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Optical signal to noise ratio improvement through unbalanced noise beating in phase-sensitive parametric amplifiers

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Abstract: We investigate the beating of signal and idler waves, which have imbalanced signal to noise ratios, in a phase-sensitive parametric amplifier. Imbalanced signal to noise ratios are achieved in two ways; first by imbalanced noise loading; second by varying idler to signal input power ratio. In the case of imbalanced noise loading the phase-sensitive amplifier improved the signal to noise ratio from 3 to 6 dB, and in the case of varying idler to signal input power ratio, the signal to noise ratio improved from 3 to in excess of 20 dB.

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References and links

1. Introduction

Phase sensitive amplifiers (PSAs) have the capability to provide the ideal optical amplifier noise figure (NF) of 0 dB [1]. This implies that under ideal conditions the input and output signal to noise ratios (SNRs) of a PSA are equal. Since in an optical communication link, the transmission reach is primarily constrained by the signal SNR [2], PSAs have attracted much attention recently. On the contrary in a phase insensitive amplifier (PIA) such as Erbium-doped fiber amplifier (EDFA), Raman amplifier (RA) etc. signal SNR is degraded by a minimum of 3 dB due to PIA’s 3-dB quantum-limited NF [3]. The fundamental principle behind PSA’s NF of 0 dB is the coherent beating of the signal with a correlated copy of itself. PSAs have been successfully demonstrated both in second order ($\chi^2$) nonlinear media and in third order ($\chi^3$) nonlinear media [4–7] and employed in wide range of applications like signal phase regeneration [8], low noise amplification [9], imaging [10] etc. In $\chi^3$ media such as optical fibers, a PSA can be realized by cascading two fiber optical parametric amplifiers (FOPAs). The first FOPA generates a correlated copy of a signal, also known as idler, through four-wave mixing (FWM) [6]. Then in the second FOPA, the signal and idler waves add coherently while uncorrelated components like stochastic noise add incoherently, leading to a gain difference between the two. The first and second FOPA are often referred to as copier and PSA, respectively. The noise from the copier however is also correlated due to the phase dependent nature of the parametric gain. This noise can be decorrelated by introducing lumped loss or a spool of single mode fiber (SMF) between the two FOPAs when the latter represents a loss-gain single-span transmission link. This loss attenuates the noise from the copier stage so that the noise from vacuum fluctuations (also known as quantum noise) dominates. This is known as a copier-loss-PSA scheme. This scheme has shown a 6-dB link NF advantage over a PI amplified link [9]. A further improvement in SNR can be achieved with increasing number of waves (modes) of the PSA. For example in [11] a 4-mode PSA showed a 12 dB SNR improvement over a PI. However in studies of PSAs as low noise amplifiers, all the waves at the PSA input are assumed to be shot noise limited, that is, the dominated noise source is from vacuum fluctuations. However, this may not be the case under some practical applications of PSAs, for example in the demonstration of PSA as an inline amplifier for multi-span experiments used PSA and erbium-doped fiber amplifier (EDFA) hybrid scheme [12]. It has been also suggested that using distributed Raman amplification (DRA) with PSAs can be a viable way to further increase the transmission reach [6]. In both previously mentioned cases, i.e., when EDFA and DRA are used in conjunction with PSA, the noise in signal and idler bands at the input of the PSA can be much higher than the quantum noise level. However this noise from either EDFA or DRA will be uncorrelated on signal and idler unlike the noise from the copier implying only half of the noise will add constructively. Recently we showed that when the noise level in the signal and the idler bands at the PSA input is significantly above the quantum noise level, then PSA provides a 3 dB optical SNR (OSNR) improvement [13]. The OSNR improvement was quantified for the level of the excess noise into the PSA. In another work PSA NF and gain performance versus the ratio of signal to idler input powers were evaluated [14]. When a PSA is used in conjunction with a PIA, its input modes can acquire imbalanced OSNR due to uneven gain and NF profiles of PIA. In this paper we complete the picture and varied both the signal and the idler OSNR at
the input of the PSA. First the signal-idler input OSNRs were unbalanced through asymmetric noise loading while keeping the signal-idler input powers equal. In this case, the OSNR improvement changes between 3 to 6 dB depending on the noise level present in signal and idler bands. Second the noise level in both bands was kept fixed and balanced while changing the signal-idler input power ratio. In that case, the OSNR improvement ranges from 3 to in excess of 20 dB. The OSNR improvement associated with imbalanced noise loading is also verified by performing bit error rate (BER) tests on a 10 Gbit/s on-off keying signal. A simple theory is formulated and matches the experimental results very well.

2. Theory

In the section we derive a simple theory for beating between waves with imbalanced OSNR. Under the large-photon-number assumption, PSAs can be modeled in a simplified manner using a semi-classical approach which uses complex amplitudes to define the optical fields of input and output waves [15]. Properties of the amplifier itself are defined via complex transfer matrix of size \(m \times m\) where \(m\) denotes the number of modes. The input-output relation of the two-mode amplifier is then defined as

\[
\begin{bmatrix}
E_{\text{out, sig}} \\
E_{\text{out, idl}}
\end{bmatrix} =
\begin{bmatrix}
\mu & \nu \\
\nu^* & \mu^*
\end{bmatrix}
\begin{bmatrix}
E_{\text{in, sig}} + n_{\text{in, sig}} \\
E_{\text{in, idl}} + n_{\text{in, idl}}
\end{bmatrix}
\]

where \(E\) denotes input and output (in/out) optical fields of signal and idler (sig/idl). \(E\) contains both deterministic and nondeterministic parts. Noise fields at the input \(n_{\text{in, sig/idl}}\) consist of two independent components

\[
n_{\text{in, sig/idl}} = n_{\text{cpl, sig/idl}} + n_{\text{vac, sig/idl}}
\]

which are the noise coupled onto each mode (also referred to as loaded noise in this study) and quantum noise originating from optical field zero-point vacuum fluctuations. The large-photon-number assumption allows the quantum noise to be treated as an additive Gaussian noise having a zero mean value, \(\langle n_{\text{vac}} \rangle = 0\), and variance of \(\langle n_{\text{vac}}^2 \rangle = hf / 2\) at a specified frequency \(f\), where \(h\) is Planck's constant and \(<>\) denotes a statistical average. The matrix coefficients satisfy the auxiliary relation \(|\mu|^2 - |\nu|^2 = 1\). They account for physical properties such as power, length and nonlinearity coefficient [16]. The pump power in the transfer matrix method is considered constant [17], i.e., there is no pump depletion. It is customary to define a PI gain as \(G = |\mu|^2\) then \(|\nu|^2 = G - 1\). The PI mode occurs when there is no idler mode at the input \(E_{\text{sig/idl}} = 0\). Assuming signal-idler pair and noise fields are mutually uncorrelated, the output powers of signal \(P_{\text{out, sig}}\) and idler \(P_{\text{out, idl}}\) using Eq. (1) are given by

\[
P_{\text{out, sig}} = GP_{\text{in, sig}} + (G-1)P_{\text{in, idl}} + 2(G-1)P_{\text{in, sig}}P_{\text{in, idl}}|\nu|^2 \cos \theta
\]

\[
P_{\text{out, idl}} = (G-1)P_{\text{in, sig}} + GP_{\text{in, idl}} + 2(G-1)P_{\text{in, sig}}P_{\text{in, idl}}|\nu|^2 \cos \theta
\]

where \(P_{\text{in, sig/idl}} = \langle |E_{\text{in, sig/idl}}|^2 \rangle\). Phase-sensitive operation is expressed through \(\theta = \phi_\mu - \phi_\nu - \phi_\sigma - \phi_{\sigma\phi}\) where \(\phi_\mu\) and \(\phi_\nu\) are phase angles of complex matrix coefficients and \(\phi_\sigma\), \(\phi_{\sigma\phi}\) are phase angles of signal and idler complex amplitudes. Since pump, signal and idler are considered ideally phase-matched then \(\theta = 0\); and we get maximum PS gain. Signal \(N_{\text{out, sig}}\) and idler \(N_{\text{out, idl}}\) noise powers are derived also from Eq. (1) (nondeterministic
part of $E$), but since we assume they are completely uncorrelated the phase-related term becomes zero.

$$N_{\text{out, sig}} = G(N_{\text{cpl, sig}} + N_{\text{vac, sig}}) + (G-1)(N_{\text{cpl, idl}} + N_{\text{vac, idl}})$$  \hspace{1cm} (5)

$$N_{\text{out, idl}} = (G-1)(N_{\text{cpl, sig}} + N_{\text{vac, sig}}) + G(N_{\text{cpl, idl}} + N_{\text{vac, idl}})$$  \hspace{1cm} (6)

All noise powers $N$ are defined as variances $N_{\text{cpl, sig}} = \langle |n_{\text{cpl, sig}}|^2 \rangle$ and $N_{\text{cpl, idl}} = \langle |n_{\text{cpl, idl}}|^2 \rangle$.

Input and output OSNR of signal or idler is then defined as

$$\text{OSNR}_{\text{in, sig/idl}} = \frac{P_{\text{in, sig/idl}}}{N_{\text{cpl, sig/idl}} + N_{\text{vac, sig/idl}}}$$  \hspace{1cm} (7)

$$\text{OSNR}_{\text{out, sig/idl}} = \frac{P_{\text{out, sig/idl}}}{N_{\text{out, sig/idl}}}$$  \hspace{1cm} (8)

If we assume high-gain ($G \gg 1$) with both input signal and idler being equal in power ($P_{\text{in, sig}} = P_{\text{in, idl}}$) and coupled noise levels being dominant (well above the quantum noise level), then the difference of the output-input OSNRs of both modes ($\Delta \text{OSNR}_{\text{sig/idl}}$) are simplified as

$$\Delta \text{OSNR}_{\text{sig/idl}} = \frac{\text{OSNR}_{\text{out, sig/idl}}}{\text{OSNR}_{\text{in, sig/idl}}} = \frac{4N_{\text{cpl, sig/idl}}}{N_{\text{cpl, sig}} + N_{\text{cpl, idl}}^2}$$  \hspace{1cm} (9)

This $\Delta \text{OSNR}$ can also be called as the NF of the optical amplifier. It can be easily seen that when the coupled noise levels of both modes are equal the OSNR at the output is 3dB higher than at the input [13]. As the noise level of one of the modes is decreased, e.g., idler, the OSNR difference of the signal ($\Delta \text{OSNR}_{\text{sig}}$) asymptotically reaches a factor of 4 (6 dB). However, based on the definitions in Eq. (7), (8) as we approach the quantum noise level with coupled noise (on both modes) the assumption of dominant coupled-noise does not hold anymore and the OSNR difference will deteriorate back to the factor of 2 (considering only signal power here). If we consider a four-mode parametric amplifier [16] and we carry out the same derivations and assumptions as with the two-mode amplifier the OSNR difference for the signal is

$$\Delta \text{OSNR}_{\text{sig}}^{\text{4-mode}} = \frac{16N_{\text{cpl, sig}}}{N_{\text{cpl, sig}} + N_{\text{cpl, idl1}} + N_{\text{cpl, idl2}} + N_{\text{cpl, idl3}}}$$  \hspace{1cm} (10)

where three other sidebands are denoted as idlers ($\text{idl1} | \text{idl2} | \text{idl3}$). In this case if all coupled noise levels are equal in power then we gain a signal OSNR difference of 4 (6 dB). On the other hand decreasing noise levels of three idler sidebands leads to an OSNR improvement of 16 (12 dB) for the signal.

3. Experiment and results

Figure 1 shows the experimental set-up. The pump seed was provided by a fixed wavelength laser source (Orbits-Lightwave) at the wavelength of 1554 nm, which was phase modulated by 2 sinusoidal tones with frequencies of 100 and 300 MHz in order to suppress the stimulated Brillouin scattering (SBS). The pump source was then boosted by an EDFA.
The signal was generated by a tunable laser source and combined with the pump using a wavelength division multiplexing (WDM) coupler. Both the pump and the signal then entered the copier and a phase conjugated copy of the signal, the idler was generated through FWM. Signal and pump power into the copier was $-10 \text{ dBm}$ and $30 \text{ dBm}$ respectively. The copier consisted of two strained Aluminum-doped highly nonlinear fibers (HNLFs) with an isolator between them [18], and provided about 10 dB of on-off gain. The total length of the HNLF was 230 m with a nonlinear coefficient of $10 \text{ W}^{-1} \text{km}^{-1}$. The average zero-dispersion wavelength (ZDWL) of the fiber was 1542 nm. After the copier, the signal-idler pair was split from the pump into two separate arms. In the signal-idler arm an optical processor (OP1, Finisar Waveshaper) was used to control temporal and phase characteristics of the signal and the idler necessary for the tuning of phase-matching condition in PSA. OP1 was also used for adjusting the desired signal-idler input power ratios at the input of the PSA. In the pump arm we used a pump recovery unit consisting of three cascaded EDFAs to boost the pump power. The signal-idler pair and the pump were recombined through a WDM and launched into the PSA. The PSA also consisted of two strained Al-doped HNLFs with an isolator in between and provided an on-off gain of 17 dB (11 dB) in PS (PI) mode. The total length of the HNLF was 300 m with a nonlinear coefficient of $10 \text{ W}^{-1} \text{km}^{-1}$. The average ZDWL of this fiber was 1539 nm. A phase locked loop (PLL) was used to keep the phase between signal, idler and pump stable against the mechanical and thermal drifts. The noise loading part consisted of a cascade of two EDFAs followed by a multi-wavelength optical filter. It was used to filter the noise in signal and idler bands only as well as to apply losses to vary signal-idler OSNR ratio at the input of the PSA. For OSNR measurements a polarizer followed by an optical spectrum analyzer (OSA) was used. The polarizer was used so that the noise only in the signal/idler polarization was measured. Figure 2 shows the ΔOSNR (OSNR at the output relative to that at the input of the PSA) for both the signal (blue circles) and the idler waves (red stars) versus the input idler – input signal OSNR difference. At point B in the Fig. 2, where the input idler – input signal OSNR difference is zero and the coupled noise level ($-45 \text{ dBm}$) is significantly higher than the quantum level ($-61 \text{ dBm} @ 0.1 \text{ nm bandwidth}$ [3]), the ΔOSNR is 2.7 dB (close to theoretical 3 dB) as also shown in [13]. The signal ΔOSNR varies from a 3 dB to a 5.5 dB as idler input OSNR is increased while keeping the signal input OSNR constant. Also, as the idler input OSNR is degraded in comparison to signal input OSNR by injecting more noise into the idler band, the signal output ΔOSNR varies from a 3 dB to a $-10 \text{ dB}$ behaving almost linearly. For the idler output ΔOSNR case as idler input OSNR is degraded the output OSNR improves from a 3 to a 6 dB. On the contrary, when idler input OSNR is improved then its output OSNR degrades from a 3 to $-10 \text{ dB}$. Solid lines in Fig. 2 were calculated based
on Eq. (9) and they depict the corresponding theoretical curves which match quite well with the experimental results.

**Fig. 2.** PSA output - input OSNR difference versus the input idler – input signal OSNR difference. Experimental points are depicted by circles (signal) and stars (idler) while solid lines are for theory using Eq. (9).

Next we study the effects of beating of waves with imbalanced OSNRs due to different signal-idler input power ratios to the PSA. Both waves had a nearly 3 dB better output OSNR when signal and idler powers were equal (each being $-27$ dBm). The idler input power at the input of the PSA was attenuated using OP1 in the signal-idler arm while keeping the signal power constant ($-27$ dBm). At the starting point (D) both waves had equal OSNR (21 dB single polarization). As the input OSNR for the idler wave was reduced the $\Delta$OSNR for the idler increased but simultaneously signal $\Delta$OSNR decreased from 3 to $-3$ dB. Using Eq. (3) and (5) the expression for signal output $\Delta$OSNR can be readily found.
as $\Delta OSNR_{\text{sig}} = 0.5(1 + P_{\text{idler}}/P_{\text{sig}} + 4(P_{\text{idler}}/P_{\text{sig}})^2)$, under the assumption that noise coupled on both waves is equal. It is clear from this equation that the signal output OSNR will improve for higher idler to signal power ratio. Even though we ignore the Raman induced noise source in our theoretical expressions they match the experimental results very well. It can be stated that in situations of having an excess noise originating from amplifiers before PSA such noise contributions can be neglected. Figure 4 shows the noise beating more visually through optical spectra of the signal taken by an optical spectrum analyzer (OSA) for three situations marked as A, B and C from Fig. 2 and for three situations marked as D, E and F from Fig. 3. At point A output OSNR is lower than the input OSNR by 5 dB, at point B output OSNR is almost 3 dB better than the input OSNR and at C the output OSNR is better by a 5 dB than the input OSNR. At point D the output OSNR is better than the input OSNR by 2.6 dB, at E the output OSNR is better than the input OSNR by 17 dB and at F the output OSNR is worse than the input OSNR by a 3 dB. As the idler power decreases towards and beyond coupled noise level, signal OSNR is deteriorated due to noise conversion into the signal band. On the other hand the idler is restored through the coherent wavelength conversion. Note that the output OSNR for the signal and the idler is always identical for all points as evident from looking at output spectra for point E and F. Therefore highly OSNR-unbalanced input waves will be balanced out after the PSA.

Fig. 4. Optical spectra corresponding to the points A, B and C in Fig. 2 and to the points D, E and F in Fig. 3. Blue dotted line is for output while red solid line is for input to the PSA.
Fig. 5. OSNR improvement versus PSA gain under unbalanced noise loading scenario. Red circles are for experimental data points and solid line is for theory from Eqs. (3-6).

The advantage of the PSA as a noiseless amplifier starts to diminish for lower gain (<10 dB) [19]. Here we have measured the OSNR improvements versus PSA gain under high and unbalanced noise loading regime. Figure 5 shows the output ΔOSNR versus the gain of the PSA. To begin with signal and idler OSNRs were unbalanced by 10 dB and PSA was set to) of input-output OSNR difference. As we reduce the gain by injecting less pump power into the PSA the improvement in OSNR starts to reduce. The theoretical curve based on Eq. (3), matches very well with the experimental data. By simplifying the equation and considering that we ignore the noise on the idler wave (as the noise on the signal is much higher) it can be easily seen that \( \Delta OSNR = 2 - 1/G + 2(1 - 1/G)^{1/2} \). Therefore to have a full advantage of the PSA it is necessary to have a minimum PSA gain of at least 20 dB.

Next we performed bit error rate (BER) tests for imbalanced noise loading. Figure 6 shows BER measurements of the signal output for four different situations: phase-insensitive (PI, black squares) and phase-sensitive with balanced noise (PS-B, blue pentagons), with
signal OSNR lower than idler OSNR by 10 dB, corresponding to pint C in Fig. 2, (PS-SigLow-IdlHigh, red stars) and with signal OSNR higher than idler OSNR by 10 dB, corresponding to pint A in Fig. 2, (PS-SigHigh-IdlLow, magenta circles). When signal and idler have balanced input OSNRs a sensitivity improvement of 2.7 dB was measured, close to value from OSNR measurements. For signal input OSNR significantly lower than idler input OSNR a sensitivity improvement of 5.8 dB was measured, again close to the measured OSNR improvement. On the other hand for signal input OSNR significantly higher than idler input OSNR a sensitivity degradation of 6 dB was measured, again close to the value from OSNR measurements.

4. Discussion

This study both theoretically and experimentally confirms that the improvement that PSA provides depends upon the input OSNR ratio of both waves (modes). Therefore it becomes important when we have PI amplifiers before PSA to choose carefully the location of the signal and the idler waves within their gain spectra. Up to now PSAs have shown inherent properties of not deteriorating signal OSNR, but here we have shown that the PSA can be efficiently used to restore a deteriorated OSNR wave.

Fig. 7. Performance of a 4-mode PSA. Signal $\Delta$OSNR vs. input idler OSNR – input signal OSNR (a); signal $\Delta$OSNR vs. input idler power – input signal power difference (b).
The working principle behind the restoration of a poor-OSNR wave is to coherently beat it with a correlated high-OSNR wave in a PSA. This is analogous to a coherent receiver where a weak signal beats with the strong local oscillator (LO). However, here the OSNR improvement takes place entirely in the optical domain. Furthermore, the OSNR improvement will increase with the number of modes. Figure 7 shows theoretical curves for the four-mode PSA case. In Fig. 7a, the input OSNR ratios are changed through different noise loading into the signal and idler bands. From Eq. (10), when $N_{\text{ap,sig}}$ is the dominant term, i.e., the noise in the signal band is much higher than the noise in the other three sidebands (idlers), signal $\Delta$OSNR is close to factor of 16 (12 dB). For the noise balanced case, meaning all four sidebands have equal noise loaded on, the OSNR improvement will be 4 (6 dB). Figure 7b shows the input and output OSNR difference with input idler and signal power difference achieved by attenuating the input powers of the sidebands. As the signal input power becomes higher than the idlers input power, signal $\Delta$OSNR decreases from 6 dB asymptotically to $-6$ dB. On the contrary when signal input power becomes lower than the idlers input power, signal output $\Delta$OSNR increases from 6 dB almost linearly with the power difference. On the other hand if the signal OSNR is much higher than the other three sidebands the output OSNR for the signal will be degraded asymptotically towards 6 dB. Generally, multi-mode PSAs can become very bandwidth-inefficient as the number of modes increases. It is considered as one of the major issues that hinders their near-future applications in real telecom systems. On the other hand, the mechanism of coherent power re-distribution among modes can find its place in systems where bandwidth resources are not as important as the reliability and the quality of received signal. Fields like free-space optics; spectroscopy or fiber optic sensors where the optical probe signal may deteriorate in OSNR due to the loss in the medium are promising candidates and can benefit from this mechanism [20, 21]. In biomedical applications a very high input SNR wave is required as the tissues can be highly absorbent [21]. For such applications PSAs can be used to restore the wave OSNR and thus relaxing the stringent requirement of high input OSNR waves. Recently in [22], a Raman amplifier was used to enhance the sensitivity of the high-speed biomedical sensors. Using PSA this type of signal wave can be restored and be used for sensing in the next medium. It is also shown that the OSNR improvement will be double to that of two mode PSAs.

5. Conclusion

We have investigated unbalanced noise as well as unbalanced signal-idler beating in a PSA and have shown the OSNR improvements for the lower input OSNR wave in excess of 20 dB. A simple theory was formulated which matches the experimental results very well. BER tests were performed to verify the OSNR measurements. This can be very useful in restoring low OSNR signals at the expense of other phase-locked sidebands.

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