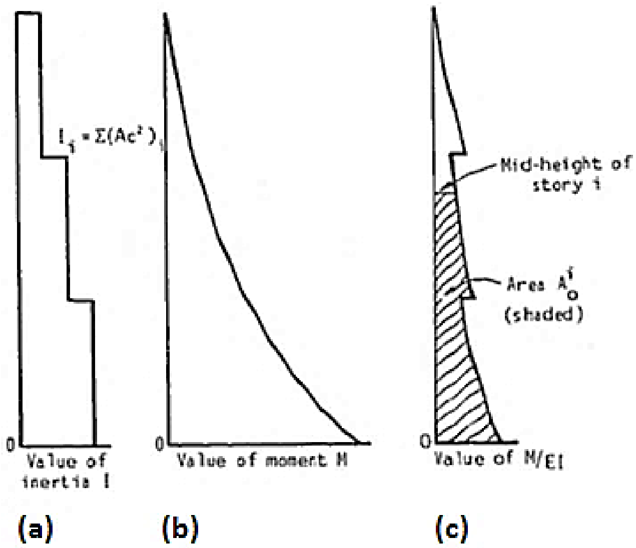

Figure 4.3: Example of global flexural deformation; (a) inertia distribution along the height of a building; (b) Moment distribution; (c) Curvature (M/EI) distribution along the height of a building. [Smith and Coull (1991)]

For a slender non-braced frame, however, the lateral deflection is calculated with regard to overall bending in addition to the two previous components of shear racking. In this case, the effect of shear racking on the upper stories becomes smaller than for the lower stories. This results in that the upper region of the structure deforms predominantly in flexure. Hence, its moment of inertia is estimated as the second moment of the column areas about their common centroid. For more precise calculation procedure the reader is referred to Smith and Coull (1991).

4.4 Special methods for core structures with outriggers

Core structures with outriggers can have uniform and non-uniform stiffness distribution along the height of the structure. The former is easier to deal with in hand calculations while latter is much more complicated. Most tall buildings have actually non-uniform stiffness distribution due to many factors such as saving material, weight and costs. The following Sections 4.4.1 and 4.4.2 explain the basic idea with how to deal with these types of structures.

4.4.1 Uniform stiffness of core and columns

According to Smith and Coull (1991), if the stiffness distribution is uniform along the height of the structure, the maximum tip deflection at the top of the structure can be obtained according to the following equation (4.4). See also Figure 4.4 which illustrates the factors in equation (4.4).

$$\delta_{\max} = \frac{wH^4}{8EI_{core}} - \frac{1}{2EI_{core}}\left[M_1\left(H^2 - x^2\right)\right] \tag{4.4}$$

where

w = the uniformly distributed wind load

$\frac{wH^4}{8EI}$ = the lateral deflection at the top of the core acting as a cantilever

and the restraining moment M_1 on the core at the location of the outrigger is

$$M_1 = \frac{w(H^3 - x^3)}{6EI_{core}} \cdot \left[\left(\frac{1}{EI} + \frac{2}{EA_{column} \cdot d^2}\right) \cdot (H - x) + \left(\frac{d}{12EI_{outrigger}}\right)\right]^{-1}$$

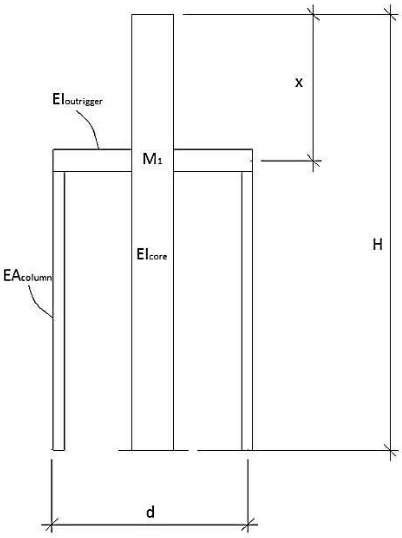

Figure 4.4: Core structure with outrigger having uniform stiffness distribution.

4.4.2 Varying stiffness of core and columns

When the structure has varying stiffness, as shown in Figure 4.5, the calculations become more complicated depending on how many levels the stiffness varies on. Equation (4.4) principally becomes longer depending on how many outriggers are provided and how many levels the stiffness varies on. For further information about how to deal with this type of problem by hand calculations, the reader is referred to Cheok, Lam and Er (2012).

In this project, commercial software was used in order to handle the effect of varying stiffness distribution. More about this is discussed in Section 9.5.

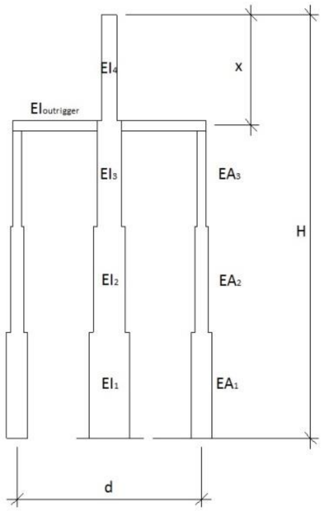

Figure 4.5: Core structure with outrigger having non-uniform stiffness distribution.

5 Stability of tall buildings

The stability of tall buildings should be assessed by checking the stability of its individual members as well as the global stability. The former is explained in detail in Eurocode while there are no detailed recommendation on how to deal with the stability of the building as a whole.

There is however a numerical solution that can be used in preliminary design of tall buildings in order to assess the stability of the structure as a whole. This method has been developed by the Italian engineer Vianello and it is called after his name Vianello's method. The method is practical and to use and its results are very accurate. Futhermore, the method can be used for structures with varying stiffness [Petersson and Sundquist (2000)].

5.1 First order analysis

In the beginning of a structural analysis of tall buildings loads are identified, such as gravity and lateral loads. Later on all internal forces are determined for the given geometry as well the lateral deflection of the structure. This approach is called first order analysis and gives the engineer a good estimation of the structural behaviour with respect to the initial geometry and material properties.

In reality, when the wind load hits the building and causes it to deform laterally, the gravity loads will act with an eccentricity in relation to the normal axis of the columns, see Figure 5.1. The structure responds to this phenomenon by deflecting further which also results in an increase of the bending moment.

There is a simple check criterion from which one can decide if second order analysis is required or not. More about the second order effect and how to consider it in analysis will be further discussed in the following Sections.

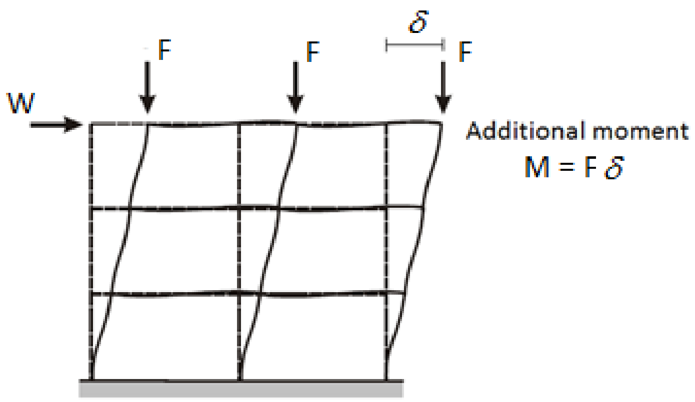

Figure 5.1: Example of a structure subjected to second order effect.

5.2 Vianello's method for estimating global buckling load

The method of Vianello is based on second order analysis and the calculated buckling load is with respect to only flexural deformation of a tall structure. In reality there is an additional deformation due to shear which, is not considered in Vianello's method. This simplification, to neglect the shear deformation, is acceptable for relatively tall and slender structures, since the part of the total deformation that arises due to shear is relatively small, except for non-braced frame structures as previously mentioned in Section 3.1. Nevertheless, it is important to consider the shear deformation for non-slender structures such as non-braced frame structures with moment resisting connections.

In the late 19th century Vianello established his method for calculating the buckling load. His ideas were based on the following considerations:

Consider a centrically loaded column by a compressive normal force Q. This column is subjected to, in addition to Q, an imposed displacement. This imposed displacement is later on removed and then three scenarios can arise [Petersson and Sundquist (2000)];

a) The column will go back to its original shape, meaning that the lateral deflection will "disappear" and the system becomes in a stable state of equilibrium. This gives a magnitude of the factor of stability $s > 1$.
b) The lateral deflection of the column will increase, meaning that the column is unstable and equilibrium is not fulfilled. This gives a magnitude of the factor of stability $s < 1$.
c) The lateral deflection remains the same as when the laterally deflected, meaning that the column is subjected to the buckling load. This gives a magnitude of the factor of stability $s = 1$.

Scenario c) when the deformation is sustained even after removal of the imposed deformation is the condition that defines the buckling load.

According to Vianello's method, the critical buckling load for a tall building subjected to uniformly distributed gravity load and having non-uniform stiffness distribution along the height of the structure can be determined as follows:

1) The building is divided into elements along its height. It is appropriate to choose an element for each storey.
2) A reasonable lateral deformation shape is assumed as u_{assu} for each element in accordance with an assumed deformed shape of the structure. This can be sketched and measured by hand.
3) The moment $M = Q \cdot u_{assu}$ and the curvature $u^{''} = \dfrac{Q \cdot u_{assu}}{EI}$ are calculated for each element.
4) The curvature $u^{''}$ is integrated twice in order to obtain a new deformed shape called u_{calc}. The procedure is simplified by assuming that the curvature is constant over an element.
5) The boundary conditions are inserted. A tall building could be modelled as a cantilever with either fixed or elastic end support.
    a. For a fixed end support the boundary condition at the support is applied as $u^{'} = 0$.

b. For an elastic end support the boundary condition at the support is applied as $u' = M \cdot \gamma_u$.

6) The assumed and the calculated deformed shapes are compared by checking that the ratio u_{assu} / u_{calc} is constant for the different elements. If this ratio is varying too much, the assumed deformed shape u_{assu} must be replaced by a new that is proportional to the recently calculated deformed shape u_{calc} and the integration is repeated in order to obtain a new deformed shape u_{calc}. The results should converge already after one or two iterations.

If the calculated deformed shape u_{calc} and the assumed deformed shape u_{assu} are equal, then is Q equal to the buckling load Q_b. If $u_{calc} < u_{assu}$ then $Q < Q_b$. The factor of stability, s, can now be calculated according to the following equation (5.1)

$$s = \frac{Q_b}{Q} = \frac{u_{assu}}{u_{calc}} \tag{5.1}$$

5.3 Second order analysis

The additional bending moment and deflection due to the second order effect is usually less than 5 % of the first order values. According to Eurocode 2, CEN (2010), if the second order effect is more than 10% of the first order, which generally means that the structure is flexible, the second order effect must be taken into account. This is because the second order effect is significant and might cause unacceptable additional deflections and moments. In worst case the additional moment might be of such a high magnitude so that the resistance or stability of a member might be insufficient resulting in structural collapse [Smith and Coull (1991)].

According to Eurocode 2, CEN (2010), global second order effects on a structure do not need to be considered, if the following condition is fulfilled

$$F_{V,Ed} \leq k_1 \cdot \frac{n_s}{n_s + 1{,}6} \cdot \frac{\sum E_{cd} \cdot I_c}{H^2} \tag{5.2}$$

where

$F_{V,Ed}$ = the total vertical load on load bearing elements

k_1 = 0.31 national parameter but recommended value

n_s = the number of stories

I_c = the moment of inertia of an uncracked bracing member

E_{cd} = the design modulus of elasticity of concrete

H = the height of the building

Note that condition (5.2) is based several assumptions that must be fulfilled, see Eurocode 2, CEN (2010).

However, when the second order effect must be considered, there is a method called the amplification factor method which is further described in the following paragraph.

The amplification factor method is an approximate method. The principle of this method is to obtain a factor that increases the lateral displacement for each storey and the moment obtained according to first order analysis. The amplification factor AF indicates that the deflection and moment for a specific storey i is increased by the magnitude of AF and it is taken to be [Gauittu and Smith (1992)]

$$AF = \frac{1}{1 - \frac{Q}{Q_b}} \tag{5.3}$$

where

Q = global total vertical load along the structure

Q_b = global critical buckling load calculated for instance according to Vianello's method described in Section 5.2

Not that if $Q < Q_b$ the structure is considered as stable. Hence, the total lateral deflection for storey i including the second order effect is given by the following equation

$$u_i = u_{i,1} \cdot AF \tag{5.4}$$

where

$u_{i,1}$ = lateral deflection of storey i according to first order analysis.

If the total lateral deflection u_{tot} fulfils the deflection criterion, described in Section 2.3.1, then the structure is considered to be appropriate with regard to serviceability in the preliminary design process. Furthermore, the design moment $M_{Ed,i}$ at storey i including the second order effect as well as the safety against instability can be determined as

$$M_{Ed,i} = M_{i,1} \cdot AF \tag{5.5}$$

where

$M_{i,1}$ = the moment at storey i according to the first order analysis.

6 Dynamic analysis

Every object has a natural frequency. If this object is subjected to a dynamic load which happens to be near or equal to the natural frequency of the object, it will start to resonate. If a structure is excited to resonance, it means that for a very small additional load the lateral deflection has a tendency to grow towards infinity.

When a building is very slender, meaning a height to width ratio of more than 5:1, then dynamic analysis is required. If the structure is non-slender, static analysis is sufficient for design and analysis to fulfil requirements [Smith and Coull (1991)].

The effects and concerns regarding earthquake action are not examined in this project, as it does not exist in the region of Sweden.

The important factors with regard to dynamic response of structures are:

- Geometry of the structure
- Mass of the structure
- Stiffness of the structure
- Damping of the structure
- The applied load on the structure

An important dynamic effect to consider in design of tall buildings is the acceleration that might arise when the building is suddenly loaded in the lateral direction. This is important because if a building is put in motion this can result in discomfort for the occupants of the building. Therefore, the acceleration must be limited.

Depending on the velocity distribution and direction of the wind, the geometry and mass of the building and its stiffness, the building can react in different ways. The effect of wind load on a building could lead to translation parallel to and perpendicular to the wind load, which can be seen in Figure 6.1.

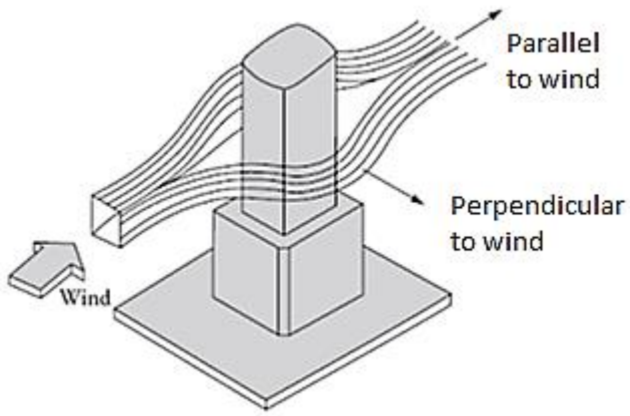

Figure 6.1: Wind load on a building leading to movements parallel and perpendicular to the wind. [Taranath (2011)]

Nowadays buildings are lighter and more slender which makes them more sensitive to dynamic effects. The dynamic effect gives larger stresses than the ones arising due to static action [Coull and Smith (1991)].

6.1 When dynamic analysis is necessary

The first few natural frequencies of a flexible structure are relatively low in comparison to the first few natural frequencies of a stiff structure. The latter stiff structure can be referred to as "static" and can be analysed assuming a static equivalent wind load. The flexible structure, on the other hand, can be referred to as "dynamic". This is due to its relatively large dynamic response when it is excited at, or close to, the structure's first few natural frequencies. This can occur if the wind load is fluctuating at one of the frequencies required for the structure to resonate. At this state the deflections are much larger than the static ones with respect to the same action of wind. In cases where a structure is referred to as "dynamic", the lateral deflection and acceleration become even more important to consider in design. In design it is also important to consider the displacement induced stresses that arise due to the large dynamic deflection as these stresses are larger than would be predicted by static analysis [Smith and Coull (1991)].

In preliminary design it is important to be able to decide whether a structure can be considered as "dynamic" or "static". This will affect the design substantially and if a dynamic behaviour is not foreseen in design, it is very expensive to adjust the structure after it has been constructed, if an adjustment is at all possible. It is actually not possible to be certain, in early design, whether dynamic analysis is required or not, but there are guidelines that can give a hint when dynamic behaviour should be considered. Smith and Coull (1991) brings up an example from the Australian Code where a building is defined as "dynamic", if the height to width ratio is larger than 5:1, considering the smallest width of the building, and that the natural frequency is smaller than 1.0 Hz in the first mode. This is compared to the Canadian Code where a building is defined as "dynamic", if its height to width ratio is larger than 4:1 or if it is higher than 120 meters. To be able to make a qualified judgment there is a need of knowledge about the damping of the structure and its first natural frequency is needed.

6.2 Along-wind induced acceleration

Acceleration is a major design factor to be considered in tall buildings in order to ensure occupants comfort. There are mainly two types of accelerations that tall buildings experience. The first and worst is the cross-wind induced acceleration which occurs at the same direction of the vortex shedding, see Figure 6.3. The second is the in the direction of the wind but is usually of smaller magnitude for slender tall buildings.

The maximum along-wind induced acceleration $\ddot{X}_{max}$ at the top of the building can be determined from the following equation [EKS 9 (2013)]

$$\ddot{X}_{\max} = k_p \cdot \frac{3 \cdot I_v(H) \cdot R \cdot q_m(H) \cdot b \cdot c_f \cdot \phi_{1,x}(z)}{m_0} \tag{6.1}$$

where

$R^2 = \dfrac{2 \cdot \pi \cdot F \cdot \phi_b \cdot \phi_h}{\delta_s + \delta_a}$ is the resonance part of response

δ_s = structural damping

δ_a = aerodynamic damping

$F = \dfrac{4 \cdot y_C}{\left(1 + 70{,}8 \cdot y_C{}^2\right)^{5/6}}$ is the wind energy spectrum

$\phi_{1,x}(z)$ = deflected modal shape of the building

$I_v(H)$ = turbulence intensity along the height of the building

$y_C = \dfrac{150 \cdot f_n}{v_m(H)}$

$k_p = \sqrt{2 \cdot \ln(v \cdot T)} + \dfrac{0{,}6}{\sqrt{2 \cdot \ln(v \cdot T)}}$ is the peak factor

$v = f_n \cdot \dfrac{R}{\sqrt{B^2 + R^2}}$ is the apparent frequency

f_n = natural frequency of the building

c_f = force coefficient factor

$B^2 = \exp\left[-0{,}05 \cdot \left(\dfrac{H}{h_{ref}}\right) + \left(1 - \dfrac{b}{H}\right) \cdot \left(0{,}04 + 0{,}01 \cdot \left(\dfrac{H}{h_{ref}}\right)\right)\right]$ is the background excitiation

H = the height the building

h_{ref} = reference height according Eurocode 1, CEN (2008)

b = the width of the building

$\phi_b = \dfrac{1}{1 + \dfrac{3{,}2 \cdot f_n \cdot b}{v_m(H)}}$ is the size effect width of the building

$\phi_h = \dfrac{1}{1 + \dfrac{2{,}0 \cdot f_n \cdot h}{v_m(H)}}$ is the size effect of the height of the building)

$q_m(H)$ = wind velocity pressure at the top of the building

$v_m(H)$ = mean wind velocity at the top of the building

According to equation (6.1), the major dynamic structural properties that affect the building acceleration are characterized as following

- Deflected mode shape
- Equivalent building mass
- Natural frequency
- Damping
- Structural geometry, i.e. height-to-width ratio.

These factors are presented in the following Sections.

6.2.1 Deflected mode shape

The deflected mode shape is influenced by the type of structural system and its height. The following equation (6.2) is based on the fact that the deflected mode shape is linear and vibrating in the first mode of a cantilever

$$\phi_{1,x}(z) = \left(\frac{z}{h}\right)^{\xi} \tag{6.2}$$

where

ξ is different depending on the type of the structural system. In this project the values are taken to be as following:

$\xi = 1.0$ = for the core structure with outrigger according to Eurocode 0, CEN (2008)

$\xi = 1.5$ = for the shear wall-braced structure according to Eurocode 0, CEN (2008).

6.2.2 Equivalent building mass

Generalized building mass must be obtained according to Eurocode 0, CEN (2008), in order to estimate the equivalent building mass as following

$$m_0 = \frac{\int_0^h m(s) \cdot \phi_1^{\,2}(s)ds}{\int_0^h \phi_1^{\,2}(s)ds} \tag{6.3}$$

where

m = the mass per unit length

h = the height of the structure

ds = the storey-height when integrating equation (6.2)

ϕ_1 = the deflected shape mode of the first mode of vibration.

6.2.3 Natural Frequency

There is a simple approach by which the natural frequency of a building can be estimated. The only parameter that must be known is the height of the building. According to Eurocode 0, CEN (2008), the natural frequency of a building can be estimated by the following equation

$$f_n = \frac{46}{H} \tag{6.4}$$

where

H = the height of the building.

When more information about the structure is available, the natural frequency can be estimated by means of a more accurate expression. As tall buildings usually are designed with varying bending stiffness and mass throughout the height of the building, there is a method called Rayleigh method, which considers such variations. According to Smith and Coull (1991), the more accurate estimation of the natural frequency can be obtained according to the following equation

$$f_n = \frac{1}{2\pi} \cdot \sqrt{\frac{g \cdot \sum F_i \cdot u_i}{\sum W_i \cdot u_i{}^2}} \tag{6.5}$$

where

g = the acceleration due to gravity = 9.81 m/s^2

F_i = the equivalent lateral load at floor i

u_i = the lateral deflection at storey i

W_i = the weight of i:th floor.

6.2.4 Damping

Every tall building structure has one or several types of damping. Principally damping results in dissipation of energy from a vibrating structure. Damping can be achieved by means of

- Structural damping
- Aerodynamic damping
- Damping due to special devices such as tuned mass dampers and sloshing tanks

These can be expressed as a logarithmic decrement of damping δ

$$\delta = \delta_s + \delta_a + \delta_d \tag{6.6}$$

δ_s = structural damping

δ_a = aerodynamic damping

δ_d = damping due to special devices (tuned mass dampers, sloshing tanks, etc).

Structural damping δ_s depends on the type of material, connections and structural system. In this project, the value of the structural damping is set to be 0.10 for reinforced concrete structure according Eurocode 1, CEN (2008).

Aerodynamic damping δ_a depends on the geometry of the structural system. For example, a building with square cross-section and sharp corners has different aerodynamic damping compared to a building with smoother corners. Furthermore, if the building is tapered along the height or contains large openings allowing the wind to pass through, this also affects the aerodynamic damping of the structure. Damping due to special devices δ_d such as tuned mass damping is also another possible solution, if the structural and aerodynamic dampings are not sufficient.

6.3 Cross-wind induced acceleration

According to Eurocode 1, CEN (2008), the effect of vortex shedding on a building does not need to be checked when expression (6.7) is fulfilled

$$v_{crit,i} > 1{,}25 \cdot v_m \tag{6.7}$$

where

$v_{crit,i}$ = the critical wind velocity of mode i

v_m = the characteristic 10 minutes mean wind velocity.

If however the vortex shedding is needed to be included in the design, then Annex E in Eurocode 1, CEN (2008), presents a procedure of how to estimate the cross-wind induced acceleration for structures not exceeding the height-to-width ratio of 30. For height-to-width ratios above 30, a detailed analysis is required according to Handa (2014). Before introducing the procedure given in Section 6.3.2, a brief theory is given in the following Section 6.3.1.

6.3.1 Vortex shedding phenomenon

There are different phenomena that might arise when a structure is subjected to wind load. Vortex shedding is one phenomenon, which leads to vibration of the structure perpendicular to the wind load. Examples of aero-elastic instabilities are flutter and galloping. Flutter is an instability phenomenon driven by coupling of fundamental transverse and torsional modes, meaning that resonance arises both in the transverse and rotational movements. Galloping is another instability phenomenon where the damping of a system is inverted to work with the vibrations induced by external loading so that, instead of counteracting the vibrations, it adds to the vibrations leading to resonance. Nevertheless these two instability phenomena – flutter and galloping – are not critical when considering tall buildings, as they occur at very high wind velocities [Hira et al. (2007)]. Vortex shedding, on the other hand, arises at lower wind velocities and therefore it is important to consider this phenomenon in this project.

In Figure 6.2 it can be seen that the wind is forced to redirect itself when approaching the building. On the opposite side of the building, where the wind flows away from the building, the wind is “pulled back” to its original direction. This induces vortices, see Figure 6.2. When the wind speed is low, these vertices are symmetrical on each side of the building. The critical situation is when the wind speed is high enough inducing vortices on each side of the building that do not act simultaneously, meaning that the wind load perpendicular to the wind direction is non-symmetric, resulting in translation perpendicular to the wind direction. The frequency of the vortices hitting the building must not match the natural frequency of the building since this would lead to resonance [Taranath (2011)].

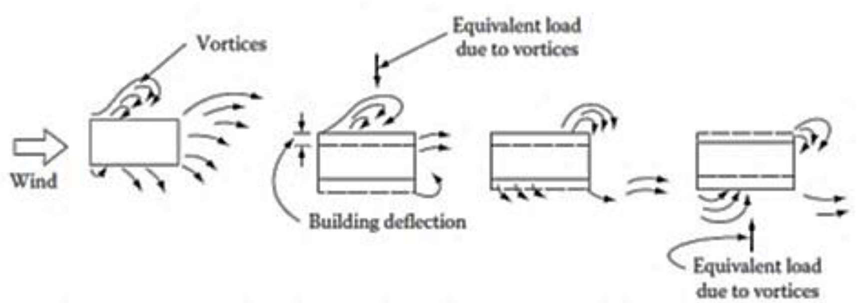

Figure 6.2: The building translates perpendicular to the wind when subjected to periodic shedding of vertices. The phenomenon is called vortex shedding. [Taranath (2011)]

The critical wind velocity of mode i is the wind velocity at which resonance is reached, meaning that the frequency of the vortex shedding is equal to the natural frequency of the structure subjected to the wind load. This critical wind load is calculated as:

$$v_{crit,i} = \frac{d \cdot f_{n,i}}{St} \tag{6.8}$$

where

d = the width of the cross-section at which resonant vortex shedding occurs and where the maximum lateral deflection arises for the structure.

$f_{n,i}$ = the natural frequency of mode i

St = the Strouhal number, which depends on the shape of the building, exemplified for buildings with rectangular cross-sections in Figure 6.3.

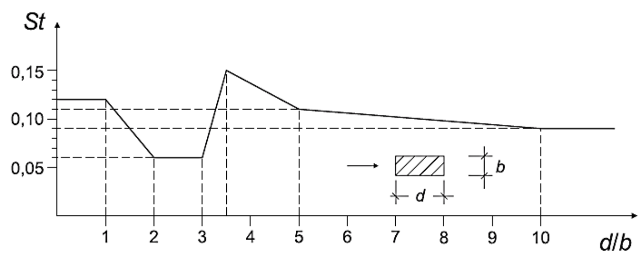

Figure 6.3: The Strouhal number for structures with sharp-edged rectangular cross-sections. [Eurocode 1, CEN (2008)]

The frequency due to the vortex shedding can be estimated from the following equation (6.9) [Taranath (2011)]

$$f_v = \frac{v_m \cdot St}{b} \tag{6.9}$$

where

v_m = mean wind speed at the top of the building

St = Strouhal number that considers the shape of the building and is dimensionless

b = width of the building.

Therefore it is of interest to adjust a structure so that it is not sensitive to effects that might arise due to vortex shedding. Examples of measures that can be taken, in order to reduce effects due to vortex shedding, are listed below [Irwin (2010)]. Note that these measures also reduce the base moment of the structure.

- Avoid sharp edges
- Tapering
- Different cross-sectional shapes at different heights
- Attach spoilers
- Provide openings

6.3.2 Estimation of forces due to vortex shedding

According to Annex E in Eurocode 1, CEN (2008), there are two approaches available in order to determine the cross-wind forces due to vortex shedding.

The vortex shedding force $F_w(s)$ is obtained as

$$F_w(s) = m(s) \cdot (2 \cdot \pi \cdot f_n)^2 \cdot \phi_{1,x}(s) \cdot y_{F,\max} \tag{6.10}$$

where

$m(s)$ = the structural mass per unit length vibrating in the first mode

f_n = the natural frequency of the structure

$\phi_{1,x}$ = the deflected mode shape of the structure

$y_{F,\max}$ = the maximum displacement at the top of the structure.

The maximum displacement $y_{F,\max}$ is determined as

$$y_{F,\max} = b \cdot \frac{1}{St^2} \cdot \frac{1}{Sc} \cdot K \cdot K_w \cdot c_{lat} \tag{6.11}$$

where

b = width of the building at which the resonant vortex shedding occur

St = Strouhal number

Sc = Scruton number

K = mode shape factor

K_w = effective correlation length factor

c_{lat} = lateral force coefficient

The susceptibility of vibrations depends on the ratio of structural mass to fluid mass as well as the structural damping. This may be expressed by the Scruton number S_c as

$$S_c = \frac{2 \cdot \delta_s \cdot m_0}{\rho \cdot b^2} \qquad (6.12)$$

where

δ_s = structural damping

m_0 = equivalent mass per unit length for vibration mode i

ρ = air density

b = reference width of the building at which resonant vortex shedding occurs.

7 Structural optimisation theory

In structural engineering optimisation is the subject about making a structure resisting loads in the best way. The optimisation subject is very important for large structures such as tall buildings, long-span structures and bridges that usually require high material usage. Therefore, if the structural engineer can come up with a solution that gives minimum material usage but at the same time satisfies set criteria, then it can be said that the structural efficiency is optimal.

For the building owners, architects and other disciplines involved in a tall building project the most interesting factor expected from the structural engineer is the structural efficiency. The most efficient structure is the one that provides least volume of material and satisfies set criteria as shown in Chapter 2. Furthermore, the more efficient the structure is, the more economical and sustainable it becomes to build it.

According to Smith and Coull (1991), the design of tall buildings is usually governed by the serviceability criteria (stiffness of a structure) rather than by the resistance criteria (load carrying capacity). Therefore, stiffness reflects the structural efficiency of a tall building, meaning that higher stiffness results in a more efficient structure. Another factor which also is included in terms of structural efficiency is the volume of material needed for a structure to resist design loads. Hence, if the designer provides a structural system that maximises the stiffness and gives least volume of material, then it is said that the structural efficiency is optimised. This was the intention of the optimisation procedure in this project.

7.1 Different types of optimisation classes

Structural optimisation is generally divided into three classes [Christensen and Klarbring (2008)]:

- Size optimisation
- Shape optimisation
- Topology optimisation

A sizing optimisation problem deals with finding the required cross-sectional area or thickness of a structural member to minimise the objective function. In this optimisation problem the variables x and y represent the cross-sectional areas, which can be the case when optimising shear wall-braced structures.

A shape optimisation problem is based on finding the optimal shape of a structure to minimise the objective function. The shape of a tall building could be curved, rectangular, quadratic, circular, triangular, tapered and so forth.

A topology optimisation problem is the most general optimisation problem for trusses for instance outriggers. It is about finding the optimal truss configuration in order to minimise the objective function. Some examples of well-known topologies are cross-bracings, diagonal bracings, K-bracings.

In this project only size optimisation was used. This was because the shape of the cross-section, which was quadratic, as well as the topology of the outriggers were not changed. Hence, shape and topology optimisation were not studied in this project.

7.2 Definition of an optimisation problem

In general, a mathematical optimisation problem of a structure can be expressed as [Adams and Essex (2010)]:

Minimise $f(x)$

Subject to $g(x) = C$

where the function $f(x)$ to be minimised is called the objective function. The equality $g(x) = C$ is called the constraint function.

Note that an optimisation problem could have several constraints. The mathematical definition stated above is a constrained extreme-value problem in which the variable x of the function $f(x)$ to be minimised must satisfy the constraint $g(x)$ [Adams and Essex (2010)].

In civil engineering and particularly in the field of structural engineering, the objective function is set to be the total volume of material to be minimised. Thus, the problem can be stated as a weight optimisation problem.

Having a structure of minimum volume of material means reduced costs as well as optimal utilisation of material [Cheng and Truman (2010)]. However, it is generally known that taking away material generally reduces the stiffness of the structure. Therefore, the equality function $g(x)$ has the task to keep the minimisation of the objective function at an acceptable level.

Concerning the constraints, they can be set as restriction of top deflection, yielding stress, buckling stress, acceleration that all must be fulfilled while minimising the objective function [Cheng and Truman (2010)].

7.3 Lagrange Multiplier technique

The Lagrange Multiplier is a mathematical optimization method that can be used in order to find a local extreme of a function subjected to constraints. The objective function and constraint function are explained in Section 7.2.

The basic idea with the Lagrange Multiplier method is to have the objective function and the constraint function continuously differentiable. Furthermore, the constraint function must have a non-zero gradient. Having these conditions fulfilled, there exists a variable, the Lagrange Multiplier λ that is unique at the local extreme, i.e. the stationary point of equation (7.1). In a general expression the Lagrangian function $L(x, \lambda)$ defines the optimisation problem as [Adams and Essex (2010)]:

$$L(x, \lambda) = f(x) + \lambda \cdot g(x) \tag{7.1}$$

where

$f(x)$ = the objective function to be minimised

$g(x)$ = the constraint function that must be satisfied

λ has a unique value at the local extremum of $f(x)$.

The stationary point of the Lagrangian function $L(x,\lambda)$ is found by partially differentiating the function with respect to the variables x and λ and setting these equations equal to zero. Thereby, in order to ensure that the objective function is minimised, the Lagrange Multiplier λ must have a value such that the gradient of the Lagrangian function $L(x,\lambda)$ according to equation (7.2) is fulfilled [Spillers and MacBain (2009)]

$$\nabla L(x,\lambda) = \nabla f(x) + \lambda^T \cdot \nabla g(x) = 0 \qquad (7.2)$$

How the Lagrange Multiplier technique has been adopted in this project is explained by the calculation procedure in Section 9.4.

8 Choice of promising solutions

8.1 Assumed conditions for a tall office building

This project concerned a fictional tall building with a height in the range of 150 to 245 m. The reason why the height was not a fixed number was that it should be possible to study how different structural parameters are influenced by different heights of tall buildings. This is brought up in the upcoming chapter 10.

The tall building that was studied in this project was imagined to be situated in the central parts of Gothenburg. The building was assumed to be used for commercial and office activities.

The height of each storey varies in different buildings, but an approximate height of a storey in this project was assumed to be 3.5 meters. This gives an approximate range of number of stories to be 43 to 70. The number of stories is relevant, since previous experiences suggest what type of structural system that is suitable for different number of stories. These suggestions are listed in Chapter 3 and they were used in this Chapter where the most promising solutions were sorted out.

8.1.1 Structure and architecture

When constructing a tall building it is of importance to consider the effect that which the structure will have on the architectural features of the building. Simultaneously, it is of importance to adjust the architectural features of the building to the structural system in situations where the building is very high and slender. This was the case in this project where the imagined building should be between 150 to 245 m, which means that the structural system of the tall building is of importance in proportion to the architecture. Nevertheless, the architectural feature is not to be neglected, but it is of less importance than for e.g. smaller residential buildings.

In this project it was assumed that the architectural features will follow the structural system. As an example the diagonals in a facade of a truss-braced structure have both a structural function and serve as an architectural feature. A real example of this case is the John Hancock Tower seen in Figure 3.20 in Section 3.4.3. So there were no requirements to have a façade that is not affected by the structural system in this project.

The base of the tall building was assumed to be square with a side length of 36 m, see Figure 8.1. With respect to the heights of interest these results in slenderness ratios, height to width from 5 to 8.3.

Due to the fact that the building should be used for office and commercial activities, it was preferred to have an open plan.

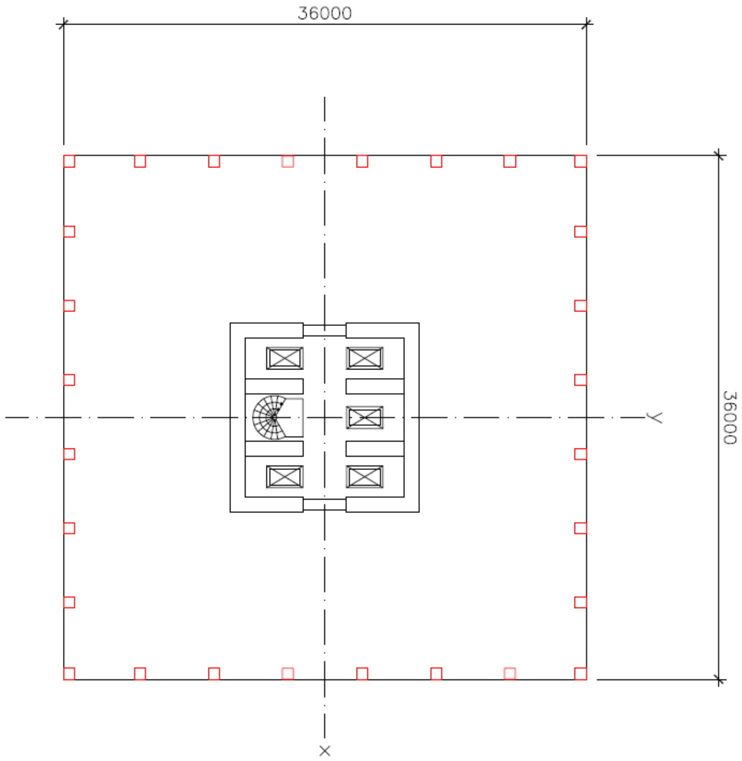

Figure 8.1: Cross-section of the assumed tall building with a façade length of 36 m.

8.1.2 Foundation

Since the building was assumed to be situated in Gothenburg, it is likely that it will be founded on clay. The consequence of this is that the building was assumed to be supported by a slab on piles.

Furthermore, since clay can be unstable, in comparison to solid bedrock, it might lead to differential settlements beneath the building. It is therefore preferable to have a structural system that is not sensitive to uneven settlements.

The foundation itself and the effects on it were not studied in this project.

8.1.3 Fire safety

General considerations about fire safety are described in Section 2.4. Given the conditions chosen in this project a structural system was developed and adjusted so that it fulfilled fire-safety requirements. The intention, when a structural system was chosen in this project, was not to choose it with regard to fire safety regulations. It was initially chosen with regard to the structural behaviour. Nevertheless, it was adjusted with regard to fire safety by ensuring that the service core was sufficiently large for evacuation and to accommodate the occupants.

8.2 Evaluation of structural systems

In Chapters 3, 4, 5 and 6 different structural systems are presented and methods for preliminary design were also described. In this Section a comparison of the different structural systems is presented. During the comparison the focus was on the structural behaviour and a little bit on the architectural features. Other important factors, which are discussed in this Section, are how the different structural systems react on the assumed soil conditions and some general fire considerations are presented.

8.2.1 Structural and architectural constraints

In the range of 40 to 70 stories it is clear to exclude the non-braced frame structure with moment resisting connections, as it is not economically feasible even though it is appealing to have an architectural freedom in the façade as well as in the interior plan. The reason why it is not economically feasible is due the fact that it requires expensive moment resisting connections. This is also confirmed by Chok (2004) who claims that when exceeding approximately 20 stories with this type of structural system, the volume of material required keeping the lateral deflection within acceptable limits is too large in comparison to the volume of material needed to carry gravity loads. Hence, the structural efficiency is limited up to approximately 20 stories.

The framed tube structure, a non-braced frame with moment resisting connections, has many structural advantages as its flexural and torsional stiffnesses are very high. However, it is discussed in Section 3.4.1 that it is not an efficient structural system since it suffers from shear-lag effect. Considering the architectural features it is a disadvantage to have closely-spaced columns and beams in the façade, since these lead to smaller openings in the façade.

A bundled tube, another non-braced frame with moment resisting connections, is also limited in the façade, where columns do not need to be as closely-spaced as in the framed tube. This is due to the fact that the bundled tube has additional interior "webs" that resist great part of the horisontal load and hence reduce the shear lag effect.

A possible disadvantage with the bundled tube might be that the volume of material needed is larger compared to a trussed tube, where the lateral loads are resisted by axial response. However, the bundled tube is still more efficient than a framed tube. The façade allows for larger openings than in a framed tube, but the freedom to design the façade is not as pronounced as for a core structure with outriggers.

However, it has been argued that the tubular structural systems are efficient for very tall buildings and therefore these excellent structural systems were considered not to be suitable for the case study in this project.

Furthermore, truss-braced structures are very efficient but do not provide fire and sound insulation in comparison to pure concrete shear walls. Therefore, although truss-braced structures are indeed potential solution, these were considered to be not as good as shear wall-braced structures.

Hence, shear wall-braced structures and core structures with outriggers were more convenient for the assumed conditions in this project. The reason why these structural systems were mostly suitable has to do with their architectural features, which were suitable for the office building assumed in this project. It also has to do with the way they lateral loads and transfer them down to the foundation. Structural systems such as core structures with outriggers resist lateral loads by axial action of the columns at the perimeter of the building. This is, as discussed earlier, the most efficient way of carrying loads, allowing full utilisation of cross-sections. This can be compared to structural systems that are idealised as flexural sticks such as shear-wall braced structures. These structures resist lateral loads predominantly in flexure, which is the next most efficient way of carrying loads. Concrete shear walls have also high in-plane stiffness, fire and sound insulation and, hence, are relatively economical in comparison to steel trusses.

The result of the evaluation of the structural behaviour showed clearly that both shear wall-braced structures and core structures with outriggers provide structural systems with minimum shear deformations, suitable architectural freedom for an office building and, hence, are chosen as suitable structural systems for the assumed conditions in this project with respect to structural and architectural constraints.

8.2.2 Foundation constraints

The reason why a non-braced frame structure was not suitable in this project was not only due to the condition of the soil but also due to its economically limited height. Since the building was assumed to be founded on clay, it would need piles to support it. Movements in the foundation might arise either due to the soil settlements or due to axial shortening of the piles. Whatever type of displacement that occurs, it is likely that the displacements are unequal in different areas of the foundation due to inhomogeneity of the soil, the piles or an unevenly distributed load on the foundation from the building. If the building is of a non-braced frame structure type, this would lead to displacement induced stresses arising in the structure, which could lead to big problems for e.g. non-structural members and cladding. Since this type of structure is sensitive to uneven settlements, in comparison to a regular simply supported column-beam system, it was not preferred in the project.

An advantage that comes with the core structure with outriggers is that the moment that must be resisted at the foundation is smaller than for a shear wall-braced structure due to the large lever arm between the columns at the perimeters.

The result of the evaluation of the foundation constraints showed clearly that both shear wall-braced structures and core structures with outriggers provide structural systems with better properties with regard to the foundation conditions than non-braced frame structures.

8.2.3 Fire constraints

Considering the core structure with outriggers the floors where the outriggers are placed can be used for installation systems and they can also be used as refuge floors to function as evacuation places in case of fire.

The structural systems that should be chosen, with respect to the conditions initially assumed, should also be adjusted so that they fulfil fire safety requirements. The intention, when a structural system was chosen in this project, was not to choose it with regard to fire safety regulations. It should initially be chosen with regard to the structural behaviour. Nevertheless, it was then adjusted a bit with regard to fire safety.

8.2.4 Choice of promising solutions

There are several structural systems that could be potential solutions for the assumed conditions in this project. This includes e.g. truss-braced structures or interacting systems. However, the assumed conditions point out that the core structure with outriggers and the shear wall-braced structure are the most promising solutions, since they possesses many advantages for the assumed conditions.

The core structure with outriggers is structurally efficient and simultaneously gives freedom in architectural planning of the interior as well as of the façade of the office building.

A positive feature of shear wall-braced structures is that the positioning of the shear walls can be adjusted to meet the architectural constraints and there is a freedom in designing the openings in the façade. Furthermore, the concrete walls function as fire and sound insulators.

A shear wall-braced structure is suitable for the lower heights while an outrigger-braced structure is suitable for the higher heights ranging from 40 stories to the very tallest. Nevertheless, it is of importance to consider that the heights that these structures are claimed to be economically suitable for are only approximate estimations seen only from structural engineering point of view. Therefore, with regard to the assumed conditions in this project, it was decided to take these two structural systems further into a comparison.

In conclusion the most promising structural systems in this project were assumed to be the shear wall-braced structure and the core structure with outriggers. These systems were further studied, optimised and compared to each other which is explained and presented in Chapters 9 and 10.

9 Calculation procedure

The following sections present the assumptions made for analytical calculations. Furthermore, special topics such as handling of the dynamic wind load and the sizing optimisation procedure are given in this chapter. The calculations themselves, which gave the results presented in Chapter 10, are presented in Appendix A.

All calculations were performed for the following specific cases:

- Shear wall-braced structure of
    - 150.5 m height (43 stories).
    - 175 m height (50 stories).
    - 210 m height (60 stories).
    - 245 m height (70 stories).
- Core structure with one outrigger of
    - 150.5 m height (43 stories).
    - 175 m height (50 stories).
    - 210 m height (60 stories).
    - 245 m height (70 stories).

The shear wall-braced structure was optimised by the Lagrange Multiplier technique, which is described in Section 7.3 and Section 9.5. The core structure with outrigger was developed by combining analytical calculations with the commercial software described in Section 9.6.

9.1 Assumed geometry of the structure and floor system

All cases were assumed to have constant cross-sectional geometry of 36x36 m^2 with a stabilising core of 12x12 m^2 placed in the centre of the building. The geometry was compiled in corporation with the architect Peric (2014) and the geometry is visualised in Figure 9.1. This is the reference geometry current for all cases listed above. If nothing else is stated, this geometry is the one referred to throughout the rest of the report.

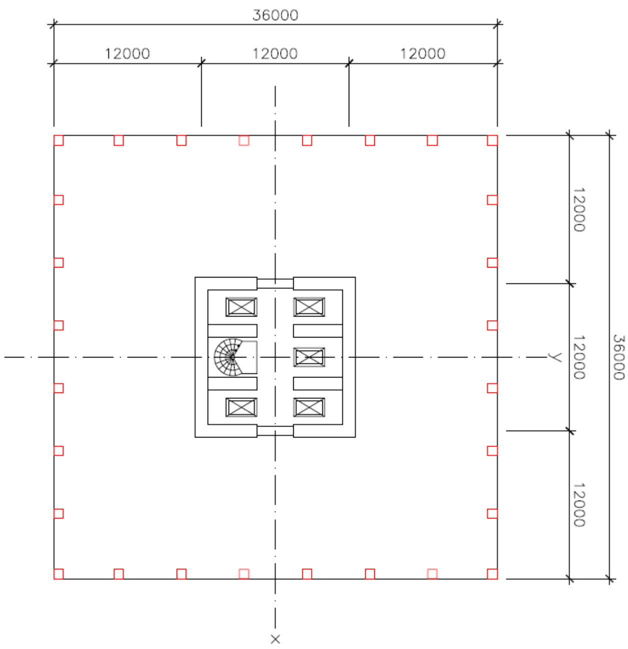

Figure 9.1: Geometry of cross-section for all analysed cases.

The distribution of the gravity load depends on the floor system. In all cases it is assumed that the floor system consists of hollow core slabs arranged according to Figure 9.2.

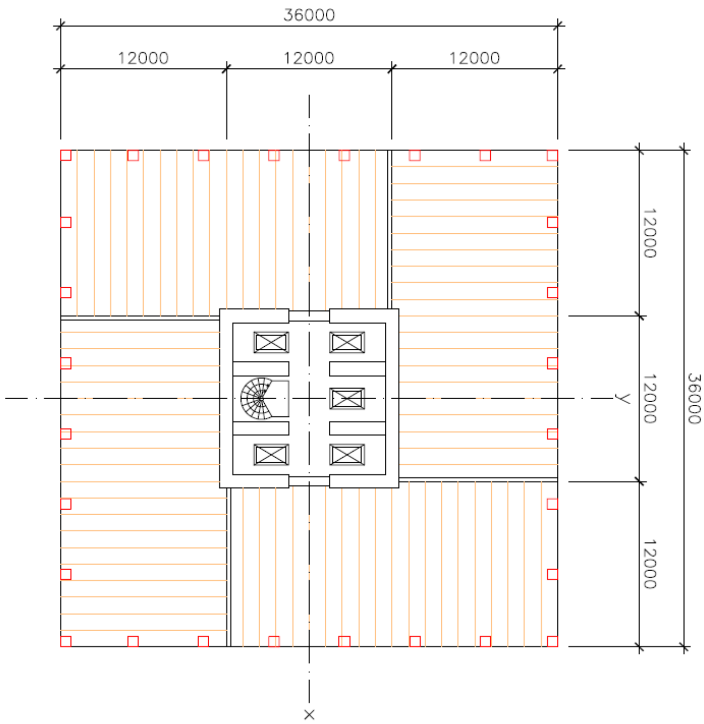

Figure 9.2: The load distribution depends on the arrangement of the floor system which consists of hollow core slabs.

The outrigger was assumed to have a constant stiffness for the four different building heights. Furthermore, the location of the outrigger was always placed slightly above mid-height of the building. The layout of the floor where the outrigger is located can be seen in Figure 9.3.

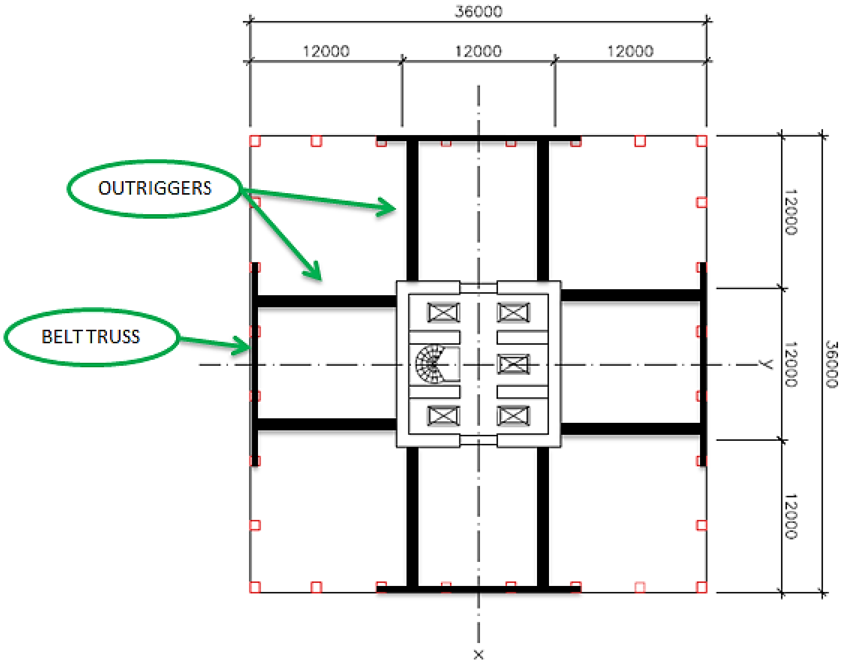

Figure 9.3: Principle layout of the outrigger floor. Outriggers are stretching between the core and the façade and belt trusses stretches along the façade to activate several columns.

9.2 Load combination in serviceability limit state

This project concerned design in the serviceability limit state as this is of major concern in design of tall buildings. Important to notice is that after designing a structure in the serviceability limit state a check of the resistance in the ultimate limit state must be performed and the structure might need to be adjusted if necessary.

The following equation (9.1) for the characteristic load combination was used in order to determine the design load in the serviceability limit state according to Eurocode 0, CEN (2010)

$$q_{d,sls} = \sum_i G_{k,i} + P_k + Q_{k,1} + \sum_{i>1} \psi_{0,j} \cdot Q_{k,i} \qquad (9.1)$$

where

$G_{k,i}$ = the permanent load

P_k = the prestressing force

$Q_{k,1}$ = the main variable load

$\psi_{0,j}$ = the reduction factor for the secondary variable load

$Q_{k,i}$ = the secondary variable load

9.3 Assumptions made in calculations

Assumptions were made, suitable for preliminary design, when performing the analytical calculations. The following sections present these assumptions.

9.3.1 Material behaviour

The material response was assumed to be linear elastic and, hence, no plastic deformations were developed. This allowed for superposition of forces and deflections which is essential in a preliminary design process.

9.3.2 Material properties

The modulus of elasticity of concrete members was reduced depending on if the section is cracked or uncracked. Eurocode provides an estimation of the modulus of elasticity of a cracked section. In Appendix H in Eurocode 1, CEN (2010), it is stated for cracked flexural stabilising members that the modulus of elasticity can be reduced by 60 %.

Smith and Coull (1991) provide an estimation of the modulus of elasticity for axially loaded members, such as columns. According to them, the modulus of elasticity can be reduced by 20 % and this estimation was used in this project.

9.3.3 Creep, temperature and shrinkage

The effects from creep, temperature and shrinkage were neglected in this project, since the focus lays on preliminary design. Nevertheless, these effects are very important to consider when performing a detailed analysis. Due to the accumulated self-weight, creep, shrinkage and temperature variation, shortening or elongation of structural members such as columns and walls is important to consider when dealing with tall buildings. However, these deformations were neglected in this project, as these effects belong to the detailed analysis.

9.3.4 Foundation

The foundation was assumed to be fully fixed with no differential settlements.

9.3.5 Axial stiffness of columns

For the core structure with outriggers, the outriggers are assumed to be rigidly connected to the core and simply supported on the façade columns. In order to capture the axial deformation of the columns supporting the outriggers, the supports were modelled as springs with the average stiffness of the columns below.

9.3.6 Deformations

Axial deformations of columns only loaded by vertical load from floors were neglected. This concerns all columns in the shear wall-braced structure. However, for the core structure with outriggers, the axial deformation of the columns below the outriggers were not neglected since these columns resist much greater load than other columns.

Shear deformations were not considered in this project. Nevertheless, in more detailed design and analysis it is of importance to consider shear behaviour for all tall building structures and especially for non-braced frame structures. The relationship between shear contribution and flexural contribution to the total lateral displacement is clarified in Chapter 3.

9.3.7 Flexural stiffness of the structure

The stiffness of non-structural elements was neglected. However, they do contribute to the overall stiffness and it is therefore recommended to take them into account.

According to Smith and Coull (1991) component stiffness of small magnitudes can be neglected. These are e.g. related to transverse bending of slabs, weak axis bending of shear walls and torsion of columns, beams and walls.

The centreline of the core walls was fixed, but the thickness of the walls was not given initially. Due to this the moment of inertia could not be estimated initially before the optimisation procedure. However, the stiffness of the core was initially described by the radius of gyration $i = \sqrt{I / A}$. In order to obtain a numerical value of the radius of gyration, which was necessary for the optimisation procedure, the moment of inertia of the core was simplified as shown in Figure 9.4 and as described in the next paragraph.

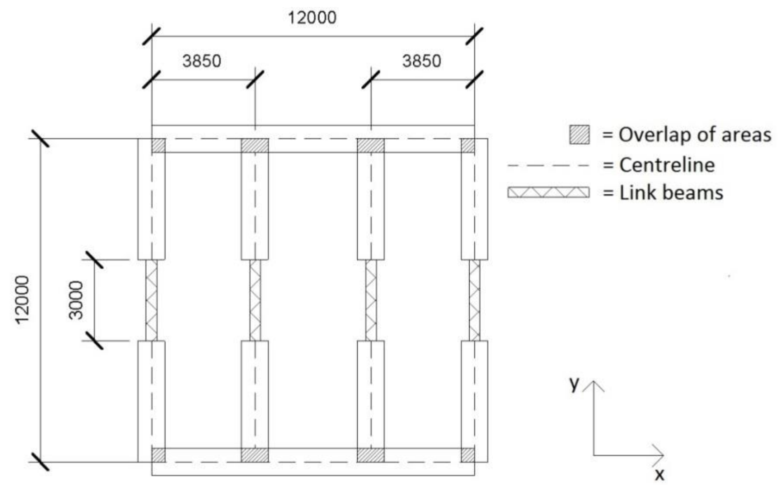

Figure 9.4: Model of the core when estimating the bending stiffness.

The whole optimisation was carried out with respect to bending in the weak direction of the core, in this case bending around the y-axis, which can be seen in Figure 9.4. The two long exterior walls of the core contributed to the stiffness by their first moment of inertia but they had no Steiner's contribution to the stiffness. The second moments of inertia of the remaining eight shorter walls around their respective centroid were neglected, since they are bending around their local weak axis. The reason for why this contribution was neglected was that the radius of gyration would not depend on the thickness of the walls. In this way a numerical value of the stiffness was obtained, which is on the safe side.

The moment of inertia and, hence, the radius of gyration were expressed based on the centreline of the core geometry seen in Figure 9.4. The areas marked as "overlapping areas" were included twice in the calculation of the stiffness of the core. This is acceptable, as the stiffness is underestimated, when the moment of inertia of the eight shorter walls around their centroid was neglected. Furthermore, since the stiffness is based on the centreline, the small corners regions of the core were excluded. Altogether, the stiffness was slightly underestimated but acceptable as it is on the safe side.

Furthermore, the contributions from the connecting beams between the shear walls, which can be seen in Figure 9.4, were neglected. This is a reasonable assumption in preliminary design as the stiffness contributions of the beams compared to the stiffness contributions from the shear walls are very small [Smith and Coull (1991)]. In detailed analysis it is recommended to take the effect of the connecting beams into account since they e.g. increase the torsional stiffness of the core.

Nevertheless, when the goal is to optimise the volume of material in a structure, it is preferable to utilise all the stiffnesses a structure provides. Other stiffnesses could also be provided by connecting beams or the floor structure. However, in preliminary design the aim is to simplify and still be on the safe side, which was the approach in this project.

9.3.8 Damping

Only structural damping is considered in the calculations according to Section 6.2.4. Damping due to special devices and aerodynamic damping were not considered in this project. The aerodynamic damping was neglected due to the fact that the assumed layout of the structural system is a regular square building with no modifications such as smoother corners or tapering. Hence, the aerodynamic damping would be relatively small in comparison to the structural damping.

9.4 Wind load including dynamic effects

According to Handa (2014-03-10), for the heights exceeding 200 m then expression for the mean wind velocity according to Eurocode 1, CEN (2008) is then no longer valid. In order to get a valid expression a term must be added to the original expression as

$$v_m = c_0(z) \cdot v_b \cdot k_r \cdot \log\left(\frac{z}{z_0}\right) + 0.01z \tag{9.2}$$

where

$c_0(z)$ = orography factor and is set to 1.0

v_b = reference wind speed

k_r = terrain factor

z = height of interest along the building

z_0 = the roughness length.

Note that the first term in equation (9.2) corresponds to equation (4.3) in Eurocode 1, CEN (2008). The second term which takes into account the wind velocity variation along the height of tall buildings is proposed by Handa (2014-03-10) and is not derived in Eurocode 1. Furthermore, the effect of the wind turbulence must also be considered when including dynamic effects in the wind load. The effect of wind turbulence is accounted in the structural factor $(c_s \cdot c_d)$. This factor takes into account the increasing effect from vibrations due to turbulence in resonance with the structure. Hence, the wind load on storey i of a cantilever structure vibrating in the first mode can be determined according to

$$F_i = (c_s \cdot c_d) \cdot c_f \cdot q_p(z_e) \cdot A_{ref} \qquad (9.3)$$

where

$(c_s \cdot c_d)$ = structural factor

c_f = force coefficient for the structure

$q_p(z_e)$ = peak wind velocity pressure

A_{ref} = reference area of the structure.

Observe that $(c_s \cdot c_d)$ should not be separated, according to EKS 9 (2013). However, this factor can be set to 1.0, if the height of the building is less than 15 m.

9.5 Sizing optimisation of bracing core

The Lagrange Multiplier technique has been performed for sizing optimisation of the bracing core according to the following procedure.

9.5.1 Lagrange Multiplier technique

According to Section 7.2 the objective function $f(A)$ was established in order to minimise the volume of material

$$f(A) = \sum_{i=1}^{n_{stories}} A_i \cdot h \qquad (9.4)$$

subjected to the maximum top lateral deflection constraint $g(A)$

$$g(A) = \sum_{i=1}^{n_{stories}} \frac{M_i}{E \cdot I_i} \cdot h \cdot x_i - u_{\max} = 0 \tag{9.5}$$

where

$\frac{M_i}{E \cdot I_i}$ = curvature at storey i

$u_{\max} = H/500$ is the maximum allowed lateral deflection

x_i = distance between storey i and the fixed end of the bracing core

H = height of the bracing core

A_i = cross-sectional area of the core walls of sub-region i

h = storey height

M_i = moment applied on sub-region i

E = modulus of elasticity

I_i = moment of inertia of sub-region i.

By formulating the Lagrangian function L according to equation (7.1), the following Lagrangian is obtained.

$$L(A, \lambda) = \sum A_i \cdot h + \lambda \cdot \left(\sum \frac{M_i}{E \cdot I_i} \cdot h \cdot x_i - u_{\max} \right) \tag{9.6}$$

Accordingly, the necessary condition for a stationary point of the Lagrangian function L according to equation (9.6) can be obtained by setting the equation equal to zero and partially differentiate with respect to the area as well as the Lagrange Multiplier.

Then with some rearrangements the following equation for the Lagrange Multiplier λ is obtained:

$$\lambda_i = \frac{M_i \cdot x_i \cdot \alpha}{E \cdot i_i^2 \cdot A_i^2} \tag{9.7}$$

where

α = shape correction factor that can be set to 1,0 for square cores

i = radius of gyration.

Equation (9.7) is interesting as the Lagrange Multiplier λ can be interpreted as contribution of a sub-region to the total top deflection. It has been shown by Baker (1992) that when λ is equal for all sub-regions throughout the height of the structure, the objective function is minimised for a given constraint resulting in a minimum volume structure.

The optimal cross-section area of each storey can be calculated by equation (9.8), which is derived from the Lagrange Multiplier technique. To understand the derivation of this equation the reader is referred to Baker (1992).

$$A_i = \frac{\sqrt{\alpha}}{u_{\max} \cdot E \cdot i} \cdot \sqrt{M_i \cdot x_i} \cdot \frac{h}{\sqrt{\alpha} \cdot i} \cdot \sum \sqrt{M_j \cdot x_j} \tag{9.8}$$

where

$u_{\max}$ = maximum allowed lateral top deflection

$\sum \sqrt{M_j \cdot x_j}$ = summation of all sub-regions j except region i

h = height of the sub-region, i.e. equal to the storey height.

i = radius of gyration.

The radius of gyration is assumed to be constant throughout the height of the structure, as long as the layout of the core is the same even though if the thickness of the core walls changes along the height.

When all the cross-sectional areas of the sub-regions of the bracing core had been solved by equation (9.8) the total volume of material needed for the stabilising core could be determined by equation (9.4). The above presented Lagrange Multiplier technique was used for flexural members only, such as concrete cores or shear walls.

9.5.2 Issues with Lagrange Multiplier

When using the Lagrange Multiplier technique it is important to be aware of the limitations of this method and how these can be handled in a practical way.

The first issue when using the Lagrange Multiplier for flexural members, such as concrete cores and shear walls, is that it always yields theoretical results, which are not always practical. This is due to the fact that the bending moment approaches zero at the top region of the cantilever acting in flexure. Accordingly, the Lagrange Multiplier interprets the calculated zero moment as there is no need of walls in that region. This is practically impossible as there are other requirements concerning the thickness of the walls. A thickness of at least 150 mm is assumed to be needed to fulfil demands with regard to fire, sound and reinforcement arrangement.

The second issue is that equation (9.6) is derived with respect to the fact that the modulus of elasticity is constant throughout the structure to be optimised. However, if a part of the structure has been cracked, while the rest of the structure is uncracked, the modulus of elasticity would vary throughout structure. The consequence would be that equation (9.6) is no longer valid and, hence, cannot be used without further modifications. A method that could be used to deal with this issue is presented in the upcoming paragraphs. Nevertheless, in this project the studied cores never experienced any cracking as the compressive normal force from gravity load always was predominant.

The solution to the first issue was achieved by dividing the flexural member into two regions and set a boundary condition for the top region, see Figure 9.5. This boundary condition was that the walls must have a minimum thickness of 200 mm. Only under this condition the Lagrange Multiplier method could be utilised for practical solutions. In the second region, which is below the top region, the Lagrange Multiplier technique could be applied in its original formulation. This approach was only applied in a minor part-study of this project, since the pure theoretical required thickness, seen in Figure 9.5a was of greater interest to study. Nevertheless, in Section 10.3 a comparison is shown between a case of not restricting the wall thickness and a case of having a minimal thickness of 200 mm.

The following conditions were set for the case study with restriction of the wall thickness:

- The wall thickness for the upper region, called region 1, of the concrete core is 200 mm due to the above mentioned practical reasons.
- The second region, called region 2, is optimised by using the Lagrange Multiplier technique.

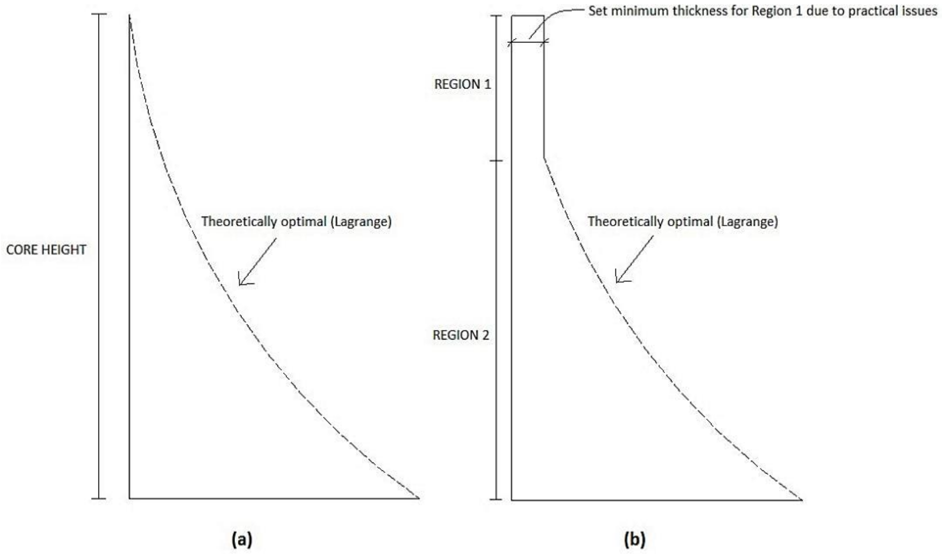

Figure 9.5: Thickness distribution of the stabilising core, (a) theoretically optimal distribution achieved by the Lagrange Multiplier technique; (b) theoretically optimal thickness distribution for region 2 and set minimum thickness for region 1 due to practical reasons.

The solution to the second issue would be to use a mean value of the modulus of elasticity of region 2. The mean value could be obtained as

$$E_{mean} = \frac{E_{cracked} \cdot (n_{cracked\_stories}) + E_{uncracked} \cdot (n_{uncracked\_stories})}{(n_{cracked\_stories} + n_{uncracked\_stories})} \tag{9.9}$$

where

$n_{cracked\_stories}$ = number of cracked stories

$n_{uncracked\_stories}$ = number of uncracked stories.

The numbers of stories that are cracked or uncracked are determined in a function file in Matlab called *cracks.m* which can be seen in Appendix A. A section is assumed to be cracked when stresses exceed the tensile strength of the concrete section.

Observe that a problem arises with such a simplification is that the real behaviour will not be fully captured. After when the structure has been sized by the method of Lagrange on the basis of the mean modulus of elasticity, the structure must be checked with regard to maximum lateral top deflection with the real modulus of elasticity at each storey, depending on whether this section is cracked or uncracked. This could result in a deflection larger than the maximum allowable value of H/500, since the modulus of elasticity is overestimated in the cracked regions when the sizing is performed with Lagrange Multiplier technique with a mean value of the modulus of elasticity. The cracked regions are normally the critical ones in the lower parts of the structure where the bending moment is largest. This issue can be resolved by increasing the stiffness of region 2 by a constant so that the maximum deflection is precisely reached giving an optimised structure with regard to lateral deflection.

Nevertheless, within the range of building heights, from 150 m up to 245 m, no cracked sections appeared because the compressive normal forces as well as the stiffness of the core were always very large.

9.6 Modelling and calculation by commercial software

WIN-Static Frame Analysis is a software developed by StruSoft. The software can be used to analyse two dimensional structures according to first and second order analyses. This software uses analytical calculations. For further information about this software the reader is referred to http://www.strusoft.com/products/win-statik.

The core structure with outrigger was modelled as a flexural stick where the whole building was divided into elements representing each storey. The model can be seen in Figure 9.6. Each storey was assigned a bending stiffness by means of the Lagrange Multiplier technique as described in Section 9.5. Outriggers were modelled as beams placed approximately at mid-height of the building. The outriggers were rigidly attached to the flexural stick and simply supported on springs representing the underlying façade columns.

The structure was loaded by the wind load calculated according to the approach presented in Section 9.4. Each storey was subjected to a concentrated load representing the resultant to the distributed wind load on that storey.

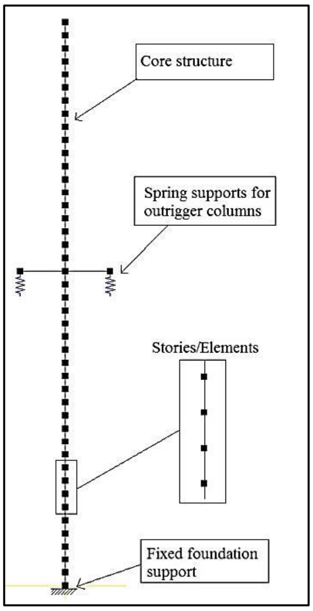

Figure 9.6: Model of core structure with outrigger in software called Frame Analysis.

9.7 Sizing optimisation of core structure with outrigger

The shear wall-braced structure was optimised by the Lagrange Multiplier technique, which is described in Section 7.3 and Section 9.5. The core structure with outrigger was analysed by combining analytical hand calculations with the commercial software described in Section 9.6. The optimisation procedure was performed according to the following steps:

1. The bending stiffness distribution of the shear-wall braced structure already examined was used as initial input data for the flexural stick in Frame Analysis program. Outriggers were provided at one storey making the whole structure stiffer. The stiffness of the spring supports of the outriggers was taken as the overall axial stiffness of the columns below the outriggers. In order to reach an optimised core structure with outriggers iterations, described in the following paragraphs, were needed.
2. A new moment distribution along the height of the flexural stick was obtained, taking into account the effect from the outriggers and the stiffness of the columns (spring supports). Also an additional axial force arose in the columns below the outriggers, which is the reaction force in the spring supports.
3. The new moment distribution was used as input data for the optimisation procedure by the Lagrange Multiplier technique in order to obtain a new stiffness distribution along the height of the core (flexural stick in frame analysis program). The columns below the outriggers were resized to carry the additional load from the outriggers.
4. The updated stiffness distribution was assigned to the model in the frame analysis program while the bending stiffness of the outriggers was kept unchanged. The stiffness of the spring supports was updated with respect to the new axial stiffness of the columns updated in step 3.

5. A new more refined moment distribution, along the height of the structure, was obtained. This was used in a final iteration in the Lagrange multiplier method to obtain a final optimised bending stiffness distribution along the core. Hence, a final optimised volume of material was obtained for the core structure with outriggers.

9.8 Check of vortex shedding

A check of the vortex shedding was performed according to Section 6.3.1 in order to ensure that vortex shedding is not a program for any of the investigated structural systems. The check was performed for the four different heights specifically studied in this project. The results showed that vortex shedding would not arise for any of the cases. This check is presented in Appendix A1.

9.9 Verification of calculations

The calculation program used in this project was verified by analysing simple cases by means of the program in order to clarify whether the program was working in the right way or not. As an example, the response was checked for a concentrated load at the top of the cantilever. This resulted in a reasonable behaviour of the cantilever with regard to moment distribution and deflection. Another test was carried out for a uniformly distributed lateral load, which also gave reasonable results. Based on these verifications the calculation program was used to analyse the selected structural systems.

The frame analysis program was used in order to verify the analytical calculations performed in Matlab. Input data was taken the Matlab program and inserted in the frame analysis program. The results of the bending moment as well as the lateral deflection wee of the same magnitude in the frame analysis program as in the calculations performed in Matlab.

Verification of the acceleration obtained by analytical calculations according to Eurocode 1, CEN (2008), was performed by comparing the results with results from another method provided by Smith and Coull (1991). The results were very similar and therefore it was concluded that the calculation procedure according to Eurocode 1 were performed appropriately.

10 Results and analysis from the parametric studies

In this chapter the results obtained from the parametric studies are presented. All calculations can be viewed in Appendix A.

10.1 Theoretical optimal thickness distribution of core walls

This study was based on the Lagrange Multiplier technique according to Section 9.5. The theoretical optimal thickness distribution of core walls was determined for two structural systems, shear wall-braced structure and core structure with one outrigger, of four different heights.

The results showed that the maximum theoretical required thickness for the 245 m tall building is approximately 2.7 m in the case of a core structure with one outrigger. The corresponding thickness for the shear wall-braced structure is approximately 3.5 m. These results can be seen in Figure 10.1.

For the case of a 150 m tall building the results showed that the maximum required thickness is approximately 0.33 m for the core structure with outrigger. The corresponding thickness for the shear wall-braced structure is 0.61 m. The thickness distributions for the two structural systems of a 150 m tall building can be seen in Figure 10.4.

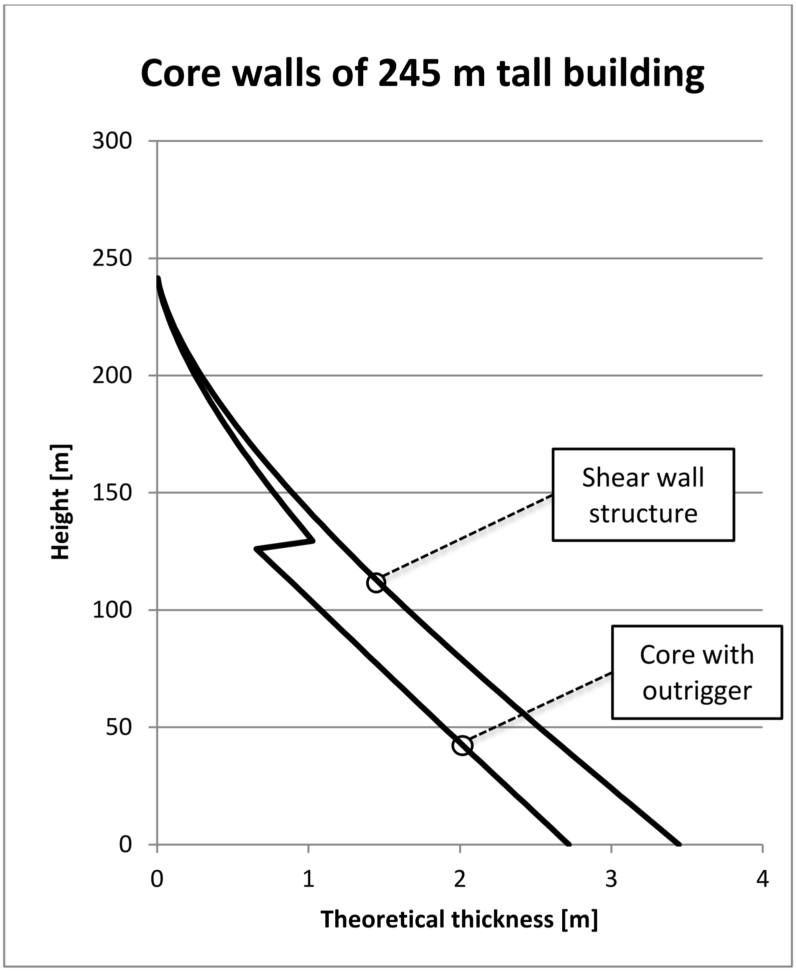

Figure 10.1: Optimised thickness distribution of core walls for two structural systems of a 245 m tall building.

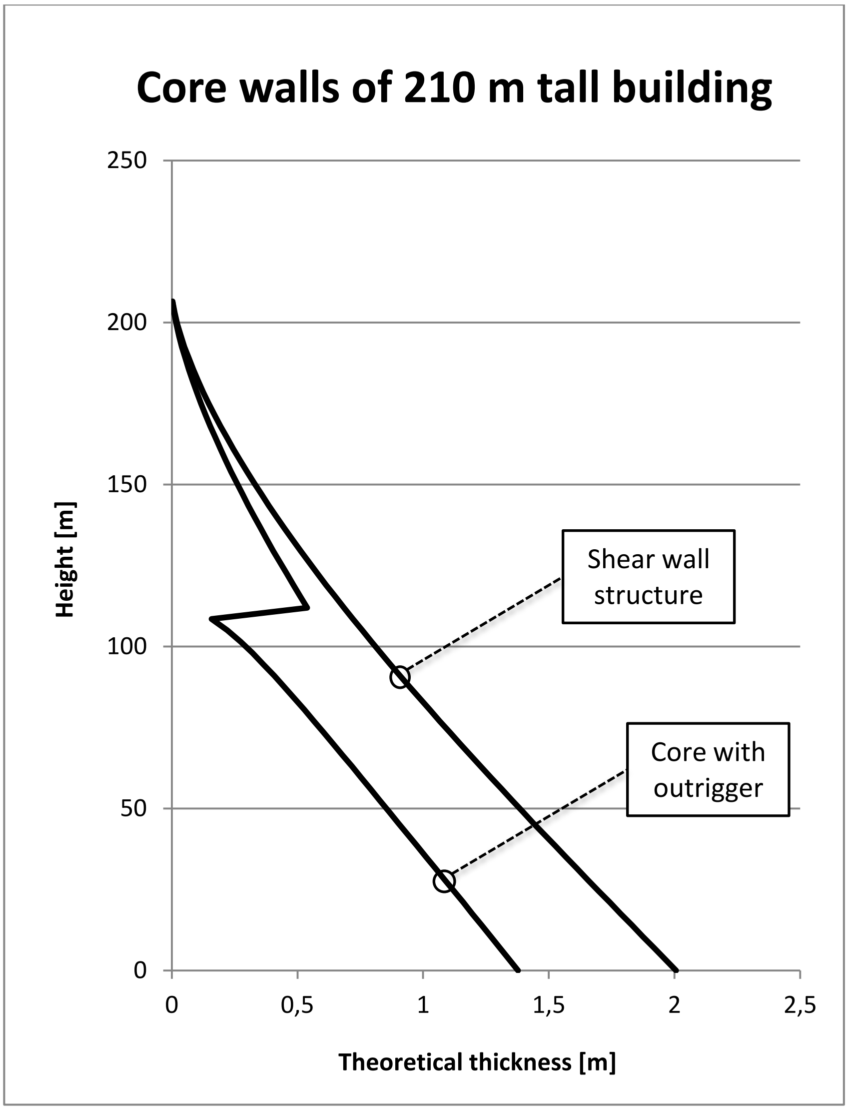

Figure 10.2: Optimised thickness distribution of core walls for two structural systems of a 210 m tall building.

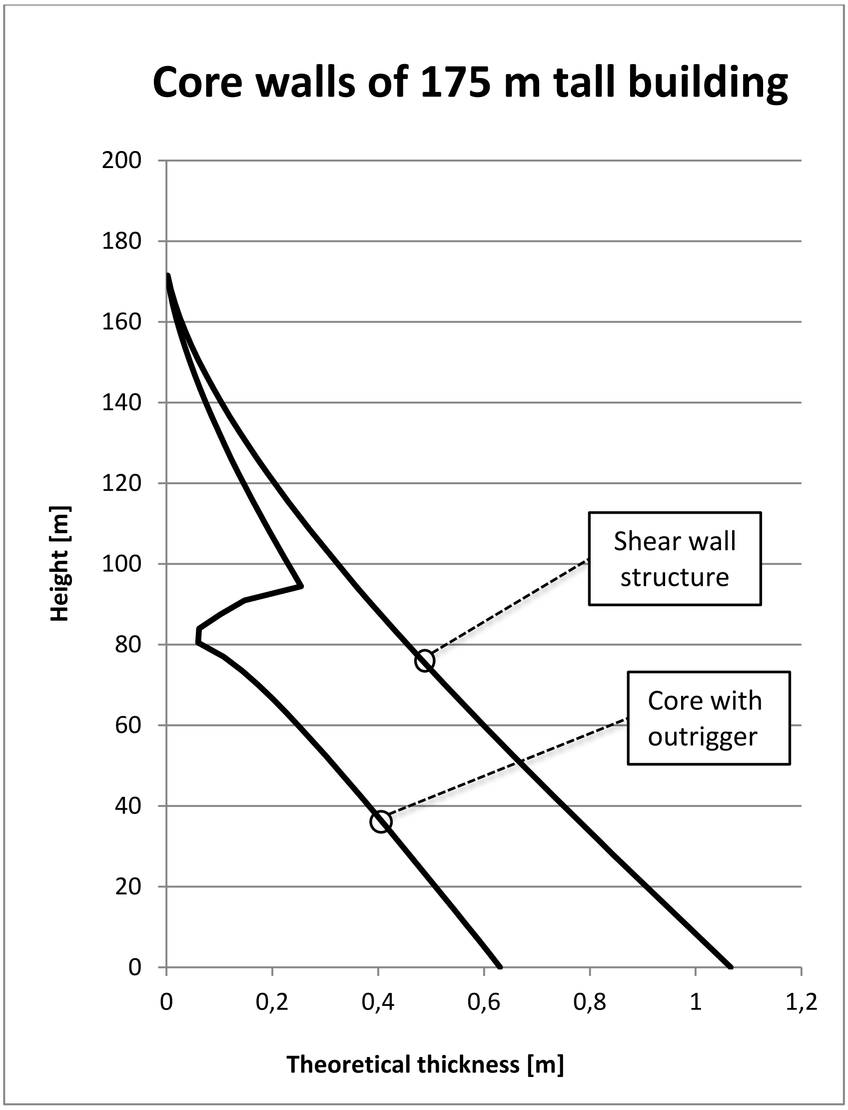

Figure 10.3: Optimised thickness distribution of core walls for two structural systems of a 175 m tall building.

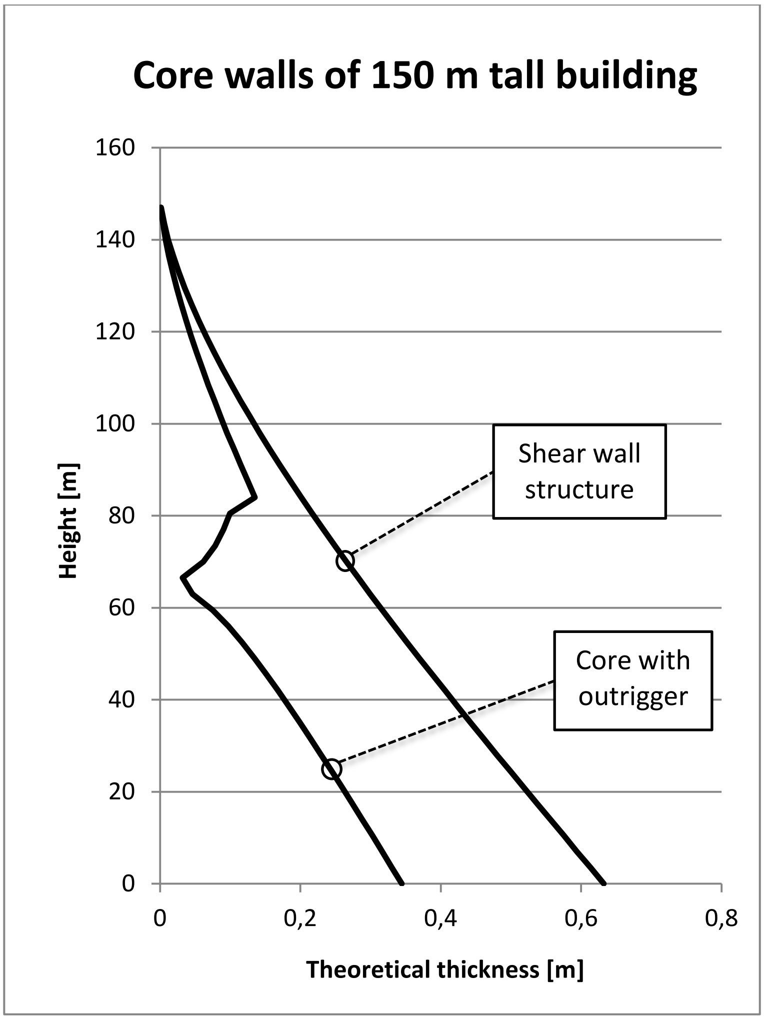

Figure 10.4: Optimised thickness distribution of core walls for two structural systems of a 150 m tall building.

For the 245 m tall building it is clear that the maximum required thicknesses of the core walls are unreasonable in practice for the both structural systems, although the core structure with one outrigger provides smaller thicknesses. This is to be compared with the 150 m tall building which resulted in more realistic thicknesses of 0.33 m and 0.61 m for the core structure with one outrigger and shear wall-braced structure respectively.

By providing one outrigger to the shear wall-braced structure reduced the maximum required thickness by almost 50 % for the case of a 150 m tall building. For the 245 m tall building the reduction is about 23 %.

However, it is important to highlight why the 245 m core structure with outrigger did not result in reasonable maximum thickness. One reason is due to the fact that the outrigger was not very stiff in relation to the stiffness of the core, which they should be in order to attract a larger portion of load from the core via the outrigger to the perimeter columns. This effect can be seen in the case of the 150 m tall building, where the maximum required thickness is reduced by 50 %. The reason is mainly due to the fact that the outrigger then is stiff in relation to the stiffness of the core compared to the stiffness relation between the outrigger and the core for the 245 m tall building.

Hence, the reason why the difference between reductions of the maximum required thickness of the 150 m and the 245 m tall buildings is so large is due to the fact that the project was limited to one outrigger of the same type and stiffness for both heights.

Note that the theoretical thickness distribution obtained by the Lagrange Multiplier technique can be adjusted so that it fulfils practical requirements. An example is illustrated in Figure 10.5.

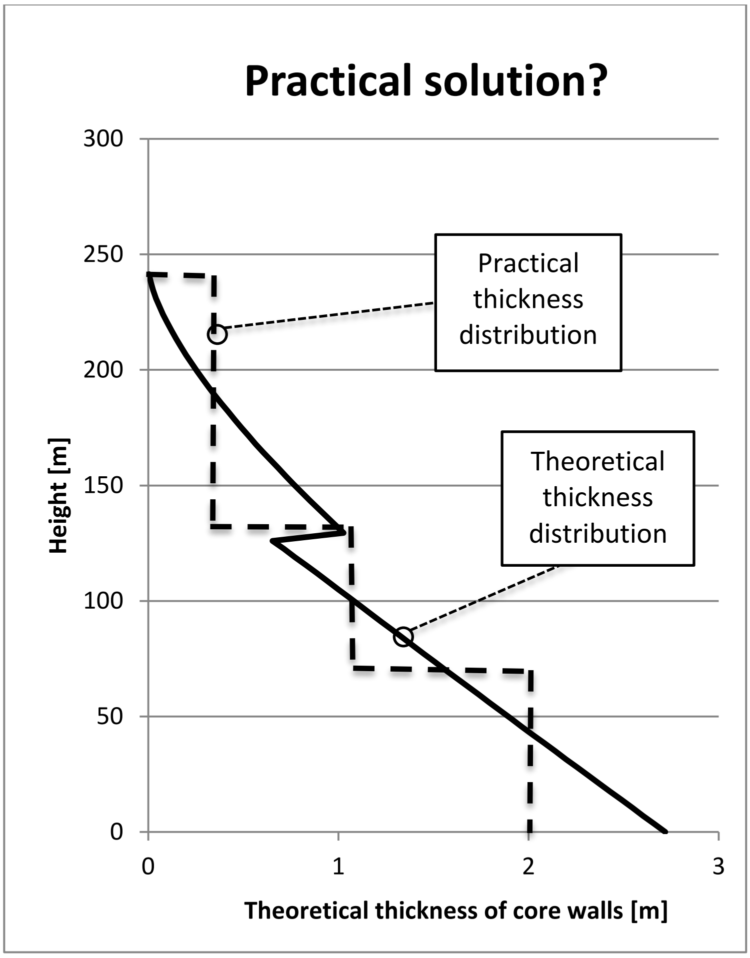

Figure 10.5: Example of practical thickness distribution of a 245 m core with reference of theoretical thickness distribution.

10.2 Influence of restriction of wall thickness on the needed volume of material

This study was based on the Lagrange Multiplier technique according to Section 9.5. The influence of restriction of the core wall thickness was investigated only for the shear wall-braced structure which is discussed in the following paragraph.

The restriction of the wall thickness to a minimum allowed thickness according to Section 9.5.2 resulted in different influences on the acceleration and the total required volume of material for the different structural systems. For the case with 150 m high building the difference is significant whether a restriction is applied or not to the wall thickness, which can be seen in Figure 10.6. Oppositely, for the cases of a taller structure, such as the case with a 245 m high structure, the acceleration and the total required volume of material did not differ much when the wall thickness was restricted to a minimum of 200 mm. This can be seen in Figure 10.7.

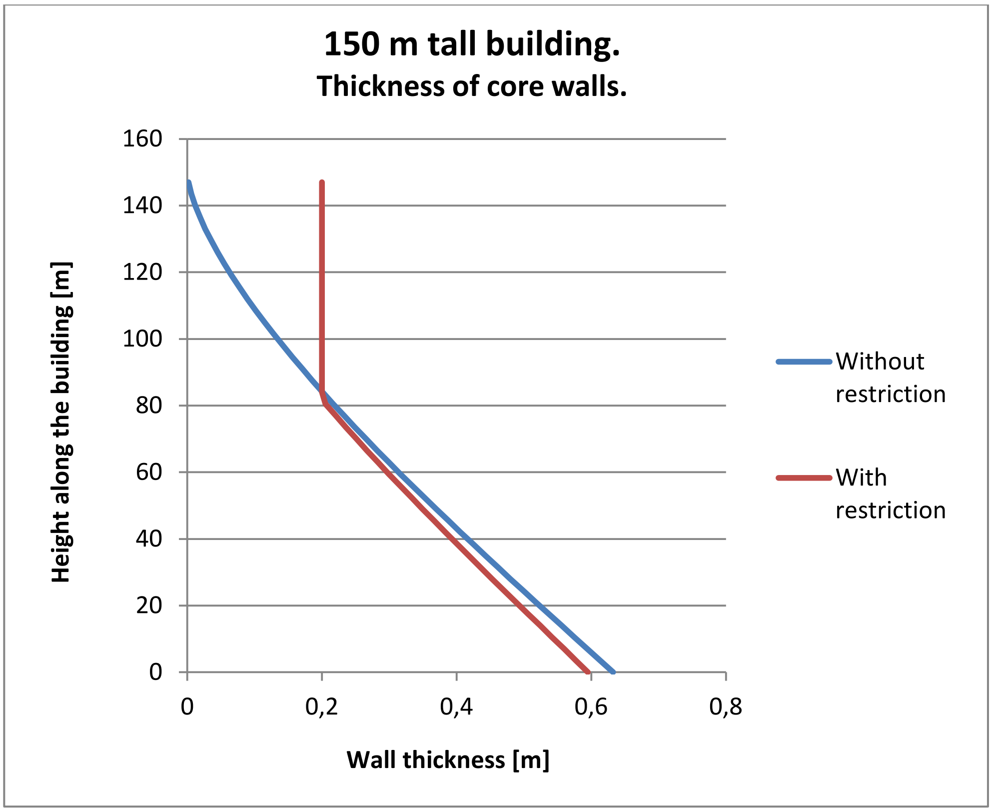

Figure 10.6: Optimal thickness distribution of the core walls of a 150 m tall shear wall-braced structure.

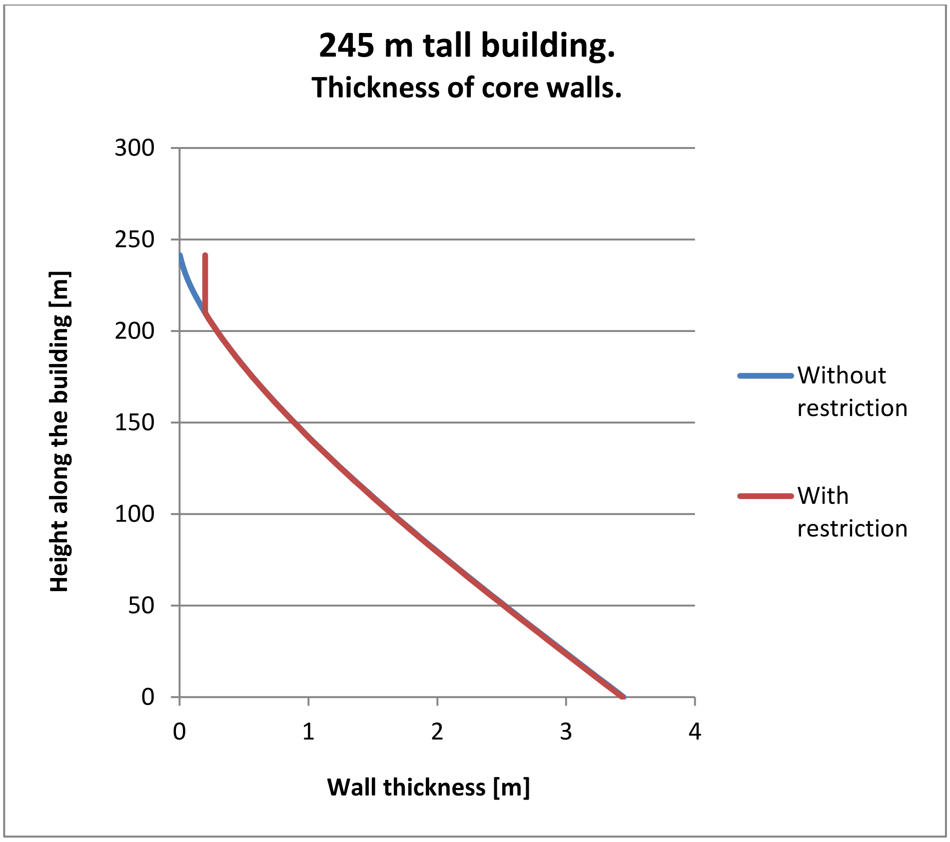

Figure 10.7: Optimal thickness distribution of the core walls of a 245 m tall shear wall-braced structure.

The results show that the influence of restricting the wall thickness to a minimum of 200 mm gives larger effects for the case of a 150 m tall building in comparison to the 245 m tall building, which can be seen in Figures 10.6 and 10.7. The reason for this is that the taller buildings require thicker walls and a very small portion of the height need to have their wall thickness adjusted to the minimum required. On the other hand, the lower buildings of this case study required thinner walls in order to fulfil the criterion of maximum top lateral deflection. This is due to the fact that the chosen geometry of the core gives relatively stiff response for the 150 m tall building as the same radius of gyration is applied for the different heights. This results in a need of thickening the walls within a larger portion of the height of the lower building in comparison to the taller one.

10.3 Required volume of material

All volumes presented in this section do not include the volume of material for the floor systems.

10.3.1 Total volume of material for the two structural systems

The total volume of material needed for shear wall-braced structures and core structure with one outrigger, respectively, is shown in Figure 10.8.

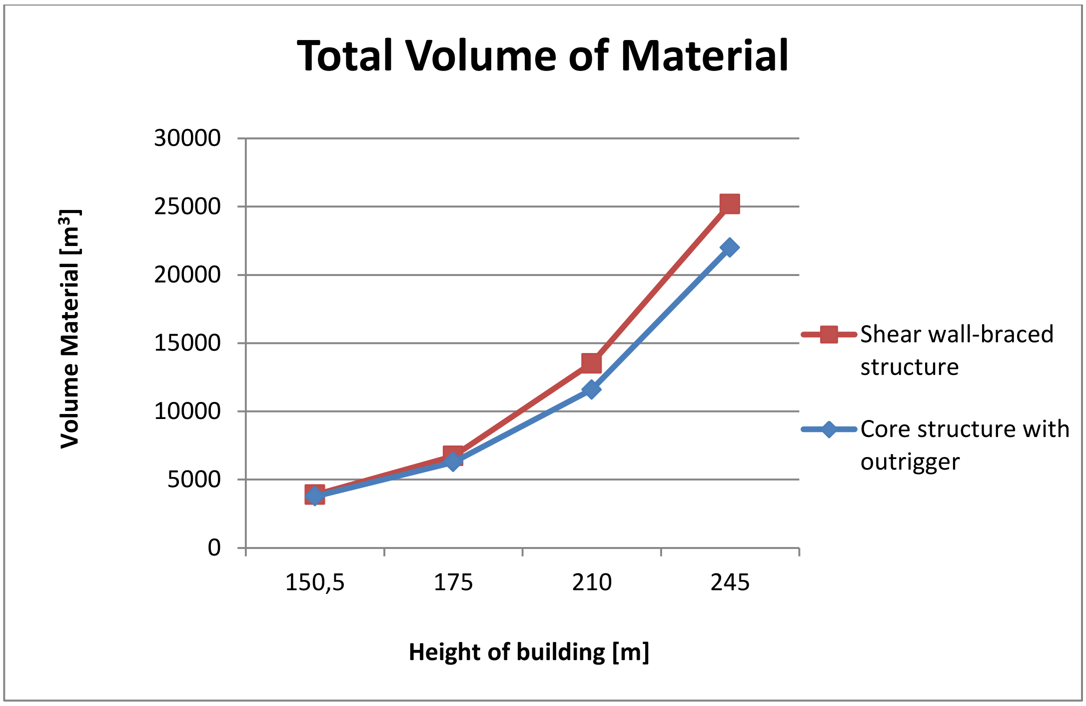

Figure 10.8: Total required volume of material for core structures with one outrigger and for shear wall-braced structures of different heights.

The results in Figure 10.8 show that the core structure with one outrigger require less total volume of material in comparison to the shear wall-braced structure. The difference becomes more significant for the taller buildings, meaning that the effects from the outrigger seem to be more pronounced the taller a structure is. This coheres with the information provided by Smith and Coull (1991).

The results also show that the effects of providing one outrigger on the total volume of material become smaller the lower a structure is.

10.3.2 Required volume of material of the bracing system

The bracing system of the shear wall-braced structure was considered to be the core, while the bracing system of the core structure with one outrigger was considered to be the core in addition to the outrigger and the columns below the outrigger. The reason why the outrigger and the columns below it were considered as parts of the bracing system is because they contribute to the lateral stiffness of the core structure with outrigger. The required volume of material of the bracing system for the two structural systems is shown in Figure 10.9.

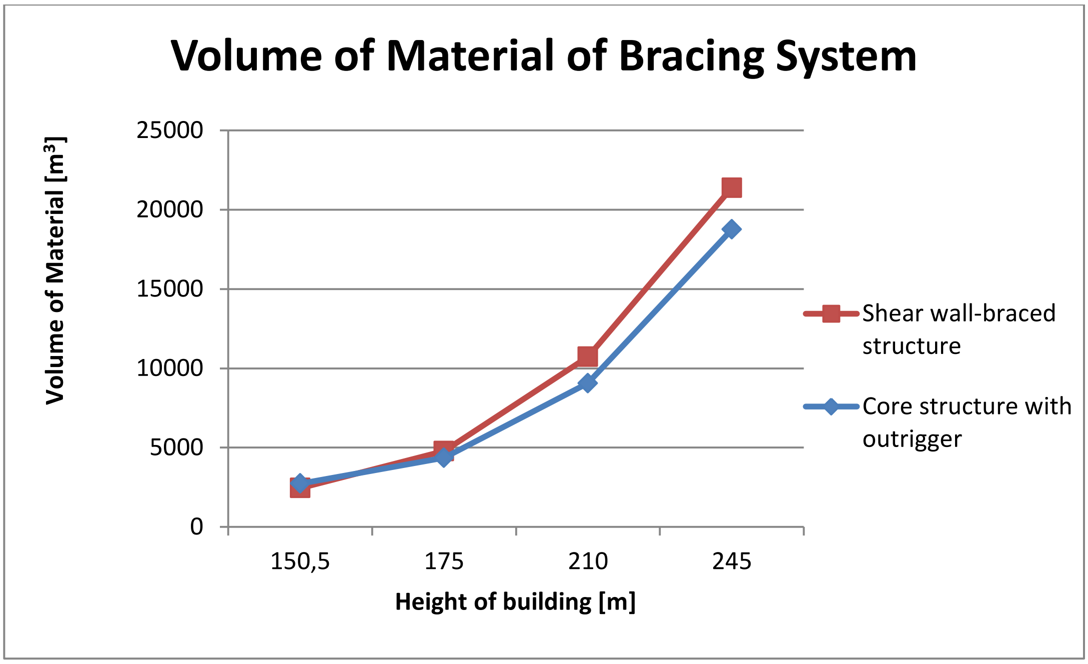

Figure 10.9: Required volume of material for the bracing system for core structures with one outrigger and for shear wall-braced structures of different heights.

The difference in volume of material of the bracing systems in the two structural systems has the same tendency as for the difference in total volume of material, which is analysed in the previous Section 10.3.1. The results are presented in Figure 10.9.

The 8 outriggers, seen in Figure 9.3, were modelled as 7 m high and 0.8 m thick concrete wall respectively. This gives a huge volume of material in relation to the volume of the core. The outriggers could have been optimised and, hence, more efficient e.g. by having trusses than concrete walls. This is why the results in Section 10.3.3 are more relevant as they show the effect an outrigger has on the stabilising core only.

10.3.3 Required volume of material of the stabilising core

The volume of material of the stabilising core for the two different structural systems is presented in Figure 10.10. In comparison to Figure 10.9, no outrigger or any columns are included here. The volume is presented for the two structural systems of four different heights as listed in Chapter 9.

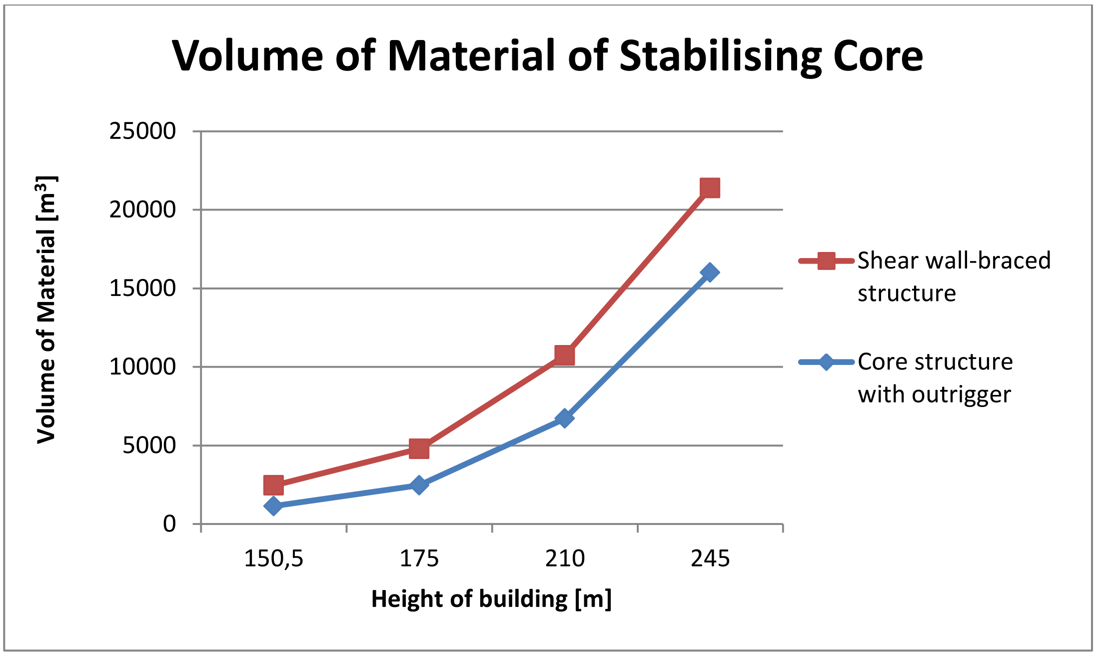

Figure 10.10: Required volume of material of the stabilising core for the two structural systems.

Concerning only the stabilising core of the two structural systems the difference in volume of material is more pronounced compared to the difference of the total volume of material, which is shown in Figure 10.10. Specifically it can be noted that the difference in volume of material of the stabilising cores is significant for the lower buildings as well as for the taller ones. Such a difference is not observed when comparing difference in total volume of material.

The reason why the volume of material of the core is decreased for all building heights is because of the large reduction of maximum moment in the core. This is an important feature of utilising outriggers.

10.3.4 Required volume of material for columns and bracing system

Figures 10.11 and 10.12 present the total volume of material of all structural components, excluding the floors, for shear wall-braced structures and core structure with one outrigger respectively. The total volume of the structure is also plotted in order to illustrate the relation between the volume of the bracing system, the volume of the columns and the total volume of the structure.

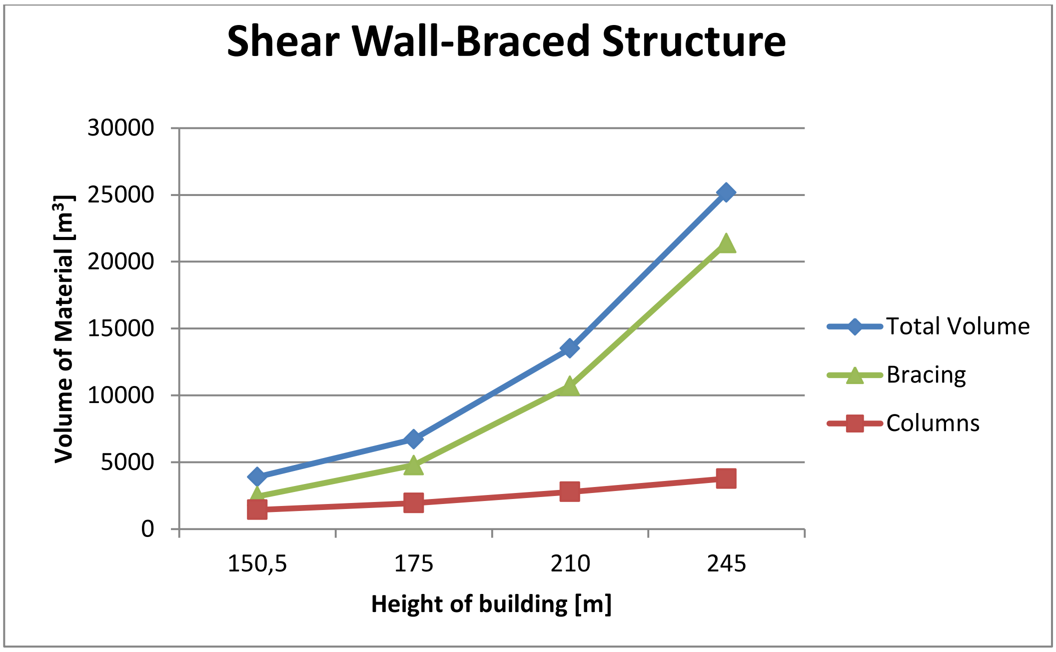

Figure 10.11: Required volume of material for columns and bracings and the total volume of material for shear wall-braced structure.

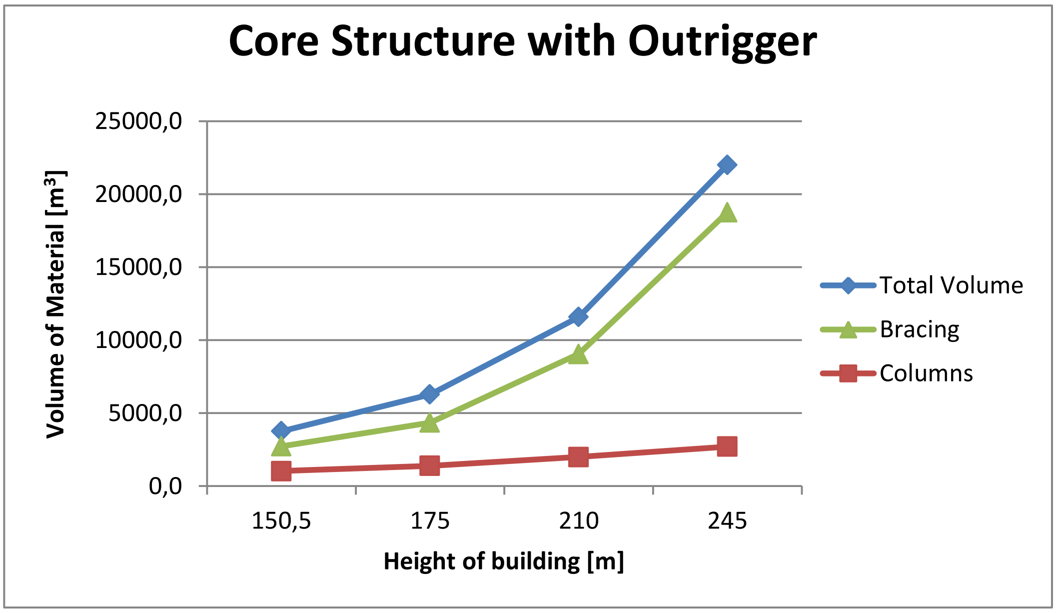

Figure 10.12: Required volume of material for columns and bracings and the total volume of material for core structure with one outrigger.

The results displayed in Figures 10.11 and 10.12 show clearly that the required volume of material of the bracing units increases nonlinearly when the height of the structure increases. This is compared with the volume of material for the columns, where the increase is almost linear and in general the increase of volume of material is small.

The tendencies observed are expected, since it is more efficient to resist load axially, which is the case for the columns, in comparison to resisting load in bending, which is the case for the stabilising system.

Even though the core structures with one outrigger require larger columns as supports for the outrigger, it is still beneficial to utilise the effects it has on the bracing system, especially for taller buildings.

10.4 Characteristic results of optimised structural systems

Tables 10.1 and 10.2 present and compare the final results obtained for shear wall-braced structures and core structures with one outrigger respectively.

Table 10-1: Results from optimisation of shear wall-braced structures.

Shear wall-braced structure	Structural mass/ equivalent mass [kg] 10^5	Natural frequency [Hz]	Structural damping [%]	Tip deflection/ Max. allowed [m]	Acceleration [m/s^2]
150m (43 stories)	432 / 8.51	0.188	0.1	0.300/0.301	0.232
175m (50 stories)	558 / 8.85	0.177	0.1	0.349/0.350	0.240
210m (60 stories)	808 / 9.53	0.164	0.1	0.419/0.420	0.246
245m (70 stories)	1183 / 10.53	0.151	0.1	0.489/0.490	0.246

Table 10-2: Results from optimisation of core structure with one outrigger.

Core structure with one outrigger	Structural mass/ equivalent mass [kg] 10^5	Natural frequency [Hz]	Structural damping [%]	Tip deflection/ Max. allowed [m]	Acceleration [m/s^2]
150m (43 stories)	388 / 8.64	0.192	0.1	0.300/0.301	0.225
175m (50 stories)	485 / 8.88	0.182	0.1	0.349/0.350	0.234
210m (60 stories)	686 / 9.54	0.168	0.1	0.419/0.420	0.240
245m (70 stories)	1019 / 10.8	0.155	0.1	0.489/0.490	0.235

There are many factors that can be examined in order to explain why the maximum accelerations were almost the same despite differences in structural properties between the shear wall-braced structures and core structures with one outrigger.

According to Table 10.1 the accelerations of the shear wall-braced structures with the four different heights showed that the difference between the accelerations was not huge with a maximum difference of only 5.7 % between the 150 m and 245 m tall buildings. A further observation was that the acceleration tended to decrease for the 245m tall building, even though the equivalent mass of the structure is increased at this height.

Table 10.1 clearly shows lower values of accelerations for the core structure with one outrigger in comparison to the shear wall-braced structure. However, the variation of accelerations between the four different cases, of four different heights of the structure, tends to be the same as for the shear wall-braced structure. The results showed that the acceleration increases from the case of a 150 m up to a 210 m tall building, but for the 245 m tall building the acceleration decreases.

A reason for this tendency is difficult to explain as there are many factors affecting the acceleration of a building. As discussed in Section 6.2, the deflected mode shape, equivalent mass and natural frequencies of the structures are major factors that affect the magnitude of acceleration. According to Tables 10.1 and 10.2 the natural frequency of the 245 m tall structure is approximately 25 % lower than for the 150 m tall structure, meaning that the taller building is more flexible. However, the equivalent mass of the 245 m tall structure is approximately 19 % larger than for the 150 m tall structure.

The equivalent mass of the different cases did not differ much in comparison to the difference in real mass of the structure. Even though a structure might be twice as heavy as another structure, the acceleration does not have to differ as much since it depends on the equivalent mass. Furthermore, the equivalent mass depends on the distribution of the mass as well as of the modal shape of the structure. Having much smaller thicknesses of the walls of the core in the higher regions of the structure, in comparison to the lower regions with much thicker walls, gives a smaller equivalent mass. This results in a larger acceleration than if more mass was distributed to the higher regions of the structure.

The difference between the calculated maximum accelerations of the two different structural systems is important to highlight. Even though the core structure with one outrigger results in a lighter structure, with much lower volume of material, the arising maximum accelerations are still lower for this structural system in comparison to the shear wall-braced structure. The reason for this is mainly the mode shape of the structures, which differs significantly. The formula for the deflected mode shape, available in Section 6.2.1, contains a factor ξ, which is larger for the core structure with one outrigger resulting in a smaller deflected mode shape than for the shear wall-braced structure. A smaller deflected mode shape increases the equivalent mass and finally it results in a smaller acceleration. A further observation, which contributed to the smaller acceleration for the core structure with one outrigger in comparison to the shear wall braced structure, is that the natural frequency of the former structure was larger than the natural frequency of the latter. The reason could be the deflected shape of the two structural systems which differs, as the outrigger changes the deflected shape of the structure.

An important factor concerning maximum acceleration is the damping of the building. In this case the aerodynamic damping did not affect the results significantly, since it was assumed to be very small. The reason is that the geometry of the façade for all cases studied in this project was limited to a square section with no rounded corners or any other adjustments to decrease the wind load on the structure or to increase the damping. Furthermore, special damping devices could have been used, which could have resulted in a more significant difference in acceleration.

Figure 10.13 presents the maximum acceleration obtained for the two types of structural systems and for four different heights. Note that both structural systems with the height of 150 m and 175 m were not fully optimised with regard to the deflection criterion H/500.

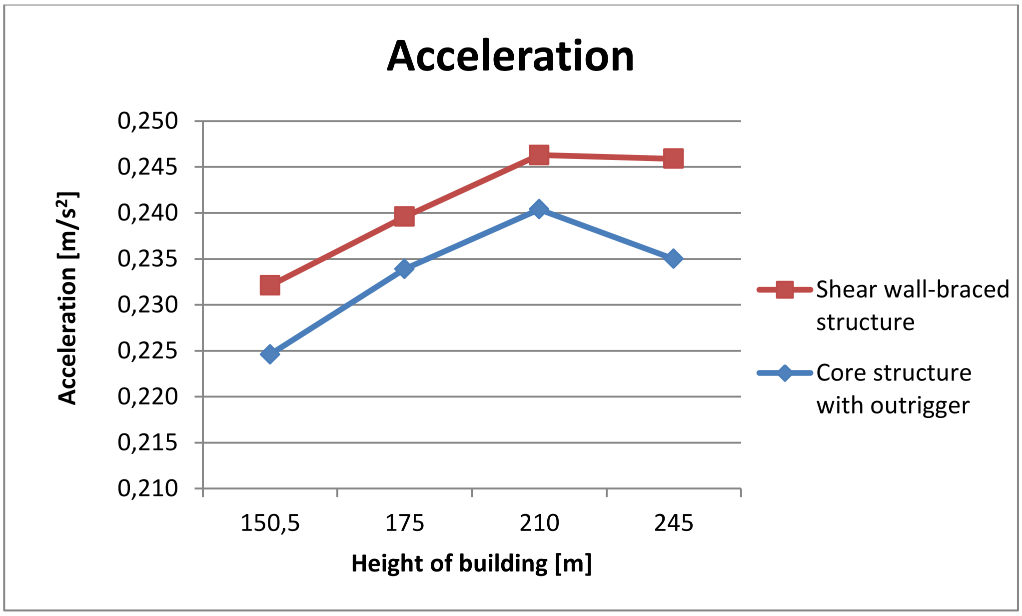

Figure 10.13: Calculated acceleration for two types of structural systems of four different heights.

Figure 10.14 shows how the natural frequency of the two types of structural systems varies for the four different heights.

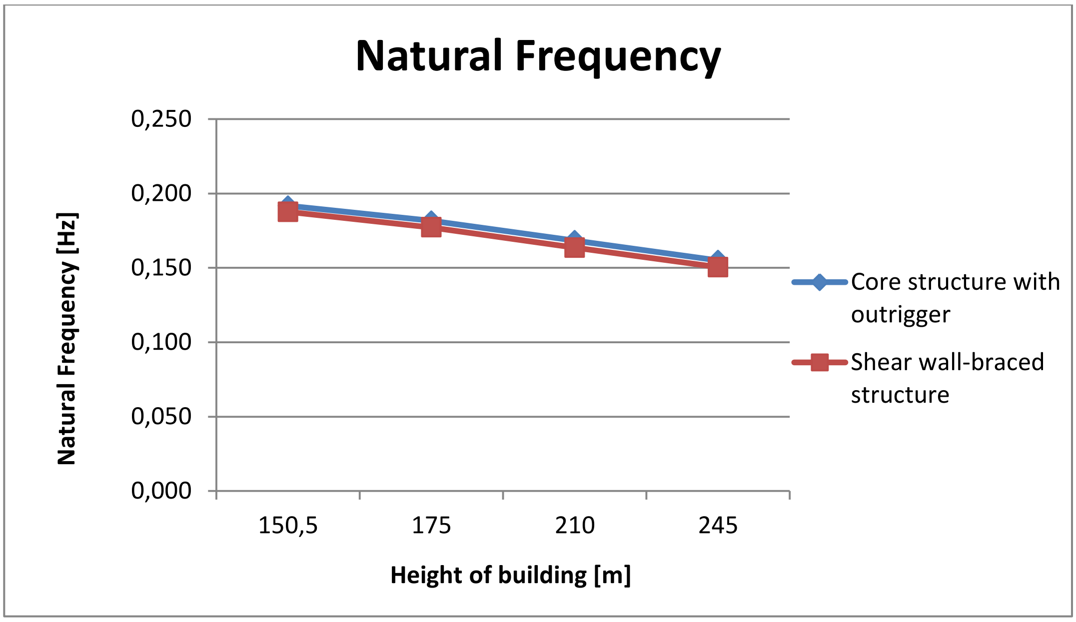

Figure 10.14: Natural frequency as a function of the building height for the two types of structural systems.

10.5 Parametric study of maximum acceleration in shear wall-braced structures

10.5.1 Varying size of core

A parametric study was performed in order to examine effect on acceleration by changing the size of the stabilising core, increase maximum allowed lateral deflection and increase damping. Note that this study was performed according to first order analysis of shear wall-braced structures.

The results of the maximum acceleration with varying size of the stabilising core are presented in Figure 10.15.

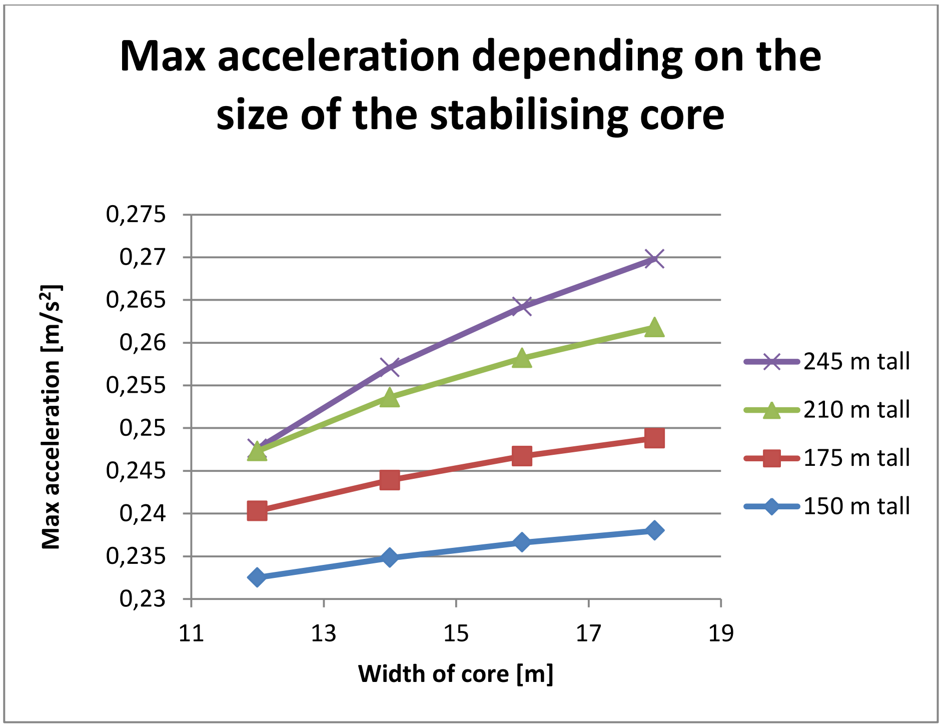

Figure 10.15: Results of parametric study on acceleration as a function of the width of the stabilising core for four different heights of shear wall-braced structures.

The core was enlarged from 12 m to 18 m. This resulted in both increased radius of gyration and bending stiffness, which decreased the required volume of material when the structure was optimised according to the Lagrange Multiplier technique as explained in Section 9.5. This resulted in a decrease of the mass of the core. A decrease of the structural mass had a significant negative impact on the maximum acceleration as the acceleration increased. It is important to note that this occurs due to the fact that the volume of material of the structure was optimised. The results would have been different, if the wall thicknesses were kept unchanged and not optimised. If the thickness of the walls would not have been changed, the volume of material would have increased resulting in a decrease of the acceleration.

In this project the optimisation was carried out with regard to one constraint, namely the maximum allowed lateral deflection. In conclusion, the approach of minimising the volume of material, with regard to the maximum allowed lateral deflection is not beneficial, if the goal is to decrease the acceleration.

10.5.2 Varying maximum allowed deflection

Decreasing the maximum allowed lateral deflection from H/500 to stricter values up to H/1000. These results are presented in Figure 10.16.

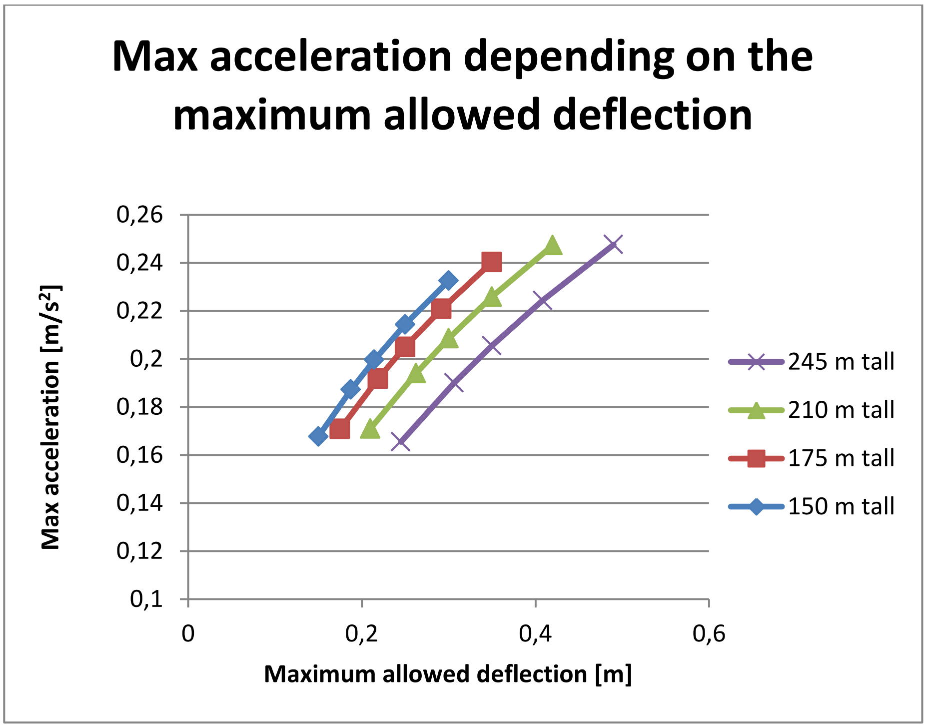

Figure 10.16: Results of parametric study on acceleration as a function of the maximum allowed lateral deflection for four different heights of shear wall-braced structures.

The criterion of the maximum allowed lateral deflection was changed from H/500 to stricter values up to H/1000. This resulted in an increase of the volume of material, as the structure was forced to be stiffer. The resulting maximum acceleration decreased as can be seen in Figure 10.16. Increasing the volume of material is obviously beneficial in order to decrease the maximum acceleration.

10.5.3 Influence of damping

Increasing the damping of the 245 m tall shear wall-braced structure from 0.1 to 0.2 decreased the acceleration significantly. The first value of structural damping was set to be 0.1 as for reinforced concrete buildings according to Eurocode 1, CEN (2008). However, when the damping was assumed to be 0.2, i.e. double the first value, for instance due to tuned mass dampers and aerodynamic damping were provided, the acceleration of the 245 m tall building decreased from 0.24 to 0.15 m/s^2. Damping obviously has great influence on the acceleration.

10.6 Effect of location of one outrigger

This study was based on two different cases of 245 m tall core structure with outrigger. The first one was with a core of optimised varying stiffness of the stabilising core. The second was with a constant stiffness of the core.

The effect of the location of the outrigger on the maximum top lateral deflection and the maximum bending moment was studied in the frame analysis program on a 70-storey core structure with one outrigger. The model was built as is illustrated in Figure 9.6. The studies were performed by varying the position of the outrigger according to the following four situations:

- Atthe top of the structure
- At three-quarter of the height
- At mid-height
- At one-quarter of the height

The results from this parametric study are summarised in Sections 10.6.1 and 10.6.2.

Note that the results obtained are with respect to a trapezoidal wind load applied on the structure. The results might differ if, for instance, the wind load would be modelled as uniformly distributed.

As seen in Figures 10.17 to 10.20 the lower the outrigger is placed, the larger the top lateral deflection arises, while the base moment is reduced. However, there are specific positions of the outrigger, along the height of the structure, which gives optimal effect with respect to top lateral deflection and base moment respectively.

As also can be seen in Figures 10.17 and 10.19, the maximum reduction in top lateral deflection arises, when the outrigger is located at a level corresponding to approximately 75 percent of the height of the building. This is to be compared with the optimal location of the outrigger with regard to the maximum reduction of the base moment, which is at 50 percent of the height of the structure. In order to gain from both of these effects, meaning to decrease both the top lateral deflection and base moment as much as possible, the optimal location of the outrigger should be somewhere in between 50 and 75 percent of the height of the structure.

In conclusion, if the design problem is stiffness driven, the outrigger should be placed closer to 75 percent of the height of the structure. However, if the design problem turns out to be resistance driven or driven by minimising the volume of material of the core, then lowering the outrigger closer to 50 percent of the height of the structure is a preferable solution.

The bending stiffness of the outrigger was not changed in this study when located at different heights. It is neither changed in the study of different tall core structures with outriggers. This is a chosen limitation in this project, even though the stiffness of the outrigger itself would affect the response of the whole building.

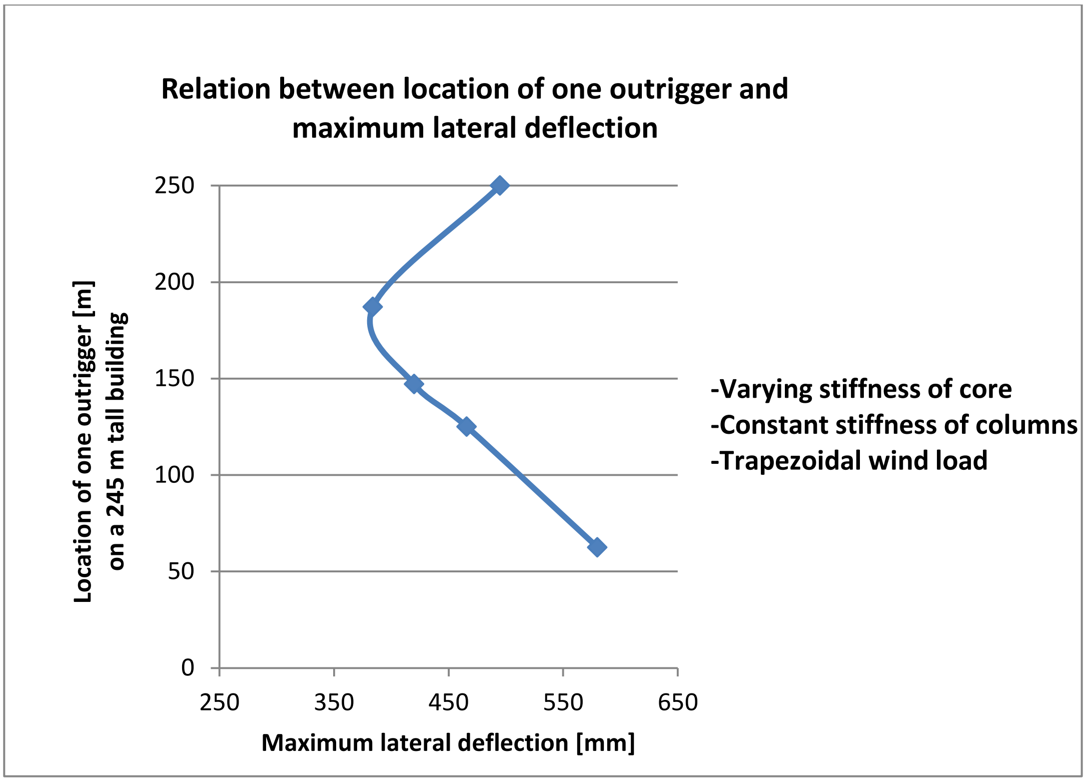

Figure 10.17: Relation between location of one outrigger and the maximum lateral deflection of a 245 m tall building with varying stiffness of the core.

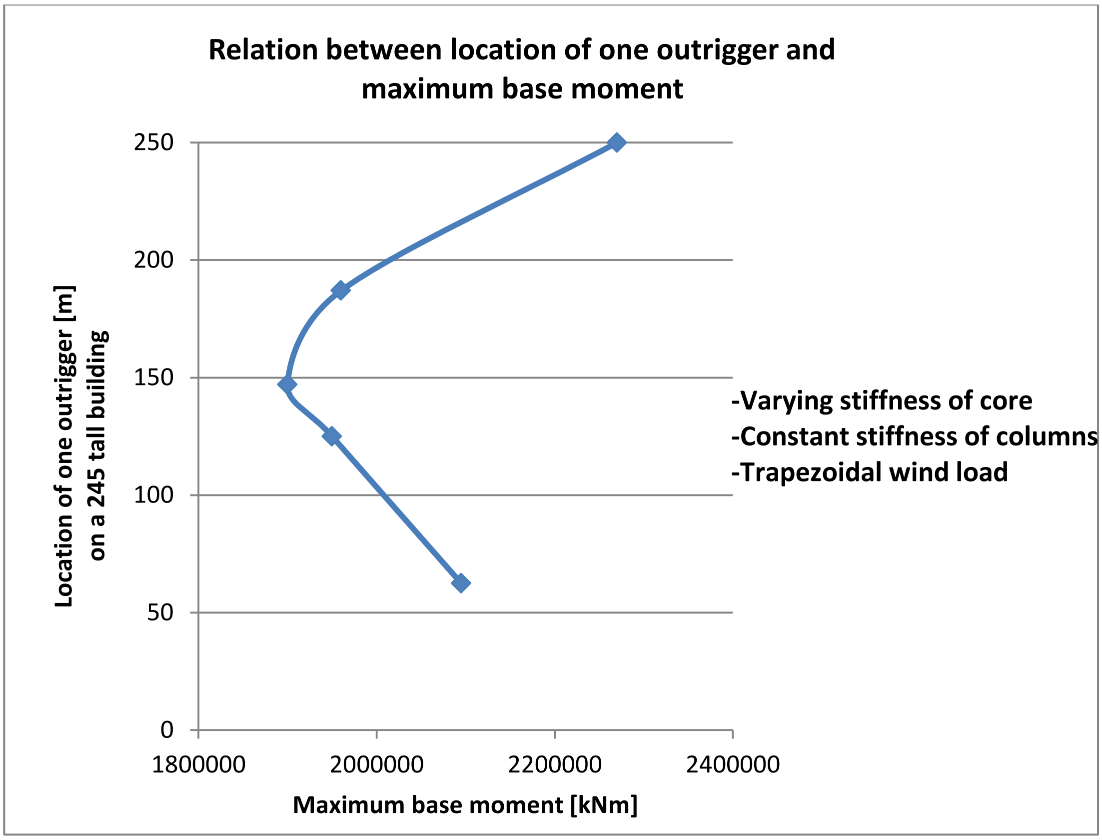

Figure 10.18: Relation between location of one outrigger and the maximum base moment of a 245 m tall building with varying stiffness of the core.

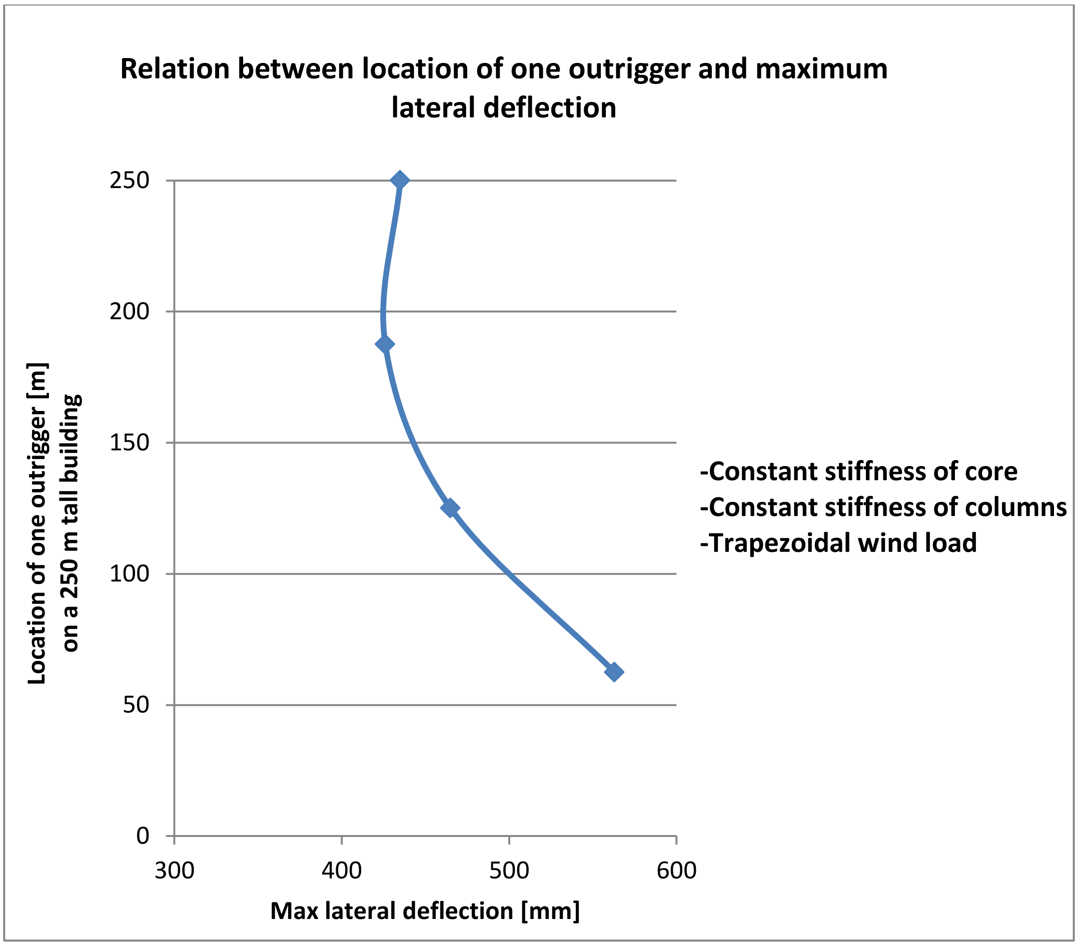

Figure 10.19: Relation between location of one outrigger and the maximum lateral deflection of a 245 m tall building with constant stiffness of the core.

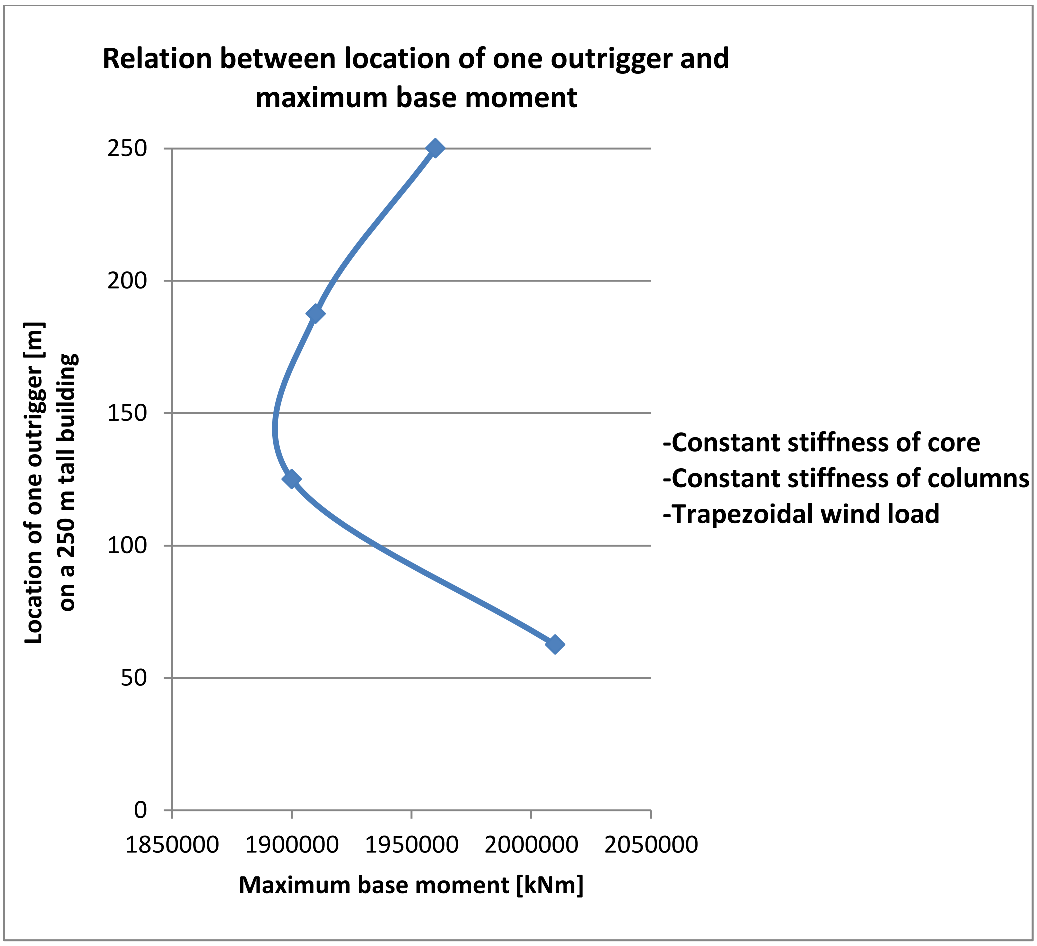

Figure 10.20: Relation between location of one outrigger and the maximum base moment of a 245 m tall building with varying stiffness of the core.

11 Analysis of literature study and results

11.1 Acceleration

With regard to the different perception levels, defined in Figure 2.2, on how acceleration is perceived by humans, it is clear that the cases studied in this project, i.e. structures optimised with regard to maximum allowed lateral deflection give rise to accelerations not acceptable for office buildings. Nevertheless, the aim in this project was to optimise the volume of material with regard to a deflection constraint and to observe tendencies of how the volume of material changed when different parameters were varied. Furthermore, it was of interest to observe the change of acceleration when changing different parameters and not necessarily to reach an acceleration that is as low as possible.

There are no precise limits for the maximum allowed acceleration in a building, but there exist approximate values concerning how humans perceive different levels of acceleration. However, it should be noted that different levels of acceleration are acceptable for different types of building. The type of activity that will take place in the a building is decisive for how high the acceleration can be, but information about exact limits are still lacking.

11.2 Wind load including dynamic effects

There are many publications providing valuable information about how to deal with dynamics of tall buildings. In this project, however, the information about how to handle the dynamic effect from wind load was mainly gathered from Eurocode 1, CEN (2008) and EKS 9 (2013) as well as Handa (2014-03-10).

It was concluded that Eurocode 1 certainly need to be updated concerning how to handle the wind load and particularly the dynamic features concerning tall buildings. One equation that needs to be updated is the formula for the mean wind velocity, which now is limited for building heights up to 200 m. Handa (2014-03-10) provided a new formula that is applicable for building heights up to 300 m.

11.3 Sizing optimisation by Lagrange Multiplier technique

The Lagrange Multiplier technique used in order to obtain an optimal stiffness distribution of the stabilising core was found to be very useful. However, the results obtained appeared to be pure theoretical, as the stiffness distribution in principle approached zero at the very top of the core. This peculiarity was not noted in the references. An example of how the theoretical results can be used in a more practical manner can be seen in Figure 10.5.

Therefore, it was concluded that the method of Lagrange Multiplier serves very well for mainly two reasons:

1. To obtain an optimal structure with respect to the volume of material in preliminary design.

2. To use this optimal structure as a reference when evaluating the efficiency of other more practically sized structures. In this way it is possible to evaluate how efficient the practical solutions are in relation to a reference optimal structure.

11.4 Economic efficiency of a structural system

In general, for a structure to be suitable for a given case it has to be economically defensible. A structure should be chosen reasonably with respect to given conditions. The total volume of material reflects the costs of the structure and hence the results obtained in this project can be used as to indicate the economic efficiency of a structure. It is not to be forgotten that there are many factors, such as production method and detailing that influence the total cost of a structural system. However, this project was limited to estimate the required volume of material, since it gives a hint of the economic efficiency of a structure. As is mentioned in the very beginning of this report, there is a major interest in minimising the required volume of material due to the environmental effects of building material has. It is important to use as little material as possible in order to minimise the carbon footprint of a structure.

Based on the assumption that a minimised volume of material gives an optimal efficiency of a structure, it is interesting to compare the results of required total volume of material for the two types of structural systems studied in this project, seen in Section 10.3.1, with the recommendations that Smith and Coull (1991) provide concerning within what ranges the structural systems are economically efficient.

As Smith and Coull (1991) suggests, the core structure with outrigger is suitable from 40 storeys. This is not a sharp boundary for whether the structure is efficient or not as this also depends on the height of each storey as well as on the radius of gyration, i.e. the geometry of the core. Altogether it depends on the slenderness of the structure.

From Figure 10.8 it is clear that in the cases studied in this project the core structure with one outrigger is almost as economically efficient as the shear wall-braced structures at a height of 43 stories, corresponding to a 150 m tall building. This is to be compared to the information provided by Smith and Coull (1991), who claim that the shear wall-braced structure is economically efficient up to 35 stories.

It is important to have in mind that there are no exact boundaries for when a structural system becomes economically efficient, but is still good to have some indications of in which range a structural system is reasonable to choose.

11.5 Optimal location of one outrigger

The optimal location of the outrigger depends mainly on how the external lateral load is applied on the structure. According to Smith and Coull (1991) the optimal location of one outrigger should be at mid-height of the structure, if the lateral load is applied as a uniformly distributed load. However, if the lateral load is applied on the structure as a trapezoidal form, which is the case in reality and also assumed in this project, the optimal location of the outrigger should be placed slightly higher than the mid-height of the structure, as shown in Section 10.6.

12 Conclusions

In the sections within this chapter conclusions are drawn with respect to the whole project. This includes conclusions from the performed study, recommendations for preliminary design of tall buildings and recommended further studies.

12.1 Conclusions from analysis of results

12.1.1 Geometry and required volume of material

The required volume of material varies for different building heights. In this project, with the specific cases studied as presented in Chapter 9, the results showed that the core structure with one outrigger is a more efficient structural system in comparison to the shear wall-braced structure. Nevertheless, it is important to keep in mind that whether the first or second structural system is the most efficient depends also on conditions set to begin with. An important factor is the slenderness of a structure, meaning the geometry that is requested. It is therefore always important for the structural engineer to be involved in the early stage of design with the architect, among others, so that strategic decisions about the structural geometry can be made early in design.

Architecture, in the sense of geometrical limitations of a structure, is the most important factor to consider regarding the efficiency of a structural system. One example of why the geometry is important for the structural efficiency is the effect of the wind load. The wind load is proportional to the width of the building and the wider a façade is, the larger is the load that must be resisted by the structure. Another example is that the maximum bending moment is proportional to the square of the height of the structure. Just a change of the geometry of the structure, by e.g. tapering, introducing a hole in the structure to allow the wind to pass through or smoothen the corners, can significantly increase the structural efficiency. The reason is that the wind load on the structure decreases by these features. Layout of the core is another important factor.

12.1.2 Maximum acceleration

The maximum acceleration for both shear wall-braced structures and core structures with one outrigger did not change for the different heights due to the fact that all cases were optimised with regard to a maximum allowed lateral deflection of H/500. As the acceleration is a function of the natural frequency, which in turn is a function of the lateral deflection, both systems are forced to deflect in the same magnitude and this resulted in no significant difference in acceleration. It is therefore concluded that in preliminary design optimisation of volume of material with regard to lateral deflection might not be sufficient, if the acceleration criterion is set high.

It is concluded that a conflict arises in optimisation of volume of material with regard to limited top lateral deflection or limited acceleration. This could be dealt with by applying stricter demands on the lateral deflection or increase damping by inserting damping devices. The issue of having a too large acceleration cannot be solved by widening the core, i.e. by increasing the radius of gyration, because this leads to a lighter structure when optimised by the Lagrange Multiplier technique.

12.1.3 Effect of location of one outrigger

In a core structure with an outrigger, larger portion of the lateral load is attracted to it in comparison to the portion lateral load that is attracted to the outrigger and the exterior columns. Hence, a stiff core reduces the efficiency of the outrigger which is not beneficial, as the main idea with an outrigger is to redirect the load to the exterior columns, so that the lateral load resisted by the core is reduced. Hence, it is a good idea to have a less stiff core and let the stiffer outriggers redirect more load. In this way a greater portion of the lateral load is redirected and resisted by the exterior columns, meaning that the load is resisted by axial action which is more efficient than when the load is resisted by flexure.

In conclusion, introducing an outrigger to a shear wall-braced structure can be a material efficient solution to reduce the top lateral deflection as well as the base moment in the core. This is due to the fact that this invokes the axial action of the perimeter columns allowing a portion of the bending moment on the core to be resisted axially by columns and, hence, reduce the base moment in the core.

A smaller core, i.e. a less wide core, leads to larger lever arm between the wall of the core and the façade columns, which the outrigger redistributes the load to. By enlarging this lever arm a greater portion of the lateral load is attracted to the façade columns. Nevertheless, it is important to keep in mind that making the core smaller reduces the shear resistance of the system. This is due to the fact that an outrigger only increases the flexural stiffness of the structure and not the shear stiffness. In conclusion, it is of interest to have a core that is not too stiff, but the optimal relation between the stiffness of the core, the stiffness of the outrigger and columns was not explored in this project.

In a case of having only one outrigger added to a core structure and for a trapezoidal wind load the optimal location of the outrigger along the height of the core is slightly above mid-height.

12.1.4 Other conclusions

The wind load increases in magnitude with the height, which can be seen in the beginning of Appendix A. This means that the higher a building is the larger load it must resist and therefore wind load is of major concern in design of tall buildings. Design of tall building is stiffness driven rather than strength driven.

Concrete is heavier than steel and is preferable for the structural system when considering the overturning moment due to wind load. This is especially of interest concerning the acceleration of a structure. The larger equivalent mass a structure possesses, the lower the acceleration becomes. For a building to be functioning properly it is preferable to have a low acceleration. Nevertheless, heavier structure can also mean larger required volume of material, higher costs and higher carbon footprint. Still it is important to note that other lighter alternatives can be more expensive, such as steel in comparison to concrete.

12.2 Recommendations

Nowadays when taller buildings are appearing more common all over the world, it is important in preliminary phases of design to investigate several alternative structural systems. It is important to not just choose a structural system based on what the designer is used to work with. Thus a more material efficient and environment friendly structural system can be achieved.

A tall building can in general be idealised as a cantilever beam and in order to make it efficient with regard to maximum lateral deflection, shear deformations should be avoided to the greatest possible extent. This means that structural systems that possess a tendency to deform in shear should be down prioritised, when a structural system is to be chosen for a tall building.

As the efficiency of tall buildings is greatly influenced by the chosen geometries, it is of great importance that structural engineers are involved in early stage design of tall building structures. This means that the structural engineer must collaborate with architects, among other professions, in order to achieve a successful final product.

Tall buildings are often "dynamic" in their nature as they are slender, meaning that their dynamic behaviour must be taken into consideration, and therefore it is of importance to handle the wind load correctly in design of tall buildings. It is not very clear neither easy to handle wind load correctly according to Eurocode 1 especially when dynamic effects should be considered.

It is important to compile the wind load that Eurocode 1 suggests and not to just take a simple case of uniformly distributed wind load that might be too large just to be conservative. This is of importance since large structures as tall building require large amount of construction material and, hence, it is of importance to catch the realistic effects as much as possible. In this way the volume of material is utilised better and the whole building can be considered as more efficient and, hence, more environment friendly.

The Lagrange Multiplier technique serves well as an optimiser to minimise the total volume of material with respect to a maximum allowed lateral deflection. However, the approach of constraining the optimisation with respect to only the maximum lateral deflection and hence minimising the volume of material increases the acceleration. This is not beneficial, since acceleration demands are just as important as the lateral deflection in order to ensure the occupants' comfort. The solution is to apply multiple constraints in the Lagrange Multiplier technique. Nevertheless, the volume distribution that this method provides, with only a single constraint of maximum lateral deflection, still gives a hint of how the material should be distributed in order to achieve an optimally sized structure.

12.3 Suggestions for further studies

In this section suggestions are made for further studies.

- The Lagrange Multiplier technique that is presented in this project was based on constant modulus of elasticity. This is useful for structures that have constant modulus of elasticity such as steel structures or uncracked concrete structures. In reality, however, concrete structures almost always crack and it is therefore of great interest to develop the Lagrange Multiplier technique to take into account different modulus of elasticity at the same time.

- A further and perhaps the most interesting development concerning the subject of sizing optimisation is to include several constraints in the Lagrange Multiplier technique. For the case of optimising with regard to the serviceability limit state it would be interesting to also have acceleration as a constraint in addition to the lateral top deflection constraint.
- In preliminary design of tall buildings it is common to neglect the stiffness from non-structural elements. This is indeed on the safe side, but in reality the non-structural elements provide some stiffness to the whole structure. It would be beneficial to take into account this additional stiffness in order to study how the sizing optimisation procedure would be affected.
- Sizing optimisation is one of three important classes within optimisation theory. The other two are shape and topology optimisation, which also play a significant role in the design of tall buildings. Therefore, it would be interesting to carry out a study that includes these two types optimisation.
- Damping of structures is very important factor to consider concerning acceleration. Structural damping has been taken into account in this project. However, the effect of aerodynamic damping and damping due to special devices such as tuned mass dampers would be good to account for as they contribute to decrease the acceleration of a building.
- The effect of torsion is important to take into account in the design of tall buildings. Torsion results in a torsional movement of the building, which in turn results in a torsional displacement in addition to the lateral displacement. It would be interesting to study this effect in serviceability limit state.
- Core structure with one outrigger has been modelled with respect to the fact that the bending stiffness of the outriggers was unchanged. However, the relation between the stiffnesses of the core, outrigger and the perimeter columns is important to study in detail. This is due to the fact that the redirection of the load at the location of the outrigger depends on the relationship between the stiffnesses of these three structural parts. Therefore, it would be interesting to gain more knowledge about what is the optimal stiffness relationship between these three.

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13.3 Interview

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Peric N. (2014-04-10): Architect in the Structural Engineering firm VBK, Gothenburg. Interview, April 2014.

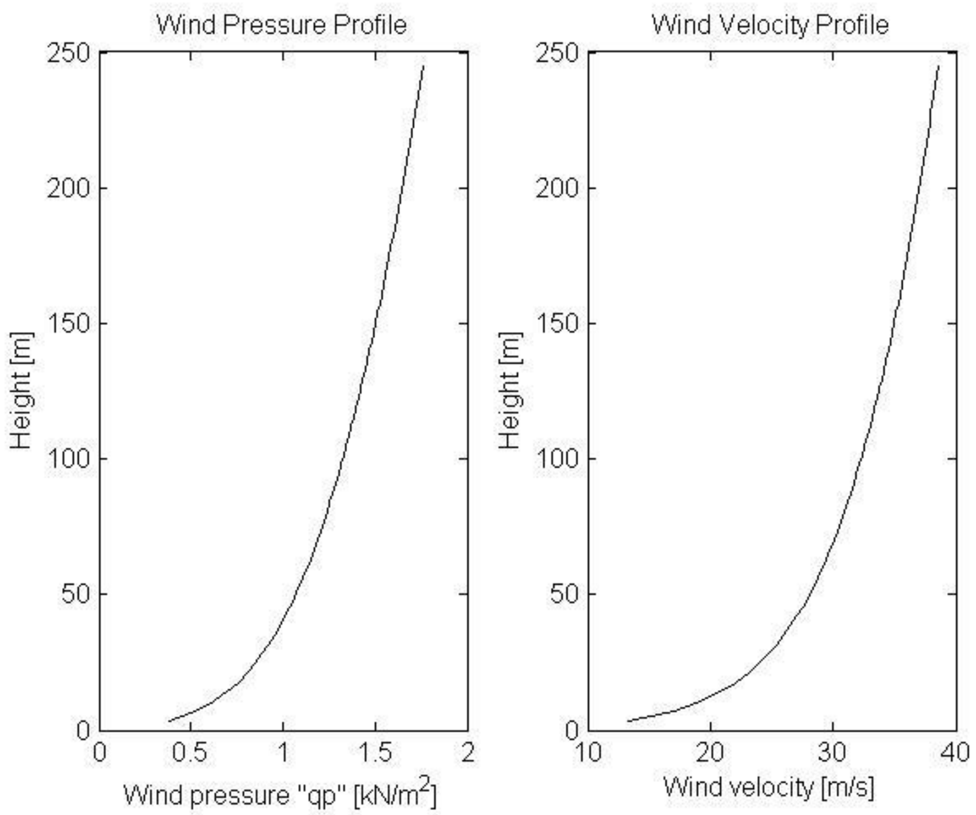
Appendix A : Matlab code

MAIN PROGRAM

```
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % %
% Program written by Ashna Zangana & Nawar Merza.
% Master Thesis, Chalmers University of Technology (2014)

% This program is built to optimise the bracing core of
% tall buildings. The acceleration and vortex shedding of
% the optimised structure is checked afterwards.
% Input data is set by the user. This main program is
% supported by several function files:
% - windload
% - normalcolumn
% - normalbracing
% - cracks
% - two_regiondefl
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % %

clc ;
clear all ;
close all ;



%% [INPUT DATA & GEOMETRIES]
% The study is to perform sizing-optimisation and compare
% structural systems of tall buildings with different
% heights.


nb_st = (1:70) ;                  % Number of stories.
st_h = 3.5 ;                      % Story height [m]
H = nb_st * st_h ;                % Height along the
                                  % building, including
                                  % roof [m]
H_st = (0:(nb_st(end)-1))*st_h ;  % Plus-height at each
                                  % story. [m]



%%
% *Geometry*

b_min = 0.2;                % Minimum allowed thickness
                            % of a shear wall [m]
% % b_max = 1.0;              % Maximum allowed thickness
% %                           % of a shear wall [m]
```

```
% Cross section of square core, from center lines [m]
Bcore = 12 ;                 % Width of core (square) [m]
B = 36 ;                     % Width of square building[m]
op = 3 ;                     % Width of opening of
                             % core to surrounding [m]
h = (Bcore-op)/2 ;           % Length of short walls [m]
htot = 2*Bcore+8*h ;         % Total length (sum) of all
                             % shear walls [m]
di = 4.3 ;                   % Distance between internal
                             % walls of core [m]

% Radius of gyration of bracing system [m]
i = sqrt((1/(htot))*(2*(Bcore^3/12+2*h*(di/2)^2+...
                        2*h*(Bcore/2)^2))) ;

% Maximum allowed lateral deflection [m]
d_max = max(H)/500 ;



%% [MATERIAL DATA]
% Properties of the stabilising structure

Gamma_cE = 1.0 ;          %  SLS According to Eurocode

% Uncracked section
E = 34e9/ Gamma_cE ;      % Modulus of elasticity [Pa]
                          % Concrete C35/45
% Cracked section
Ecr = 0.4 * E  ;          % Approximate design modulus of
                          % elasticity of cracked section
                          % [Pa]. EN 1992-1-1 (Appendix H)
rho_ccrt = 25e3 ;         % Density reinforced concrete
                          % C35/45 [N/m^3]
fcm = 43e6 ;              % Mean compressive strength [Pa]
fctm = 3.2e6 ;            % Mean tensile strength [Pa]



%% [PARTIAL SAFETY FACTORS]

% Partial safety factors (ULTIMATE LIMIT STATE)
gm_G_uls = 1.35 ;         % Unfavourable permanent action
gm_Q_uls = 1.5  ;         % Unfavourable variable action

% Partial safety factors (SERVICEABILITY LIMIT STATE)
gm_G_sls = 1.0 ;          % Unfavourable permanent action
gm_Q_sls = 1.0 ;          % Unfavourable variable action
```

```
% Partial safety factors (LOAD ACTION) ;
psi_0_imp = 0.7 ;          % Imposed load, office building.
psi_0_wind = 0.3 ;         % Wind load
psi_0_snow = 0.6 ;         % Snow load



%% [HORISONTAL LOAD]
%%
% *Wind load [EN 1991-1-4:2005]*
z0 = 0.3 ;                % Terrain type III [m]
zmax = 250 ;              % zmax = 200, Eurocode. [m]
                          % Handa provide method for higher
                          % heights.
z0II = 0.05 ;             % Assuming terrain type III. [m]
zmin = 5 ;                % Terrain type III ...
                          % ... [Table 4.1 EN 1991-1-4] [m]
vb = 25 ;                 % Reference wind speed [m/s]
                          % Gothenburg
rho_air = 1.25 ;          % Density air [kg/m^3]
qb =(0.5*vb^2*rho_air) ;% Reference mean wind pressure
                          % [N/m^2]
kI = 1.0 ;                % Turbulence factor
c0 = 1.0 ;                % Topography factor assuming no
                          % significant height differences
                          % in the surrounding.


%%
% *Function file "windload" to estimate the wind load.*
[Iv,Qp,v_mean,CsCd,RR,BB,qp,kp,fL,d] = ...
            windload(z0,z0II,vb,rho_air,kI,c0,H,B,st_h);
```

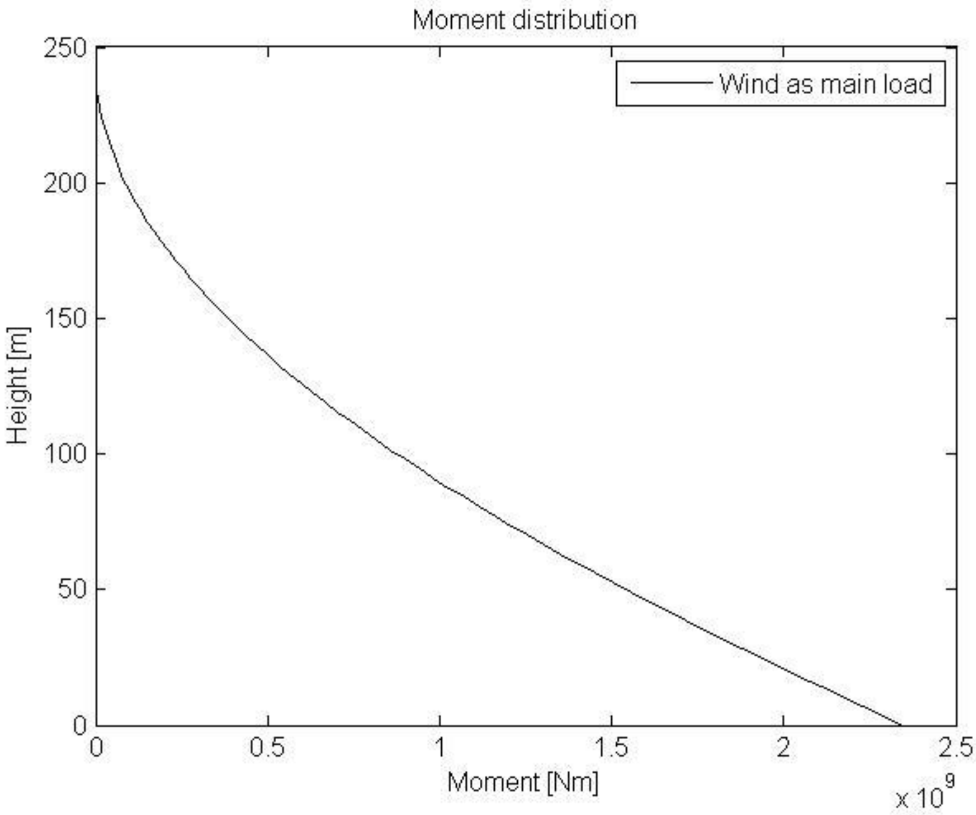



```
%%
% *SLS. Main load: WIND*
Qp_wind = gm_Q_sls * Qp * CsCd;    % [N] Concentrated load
                                   %     on each story.
qp = gm_Q_sls * qp * CsCd ;        % [N/m^2] Distributed
                                   %         load.


%%
% *Permanent load*
g_hc = 3.6e3 ;                  % [N/m^2] Hollow core slabs.
g_aff = 0.05 * rho_ccrt ;       % [N/m^2] Affusion to even
                                %         out deformed slab.
g_inwall = 0.5e3 ;              % [N/m^2] Non-structural
                                %         walls.
g_inst = 0.5e3 ;                % [N/m^2] Installations and
                                %         ceiling.
% Total permanent load [N/m^2]
% except self-weight of bracing and columns.
g_tot = g_hc + g_aff + g_inwall + g_inst ;


%%
```

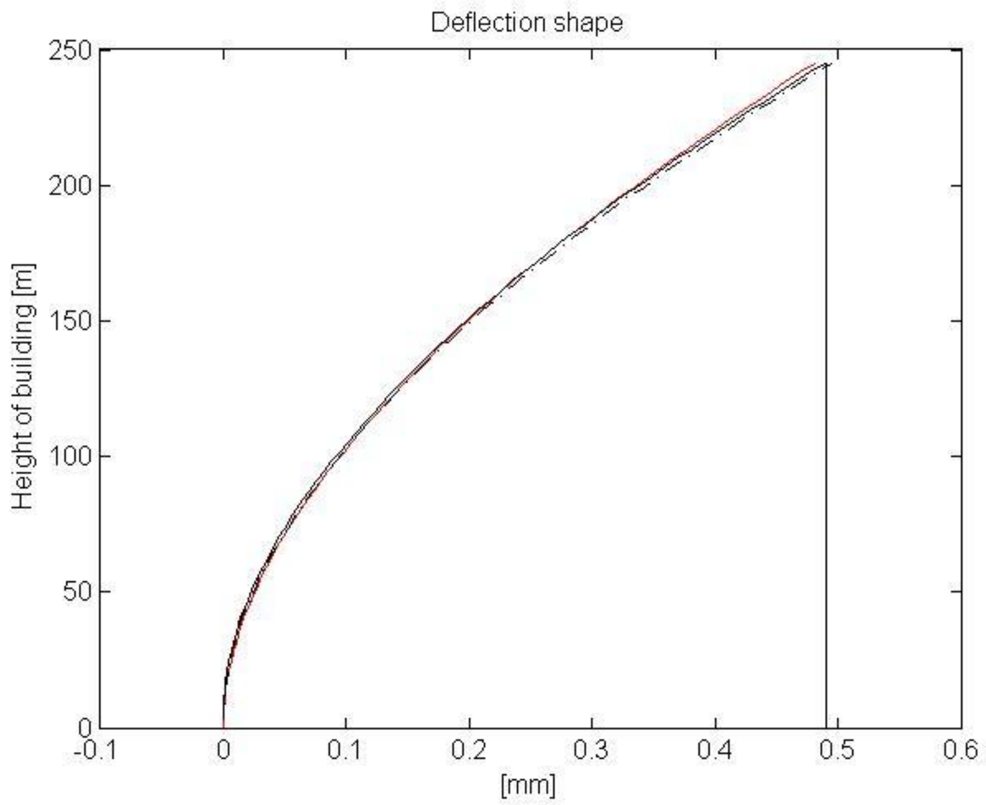
```
% *Snow load [EN 1991-1-3:2005]*
sk = 1.5e3 ;            % Reference snow load [N/m^2] Gbg
ce = 1.0 ;              % Exposure coefficient
ct = 1.0 ;              % Thermal coefficient
mi = 0.8 ;              % Shape coefficient
s =mi * ce * ct * sk ;  % Characteristic snow load [N/m^2]



%%
% *Variable loads*
% Imposed load (characteristic) in office buildings
q_imp = 2.5e3 ;                    % [N/m^2]



%% [NORMAL FORCE ON COLUMNS]

% Due to the hollow core slab system, the columns in the
% facade are differently loaded. There will be three
% differently loaded columns, meaning three
% different sizings of columns would be needed.
% Nevertheless, the total normal force on the columns at
% each story is needed , hence an assumed line load along
% the facade is calculated.

% Tributary area on columns of one side (one facade) of
% the building. [m^2]
A_fac = (B-5.12) * (B-Bcore)/2/2 ;

[N_fac_w,N_fac_i] = normalcolumn(A_fac,Qp,q_imp,...
                    psi_0_imp,g_tot,H_st,s,psi_0_snow);



%% [UNINTENDED INCLINATION]
% SS-EN 1992-1-1 (Section 5.2)

n_st_col = 28 ;                    % Number of columns at
                                   % each story.
m = (n_st_col + 1) * nb_st(end);   % Total nbr of vertical
                                   % structural members in
                                   % whole building.
                                   % (columns + 1 core).
phi_0 = 1/200 ;                  % Basic value (angle).
alpha_h = 2/sqrt(st_h) ;         % Reduction factor for
                                 % column height.

if alpha_h > 1
   alpha_h = 1 ;                 % 2/3 < alpha_h < 1
end
if alpha_h < 2/3
```

```
    alpha_h = 2/3 ;
end

alpha_m = sqrt(0.5*(1+1/m)) ;    % Reduction factor for
                                 % number of columns.
phi_i = phi_0 *alpha_h*alpha_m ;% Unintended inclination.



%%
% *Horisontal resultant due to unintended inclination*
% *SLS. Main load: WIND*
% The columns are assumed to be unbraced

A_tot = B*B ;

H_incl_w = zeros(1,length(N_fac_w));
H_incl_w(1) = phi_i * s * psi_0_snow      % Snow at roof.

for j = 1:length(H_incl_w)
    H_incl_w(j) = H_incl_w(j) + ...
             phi_i * A_tot * (q_imp * psi_0_imp + g_tot);
end

H_incl_w = flip(H_incl_w);

figure(3)
plot(H_incl_w, H_st, 'k')
legend('Wind as main load')
title('Horisontal force on Bracing system due to',...
      'Gravity load on columns')
xlabel('Horisontal force [N]')
ylabel('Height [m]')



%% [MOMENT DISTRIBUTION]
%%
% * Moment due to wind load and unintended inclination*

Horisontal = gm_Q_sls * [0 ; Qp_wind] + [H_incl_w 0]';
                                  % Partial factor already
                                  % included for H_incl_w.

% Testing program by inserting concentrated load:
% % % Horisontal(1:(end-1)) = 0;
% % % Horisontal  = Horisontal*60 ;

% Testing program by inserting evenly distributed load:
```

```
% % % for j = 1:length(Horisontal)
% % %     Horisontal(j) = Horisontal(end) ;
% % % end

hh = [0 H] ;
M_wind = zeros((length(hh)),1) ;
% Moment distribution [Nm]
for jj = 1:length(M_wind)

    n = 1:length(M_wind) ;
    n = n-jj ;
    n(1:jj) = 0 ;

    for j =1:length(M_wind)
        M_j(j) = Horisontal(j)*n(j)*st_h ;
    end
    M_wind(jj) = sum(M_j) ;
    M_j = [] ;
end
M = M_wind ;

% Plotting the moment distribution along the building
figure(4)
plot(M_wind,hh,'k')
legend('Wind as main load')
title('Moment distribution')
xlabel('Moment [Nm]')
ylabel('Height [m]')
```

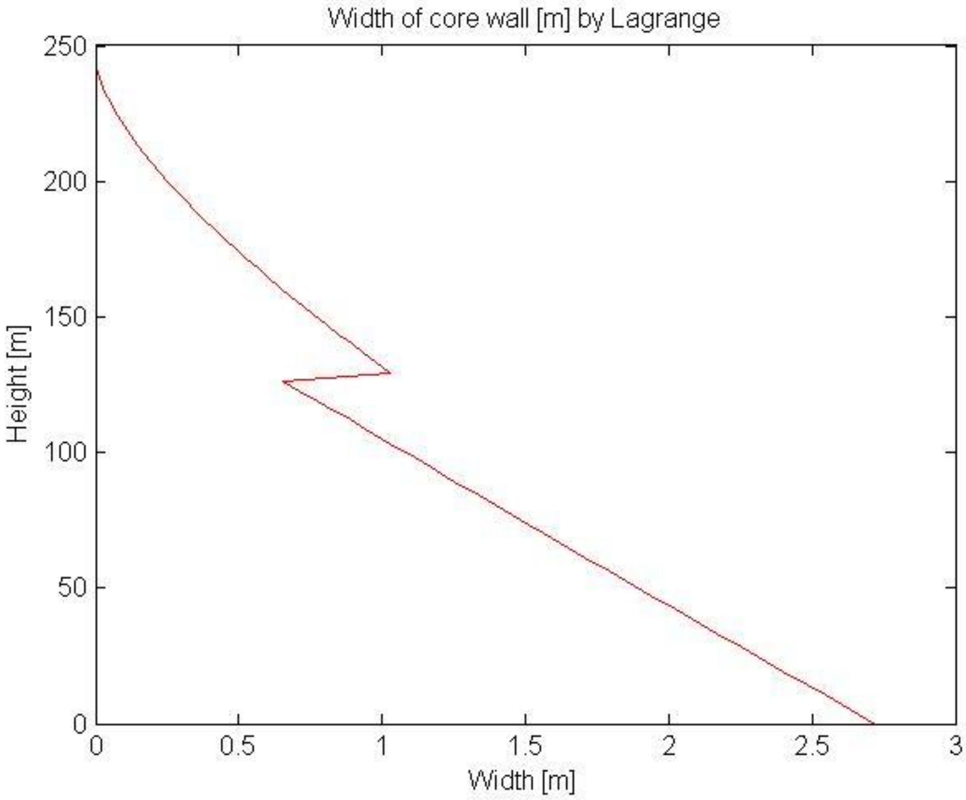

Moment distribution
Wind as main load
Height [m]
Moment [Nm]
x 10^9
250
200
150
100
50
0
0
0.5
1
1.5
2
2.5

```
%%
% *Moment and lateral deflection; core with outrigger*
% The moment distribution and the displacement of the
% whole structure are imported from an excel file.
% The values originally obtained from Frame Analysis.

M_u_out = xlsread('fourty_iteration2_OUT_2nd') ;

u_out = M_u_out(:,2)/1000000 ;              % Deflection [m]

M_out = M_u_out(:,1)*(-1) ;                 % Moment [Nm]
M_wind = M_out ;
M_wind = abs(M_wind) ;

% Plotting the deflected shape
figure(20)
plot(u_out,hh','r')
title('Deflection shape')
hold on
plot([max(H)/500 max(H)/500],[0 max(H)] ,'k')
```



```
%% [PRELIMINARY SIZING OPTIMISATION OF BRACING SYSTEM]
% Integration of curvature in the Lagrange Multiplier
% method is used to obtain optimal material distribution
% of the corealong the height of the building.


%% SETTING BOUNDARIES IN THE OPTIMISATION PROCEDURE
% The optimisation can be carried out for a case with
% restricted boundaries, meaning that the thickness of
% the walls of the core must be within a certain range.
% Furthermore, it is carried out for a load case when the
% wind load is set as the main load. This is due to the
% fact that the optimal material distribution is
% calculated with respect to the maximum allowed lateral
% deflection due to
% lateral loading. It is common for tall buildings that
% the lateral deflection is the dominating design issue.

a = 1.0 ;                   % Geometry correction factor,
                            % provided by Baker.
                            % Rectangular shapes = 1.0
                            % Other Shapes = 1.3

% Lever arm, used in Conjuagate beam method
l_arm = zeros(length(hh),1);
for j = 1:length(hh)
    l_arm(j) = hh(end)-hh(j)-st_h/2 ;
end



%%
% *SLS. Main load: WIND*
Aw = zeros(length(hh),1) ;

% Optimal cross sectional area at each story of the core,
% according to the Lagrange Multiplier technique.
for j = 1 : length(hh)
    Aw(j) = 1/(d_max*E) * sqrt(M_wind(j)*...
            l_arm(j)*a)/i * st_h/(sqrt(a)*i)*...
            (sum(sqrt(M_wind.*l_arm))-sqrt(M_wind(j)*...
            l_arm(j))) ;   % Baker
end
b_i_w = Aw / (htot) ;      % [m]

b_i1 = b_i_w ;
b_i1(end) = [] ;           % Roof story has no thickness.
```

```
%%
% *Adjusting thickness distribution to minimal thickness*
% This could be done if desired by the user. Minimal
% thickness is required due to practical issues such as
% reinforcement arrangement.
% % % b = zeros(length(hh),1) ;
% % % for j = 1:length(b)
% % %     if b_i1(j) < b_min
% % %        b(j) = b_min ;
% % %     else
% % %        b(j) = b_i1(j) ;
% % %     end
% % % end
% % %
% % % A = htot * b ;                % [m^2]


% Plot thickness development (optimisation)
figure(5)
plot(b_i1,H_st,'k')
xlabel('Width [m]')
ylabel('Height [m]')
title('Width [m] along height by Lagrange')
hold on
```

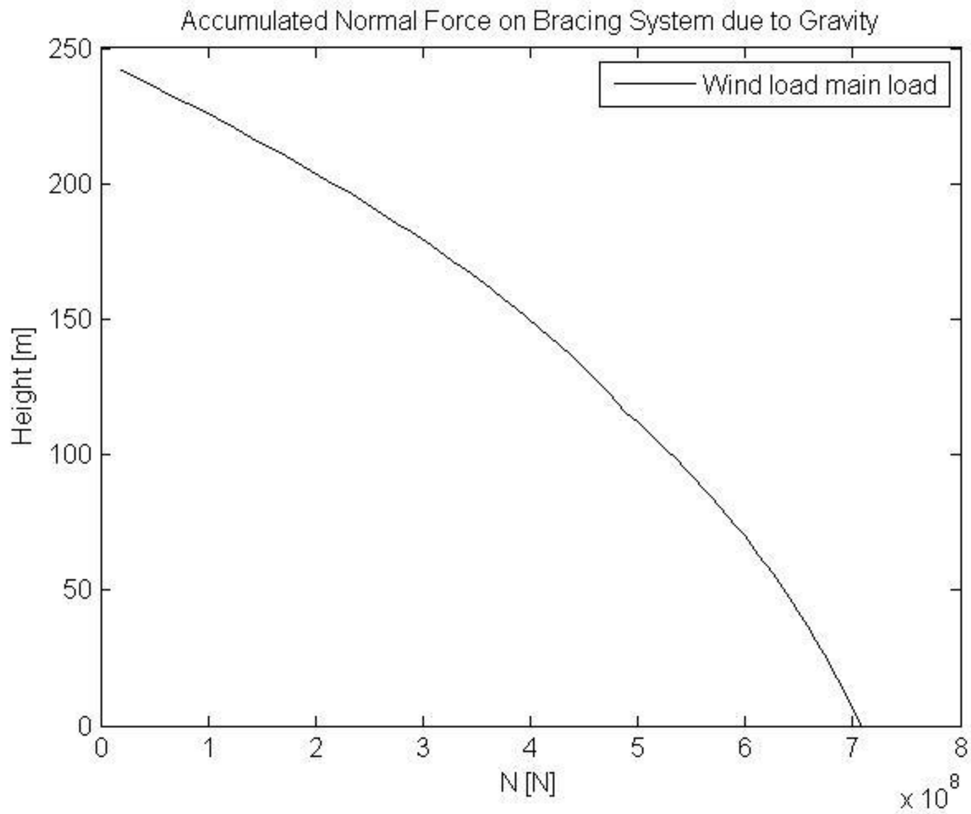



```
% % % figure(6)
% % % plot(b_i1,hh,'k')
% % % hold on
```

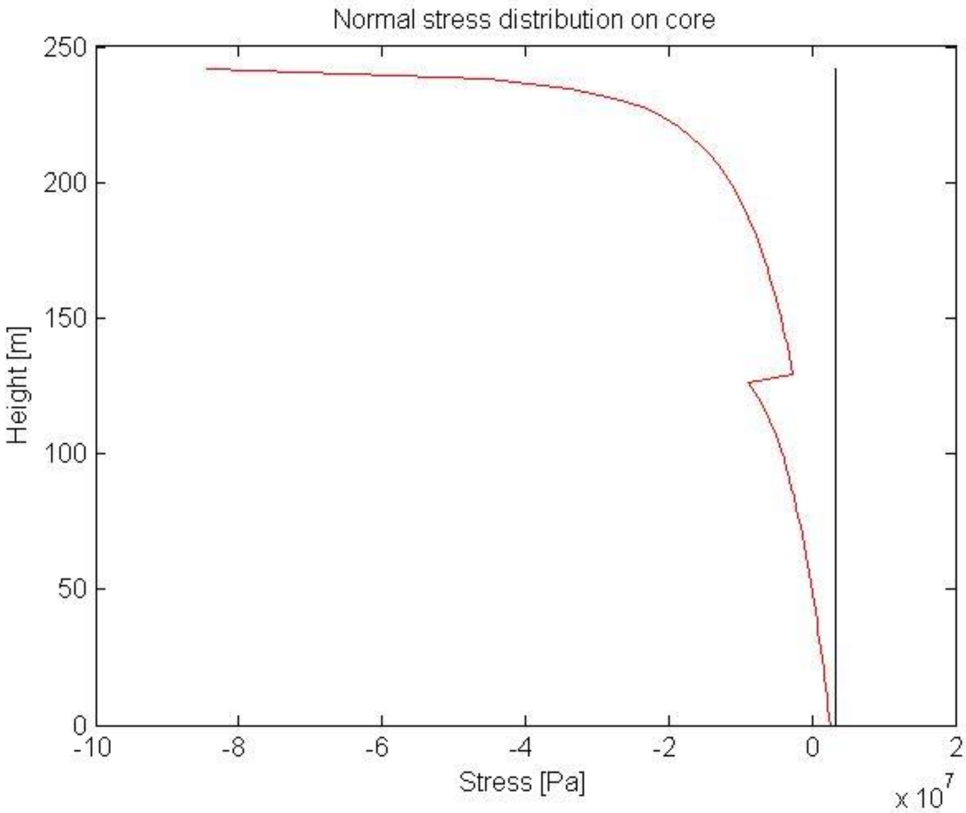
```
% % % plot(b,hh,'k-.')
% % % legend('Initial width','Adjusted width wrt BC')
% % %
% % % xlabel('Width [m]')
% % % ylabel('Height [m]')
% % % title('Width [m] along height by Lagrange,',...
% % %       'initially and adjusted to BC')
% % % hold on


defl = zeros((length(H)),1) ;
l_arm_u = zeros((length(H)),1) ;
w_u = zeros((length(H)),1) ;

Aw(end) = [] ;
EI = E*(Aw*i^2) ;

M_wind(end) = [] ;
M_flip = flip(M_wind) ;  % Flipped (prepared) for
EI_flip = flip(EI) ;     % integration of curvature.

for jj = 1:length(H)

    n = 0:length(H) ;
    n = n + 1/2 ;
    n = n - jj + 1/2 ;
    n(1:(jj-1)) = 0 ;

    for j =1:length(H)
        l_arm_u(j) = n(j)*st_h;     % "lever arm" [m]
        % Deflection by integration of curvature [m]
        w_u(j) = (M_flip(j)/EI_flip(j))...
                  *st_h * l_arm_u(j) ;
    end
    defl(jj) = sum(w_u) ;           % Sum of contributions
    w_u = [] ;                      % from each story.
end
defl = flip(defl) ;

% Plotting the deflected shape after adjusting the
% thickness distribution wrt minimal required thickness.
figure(20)
plot(defl,H,'k-.')
title('Deflection shape')
hold on
plot([max(H)/500 max(H)/500],[0 max(H)] ,'k')
```

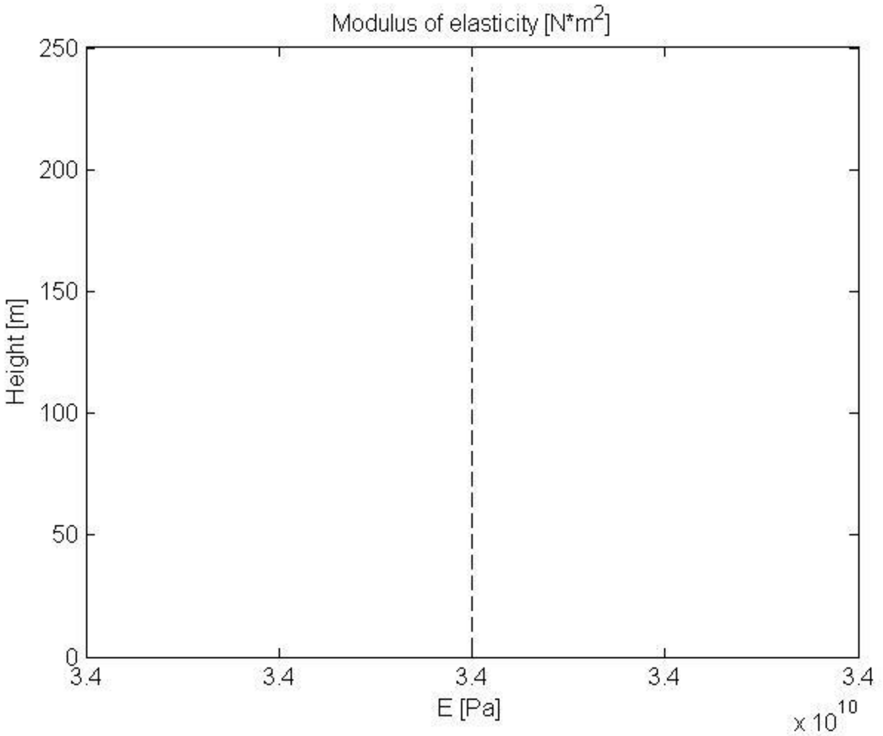
```
%% [PERMANENT (GRAVITY) LOAD]

% Permanent loads defined earlier in this main program
g_hc ;                        % [N/m^2] Hollow core slabs.
g_aff                         % [N/m^2] Affusion to even
                              %         out deformed slab.
g_inwall ;                    % [N/m^2] Non-structural
                              %         walls.
g_inst = ;                    % [N/m^2] Installations and
                              %         ceiling.
% Total permanent load [N/m^2]
% except self-weight of bracing and columns.
g_tot = g_hc + g_aff + g_inwall + g_inst ;


% Imposed load (characteristic) in office buildings,
% defined earlier in the program.
q_imp ;                       % [N/m^2]



%% [DEVELOPMENT OF MATERIAL DISTRIBUTION]
%%
% *Deflection contribution from different regions*
% As the concrete may crack it will have different
% properties in cracked regions in comparison to
% uncracked regions. In this section, the cracked regions
% are identified and considered in an iteration of the
% optimisation procedure.
lt = 1 ;                 % Initial linetype.

% If restriction of wall thicknesses is applied, the
% inactive part below must be used. Otherwise, use the
% subsequent part.
% % % findbmin = 1 ;
% % % while isempty(findbmin) == 0
% % %
% % %
% % % [ACCUMULATED GRAVITY LOAD ON BRACING SYSTEM]
% % % % Normal force distribution on the BRACING UNITS
% % % % due to gravity is calculated in this section.
% % %
% % % [N_tot_wind,N_wind,Ng_ext_w,Ng_int_w,A_trib]...
% % %       = normalbracing(B,Bcore,H_st,s,psi_0_snow,...
% % %         psi_0_imp,gm_G_sls,q_imp,g_tot,st_h,...
% % %         rho_ccrt,gm_Q_sls,h,b) ;
% % %
% % % [CRACKED REGIONS ARE IDENTIFIED]
% % % [I_cr,I_uncr,EE,crack_st,EI,sig_w,sig_comp,I,y,...
```

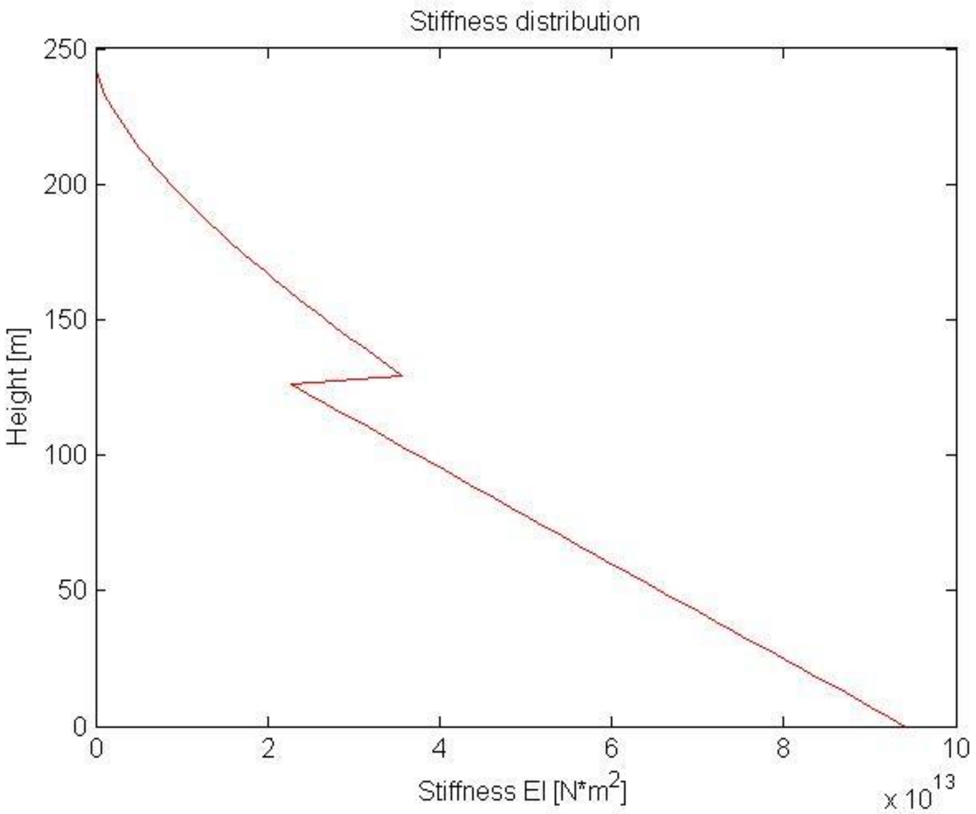
```
% % %        Acore] = cracks(i,A,Bcore,b,M_wind,fctm,...
% % %        fcm,N_tot_wind,E,Ecr,H_st,lt,op) ;
% % %
% % % [DEFLECTION OF THE CRACKED & UNCRACKED REGIONS]
% % % [b_new,b,findbmin,defl1,d_max_new,reg1,reg2,h_1...
% % %        ,h_2,l_arm1,l_arm2,M_1,M_2]=...
% % %        two_regiondefl(H,H_st,b,b_min,a,M_wind,EI,...
% % %        EE,E,Ecr,i,st_h,l_arm,crack_st,htot,lt) ;
% % %      A = b * htot ;
% % %      lt = lt + 1 ;
% % % end


%% No restriction of wall thickness:
b = b_i1 ;
A = b * htot ;

% *[ACCUMULATED GRAVITY LOAD ON BRACING SYSTEM]*
% Normal force distribution on the BRACING UNITS
% due to gravity is calculated in this section.
[N_tot_wind,N_wind,Ng_ext_w,Ng_int_w,A_trib,Ng_core,...
 Vtot_core,N_snow,N_perm,N_imp] = ...
    normalbracing(B,Bcore,H_st,s,psi_0_snow,psi_0_imp,...
                   gm_G_sls,q_imp,g_tot,st_h,rho_ccrt,...
                   gm_Q_sls,h,b) ;
```



```
% *[CRACKED REGIONS ARE IDENTIFIED]*
[I_cr,I_uncr,EE,crack_st,EI,sig_w,sig_comp,I,y,Acore,...
 crush1,crush2] = cracks(i,A,Bcore,b,M_wind,fctm,fcm,...
                        N_tot_wind,E,Ecr,H_st,lt,op) ;

% [DEFLECTION OF THE CRACKED & UNCRACKED REGIONS]
[b,findbmin,defl1,d_max_new,reg1,reg2,h_2,l_arm2,M_2]...
    = two_regiondefl(H,H_st,b,b_min,a,M_wind,EI,EE,E,...
                    Ecr,i,st_h,l_arm,crack_st,htot,lt) ;

% Plotting the new thickness distribution wrt cracked and
% uncracked regions.
figure(6)
plot(b,H_st,'r')

% Plotting the stress distribution along the height of
% the core
figure(8)
plot(sig_w,H_st,'r')

% Plotting the bending stiffness of the core
figure(9)
plot(EI,H_st,'r')
```




Modulus of elasticity [N*m2]
Height [m]
250
200
150
100
50
0
3.4
3.4
3.4
3.4
3.4
E [Pa]
x 10^10


Stiffness distribution
Height [m]
250
200
150
100
50
0
0
2
4
6
8
10
Stiffness EI [N*m2]
x 10^13

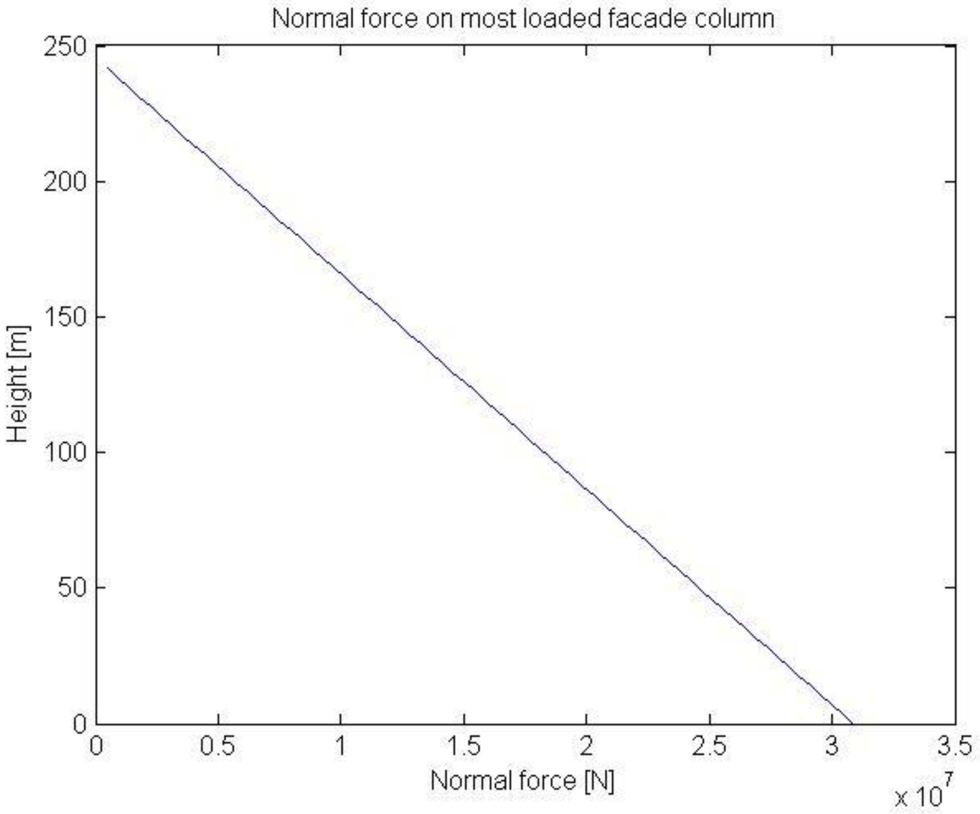
```
%%
% *New deflection is calculated wrt cracked regions*

defl_2nd = zeros((length(H)),1) ;
l_arm_u_2nd = zeros((length(H)),1) ;
w_u_2nd = zeros((length(H)),1) ;

M_flip = flip(M_wind) ;        % Flipped (prepared) for
EI_flip = flip(EI) ;           % integration of curvature.

for jj = 1:length(H)

    n = 0:length(H) ;
    n = n+1/2 ;
    n = n - jj ;
    n(1:(jj-1)) = 0 ;

    for j =1:length(H)
        l_arm_u_2nd(j) = n(j)*st_h;      % "lever arm" [m]
        % Deflection by integration of curvature
        w_u_2nd(j) = (M_flip(j)/EI_flip(j))...
                  *st_h * l_arm_u_2nd(j) ;
    end
    defl_2nd(jj) = sum(w_u_2nd) ;  % Sum of contributions
    w_u_2nd = [] ;                 % from each story.
end
defl_2nd = flip(defl_2nd) ;

% Plotting the new deflected shape wrt cracked and
% uncracked regions.
figure(20)
plot(defl_2nd,H,'k')
title('Deflection shape')
hold on
plot([max(H)/500 max(H)/500],[0 max(H)] ,'k')

%% [DEFLECTION SHAPE]

w = zeros(length(M_wind),1);
l_arm = zeros(length(M_wind),1);
for j = 1:length(H)
    l_arm(j) = H(end)-H(j)+st_h/2 ;
    w(j) = (M_wind(j)/EI(j)) *st_h * l_arm(j) ;
end
w_max=sum(w) ;



%% [ADJUSTING THICKNESS DISTRIBUTION WRT NEW DEFLECTION]
```

```
% *This section is needed if any sections are cracked.*

% When calculating the optimal material distribution with
% Lagrange Multiplier technique, the modulus of
% elasticity used is a mean value of the cracked and
% uncracked regions. This results in an final tip
% deflection not equal to H/500. In a final step the
% thickness of region two, where the method of Lagrange
% is applied, is adjusted in a loop .

% % % tipdefl = defl(end) ;        % Max deflection before
% % %                              % final adjustment.
% % % adj = 1.00001 ;              % Factor to
% % %                              % increase/decrease the
% % %                              % thickness.
% % % if tipdefl < max(H)/500      % Start value to start
% % %                              % the while-loop.
% % %     tipdefl_fin = (max(H)/500) / 1.2 ;
% % % elseif tipdefl > max(H)/500
% % %     tipdefl_fin = (max(H)/500) * 1.2 ;
% % % end
% % % iter = 0 ;                   % Keep track of number
% % % s = 1 ;                      % of iterations.
% % %
% % % b_new(end) = [];
% % %
% % % if isempty(reg2) == 0
% % %     f = reg2(end) ;
% % %
% % %     if tipdefl < max(H)/500
% % %         % If tip deflection is smaller than maximum
% % %         % allowed deflection:
% % %         while tipdefl_fin < max(H)/500
% % %             % Increase thickness:
% % %             b_new(s:f) = b_new(s:f) ./ adj ;
% % %             A = b_new*htot; % New area.
% % %             I = i^2 * A ;    % New moment of inertia
% % %             EI = EE' .* I ; % New stiffness.
% % %
% % %             w2 = zeros(length(M_wind),1);
% % %             l_arm2 = zeros(length(M_wind),1);
% % %             for j = 1:length(H)
% % %                 l_arm2(j) = H(end)-H(j)+st_h/2 ;
% % %            % Deflection by integration of curvature
% % %                 w2(j) = (M_wind(j)/EI(j)) *st_h *...
% % %                       l_arm2(j);
% % %             end
% % %             % Maximum deflection after increase of
% % %             % thickness.
% % %             tipdefl_fin = sum(w2) ;
% % %
```

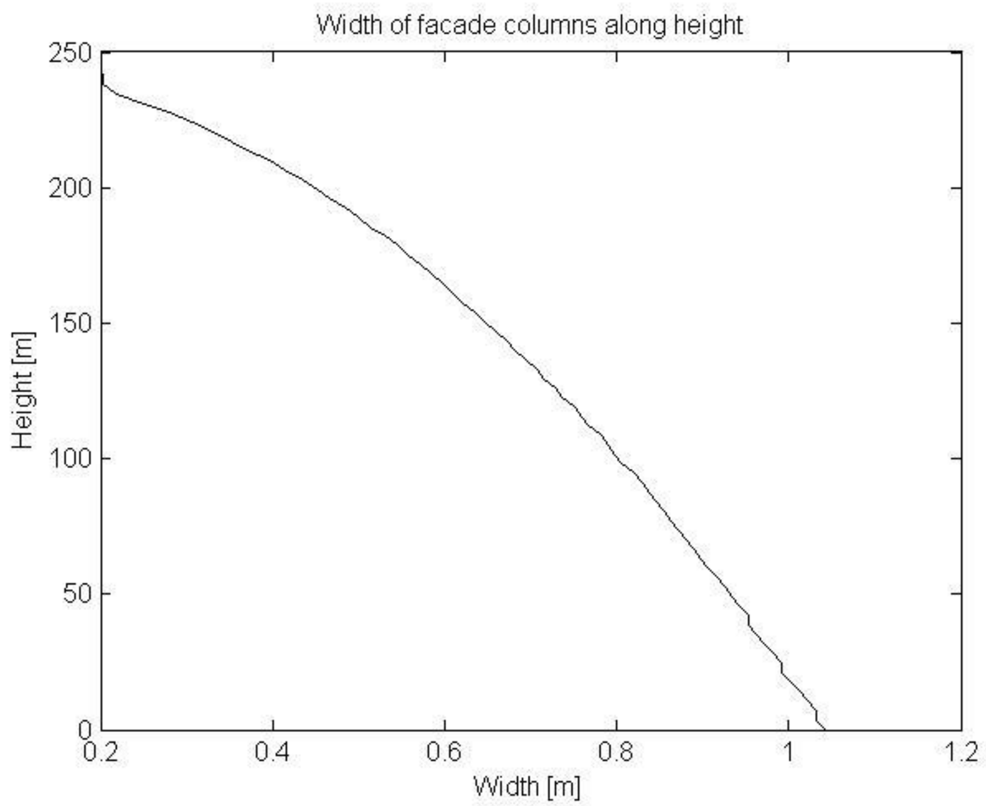
```
% % %             % Number of iterations:
% % %             iter = iter + 1 ;
% % %
% % %         end
% % %
% % %     elseif tipdefl > max(H)/500
% % %         % If tip deflection is larger than maximum
% % %           allowed deflection:
% % %         while tipdefl_fin > max(H)/500
% % %             % Increase thickness.
% % %             b_new(s:f) = b_new(s:f) .* adj ;
% % %             A = b_new*htot; % New area.
% % %             I = i^2 * A ;    % New moment of inertia
% % %             EI = EE' .* I ; % New stiffness.
% % %
% % %             w2 = zeros(length(M_wind),1);
% % %             l_arm2 = zeros(length(M_wind),1);
% % %             for j = 1:length(H)
% % %                 l_arm2(j) = H(end)-H(j)+st_h/2 ;
% % %            % Deflection by integration of curvature
% % %                 w2(j) = (M_wind(j)/EI(j)) *st_h...
% % %                         * l_arm2(j);
% % %             end
% % %             % Maximum deflection after increase of
% % %             % thickness.
% % %             tipdefl_fin = sum(w2) ;
% % %
% % %             % Number of iterations:
% % %             iter = iter + 1 ;
% % %         end
% % %
% % %
% % %         %% [FINAL DEFLECTION SHAPE]
% % %
% % %         defl = zeros((length(hh)),1) ;
% % %         l_arm_u = zeros((length(hh)),1) ;
% % %         w_u = zeros((length(hh)),1) ;
% % %
% % %         M_flip = flip(M_wind) ;
% % %         EI_flip = flip(EI) ;
% % %
% % %
% % %         for jj = 1:length(H)
% % %
% % %             n = 0:length(H) ;
% % %             n = n+1/2 ;
% % %             n = n - jj+1/2 ;
% % %             n(1:(jj-1)) = 0 ;
% % %
```

```
% % %               for j =1:length(H)
% % %                   l_arm_u(j) = n(j)*st_h;
% % %              % Deflection by integration of curvature
% % %                   w_u(j) = (M_flip(j)/EI_flip(j))...
% % %                             *st_h * l_arm_u(j) ;
% % %               end
% % %               % Sum of contributions from each story.
% % %               defl(jj) = sum(w_u) ;
% % %               w_u = [] ;
% % %           end
% % %           defl = flip(defl) ;
% % %
% % % % EI(end) = 0.000001 ;
% % %
% % %
% % %           % Plot development
% % %           figure(20)
% % %           plot(defl,hh,'k')
% % %
% % %           figure(6)
% % %           plot(b_new,H_st,'m')
% % %
% % %           figure(9)
% % %           plot(EI,H_st,'m')
% % %       elseif isempty(reg2) == 1
% % %       end
% % % end



%% [DESIGN COLUMNS, ULS. Main load: Imposed]
% Concrete columns are used since steel columns will be
% too expensive when having large normal forces as in
% this case. Furthermore it is preferable to have large
% self weight in order to resist overturning moment and
% also for acceleration criteria.

% Material properties & geometry
Ec = 0.8*E ;                % Modulus of elasticity [Pa] for
                            % axially loaded members
                            % (Smith & Coull, 1996)
fcm ;                       % Mean compressive strength [Pa]
fctm ;                      % Mean tensile strength [Pa]
gamma_c = 1.5 ;             % Partial factor, concrete ULS

st_h ;                      % Length of a column

N = ones(1,length(H_st)) ;

% Initial widths and thicknesses of column cross
% sections, which is quadratic with side b_c, at each
```

```
% story.
b_c = 0.200 * ones(1,length(N)) ;

A_c = b_c .* b_c ;          % Quadratic cross section [m^2]

% Tributary area for the most loaded column:
A_c_trib = (10.2/2 + 5.1/2) * (B-Bcore)/2/2 ;


%%
% *ULS. Main load: IMPOSED*

N_c = N * s * psi_0_snow * A_c_trib ;% Snow load
                                     % applied at roof.
for j = 1:length(N_c)                % Starting from top.
    N_c(j) = N_c(j) +...             % [N]
        j * q_imp * gm_Q_uls * A_c_trib + ...%Snow+Impose
        j * g_tot * gm_G_sls * A_c_trib ;% Permanent load
end

figure(15)
plot(flip(N_c), H_st)
title('Normal force on most loaded facade column')
xlabel('Normal force [N]')
ylabel('Height [m]')
```



```
Ic = zeros(1,length(N_c)) ;      % Predefine
NRd = N_c/2 ;                    % Initiate NbRd so that
                                 % while-loop starts.
cff = 1.01 ;                     % Initial value, changing
                                 % factor in loop below.
index = zeros(1,length(N_c)) ;   % Number of loops in
                                 % while-loop documented.
for j = 1:length(N_c) ;
    while NRd(j) < N_c(j)

          % Adjusting geometry
          b_c(j) = cff * b_c(j) ;

          A_c(j) = b_c(j) * b_c(j) ;
          Ic(j) = (b_c(j)*b_c(j)^3)/12 ; %[m^4]

          % Axial resistance of columns at each story
          NRd(j) = A_c(j)*fcm/gamma_c ;

          % Number of loops
          index(j) = index(j) + 1 ;
    end
end
b_c = flip(b_c) ;
A_c = flip(A_c) ;


figure(16)
plot(b_c, H_st,'k')
title('Thickness of facade columns along height')
xlabel('Thickness [m]')
ylabel('Height [m]')
```




```
% figure(17)
% plot(A_c, H_st,'k')
% title('Area distribution of facade columns')
% xlabel('Area [m^2]')
% ylabel('Height [m]')

%% [OUTRIGGER]
%%
% *Initial stiffness of columns below outriggers*
% Columns supporting outrigger modeled in Frame Analysis
% as springs.

E_col = 0.8 * 34e9/1.0 ;% Modulus of elasticity for
                        % columns, reduction as
                        % compressive member.
EA = E_col * A_c ;      % Column stiffness.

% st_outr is the story where the outrigger is placed.
if mod(max(nb_st),2) == 1
    st_outr = mean(nb_st) + 3 ;
else
    st_outr = ceil(mean(nb_st)) + 3 ;
end
H_out = H_st(st_outr) ;
% Mean value of stiffness of columns below an outrigger.
```

```
EA_mean = mean(EA(1:st_outr))/((st_outr-1)*st_h) ;



%% [COLUMNS SUPPORTING OUTRIGGERS]
% Additional axial force (iterate) due to outriggers:
F_out = 30764000 / 4 ; % For Each Column. Total 4 columns
                       % on each side of core activated
                       % due to outrigger + belt truss.

% Normal force on columns below an outrigger.
FN_out = N_c((end-st_outr+1):end) + F_out ;



%%
% *Dimensions of columns below outriggers*

% Initial width and thickness of column cross section,
% which is quadratic with side b_c_out.

b_c_out = 0.200 * ones(1,length(FN_out)) ;

A_c_out = b_c_out .* b_c_out ;       % Quadratic cross
                                     % section [m^2]
Ic_out = zeros(1,length(FN_out)) ;   % Predefine
NRd_out = FN_out/2 ;                 % Initiate NbRd so
                                     % while-loop starts.
cff = 1.01 ;                         % Initial value,
                                     % changing factor
                                     % in loop below.
index = zeros(1,length(FN_out)) ;    % Number of loops in
                                     % documented.
for j = 1:length(FN_out) ;
    while NRd_out(j) < FN_out(j)

          % Adjusting geometry
          b_c_out(j) = cff * b_c_out(j) ;

          A_c_out(j) = b_c_out(j) * b_c_out(j) ;
          Ic_out(j) = (b_c_out(j)*b_c_out(j)^3)/12 ;

          % Axial resistance of columns at each story
          NRd_out(j) = A_c_out(j)*fcm/gamma_c ;

          % Number of loops
          index(j) = index(j) + 1 ;
    end
end
b_c_out = flip(b_c_out) ;
```

```
A_c_out = flip(A_c_out) ;

% The inactive part below can be used if the user wants
% to increase the stiffness of the outriggers manually:

% % % %%
% % % *Adjust manually, to increase stiffness of...
% % % supports for the outriggers*
% % % b_c_out = b_c_out + 0.145 ;
% % % A_c_out = b_c_out .* b_c_out ;



%%
% *Adjusted stiffness of columns below outriggers*
% Columns supporting outrigger modeled in Frame Analysis
% as springs.

EA_2 = E_col * A_c_out ;          % Column stiffness.
EA_mean_2 = mean(EA_2(1:st_outr))/((st_outr-1)*st_h) ;



%% [VOLUME OF MATERIAL]


% Bracing core
v_core = Acore * st_h ;           % [m^3] Reinforced
                                  % concrete at each story
g_core = v_core * rho_ccrt ;      % [N]   Weight at each
                                  % story.
Vcore = sum( Acore * st_h ) ;     % [m^3] Reinforced
                                  % conrete Total.
Gcore = Vcore * rho_ccrt ;        % [N]   Weight Total.

n_st_col = n_st_col-16/2 ;


% Columns in facade
v_column = A_c * st_h *n_st_col ;% [m^3] Reinforced
                                  % concrete at each story
g_column = v_column * rho_ccrt ; % [N]   Weight at each
                                  % story.
Vcolumn = sum( v_column ) ;       % [m^3] Reinforced
                                  % conrete Total.
Gcolumn = Vcolumn * rho_ccrt ;    % [N]   Weight Total.


% Volume columns below outrigger
```

```
v_co_out = A_c_out * st_h * 16 ; % [m^3] Reinforced
                                 % concrete at each story
Vcoout = sum(v_co_out) ;         % [m^3] Reinforced
                                 % conrete Total.

% Geometry & Volume outriggers (concrete walls)
h_out = 7 ;                         % [m] Height of a
                                    % outrigger.
t_out = 0.8 ;                       % [m] Width of an
                                    % outrigger.
l_out = (B-Bcore)/2 ;               % [m] Length of an
                                    % outrigger.
V_out = 8 *h_out*t_out*(B-Bcore)/2 ;% [m^3] Volume of all
                                    % outriggers.
g_out = zeros(1,length(H_st)) ;
g_out(st_outr) = V_out*rho_ccrt;    % [N] Weight of
                                    % outriggers put at
                                    % st_outr.

% Permanent loads; hollow core slabs, affusion,
% non-structural walls & installations.
Afloor = B * B ;
g_perm = Afloor * g_tot ;      % [N] Weight at each story.
Gperm = nb_st(end) * g_perm ; % [N] Weight Total.

% Total mass at each story
mass = (g_core + g_column + g_perm + g_out)/9.81 ; % [kg]
% Total weight
Weight = Gcore + Gcolumn + Gperm + sum(g_out) ;    % [N]
% Total mass
Mass = (Gcore + Gcolumn + Gperm)/9.81 ;            % [kg]

%%
% *Volume gravity system*
V_grav = Vcolumn ;

%%
% *Volume bracing system*
V_bracing = Vcore + V_out + Vcoout ;

%% [NATURAL FREQUENCY, MORE ACCURATE]

F = Qp_wind +  H_incl_w' ;     % Horisontal concentrated
                               % load at each story.
w = zeros(1,length(F)) ;       % Static lateral deflection
                               % contribution [m] from
                               % each story.
% Vertical load on each story [N].
W = mass * 9.81;

% fn = natural frequency
```

```
fn = 1/(2*pi) * sqrt(9.81 * sum(F.*defl_2nd)/sum(W'.*...
    (defl_2nd.^2))) ;

fn = fn



%% [RESONANCE RESPONSE FACTOR R^2]
Lz = 150 ;                      % Turbulence length scale
                                % L(z) (EKS9)

yc = fn * Lz / v_mean(end) ;% Non-dimensional frequency
                                % (EKS9, 6.1.3(1))
F = 4 * yc / ...                % Power spectral density
     (1+70.8*yc^2)^(5/6) ;     % function (dimensionless).

phi_h = 1/(1+2*fn*max(H)/v_mean(end)) ;
phi_b = 1/(1+3.2*fn*B/v_mean(end)) ;

% Resonance response factor R^2, called RR here.
RR = 2 * pi * F * phi_b * phi_h / d ;



%% [ACCELERATION]

ksi = 1.0 ;                  % Concrete core + column facade.

 % 1st mode shape phi_1 (EN 1991-1-4, F.3)
phi_1 = (H_st./max(H_st)).^ksi ;


% *Equivalent Mass*
m_e = sum((mass).*phi_1.^2 * st_h) / sum(phi_1.^2.*st_h);

cf0 = 2.1 ;                  % Square building.
                             % (EN 1991-1-4 (Figure 7.23))
lambda = 1.4*max(H)/B ;
if nb_st(end) == 70          % Solid. Figure 7.36 EN 1991-1-4
    psi_lambda = 0.698 ;
elseif nb_st(end) == 60
    psi_lambda = 0.690 ;
elseif nb_st(end) == 50
    psi_lambda = 0.68 ;
elseif nb_st(end) == 43
    psi_lambda = 0.675 ;
elseif nb_st(end) == 80
    psi_lambda = 0.70 ;
end
```

```
psi_r = 1 ;                        % No rounded corners, r=0
                                   % (Fig. 7.24)
cf = cf0 * psi_lambda * psi_r ;    % Force coefficient
                                   % Eurocode equ. 7.9.
                                   % (EN 1991-1-4:2005)
                                   % (Figure 7.23)
% Maximum acceleration, see EKS9 6.3.2(1)
acc_max = kp * 3 * Iv(end) * sqrt(RR) * qp(end) * B ...
          * cf * phi_1 / (m_e) ;
max_acc = max(acc_max)



%% [VORTEX SHEDDING]

St = 0.12 ;             % Strouhal number [EN 1991-1-4 Fig.E1]
                        % for a square cross section.

v_crit = B*fn/St ; % Critical wind velocity (E.1.3.1)
if v_crit > 1.25*v_mean(end)
    ('No need for checking vortex shedding')
else
    % Frequency of vortex shedding.
    fv = v_mean(end)*St/B ;
    if abs(fv-fn)/fn > 0.05 % Taranath (2011)
        ('No risk for vortex shedding')
    else
        ('Risk for vortex shedding')
    end
end
```

WIND LOAD

```
function [Iv,Qp,v_mean,CsCd,RR,BB,qp,kp,yc,d,v] = ...
            windload(z0,z0II,vb,rho_air,kI,c0,H,B,st_h)

%% windload.m
% This function file calculates the wind load that is
% used in the main program.

%%
% *Iv = Turbulence intensity*
%%
% *v_mean = Mean wind velocity profile valid for up to
%           300 m [Handa(2014)]*
%%
% *qp = Peak velocity pressure.*

kr = 0.19 * (z0/z0II)^0.07 ;

% Predefining vectors for faster execution loops.
Iv = zeros (length(H),1) ;
qp = zeros (length(H),1) ;
v_mean = zeros (length(H),1) ;


for j = 1:length(H)
    Iv(j) = kI / (c0 * log(H(j)/z0)) ;
    v_mean(j)= vb * kr * log(H(j) / z0)...
               + 0.01 * H(j);                % 0.01*[1/s]
    qp(j) = (1 + 6*Iv(j)) * (v_mean(j))^2 ...
             * 0.5 *rho_air;                 % [N/m^2]
end

% Plot: the wind pressure along height of the building

figure(1)
subplot(1,2,1)
plot(qp/1000,H,'k')
xlabel('Wind pressure "qp" [kN/m^2]')
ylabel('Height [m]')
title('Wind Pressure Profile')
hold on

% Plot: the mean wind velocity profile
subplot(1,2,2)
plot(v_mean,H,'k');
xlabel('Wind velocity [m/s]'); ylabel('Height [m]');
title('Wind Velocity Profile')
```

```
%%
% *Wind load on each story*

% The wind load is assumed to be applied as a
% concentrated load at each story instead of a
% distributed area load. The bottom story and the roof
% story is subjected to half the wind (line)load as the
% other intermediate stories are subjected to.

Qp = zeros(length(H),1) ;
Qp(1) = qp(1)* st_h/2 * B ;
Qp(end) = qp(end)*st_h/2 *B ;


for j = 2:(length(Qp)-1)
    % Concentrated load on each story.
    Qp (j) = qp(j) * st_h * B ;       % [N]
end




%% [STRUCTURAL FACTOR CsCd]
%%
% *EN 1991-1-4 (6.3.1)*
% When calculating CsCd, Iv(Zs) is required. Zs is the
% reference height which, in this case, is equal to the
% height of the structure.

% Turbulence length scale L(z)
Lt = 300 ;                    % Reference length scale [m]
zt = 200 ;                    % Reference height [m]
alpha_w = 0.67+0.05*log(z0);
Lz = 150 ;                    % Turbulence length scale L(z)
                              % (EKS9)



%%
% *Dynamic characteristics of the structure*

% Rough initial estimation of the natural frequency of
% the structure (EN 1991-1-4, Annex F.2).
n1 = 46/max(H) ;


% Logarithmic decrement of damping, EN1991-1-4, Annex F.5
ds = 0.10 ;     % Reinforced concrete structure
                % Table F.2 (Structural damping)
```

```
dd = 0 ;          % No special devices added,
                  % such as dampers.

da = 0 ;          % Aerodynamic damping, depends on shape of
                  % the structure and similar properties.
                  % Since the facade is not adjusted to
                  % resist dynamic effects we consider da=0.

d = ds + da + dd ;           % Total logarithmic decrement
                             % of damping.
yc = n1 * Lz / v_mean(end) ;% Non-dimensional frequency
                             % (EKS9, 6.1.3(1))
F = 4 * yc / ...             % Power spectral density
     (1+70.8*yc^2)^(5/6) ;  % function (dimensionless).


phi_h = 1/(1+2*n1*max(H)/v_mean(end)) ;
phi_b = 1/(1+3.2*n1*B/v_mean(end)) ;


% Resonance response factor R^2, called RR here.
RR = 2 * pi * F * phi_b * phi_h / d ;

% Background factor B^2, called BB here.
href = 10 ;                  % Reference height 10 m.
BB = exp(-0.05*(max(H)/href)+(1-B/max(H))*...
                             (0.04+0.01*(max(H)/href)));


% Up-crossing frequency [Hz] =
% Appearent ("artificial") frequency
v = n1 * sqrt(RR)/sqrt(BB+RR) ;
if v < 0.08
   v = 0.08 ;
end


T = 600 ;                % [s] Avarage time for mean wind
                         %     velocity.

% Peak factor kp = ratio of max value of fluctuating part
% of the response to its standard deviation.
kp = sqrt(2*log(v*T)) + 0.6/(sqrt(2*log(v*T))) ;
if kp < 3
   kp = 3 ;
end

CsCd = (1+2*kp*Iv(end)*sqrt(BB + RR)) / (1 + 6*Iv(end)) ;
```

```
if CsCd < 0.85
   CsCd = 0.85 ;
end
```

NORMAL FORCE ON THE CORE DUE TO VERTICAL LOAD

```
function [N_tot_wind,N_wind,Ng_ext_w,Ng_int_w,A_trib,...
          Ng_core,Vtot_core,N_snow,N_perm,N_imp] = ...
          normalbracing(B,Bcore,H_st,s,psi_0_snow,...
                        psi_0_imp,gm_G_sls,q_imp,...
                        g_tot,st_h,rho_ccrt,gm_Q_sls,h,b)

%% normalbracing.
% Normal force distribution on the BRACING UNITS due to
% gravity is calculated in this program.

% The tributary area is for one side of the exterior
% walls of the core.
L_trib1 = Bcore + (B-Bcore)/2 ;   % Tributary length [m]
W_trib1 = Bcore + (B-Bcore)/2 ;   % Tributary width [m].
                                  % (Symmetric core)
A_trib = L_trib1 * W_trib1 ;      % Tributary area for
                                  % core [m^2] from
                                  % imposed load.


% The vector N represents the accumulated normal force
% along the height of the building. At each story j the
% normal forces on the bracing unit is summarised to a
% concentrated load N(j).
N = ones(1,length(H_st)) ;

%%
% *SLS. Main load: WIND*
% Observe:
% Normal forces calculated here are concentrated loads
% acting on one side of the core. Each one of these
% normal forces acts on each side of the square core. The
% concentrated loads are assumed to act in the center of
% each side of the core.

N_wind = N * s * psi_0_snow * A_trib ;   % Snow load
                                         % at roof.
for j = 1:length(N)
    N_wind(j) = N_wind(j) +...           % [N]
     j * q_imp * psi_0_imp * A_trib + ...% Snow+Imposed
     j * g_tot * gm_G_sls * A_trib ;     % Permanent load
end
N_wind = flip(N_wind) ;

% Accumulated self weight of exterior walls of core. A
% concentrated load for each story is calculated.
Ng_ext_w = zeros(1,length(H_st)) ;
```

```
Vext = zeros(1,length(H_st)) ;

for j = 1:length(N)
    % Vext = volume of one wall of one floor.
    Vext(j) = 2 *Bcore*b(j)*st_h ;       % 2 long exterior
                                         % walls.
    Ng_ext_w(j) = Vext(j) * rho_ccrt ;   % [N]
end
for j = 2:length(N)
    % Accumulated normal force.
    Ng_ext_w(j) =  Ng_ext_w(j-1) + Ng_ext_w(j);
end
Ng_ext_w = flip(Ng_ext_w) ;

% Accumulated self weight of interior walls of core.
Ng_int_w = zeros(1,length(H_st)) ;
Vint = zeros(1,length(H_st)) ;

for j = 1:length(N)
    % Vint = volume of one wall of one floor.
    Vint(j) = 8*h*b(j)*st_h ;            % 8 Shorter walls
    Ng_int_w(j) = Vint(j) * rho_ccrt ;   % [N]
end
for j = 2:length(N)
    % Accumulated normal force.
    Ng_int_w(j) =  Ng_int_w(j-1) + Ng_int_w(j);
end
Ng_int_w = flip(Ng_int_w) ;

Vtot_core = Vint + Vext ;

%% [ACCUMULATED VERTICAL LOAD ON BRACING SYSTEM]
% Total normal force at each story, assumed to be acting
% on the whole cross-section of the core.
N_tot_wind = N_wind + Ng_ext_w + Ng_int_w ;

% Tot self weight of core.
Ng_core = Ng_ext_w + Ng_int_w ;



% %% Load on core for Frame Analysis
% % No load combination here, since this is made in the
% % program Frame Analysis.
%
% N_snow = s * A_trib ;       % Snow load applied at roof.
% N_imp = q_imp * A_trib ;    % Imposed load at each story.
% N_perm = g_tot * A_trib ;   % Permanent load at each
%                             % story
%
```

```
%%
% Plot: Normal force, due to gravity, on BRACING SYSTEM
figure(7)
plot(N_tot_wind,H_st,'k')

legend('Wind load main load')
title('Accumulated Normal Force on Bracing System',...
      'due to Gravity')
xlabel('N [N]')
ylabel('Height [m]')
```

CRACKED AND UNCRACKED REGIONS

```
function [I_cr,I_uncr,EE,crack_st,EI,sig_w,sig_comp,...
          I,y,Acore,crush1,crush2] = ...
                   cracks(i,A,Bcore,b,M_wind,fctm,fcm,...
                          N_tot_wind,E,Ecr,H_st,lt,op)

%% cracks.m
% This function file identifies which sections that are
% cracked v.s. uncracked. Cross-sectional properties are
% provided afterwards.

%% [NORMAL STRESS]
% Assuming that the total normal force acts uniformly on
% the whole cross section of the core. In reality, the
% external walls of the core are subjected to a greater
% part of the normal force than the internal walls.
I = zeros(1,length(H_st)) ;        % Moment of inertia at
                                   % each story [m^4]
%%
% *SLS. Main load: WIND*
sig_w = zeros(1,length(H_st)) ;    % Tensile stress at
                                   % each story [Pa]
sig_comp = zeros(1,length(H_st)); % Compressive stress at
                                   % each story [Pa]
Acore = zeros(1,length(H_st)) ;    % Cross-sectional area
                                   % of core at each floor

for j = 1:length(H_st)
    y = (Bcore/1.5+b(j)/2) ;
    Acore(j) = 2*(Bcore*b(j)) + 8*((Bcore-op)/2*b(j)) ;
    I(j) = (i^2) * Acore(j) ;

    % Naviers formula
    % Tension side
    sig_w(j) = M_wind(j) / I(j) * y - ...
               N_tot_wind(j)/Acore(j); % [Pa] [N/m^2]
    % Compression side
    sig_comp(j) = M_wind(j) / I(j) * y +...
                  N_tot_wind(j)/Acore(j);
end
% The normal force is the one obtained from a load
% combination where wind load is the main load. This is
% reasonable since the vertical load is, in this case,
% favorable and, hence, the smaller one must be chosen.



%%
% *Check if any section is crushed (sigma > fcm)*
```

```
crush1 = find(sig_w <= -fcm) ;   % Find storeys where the
                                 % stress is larger than
                                 % fcm. (Concrete crushed)
crush2 = find(sig_comp >= fcm) ;

if isempty(crush1) == 0
   error('Concrete crushed')
end
if isempty(crush2) == 0
   error('Concrete crushed')
end
%%
% *Iteration wrt cracked and uncracked regions*

% Sort out cracked elements.
sig_t = zeros(1,length(sig_w)) ;
for j = 1 : length(sig_w)
    if sig_w(j) >= fctm
       sig_t(j) = sig_w(j) ;
    else
       sig_t(j) = 0 ;
    end
end

sig_t = find(sig_t);
crack_st = max(sig_t) ;      % Up to story "crack_st", a
                             % reduced modulus of
                             % elasticity must be used
                             % since these stories are
                             % cracked.

%%
% *Vector EI: the stiffness distribution along height.*
EI = ones(1,length(b)) ;
EE = ones(1,length(b)) ;
I_cr = ones(1,crack_st) ;
I_uncr = ones(1,(length(b)-crack_st)) ;

E ;                         % Modulus of elasticity C35/45
Ecr ;                       % Reduced E due to cracking.
                            % EN 1992-1-1 (Appendix H)
if isempty(crack_st) == 0
    % Cracked region:
    for j = 1:crack_st
        I_cr(j) = i^2 * Acore(j) ;
        EE(j) = Ecr ;
        EI(j) = Ecr * I_cr(j) ;
    end
```

```
    % Uncracked region
    for j = (crack_st+1):length(b)
        I_uncr(j-crack_st) = i^2 * Acore(j) ;
        EE(j) = E ;
        EI(j) = E * I_uncr(j-crack_st) ;
    end
else
    I = i^2*Acore ;
    EE = E*EE ;
    EI = EE.*I ;
end



%%
% *Plot: The maximum  tensile stress at each story*
% Different linetypes for each new adjusted distribution
% (each loop).
if      lt == 1 || lt == 4 || lt == 7
        line = 'k--';
elseif  lt == 2 || lt == 5 || lt == 8
        line = 'k:' ;
elseif  lt == 3 || lt == 6
        line = 'k-*';
else
        line = 'm' ;
end
figure(8)
plot(sig_w,H_st,line)
hold on
xlabel('Stress [Pa]')
ylabel('Height [m]')
title('Normal stress distribution')
hold on

fctm  ;                     % Tensile strength C35/45
plot([fctm fctm],[0 H_st(end)],'k')

figure(9)
plot(EI,H_st, line)
xlabel('Stiffness EI [N*m^2]')
ylabel('Height [m]')
title('Stiffness distribution')
hold on

figure(10)
plot(EE,H_st, line)
xlabel('E [Pa]')
ylabel('Height [m]')
title('Modulus of elasticity [N*m^2]')
hold on
```

PROPERTIES AND DEFLECTION OF THREE REGIONS

```
function [b_new,findbmin,defl1,d_max_new,reg1,reg2,...
          h_2,l_arm2,M_2]=...
          two_regiondefl(H,H_st,b,b_min,a,M_wind,EI,EE...
                        ,E,Ecr,i,st_h,l_arm,crack_st,...
                         htot,lt)
%%
% *Deflection contribution from different regions*

% reg1 = Upper region where b = b_min.
% reg2 = Region between reg1 and reg3 where
% b_min < b < b_max mm.

% reg2 will now be adjusted so that the stiffnesses from
% reg1 is taken into account with respect to the boundary
% of b_min thicknesses.

% defl1 = deflection contribution from region 1.
% defl2 = deflection contribution from region 2.

% defl1 + defl2 = d_max
% The total deflection, consisting of contributions from
% the two different regions, must be equal to the maximum
% allowed deflection. This gives an optimal material
% distribution with respect to a boundary condition.

hh = [0 H] ;

%%
% *Deflection contribution from region 1:*
reg1 = find(b == b_min) ;
if isempty(reg1) == 0
    b_1 = b(reg1(1):reg1(end)) ;       % Thickness
                                       % distribution of
                                       % reg1.
    h_1 = hh((reg1(1)):(reg1(end))) ; % Heights of reg1.
    M_1 = M_wind(reg1(1):reg1(end)) ; % Moment
                                       % distribution in
                                       % reg1.
    EI = [EI 0.00001] ;
    EI_1 = EI(reg1(1):reg1(end)) ;     % Stiffness
                                       % distribution of
                                       % reg1.
    l_arm1 = zeros(1,length(reg1)) ;   % Lever arm


    defl1 = zeros(1,length(reg1)) ;    % Predefine vector.

    M_1 = M_1' ;
    for j = 1:length(reg1)
        l_arm1(j) = h_1(end)-h_1(j)+st_h/2 ;
```

```
        % Deflection by integration of curvature
        defl1(j) = M_1(j)/EI_1(j) * st_h * l_arm1(j) ;
    end

    % Sum of deflection arising in region 1:
    defl1 = sum(defl1) ;


    if defl1 > max(H)/500
        error(['Maximum deflection already exceeded',...
            ' by the sum of deflection contribution',...
            ' of region 1. Either change b_min or',...
            ' consider system not working.'])
    end
else
    defl1 = 0 ;
end

% Region 2 must contibute with a maximum deflection of
% magniture d_max_new.
d_max_new = max(H)/500 - defl1 ;

%%
% *Region 2 designed by Lagrange*
if isempty(reg1) == 0
    reg2 = 1:(min(reg1)-1) ;
else
    reg2 = 1:length(b) ;
end

if isempty(reg2) == 0

    h_2 = H((reg2(1)):(reg2(end))) ;        % Heights of
                                            % reg2.
    l_arm2 = l_arm((reg2(1)):(reg2(end))); % Lever arm.
    M_2 = M_wind(reg2(1):reg2(end)) ;       % Moment
                                            % distribution
                                            % in reg2.
    if numel(reg2) > 1
        EI_2 = EI(reg2(2):reg2(end)) ;      % Stiffness
                                            % distribution
                                            % of reg2.
    elseif numel(reg2) == 1
        EI_2 = EI(reg2) ;
    end
    %%
    % *SLS. Main load: WIND*
    A2 = zeros(length(reg2),1) ;
```

```
    % Modulus of elasticity for region 2.
    E2 = E * ones(1,length(reg2)) ;

    for j = 1 : crack_st
        E2(j) = Ecr ;
    end


% Nevertheless, the method for optimising region 2
% (method of Lagrange) requires that the modulus of
% elasticity is constant in theregion. In this project we
% choose to assume a mean value of the different modulus
% of elasticities for the different storys in region 2.
    E2 = mean(E2) ;

    for j = 1 : length(reg2)
        A2(j) = 1/(d_max_new*E2) * sqrt(M_2(j)*...
               l_arm2(j)*a)/i * st_h/(sqrt(a)*i)*...
               (sum(sqrt(M_2.*l_arm2))-...
               sqrt(M_2(j)*l_arm2(j))) ;
    end
        b_2 = A2 / (htot) ;          % [m]

end


if  isempty(reg2) == 0 && isempty(reg1) == 0
    b_new = [ b_2 ; b_1] ;
elseif isempty(reg2) == 1 && isempty(reg1) == 0
    b_new =  b_1 ;
elseif isempty(reg2) == 0 && isempty(reg1) == 1
    b_new = b_2 ;
end

% Plot: New thickness distribution with different
% linetypes for each new adjusted distribution
% (each loop).
if      lt == 1 || lt == 4 || lt == 7
        line = 'k--';
elseif  lt == 2 || lt == 5 || lt == 8
        line = 'k:' ;
elseif  lt == 3 || lt == 6
        line = 'k-*';
else
        line = 'm' ;
end
figure(6)
plot(b_new, H_st, line)
title('New thickness distribution')

findbmin = find(b_new < b_min) ;
```

```
% % findbmax = find(b_new > b_max) ;

%%
% *Adjusting the thicknesses to min. & max. thickness*
% Required due to practical issues (reinforcement,
% architectural considerations etc.).
% % %
% % % Initial thickness, second loop.
% % % b_adj = zeros(length(M_wind),1) ;
% % % for j = 1:length(b)
% % %     if b_new(j) < b_min
% % %        b_adj(j) = b_min ;
% % %     else
% % %        b_adj(j) = b_new(j) ;
% % %     end
% % % end
% % % for j = 1:length(b)
% % %     if b_new(j) > b_max
% % %        b_adj(j) = b_max ;
% % %     end
% % % end

% Required area of bracing unit at each story.
A = htot * b_new ;       % [m^2]
```

NORMAL FORCE ON COLUMNS DUE TO GRAVITY LOAD

```
function [N_fac_w,N_fac_i] = normalcolumn(A_fac,Qp,...
                             q_imp,psi_0_imp,...
                             g_tot,H_st,s,psi_0_snow)
%% normalcolumn.m
% This function file calculates the normal force that the
% facade columns are subjected to due to vertical load.

N = ones(1,length(H_st)) ;
N_fac = N * s * psi_0_snow * A_fac;  % Snow load at roof.

%%
% *SLS. Main load: WIND*

% Summarised concentrated laod [kN] for columns on each
% side, on each story.
N_fac_w = (q_imp * psi_0_imp + g_tot )* A_fac * N ; % [N]
N_fac_w = N_fac_w + N_fac ;


% Accumulating normal force on columns. Observe; one side
% of building (one side of facade), summarised for all
% columns on the side in mind.
for j = 2:length(N)
    % Accumulated normal force.
    N_fac_w(j) =  N_fac_w(j-1) + N_fac_w(j);
end


% Plot: The normal force from vertical load on COLUMNS
figure(2)
plot(N_fac_w,updown,'k')
legend('Wind load main load')
title('Normal Force on Columns due to Gravity')
xlabel('N [N]')
ylabel('Height [m]')
```