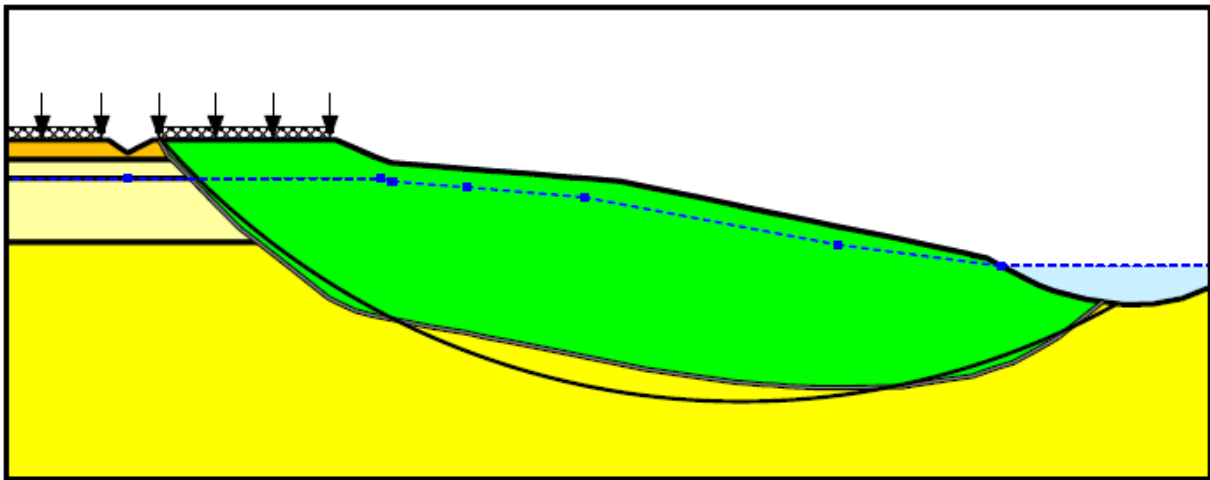




Applicability of Optimised Slip Surfaces

Evaluation of a software's optimisation function for generating composite slip surfaces, applied on stability analysis of clay slopes

Master of Science Thesis in the Master's Program Infrastructure and Environmental Engineering

JENNY GUSTAFSSON
MATILDA LINDSTRÖM

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CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2014
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Examensarbete / Institutionen för bygg- och miljöteknik,
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Illustration of a composite slip surface generated by optimising a circular slip surface. Picture
extracted from SLOPE/W.

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Abstract

The aim of this master's thesis is to investigate the applicability of the slip surface optimisation function in the software SLOPE/W, which generates composite slip surfaces through mathematical minimisation. The results obtained, when applying the Optimise function, are analysed with respect to the computed factor of safety as well as to the shape and position of the slip surface. The methodology consists of a literature survey, selection of slope models and calculations including evaluation and verification of the results. The study is limited to clay slopes with strength properties typical for western Sweden. Initially, the case of a load of the same magnitude as the cohesion, applied on a horizontal ground surface, is investigated. This is followed by the analyses of three characteristic geometries: one horizontal, one elongated and one steep slope, with the load applied on embankments. The models are altered with regard to the load and the drainage conditions. A case of a steep, layered slope with a stiff dry crust is also analysed, as well as the effect of changing selected settings for the Optimise function for all of the modelled slopes. The results are compared to the results of models created in the finite element software PLAXIS 2D. The results indicate that the Optimise function is applicable for the simple case of an elongated slope, but not for the case of a horizontal or steep slope. The conclusion is that, since the Optimise function only considers the mathematical convergence and not kinematic admissibility, the Optimise function should merely be used as a tool for investigating composite slip surfaces.

KEYWORDS: Slope stability analysis, SLOPE/W, optimised slip surfaces, composite slip surfaces, Optimise function, clay slopes

Tillämpbarhet av optimerade glidytor

Utvärdering av en programvaras optimeringsfunktion för generering av sammansatta glidytor vid stabilitetsutredning av lerslänter

Examensarbete inom masterprogrammet Infrastructure and Environmental Engineering

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Institutionen för bygg- och miljöteknik

Avdelningen för geologi och geoteknik

Chalmers tekniska högskola

Sammanfattning

Syftet med examensarbetet är att utreda tillämpbarheten av optimeringsfunktionen i programvaran SLOPE/W, vilken genererar sammansatta glidytor genom en matematisk minimeringsprocess. Resultatet som fås genom att använda optimeringsfunktionen analyseras med hänsyn till värdet på den beräknade säkerhetsfaktorn samt till glidyternas form och position. Metoden för denna studie består utav en litteraturstudie, val av släntmodeller och beräkningsmetoder, samt hur resultatet ska utvärderas och presenteras. Studien behandlar endast lerslänter med egenskaper typiska för västsvenska förhållanden. Inledningsvis undersöks en modell där ett horisontellt lerlager utsätts för en last, där lasten har samma magnitud som lerans kohesion. Därefter analyseras tre typslänter; en horisontell, en långsträckt slänt samt en brant slänt, som alla utsätts för en last som verkar på en bank. Flera olika analyser görs för typslänterna där lasten och dräneringsförhållandena varieras. Slutligen studeras en brant slänt med en hård torrskorpa. Vidare undersöks hur inställningarna för optimeringsfunktionen påverkar resultatet för samtliga slänter. Resultaten från SLOPE/W jämförs sedan med beräkningar gjorda i det finita element programmet PLAXIS 2D. Resultatet av studien visar att optimeringsfunktionen är tillämpbar på den långsträckta slänten. Slutsatsen är att eftersom optimeringsfunktionen endast tar hänsyn till matematisk konvergens och inte till kinematisk korrekta glidytor, bör optimeringsfunktionen endast användas som ett verktyg för att undersöka sammansatta glidytor.

NYCKELORD: Släntstabilitetsutredning, SLOPE/W, optimerade glidytor, sammansatta glidytor, optimeringsfunktionen, lerslänter

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Preface

We would like to thank our supervisor at Chalmers, Professor Göran Sällfors, for his encouragement and support throughout this work. We would also like to thank our supervisor at Golder Associates, Urban Högsta, for all the good advice and suggestions. Lastly, we would also like to direct many thanks to the other geotechnical engineers at Golder Associates for your help and for the pleasant working environment.

Jenny Gustafsson and Matilda Lindström
Gothenburg, June 2014

Notations

ROMAN

E	Interslice normal force	[kN]
E_Y	Young's modulus	[kPa]
F	Factor of safety	[-]
F_{circ}	Factor of safety for a circular slip surface	[-]
F_f	Factor of safety with respect to force equilibrium	[-]
F_m	Factor of safety with respect to moment equilibrium	[-]
F_{opt}	Factor of safety for an optimised slip surface	[-]
F^{JG}	Factor of safety obtained using Janbu's Generalised calculation method	[-]
F^{MP}	Factor of safety obtained using Morgenstern-Price calculation method	[-]
F^{PLAXIS}	Factor of safety obtained from PLAXIS 2D	[-]
N	Bearing capacity factor	[-]
R	Radius	[m]
X	Interslice shear force	[kN]
W	Weight of a slice	[kN]
c	Cohesion	[kPa]
c'	Apparent cohesion	[kPa]
c_u	Undrained cohesion	[kPa]
w_L	Liquid limit	[-]
q_b	Surrounding surcharge load	[kPa]

GREEK

α	Angle of a circle sector	[°]
γ	Unit weight	[kN/m ³]
ν	Poisson's ratio	[-]
ϕ	Friction angle	[°]
ϕ'	Effective friction angle	[°]
σ'	Effective stress	[kPa]
σ'_c	Preconsolidation pressure	[kPa]
τ_f	Shear strength at failure	[kPa]
τ_{fd}	Shear strength at failure, drained condition	[kPa]
τ_{fu}	Shear strength at failure, undrained condition	[kPa]
τ_{mob}	Mobilised shear stress	[kPa]
ψ	Dilatancy angle	[°]

1 Introduction

This chapter introduces the background, aim and limitations of this master's thesis study. The aim is elaborated and clarified by the objectives.

1.1 Background

The theories which modern, computer aided, slope stability analysis rely on were developed when only manual calculation methods were available. The Ordinary Method of Slices, for example, was introduced by Fellenius in 1936 (Krahn, 2003). Since then, the calculation methods have been refined and the benefits of today's modelling tools include the possibility of calculating the factor of safety for a large number of slip surfaces, and performing a large number of iterations for each of those. Finding the critical slip surface has become easier and faster, but perhaps for the price of losing the fundamental understanding of the very basic statics that these calculation methods consist of. And without this knowledge, how can we determine whether these easily produced results are reasonable or not? Are we at risk of ignoring vital limitations of these, by today's standard, crude methods in the translation process into user interface via software code?

The commercial software SLOPE/W is a commonly used computational tool for slope stability analyses, and has been on the market since 1977 (GEO-SLOPE International Ltd., 2008). With iterative calculations, using limit equilibrium equations, SLOPE/W identifies the critical slip surface and the corresponding factor of safety. Recent versions of SLOPE/W include an optimisation function which refines the critical slip surface further, leading to a lower value of the computed factor of safety (GEO-SLOPE International Ltd., 2008). As lower factors of safety result in more extensive measures to increase stability, and subsequently more expensive projects, concerns have been expressed about the optimisation function being used too casually. The question has been raised whether or not the optimised factor of safety is reliable enough, to be deemed decisive with the associated cost in mind. The idea of this thesis is therefore to investigate the limitations of the Optimise function, and for what cases it should be utilised.

1.2 Aim

The aim of this master's thesis is to investigate the applicability of the slip surface optimisation function in the software SLOPE/W. The results obtained, when applying the Optimise function, are analysed with regard to the computed factor of safety as well as to the shape and position of the slip surface.

1.3 Objectives

Two main problems with the Optimise function have been observed, and can be summarised as follows:

- The optimised slip surface derives strongly from the critical circular slip surface, with a "strange" and physically inadmissible shape.
- The optimised slip surface is very similar to the critical circular slip surface in shape and position, but with a relatively large and seemingly inexplicable difference in the value of the factor of safety.

In order to fulfil the aim, the below stated questions should be answered:

1. For which typical slope stability modelling scenarios can the Optimise function be considered reliable and for which scenarios should it be used with caution, presuming that sound geotechnical engineering judgement is applied?
2. What are the limitations of the Optimise function and how should it be used with those limitations in mind?

1.4 Limitations

This master's thesis is conducted in collaboration with the geotechnical division of the consultant company Golder Associates in Gothenburg. The aspects of slope stability analysis accounted for in the thesis are such that coincide with real stability problems within the region of western Sweden. As this is closely associated with massive clay layers, this master's thesis focuses on slopes within cohesive soils.

The extent of the thesis is primarily limited by the scenarios to be investigated. The characteristic slopes are mainly distinguished through their geometry; the height and width of the slope and the corresponding inclination. All modelled slopes are thought to have similar conditions with respect to pore water pressures and soil properties. The effect, on the result of using the Optimise function, of altering these parameters is not investigated. Therefore, the soil parameters are not changed within the analysis of a slope, and neither are the water conditions.

Although no slopes of purely frictional soil are included, and thus no strictly drained conditions, the thesis will include combined analysis. The calculations in PLAXIS 2D are limited to undrained analysis, since this is the main focus of the thesis. This means that the combined analyses of the characteristic slopes, performed in SLOPE/W, do not have any PLAXIS 2D counterparts for comparison.

Hydrostatic pore water pressure is assumed for all slopes, and the water level of open surface water is considered constant. The possibility of anisotropic strength conditions is neglected as well as the effect of tension cracks. Reinforcements as a measure for increased stability are not considered. The Mohr-Coulomb failure criteria, and thereby a perfectly plastic behaviour of the soil, is assumed.

The thesis is focused on using the Optimise function for obtaining a composite slip surface from the critical circular slip surface generated with the Grid and Radius method. The Fully Specified slip surface method is used to reproduce and recalculate optimised slip surfaces. None of the remaining slip surface generation methods available in SLOPE/W are utilised in this thesis. Likewise, in SLOPE/W, the Morgenstern-Price calculation method and Janbu's Generalised calculation method are the only calculation methods out of the available that are included in the calculations.

The theoretical background include a few additional calculation models from which Morgenstern-Price and Janbu's Generalised calculation methods were developed, although focus lies on the Morgenstern-Price calculation method. Within the Morgenstern-Price calculation method, the interslice shear stress function of half-sinus is considered exclusively.

Although modern safety philosophy is moving towards a different approach, namely that of partial safety factors, this thesis will be limited to computing and evaluating global factors of safety. This is consistent with the general practise, within industry and research, regarding stability analysis of

existing slopes. Therefore, partial factors of safety are excluded.

Ultimately, this thesis does not aim to compare SLOPE/W to other computational software available for slope stability analysis. Neither does it aim to challenge the theory or the mathematical algorithms on which the programming of the Optimise function relies, nor to compare the different Monte Carlo methods developed for slip surface optimisation. The theoretical section devoted to this subject is simply included for the purpose of investigating the inherent limitations of the mathematical formulations. This is done with the objective of evaluating the applicability of the Optimise function for the slopes selected for this thesis.

2 Method

This chapter aims to thoroughly declare the methodology used to conduct this master's thesis. The work is divided into the following phases: a literature survey combined with field and laboratory observations, selection of slopes for which to perform the calculations and finally calculations including evaluation and verification of the results.

2.1 Literature Survey

The literature survey covers the foundations of slope stability theory required for conducting this thesis. The relevant information gathered through the literature study can be found in Chapter 3. It includes soil mechanic theory regarding shear strength and the governing soil parameters, the derivation of the bearing capacity factors for different slip surface shapes, a brief summary of the relevant features of the software SLOPE/W and ultimately the necessary theory for modelling with the software PLAXIS 2D.

The literature survey is complemented with participation in field and laboratory work. Additional sources of information included in the literature survey are conversations with the supervisors of the thesis and an e-mail conversation with technical support of GEO-SLOPE International Ltd, the creator and owner of the software SLOPE/W.

2.2 Selection of Modelled Slopes

In order to demonstrate the two main problems that have previously been observed when using the Optimise function, which are stated in Chapter 1.3, five slopes are created. These slopes aim to represent frequently occurring slope stability cases, and they are simplified with regard to the intended focus of the thesis.

Initially, a bearing capacity test is conducted for a horizontal surface with an applied surcharge load of the same magnitude as the cohesive strength of the soil. This test is performed in order to investigate how the Optimise function treats a simple bearing capacity problem. The resulting factor of safety and the shape of the corresponding slip surface are compared to the bearing capacity factors for different slip surface shapes, derived in Chapter 3.3. The geometry of the slope and the calculation input data are described in Chapter 4.1.

Three characteristic slopes are created, whereof the first one has a horizontal geometry with horizontal soil layers, the second slope is an elongated slope and the third is a steep slope. The characteristic slopes consist of one or two different layers of homogeneous clay with properties that are typical for west Swedish conditions. All characteristic slopes include embankments with an applied traffic load, which in SLOPE/W is modelled as a surcharge load. Two different loading scenarios are applied to the characteristic slopes to represent road and railroad traffic respectively. A more detailed description of the characteristic slopes is presented in Chapter 4.2.

A steep slope with a stiff dry crust is included with the purpose of exemplifying a more complex case. Hydrostatic conditions are assumed for all slopes, and the elongated slope, the steep slope and the steep slope with dry crust include open surface water. The expectation is that the selected slopes, which have similar material properties and applied loads but different geometries, will give

an indication of which slope geometries it is suitable to apply the Optimise function when analysing slopes in clay soils.

2.3 Calculations

The calculations of the modelled slopes are mainly performed in SLOPE/W using the rigorous Morgenstern-Price calculation method, as this method is regularly used by consultants performing slope stability analyses. For the characteristic slopes, the slip surfaces obtained with the Morgenstern-Price calculation method are also calculated with Janbu's Generalised calculation method as comparative calculations of the results. The 2012 version of SLOPE/W is used.

Selected SLOPE/W models are recreated in PLAXIS 2D as an additional measure of comparing the SLOPE/W results. Unlike the calculations with Janbu's Generalised method, the PLAXIS 2D models are not controlled in order to recreate a specific slip surface. The 2012 version of PLAXIS 2D is used. A schematic overview of the performed calculations for each model can be seen in Table 2.1. The calculation procedures in SLOPE/W and PLAXIS 2D are explained in detail in this chapter, as well as the method for interpretation of the results.

Table 2.1: Schematic view of which scenarios that are modelled for the different slopes, in the different software.

	Bearing capacity test	Characteristic slopes	Steep slope with dry crust
SLOPE/W			
Undrained analysis, load 15 kPa			X
Undrained analysis, load 20 kPa	X	X	
Undrained analysis, load 43 kPa		X	
Combined analysis, load 20 kPa		X	
Optimise settings: starting-ending	X	X	
Optimise settings: concave angles	X	X	X
PLAXIS 2D			
Undrained analysis, load 15 kPa			X
Undrained analysis, load 20 kPa	X	X	
Undrained analysis, load 43 kPa		X	

2.3.1 Calculations in SLOPE/W

The critical circular slip surface and the corresponding factor of safety are computed in SLOPE/W using the Morgenstern-Price calculation method, described in Chapter 3.5.2. To generate the critical circular slip surface and the corresponding factor of safety F_{circ}^{MP} , the Grid and Radius method is used, which is described in Chapter 3.4.1. To ensure that the critical circular slip surface is obtained, the search area of the grid and radius are refined until the minimum obtainable value of the factors of safety is found.

The Optimise function is applied to the critical slip surface, i.e. the circular slip surface that corresponds to the lowest factor of safety. The Optimise function is activated by ticking a box in the project settings. When it is activated it automatically generates one optimised slip surface,

based on the critical circular slip surface, at the end of the iterative slip surface search. This optimisation process results in a composite slip surface shape with a corresponding value for the factor of safety, F_{opt}^{MP} , that is always lower than the value of F_{circ}^{MP} . A description of the optimisation procedure in SLOPE/W can be found in Chapter 3.6.

When performing undrained analysis in SLOPE/W, the cohesive soils are modelled with the material models $S=f(depth)$ or $S=f(datum)$. This enables specification of an increase of shear strength with depth, starting at the top of the layer or at a specified depth. The same principle is applied when performing combined analysis, in which the cohesive soils are modelled with the material model *Combined*, $S=f(depth)$ or *Combined*, $S=f(datum)$. The friction material of the embankments are modelled with the *Mohr-Coulomb* material model.

The main case investigated for the characteristic slopes is the undrained analysis of the slopes when submitted to a surcharge load of 20 kPa, as described in Chapter 4.2. Further, the characteristic slopes are investigated for undrained analysis with a surcharge load of 43 kPa and combined analysis for a surcharge load of 20 kPa which is explained in Chapter 4.2. In the calculations performed within this thesis, all surcharge loads are entered as the desired value in kPa, in the vertical direction and with the assigned the height of 1 m.

In addition, all slopes are investigated for the case of undrained analysis with an applied surcharge load of 20 kPa, but with altered Optimise settings; the number of vertices and the maximum concave angles which are described in Chapter 4.5. The purpose of these calculations is to investigate how the factor of safety and the corresponding shape of the slip surface behaves when the default values of the Optimise settings are changed within the permitted limits.

To summarise, the following types of models are created in SLOPE/W for each characteristic slope:

- **Undrained analysis, surcharge load of 20 kPa**
- **Changed optimised settings**
 - Number of starting and ending points
 - Maximum concave angles, for the driving and the resisting side
- **Undrained analysis, surcharge load of 43 kPa**
- **Combined analysis, surcharge load of 20 kPa**

Janbu's Generalised calculation method, described in Chapter 3.5.1, is used as a comparison to the Morgenstern-Price calculations that corresponds to the optimised slip surface. This factor of safety is denoted F_{opt}^{JG} . Since these are two different methods, the exact same value of the factors of safety should not be expected. To assess and account for this expected difference between the two calculation models, the factor of safety corresponding to the circular slip surface is also recalculated with Janbu's Generalised calculation method. This factor of safety is denoted F_{circ}^{JG} and is included to give an indication of the expected difference between the results of the two methods and to ensure that the computed factors of safety are within the same range regardless of which calculation method that is used.

If the exact same slope is calculated with both the Morgenstern-Price method and Janbu's Generalised method, they might not identify the same critical circular nor optimised slip surface. When using Janbu's Generalised method for calculating F_{circ}^{JG} , the position of the grid and radius are therefore specified as a single rotation point and a single radius line, i.e. the rotation point and

radius obtained from the critical slip surface of the Morgenstern-Price calculation. The characteristic slopes are the most thoroughly investigated slopes, out of the slopes included in this thesis, and the comparative calculations with Janbu's Generalised method are only performed for those. Likewise, the alternative loading case as well as combined analysis is only included for the characteristic slopes.

To enable SLOPE/W to calculate the exact same optimised slip surface as obtained with the Morgenstern-Price calculation method with Janbu's Generalised method, and not optimise a new circular slip surface, the slip surface is defined using the Fully Specified slip surface generation method which is described in Chapter 3.4.2. In cases where it is not possible to compute the factor of safety with Janbu's Generalised calculation method, i.e. the factor of safety results in an error code, the convergence tolerance for the factor of safety is decreased. The relevant error codes are explained in the results as they occur, in Chapter 5.

The notations used for the different factors of safety calculated in SLOPE/W are summarised in Table 2.2, along with the respective calculation and slip surface generation methods.

Table 2.2: Summation of how the different factors of safety are calculated and denoted, and how the corresponding slip surfaces are generated in SLOPE/W.

Calculation method	Slip surface generation method	Notation
Morgenstern-Price	Grid and Radius - critical circular slip surface	F_{circ}^{MP}
Morgenstern-Price	Grid and Radius - optimised slip surface	F_{opt}^{MP}
Janbu's Generalised	Grid and Radius - single point and radius line from F_{circ}^{MP}	F_{circ}^{JG}
Janbu's Generalised	Fully Specified - same slip surface shape as for F_{opt}^{MP}	F_{opt}^{JG}

2.3.2 Calculations in PLAXIS 2D

The recalculations in SLOPE/W with Janbu's Generalised calculation method can only be used as a comparison to the values of the factors of safety, and not the shape and position of the corresponding slip surface, as they are deliberately fixed to the slip surfaces obtained with the Morgenstern-Price calculation method. The shape and position of the slip surface for the undrained analysis of each characteristic slope for the two loading scenarios, as well as the bearing capacity test and the steep slope with a dry crust, are compared to the slip surface generated in the finite element software PLAXIS 2D.

The slip surface obtained in PLAXIS 2D serves, by the means of ocular inspection, as a verification tool. The purpose of performing the PLAXIS 2D calculations is to enable evaluation of the shape of the two slip surfaces obtained for each slope with SLOPE/W; the critical circular slip surface and the optimised slip surface. For example, if the slip surface obtained with PLAXIS 2D is more similar to the optimised slip surface in SLOPE/W than to the circular slip surface, the calculation for verification will be considered as an indication that the Optimise function is applicable when modelling this specific case using SLOPE/W. The values of the factor of safety obtained with PLAXIS 2D are used as an additional comparison of the factor of safety obtained with SLOPE/W.

PLAXIS 2D models are not created for each case modelled in SLOPE/W. Focus lies on the characteristic slopes and only undrained analyses are performed in PLAXIS 2D. The remaining slopes are modelled once in PLAXIS 2D, as they are only subjected to one loading case each.

To summarise, the following types of models are created in PLAXIS 2D for each characteristic slope:

- **Undrained analysis, surcharge load of 20 kPa**
- **Undrained analysis, surcharge load of 43 kPa**

In PLAXIS 2D, the slopes are modelled in using the *Classical mode* which is the default calculation mode. The slopes are defined with the same geometry and soil properties as in SLOPE/W, and the material model *Mohr-Coulomb*, based on the Mohr-Coulomb failure criterion described in Chapter 3.2, is used for all soils. Drainage conditions needs to be defined for each soil. The clays are modelled as undrained using the drainage condition *Undrained(C)*, whereas embankments are modelled as drained using drainage condition *Drained*. The material model Mohr-Coulomb in PLAXIS 2D requires values for Young's modulus, E , Poisson's ratio, ν and dilatancy angle, ψ . The values of these parameters are assumed by consulting literature.

The input parameters are presented in Chapter 3.7. The mesh is consistently set to the default coarseness; medium, and no refinement is made. The calculations are performed in three phases; *Gravity loading*, *Plastic* and *Safety*, described in Chapter 3.7. The factor of safety obtained with PLAXIS 2D is denoted F^{PLAXIS} .

2.3.3 Interpretation of the Results

The values of the factor of safety, in the calculations result, are not to be evaluated with regard to the demands set by the Swedish Commission of Slope Stability. Instead, focus lies on the magnitude of difference between the values of the factors of safety corresponding to the circular and the optimised slip surfaces for each case of each slope. However, the demands provide some guidelines for what magnitude of differences that should be accounted for when calculating a global factor of safety with regard to slope stability. The range of the required factors of safety, stated in Chapter 3.1.2, may also give some direction of what differences that are to be expected when comparing a circular and a composite slip surface within the same slope, for both undrained and combined analyses.

The main result is the comparison of the factors of safety corresponding to the circular and the optimised slip surface obtained with the Morgenstern-Price calculation method. The difference in the factors of safety, ΔF_1 , is consistently expressed as the percentage of decrease of the optimised factor of safety compared to the factor of safety of the circular slip surface, see Equation 2.1.

$$\Delta F_1 = \frac{F_{circ}^{MP} - F_{opt}^{MP}}{F_{circ}^{MP}} \quad (2.1)$$

The same principal is applied when comparing the factor of safety for the circular slip surface and the optimised factor of safety respectively with the calculations performed with Janbu's Generalised calculation method, as in Equations 2.2 and 2.3. By this definition, a negative value of ΔF_2 or ΔF_3 expresses an increase in the factor of safety for the calculations performed with Janbu's Generalised calculation method, compared to those performed with the Morgenstern-Price calculation method.

$$\Delta F_2 = \frac{F_{circ}^{MP} - F_{circ}^{JG}}{F_{circ}^{MP}} \quad (2.2)$$

$$\Delta F_3 = \frac{F_{opt}^{MP} - F_{opt}^{JG}}{F_{opt}^{MP}} \quad (2.3)$$

When comparing the factor of safety obtained with PLAXIS 2D, with the Morgenstern-Price calculations in SLOPE/W, the percental difference is expressed as in Equations 2.4 and 2.5. As an example; a positive value of ΔF_4 and a negative value for ΔF_5 means that F^{PLAXIS} is lower than F_{circ}^{MP} but higher than F_{opt}^{MP} .

$$\Delta F_4 = \frac{F_{circ}^{MP} - F^{PLAXIS}}{F_{circ}^{MP}} \quad (2.4)$$

$$\Delta F_5 = \frac{F_{opt}^{MP} - F^{PLAXIS}}{F_{opt}^{MP}} \quad (2.5)$$

The results of altering the settings for the Optimise function are presented in tables which include the total volume and weight of the potentially sliding soil mass which can be used as an indication of relevant changes in the shape and position of the slip surfaces. Selected results are presented as figures which can be compared to the results of the calculations performed with the default settings and the results from PLAXIS 2D.

3 Theoretical Background

This chapter presents the result of the literature survey, and contains a brief summary of the theoretical background considered relevant for evaluating the Optimise function in the software SLOPE/W. This begins with the general definition of the factor of safety with respect to slope stability, and the required values for such according to Swedish practise. This is followed by a sub chapter devoted to the concept of shear strength, and its dependency on the drainage relationship, which includes the relevant soil parameters and the assessment of those. The third sub chapter aims to explain the connection between the shape of the slip surface and the factor of safety. The theorem of bearing capacity is used to introduce the concept of kinematics related to the slip surface of a potentially moving soil mass.

In the second part of this chapter, Chapter 3.4-3.6, focus is shifted from general slope stability theory to the specific theory implemented in SLOPE/W. The slip surface generation methods used in the calculations are explained, followed by the relevant calculation methods. It should be noted that the calculation methods are explained as they are programmed in this specific software, and that this may vary from the original methods as well as from implementations of the same methods in other software. Thereafter, the Optimise function in SLOPE/W is explained. The final sub chapter contains a brief summary of the theory of the software PLAXIS 2D.

3.1 Factor of Safety

In western Sweden, the soil masses that are susceptible to instability are mainly constituted by declining areas of clay soil since such slopes may be subjected to elevated pore water pressures and shear stresses. Generally, for natural slopes without any altering constructions, only slopes with an inclination larger than 1:10 are considered at risk of land sliding (Skredkommissionen, 1995).

For soils consisting of granular materials such as silt and sand, a steeper inclination is usually required for sliding to occur. Slopes that are in contact with rivers or streams are considered extra susceptible to sliding, since the water causes erosion on the resisting masses of the slope. To assess the risk of instability a factor of safety is used.

3.1.1 Definition of the Factor of Safety

The factor of safety, with regard to slope stability, is commonly defined as the ratio between average shear strength, τ_f , of the soil along a possible slip surface and the corresponding mobilised shear stress, τ_{mob} , as seen in Equation 3.1 (Sällfors, 2009).

$$F = \frac{\tau_f}{\tau_{mob}} \quad (3.1)$$

The factor of safety can equivalently be described in terms of either forces or moments acting on the soil mass, as seen in Equation 3.2, where the driving forces consist of the gravity loads of the landslide mass including any external loads, and the resisting forces consist of the shear strength of the soil (Cornforth, 2005).

$$F = \frac{\text{Resisting forces}}{\text{Driving forces}} = \frac{\text{Resisting moments}}{\text{Driving moments}} \quad (3.2)$$

The mathematical solution for a stability problem is therefore to calculate the driving and resisting moments of the soil mass. Figure 3.1 illustrates a simple slope without any external load or water. The corresponding formulation for the factor of safety can be seen in Equation 3.3 where R is the radius, α is the angle of the circle sector, W is the weight of the soil mass and τ_{mob} is the mobilised shear strength. The length of the slip surface arc is equal to αR .

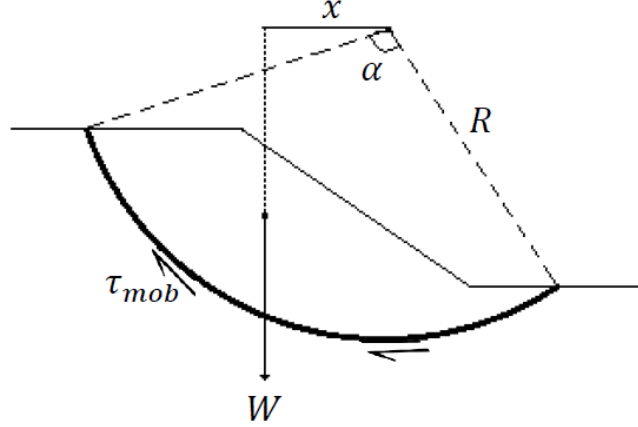


Figure 3.1: A slope with a potentially moving circular slip surface.

$$F = \frac{\text{Resisting moments}}{\text{Driving moments}} = \frac{\tau_{mob}\alpha R}{Wx} \quad (3.3)$$

According to these definitions, a factor of safety equal to 1 indicates that the slope is at limit equilibrium. A factor of safety below 1 indicates an unstable slope that theoretically already should have failed, and consequently a factor of safety greater than 1 indicates stability.

This definition of the factor of safety refers to the concept of global factors of safety (Skredkommissionen, 1995). It infers that the safety margin that is expressed by factors of safety greater than 1, should account for all of the uncertainties inherently associated with the calculations. This is the traditional approach to safety assessment with regard to slope stability.

3.1.2 Required Factor of Safety

In Sweden, the Commission of Application of European Standards for Geotechnics, IEG, provides the recommendations for the demands of the required factor of safety for different types of analyses and for different levels of detail within those analyses (Hultén et al., 2011). The general idea is that calculations performed with more extensive in-data, and thereby containing less uncertainty, should require lower values for the factor of safety. This is due to the fact that the safety margin of $F - 1$ is thought to account for these same uncertainties, and thus the requirements for the factor of safety should descend with increased level of detail.

The required factor of safety varies depending on what type of investigation that is to be executed, if it concerns a new exploitation of land or an existing construction, as well as with regard to the consequences of a landslide at a specific site (Hultén et al., 2011). As this thesis focuses on existing constructions, only such cases will be described in this chapter.

IEG, as well as the industry, distinguish between three phases of investigations that are different in level of detail of the calculations and the extent of the necessary input data. The phases are known

as pre-investigation, detailed investigation and in-depth investigation (Hultén et al., 2011). Within these phases, the requirements may vary between an undrained and a combined analysis. IEG also state that, for cohesive soils, a combined analysis must be performed in addition to the undrained analysis and that the required factors of safety for both these analyses must be satisfied. The importance of the drainage conditions and their significant differences are explained in Chapter 3.2.1.

For the rough calculations of the pre-investigation, the required factors of safety should exceed 2.0 and 1.5 for the undrained and combined analyses respectively (Hultén et al., 2011). For the detailed investigation, the stability is deemed satisfactory when the factor of safety has a value equal to or greater than 1.5-1.3 for combined analyses and 1.7-1.5 for undrained analyses. For sand slopes however, the value of the factor of safety for the undrained analysis should have a minimum value of 1.3.

For the in-depth investigation, the value of the factor of safety should be equal to or greater than 1.4-1.3 for undrained analyses, and 1.3-1.2 for the combined analyses (Hultén et al., 2011). For sand slopes the value for the factor of safety for an undrained analysis should again be greater than 1.3. Exceptions from these requirements can be permitted in cases where they can be adequately justified, for example when the calculated value is close to satisfy the minimum and the measures necessary for raising the value would be disproportionate to the monetary consequences.

3.2 Shear Strength

According to the Mohr-Coulomb failure criterion for soil, the shear strength of a soil at failure, τ_f , can be described by Equation 3.4, where c' is the effective cohesion, σ' is the effective stress and ϕ' is the effective friction angle (Knappett and Craig, 2012). Graphically, the stress states in the soil can be illustrated as in Figure 3.2.

$$\tau_f = c' + \sigma' \tan(\phi') \quad (3.4)$$

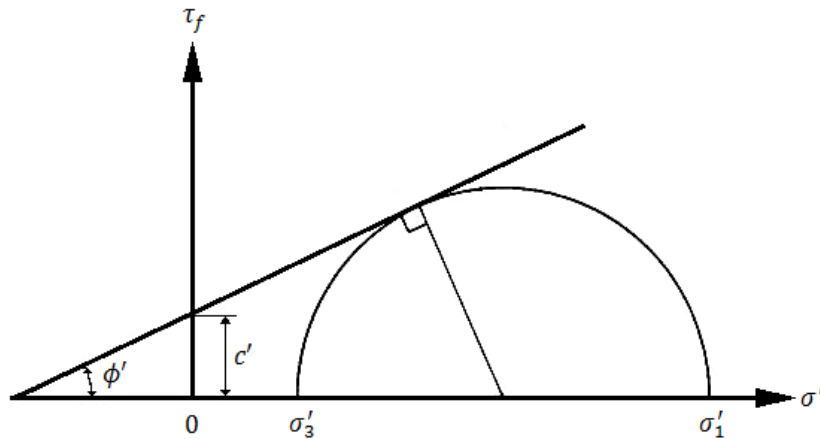


Figure 3.2: The Mohr-Coulomb failure envelope.

The Mohr's circle represents all possible stress states on all planes within a soil element (Knappett and Craig, 2012). Failure occurs when the soil is subjected to any critical combination of shear strength and effective normal stress, which is illustrated by the straight line known as the failure envelope. The points below the failure envelope are in equilibrium which entails that only elastic deformations, and thus no failure, will occur.

3.2.1 Drainage Conditions

When determining the shear strength of a soil, the drainage conditions are of great importance as they largely influence the behaviour of the soil (Skredkommissionen, 1995). Slope stability is therefore divided into drained and undrained analyses. For many real cases both the drained and the undrained shear strengths need to be evaluated, which is referred to as combined analysis.

Undrained conditions refer to a stage where the excess pore water pressure is at its initial conditions, i.e. before the consolidation process has started (Knappett and Craig, 2012). For saturated cohesive soils the friction angle is zero which makes the undrained shear strength independent of effective stress (Skredkommissionen, 1995), see Figure 3.3, and the undrained shear strength at failure, τ_{fu} , can therefore be expressed as in Equation 3.5.

$$\tau_{fu} = c_u \quad (3.5)$$

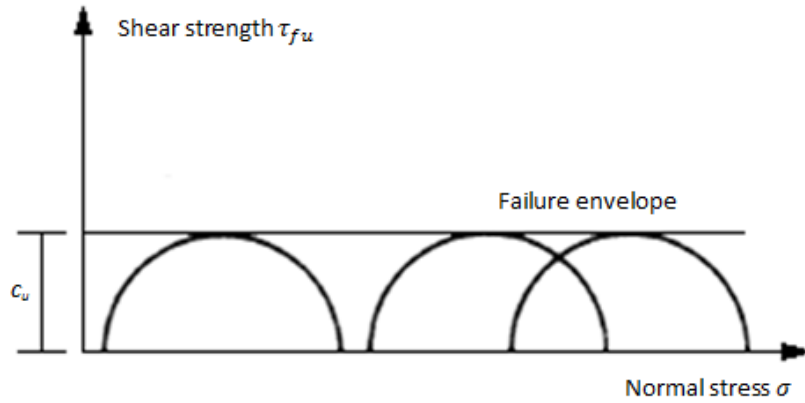


Figure 3.3: The Mohr-Coulomb failure envelope in undrained conditions (Knappett and Craig, 2012) (modified by the authors).

Drained conditions refer to a stage where the consolidation is completed, meaning that the reduction of excess pore water pressure has started and finished (Knappett and Craig, 2012). This state is primarily associated with friction soils, although some important exceptions exist. The drained shear strength can be of relevance for cohesive soils subjected to extremely rapid loading as well as for overconsolidated cohesive soils, namely those with an overconsolidation ratio greater than 2 (Larsson, 2008).

The drained shear strength at failure, τ_{fd} , is directly dependent on the friction angle of the soil and can be expressed by Equation 3.6, where c' is the effective drained cohesion, σ' is the effective stress and ϕ' is the effective friction angle (Skredkommissionen, 1995).

$$\tau_{fd} = c' + \sigma' \tan(\phi') \quad (3.6)$$

Overconsolidated fine grained soils and soil profiles containing high pore water pressures should be evaluated with regard to drained shear strength (Skredkommissionen, 1995). The more overconsolidated the soil is, the more misleading is the undrained shear strength. Moreover, the occurrence of draining layers of silt or other more granular materials as well as tension cracks providing drainage may strongly progress the consolidation process, giving relevance to the drained shear strength.

As opposed to a normally consolidated cohesive soil, an overconsolidated cohesive soil is a dilatant material and thus strives towards a volume increase when subjected to shear stress (Larsson, 2008). Since no instantaneous volume change is possible for a cohesive soil, there will instead be a decrease in pore water pressure, why it within slope stability analysis should be evaluated for drained shear strength.

In order to acknowledge the risk of the drained shear strength being decisive at any part of the possible slip surface, a combined analysis can be performed (Larsson et al., 2007). In the combined analysis both drained and undrained shear strength is evaluated for each slice throughout the possible slip surface. The critical value of the two is selected for each slice and thereby the factor of safety of the combined analysis is always the lowest, which also can be illustrated with the Mohr-Coulomb failure criterion for a combined analysis as in Figure 3.4.

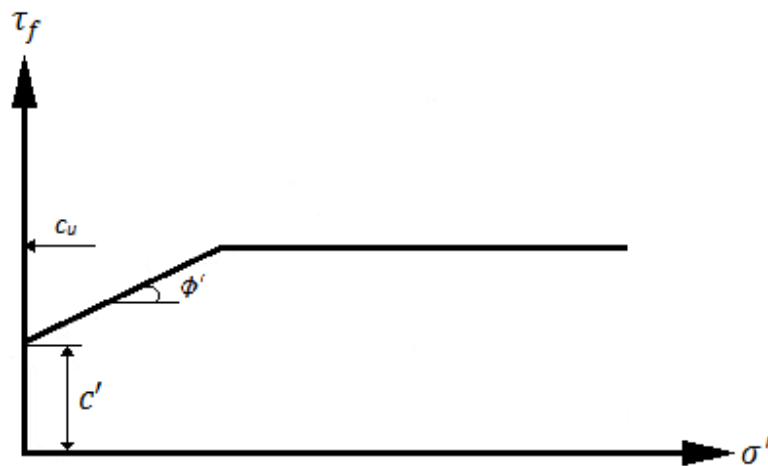


Figure 3.4: The Mohr-Coulomb failure envelope in a combined analysis.

Combined analysis should be performed for slopes in fine grained and intermediate soils, where it is not obvious what drainage condition that applies, as demanded by IEG (Hultén et al., 2011). The main reason for combined analysis being implemented in Sweden is that landslides often occur in autumn when the pore water pressures are relatively elevated, compared to the rest of the year.

3.2.2 Assessment of Soil Parameters

According to the Mohr-Coulomb failure criterion, the same soil sample will exhibit different strengths depending on if a drained or an undrained test is performed (Knappett and Craig, 2012). It is crucial to determine the shear strength of the soil with adequate methods. In addition to the method of investigation, the method for selecting the characteristic values, the anisotropy of the soil and the pore water pressures also affect the result.

The determination of the values of the undrained shear strength is to a large extent based on in situ vane shear tests and laboratory tests such as fall cone tests and direct shear tests (Skredkommissionen, 1995). Values obtained from vane shear tests and fall cone tests need to be adjusted with a factor based on the liquid limit, w_L , while values obtained from direct shear tests can be used directly.

Empirically known relationships of the undrained shear strength suggest that it is primarily a function of the preconsolidation pressure and the liquid limit (Skredkommissionen, 1995).

Anisotropic effects make these relationships dependent on the specific loading cases, and it becomes necessary to distinguish between active shearing, direct shearing and passive shearing, as illustrated in Figure 3.5. The corresponding laboratory tests are active (increasing the vertical stress) triaxial test for the active zone, direct shear test for the direct shear zone and passive (increasing the vertical strain) triaxial test for the passive zone respectively. The active shearing zone occurs on what is also known as the driving side of the slope, whereas the passive shearing zone occurs on the resisting side.

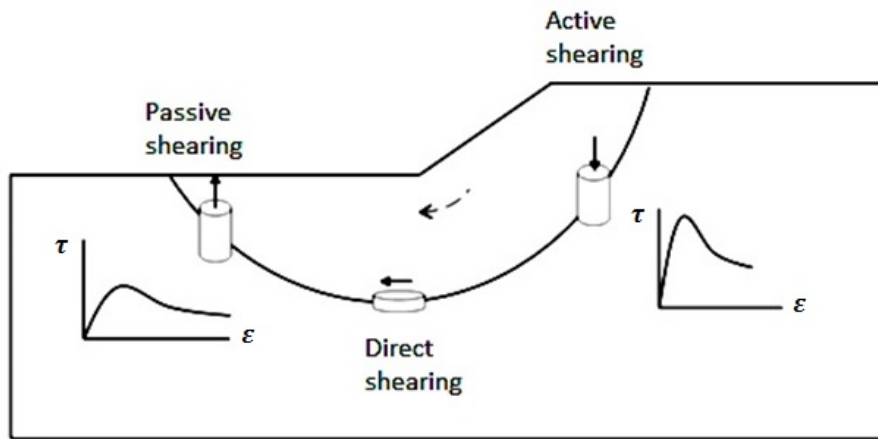


Figure 3.5: The different shearing zones of a slope; passive shearing (strain), direct shearing and active shearing (stress) (Westerberg et al., 2012) (modified by the authors).

As previously mentioned in Chapter 3.2.1, a drained analysis may be of importance particularly in the case of overconsolidated cohesive soils. The overconsolidation ratio can be determined through constant rate of strain, CRS, test in laboratory (Skredkommissionen, 1995).

For overconsolidated cohesive soils, the cohesion and the friction angle can be assessed by empirically known correlations to the undrained parameters. The apparent cohesion, c' , is typically set to ten percent of the undrained cohesion, c_u , or to three percent of the preconsolidation pressure, σ'_c (Larsson et al., 2007). The effective friction angle, ϕ' , is usually set to 30° .

Anisotropy can have a notable effect on the stability in steep slopes where the active zone is considerably bigger than the passive zone, creating a relatively large direct shear zone (Skredkommissionen, 1995). Taking the anisotropy into account always raises the calculated factor of safety in cohesive soils. For steep slopes and soils with a lower liquid limit the effects of anisotropy is especially evident. Effects of anisotropy can be verified through triaxial tests on undisturbed samples that have been reconsolidated to the in situ stresses.

Measurements of the pore water pressures are important since hydrostatical conditions in slopes are rare (Skredkommissionen, 1995). In pervasive layers, an open measurement system should be used, while for low permeable layers it is recommended to use a closed measurement system. Seasonal changes must be taken into account and thought should be given to which extremes of the water levels that, in combination with the other conditions, would cause the worst case scenarios.

In stability calculation for slopes in cohesive soil, the lowest water level of the open surface water if such exists, in combination with the highest pore water pressure, is normally the critical state. If the cohesive soil is overconsolidated, as well as for friction soil or combinations of soil layers, the critical combination of water level and pore pressure gives the design values.

3.3 Slip Surface Shape and Bearing Capacity

When determining the factor of safety through the formulation of a statically correct slip surface, the true solution is approached from the safe side and the value of the factor of safety will be lower than the real value. This is because the statically correct slip surface does not account for the work required for a rotational movement to occur. With a kinetically correct slip surface on the other hand, one approaches the true solution from the unsafe side, and the value of the factor of safety is higher than the true value. To obtain a solution as close as possible to the true factor of safety, the formulation should account for both static and kinematic moment equilibrium.

This chapter introduces the theory behind bearing capacity for shallow foundations in a two-dimensional analysis. The expression and value for the bearing capacity factors, N , are mathematically derived for four different slip surface shapes. The first case is a statically correct slip surface, the second case is a kinematically correct slip surface, the third case is the kinematically correct slip surface with a minimised factor of safety and the fourth case represents a slip surface that is both statically and kinematically correct. The last sub chapter contains a summary of the obtained bearing capacity factors, as well as a brief explanation of the connection between the bearing capacity factor and the factor of safety.

The following mathematical formulations rely on a number of assumptions; the foundation has a constant width, b , and is placed directly on a horizontal ground surface (Sällfors, 2009). The foundation is submitted to a centric, vertical load, Q , per meter and is surrounded by a surcharge load q_0 . The soil below the foundation is homogeneous and isotropic with constant soil properties, c' and ϕ' or c_u , and constant effective unit weight, γ . With these assumptions, the general equation for the bearing capacity can be expressed as seen in Equation 3.7. The factor of safety is assumed to be 1, as the bearing capacity can only be derived from loading cases at moment equilibrium.

$$q_b = \tau_f N_c + q_0 N_{q_0} + 0.5 \gamma b N_\gamma \quad (3.7)$$

For undrained conditions i.e. ϕ' is zero, which are considered in this chapter, the bearing capacity factor N_γ is equal to zero and N_q is equal to 1 (Sällfors, 2009). The surrounding surcharge load, q_0 , is in this case set to zero, meaning that the shear strength of the soil is the only force resisting the failure. With these assumptions, Equation 3.8 represents the general equation for the bearing capacity factor for undrained conditions.

$$q_b = \tau_f N_c \Rightarrow N_c = \frac{q_b}{\tau_f} \quad (3.8)$$

For undrained conditions the failure mechanism within the soil mass consists of slip lines that are either straight lines, circular arcs or a combination of the two (Knappett and Craig, 2012). The bearing capacity factor, for the undrained case, expresses the ratio of q_b over τ_f necessary for moment equilibrium for the different types of slip lines.

3.3.1 Statically Correct Slip Surface

A statically correct slip surface consists of two straight slip lines, illustrated in Figure 3.6, where the entry and exit angles are $45^\circ + \phi'/2$ and $45^\circ - \phi'/2$ respectively (Terzaghi, 1959). Since ϕ' is zero for undrained conditions, both angles are 45° . By assuming moment equilibrium moment around point A, as in Equation 3.9, the obtained value of N_c is 4.00.

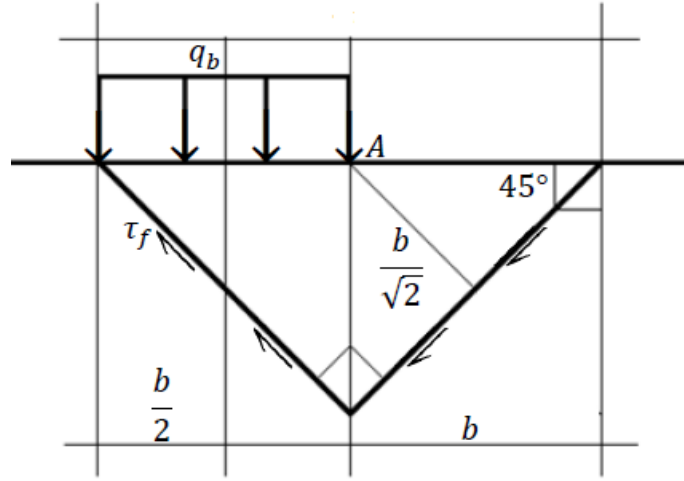


Figure 3.6: A statically correct slip surface shape consisting of two straight lines.

$$M_A : q_b b \frac{b}{2} - 2(\tau_f b \sqrt{2} \frac{b}{\sqrt{2}}) = 0 \quad (3.9)$$

$$\Leftrightarrow q_b \frac{b^2}{2} = 2\tau_f b^2 \Leftrightarrow q_b = 4\tau_f \Rightarrow N_c = 4$$

3.3.2 Kinematically Correct Slip Surface

A kinematically correct slip surface consists of a circular slip line (University of the West of England, 2001), as seen in Figure 3.7. By calculating moment equilibrium around point A, as in Equation 3.10, the obtained value of N_c is 6.28.

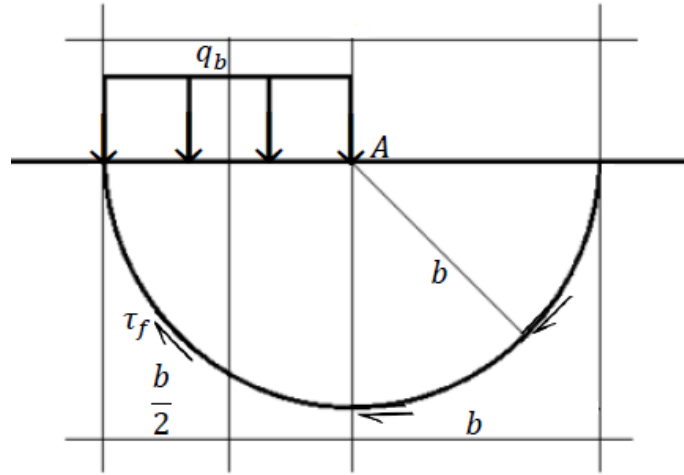


Figure 3.7: A kinematically correct slip surface shape, consisting of a half circle.

$$M_A : q_b b \frac{b}{2} - \tau_f \pi b b = 0 \quad (3.10)$$

$$\Leftrightarrow q_b \frac{b^2}{2} = \tau_f \pi b^2 \Leftrightarrow q_b = 2\pi\tau_f = 6.28\tau_f \Rightarrow N_c = 6.28$$

By searching for the location of the centre of the circle resulting in the lowest factor of safety, a minimum N_c can be obtained. This can be done by introducing the angle α as unknown and deriving the factor of safety with respect to α (University of the West of England, 2001).

By moving up the centre of the circle, the rotation point A, as seen in Figure 3.8, the expression in Equation 3.10 can be minimised. The radius, R , depending on α , then becomes unknown. By calculating the derivative of the expression for the factor of safety, F , and solving for zero the value of α that results in the lowest F can be obtained.

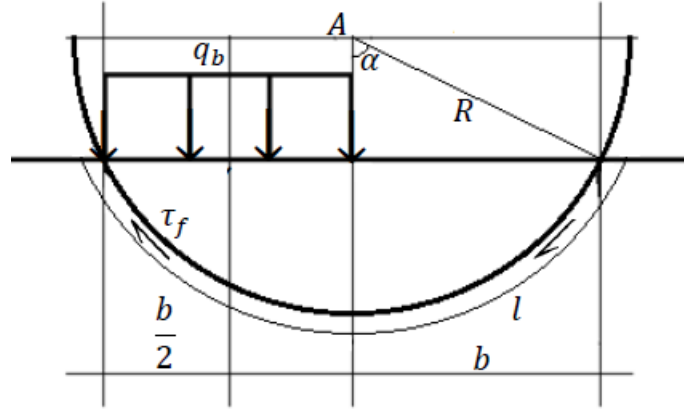


Figure 3.8: A critical kinematically correct slip surface shape, consisting of a shallow circular arc.

The equilibrium equation for the counter-clockwise moment around point A, in Figure 3.8, is set up as in Equation 3.11.

$$M_A : q_b b \frac{b}{2} - l \tau_f R = 0 \quad (3.11)$$

Using the geometric relationships in Figure 3.8, the unknown length of the circular slip line is expressed in terms of α and R as in Equation 3.12. With trigonometric relationships, an expression for R as a function of α is obtained, see Equation 3.13.

$$l = R 2\alpha \quad (3.12)$$

$$\sin \alpha = \frac{b}{R} \Rightarrow R = \frac{b}{\sin \alpha} \quad (3.13)$$

As stated in Chapter 3.1.1, the factor of safety can be defined as the ratio between the resisting moments and the driving moments as in Equation 3.2. In the case of the critical kinematically correct slip surface, this ratio can be expressed as in Equation 3.14.

$$F = \frac{l \tau_f R}{q_b b(b/2)} \quad (3.14)$$

$$\Rightarrow F = \frac{2\alpha R \tau_f R}{q_b b^2/2} = \frac{4R^2 \alpha \tau_f}{q_b b^2}$$

$$\Leftrightarrow F = \frac{4 \frac{b^2}{\sin^2 \alpha} \alpha \tau_f}{q_b b^2} = \frac{4 b^2 \alpha \tau_f}{q_b (\sin^2 \alpha) b^2} = \frac{4 \alpha \tau_f}{q_b (\sin^2 \alpha)} = \frac{4 \tau_f}{q_b} \left(\frac{\alpha}{\sin^2 \alpha} \right)$$

Differentiating the expression in Equation 3.14, with respect to α , and solving Equation 3.15 for $F_{min}(\alpha)$ gives the critical value of α .

$$\frac{dF}{d\alpha} = 0 \quad (3.15)$$

$$\Rightarrow \frac{dF}{d\alpha} = \frac{4 \tau_f}{q_b} \left(\frac{\sin^2 \alpha - 2 \alpha \sin \alpha \cos \alpha}{\sin^4 \alpha} \right) = \frac{4 \tau_f}{q_b} \left(\frac{\sin \alpha - 2 \alpha \cos \alpha}{\sin^3 \alpha} \right) = 0$$

$$\Rightarrow \sin \alpha - 2 \alpha \cos \alpha = 0 \Leftrightarrow 2 \alpha \cos \alpha = \sin \alpha \Leftrightarrow 2 \alpha = \frac{\sin \alpha}{\cos \alpha} = \tan \alpha$$

$$\Rightarrow 2 \alpha = \tan \alpha \Rightarrow \alpha = 1.166 \text{ rad} = 66.8^\circ$$

Finally, the calculated value of α is inserted to the equation for the factor of safety, Equation 3.14. Equation 3.16 expresses the critical factor of safety for a kinematically correct slip surface.

$$F = \frac{4 \tau_f}{q_b} \left(\frac{\alpha}{\sin^2 \alpha} \right) = \frac{4 \tau_f}{q_b} \left(\frac{1.166}{\sin^2 1.166} \right) = \frac{5.52 \tau_f}{q_b} \quad (3.16)$$

Since moment equilibrium is assumed, F equals 1 and, the bearing capacity for the critical kinematically slip surface has a value of 5.52 as seen in Equation 3.17.

$$F = 1 \Rightarrow q_b = 5.52 \tau_f \Rightarrow N_c = 5.52 \quad (3.17)$$

3.3.3 Statically and Kinematically Correct Slip Surface

A slip surface that is both statically and kinematically correct has a combination of straight and circular slip lines, as seen in Figure 3.9. Moment equilibrium around point A results in a bearing capacity factor with the value 5.14, see Equation 3.18.

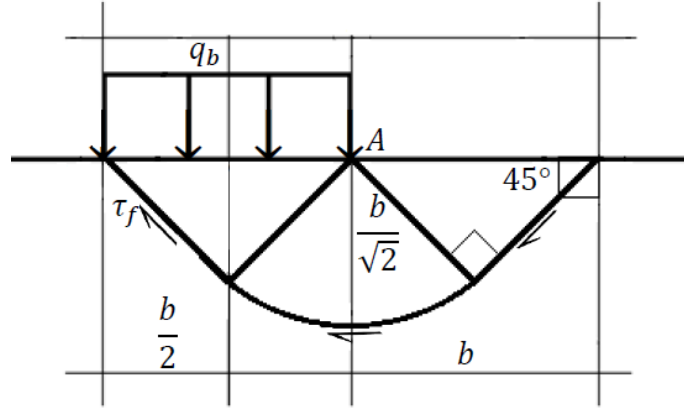


Figure 3.9: A statically and kinematically correct slip surface shape for cohesive soils according to Prandtl's theorem.

$$M_A : q_b b \frac{b}{2} - 2(\tau_f \frac{b}{\sqrt{2}} \frac{b}{\sqrt{2}}) - \tau_f \pi \frac{b}{\sqrt{2}} \frac{b}{\sqrt{2}} = 0 \quad (3.18)$$

$$\Leftrightarrow q_b \frac{b^2}{2} = 2\tau_f \frac{b^2}{2} + \tau_f \pi \frac{b^2}{2}$$

$$\Rightarrow q_b = (2 + \pi)\tau_f \Rightarrow q_b = 5.14\tau_f \Rightarrow N_c = 5.14$$

3.3.4 Bearing Capacity Factors

The results from the derivations of the four different slip surface shapes are summarised in Table 3.1.

Table 3.1: Summary of the bearing capacity factors for the different slip surface shapes.

Slip surface shape	N_c
Statically correct	4.00
Kinematically correct	6.28
Critical kinematically correct	5.52
Statically and kinematically correct	5.14

The bearing capacity factor can be directly compared to the factor of safety when performing calculations of slope stability analysis. If a safety margin is included in the surcharge load q_b in terms of a factor of safety, q_b is equal to qF as in Equation 3.19. By setting the value of q equal to the value of τ_f , N_c is equal to F .

$$N_c = \frac{q_b}{\tau_f} \quad (3.19)$$

$$q_b = qF$$

$$\Rightarrow N_c = \frac{qF}{\tau_f}, \quad q = \tau_f \Rightarrow N_c = F$$

3.4 Slip Surface Generation Methods in SLOPE/W

Determining the shape and position of the critical slip surface is a crucial step in executing a slope stability analysis. In SLOPE/W, the critical slip surface is located through the calculation of several trial slip surfaces that can be more or less controlled by the user. The software computes the factor of safety for the trial slip surfaces whereupon the slip surface generating the minimum factor of safety is considered as the critical.

In SLOPE/W it is possible to display all of the computed factors of safety and their respective slip surfaces. This allows the user to evaluate the plausibility of the various slip surfaces with regard to the information available about the specific slope. It is important that the SLOPE/W user understands the physics behind the computed factor of safety in order to disregard unreasonable results (GEO-SLOPE International Ltd., 2008).

Although physically inadmissible slip surfaces may obstruct the software from computing the factor of safety, this is simply due to poor mathematical convergence of the solving algorithm (GEO-SLOPE International Ltd., 2008). SLOPE/W then displays one of several error codes in place of the factor of safety. The error codes are easily distinguished from the factors of safety by that they are within the interval of E983 to E999. Nonetheless, the risk remains of obtaining other types of physically invalid slip surfaces, which are still mathematically possible within the computational algorithms of the software.

The number of vertical slices that the slip surface is divided into can be specified by the user, although this may be overridden by the software. The minimum slip surface depth can be adjusted in order to force the software to penetrate a certain layer. The default convergence settings in SLOPE/W are listed in Table 3.2.

Table 3.2: Default values of the convergence settings in SLOPE/W (GEO-SLOPE International Ltd., 2008).

Convergence settings	Default value
Number of slices	30
Minimum slip surface depth	0.1 m
Maximum number of iterations	100
Factor of safety tolerance	0.001

The shape and position of the critical slip surface depend on the loading case, the generated pore water pressures, the layering of the soil and the soil strength parameters. In order for SLOPE/W to identify the most critical slip surface it is required by the user to choose an, for the specific conditions of the examined slope, appropriate method for generating the trial slip surfaces.

There are several methods for generating the trial slip surfaces in SLOPE/W; Grid and Radius for circular slip surfaces, Grid and Radius combined with an impermeable layer for composite slip surfaces, Fully Specified slip surfaces through a series of coordinates, Block Specific slip surfaces and Entry and Exit specification (GEO-SLOPE International Ltd., 2008). The user is required to specify which part of the slope profile that should be considered the driving side i.e. if the slip surface should run from left to right or from right to left.

Additionally, the Optimise function can be applied to all of these slip surface generation methods, whereupon the optimised slip surfaces is stored as the last slip surface. The Optimise function is described in detail in Chapter 3.6. This chapter is focused on the two slip surface generation methods used in this thesis, namely the Grid and Radius method and the Fully Specified method.

3.4.1 Grid and Radius

The variables in a perfectly circular slip surface are, as seen in Chapter 3.3.2, the radius of the circle arc and the location of the point around which to calculate moments. By altering these variables, endless variations of the circular slip surface could theoretically be calculated, and a large number of these can be computed in SLOPE/W by using the Grid and Radius slip surface generation method.

The Grid and Radius method generates a number of circular slip surfaces and identifies the critical one. In this method the user defines the location, shape, size and coarseness of the search grid above the slope for locating the point around which the slip surface is assumed to rotate in the event of land sliding (GEO-SLOPE International Ltd., 2008). The grid is coupled with a search area for the slip surface radius tangent line, as illustrated in Figure 3.10.

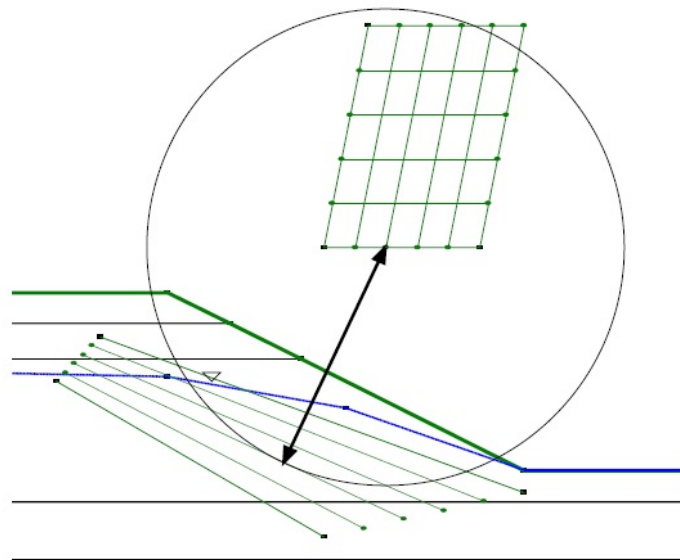


Figure 3.10: A circular slip surface obtained with the Grid and Radius method (GEO-SLOPE International Ltd., 2008).

The coarseness of the search areas determine how many circular slip surfaces that are to be computed (GEO-SLOPE International Ltd., 2008). For example, a grid of 5×5 rotation points and 5 tangent lines generates $5 \times 5 \times 5 = 125$ slip surfaces. A good practise is to refine the coarseness of the search area, both the rotational point and the tangent lines, until no apparent changes in the value of the factor of safety occur.

The search area can preferably be extensive to begin with, to get an indication of where the critical rotation point and tangent line is located. The user should make sure that, if possible, the critical rotation point and tangent line are located well within the borders of the search areas. If they are located on the edge of the search area, this indicates that there might exist a critical slip surface outside of the search area that will not be computed by the software.

If calculating a known circular slip surface, for example when performing back- or recalculations, it is possible to generate this specific slip surface by entering one single rotational point and tangent line respectively. If a composite slip surface is known, the Fully Specified method, described below, may be the favourable. However, when little is known about the location of the slip surface, a great advantage of the Grid and Radius method is that it enables searching through the entire slope. When combined with the Optimise function, it can also be a suitable method for identifying composite slip surfaces.

In the case of a surcharge load on a horizontal ground surface, as in a bearing capacity problem, the exit angle for the slip surface is known from theory as $45^\circ - \phi/2$ and the entry angle is known as $45^\circ + \phi/2$ (Terzaghi, 1959), as illustrated in Figure 3.11. This leads to a slip surface shape that is not circular but composite. With the Grid and Radius method, the projection angle can be specified (GEO-SLOPE International Ltd., 2012). Moreover, as the slip surface should include the bottom left corner of the footing, the radius should be defined as a single point located in this corner.

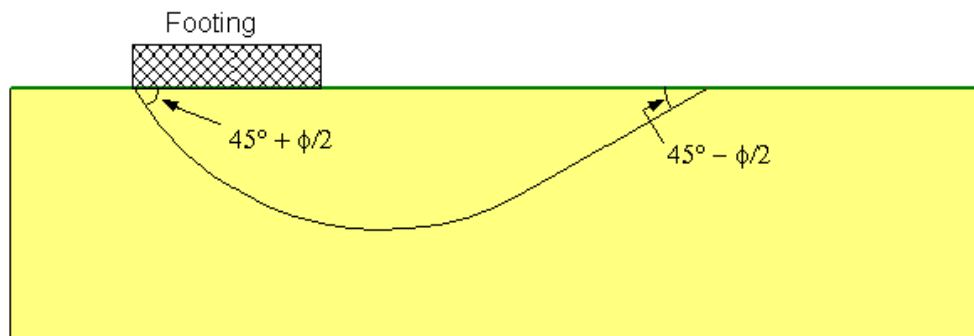


Figure 3.11: A circular slip surface with a specified projection exit angle.(GEO-SLOPE International Ltd., 2012) (modified by the authors).

3.4.2 Fully Specified

The Fully Specified method allows the user to define a slip surface with specified points as can be seen in Figure 3.12 (GEO-SLOPE International Ltd., 2008). It is recommended that the starting and ending points are set outside the geometry in order to allow the software to compute the intersection points. This is preferable in order to avoid numerical misperception.

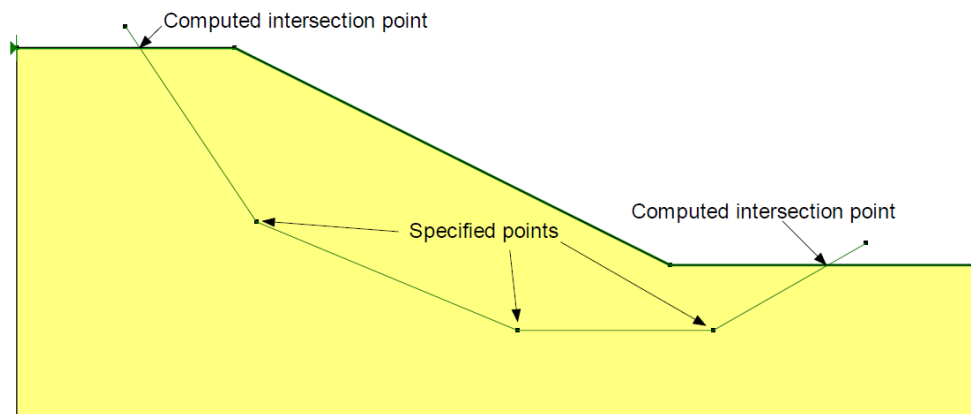


Figure 3.12: A fully specified slip surface (GEO-SLOPE International Ltd., 2008).

An axis point should also be defined, around which to take moments when performing the calculations. If this point is not specified by the user, SLOPE/W estimates the axis point by taking into account the geometry of the slope and the specified slip surface (GEO-SLOPE International Ltd., 2008). When using rigorous calculation methods that satisfies both moment and force equilibrium, such as the Morgenstern-Price calculation method, the calculations are not sensitive to the position of the axis point. When using a calculation method that does not take both moment and force equilibrium into account, the position of the axis point is of greater importance.

When using the Fully Specified method, a single factor of safety is computed. If the Optimise function is activated this slip surface is optimised leading to one additional factor of safety. The method is therefore useful when investigating a specific slip surface; however it is not suitable when searching for a large number of critical slip surfaces (GEO-SLOPE International Ltd., 2008) as they all have to be defined individually by the user. It should be noted that, if the Fully Specified method is used to recreate a circular slip surface, the computed factor of safety would be similar, presuming that the same calculation method is used.

3.5 Calculation Methods in SLOPE/W

There are several different methods available in SLOPE/W for computing the factor of safety. All of these calculation methods, except for one using a finite element method, are based on limit equilibrium formulations (GEO-SLOPE International Ltd., 2008). Except for the basic principle of limit equilibrium and the Ordinary Method of Slices, from which these methods derive, this chapter only describes the methods that are used for the calculations in this thesis.

As stated in Chapter 3.1.1, the factor of safety regarding slope stability is equal to the ratio of the driving forces over the resisting forces of a potentially moving soil mass usually calculated in two dimensional analysis. In 1936, Fellenius introduced the Ordinary Method of Slices (GEO-SLOPE International Ltd., 2008), in which the circular slip surfaces is divided into vertical slices with bases that are assumed to be straight lines. The normal force acting on each slice can be calculated, and thereby also the available shear strength as it is the perpendicular counterpart.

The simplest formulation of the Ordinary Method of Slices, for a circular slip surface without any pore-water pressures, can be seen in Equation 3.20 (GEO-SLOPE International Ltd., 2008). Force equilibrium is calculated by adding all of the resisting and the driving forces respectively, and interslice forces are ignored. This calculation can be executed by hand or in a spread sheet, but it is also included in SLOPE/W.

$$F = \frac{\sum (c\beta + N\tan\phi)}{\sum W\sin\alpha} \quad (3.20)$$

The variables in Equation 3.20 are defined as follows:

- c = cohesion
- β = slice base length
- N = base normal ($W \cos\alpha$)
- ϕ = friction angle
- W = slice weight
- α = slice base inclination

Since the introduction of the Ordinary Method of Slices, a number of limit equilibrium methods have been developed. The differences between the methods lies within which equations that are used, if the interslice normal and shear forces are included and if so, what relationship that is assumed between them (GEO-SLOPE International Ltd., 2008). Different methods will compute different factors of safety, although they may coincide for many cases.

A free body diagram of a typical slice in a potential sliding mass is illustrated in Figure 3.13, where the interslice normal force E and shear force X are acting on the sides of the slice and W is the weight of the slice.

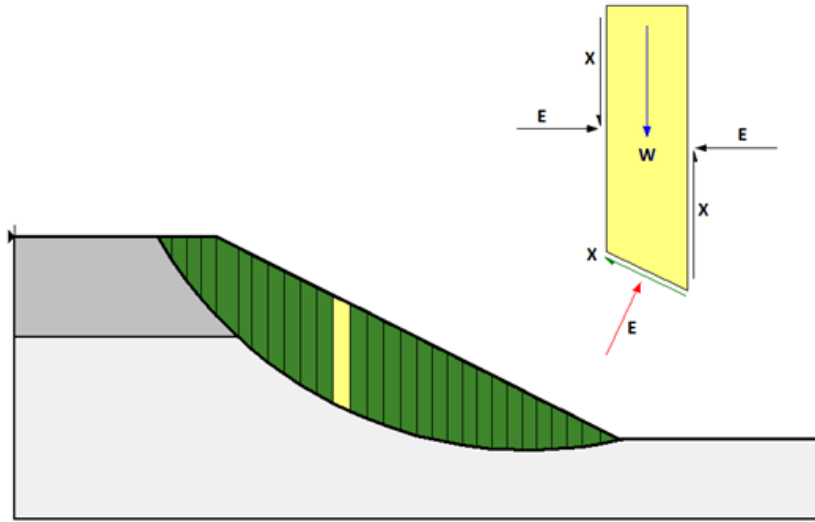


Figure 3.13: A circular slip surfaces and the interslice forces of a selected slice together with a free body diagram (GEO-SLOPE International Ltd., 2008).

The interslice shear force, X , is a function of the interslice normal force, E , the specified force function, $f(x)$, and the percentage of that function that is used, λ , as seen in Equation 3.21 (GEO-SLOPE International Ltd., 2008). Arc tan of $X/100$ is the inclination, in degrees from the horizontal, of the resultant interslice force. Figure 3.14 illustrate how the lambda value may vary along the slices.

$$X = E\lambda f(x) \quad (3.21)$$

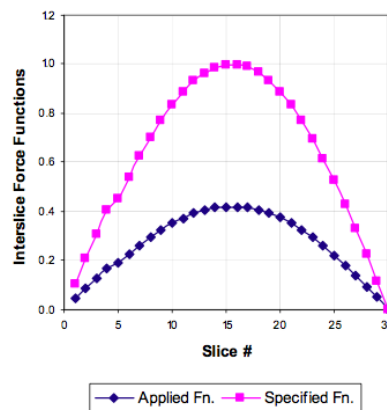


Figure 3.14: Illustration of λ for a half-sine interslice function, and the variation along the slices of a slope (GEO-SLOPE International Ltd., 2008).

The significance of the interslice force function depends mainly on the amount of contortion that the potential sliding mass must be submitted to in order for movement to occur (GEO-SLOPE International Ltd., 2008). In the case of a perfectly circular slip surface, the moment equilibrium is independent of the interslice shear forces as the sliding mass body can rotate without any relative movement between the slices. This is illustrated in by the moment equilibrium plot in Figure 3.15.

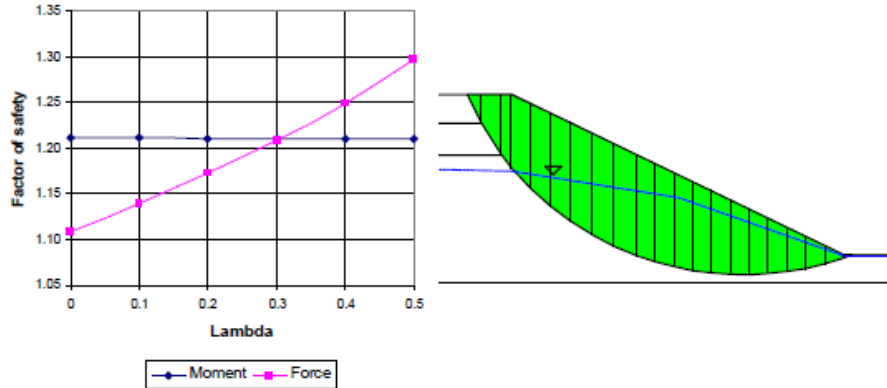


Figure 3.15: A circular slip surface and the corresponding F - λ -plot. Not that the factor of safety with respect to moment equilibrium is constant (GEO-SLOPE International Ltd., 2008).

On the contrary, for the same slip surface, the horizontal force equilibrium is sensitive to interslice shear forces as substantial relative slippage between the slices is inevitable in the occurrence of lateral movement (GEO-SLOPE International Ltd., 2008). For planar slip surfaces, the relationships are the opposite; force equilibrium is dependent of interslice shear while moment equilibrium is not. This can be seen in Figure 3.16.

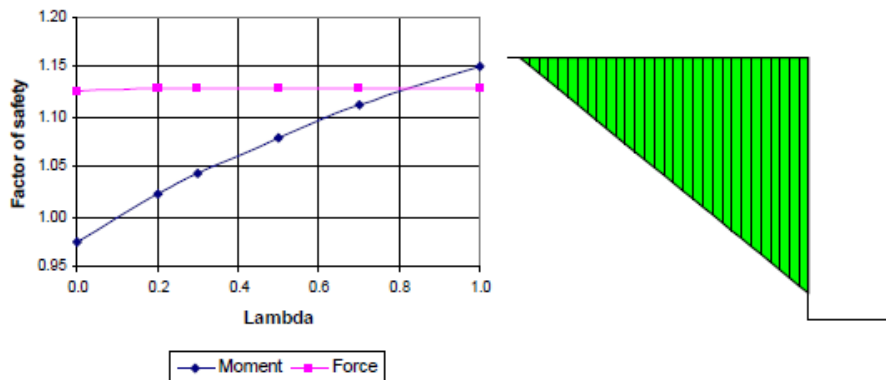


Figure 3.16: A planar slip surface and the corresponding F - λ -plot. Not that the factor of safety with respect to force equilibrium is constant (GEO-SLOPE International Ltd., 2008).

In the case of a composite slip surface, which essentially is a slip surface composed by both circular and planar line segments, both force and moment equilibrium are dependent of the interslice forces. This is illustrated in Figure 3.17.

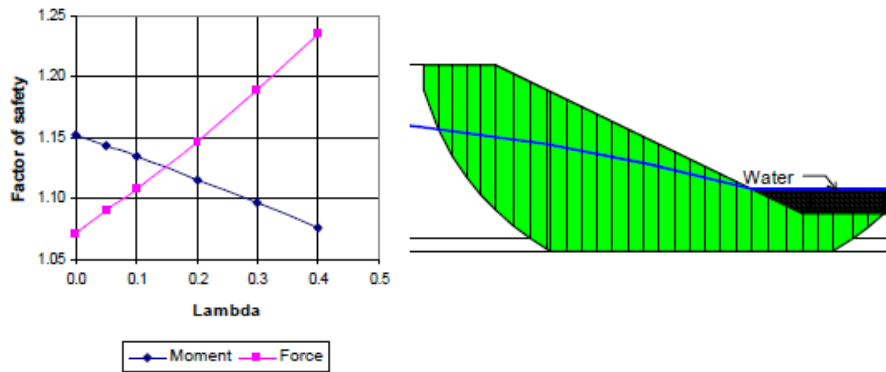


Figure 3.17: A composite slip surface and the corresponding F - λ -plot (GEO-SLOPE International Ltd., 2008).

3.5.1 Janbu's Generalised

Janbu's Generalised calculation method is a development of Janbu's Simplified calculation method, which in turn is developed from the Ordinary Method of Slices. Janbu's Simplified method considers the interslice normal forces but not the interslice shear forces and λ , in Equation 3.21, is set to zero (GEO-SLOPE International Ltd., 2008). Horizontal equilibrium is satisfied in Janbu's Simplified calculation method, whereas moment force equilibrium is not. For circular slip surfaces, which are more sensitive to horizontal equilibrium than to moment equilibrium this method results in an overly conservative value for the factor of safety (GEO-SLOPE International Ltd., 2008).

The factor of safety for Janbu's Generalised methods is calculated in an iterative process, where the vertical forces on each slice are a function of the factor of safety. As a consequence of this, the base normal also becomes a function of the factor of safety, leading to that the factor of safety becomes a nonlinear equation, and an iterative procedure is required to compute the factor of safety. Moreover, the variable m_α , which is a function of the inclination of the base of a slice, α , and the relation between the friction angle and the factor of safety, $\tan \phi' / F$, is included in the equation for the factor of safety.

As a result the shape of the slip surface will affect the computed value of the factor of safety. An initial value for the factor of safety needs to be assumed in order to start the iteration process (GEO-SLOPE International Ltd., 2008). With this assumed value, m_α can be calculated which results in a new value for the factor of safety. The new value for the factor of safety is compared with the assumed value, and the iteration is continued until the two values converge.

Janbu's Generalised method, as opposed to Janbu's Simplified method, does account for interslice shear forces as well as interslice normal forces (GEO-SLOPE International Ltd., 2008). The main difference between Janbu's Generalised method and other limit equilibrium methods, for example the Morgenstern-Price method, is that the stress distribution in between the slices is defined by a line of thrust. The interslice shear forces are modelled by a line of thrust which typically intersects the slices at approximately the lower third.

In the SLOPE/W implementation of this method, the line of thrust is fixed to this position, whereas when performing hand calculation it is possible to alter it for the different slices, as one believes that the real line of thrust would occur in the slope. The method thereby satisfies moment equilibrium on slice level, but not for the overall factor of safety. By fixating the line of thrust to the lower third of the slices, accurate results of the Janbu's Generalised method as implemented in SLOPE/W is limited to cases where it is reasonable to assume that the interslice shear forces are actually acting

at this position. For simple cases, with no sharp corners along the slip surface or high concentrated loads, this method will compute a factor of safety with a value similar to those of rigorous methods.

3.5.2 Morgenstern-Price

The Morgenstern-Price method is a rigorous method also based on limit equilibrium formulation (GEO-SLOPE International Ltd., 2008). The method considers both shear normal interslice forces and satisfies both moment and force equilibrium. Two equations are used for calculating the factor of safety; one with respect to moment equilibrium, F_m , and the other with respect to horizontal force equilibrium, F_f , see Equation 3.22 and 3.23. All input parameters described below and also illustrated in Figure 3.18.

$$F_m = \frac{\sum (c' \beta R + (N - u \beta) R \tan \phi')}{\sum Wx - \sum Nf + \sum kW_e \pm \sum Dd \pm \sum Aa} \quad (3.22)$$

$$F_f = \frac{\sum (c' \beta \cos \alpha + (N - u \beta) \tan \phi' \cos \alpha)}{\sum N \sin \alpha + \sum kW - \sum D \cos \omega \pm \sum A} \quad (3.23)$$

The variables are defined as follows:

- c' = effective cohesion
- ϕ' = effective friction angle
- u = pore water pressure
- W = the total weight of a slice
- N = the total normal force on the base of a slice
- D = an external point load
- kW = the horizontal seismic load applied through the centroid of each slice
- R = the radius for a circular slip surface
- f = the perpendicular offset of the normal force from the centre of rotation
- x = the horizontal distance from the centerline of each slice to the centre of rotation
- e = the vertical distance from the centroid of each slice to the centre of rotation
- d = the perpendicular distance from a point load to the centre of rotation
- a = the perpendicular distance from the resultant water force to the centre of rotation
- A = the resultant external water forces
- ω = the angle of the point load from the horizontal
- α = the angle between the tangent to the centre of the base of each slice and the horizontal
- β = the base length of each slice

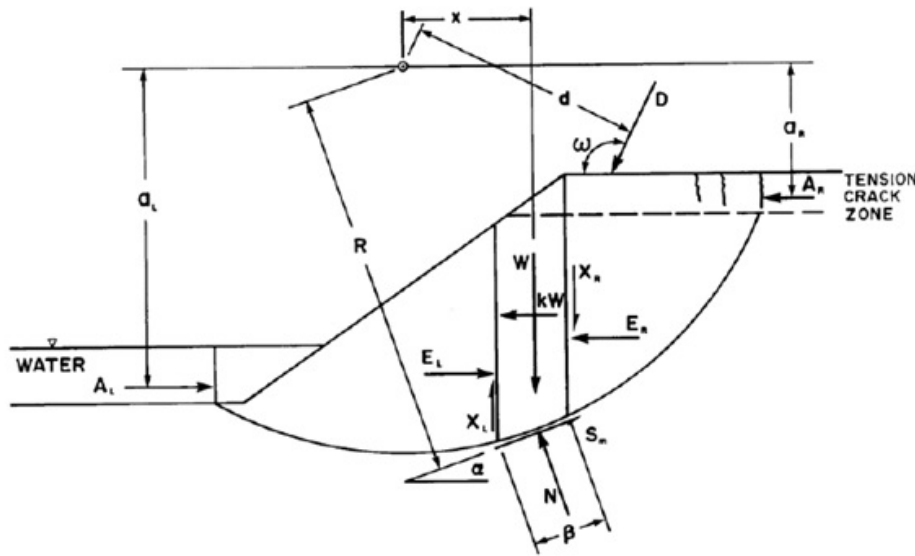


Figure 3.18: The forces acting on a slice for a circular slip surface. (GEO-SLOPE International Ltd., 2008).

The ratio between the interslice forces is iterated until the two factors of safety, F_m and F_f , are equal (GEO-SLOPE International Ltd., 2008). In the case of composite slip surface, a rigorous method generally produce more accurate results that also err on the safe side. This is illustrated in Figure 3.19, which shows a slice force polygon with good closure.

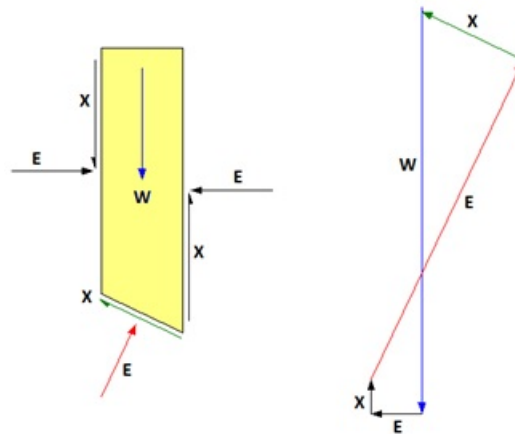


Figure 3.19: A free body diagram of a slice and, to the right, the corresponding force polygon (GEO-SLOPE International Ltd., 2008) (modified by the authors).

The Morgenstern-Price method allows for the user to select the interslice force function (GEO-SLOPE International Ltd., 2008). There are five different interslice force functions available in SLOPE/W: the constant function, the half-sine function, the clipped-sine function, the trapezoidal function and the data-point specified function. As stated in Chapter 1.4, only the half-sine function is considered in this thesis.

3.6 The Optimise Function in SLOPE/W

For cases such as elongated slopes, stratified slopes and slopes in heterogeneous soils, a composite slip surface may describe the possible slip surface more realistically. Instead of for example using a

fully specified slip surface, and thereby lose the part of the circular slip surface that seems reasonable, the critical slip surface can be refined by iteratively altering parts of it. This is referred to as an optimisation of the slip surface, and is possible in SLOPE/W with the Optimise function (GEO-SLOPE International Ltd., 2008).

If initially, as in this thesis, a circular trial slip surface is utilised, the software searches for any local conditions in the slope that would cause the slip surface to derive from its initial perfect circular shape and result in a lower factor of safety. If the Optimise function does not find any altered slip surface with a lower factor of safety than the critical trial slip surface, and $F_{opt} \geq F_{circ}$, then no optimised slip surface is stored.

The optimised, composite, slip surface is obtained by minimising the mathematical function that computes the factor of safety. This is essentially the same that is done for the both statically and kinematically correct slip surface in Chapter 3.3.3, although it is more complex with regard to both the mathematical expression itself and the degree of freedom of the slip surface line. Whereas said slip surface consists of three slip surface lines that are all functions of one angle, the optimised slip surface in SLOPE/W consists of an unlimited number of line segments that can move relative to each other in several directions.

The optimisation function implemented in SLOPE/W is based on two theories developed by Greco (1996) and Husein Malkawi et al. (2001), in which a Monte Carlo method with a statistical random walking procedure is used to optimise the critical slip surface (GEO-SLOPE International Ltd., 2008). These two methods differ slightly in the mathematical formulation of the solution but the formulation of the problem, i.e. the position of the critical slip surface, is the same for both methods. The formulation of the factor of safety within the Optimise function is always identical to the chosen calculation method that generates the trial slip surface i.e. the critical circular slip surface (Support, GEO-SLOPE International Ltd. 2014, pers.comm., 26 February).

3.6.1 Description of the Optimisation Process

The optimisation process in SLOPE/W is initiated by dividing the critical trial slip surface into a number of straight line segments, as illustrated in Figure 3.20. The number of line segments is controlled by the number of ending points, which has the default value of 16, see Table 3.3. The vertices of each of these line segments are then relocated in a Monte Carlo based statistically random routine within an adjacent elliptical search area, also seen in Figure 3.20, to search for a potentially existing lower factor of safety (GEO-SLOPE International Ltd., 2008).

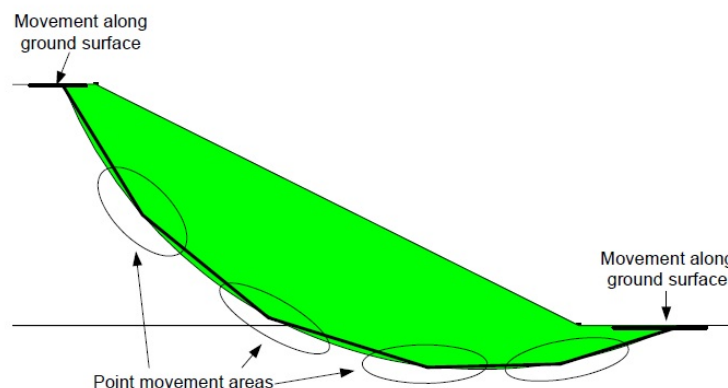


Figure 3.20: The elliptical search areas around the vertices of the broken line that compose the slip surface at the beginning of the optimisation procedure (GEO-SLOPE International Ltd., 2008).

As the user also specifies the number of starting points, or use the default value of 8, this setting combined with the number of ending points control to what extent the optimised slip surface is able to deviate from the circular slip surface from which it is constructed. A high value of the number of starting points results in the starting slip surface, made out of the straight line segments, that is more similar to the circular slip surface since it is allowed to more closely follow the circular arc.

Table 3.3: Default and limit values of the controlling parameters for the Optimise function in SLOPE/W.

Parameter	Default value	Limits
Maximum numbers of iterations	2 000	-
Convergence tolerance for the factor of safety	10^{-7}	-
Number of starting points on slip surface	8	> 0
Number of ending points on slip surface	16	$\geq$ starting points
Number of complete passes per point insertion	1	-
Slip surface maximum concave angle on the driving side, β_d	5°	$0^\circ < \beta_d \leq 30^\circ$
Slip surface maximum concave angle on the resisting side, β_r	1°	$0^\circ < \beta_r \leq 10^\circ$

The first vertex to be relocated is the one in which the slip surface enters the ground surface (GEO-SLOPE International Ltd., 2008). This point is moved randomly along the ground surface, and not in an elliptical search area as the ground surface is one of the defined fixities described in Chapter 3.6.2, until the minimum local factor of safety is found. Next, the vertices along the slip surface are successively moved within an elliptical search area until the coordinates that correspond to the lowest factor of safety is found for each vertex. Finally, the exit point is moved randomly along the ground surface.

When all of the vertices have been potentially relocated, the longest line segment is subdivided into two parts which creates a new vertex to be randomly moved until finding the critical location (GEO-SLOPE International Ltd., 2008). As stated above, the number of starting and ending points i.e. the vertices can be defined by the user, the default values set by SLOPE/W are specified in Table 3.3. If the value is the same for both the starting and ending points, then the movement within the search area is the only manner in which the slip surface is altered. In the case of the default settings the number of straight line segments is doubled.

The setting *Number of complete passes per point insertion* controls how many random walks that are generated for each vertex (Support, GEO-SLOPE International Ltd. 2014, pers.comm., 26 February). The iterative optimisation procedure is repeated until changes in the factor of safety do not exceed a specified tolerance interval or until the process reaches the maximum number of optimisation trials (GEO-SLOPE International Ltd., 2008). The default values set by in SLOPE/W are specified in Table 3.3. The Optimise function is purely a minimisation process, and does not put any constraints on the slip surface, apart from those stated in Table 3.3.

As always, when using computing software such as SLOPE/W, the physical plausibility of the obtained slip surface must be evaluated by the user. In SLOPE/W it is possible to specify the maximum concave angles that the software should allow for the driving and resisting masses respectively (GEO-SLOPE International Ltd., 2008). The default values are specified in Table 3.3.

This setting is the only known limitation to what shape the optimised slip surface will obtain. It is not unheard of composite slip surfaces with some concave angles, which is indicated by the limitations of this setting i.e. that the maximum concave angles cannot be set to 0° , but most slip surface with distinct concave angles look peculiar and kinematically incorrect.

Figure 3.21 illustrates an example of a composite slip surface obtained by optimising a circular slip surface. By performing the optimisation process, SLOPE/W may detect and adjust for weak layers in the slope which usually result in a lower value of the factor of safety (GEO-SLOPE International Ltd., 2008). However, the prosperity of the Optimise function depends to some extent on how well the trial slip surface is selected. Furthermore, even with a suitable choice of trial slip surface, the optimisation may result in a physically inadmissible slip surface.

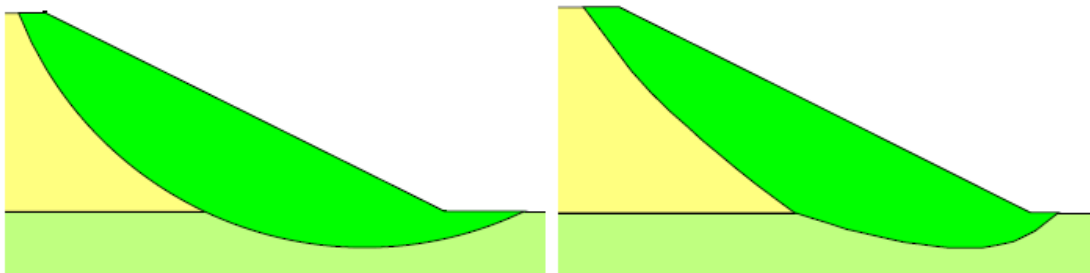


Figure 3.21: Illustration of a circular slip surface (left) and the composite slip surface obtained when applying the Optimise function (right) (GEO-SLOPE International Ltd., 2008).

When using the Fully Specified slip surface generation method for creating the trial slip surface, there is the option of specifying one or several points as fixed (GEO-SLOPE International Ltd., 2008). The software will then simply skip the relocation of these points when running the optimisation procedure. This is useful when the user has sufficient information about the soil properties, in certain points or layers, to determine that certain points ought to be included in the final critical slip surface.

3.6.2 Mathematical Formulation

As previously mentioned, the mathematical formulation of the Optimise function is based on two articles written by Greco (1996) and Husein Malkawi et al. (2001) on the subject of locating composite slip surfaces. However, these articles describe the process of locating a composite slip surface without starting from a circular nor any other type of trial slip surface. The information about the exact implementation of these articles in the programming of the Optimise function in SLOPE/W, except that there are no additional constraints other than the settings mentioned above (Support, GEO-SLOPE International Ltd. 2014, pers. comm., 26 February), is not available as it is protected by the creator of the software. Below, the optimising procedure suggested by Greco (1996) and Husein Malkawi et al. (2001) are summarised.

To formulate the problem, mathematical functions are used in the xy -plane to describe the topography of the soil layers, the slip surface and the water table as seen in Figure 3.22. Both Greco (1996) and Husein Malkawi et al. (2001) use the same functions to describe the geometrical boundaries, however the notations from Husein Malkawi et al. are used in this chapter. Equations 3.24, 3.25, 3.26, 3.27 and 3.28 describe the geometrical boundaries that can be seen in Figure 3.22: the topographic profile of the soil, the discontinuity surface in a layered soil, the lower boundary, the slip surface and the water table (Husein Malkawi et al., 2001).

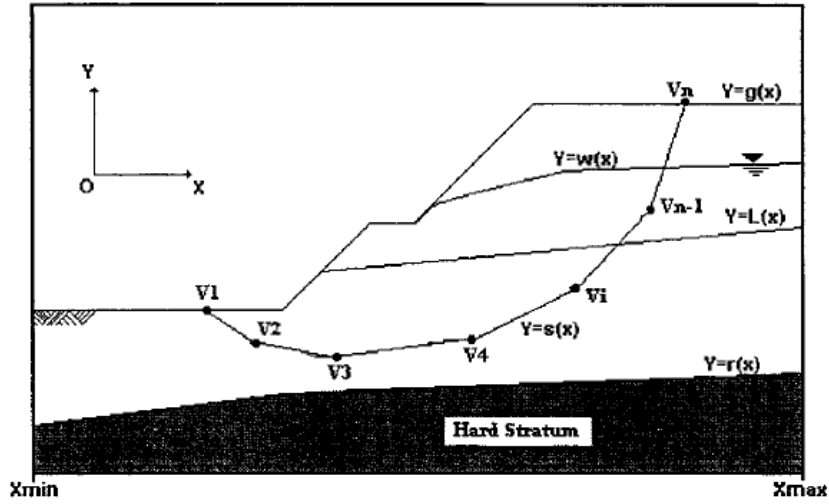


Figure 3.22: Geometrical boundaries expressed by functions and the slip surface line expressed by its vertices (Husein Malkawi et al., 2001) (modified by the authors).

$$Y = g(x) \quad (3.24)$$

$$Y = L(x) \quad (3.25)$$

$$Y = r(x) \quad (3.26)$$

$$Y = s(x) \quad (3.27)$$

$$Y = w(x) \quad (3.28)$$

The search for the critical slip surface can be divided into two phases; the exploration phase and the extrapolation phase. As mentioned in Chapter 3.6.1, the slip surface is divided into $n - 1$ straight line segments, as illustrated in Figure 3.20, and represented by n vertices $[V_1, V_2, \dots, V_n]$ with coordinates $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ (Husein Malkawi et al., 2001). The coordinates are the unknown variables of the function describing the slip surface and the optimisation consists in searching, by using Monte Carlo random walking displacement, for the coordinates that corresponds to the minimum value of that same function (Greco, 1996). As also stated in Chapter 3.6.1, the number of vertices can be controlled, which in term control the degree of approximation of a slip surface as it increases with the number of vertices (Greco, 1996).

The random walking method generates a random slip surface based on the previous; the i th slip surface is modified and used as a base for the $i + 1$ slip surface (Husein Malkawi et al., 2001). The slip surface is mathematically expressed by coordinates in a $2n$ -dimensional array in the xy -plane, $s(x)$, as in Equation 3.29.

$$S = [x_1, y_1, x_2, y_2, \dots, x_n, y_n]^T \quad (3.29)$$

As previously mentioned, the optimisation consists of minimising the factor of safety that corresponds to the vector $\mathbf{S}$, i.e. the function $F(\mathbf{S})$, as in Equation 3.30. As the problem is solved iteratively, the optimisation process follows the pattern expressed in Equation 3.31 (Greco, 1996).

$$\min F(\mathbf{S}) \quad (3.30)$$

$$F(\mathbf{S}^0) > F(\mathbf{S}^1) > \dots > F(\mathbf{S}^k) > F(\mathbf{S}^{k+1}) > \dots \quad (3.31)$$

To ensure a geometrically feasible result, three constraints are implemented on the coordinates of the above stated functions for the geometrical boundaries (Husein Malkawi et al., 2001). The constraint expressed in Equation 3.32 ensures that the vertices remain ordered, and the constraints expressed in Equation 3.33 and 3.34 guarantee that the entering and exiting vertices stay along the ground surface while all of the remaining vertices stay below it.

$$x_i < x_{i+1} \quad (3.32)$$

for $i = 1$ to $n - 1$

$$y_i = g(x_i) \quad (3.33)$$

for $i = 1$ and $i = n$

$$r(x_i) < s(x_i) < g(x_i) \quad (3.34)$$

for $i = 2$ to $n - 1$

In the exploration phase, each vertex of the slip surface S is randomly moved, starting with the vertex furthest on the driving side where the slip surface enters the ground surface (Greco, 1996). The vertex i in point (x_i^k, y_i^k) is moved to point (x_i^{k+1}, y_i^{k+1}) , and the new coordinates of the vertex are obtained as expressed in Equations 3.35, 3.36 and 3.37.

$$x_i^{k+1} = x_i^k + \xi_i^k \quad (3.35)$$

$$y_1^{k+1} = g(x_1^{k+1}) \quad (3.36)$$

for $i = 1$ and $i = n$

$$y_i^{k+1} = y_i^k + \eta_i^k \quad (3.37)$$

for $i = 2$ to $n - 1$

Here, ξ_i^k and η_i^k are the random displacements of vertex i in directions x and y respectively (Greco, 1996). The displacements are given by the expressions 3.38 and 3.39, creating an elliptical search area around the vertex, as seen in Figure 3.23.

$$\xi_i^k = N_x R_x D x_i^k \quad (3.38)$$

$$\eta_i^k = N_y R_y D y_i^k \quad (3.39)$$

In the equations above, R_x and R_y are numbers randomly extracted from a uniformly distributed population in the range $[-0.5, 0.5]$ (Greco, 1996). Dx_i^k and Dy_i^k are the widths of the search steps in directions x and y for vertex i at stage k and N_x and N_y are defined numbers whose combinations generates various displacements of vertex i for the same pair of random numbers R_x and R_y .

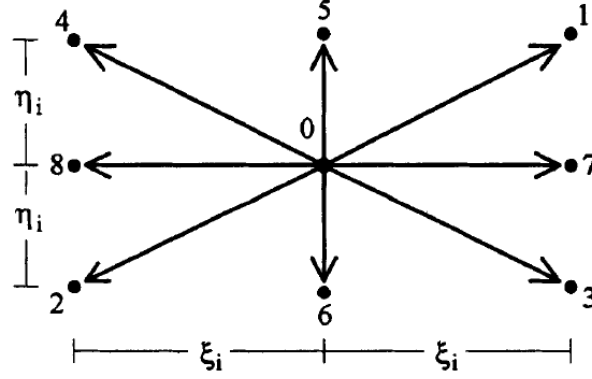


Figure 3.23: The elliptical search area for a vertex i (Greco, 1996).

If one of these trial displacements generate a lower factor of safety, no further trials are made for this vertex, and the width of the search step is increased as expressed in Equations 3.40 and 3.41 (Greco, 1996).

$$Dx_i^{k+1} = Dx_i^k + |x_i^{k+1} - x_i^k| \quad (3.40)$$

$$Dy_i^{k+1} = Dy_i^k + |y_i^{k+1} - y_i^k| \quad (3.41)$$

If no trial is successful for vertex i , then the width of the search step for the next step $k+1$ is reduced as expressed in Equations 3.42 and 3.43 (Greco, 1996).

$$Dx_i^{k+1} = Dx_i^k(1 - \epsilon) \quad (3.42)$$

$$Dy_i^{k+1} = Dy_i^k(1 - \epsilon) \quad (3.43)$$

In the equations above, ϵ is a number between 0 and 1 that should be chosen with regard to the corresponding computational time (Greco, 1996). The exploration phase is followed by the extrapolation phase in which a new slip surface, with the total displacements obtained in the exploration phase, is generated as expressed in Equations 3.44, 3.45 and 3.46.

$$x_i^e = 2x_i^{k+1} - x_i^k \quad (3.44)$$

for $i = 1$ to n

$$y_i^e = 2y_i^{k+1} - y_i^k \quad (3.45)$$

for $i = 2, \dots, n-1$

$$y_i^e = g(x_i^e) \quad (3.46)$$

for $i = 1$ or $i = n$

This iterative procedure is repeated until the current vector $\mathbf{S}^{k+1}$ simultaneously fulfils the two criteria expressed in Equations 3.47 and 3.48 (Greco, 1996), where Δ is the lowest admissible width for the search area and δ is the tolerance value for the factors of safety (Greco, 1996).

$$Dx_i^{k+1} < \Delta \quad (3.47)$$

and

$$Dy_i^{k+1} < \Delta$$

$$\forall i : i = 1 \text{ to } n$$

$$|F(\mathbf{S}^k) - F(\mathbf{S}^{k+1})| \leq \delta \quad (3.48)$$

Again, it should be remembered that it is not known exactly how the Optimise function in SLOPE/W is implemented, only that the mathematical formulation is based on the two articles by Greco (1996) and Husein Malkawi et al. (2001). Also, the mathematical formulation described here is a summary of the two articles and not a complete declaration.

3.7 Slope Stability in PLAXIS 2D

PLAXIS 2D is a two-dimensional finite element software for geotechnical applications such as stability analysis (PLAXIS 2D Reference manual, 2012). Since PLAXIS 2D is used as a tool for comparison of the results obtained with SLOPE/W, the theory behind the software will only be briefly described.

When performing a safety calculation in PLAXIS 2D, also known as a phi/c reduction, the strength parameters ϕ and c are reduced until failure occur (PLAXIS 2D Reference manual, 2012). The factor of safety for stability analysis in PLAXIS 2D is defined as the ratio of the available strength over the strength at failure, and is denoted $\sum M_{sf}$, see Equation 3.49.

$$F^{PLAXIS} = \sum M_{sf} = \frac{\text{Available strength}}{\text{Strength at failure}} \quad (3.49)$$

After defining the geometry of the model and all material properties, the geometry is divided into finite element nodes, also known as a mesh (PLAXIS 2D Reference manual, 2012). Different phases are then used to define the calculation procedure. In this thesis, three different phases are used. In phase 1, *Gravity loading*, the initial stresses are generated, in phase 2, *Plastic*, the load is applied and in phase 3, *Safety*, the factor of safety is calculated as described above.

In the calculation phase *Safety*, PLAXIS 2D uses the reduction of shear strength method i.e. reducing the friction angle and the cohesion until failure occur. A potential slip surface is shown in PLAXIS 2D by the deformation within the soil when failure occurs (PLAXIS 2D Reference manual, 2012), in this thesis by using a plot of the incremental displacements. As opposed to SLOPE/W, where the slip surface direction is defined by the user, PLAXIS 2D may show displacements anywhere in the mesh. For horizontal geometries PLAXIS 2D will generate symmetric displacements on both sides of an applied load.

There are several different material models available in PLAXIS 2D out of which Mohr-Coulomb model, described in Chapter 3.2, is selected. This material model requires five input parameters; Young's modulus, E_Y , and Poisson's ratio, ν , for elasticity, the friction angle, ϕ , and the cohesion,

c , for soil plasticity, and the dilatancy angle, ψ (PLAXIS 2D Material Models Manual, 2012).

Undrained conditions of a soil can be modelled in PLAXIS 2D with three different kinds of modelling schemes; *Undrained (A)*, *Undrained (B)* and *Undrained (C)* (PLAXIS 2D Reference manual, 2012). The main differences between these modelling schemes are that *Undrained (A)* uses the effective parameters for stiffness and strength whereas *Undrained (B)* uses effective parameters for stiffness but undrained strength parameters, and both models generates pore pressures. The model *Undrained (C)* however simulates the undrained behaviour by using a total stress analysis with undrained parameters and pore pressures are not generated.

For soils that are completely undrained, ν is by definition equal to 0.5 as no volume change can occur when the soil is exposed to stresses (Knappett and Craig, 2012). Thereby, drainage condition *Undrained (C)* is used for all of the clays. However, it is not possible to use the value of exactly 0.5 in PLAXIS 2D, and instead ν is set to 0.4990 (PLAXIS 2D Reference manual, 2012). When using *Undrained (C)*, ψ is automatically set to 0. Embankments are modelled as *Drained*.

4 Calculation Input

This chapter describes all of the studied models, starting with the bearing capacity test, followed by the characteristic slopes and lastly the steep slope with a dry crust. The geometry, soil properties and applied loads of the models are stated and illustrated in figures. The models recreated in PLAXIS 2D are also displayed in this chapter, and the alterations made of the optimise settings are explained. The calculation procedures are previously explained in Chapter 2.

4.1 Bearing Capacity Test

The bearing capacity test demonstrates bearing capacity in relation to the slip surface shape, previously described in Chapter 3.3. It consists of a surcharge load of 20 kPa, acting on 5 m and applied directly on a single horizontal clay layer, Clay 1 ud, which has a cohesive strength of 20 kPa, a unit weight of 16 kN/m³ and a depth of 8 m. The surcharge load is intentionally set to the same value as the cohesive strength of the clay. The analysis is performed for undrained conditions exclusively, and the water table is situated 0.5 m below the ground surface.

The critical circular slip surface and the corresponding factor of safety, F_{circ}^{MP} is calculated with the Grid and Radius slip surface generation method, using the Morgenstern-Price calculation method. The Optimise function is applied for the critical circular slip surface, and an optimised slip surface with corresponding factor of safety, F_{opt}^{MP} , is thereby obtained.

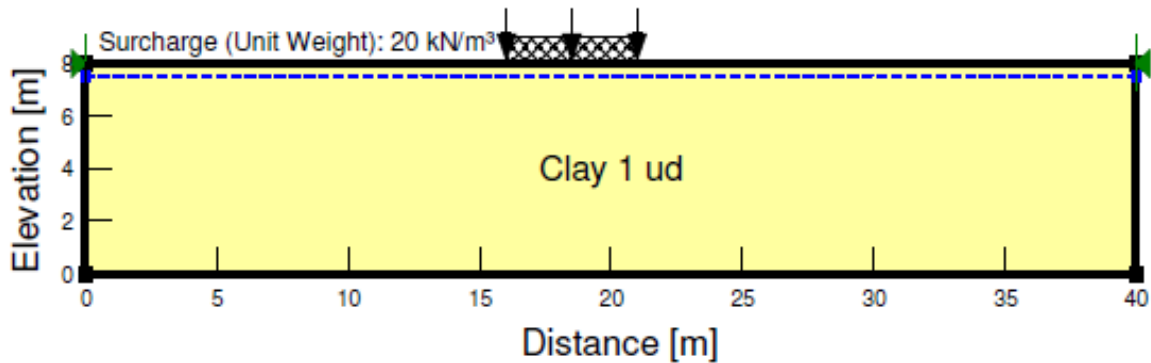


Figure 4.1: Geometry of the bearing capacity test in SLOPE/W.

4.2 Characteristic Slopes

The three characteristic slopes selected for this thesis, seen in Figure 4.2, are chosen to represent real, but slightly simplified, recurring situations encountered within stability investigations in western Sweden. The possible features of the characteristic slopes include; one or two different soil regions, open surface water and surcharge loads applied on an embankment. Two different surcharge loads are applied; 20 kPa and 43 kPa that represents traffic and railway load respectively (Trafikverket, 2011). Undrained analysis is carried out for all the different cases, and for the cases with surcharge load of 20 kPa combined analysis is also executed. The pore water pressures are generated by a piezometric line and hence hydrostatic conditions are assumed for all slopes.

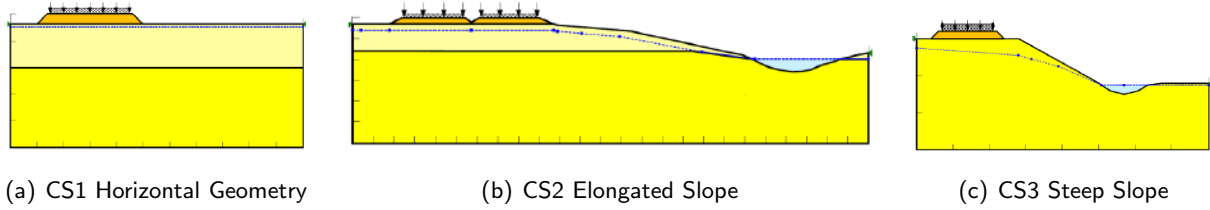


Figure 4.2: The three characteristic slopes to be modelled in SLOPE/W.

The characteristic slopes are developed to resemble typical slopes encountered in western Sweden. All of the soil layers, except for the embankment fill, are clays with a unit weight, γ , of 16 kN/m^3 and an undrained shear strength, c_u , of 20 kPa . The effective cohesion, c' , is consistently set to 10 % of the undrained shear strength. The effective friction angle, ϕ' , of all clays is 30° . The upper clay layer, Clay 1, has a thickness in the range of six to eight meters. The underlying clay layer, Clay 2, has an increase in c_u of 2 kPa per vertical meter. The embankment fill material has a γ of 18 kN/m^3 and a ϕ' of 35° . All material properties are listed in Table 4.1.

Table 4.1: Material properties for the characteristic slopes in SLOPE/W.

Material	Clay 1 ud	Clay 2 ud	Clay 1 comb	Clay 2 comb	Embankment
$\gamma \text{ [kN/m}^3\text{]}$	16	16	16	16	18
$c_u \text{ [kPa]}$	20	$20+2z$	20	$20+2z$	0
$c' \text{ [kPa]}$	-	-	$0.1(20)$	$0.1(20+2z)$	0
$\phi' \text{ [}^\circ\text{]}$	-	-	30	30	35

4.2.1 Characteristic Slope 1: Horizontal Geometry

Characteristic slope 1, CS1, consists of a horizontal clay layers that stretches 55 m in the x-direction and 23 m in the y-direction as illustrated in Figure 4.3. The slope consists of two clay layers, Clay 1 and Clay 2, with thicknesses of 8 m and 15 m respectively. The material models used in SLOPE/W for the two clays are the models $S=F(\text{depth})$ for the undrained analyses and $Comb, S=F(\text{depth})$ for the combined analysis. The groundwater table is situated 0.5 m below the ground surface. An embankment with traffic load is applied on the soil surface. The embankment has a height of 2 m and a base of 20 m and the material model *Mohr-Coulomb* was used. The surcharge load is acting on a width of 16 m.

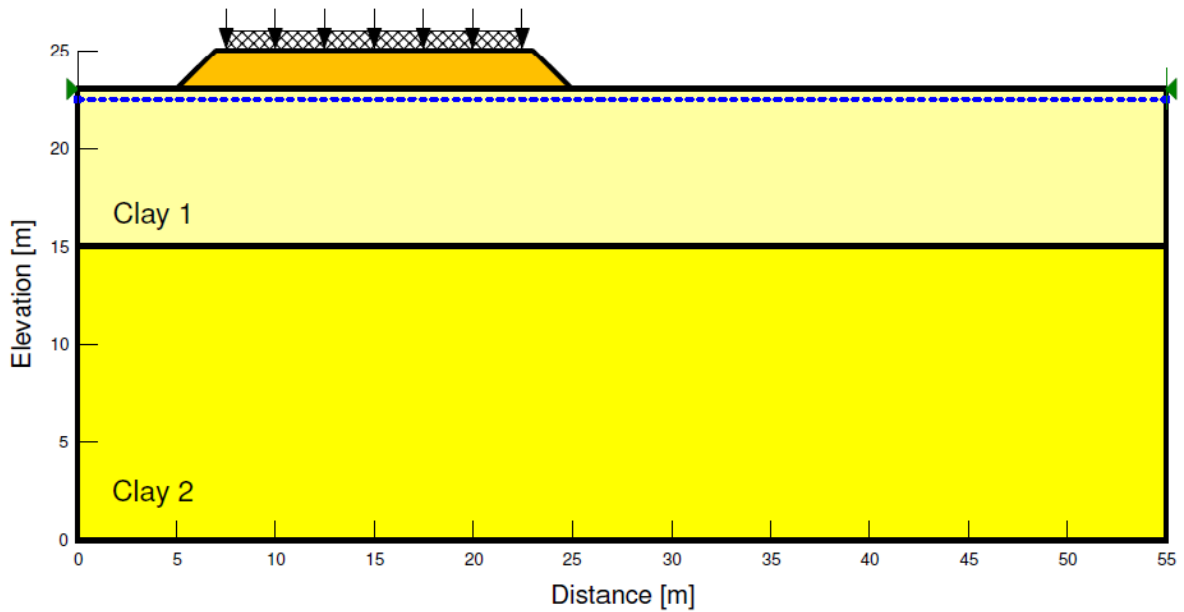


Figure 4.3: Illustration of CS1: Horizontal geometry, as defined in SLOPE/W.

4.2.2 Characteristic Slope 2: Elongated Slope

Characteristic slope 2, CS2, is an elongated slope with geometry properties as illustrated in Figure 4.3. The inclination of the slope is approximately 11° . CS2 consists of two clay layers, Clay 1 and Clay 2, with thicknesses of 6.5 m and 22 m respectively. The material models used in SLOPE/W for the two clays are the models $S=F(\text{depth})$ for the undrained analyses and $Comb, S=F(\text{depth})$ for the combined analysis. The groundwater table is situated 1.5 m below the ground surface, and flows out into open surface water region, with a depth of 3 m, at the toe of the slope.

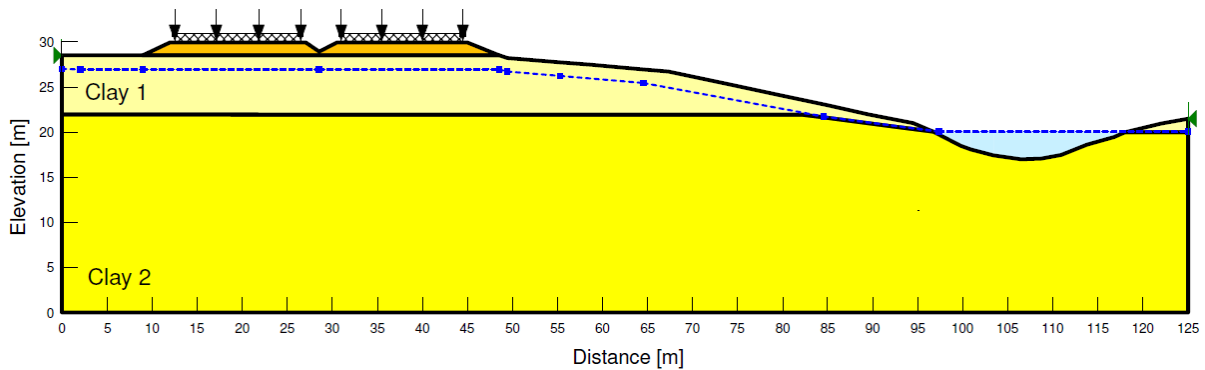


Figure 4.4: Illustration of CS2: Elongated slope, as defined in SLOPE/W.

At the crest of the slope, traffic load is applied on an embankment. The embankment is 1.5 m high and has a base of approximately 40 m, the material model used in SLOPE/W for the embankment is the model *Mohr-Coulomb*. The traffic load is modelled as two surcharge loads acting on 14 m each. The geometry for the embankment and the loads are based on a real example given by the geotechnical division at Golder Associates in Gothenburg.

4.2.3 Characteristic Slope 3: Steep Slope

Characteristic slope 3, CS3, illustrated in Figure 4.5, is a steep slope, with an inclination of approximately 27° . CS3 consists of a 15 m deep clay layer, Clay 2. For this case, the material models used in SLOPE/W for the clay are the model $S=F(datum)$ for the undrained analyses and $Comb, S=F(datum)$ for the combined analysis. These models were chosen to be able to recreate the same material behaviour in PLAXIS 2D. An embankment with applied surcharge load is acting at the crest of the slope. The embankment is 1 m high and has a base of 10 m and it is modelled with the material model *Mohr-Coulomb* in SLOPE/W. The surcharge load is acting on a width of 8 m. The groundwater table is situated approximately 1.5 m below ground surface and follows the geometry of the slope. The groundwater flows out into a open surface water region at the toe of the slope.

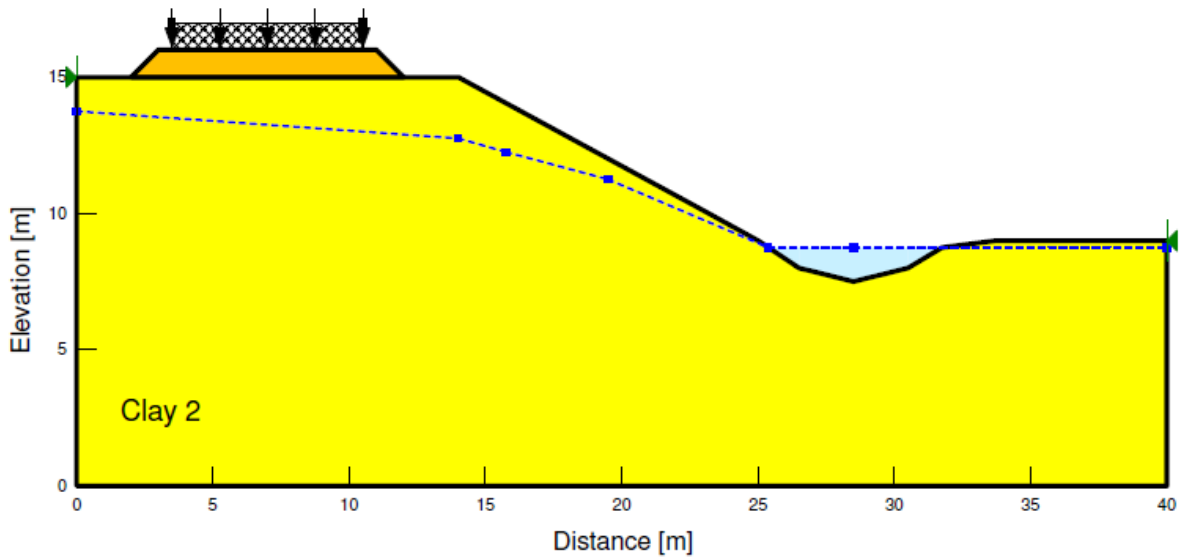


Figure 4.5: Illustration of CS3: Steep slope, as defined in SLOPE/W.

4.3 Steep Slope with Dry Crust

This slope, illustrated in Figure 4.6, is a version of CS3 prior to simplification that more accurately resemble the real case out of which CS3 was constructed. Similar to CS3, this slope has an inclination of approximately 31° but it is subjected to a surcharge load of 15 kPa acting on 8 m.

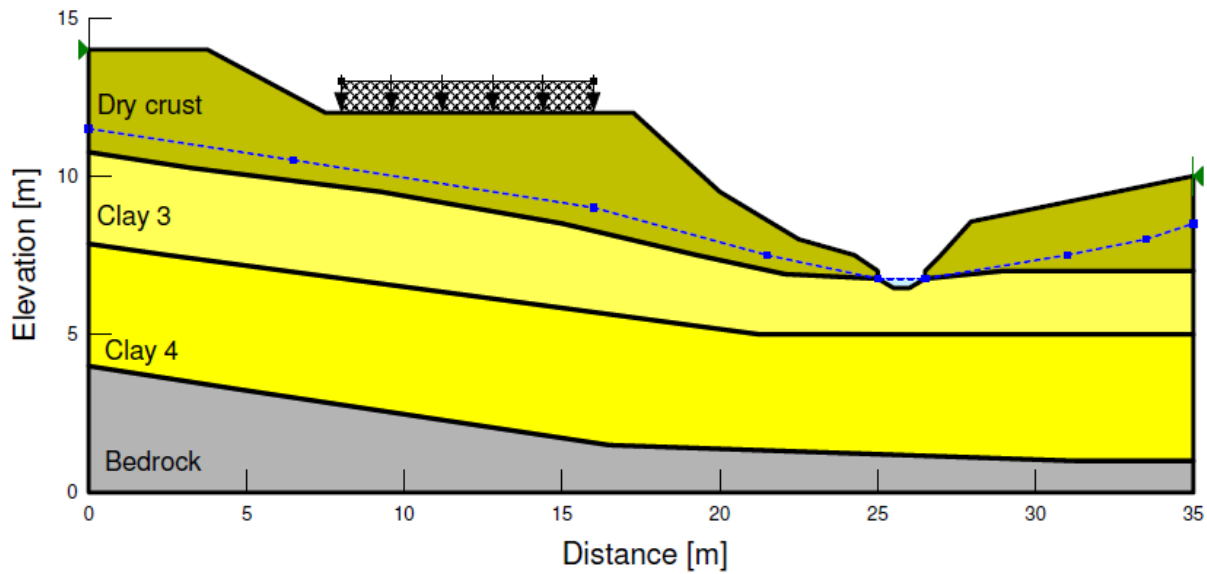


Figure 4.6: Illustration of the steep slope with a dry crust, as defined in SLOPE/W.

The upper layer of the slope is a dry crust with an approximate depth of 3 m, c_u of 50 kPa and γ equal to 19. The slope has two different clay layers, Clay 3 and Clay 4, with γ of 19 and 19.5 kN/m³ respectively. The value of c_u is 22 kPa for both clays, but with an increase of 2 kPa per vertical m for Clay 4. The material properties for the different layers can be seen in Table 4.2. The groundwater table is situated in the lower part of the dry crust and flows out into open surface water at the toe of the slope. Undrained analysis is considered exclusively.

Table 4.2: Material properties for steep slope with dry crust

Material	Clay 3	Clay 4	Dry crust	Bedrock
Material model	S=f(depth)	S=f(depth)	Undrained (Phi=0)	Bedrock (Impenetrable)
γ [kN/m ³]	19	19.5	19	-
c_u [kPa]	22	22+2z	50	-

The model is submitted to same procedure, of altering the maximum allowed concave angles on the driving and resiting side of the slope, as the bearing capacity test and the characteristic slopes. The maximum concave angles are schematically altered according to Table 4.5 in Chapter 4.5.

4.4 Models in PLAXIS 2D

The undrained analyses of all slopes are recreated in PLAXIS 2D, primarily for a verification of the slip surfaces shape, but also to compare the obtained values for the factors of safety. The characteristic slopes are modelled with both surcharge loads of 20 and 43 kPa. The PLAXIS 2D models are schematically shown in Figure 4.7. As can be seen in Figure 4.7 (e) and (f), the steep slope with dry crust is modelled with two slightly different geometries. This is a rationalisation made in order to obtain a more defined slip surface shape in PLAXIS 2D.

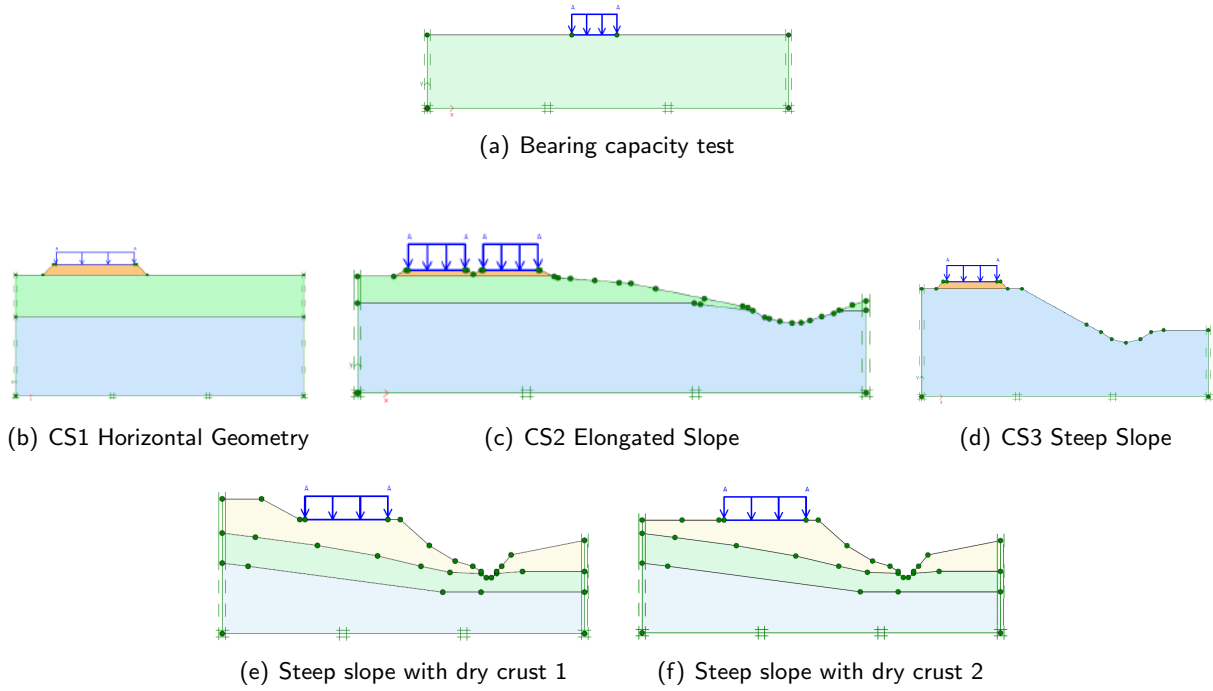


Figure 4.7: Overview of the slopes as defined in PLAXIS 2D.

The material model used in PLAXIS 2D is *Mohr-Coulomb*, which except from the input parameters used in SLOPE/W requires values of Young's modulus, E_Y , Poisson's ratio, ν and dilatancy angle, ψ . According to Knappett and Craig (2012) ν is equal to 0.5 for undrained conditions and, with this assumption, drainage type *Undrained(C)* is chosen for all clays as described in Chapter 3.7. ψ is automatically set to zero when using *Undrained(C)*. E_Y are set to $10 \cdot 10^3$ kN/m³ and $10 \cdot 10^3$ kN/m³ with an increase with depth of $2 \cdot 10^3$ kN/m³ per m for Clay 1 and Clay 2 respectively. For Clay 3 and Clay 4 E_Y are set to $20 \cdot 10^3$ kN/m³ and $20 \cdot 10^3$ kN/m³ with an increase of $2 \cdot 10^3$ kN/m³ per m to represent a stiffer clay. For the dry crust E_Y is also set to $20 \cdot 10^3$ kN/m³. The values for soil unit weight, γ , cohesion, c_{ref} , and friction angle, ϕ , are for the clays the same as in SLOPE/W. The input parameters used in PLAXIS 2D for the different clays are in Table 4.3, and the remaining input values are default values.

The embankment is modelled as a drained material with drainage type *Drained*. According to Knappett and Craig (2012), the value of ν is normally between 0.2 and 0.4 for soils under fully drained conditions. In this case, ν is set to 0.3 for the embankment. Also for the embankment ψ is set to zero. E_Y is set to $50 \cdot 10^3$ kN/m³ (Stanford University, 2010). To obtain a result in PLAXIS 2D, with a distinct slip surface and a corresponding factor of safety, the embankment needs to be defined with cohesion greater than zero to avoid soil collapse. To resemble the model in SLOPE/W, where cohesion for the embankments are equal to zero, as low a value as possible is chosen to avoid a slip surface in the embankment itself. For CS1 the cohesion is set to 5 kN/m², for CS2 10 kN/m² and for CS3 the cohesion is set to 2 kN/m³. The most relevant input values for the embankment are listed in Table 4.3, the remaining input values are default values.

Table 4.3: Material properties in PLAXIS 2D for the modelled slopes.

Material	Clay 1	Clay 2	Clay 3	Clay 4	Dry crust	Embankment
Material model	M-C	M-C	M-C	M-C	M-C	M-C
Drainage type	UD(C)	UD(C)	UD(C)	UD(C)	UD(C)	Drained
γ_{unsat} [kN/m ³]	16	16	19	19.5	19	18
γ_{sat} [kN/m ³]	16	16	19	19.5	19	20
c_{ref} [kN/m ²]	20	20+2z	22	22+2z	50	2 - 10
ϕ [°]	0	0	0	0	0	35
E_Y [kN/m ²]	20·10 ³	(20+2z)·10 ³	20·10 ³	(20+2z)·10 ³	20·10 ³	50·10 ³
ν [-]	0.4990	0.4990	0.4990	0.4990	0.4990	0.3
ψ [°]	0	0	0	0	0	0

4.5 Optimise Settings

In these calculations, the models are modified by changing two selected settings for the Optimise function: the number of starting and ending points and the maximum allowed concave angles on the driving and resisting side. The remaining settings concern the iteration process and are not considered vital for what slip surface shape that is generated by the Optimise function.

The starting and ending points control the number of vertices to relocate in the optimising process, and influence the optimised slip surface, as described in Chapter 3.6.1. To investigate the visible effect of changing this setting, as well as any notable changes of the resulting factor of safety, the number of starting and ending points are altered differently in Cases (a) through (g) according to the schematic pattern stated in Table 4.4.

Table 4.4: The investigated cases of altering the number of starting and ending points.

Number of starting and ending points			
Case	Description	Starting	Ending
Default	Default settings	8	16
a	Increase starting points	16	16
b	Decrease starting points	1	16
c	Increase ending points	8	30
d	Decrease ending points	8	8
e	Increase starting and ending points	30	30
f	Decrease starting and ending points	1	1
g	Decrease starting points, increase ending points	1	30

The setting for changing the maximum allowed concave angles, for the driving and resisting side of the optimised slip surface, is a mean of avoiding rare and unreasonable shapes. To investigate the visible effect of changing this setting, as well as any notable changes of the resulting factor of

safety, the maximum allowed concave angles are altered in Cases (a) through (h) according to the scheme stated in Table 4.5.

Unlike the number of starting and ending points, the concave angles are not specified as a singular value but as a maximum value. This means that concave angles are allowed to vary in the range of zero to the specified value. Consequently, if the shape of optimised slip surface generated with the default values do not consist of any notable concave angles, changing these settings may have little to no effect.

Table 4.5: The investigated cases of altering the maximum concave angles of the driving and resisting side of the slope.

Driving and resisting maximum concave angles			
Case	Description	Driving [°]	Resisting [°]
Default	Default settings	5	1
h	Increase on driving side	30	1
i	Decrease on driving side	0.001	1
j	Increase on resisting side	5	10
k	Decrease on resisting side	5	0.001
l	Increase on driving and resisting side	30	10
m	Decrease on driving and resisting side	0.001	0.001
n	Decrease on driving side, increase on resisting side	0.001	10
o	Increase on driving side, decrease on resisting side	30	0.001

The remaining setting for the Optimise function; the maximum numbers of iterations, the convergence tolerance for the factor of safety and the number of complete passes per point, do not directly control the shape of the slip surface but instead concern the extent of the iterations and the convergence. Therefore, the effects of altering these settings were not considered vital to investigate.

5 Calculation Result

In this chapter the results of the calculations stated in Chapter 4 are presented. The results are interpreted by reviewing the percental differences between the factors of safety corresponding to the different calculations, as explained in Chapter 2.3.3. Results from altering the settings for the Optimise function are presented in tables and selected figures.

5.1 Bearing Capacity Test

The following results for the bearing capacity test are presented in this chapter: the undrained analyses performed in SLOPE/W and PLAXIS 2D together with a comparison to the bearing capacity theory, and the results when altering the optimise settings in SLOPE/W.

Undrained Analysis, Load 20 kPa

The result from the bearing capacity test performed in SLOPE/W can be seen in Figure 5.1 and Figure 5.2, showing the circular and the optimised slip surfaces. The circular slip surface corresponds to a factor of safety, F_{circ}^{MP} , of 5.52 whereas the optimised slip surface correspond to a factor of safety, F_{opt}^{MP} , with a value of 3.79.

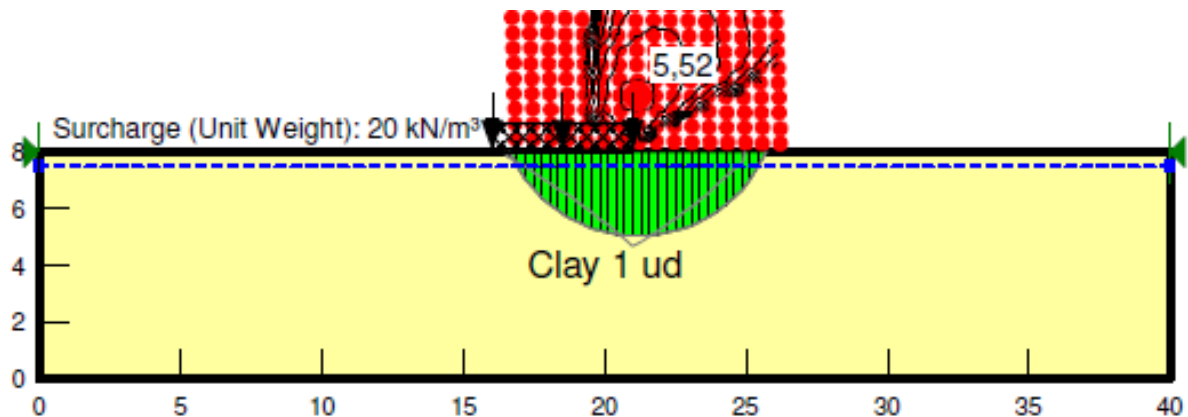


Figure 5.1: The circular slip surface of the bearing capacity test, $F_{circ}^{MP} = 5.52$. The shape of the optimised slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

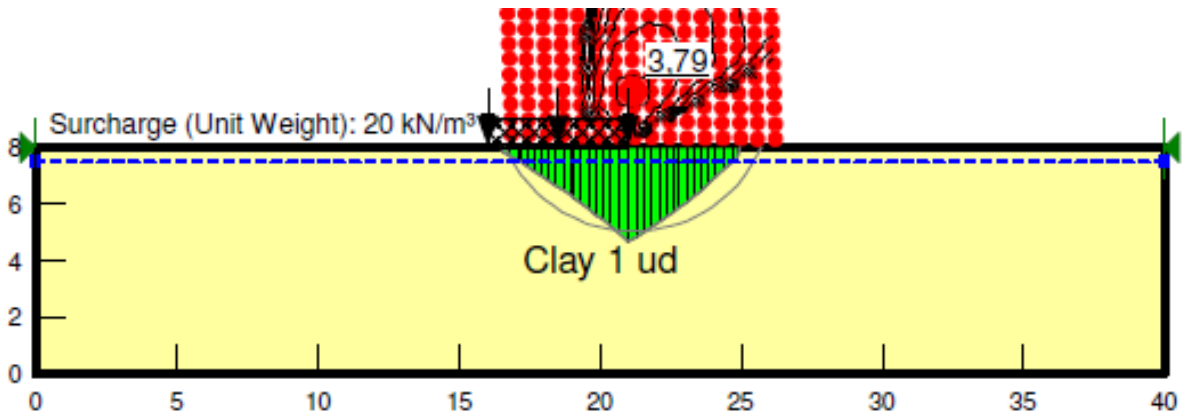


Figure 5.2: The optimised slip surface of the bearing capacity test, $F_{opt}^{MP} = 3.79$. The shape of the circular slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

The calculation performed in PLAXIS 2D, corresponding to the undrained analysis in SLOPE/W, results in a slip surface shape as shown in Figure 5.3. The factor of safety, F_{PLAXIS} is calculated to 5.15.

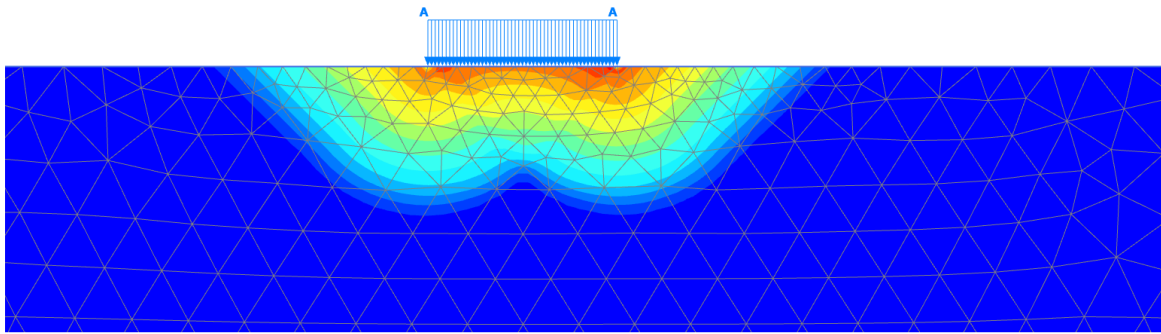


Figure 5.3: The slip surface obtained with PLAXIS 2D, $F_{PLAXIS} = 5.15$.

According to Chapter 3.3, where statically and kinematically correct slip surfaces are derived from theory, the resulting slip surfaces should have shapes as in Figure 5.4. Comparing this to the results obtained from the bearing capacity test performed in SLOPE/W and PLAXIS 2D, it can be seen that the critical circular slip surface shape from SLOPE/W has a shape that is similar to a critical kinematically correct slip surface shape as in Figure 5.4 (b). The optimised slip surface obtained from SLOPE/W has a shape that is comparable to a statically correct slip surface as in Figure 5.4 (a). The slip surface shape obtained from PLAXIS 2D on the other hand, is more similar to a statically and kinematically correct slip surface as in Figure 5.4 (c), consisting of both straight and circular arcs.

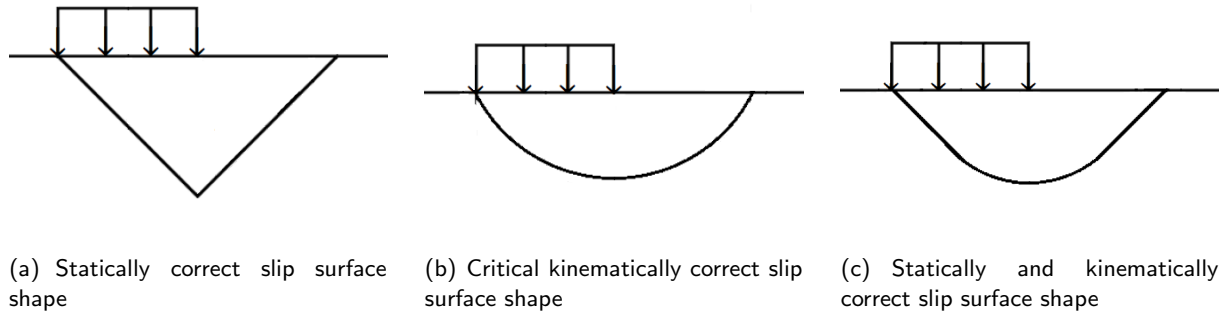


Figure 5.4: Slip surface shapes according to the bearing capacity theory.

As stated in Chapter 3.3.4, the bearing capacity factor, N_c , can be directly compared to the factor of safety, with the obtained values listed in Table 5.1. The factor of safety that corresponds to the circular slip surface shape in SLOPE/W is identical to the bearing capacity factor for a critical kinematically correct slip surface, both with a value of 5.52. The factor of safety that corresponds to the optimised slip surface in SLOPE/W is most similar to the bearing capacity factor for a statically correct slip surface, with values of 3.79 and 4.00 respectively. The factor of safety obtained from PLAXIS 2D, with a value of 5.15 is similar to a the bearing capacity factor for a slip surface that is both statically and kinematically correct, which has a value of 5.14.

Table 5.1: Summation of the factors of safety obtain in SLOPE/W and PLAXIS 2D for the bearing capacity test, and bearing capacity factors derived from theory.

Software calculation	F	Slip surface shapes from theory	N_c
SLOPE/W, Circular	5.52	Statically correct	4.00
SLOPE/W, Optimised	3.79	Critical kinematically correct	5.52
PLAXIS 2D	5.15	Statically and kinematically correct	5.14

Altered Optimise Settings

The resulting value of the factor of safety, as well as the volume and the weight of the slip surface, when altering the starting and ending points are presented in Table 5.2. The critical circular slip surface is included as a comparison. For Cases (b), (f) and (g), no optimisation of the slip surface was generated and the critical circular slip surface was the one with the lowest factor of safety.

For Case (c), with the resulting slip surface shape illustrated in Figure 5.5, the factor of safety is closer to 4.00, which according to the theory should be the correct value for a statically correct slip surface. The slip surface shape is only slightly changed from the default optimised slip surface shape, as seen in Figure 5.2.

Case (e), illustrated in Figure 5.6, results in a factor of safety of 5.12, which is the case that is closest to the bearing capacity factor for a statically and kinematically correct slip surface. The shape of the slip surface for Case (e) is also similar to the circular slip surface.

Table 5.2: The results of changing the number of starting and ending points for the Optimise function, within the Bearing capacity test.

Bearing capacity test - Number of starting and ending points					
Case	Starting	Ending	F^{MP}	Volume [m ³]	Weigth [kN]
Circular	-	-	5.52	19	309
Default optimised	8	16	3.79	15	237
a	16	16	4.51	16	246
b	1	16	5.52	19	309
c	8	30	3.91	15	242
d	8	8	3.83	13	206
e	30	30	5.12	18	293
f	1	1	5.52	19	309
g	1	30	5.52	19	309

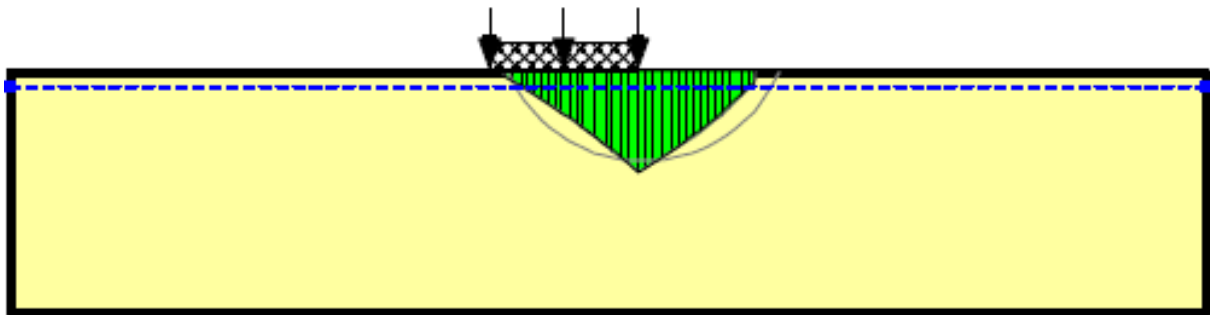


Figure 5.5: Case (c) - The optimised slip surface of the bearing capacity test, generated with 8 starting points and 30 ending points, $F^{MP} = 3.91$. The shape of the circular slip surface is also marked.

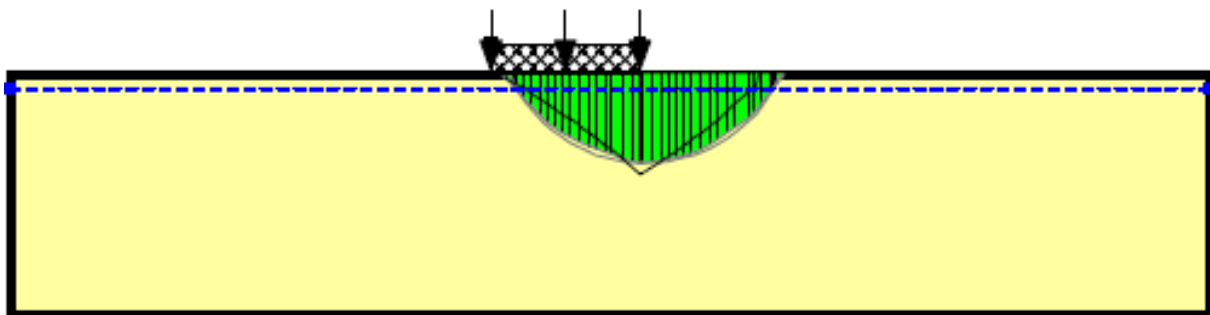


Figure 5.6: Case (e) - The optimised slip surface of the bearing capacity test, generated with 30 starting points and 30 ending points, $F^{MP} = 5.12$. The shape of the default optimised slip surface and the circular slip surface are also marked.

The resulting value for the factor of safety, as well as the volume and the weight of the slip surface, when altering the maximum concave angles on the driving and resisting sides are presented in Table 5.3. All different Cases resulted in optimised slip surfaces, but the slip surface shapes only change slightly from the default optimised slip surface. However, the factor of safety varies from 2.83 (Case (l)) to 4.38 (Case (m)). In Figure 5.7 the slip surface shape from Case (m) can be seen. The value of the factor of safety is still closest to a statically correct slip surface shape with regards to both shape and factor of safety.

Table 5.3: Results of changing, within the bearing capacity test, the driving and resisting maximum concave angles.

Bearing capacity test - Driving and resisting maximum concave angles					
Case	Driving [°]	Resisting [°]	F^{MP}	Volume [m ³]	Weigth [kN]
Circular	-	-	5.52	19	309
Default optimised	5	1	3.79	15	237
h	30	1	3.93	15	245
i	0.001	1	4.33	16	253
j	5	10	3.58	15	239
k	5	0.001	4.12	15	424
l	30	10	2.83	14	232
m	0.001	0.001	4.83	16	262
n	0.001	10	4.17	15	244
o	30	0.001	3.85	13	215

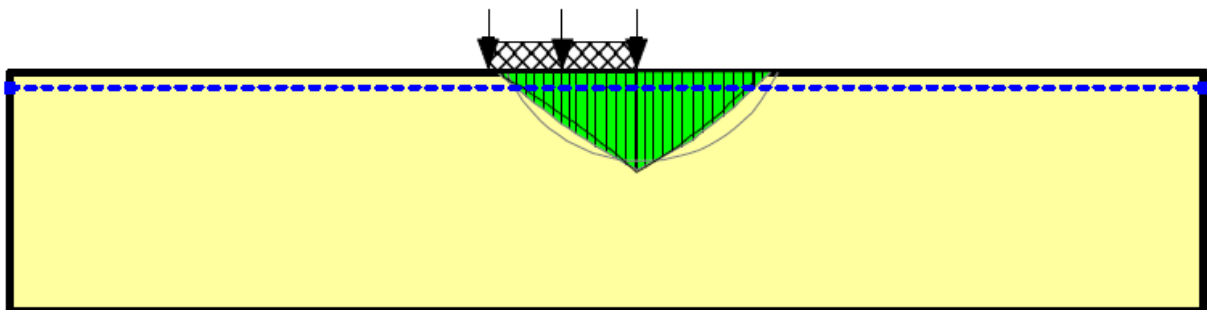


Figure 5.7: Case (m) - The optimised slip surface for the bearing capacity test, with maximum concave angles set to 0.001 for both the driving and resisting side, $F^{MP} = 4.38$. The shape of the circular slip surface is also marked with a line.

5.2 Characteristic Slopes

The results presented below correspond to the calculations described in Chapter 4.2. The differences in the values of the factors of safety are presented as explained in Chapter 2.3.3. The shape and position of the optimised slip surfaces are evaluated by comparing them to the results from the calculations performed in PLAXIS 2D calculations, described in Chapter 4.4. Further, the results from altering the settings for the Optimise function are included for each characteristic slope.

5.2.1 Characteristic Slope 1: Horizontal

The following results for CS1 are presented in this chapter: the undrained analyses performed in SLOPE/W and PLAXIS 2D with a surcharge load of 20 kPa, the results when altering the optimise settings in SLOPE/W, the undrained analysis with a surcharge load of 43 kPa performed in SLOPE/W and PLAXIS 2D, and the combined analysis with a surcharge load of 20 kPa performed in SLOPE/W.

Undrained Analysis, Load 20 kPa

The results for the undrained analysis of CS1, submitted to a surcharge load of 20 kPa, are presented in Table 5.4. The value of F_{circ}^{MP} was calculated to 1.91, which corresponds to the circular slip surface in Figure 5.8. The value of F_{opt}^{MP} was calculated to 1.68, giving a ΔF_1 of 12%. The corresponding optimised slip surface can be seen in Figure 5.9. The circular and the optimised slip surface from SLOPE/W can be compared to Figure 5.10, which illustrates the slip surface generated in PLAXIS 2D. This slip surface corresponds to F^{PLAXIS} which has a value of 1.87, giving a ΔF_4 of 2% and a ΔF_5 of -11%.

Table 5.4: Results for CS1, undrained analysis, submitted to a surcharge load of 20 kPa.

CS1 - Undrained analysis - Load 20 kPa				
F	Value	Conv. tol.	ΔF	Value
F_{circ}^{MP}	1.91	0.001	ΔF_1	$\frac{1.91-1.68}{1.91} = 12\%$
F_{opt}^{MP}	1.68	0.001	ΔF_2	$\frac{1.91-1.77}{1.91} = 7\%$
F_{circ}^{JG}	1.77	0.06	ΔF_3	$\frac{1.68-0.25}{1.68} = 85\%$
F_{opt}^{JG}	0.25	0.006	ΔF_4	$\frac{1.91-1.87}{1.91} = 2\%$
F^{PLAXIS}	1.87	-	ΔF_5	$\frac{1.68-1.87}{1.68} = -11\%$

The calculations with Janbu's Generalised method both generated an error code; E999, which occurs when the slip surface does not have a converged solution (GEO-SLOPE International Ltd., 2008). Converged solutions for F_{circ}^{JG} and F_{opt}^{JG} were obtained with the convergence tolerance set to 0.06 and 0.006 respectively. The value of F_{circ}^{JG} was then calculated to 1.77, giving a ΔF_2 of 7%. The value of F_{opt}^{JG} was calculated to 0.25, giving a ΔF_3 of 85%.

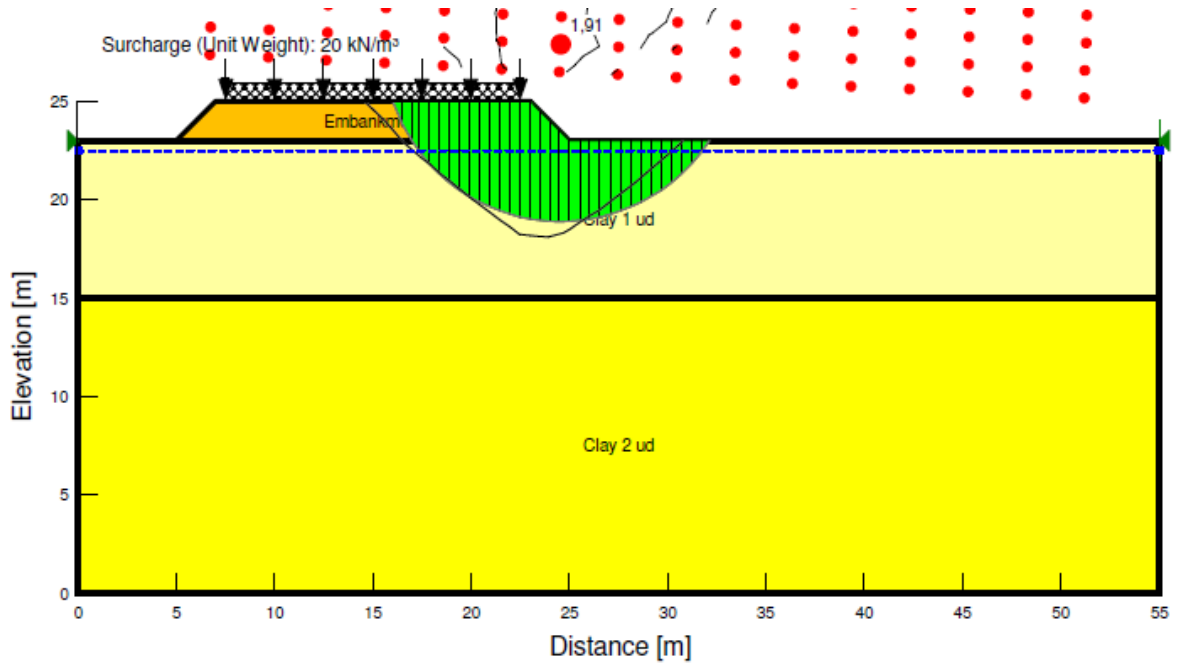


Figure 5.8: The circular slip surface of the undrained analysis of CS1, with a surcharge load of 20 kPa, $F_{circ}^{MP} = 1.91$. The shape of the optimised slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

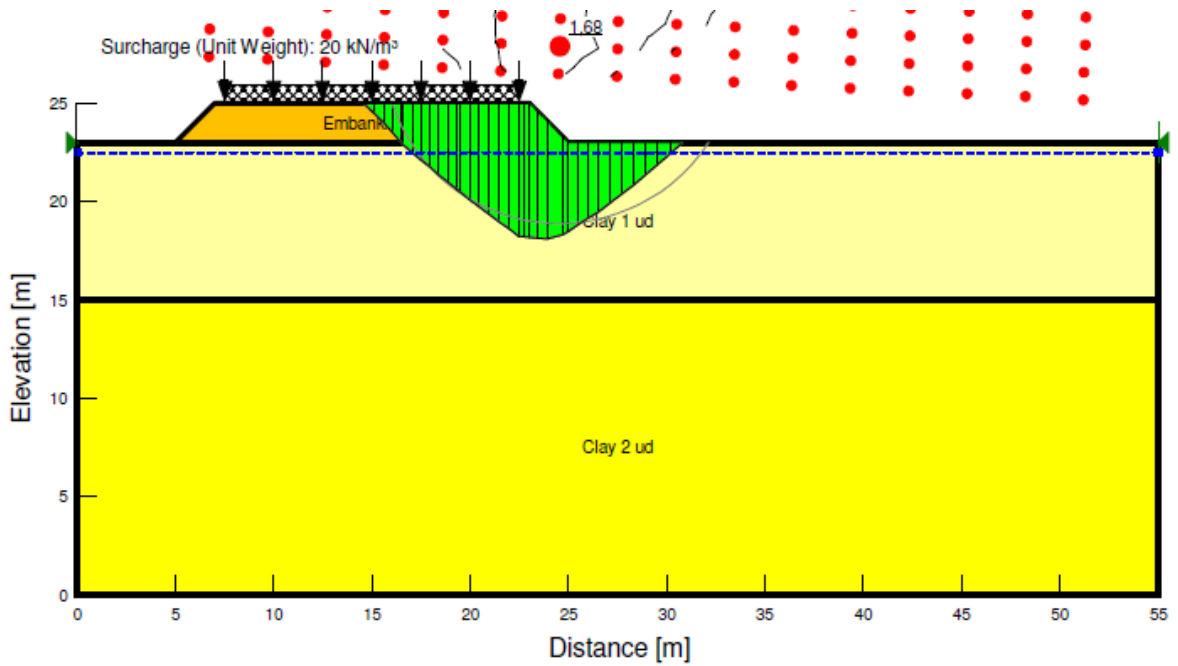


Figure 5.9: The optimised slip surface of the undrained analysis of CS1, with a surcharge load of 20 kPa, $F_{opt}^{MP} = 1.68$. The shape of the circular slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

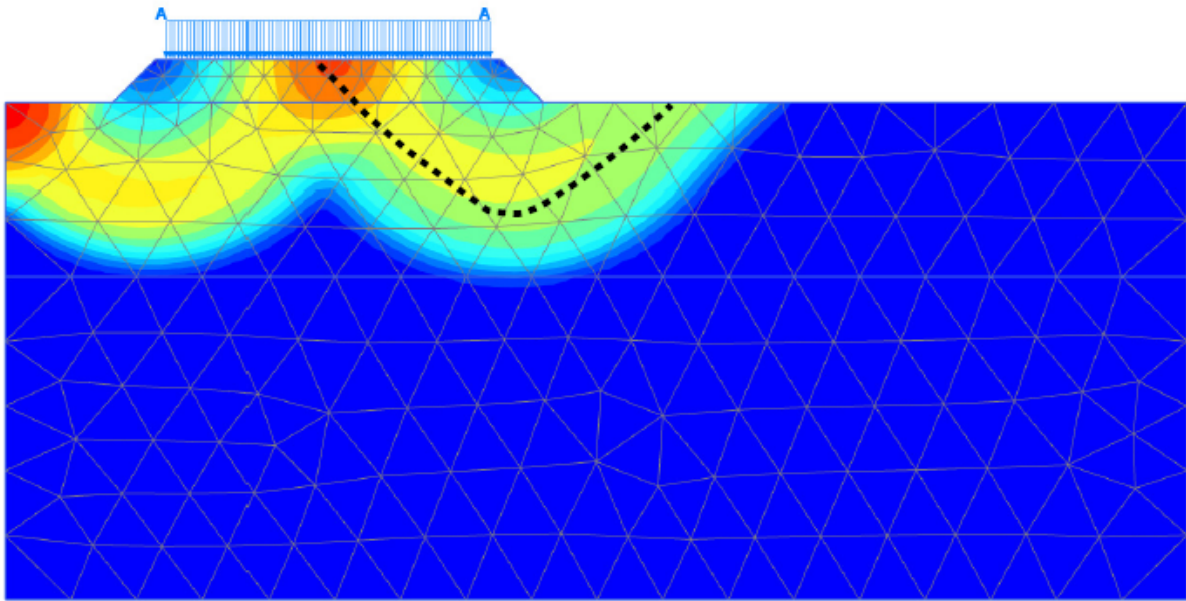


Figure 5.10: The slip surface obtained in PLAXIS 2D, when applying a surcharge load of 20 kPa to CS1, $F^{PLAXIS} = 1.87$. The optimised slip surface obtained with SLOPE/W is marked as a dotted line.

Altered Optimise Settings

The results of the changed settings for the number of starting and ending points, within the CS1 model for undrained conditions and with the applied surcharge load of 20 KPa, are presented in Table 5.5.

Table 5.5: The results of changing the number of starting and ending points for the Optimise function, within the undrained analysis of CS1 when applying a surcharge load of 20 kPa.

CS1 - Number of starting and ending points					
Case	Starting	Ending	F^{MP}	Volume [m ³]	Weigth [kN]
Circular	-	-	1.91	59	969
Default optimised	8	16	1.69	57	953
a	16	16	1.71	59	972
b	1	16	1.91	59	969
c	8	30	1.68	57	948
d	8	8	1.66	74	1 225
e	30	30	1.80	50	959
f	1	1	1.91	59	969
g	1	30	1.91	59	969

In Case (d), the value of factor of safety is 1.66 and the corresponding slip surface is seen in Figure 5.11. In Case (e), the factor of safety was calculated to 1.80 and the corresponding slip surface can be seen in 5.12. In Cases (b), (f) and (g), SLOPE/W did not generate any optimised slip surface. In Case (a) and (c), the factor safety was calculated to 1.71 and 1.69 respectively.

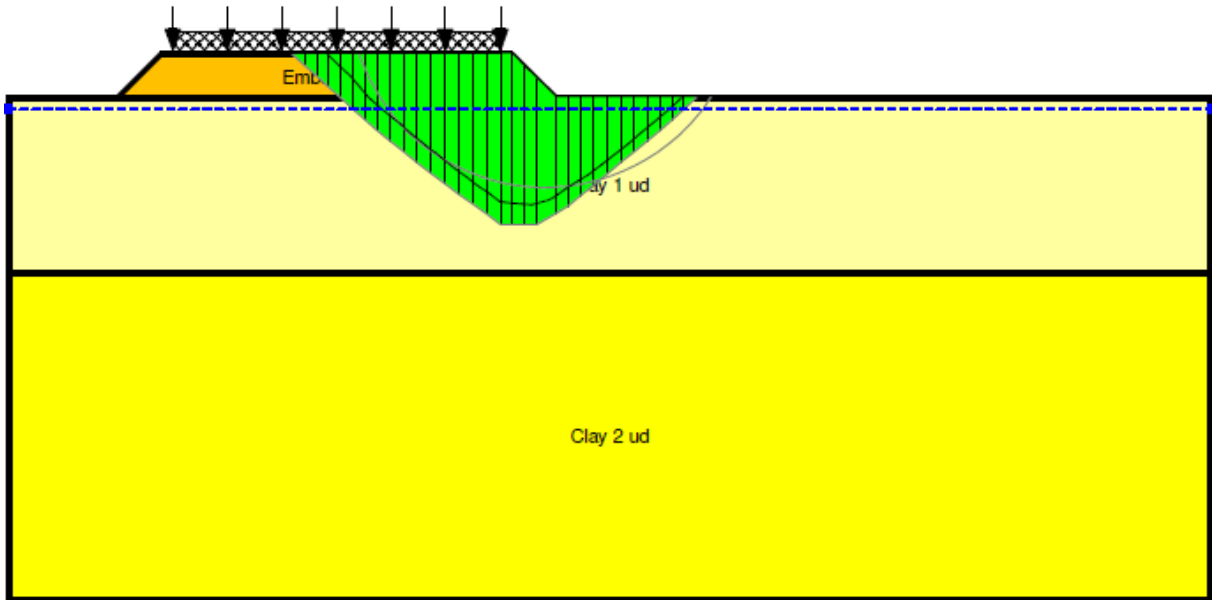


Figure 5.11: Case (d) - The optimised slip surface in the undrained analysis of CS2, generated with 8 starting points and 8 ending points, resulting in $F^{MP} = 1.66$. Both the circular and default optimised slip surfaces are marked.

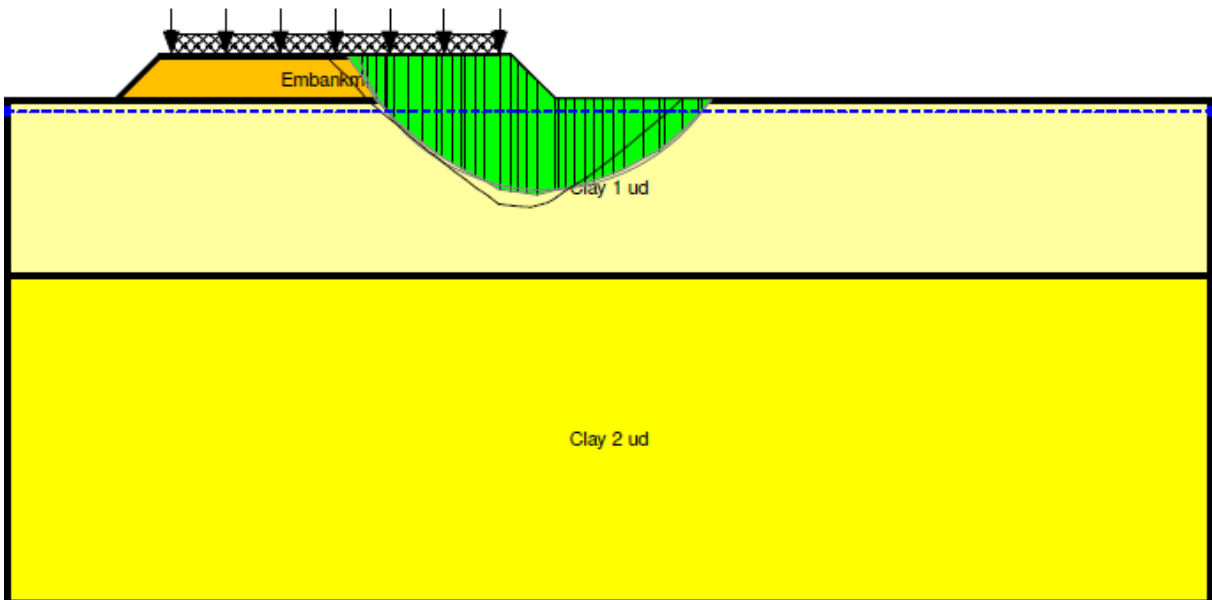


Figure 5.12: Case (e) - The optimised slip surface of the undrained analysis of CS2, optimised with 30 starting points and 30 ending points, resulting in $F^{MP} = 1.80$. Both the circular and default optimised slip surfaces are marked.

The results of the changed settings for the maximum concave angles, within the CS1 model for undrained conditions and with the applied surcharge load of 20 KPa, are presented in Table 5.6. The factors of safety vary in the range of 1.67, as for Case (l), and 1.69, as for Cases (k) and (n).

Cases (h), (i), (j), (m) and (o), all have a value of the factor of safety of 1.68 which is the same as for the case of the default settings.

Table 5.6: Results of changing, within the undrained CS1 model with an applied surcharge load of 20 kPa, the driving and resisting maximum concave angles in the Optimise settings.

CS1 - Driving and resisting maximum concave angles					
Case	Driving [°]	Resisting [°]	F^{MP}	Volume [m ³]	Weigth [kN]
Circular	-	-	1.91	59	969
Default optimised	5	1	1.68	57	953
h	30	1	1.68	60	999
i	0.001	1	1.68	61	1 006
j	5	10	1.68	58	960
k	5	0.001	1.69	57	943
l	30	10	1.67	61	1 008
m	0.001	0.001	1.68	58	957
n	0.001	10	1.69	58	957
o	30	0.001	1.68	62	1 019

Undrained Analysis, Load 43 kPa

The results for the undrained analysis of CS1, here submitted to a surcharge load of 43 kPa, are presented in Table 5.7. The value of F_{circ}^{MP} was calculated to 1.37, which corresponds to the circular slip surface in Figure 5.13. The value of F_{opt}^{MP} was calculated to 1.15, giving a ΔF_1 of 16%. The corresponding optimised slip surface can be seen in Figure 5.14. The circular and the optimised slip surface from SLOPE/W can be compared to Figure 5.15, which illustrates the slip surface generated in PLAXIS 2D. This slip surface corresponds to F^{PLAXIS} which has a value of 1.33, which gives a ΔF_4 of 3% and a ΔF_5 of -16%.

Table 5.7: Results for CS1, undrained analysis, submitted to a surcharge load of 43 kPa.

CS1 - Undrained analysis - Load 43 kPa				
F	Value	Conv. tol.	ΔF	Value
F_{circ}^{MP}	1.37	0.001	ΔF_1	$\frac{1.37-1.15}{1.37} = 16\%$
F_{opt}^{MP}	1.15	0.001	ΔF_2	$\frac{1.37-1.24}{1.37} = 9\%$
F_{circ}^{JG}	1.24	0.008	ΔF_3	$\frac{1.15-0.11}{1.15} = 90\%$
F_{opt}^{JG}	0.11	0.09	ΔF_4	$\frac{1.37-1.33}{1.37} = 3\%$
F^{PLAXIS}	1.33	-	ΔF_5	$\frac{1.15-1.33}{1.15} = -16\%$

The calculations with Janbu's Generalised method both generated error code E999; which occurs when the slip surface does not have a converged solution (GEO-SLOPE International Ltd., 2008). Converged solutions for F_{circ}^{JG} and F_{opt}^{JG} were obtained with the convergence tolerance set to 0.008 and 0.09 respectively. The value of F_{circ}^{JG} was then calculated to 1.24, giving a ΔF_2 of 9%. The value of F_{opt}^{JG} was calculated to 0.11, giving a ΔF_3 of 90%.

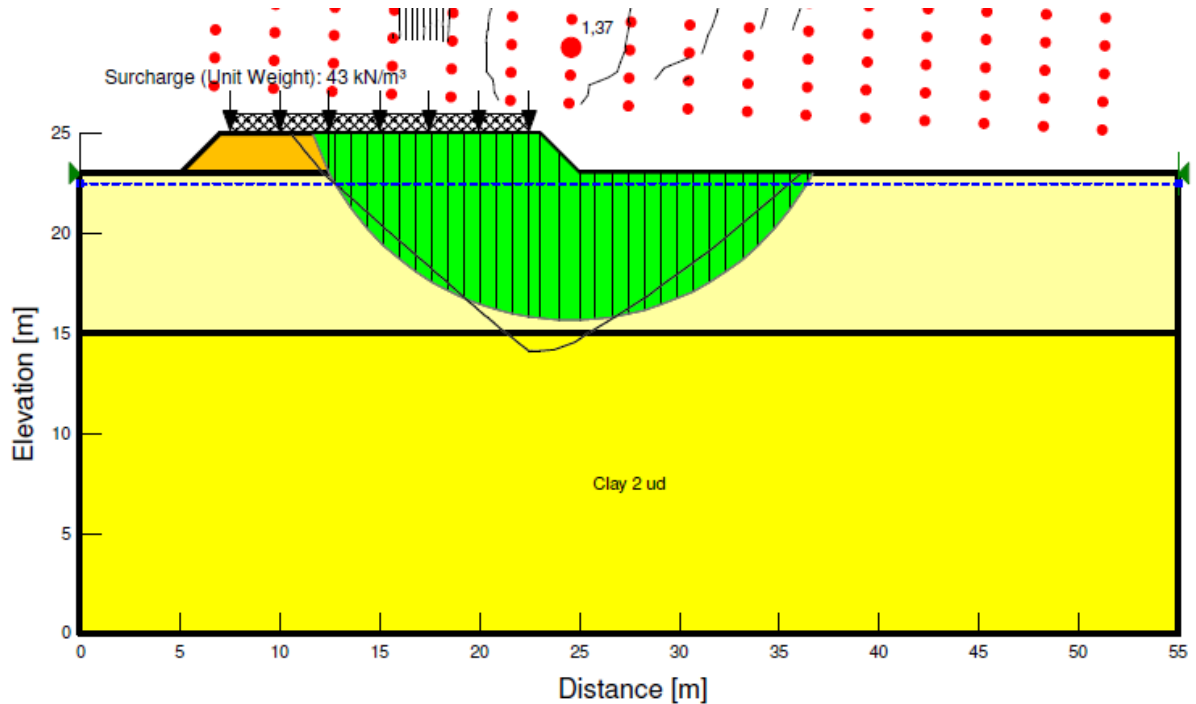


Figure 5.13: The circular slip surface of the undrained analysis of CS1, with a surcharge load of 43 kPa, $F_{circ}^{MP} = 1.37$. The default optimised slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

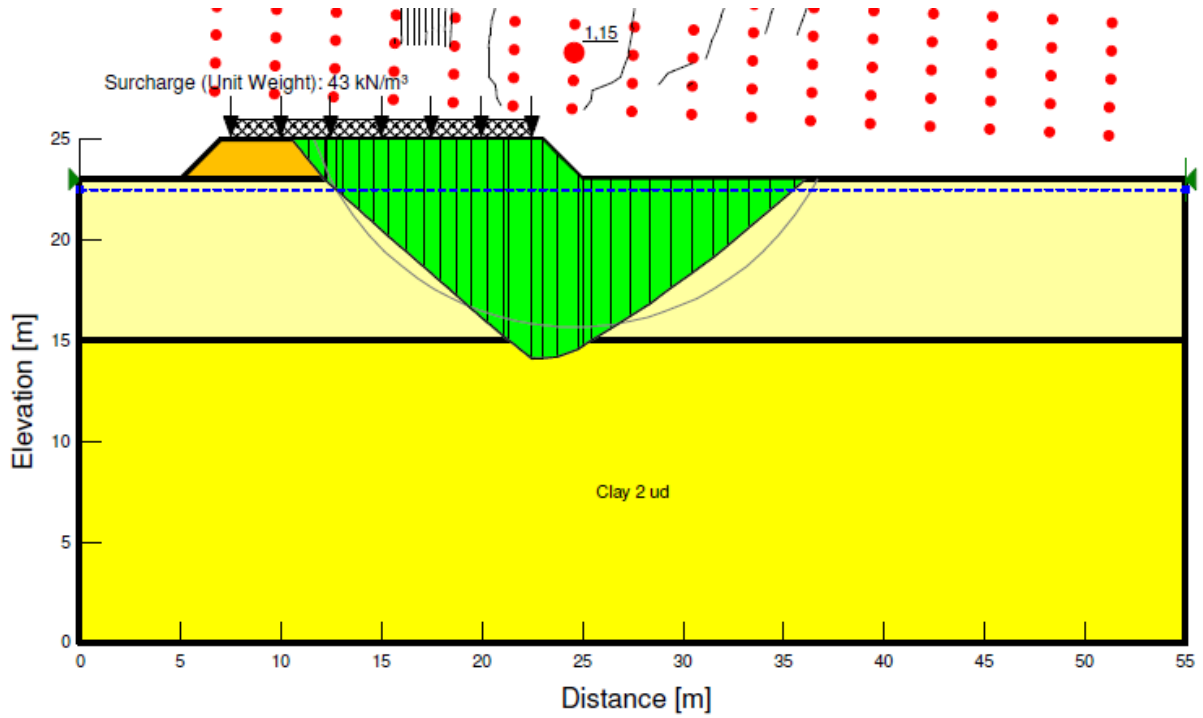


Figure 5.14: The optimised slip surface of the undrained analysis of CS1, with a surcharge load of 43 kPa, $F_{opt}^{MP} = 1.15$. The default optimised slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

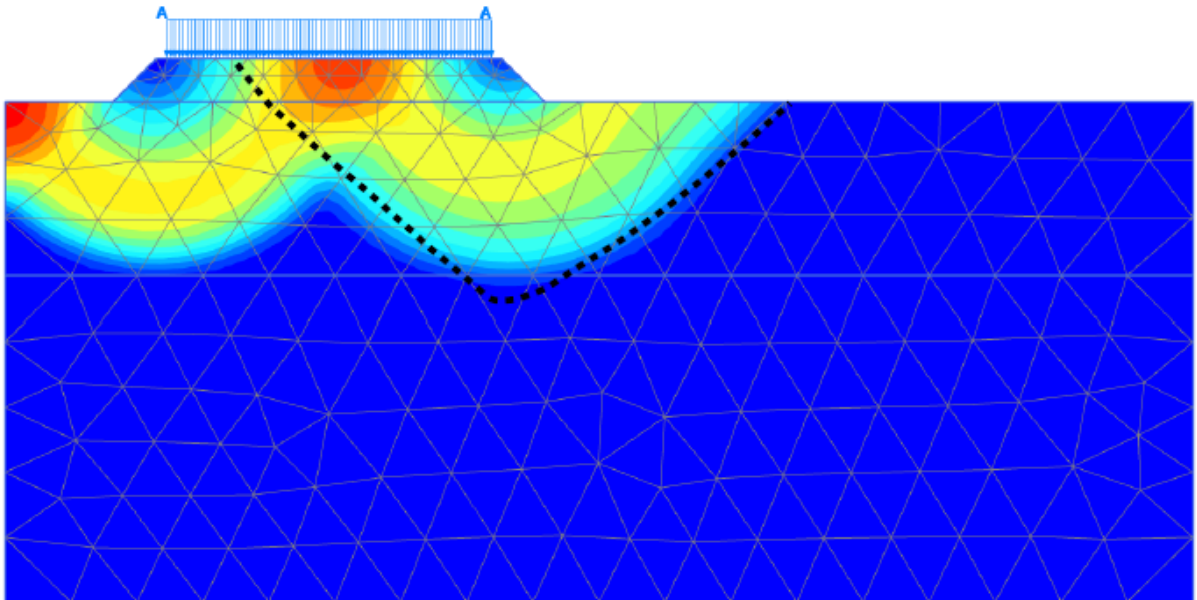


Figure 5.15: The slip surface obtained in PLAXIS 2D, when applying a surcharge load of 43 kPa to CS1, $F^{PLAXIS} = 1.33$. The optimised slip surface obtained with SLOPE/W is marked as a dotted line.

Combined Analysis, Load 20 kPa

The results for the combined analysis of CS1, submitted to a surcharge load of 20 kPa, are presented in Table 5.8. To avoid the slip surface to only intersect in the embankment, the maximum slip surface depth was changed to 4 m. For the combined analysis, the value of F_{circ}^{MP} was calculated to 1.69 and

the corresponding circular slip surface can be seen in Figure 5.16. The value of F_{opt}^{MP} was calculated to 1.59, with a corresponding optimised slip surface seen in Figure 5.17.

Table 5.8: Results for CS1, combined analysis, submitted to a surcharge load of 20 kPa.

CS1 - Combined analysis - Load 20 kPa				
F	Value	Conv. tol.	ΔF	Value
F_{circ}^{MP}	1.69	0.001	ΔF_1	$\frac{1.69-1.59}{1.69} = 6\%$
F_{opt}^{MP}	1.59	0.001	ΔF_2	$\frac{1.69-1.45}{1.69} = 14\%$
F_{circ}^{JG}	1.45	0.008	ΔF_3	$\frac{1.59-0.93}{1.59} = 42\%$
F_{opt}^{JG}	0.93	0.08		

To enable calculation of F_{opt}^{JG} the convergence tolerance needed to be changed to 0.08, in order to avoid error code E999, resulting in factor of safety of 0.93 and a ΔF_3 of 42%. F_{circ}^{JG} displayed error code E997 for the default convergence settings, which indicates that the slip surface exit angle is too steep (GEO-SLOPE International Ltd., 2008). When changing the convergence tolerance to 0.008, F_{circ}^{JG} was calculated to 1.45, giving a ΔF_2 of 14%.

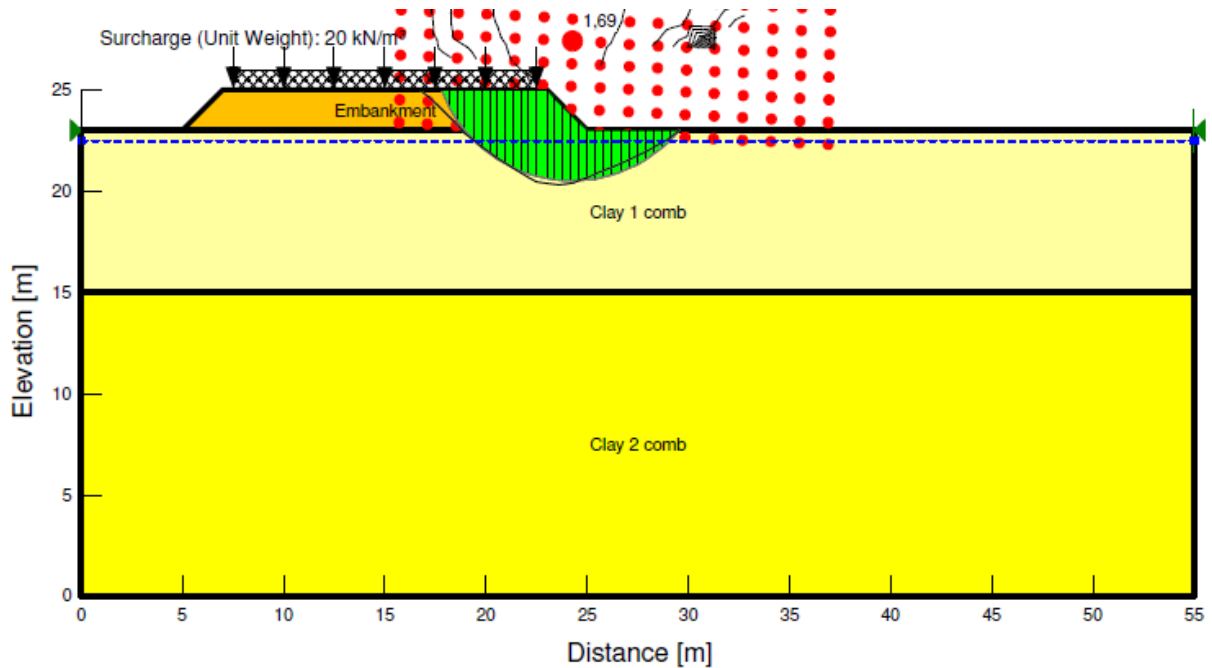


Figure 5.16: The circular slip surface of the combined analysis of CS1, with a surcharge load of 20 kPa, $F_{circ}^{MP} = 1.69$. The optimised slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

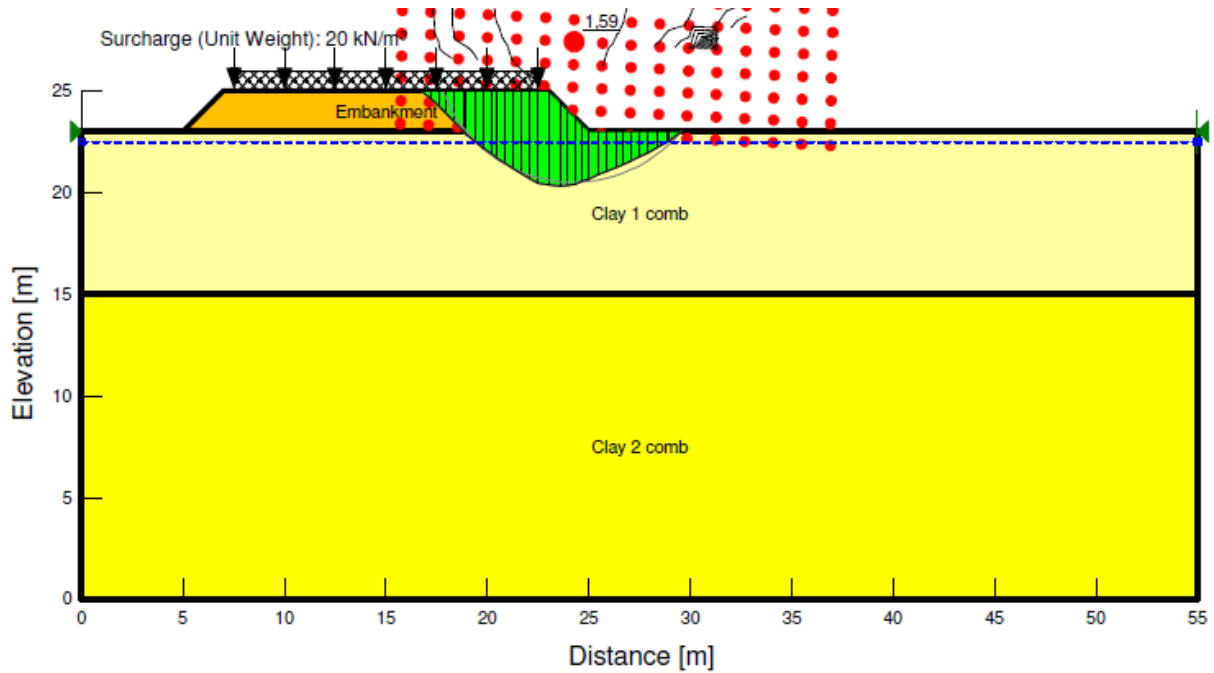


Figure 5.17: The optimised slip surface of the combined analysis of CS1, with a surcharge load of 20 kPa, $F_{opt}^{MP} = 1.59$. The circular slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

5.2.2 Characteristic Slope 2: Elongated

The following results for CS2 are presented in this chapter: the undrained analyses performed in SLOPE/W and PLAXIS 2D with a surcharge load of 20 kPa, the results when altering the optimise settings in SLOPE/W, the undrained analysis with a surcharge load of 43 kPa performed in SLOPE/W and PLAXIS 2D, and the combined analysis with a surcharge load of 20 kPa performed in SLOPE/W.

Undrained Analysis, Load 20 kPa

The calculated results for the undrained analysis of CS2, submitted to a surcharge load of 20 kPa, can be seen in Table 5.9. The value of F_{circ}^{MP} was calculated to 1.39, which corresponds to the circular slip surface in Figure 5.18. The value of F_{opt}^{MP} was calculated to 1.34, giving a ΔF_1 of 4%. The corresponding optimised slip surface can be seen in Figure 5.19. The circular and the optimised slip surface from SLOPE/W can be compared to Figure 5.20, which illustrates the slip surface generated in PLAXIS 2D. This slip surface corresponds to F^{PLAXIS} which has a value of 1.35, giving a ΔF_4 of 3% and a ΔF_5 of 0%.

Table 5.9: Results for CS2, undrained analysis, submitted to a surcharge load of 20 kPa.

CS2 - Undrained analysis - Load 20 kPa				
F	Value	Conv. tol.	ΔF	Value
F_{circ}^{MP}	1.39	0.001	ΔF_1	$\frac{1.39-1.34}{1.39} = 4\%$
F_{opt}^{MP}	1.34	0.001	ΔF_2	$\frac{1.39-1.39}{1.39} = 0\%$
F_{circ}^{JG}	1.39	0.05	ΔF_3	$\frac{1.34-1.30}{1.34} = 3\%$
F_{opt}^{JG}	1.30	0.07	ΔF_4	$\frac{1.39-1.35}{1.39} = 3\%$
F^{PLAXIS}	1.35	-	ΔF_5	$\frac{1.34-1.35}{1.34} = 0\%$

The calculations with Janbu's Generalised method both generated an error code; E999, which occurs when the slip surface does not have a converged solution (GEO-SLOPE International Ltd., 2008). Converged solutions for F_{circ}^{JG} and F_{opt}^{JG} were obtained with the convergence tolerance set to 0.05 and 0.07 respectively. The value of F_{circ}^{JG} was then calculated to 1.39, giving a ΔF_2 of 0%. The value of F_{opt}^{JG} was calculated to 1.30, giving a ΔF_3 of 3%.

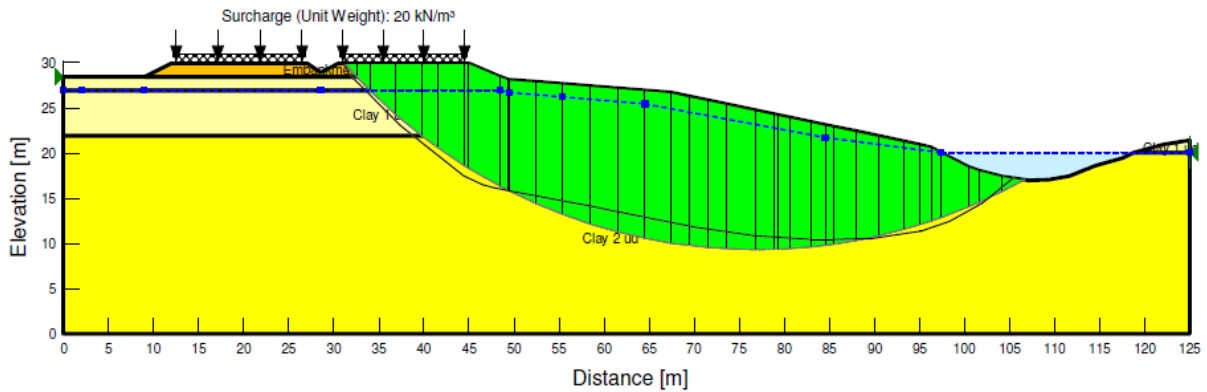


Figure 5.18: The circular slip surface of the undrained analysis of CS2, surcharge load of 20 kPa, $F_{circ}^{MP} = 1.39$. The optimised slip surface is also marked.

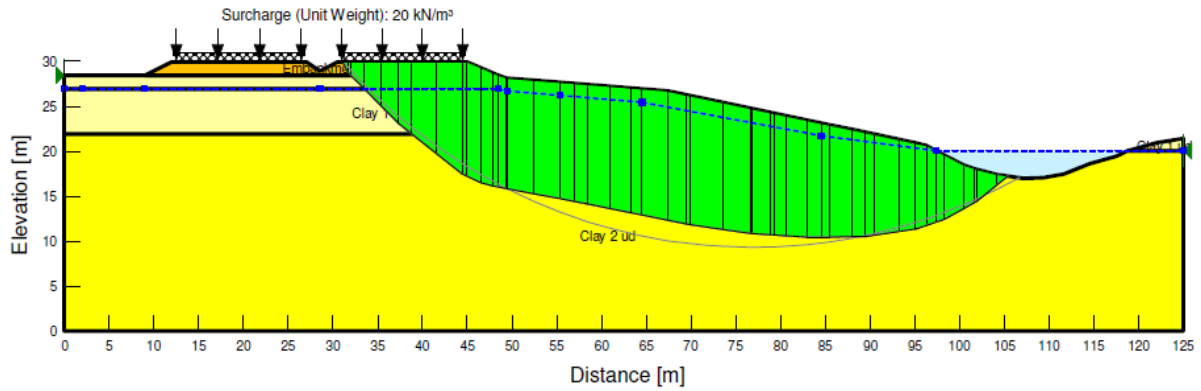


Figure 5.19: The optimised slip surface of the undrained analysis of CS2, with an applied surcharge load of 20 kPa, $F_{opt}^{MP} = 1.34$. The circular slip surface is also marked.

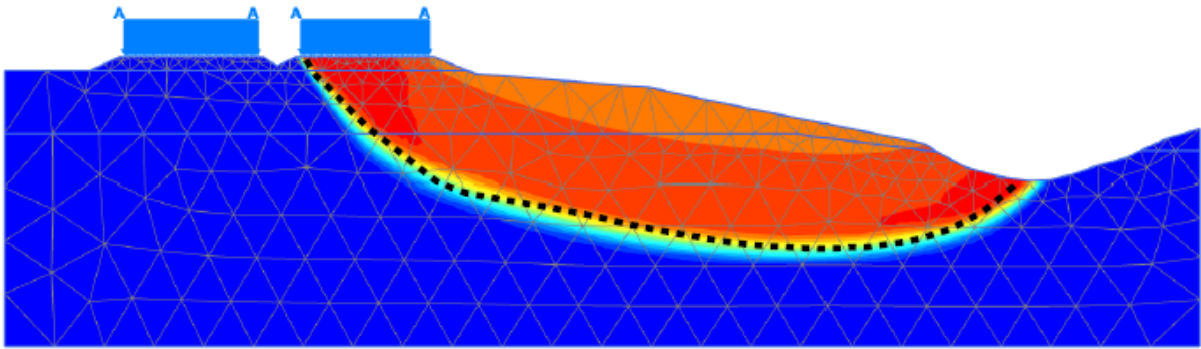


Figure 5.20: The slip surface obtained in PLAXIS 2D, when applying a surcharge load of 20 kPa to CS2, $F^{PLAXIS} = 1.35$. The optimised slip surface obtained with SLOPE/W is marked as a dotted line.

Altered Optimise Settings

The results of the changed settings for the number of starting and ending points, within the CS2 model for undrained conditions and with the applied surcharge load of 20 KPa, are presented in Table 5.10.

Table 5.10: The results of changing the number of starting and ending points for the Optimise function, within the undrained analysis of CS2 when applying a surcharge load of 20 kPa.

CS2 - Number of starting and ending points					
Case	Starting	Ending	F^{MP}	Volume [m ³]	Weigth [kN]
Circular	-	-	1.39	842	13 523
Default optimised	8	16	1.34	802	12 870
a	16	16	1.34	830	13 327
b	1	16	1.38	608	9 766
c	8	30	1.34	802	12 885
d	8	8	1.35	793	12 727
e	30	30	1.36	840	13 482
f	1	1	1.39	842	13 523
g	1	30	1.39	596	9 586

The values of F^{MP} vary in the range of 1.34 and 1.39, which coincides with the values of F^{MP} for the default settings for the Optimise function and the circular slip surface. In Case (b), the value of the factor of safety is 1.38 and the corresponding slip surface is seen in Figure 5.21. In Case (d), the factor of safety was calculated to 1.35 and the corresponding slip surface can be seen in Figure 5.22. In Case (e), the value of the factor of safety is 1.36, with a slip surface as seen in Figure 5.23. In Case (f), SLOPE/W did not generate any optimised slip surface. In Cases (a) and (c), the factors safety was calculated to 1.34 and in Case (g) to 1.39.

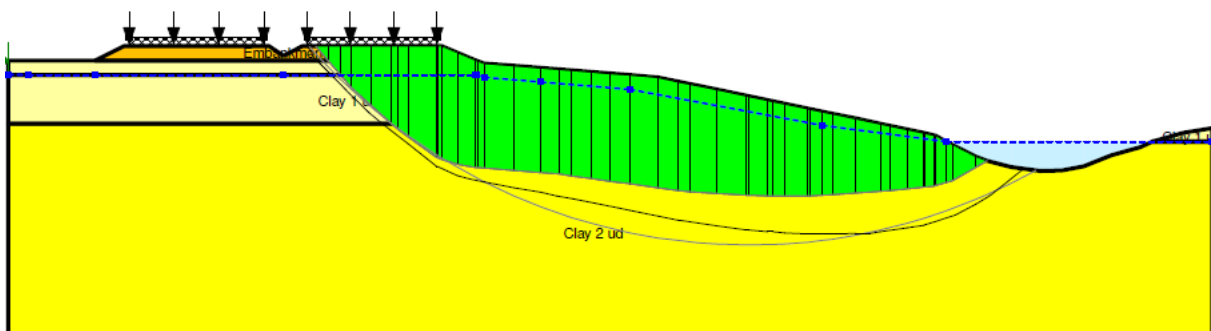


Figure 5.21: Case (b) - The optimised slip surface in the undrained analysis of CS2, generated with 1 starting point and 16 ending points, resulting in $F^{MP} = 1.38$. The circular and the default optimised slip surfaces are also marked.

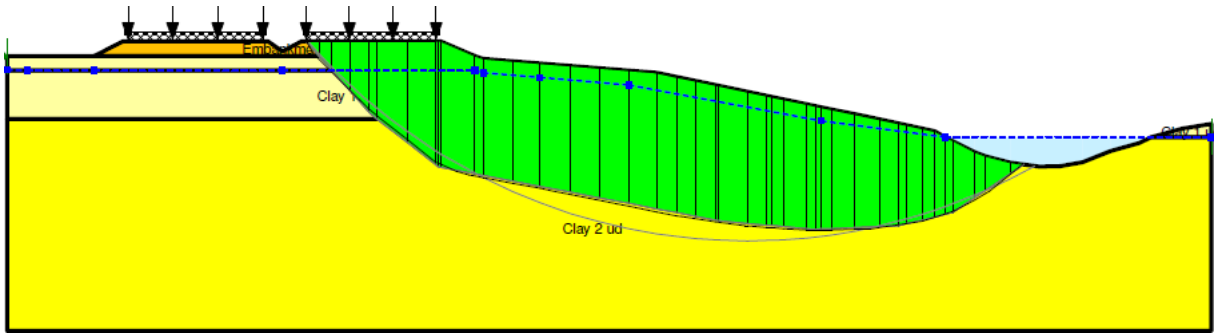


Figure 5.22: Case (d) - The optimised slip surface in the undrained analysis of CS2, generated with 8 starting points and 8 ending points, resulting in $F^{MP} = 1.35$. The circular and the default optimised slip surfaces are also marked.

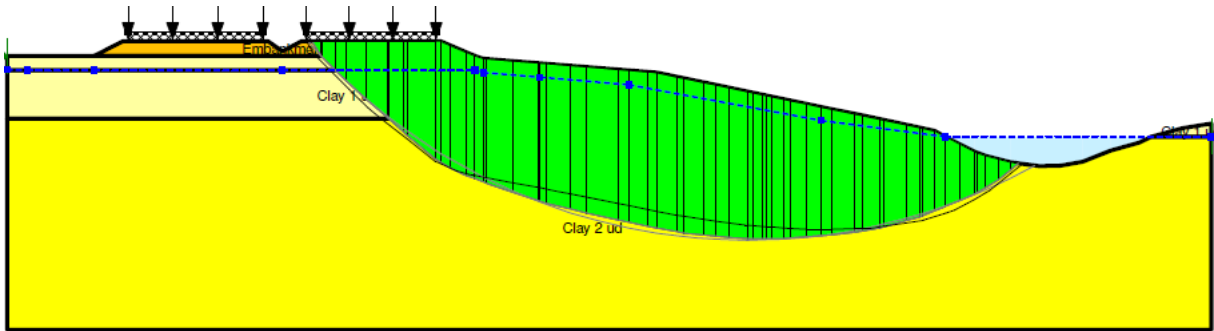


Figure 5.23: Case (e) - The optimised slip surface in the undrained analysis of CS2, generated with 30 starting points and 30 ending points, resulting in $F^{MP} = 1.36$. The circular and the default optimised slip surfaces are also marked.

The results of the changed settings for the maximum concave angles, within the C21 model for undrained conditions and with the applied surcharge load of 20 KPa, are presented in Table 5.11. The factors of safety for all cases, (h) through (o), is calculated to 1.34, despite changing the settings for the maximum concave angles.

Table 5.11: Results of changing, within the undrained CS2 model with an applied surcharge load of 20 kPa, the driving and resisting maximum concave angles in the Optimise settings.

CS2 - Driving and resisting maximum concave angles					
Case	Driving [°]	Resisting [°]	F^{MP}	Volume [m ³]	Weigth [kN]
Circular	-	-	1.39	842	13 523
Default optimised	5	1	1.34	802	12 870
h	30	1	1.34	802	12 870
i	0.001	1	1.34	828	13 292
j	5	10	1.34	802	12 870
k	5	0.001	1.34	802	12 870
l	30	10	1.34	802	12 870
m	0.001	0.001	1.34	828	13 292
n	0.001	10	1.34	828	13 292
o	30	0.001	1.34	802	12 870

Undrained Analysis, Load 43 kPa

The results for the undrained analysis of CS2 when submitted to a surcharge load of 43 kPa can be seen in Table 5.12. The value of F_{circ}^{MP} was calculated to 1.28, which corresponds to the circular slip surface in Figure 5.24. The value of F_{opt}^{MP} was calculated to 1.20, giving a ΔF_1 of 6%. The corresponding optimised slip surface can be seen in Figure 5.25. The circular and the optimised slip surface from SLOPE/W can be compared to Figure 5.26, which illustrates the slip surface generated in PLAXIS 2D. This slip surface corresponds to F^{PLAXIS} which has a value of 1.22, which gives a ΔF_4 of 5% and a ΔF_5 of -2%.

Table 5.12: Results for CS2, undrained analysis, submitted to a surcharge load of 43kPa.

CS2 - Undrained analysis - Load 43 kPa				
F	Value	Conv. tol.	ΔF	Value
F_{circ}^{MP}	1.28	0.001	ΔF_1	$\frac{1.28-1.20}{1.28} = 6\%$
F_{opt}^{MP}	1.20	0.001	ΔF_2	$\frac{1.28-1.26}{1.28} = 2\%$
F_{circ}^{JG}	1.26	0.06	ΔF_3	$\frac{1.20-1.14}{1.20} = 5\%$
F_{opt}^{JG}	1.14	0.07	ΔF_4	$\frac{1.28-1.22}{1.28} = 5\%$
F^{PLAXIS}	1.22	-	ΔF_5	$\frac{1.20-1.22}{1.20} = -2\%$

The calculations with Janbu's Generalised method both generated an error code; E999, which occurs when the slip surface does not have a converged solution (GEO-SLOPE International Ltd., 2008). Converged solutions for F_{circ}^{JG} and F_{opt}^{JG} were obtained with the convergence tolerance set to the 0.06 and 0.07 respectively. The value of F_{circ}^{JG} was then calculated to 1.26, giving a ΔF_2 of 2%. The value of F_{opt}^{JG} was calculated to 1.14, giving a ΔF_3 of 5%.

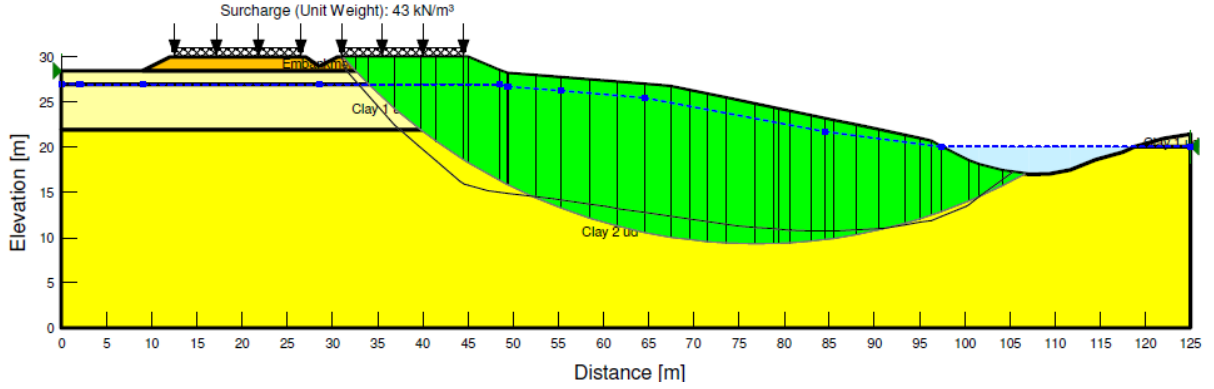


Figure 5.24: The circular slip surface of the undrained analysis of CS2, with a surcharge load of 43 kPa, $F_{circ}^{MP} = 1.28$. The shape of the optimised slip surface is also marked.

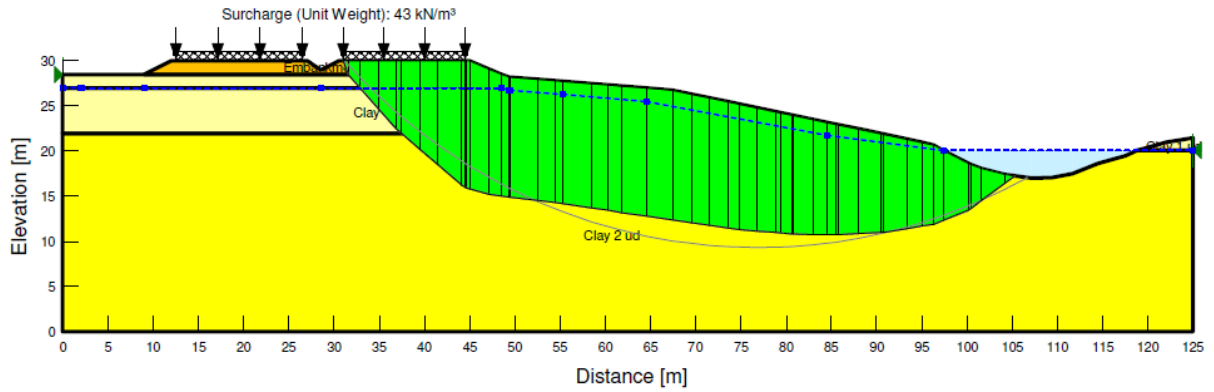


Figure 5.25: The optimised slip surface of the undrained analysis of CS2, with a surcharge load of 43 kPa, $F_{opt}^{MP} = 1.20$. The shape of the circular slip surface is also marked.

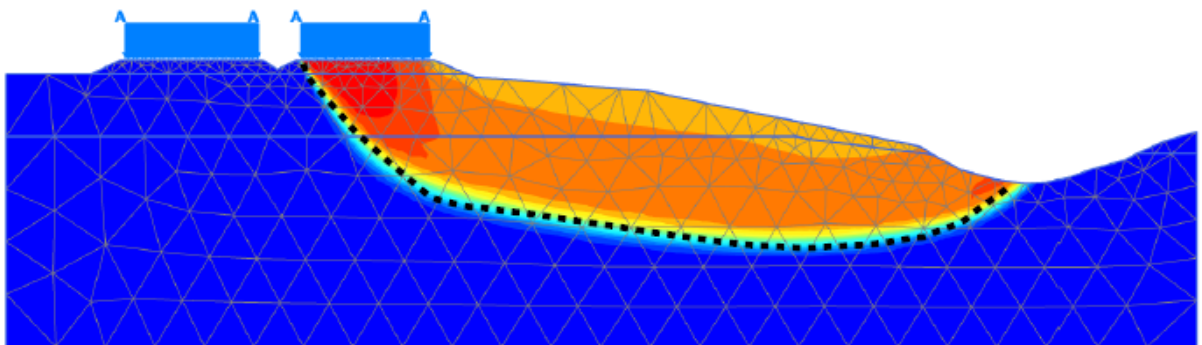


Figure 5.26: The slip surface obtained in PLAXIS 2D, when applying a surcharge load of 43 kPa to CS2, $F^{PLAXIS} = 1.22$. The shape of the optimised slip surface, obtained with SLOPE/W, is marked as a dotted line.

Combined Analysis, Load 20 kPa

The results for the combined analysis of CS2, submitted to a surcharge load of 20 kPa, are presented in Table 5.13. For the combined analysis, the value of F_{circ}^{MP} was calculated to 1.35 and the corresponding circular slip surface can be seen in Figure 5.27. The value of F_{opt}^{MP} was calculated to 1.31, with a corresponding optimised slip surface seen in Figure 5.28.

Table 5.13: Results for CS2, combined analysis, submitted to a surcharge load of 20 kPa.

CS2 - Combined analysis - Load 20 kPa				
F	Value	Conv. tol.	ΔF	Value
F_{circ}^{MP}	1.35	0.001	ΔF_1	$\frac{1.35-1.31}{1.35} = 3\%$
F_{opt}^{MP}	1.31	0.001	ΔF_2	$\frac{1.35-1.31}{1.35} = 3\%$
F_{circ}^{JG}	1.31	0.004	ΔF_3	$\frac{1.31-1.22}{1.31} = 7\%$
F_{opt}^{JG}	1.22	0.002		

The calculations with Janbu's Generalised method both generated an error code; E999, which occurs when the slip surface does not have a converged solution (GEO-SLOPE International Ltd., 2008). Converged solutions for F_{circ}^{JG} and F_{opt}^{JG} were obtained with the convergence tolerance set to 0.004 and 0.002 respectively. The value of F_{circ}^{JG} was then calculated to 1.31, giving a ΔF_2 of 3%. The value of F_{opt}^{JG} was calculated to 1.22, giving a ΔF_3 of 7%.

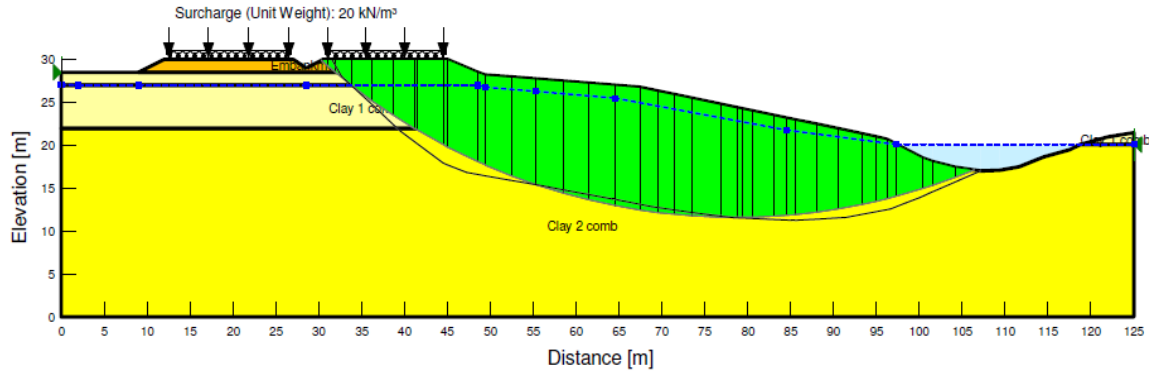


Figure 5.27: The circular slip surface of the combined analysis of CS2, with a surcharge load of 20 kPa, $F_{circ}^{MP} = 1.35$. The shape of the optimised slip surface is also marked.

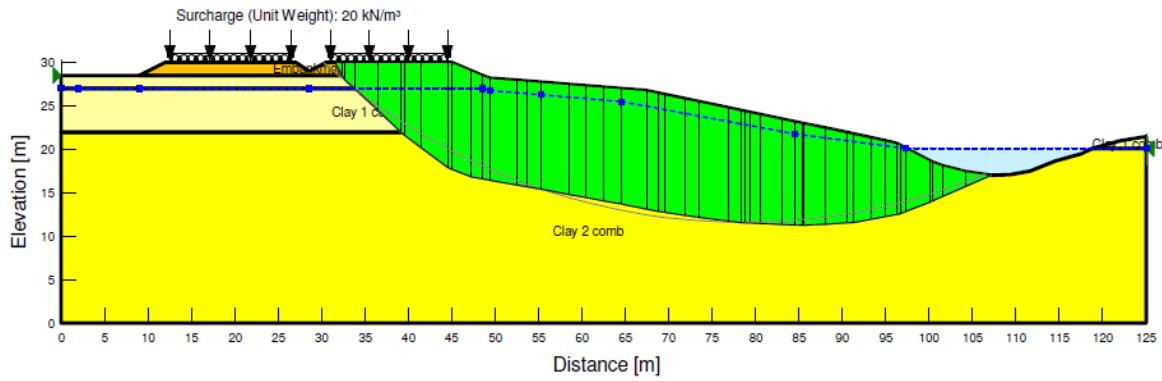


Figure 5.28: The optimised slip surface of the combined analysis of CS2, with a surcharge load of 20 kPa, $F_{opt}^{MP} = 1.31$. The shape of the circular slip surface is also marked.

5.2.3 Characteristic Slope 3: Steep

The following results for CS3 are presented in this chapter: the undrained analyses performed in SLOPE/W and PLAXIS 2D with a surcharge load of 20 kPa, the results when altering the optimise settings in SLOPE/W, the undrained analysis with a surcharge load of 43 kPa performed in SLOPE/W and PLAXIS 2D, and the combined analysis with a surcharge load of 20 kPa performed in SLOPE/W.

Undrained Analysis, Load 20 kPa

The results for the undrained analysis of CS3, submitted to a surcharge load of 20 kPa, are presented in Table 5.14. The value of F_{circ}^{MP} was calculated to 1.40, which corresponds to the circular slip surface in Figure 5.29. The value of F_{opt}^{MP} was calculated to 1.38, giving a ΔF_1 of 1%. The corresponding optimised slip surface can be seen in Figure 5.30. The circular and the optimised slip surface from SLOPE/W can be compared to Figure 5.31, which illustrates the slip surface generated in PLAXIS 2D. This slip surface corresponds to F^{PLAXIS} which has a value of 1.41, giving a ΔF_4 of -1% and a ΔF_5 of -2%.

Table 5.14: Results for CS3, undrained analysis, submitted to a surcharge load of 20 kPa.

CS3 - Undrained analysis - Load 20 kPa				
F	Value	Conv. tol.	ΔF	Value
F_{circ}^{MP}	1.40	0.001	ΔF_1	$\frac{1.40-1.38}{1.40} = 1\%$
F_{opt}^{MP}	1.38	0.001	ΔF_2	$\frac{1.40-1.32}{1.40} = 6\%$
F_{circ}^{JG}	1.32	0.09	ΔF_3	$\frac{1.38-1.43}{1.38} = -4\%$
F_{opt}^{JG}	1.43	0.03	ΔF_4	$\frac{1.40-1.41}{1.40} = -1\%$
F^{PLAXIS}	1.41	-	ΔF_5	$\frac{1.38-1.41}{1.38} = -2\%$

The calculations with Janbu's Generalised method both generated an error code; E999, which occurs when the slip surface does not have a converged solution (GEO-SLOPE International Ltd., 2008). Converged solutions for F_{circ}^{JG} and F_{opt}^{JG} were obtained with the convergence tolerance set to 0.09 and 0.03 respectively. The value of F_{circ}^{JG} was then calculated to 1.32, giving a ΔF_2 of 6%.

The value of F_{opt}^{JG} was calculated to 1.43, giving a ΔF_3 of -4%.

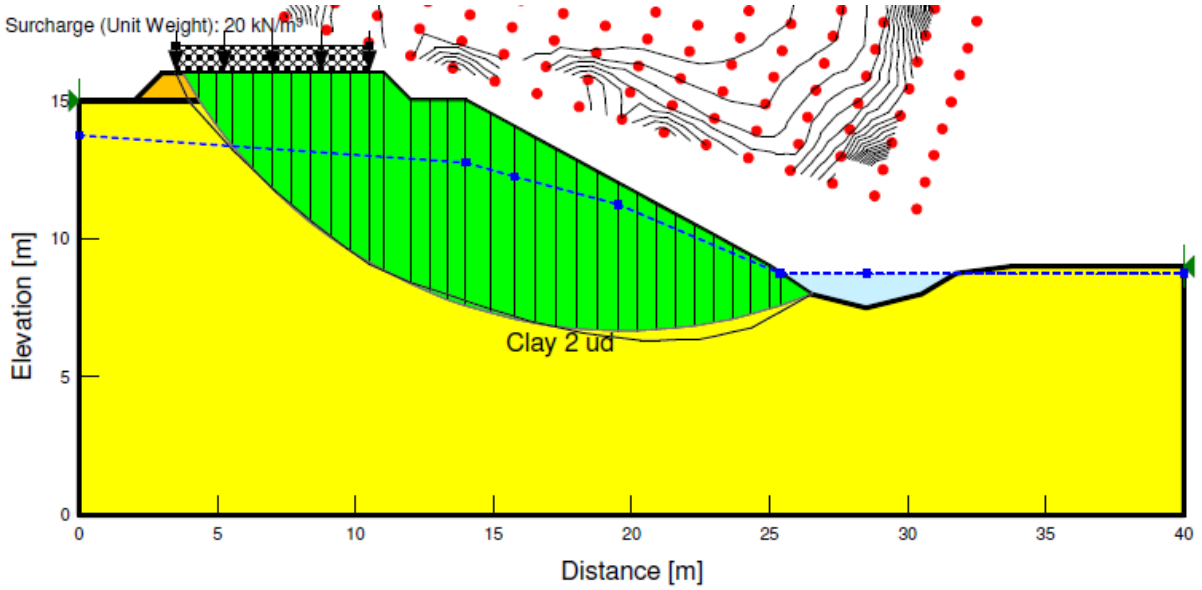


Figure 5.29: The circular slip surface of the undrained analysis of CS3, with a surcharge load of 20 kPa, $F_{circ}^{MP} = 1.40$. The shape of the optimised slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

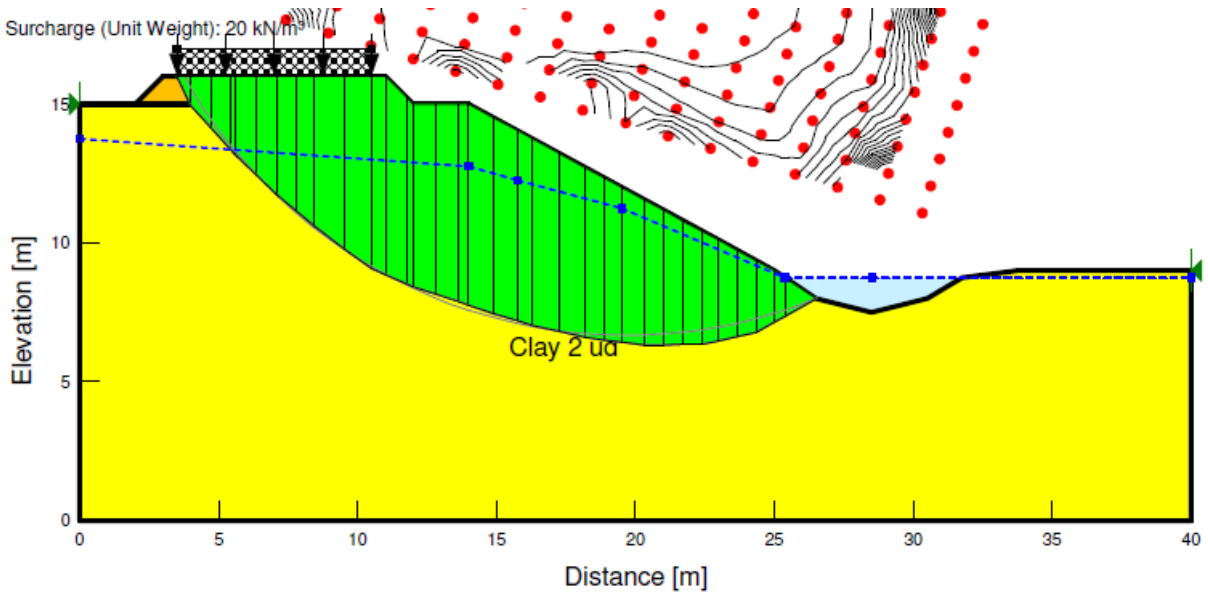


Figure 5.30: The optimised slip surface of the undrained analysis of CS3, with a surcharge load of 20 kPa, $F_{opt}^{MP} = 1.38$. The shape of the circular slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

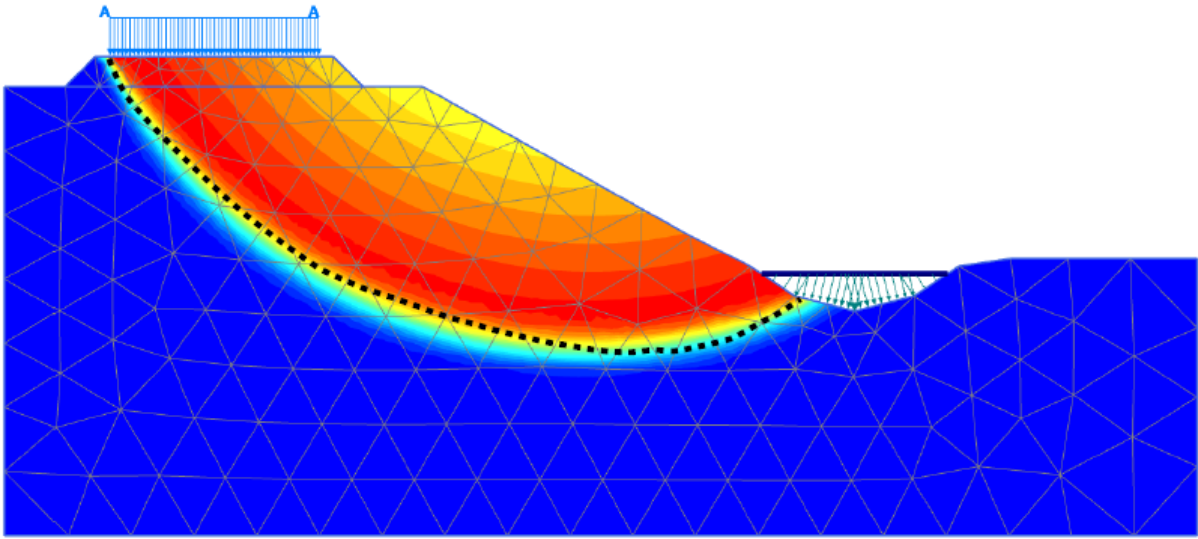


Figure 5.31: The slip surface obtained in PLAXIS 2D, when applying a surcharge load of 20 kPa to CS3, $F^{PLAXIS} = 1.41$. The shape of the optimised slip surface, obtained with SLOPE/W, is marked as a dotted line.

Altered Optimise Settings

The results of the changed settings for the number of starting and ending points, within the CS3 model for undrained conditions and with the applied surcharge load of 20 kPa, are presented in Table 5.15.

Table 5.15: The results of changing the number of starting and ending points for the Optimise function, within the undrained analysis of CS3 when applying a surcharge load of 20 kPa.

CS3 - Number of starting and ending points					
Case	Starting	Ending	F^{MP}	Volume [m ³]	Weight [kN]
Circular	-	-	1.40	108	1 748
Default optimised	8	16	1.38	111	1 798
a	16	16	1.37	112	1 807
b	1	16	1.38	106	1 707
c	8	30	1.37	111	1 795
d	8	8	1.39	113	1 823
e	30	30	1.39	109	1 754
f	1	1	1.40	108	1 748
g	1	30	1.38	106	1 707

Case (b) and Case (g) both results in a value of the factor of safety of 1.38, which is equal to the factor of safety for the default settings. The corresponding slip surfaces have however a slightly

different shape as seen in Figure 5.32. In Cases (a) and (c) the factor of safety also has a value of 1.37 but the slip surface shape is very similar to the default slip surface shape. For Cases (d), (e) and (f) the factors of safety increases compared to the default value, but the slip surface shapes are also for these cases more or less identical to the default shape.

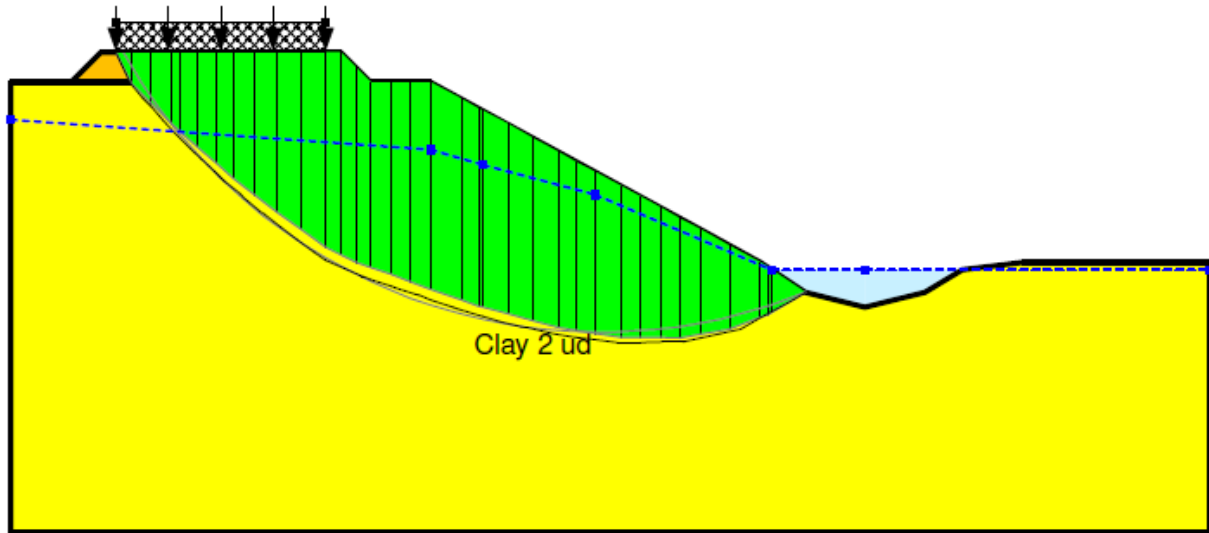


Figure 5.32: Case (b) and Case (g) - The optimised slip surface in the undrained analysis of CS2, generated with 1 starting point and 16 ending points for Case (b) and 1 starting point and 30 ending points for Case (g), resulting in $F^{MP} = 1.38$. Both the circular and the default optimised slip surfaces are marked.

The results of the changed settings for the maximum concave angles, within the CS1 model for undrained conditions and with the applied surcharge load of 20 kPa, are presented in Table 5.16. The factors of safety for all cases, (h) through (o), is calculated to 1.38, despite changing the settings for the maximum concave angles.

Table 5.16: Results of changing, within the undrained CS3 model with an applied surcharge load of 20 kPa, the driving and resisting maximum concave angles in the Optimise settings.

CS3 - Driving and resisting maximum concave angles					
Case	Driving [°]	Resisting [°]	F^{MP}	Volume [m ³]	Weigth [kN]
Circular	-	-	1.40	108	1 748
Default optimised	5	1	1.38	111	1 798
h	30	1	1.38	111	1 798
i	0.001	1	1.38	114	1 840
j	5	10	1.38	111	1 798
k	5	0.001	1.38	111	1 798
l	30	10	1.38	111	1 798
m	0.001	0.001	1.38	114	1 840
n	0.001	10	1.38	114	1 840
o	30	0.001	1.38	111	1 798

Undrained Analysis, Load 43 kPa

The results for the undrained analysis of CS3, here submitted to a surcharge load of 43 kPa, are presented in Table 5.17. The value of F_{circ}^{MP} was calculated to 1.20, which corresponds to the circular slip surface in Figure 5.33. The value of F_{opt}^{MP} was calculated to 1.18, giving a ΔF_1 of 2%. The corresponding optimised slip surface can be seen in Figure 5.34. The circular and the optimised slip surface from SLOPE/W can be compared to Figure 5.35, which illustrates the slip surface generated in PLAXIS 2D. This slip surface corresponds to F^{PLAXIS} which has a value of 1.20, which gives a ΔF_4 of 0% and a ΔF_5 of -2%.

Table 5.17: Results for CS3, undrained analysis, submitted to a surcharge of 43 kPa.

CS3 - Undrained analysis - Load 43 kPa				
F	Value	Conv. tol.	ΔF	Value
F_{circ}^{MP}	1.20	0.001	ΔF_1	$\frac{1.20-1.18}{1.20} = 2\%$
F_{opt}^{MP}	1.18	0.001	ΔF_2	$\frac{1.20-1.14}{1.20} = 5\%$
F_{circ}^{JG}	1.14	0.08	ΔF_3	$\frac{1.18-1.05}{1.18} = 11\%$
F_{opt}^{JG}	1.05	0.09	ΔF_4	$\frac{1.20-1.20}{1.20} = 0\%$
F^{PLAXIS}	1.20	-	ΔF_5	$\frac{1.18-1.20}{1.18} = -2\%$

The calculations with Janbu's Generalised method both generated an error code; E999, which occurs when the slip surface does not have a converged solution (GEO-SLOPE International Ltd., 2008). Converged solutions for F_{circ}^{JG} and F_{opt}^{JG} were obtained with the convergence tolerance set to 0.08 and 0.09 respectively. The value of F_{circ}^{JG} was then calculated to 1.14, giving a ΔF_2 of 5%. The value of F_{opt}^{JG} was calculated to 1.05, giving a ΔF_3 of 11%.

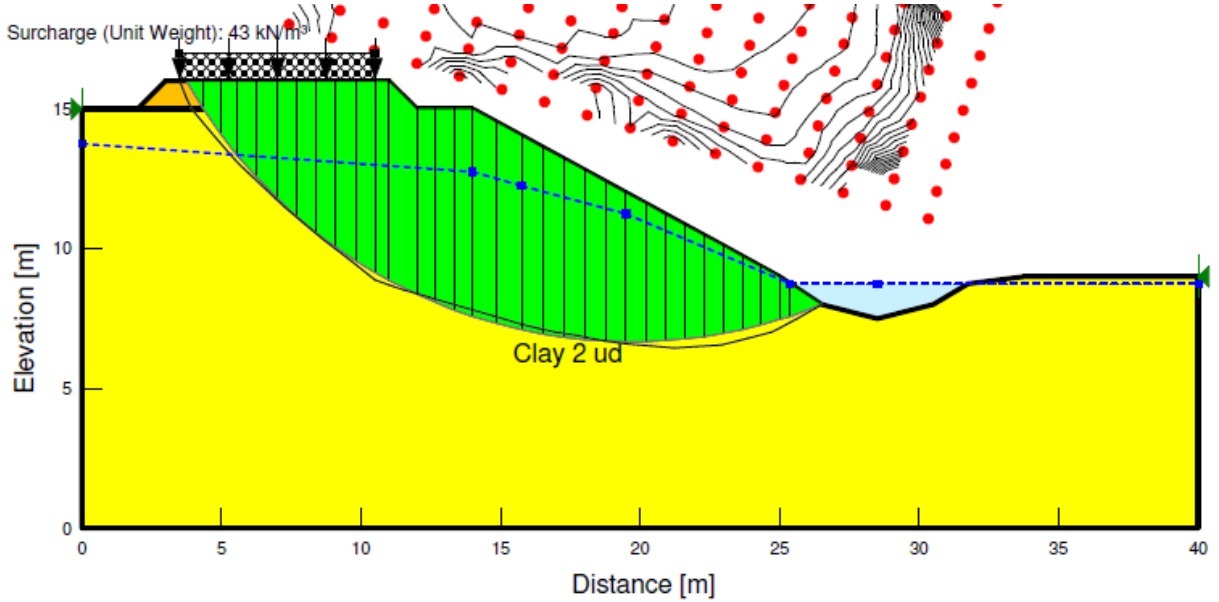


Figure 5.33: The circular slip surface of the undrained analysis of CS3, with a surcharge load of 43 kPa, $F_{circ}^{MP} = 1.20$. The shape of the optimised slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

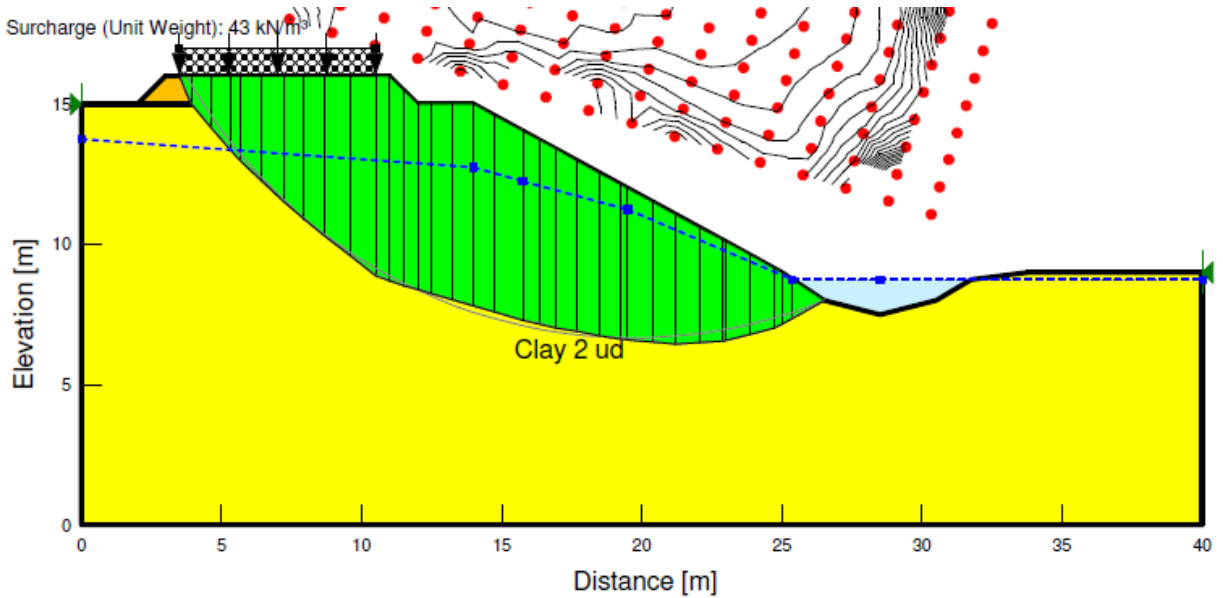


Figure 5.34: The optimised slip surface of the undrained analysis of CS3, with a surcharge load of 43 kPa, $F_{opt}^{MP} = 1.18$. The shape of the circular slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

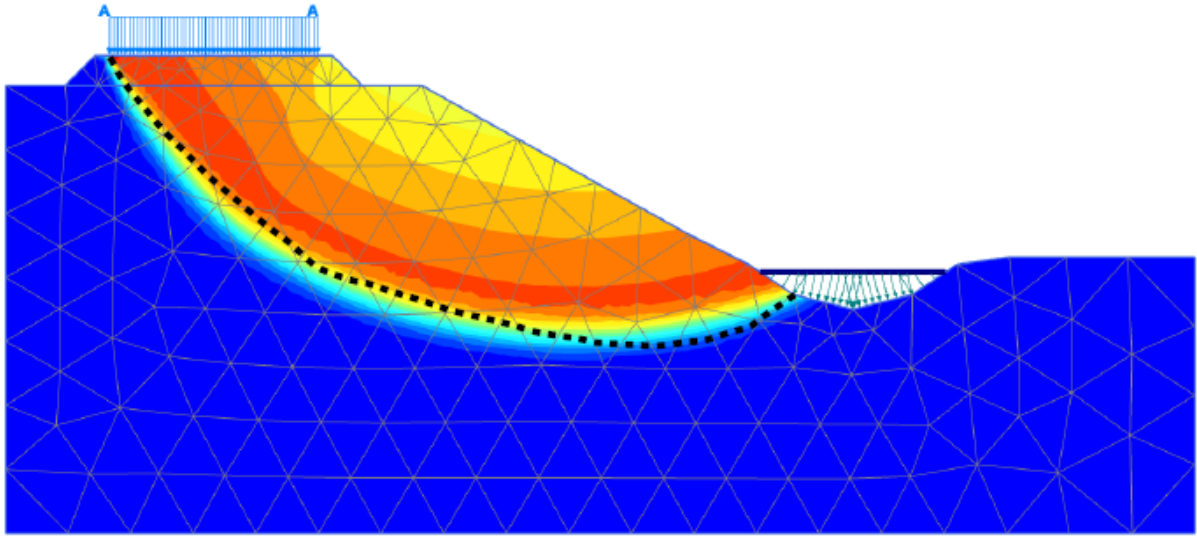


Figure 5.35: The slip surface obtained in PLAXIS 2D, when applying a surcharge load of 43 kPa to CS3, $F^{PLAXIS} = 1.20$. The shape of the optimised slip surface, obtained with SLOPE/W, is marked as a dotted line.

Combined Analysis, Load 20 kPa

The results for the combined analysis of CS3, submitted to a surcharge load of 20 kPa, are presented in Table 5.18. The inclination of the slope required that the minimum allowed slip surface depth was changed to 4 m, in order to avoid a slip surface that did not intersect the embankment.

Table 5.18: Results for CS3, combined analysis, submitted to a surcharge load of 20 kPa.

CS3 - Combined analysis - Load 20 kPa				
F	Value	Conv. tol.	ΔF	Value
F_{circ}^{MP}	1.00	0.001	ΔF_1	$\frac{1.00-0.99}{1.00} = 1\%$
F_{opt}^{MP}	0.99	0.001	ΔF_2	$\frac{1.00-0.98}{1.00} = 2\%$
F_{circ}^{JG}	0.98	0.001	ΔF_3	$\frac{0.99-0.90}{0.99} = 9\%$
F_{opt}^{JG}	0.90	0.02		

For the combined analysis, the value of F_{circ}^{MP} was calculated to 1.00 and the corresponding circular slip surface can be seen in Figure 5.36. The value of F_{opt}^{MP} was calculated to 0.99, with a corresponding optimised slip surface seen in Figure 5.37. F_{circ}^{JG} could be calculated with the default value of 0.001, resulting in a value of 0.98 and a ΔF_2 of 2%. F_{opt}^{JG} displayed error code E999, meaning that the calculation did not converge (GEO-SLOPE International Ltd., 2008). When changing the convergence tolerance to 0.02, F_{circ}^{JG} was calculated to 0.90, giving a ΔF_3 of 9%.

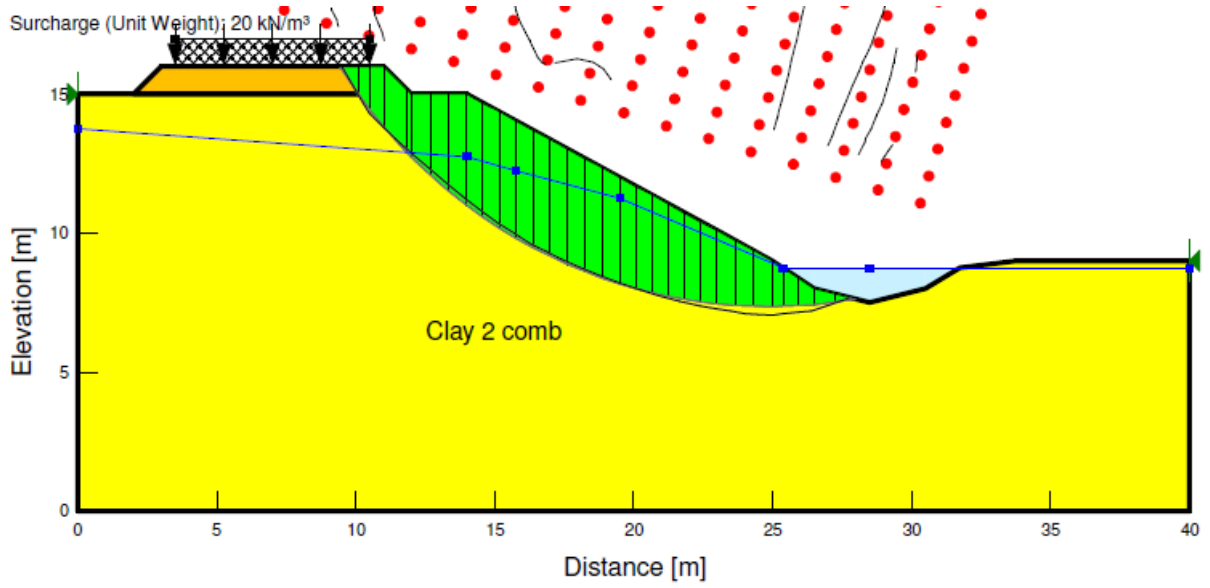


Figure 5.36: The circular slip surface of the combined analysis of CS3, with a surcharge load of 20 kPa, $F_{circ}^{MP} = 1.00$. The shape of the optimised slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

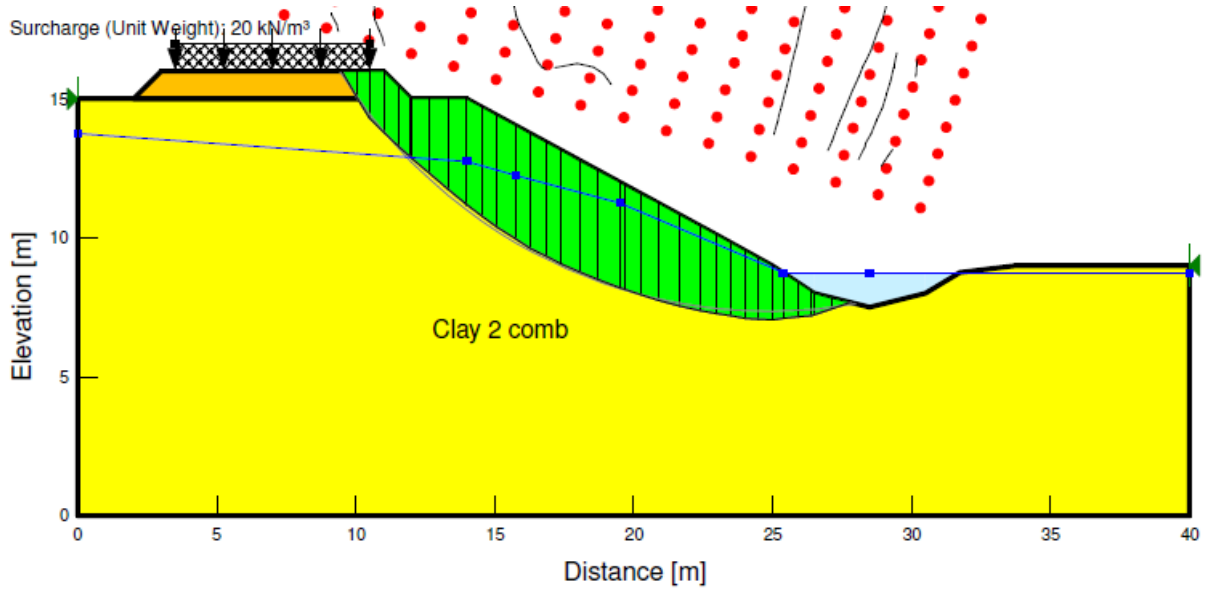
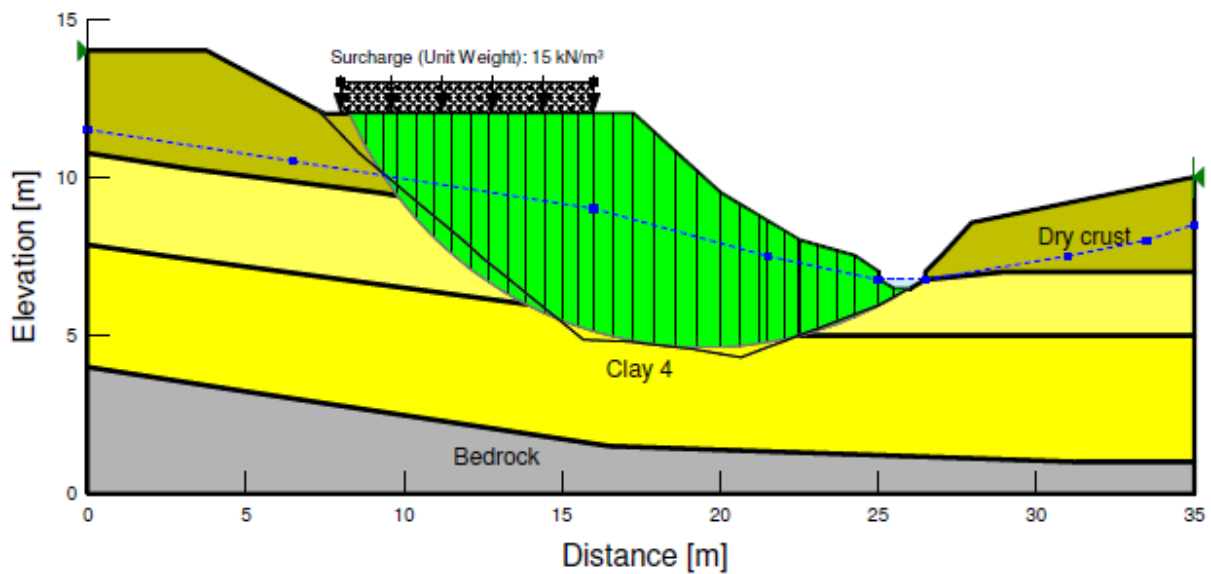


Figure 5.37: The optimised slip surface of the combined analysis of CS3, with a surcharge load of 20 kPa, $F_{opt}^{MP} = 0.99$. The shape of the circular slip surface is also marked. The dots above the slope represent the rotation points in the search grid.

5.3 Steep Slope with Dry Crust

The results for the steep slope with a dry crust is presented in Table 5.19. The circular slip surface shape for the slope with the dry crust is illustrated in Figure 5.38, with a corresponding value for F_{circ}^{MP} of 1.34. With the default settings for the maximum concave angles for the driving and resisting side, the optimised slip surface result in a slip surface shape as illustrated in Figure 5.39, with a corresponding F_{opt}^{MP} of 1.15. For the optimised case, it can be seen that the slip surface has a distinct concave shape at its deepest part. The value of ΔF_1 is calculated to 14%.

Steep slope with dry crust			
F	Value	ΔF	Value
F_{circ}^{MP}	1.34	ΔF_1	$\frac{1.34-1.15}{1.34} = 14\%$
F_{opt}^{MP}	1.15	ΔF_4	$\frac{1.34-1.31}{1.34} = 2\%$
F^{PLAXIS}	1.31	ΔF_5	$\frac{1.15-1.31}{1.15} = -14\%$



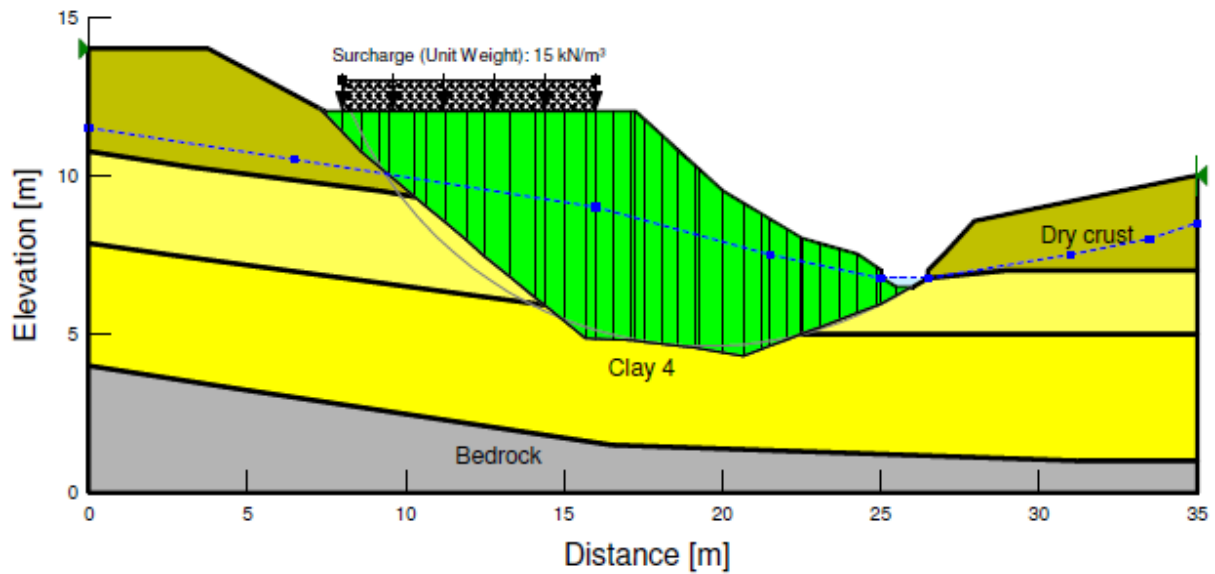


Figure 5.39: The optimised slip surface of the steep slope with a dry crust, using the default settings for the concave angles, with a factor of safety of 1.15. The circular slip surface is also marked.

The resulting slip surface, when modelling the slope in PLAXIS 2D, is illustrated in Figure 5.40. As the figure shows, the slip surface reaches over a greater area than it does in SLOPE/W. Therefore, the geometry was modified where the ground surface was made horizontal. Figure 5.41 shows the slip surface shape for this case. The resulting F^{PLAXIS} is computed to 1.31 for both cases, and no concave slip surface shape was obtained in either of the two models in PLAXIS 2D. ΔF_4 and ΔF_5 are calculated to 2% and -14% respectively.

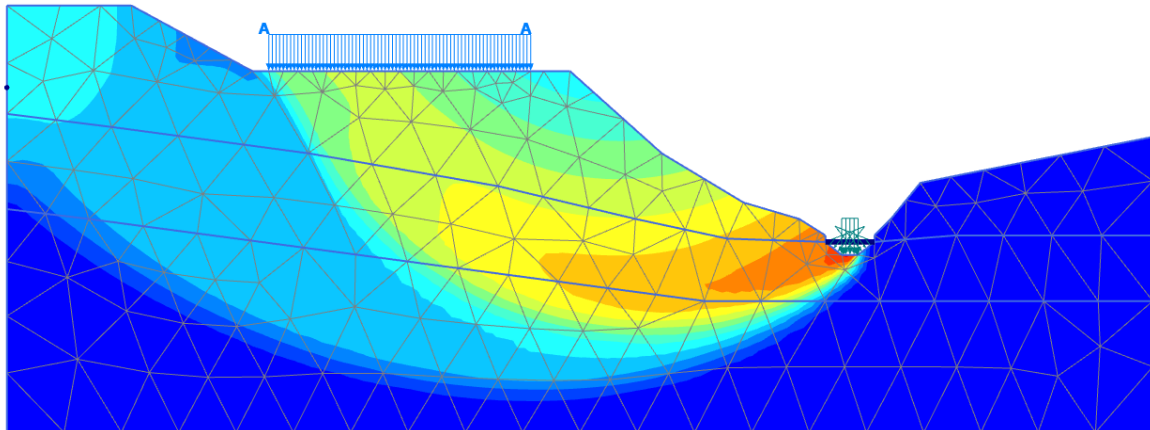


Figure 5.40: The slip surface obtained for the steep slope with a dry crust in PLAXIS 2D, the obtained factor of safety is 1.31.

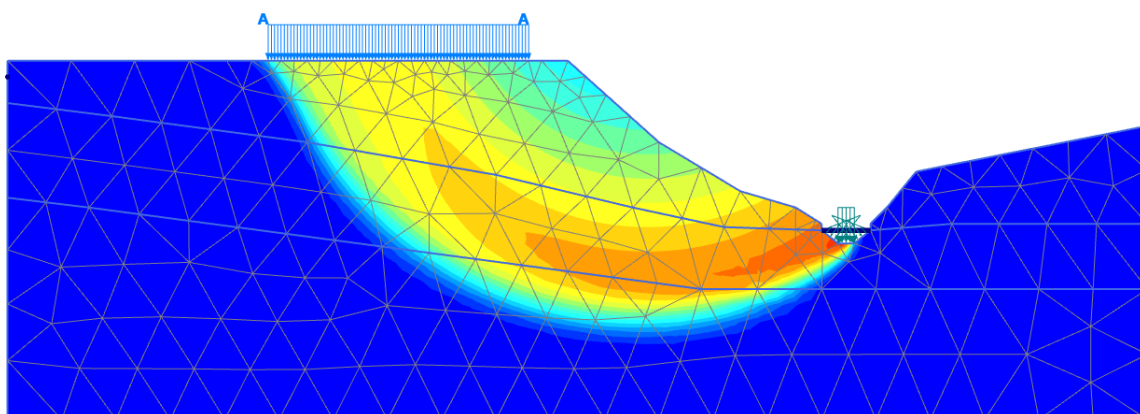


Figure 5.41: The slip surface obtained for the steep slope with a dry crust in PLAXIS 2D with a modified geometry, the obtained factor of safety is 1.31.

The results, in terms of values of factors of safety and volume and weight of slip surfaces, when altering the maximum angles for the driving and resisting sides are presented in Table 5.20. As can be seen, the values of the factors of safety vary between 1.15 (Case (k)) and 1.22 (Cases (i) and (n)). The volumes and the weights for the different cases indicates only small variations of the slip surface.

Table 5.20: Results of changing the driving and resisting maximum concave angles.

Steep slope dry crust - Driving and resisting maximum concave angles					
Case	Driving [°]	Resisting [°]	F^{MP}	Volume [m ³]	Weigth [kN]
Circular	-	-	1.34	76	1 443
Default optimised	5	1	1.15	75	1 425
h	30	1	1.16	74	1 413
i	0.001	1	1.22	75	1 419
j	5	10	1.19	74	1 404
k	5	0.001	1.15	75	1 424
l	30	10	1.17	74	1 410
m	0.001	0.001	1.20	76	1 438
n	0.001	10	1.22	75	1 429
o	30	0.001	1.16	74	1 413

When lowering the value for the maximum value on the driving side, as in Case (i), illustrated in Figure 5.42, no concave shape is obtained and the factor of safety, with a value of 1.22, is higher than for the default optimised slip surface. If instead lowering the value for the maximum concave angle on the resisting side, as in Case (k), the concave shape of the slip surface appears again as can

be seen in Figure 5.43, and the factor of safety is computed to 1.15. If lowering the concave angle on both the driving and resisting side, as in Case (m), the slip surface obtains a shape that is not concave, as in Figure 5.44, and the factor of safety is computed to 1.20.

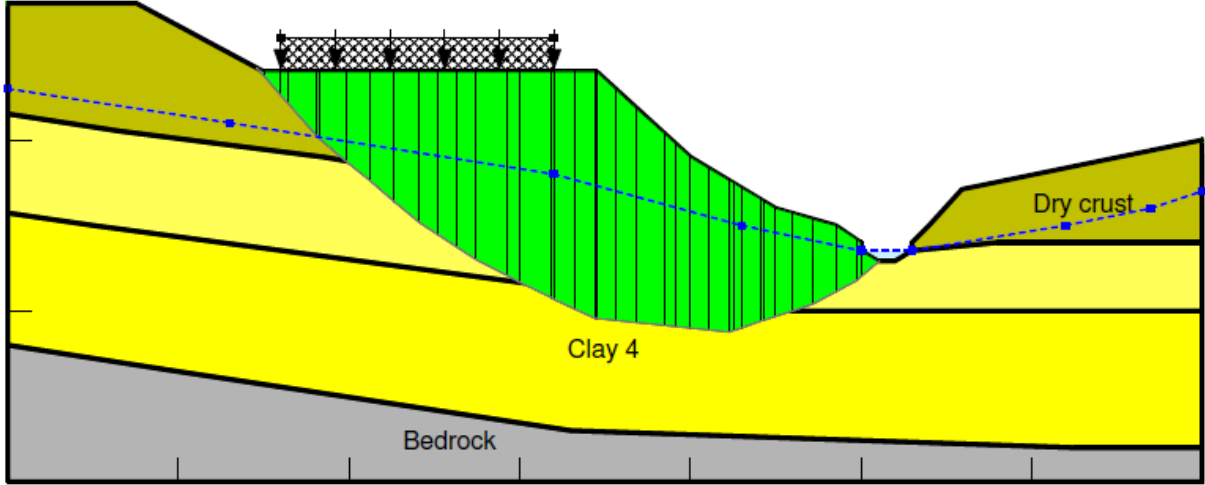


Figure 5.42: Case (i) - The optimised slip surface of the steep slope with dry crust, generated with the maximum concave angles set to 0.001° and 1° for the driving and resisting side respectively, $F^{MP} = 1.22$.

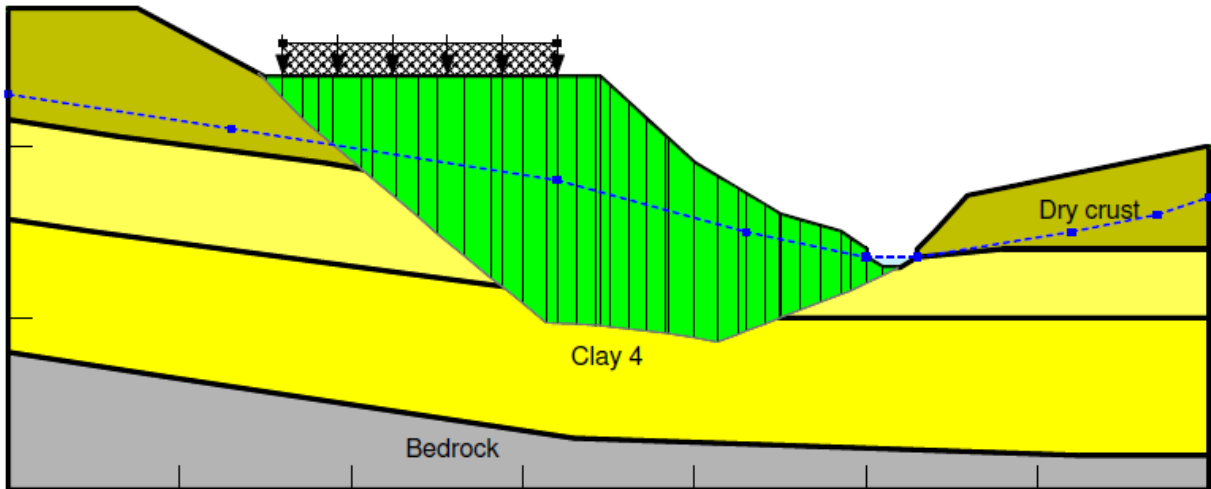


Figure 5.43: Case (k) - The optimised slip surface of the steep slope with dry crust, generated with the maximum concave angles set to 5° and 0.001° for the driving and resisting side respectively, $F^{MP} = 1.15$.

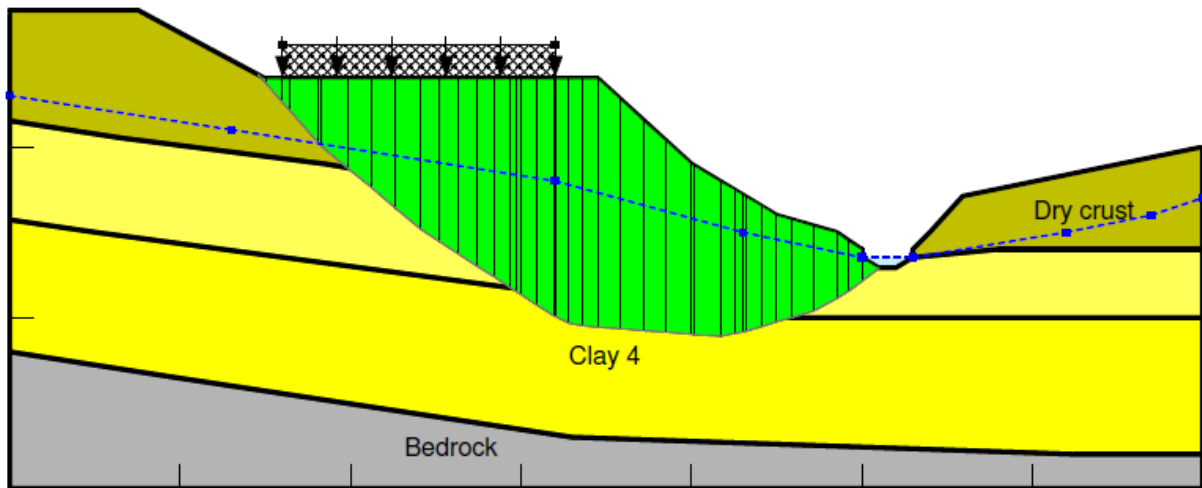


Figure 5.44: Case (m) - The optimised slip surface of the steep slope with dry crust, generated with the maximum concave angles set to 0.001° and 0.001° for the driving and resisting side respectively, $F^{MP} = 1.20$.

When lowering the concave angle on the driving side and set the concave angle on the driving side to its maximum, as in Case (g), no concave shape occurs as can be seen in Figure 5.45. The factor of safety is for this case 1.22. When doing the opposite, as for Case (h), the slip surface adopts an uneven concave shape as illustrated in Figure 5.46 and the factor of safety decreases to a value of 1.16.

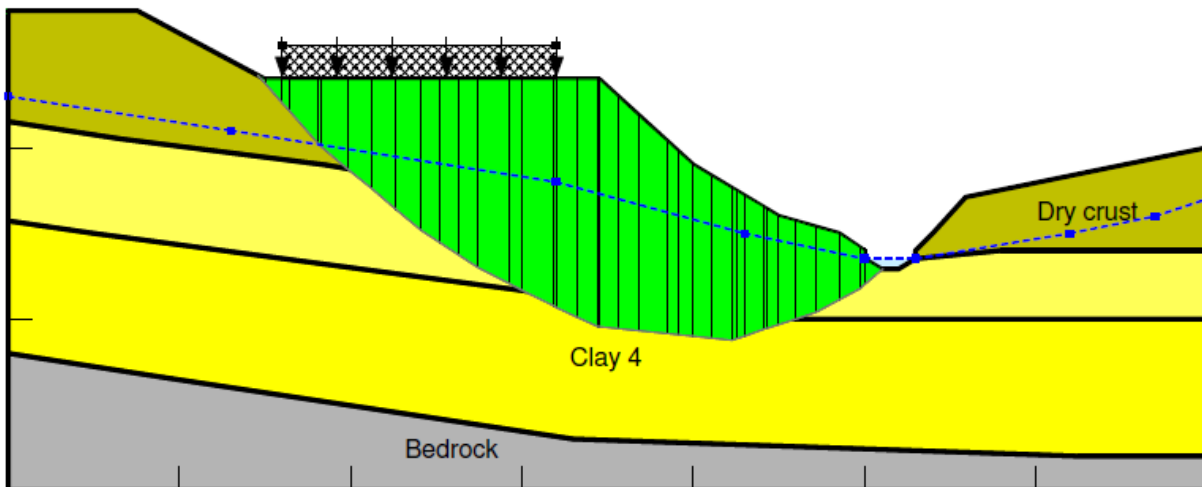


Figure 5.45: Case (n) - The optimised slip surface of the steep slope with dry crust, generated with the maximum concave angles set to 0.001° and 10° for the driving and resisting side respectively, $F^{MP} = 1.22$.

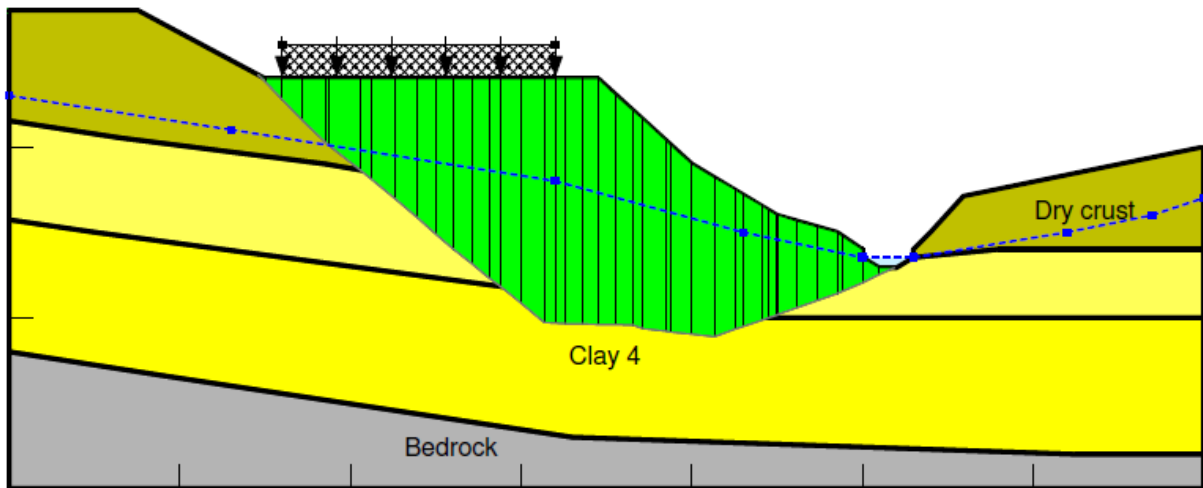


Figure 5.46: Case (o) - The optimised slip surface of the steep slope with dry crust, generated with the maximum concave angles set to 30° and 0.001° for the driving and resisting side respectively, $F^{MP} = 1.16$.

Case (l), seen in Figure 5.47, illustrates the shape of the optimised slip surface when the maximum concave angles are raised to the maximum permitted values. The corresponding factor of safety is calculated to 1.17.

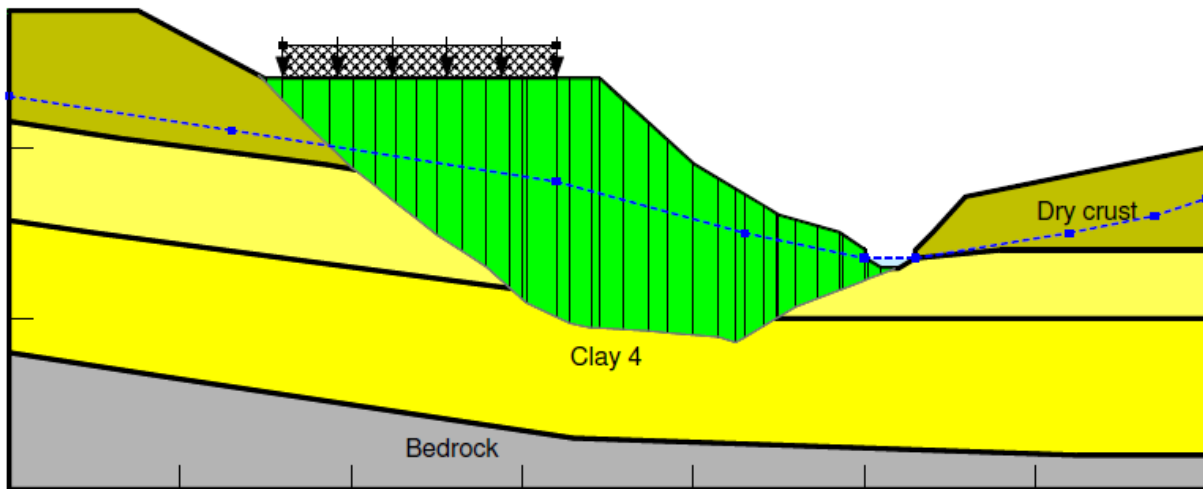


Figure 5.47: Case (l) - The optimised slip surface of the steep slope with dry crust, generated with the maximum concave angles set to 30° and 10° for the driving and resisting side respectively, $F^{MP} = 1.17$.

6 Discussion

In this chapter, the results of each modelled slope are discussed following the same order as in Chapter 5. The general applicability of the Optimise function is then discussed based on the overall observations. This is followed by a discussion about the uncertainties within the study and the potential sources of error and, lastly, suggestions for further investigations on this subject.

6.1 Bearing Capacity Test

The bearing capacity test corresponds to the derivation of the bearing capacity for undrained analysis, and the calculated factor of safety is compared to the bearing capacity factor. The results show that, for the circular slip surface, SLOPE/W successfully identifies the critical kinematically correct slip surface for both the shape and the value of the corresponding factor of safety. For the optimised slip surface however, the result corresponds to a statically correct slip surface with a value for the factor of safety even lower than the value for a slip surface with that shape according to the bearing capacity theory.

These results demonstrate the main weakness of the Optimise function; that it does not take kinematics or any other physical limitations into account. This explains the overly conservative results that the Optimise function sometimes produces. Instead of this behaviour, it would be desirable that the Optimise function, for this geometry and loading case, would identify the both static and kinematic correct slip surface and calculate a factor of safety close to the derived value of 5.14. This is supported by the calculation of verification of the shape of the slip surface in PLAXIS 2D, which computes a factor of safety of 5.15.

When systematically changing the number of starting and ending points in the Optimise function settings, the results show that all of the alterations generate a higher value of F^{MP} than for optimising with the default settings. Case (e) generates the value closest to the both statically and kinematically correct bearing capacity factor, with a F^{MP} of 5.12. In this case, both the number of starting and ending points is set to 30 which makes the shape of the optimised slip surface more fixed to the shape of the circular slip surface. Therefore, the resulting slip surface lacks the correct entry and exit angles that it should have according to the bearing capacity theory and is instead more circular in its shape. As it was known what the correct value for the factor of safety is for this case, it was possible to change the optimise settings to obtain that value. Such adjustments are however difficult to make when the value of the correct factor of safety, as in most cases, is unknown.

The slip surface obtained with the default settings for the Optimise function has a distinct concave shape on the driving side, and a slightly convex shape on the resisting side. By controlling the maximum concave angles, this shape disappears and the factor of safety can be raised to some extent. It is however not possible to obtain the desired value of F^{MP} of 5.14.

6.2 CS1 Horizontal Geometry

CS1, the horizontal slope, can be seen as an extended version of the bearing capacity test, where an embankment is added to the horizontal ground surface. This slope represents a more commonly performed stability analysis as embankments on a horizontal ground surface are often occurring

stability problem within the design of roads and railways.

The undrained analysis of CS1, with a surcharge load of 20 kPa, performed in SLOPE/W shows similar results as obtained for the bearing capacity test. The optimised slip surface for CS1 has a shape that recalls of a statically correct shape, however it consists of a circular arc at the deepest part. The difference in value of the factor of safety for the circular and optimised slip surface, ΔF_1 , is 12%, which can be regarded as a significant difference. When calculating F for the two slip surfaces shape with Janbu's Generalised calculation method, two different types of situations occur. For F_{circ}^{JG} the convergence tolerance has to be changed to 0.06 to obtain a result. However, for F_{opt}^{JG} , the calculation converge at 0.006 but the obtained value is only 0.25, with ΔF_3 of 85%. This can be seen as an indication of that the optimised slip surface shape is incorrect for this case. Furthermore, the convergence difficulty itself indicate that calculations performed with the Morgenstern-Price method are mathematically correct, but not necessarily physically plausible.

The same analysis performed in PLAXIS 2D results in a slip surface shape that lies somewhere between the two shapes from SLOPE/W; the slip surface is almost circular but the exit part consist of an almost straight line. It can be noted that the slip surface obtained in PLAXIS 2D is deeper than the slip surfaces in SLOPE/W. F^{PLAXIS} is computed to 1.87, which is closer to F_{circ}^{MP} than to F_{opt}^{MP} . This indicates that the optimisation in SLOPE/W is not reliable in this case; it is rather a statically than a kinematically correct slip surface shape, but also that there might exist a critical composite slip surface.

Altering the number of starting and ending points for the optimise settings within the same scenario does not generate any notable results in terms of a more kinematically correct slip surface shape. For Case (d), the resulting slip surface has a very similar shape as for the default case, but it is deeper and the factor of safety becomes slightly lower. For Case (e), the slip surface is locked to the circular shape as the starting and ending points are set to the maximum value of 30. With these adjustments, it can be seen that the ending and exiting corners gets a more straight shape, which is more similar to the slip surface obtained in PLAXIS 2D, but most interesting is that the factor of safety is calculated to 1.80, which almost exactly halfway between F_{circ}^{MP} and F_{opt}^{MP} .

Altering the driving and resisting maximum concave angles did not result in any notable changes, which were expected as the default slip surface shape does not demonstrate any concave angles.

When increasing the surcharge load to 43 kPa, both slip surfaces expand in volume while maintaining the same shapes as for the lower load. The optimised slip surface is then deeper and intersects the second clay layer. This intersection is notable since this clay has higher shear strength than the overlying clay and a real composite slip surface probably would not penetrate this layer, which is also supported by the slip surface obtained in PLAXIS 2D.

The calculated factors of safety have similar differences as for the lower load, with a ΔF_1 of 16% and a F^{PLAXIS} again closer to F_{circ}^{MP} than to F_{opt}^{MP} . The calculations performed with Janbu's Generalised method show similar values in terms of ΔF_2 and ΔF_3 , but the convergence tolerance needed to be changed to 0.008 and 0.09 for F_{circ}^{JG} and F_{opt}^{JG} respectively, which is the opposite behaviour as for the case with the lower load. There is no obvious explanation for this, except that the value for F_{opt}^{JG} is too low and clearly incorrect.

The combined analysis for CS1 performed in SLOPE/W, with a surcharge load of 20 kPa, resulted in a slip surface that only intersects the embankment by the default value of the minimum slip surface depth. To obtain a deeper slip surface, the minimum slip surface depth was set to 4 m. This resulted

in an optimised slip surface shape that visibly seems realistic, with a ΔF_1 of 6%. The calculations performed with Janbu's Generalised calculation method did not converge with the default settings. With the convergence tolerance changed F_{circ}^{JG} and F_{opt}^{JG} is lower than when using Morgenstern-Price calculation method, with ΔF_2 and ΔF_3 of 14% and 42%. The result of error code E997 for the circular slip surface, which indicates that the slip surface exit angle is too steep, is unexpected as the angle does not look very steep.

6.3 CS2 Elongated Slope

For the elongated slope, CS2, the results for the undrained analysis, with a surcharge load of 20 kPa, shows that the Optimise function in this case gives a reasonable value for F_{opt}^{MP} when comparing to F_{circ}^{MP} with a value for ΔF_1 of 4%. The optimised slip surface shape is seemingly realistic, as an elongated slope is presumed to have a composite slip surface shape rather than a circular. The shape of the optimised slip surface is also seems kinematically correct. The calculations performed with Janbu's Generalised method resulted in an identical value for the factor of safety for the circular slip surface shape, with a ΔF_2 of 0%, and a value for the optimised slip surface within the same range with a ΔF_1 of 3%.

In the analysis performed in PLAXIS 2D for this case, with F^{PLAXIS} calculated to 1.35, results in a ΔF_5 of 0%. The slip surface shape obtained in PLAXIS 2D is also very similar to the optimised slip surface shape in SLOPE/W and is clearly a composite slip surface shape. The results indicate that the Optimise function manages to identify an accurate composite slip surface shape and a correct value for the factor of safety for this elongated slope.

Altering the starting and ending points in, in the optimise settings, results in a few cases of questionable slip surface shapes. For Cases (b) and (g), which are the same types of situations where the starting points are decreased to 1 and the ending points are set to the default value of 16 and increased to 30 respectively, result in a factor of safety that is close or equal to the default calculation of F_{opt}^{MP} . The shapes of the slip surfaces are similar to the optimised slip surface but significantly shallower. Altering the maximum concave angles on the driving and resisting side resulted in a factor of safety equal to F_{opt}^{MP} for all cases, with only slightly differences in volume and weights.

When increasing the load within the undrained analysis to 43 kPa, the differences in terms of ΔF are only slightly changed when comparing this to the results of the previous loading case of 20 kPa. As for the case with a lower load, the optimised slip surface shape and factor of safety seems reliable. Also for the combined analysis the result for the optimise function appear reasonable, and the factors of safety are as expected lower than for the undrained analysis.

6.4 CS3 Steep Slope

The steep slope, CS3, shows a seemingly reasonable result for the optimised slip surface, within the undrained analysis of applying a surcharge load of 20 kPa, as ΔF_1 is only 1%. Although the overall shape is expected, the slip surface would probably be smoother in a real case of failure. When calculating the two slip surfaces with Janbu's Generalised method, the convergence tolerance had to be increased after which F_{opt}^{JG} was calculated to 1.43 which is a higher value than for F_{circ}^{MP} of 1.32. This is notable, but also possible since F_{opt}^{JG} is not actually an optimisation of F_{circ}^{JG} , but a recalculation of F_{opt}^{MP} .

When comparing the optimised and the circular slip surfaces to the slip surface obtained in PLAXIS 2D, it seems as if the circular slip surface more accurately capture the resisting side of the slip surface while the optimised capture the entry angle on the driving side. The result from PLAXIS 2D shows, besides a steep entry angle, a concave shape on the driving side which might be due to the coarseness of the mesh. The fact that the value of F^{PLAXIS} is higher than both F_{circ}^{MP} and F_{opt}^{MP} , with a ΔF_4 of -1% and ΔF_5 of -2%, indicates that there does not exist any critical composite slip surface in reality. Altering the optimise settings did not result in any significant changes neither for altering the number of starting and ending points nor changing the maximum concave angles.

When applying a surcharge load of 43 kPa within the undrained analysis, the results follow a similar pattern as for loading with 20 kPa. The combined analysis results in more shallow slip surfaces, due to the inclination of the slope, since the shear strength in this case is partly dependent on the friction angle. ΔF_1 is only 1%, and it is not possible nor a priority, to deem either slip surface as the more accurate. The overall conclusion from CS3 is that the Optimise function is not applicable, nor necessary, for this case.

6.5 Steep Slope with Dry Crust

This slope was created to investigate how the settings of the Optimise function, that control the maximum concave angles of the driving and resisting side, influence the result of the optimisation. The slip surface obtained with the default optimise settings shows a distinct concave shape at the bottom, which strongly compromises kinematics. The result from altering the concave angles shows that this unreasonable shape could be managed to some extent by changing the angles to the value of 0.001° on the driving side as in Cases (i) and (l) sides, or which results in the smoothest slip surface shape, use the value of 0.001° on both the driving and resisting sides as in Case (m).

The obtained value of F_{opt}^{MP} , with the default values for the angles, can also be considered as too conservative as the value of ΔF_1 is 14%, which is a significantly high difference. If changing the settings for the Optimise function, the factors of safety are for all cases slightly higher than F_{opt}^{MP} with the default settings but still notably low.

The result obtained from PLAXIS 2D confirms that the Optimise function is not applicable for this slope. The slip surface shape is similar to the circular slip surface shape in SLOPE/W, and the value of F^{PLAXIS} indicates that F_{opt}^{MP} is too low as ΔF_5 is -14% whereas ΔF_4 is only 2%. The adjustments made for Case (m) generates a slip surface visibly closest to the result from PLAXIS 2D, with an almost circular slip surface shape except for the slightly steeper entry angle, however the resulting value of the factor of safety of 1.20 is still low.

When the maximum concave angles are permitted to reach the maximum value of 30° and 10° on the driving and resisting side respectively, as in Case (l), the result further indicates that these settings can have a significant effect and that altering them can be useful.

6.6 General Applicability of the Optimise Function

The inconsistency of the values of ΔF_1 are to be expected since it is known that the need to analyse composite slip surfaces, i.e. to optimise the slip surfaces, vary for different types of slopes. For example, for a slope in a deep homogeneous clay layer, the critical slip surface would be expected to be more or less circular and analysis of a composite slip surface should not be considered required.

As a software based on limit equilibrium, SLOPE/W does not account for physical admissibility. The algorithm simply concerns convergence of the mathematical expression for force equilibrium and/or moment. When using the Grid and Radius slip surface generation method, this is not a problem as the user already knows and approves of the shape of slip surfaces to be computed. These two components; the mathematics and physics, together form a method that generates reasonable results.

When using the Optimise function, no further constraints are put on the slip surface and the shape is allowed to take any, mathematically converging, shape. Therefore the Optimise function in SLOPE/W cannot, and is neither supposed to, be regarded as a method for slip surface generation as it only satisfies the mathematical aspect of the moment of the slip slice and not the physical i.e. the kinematic aspect. However, the Optimise function should be regarded as a tool. Further, when using this tool, it might actually be preferable that the Optimise function generates unreasonable results that the user can disregard than that the user misses a real slip surface because of constraints.

The bearing capacity test clearly shows, that even if a composite slip surface is known and expected, one cannot rely on the Optimise function to identify this particular slip surface but may surpass it if a mathematically more critical slip surfaces exists. As mentioned above, this is an inherit limitation of the Optimise function.

The only case where the Optimise function seems directly applicable is for the elongated slope, CS2. However, as the Optimise function is not applicable to horizontal ground surfaces with embankments as in CS1 nor for steep slopes such as CS3, it is still undetermined at which inclination the Optimise function becomes feasible.

When increasing the load, as done for the characteristic slopes, the results follow the same pattern of slightly higher ΔF_1 except for the steep slope where ΔF_1 remain the same for the two loading cases. The shape of the optimised slip surface, except for as expected being deeper for the greater load, are essentially the same. This indicates that the loading case have less influence on the applicability of the Optimise function than the geometry of the slopes. The same can be said for the combined analyses.

Altering the optimise settings of the maximum concave angles can have an improving effect on the slip surface shape and corresponding factor safety, in cases where optimising with the default settings generate a questionable slip surface shape. The greatest challenge however, as for all stability modelling, is the evaluation of the plausibility of the generated slip surfaces. Subsequently, evaluating the effects of the setting alterations is equally or even more difficult. With that said, the programming of the Optimise function certainly leaves room for experimenting with the settings. As mentioned above, the Optimise function is primarily a tool for experimenting with composite slip surfaces, in cases where such may be expected. Besides the attention to unreasonable results that is always required, when applying the optimise function, extra focus should be directed to kinematic admissibility.

Altering the starting and ending points for the optimise settings for the different models does not demonstrate any consistent pattern for the results. The different cases for the selected slopes behave contrarily; the value for factor of safety increase and decrease rather depending on the geometry conditions than the number of starting and ending points.

The current, limited transparency of how the Optimise function is programmed can cause difficulty

for the user to evaluate the results. The Optimise function could be a more powerful, effective and reliable tool for detecting critical composite slip surfaces if additional guidance for its application was available. It is therefore suggested to raise the question to GEO-SLOPE International Ltd. of more extensive user guidance specific to the Optimise function.

It would be inconsistent to expect the Optimise function to be completely reliable i.e. to never generate questionable slip surface shapes and values for the factor of safety. The optimise function has potential of being, and is already being used as, a useful tool for identifying composite slip surface where such are expected to be critical. However, as it does not consider kinematics, it lacks the ability of disregarding unreasonable results and should therefore be used with great caution.

6.7 Uncertainties and Potential Sources of Error

The fact that the results of the calculations of verification, ΔF_2 and ΔF_3 , also vary significantly is less expected. It is known that the two calculation methods, the Morgenstern-Price method and the Janbu's Generalised method, produce slightly different results. However, these differences were expected to be more consistent in their magnitude. The results from this study are not sufficient to draw such conclusions but the convergence difficulties, when recreating the slip surfaces obtained with the Morgenstern-price calculations method with Janbu's Generalised calculation method, may indicate that. If that is the case, this has a greater influence on the optimised slip surface than the circular, as ΔF_3 is consequently larger than ΔF_2 . More likely however, is that the Janbu's Generalised calculation method struggles with the convergence because the line of thrust is fixated to the lower thirds of the slices in combination with forcing the method to calculate the exact same shapes as obtained with the Morgenstern-Price method. For complicated cases such as composite slip surface, it is unlikely that the line of thrust acts at that point for all slices.

Using Janbu's Generalised calculation method for comparison was not ideal, as Janbu's Generalised calculation method often had problems converging when recalculating the slip surfaces obtained with the Morgenstern-Price calculation method. The comparison between the two methods contributed less than desired to the study. However, no better measure for verification was found.

When using the Grid and Radius method in SLOPE/W, the position and coarseness of the inserted grid and radius tangent line search areas largely influence the result of the calculation. Even with the method used for refinement of the search areas, errors with effect on the shape and position of the slip surface and the corresponding factor of safety might occur. A related observation is that the factor of safety corresponding to the optimised slip surface seems to vary more than the circular slip surface, for the same changes of grid and the radius tangent lines. This was unexpected since the optimised slip surface

PLAXIS 2D generates slip surface shapes that in most cases lie somewhere in between the circular and the optimised slip surface shapes, also with respect to the corresponding factor of safety. When defining the material properties in the PLAXIS 2D models, more input parameters are required than for the SLOPE/W models. As PLAXIS 2D is a more complex software than SLOPE/W, a direct comparison entails many uncertainties. No mesh refinements were made in the PLAXIS 2D models, which further induce uncertainties in the results.

6.8 Suggestions for Further Investigation

It would have been desirable to perform further tests that did not change settings, loads, geometries and soil properties for all the analysed slopes. The results from this thesis indicate that it is feasible

to study the applicability of the Optimise function for certain geometries. For example, it could be useful to investigate for what range of inclinations that the Optimise function is reliable and to include a more extensive investigation of the Optimise functions behaviour in combined analyses.

7 Conclusion

The greatest, and so far the only indisputable, advantage of using the Optimise function is that it can give an indication of if a composite slip surface should be considered within a specific stability analysis. As the results of the bearing capacity test show, the Optimise function only concerns the mathematical convergence and does not consider kinematic admissibility. When applying the Optimise function great caution must therefore be taken with overly conservative results, that corresponds to statically correct slip surfaces. However, as the results for the elongated slope show, it do serve its purpose in regard to this uncomplicated case of an elongated slope. The Optimise function should merely be used as a tool, and not a complete method, for generating and investigating composite slip surfaces.

References

- Cornforth, D. (2005), *Landslides In Practice: Investigation, Analysis, and Remedial/Preventive Options in Soils*, N.J.: J. Wiley, Hoboken.
- GEO-SLOPE International Ltd. (2008), *Stability Modeling with SLOPE/W 2007*, GEO-SLOPE International Ltd., Calgary, Alberta, Canada.
- GEO-SLOPE International Ltd. (2012), *Footing stability*, <http://www.geo-slope.com/support/geostudio2012/examples/detail.aspx?id=examples:slopes:footingstability>, (Accessed 26 May 2014).
- Greco, V. R. (1996), 'Efficient Monte Carlo Technique for Locating Critical Slip Surface', *Journal of Geotechnical Engineering*, **122**(7), ss. 517–525.
- Hultén, C., Högsta, U., Bengtsson, P.-E., Franzén, G. and Larsson, R. (2011), *Tillståndsbedömning/klassificering av naturliga slänter och slänter med befintlig bebyggelse och anläggningar*, Stockholm: IEG. (IEG Rapport 4:2010).
- Husein Malkawi, A. I., Hassan, W. F. and Sarma, S. (2001), 'Global Search Method for Locating General Slip Surface Using Monte Carlo Techniques', *Journal of Geotechnical and Geoenvironmental Engineering*, **127**(8), ss. 688–698.
- Knappett, J. A. and Craig, R. F. (2012), *Craig's Soil Mechanics*, Spon Press, New York.
- Krahn, J. (2003), 'The 2001 R.M Hardy Lecture: The limits of limit equilibrium analyses', *Canadian Geotechnical Journal* **40**(3), ss. 643–660.
- Larsson, R. (2008), *Jords Egenskaper*, Linköping: Swedish Geotechnical Institute. (SGI Information 1).
- Larsson, R., Sällfors, G., Bengtsson, P.-E., Alén, C., Bergdahl, U. and Eriksson, L. (2007), *Skjuvhållfasthet - utvärdering i kohesionsjord*, Linköping: Swedish Geotechnical Institute and Chalmers University of Technology. (SGI Informtion 3).
- PLAXIS 2D Material Models Manual (2012), *PLAXIS 2D Material Models Manual*, <http://www.plaxis.nl/files/files/2D2012-3-Material-Models.pdf>, (Accessed 8 April 2014).
- PLAXIS 2D Reference manual (2012), *PLAXIS 2D Reference Manual*, <http://www.plaxis.nl/files/files/2D2012-2-Reference.pdf>, (Accessed 8 April 2014).
- Skredkommissionen (1995), *Anvisningar för släntstabilitetsutredningar*, Rapport 3:95, Ingenjörsvetenskapsakademin, Linköping.
- Stanford University (2010), *Some Useful Numbers on the Engineering Properties of Materials (Geologic and Otherwise)*, <http://www.stanford.edu/~tyzhu/Documents/Some%20Useful%20Numbers.pdf>, (Accessed 7 April 2014).
- Sällfors, G. (2009), *Geoteknik: Jordmateriallära - Jordmekanik*, Chalmers tekniska högskola, Göteborg.

- Terzaghi, K. (1959), *Theoretical Soil Mechanics*, John Wiley and Sons, United States of America.
- Trafikverket (2011), *TRV Geo, Trafikverkets tekniska krav för geokonstruktioner*, Trafikverket. (TK Geo 11).
- University of the West of England (2001), *Bearing Capacity*,
<http://environment.uwe.ac.uk/geocal/foundations/founbear.htm>, (Accessed 4 April 2014).
- Westerberg, B., Hov, S. and Holmén, M. (2012), *Triaxialförsök - En vägledning*, Linköping: Swedish Geotechnical Society. (SGF Rapport 2:2012).