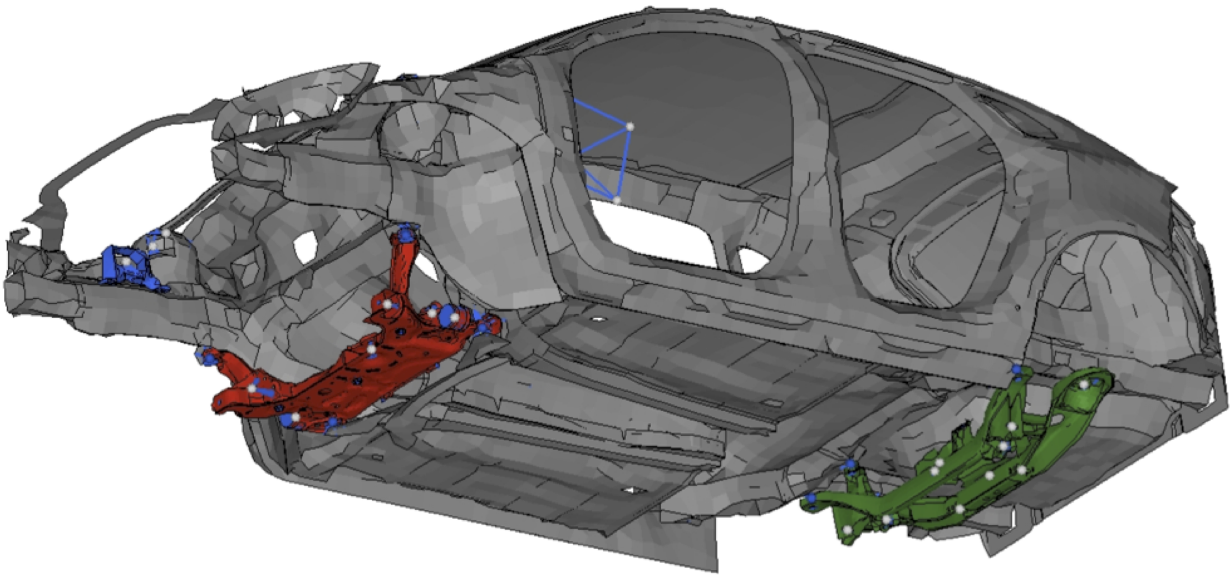

CMS methods in complete NVH analysis

Master's thesis in Applied Mechanics

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Department of Applied Mechanics
Division of Dynamics
CHALMERS UNIVERSITY OF TECHNOLOGY
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Cover:
Volvo V40 trim body with front and rear subframes.

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Abstract

Nowadays in engineering problems the use of finite element models is a common rule and the use of analysis tools in predicting the dynamic behavior of a structure is a common practice. Specifically, in the automotive industry the level of details included in a full model has been increased constantly followed by the need for higher computational power capable in order to handle such huge models and to keep the simulation time on a reasonable level. To fight this trend many different techniques exist in order to fill the need for detailed dynamic analysis and short analysis time.

In this thesis the *Dynamic Substructuring* (DS) approach will be proposed with a focus on noise and vibration analysis within the automotive engineering field. Dynamic subtracting is based on the principle of dividing our large model in smaller subsystems, easier to study and analyze. Then the total structure dynamics is calculated assembling all the small components dynamics. Since this topic is very broad, the thesis will focus on one theory of *Component Mode Synthesys* (CMS), one of the many techniques within the DS.

The thesis in divided into three main parts: firstly, the theory behind the DS is covered. From the analysis of linear dynamic systems to the two CMS reduction methods (Craig-Bampton and Craig-Cang methods) that are compared in the next parts. Secondly a dynamic substructuring analysis is performed on a simple model, where both reduction methods are applied and compared. Finally a deeper analysis of a real passenger car, in this case a Volvo V40, is carried out. The influence of the reduction basis, of connections type on the response accuracy and simulation time are studied.

From the results obtained it is possible to asses the powerfulness of the Dynamic Substructuring approach. The reduction method that performed better is the Craig-Chang method when modes are retained up to one time and a half (1.5) the maximum frequency response studied. For this method the error between the response of the full model and the response of the CMS model was less than 5%. Moreover no particular influence of the connection, bolt or bushings, have been found in choosing one or the other method. Finally the simulation time has been reduced by 1/3 if comparing the the response of the full model using a modal approach and the the response of the CMS model.

Keywords: Dynamic Substructuring, Component Mode Syntesis, Craig-Bampton, Craig-Chang.

Contents

1

Introduction

In complete vehicle noise and vibration (NVH) analysis modal sub-structuring or Component Mode Synthesis is commonly used at Volvo Cars. The benefit of using modal sub-structuring is to reduce the number of degrees of freedom for a complete vehicle FE-model and by doing that to reduce the analysis time.

1.1 Purpose

Dealing with a complete vehicle FEM model requires high computational power and time, since a full model usually is made up of millions of degrees-of-freedom. The thesis that will be presented focuses on Component Mode Synthesis, a modal sub-structuring technique aimed to reduce the full set of DoF into a smaller one and in doing that to reduce the simulation time and the computational power required. Two CMS methods (Craig-Bampton and Craig-Chang) will be applied to the V40 trim-body, front and rear subframes. The influence of reduction basis, connections type with respect the two methods will be investigate to find criteria when different techniques should be used.

1.2 Approach

First of all the theory behind the vibration of dynamic system, the sub-structuring dynamics (DS) approach and the component mode synthesis (CMS) methods will be covered in order to have a theoretical background. Then a very simple system will be analyzed. The effect of modes truncation will be studied, then a comparison between the two methods and the influence of different connections will be performed. Finally a deeper analysis of a full trim vehicle body with front and rear subframes will be carried out.

2

Theory Background

In this chapter the theory behind the thesis will be covered. The first part concerns vibrations in linear dynamic systems. After a general introduction, the basic properties will be explained starting with the simplest linear dynamic system,the mass-damper-spring system with 1-DoF. The free and forced response for damped/undamped system will be examined. The same analysis will be performed for the most general case of n-DoF.

In the second part the Dynamic Substructuring will be introduced. The basic idea behind the sub structuring technique will be highlighted with a focus on the component mode synthesis (CMS). Two reduction methods will be examined: the Craig-Bampton method and the Craig-Chang method. For each method the type of modes used in the reduction will be discussed and it will be shown how to calculate the transformation matrix for both methods. Finally a comparison between the two methods will be carried out and differences and similarities will be discussed.

Vibration Theory

The vibration phenomenon is usually related to oscillations in mechanical dynamic systems. Vibratory systems comprises elements for storing potential energy (spring), elements for storing kinetic energy (mass) and elements that can dissipate energy (damper). The chapter consider only lumped elements, hence ideal elements and rigid bodies (no deformations occur) are considered. Vibrations occur any time there is an alternating transfer of kinetic energy into potential energy and viceversa. In damped systems a part of the energy is dissipated and then an external force must be applied in order to have a steady vibration. Mainly two type of vibrations exist: free and forced vibrations. Free vibration occurs when a mechanical system is displayed from its equilibrium point and allowed to vibrate freely. This system will vibrate at one or more of its *natural frequencies*. Forced vibrations occurs when a time-variant force, also called disturbance, is applied to a mechanical system. The force can be periodic, steady-state, transient or random. If we consider a linear dynamic system excited by a steady-state harmonic force, it will vibrate at the same frequency of the applied force but the amplitude of the response depend on the system characteristics.

2.1 1-DoF System

A dynamic system can be described as a system with a mass **m**, a spring with a stiffness **k** and a damper with a damping coefficient **c**, figure 2.1.

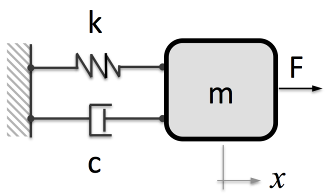

Figure 2.1: 1-DoF mass-spring-damper system.

The spring is considered to have no mass, the mass is a rigid body and for the damper we consider a viscous damping, proportional to the velocity of its connection point. This system can move just in one direction and for this reason usually it is called *single degree-of-freedom* system.

$$F(t) = m\ddot{x}(t) \tag{2.1}$$

$$F(t) = kx(t) \tag{2.2}$$

$$F(t) = c\dot{x}(t) \tag{2.3}$$

Then, considering the resultant of all the forces acting on the system shown in figure 2.1, it is possible to derive the linear dynamic equation 2.4 representing the evolution over time, since the state x and its derivative are functions of time.

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t) \tag{2.4}$$

As stated previously, this system can experience *free vibration* if the mass is displaced from its equilibrium point or *forced vibration* if the mass is excited by an external force. Next it will be discussed free and force vibration for an undamped and damped system.

2.1.1 Free Respone

Undamped System

Starting with the simplest system consisting of a mass constrained to a rigid support by means of a spring, figure 2.2.

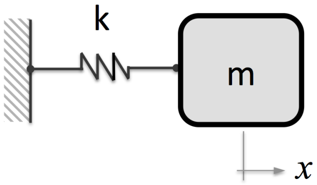

Figure 2.2: 1-DoF mass-spring system

The equation 2.4 can be simplified, taking into account the absence of the damper and the external force, leading to the equation 2.5, where the force extorted by the mass is balanced by the force applied by the spring.

$$m\ddot{x} + kx = 0 \tag{2.5}$$

The solution for the differential equation 2.5 is:

$$x(t) = Acos\sqrt{\frac{k}{m}}t + Bsin\sqrt{\frac{k}{m}}t \tag{2.6}$$

where $w_n = \sqrt{k/m}$ is the the natural frequency and the term A and B are defined from the *initial conditions* of the system. Indeed A is equal to the initial mass displacement x_0

at time t_0, while B is equal to the initial mass speed divided per the natural frequency, $B = \dot{x_0}/w_n$ at time t_0. Therefore the motion of the system is a consequence of the initial conditions. If the system is in the equilibrium point then no motion occurs, i.e. $x_0 = 0, \dot{x_0} = 0$.

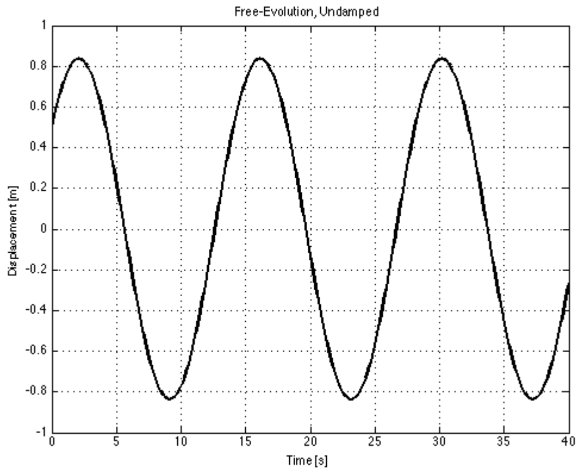

Figure 2.3: Free evolution, undamped system. (m=100 kg, k=20 Nm, $x_0 = 0.5$ m, $\dot{x_0} = 0.3$ m/s, $w_n = 0.4472$ rad/s.

Damped System

Different type of damping model can be used to describe the damping of a dynamic system.

In this case it has been used a *viscous damping*, i.e. proportional to the velocity, to show the effect of the damping in a free-evolution motion. The equation 2.4 in absence of external force become simply:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = 0 \tag{2.7}$$

An important parameter for a damped system is the *critical damping coefficient* c_c:

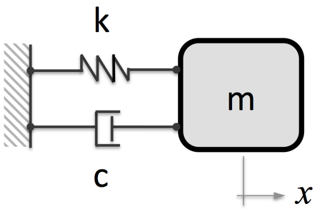

Figure 2.4: 1-DoF mass-spring-damper system

$$c_c = 2\sqrt{km} \tag{2.8}$$

The ratio $\zeta = c/c_c$ between the actual damping and the critical is called *damping ratio* or *damping factor*. Depending on the value of ζ the equation of motion of the free-evolution of a damped system varies.

Less-than-critical Damping. If the actual damping of the system is less than the critical, i.e. $\zeta < 1$, then the solution of the equation 2.7 is:

$$x(t) = e^{-ct/2m}(Asin\omega_d t + Bcos\omega_d t) \tag{2.9}$$

where ω_d is called *damped natural frequency* and is linked to the undamped natural frequency by the equation 2.10.

$$\omega_d = \omega_n(1-\zeta^2)^{1/2} \tag{2.10}$$

Critical Damping. If the actual damping of the system is equal to the critical, i.e. $c = c_c$, no oscillations occur in the response and the solution of the equation of motion 2.7 is:

$$x(t) = e^{-ct/2m}(A + Bt) \tag{2.11}$$

Greater-than-critical Damping. If the actual damping is greater than the critical, i.e. $\zeta > 1$, then the solution of the equation of motion 2.7 is:

$$x(t) = e^{-ct/2m}(Ae^{\omega_n\sqrt{z^2-1}t} + Be^{-\omega_n\sqrt{z^2-1}t}) \tag{2.12}$$

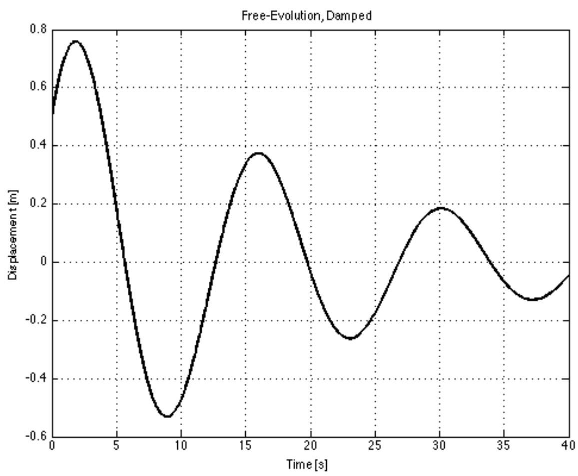

Figure 2.5: Free evolution, damped system. (m=100 kg, k=20 Nm, c=10 m/s, $x_0 = 0.5$ m, $\dot{x_0} = 0.3$ m/s, $w_n = 0.4472$ rad/s, $\zeta = 0.11$ (less-than-critical).

2.1.2 Forced Response

Undamped System

Considering a harmonic force, $F = F_0 cos(w_0 t)$, the equation 2.4 can be simplified in the following:

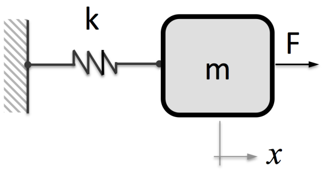

Figure 2.6: 1-DoF mass-spring system, with external force.

$$m\ddot{x} + kx = F_0 cos(w_0 t) \tag{2.13}$$

The solution of this equation is made up of a transient solution, the same as the free response, plus a steady-steady oscillation at the *forcing frequency* $\omega_{\mathbf{0}}$.

$$x(t) = X_0 + X_p = (Acos\omega_n t + Bsin\omega_n t) + \frac{F_0(cos\omega_0 t - cos\omega_n t)}{m(\omega_n^2 - \omega_0^2)} \tag{2.14}$$

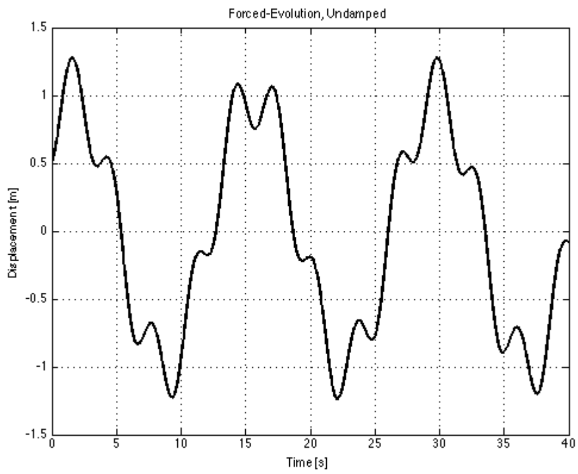

Figure 2.7: Forced evolution, undamped system. (m=100 kg, k=20 Nm, x_0 = 0.5 m, $\dot{x_0}$ = 0.3 m/s, w_n = 0.4472 rad/s, F=100cost(2t).

Damped System

Finally including the viscous damping the general equation stated at the beginning of the chapter, equation 2.4, will be analyzed. In this case, figure 2.1, the solution is described by a transient term and a steady-state term as descried in equation

$$x = X_0 + X_p \tag{2.15}$$

The terms X_0 is exactly the same solution of homogeneous equation 2.7 that depends on the damping factor. The particular solution X_p is the solution of the full non-homogeneous equation and then represent the response od the system due to the external force.

$$X_p = \frac{F_0}{(k - m\omega_0^2)^2 + c^2\omega_0^2)}(k - m\omega_0^2)cos(\omega_0 t) + c\omega_0 sin(\omega_0 t) \tag{2.16}$$

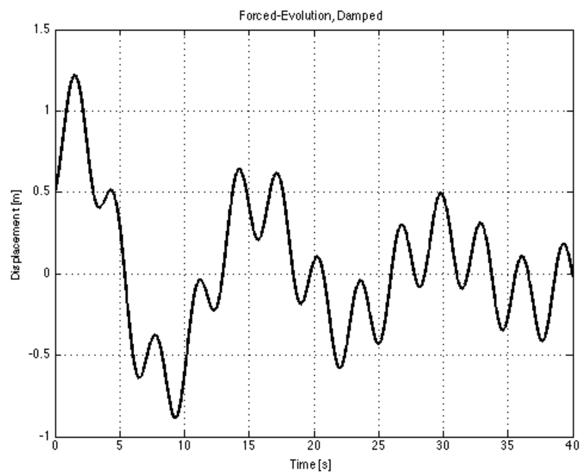

Figure 2.8: Forced evolution, undamped system. (m=100 kg, k=20 Nm, c_c = 10 m/s, $x_0 = 0.5$ m, $\dot{x_0} = 0.3$ m/s, $\zeta = 0.11$, $w_n = 0.4472$ rad/s, $F = 100cost(2t)$).

Transfer function

Since the transient response decay after a while, it assumes a particular importance the steady-state amplitude of the response due to a unit harmonic excitation. In this sense it is possible to plot this factor as a function of the excitation frequency. This is called frequency response function or steady-state amplification factor or transfer function in most of the cases.

$$H(\omega) = \frac{F(\omega)}{X(\omega)} \tag{2.17}$$

Considering the system shown in figure 2.6, made of a mass and a spring excited by a harmonic force and recalling equation 2.14, it is possible to derive the amplification factor:

$$H(\omega) = \frac{1}{m(\omega_n^2 - \omega^2)} \tag{2.18}$$

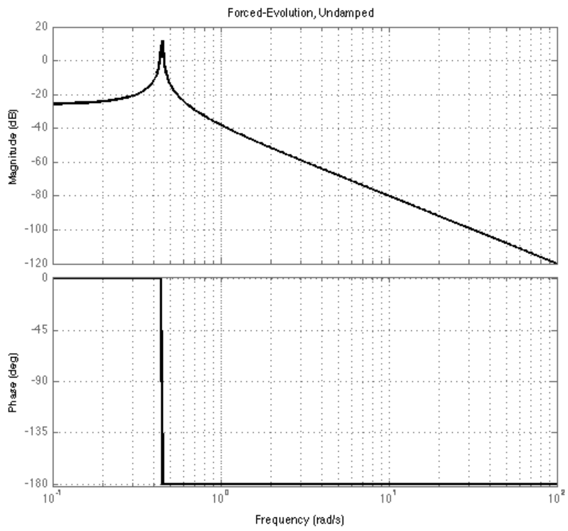

Figure 2.9: Magnitude and phase. (m=100 kg, k=20 Nm, $w_n = 0.4472$ rad/s.

It possible to notice that at $\omega = 0 \rightarrow H(\omega) = 1/k$, for $\omega = \infty \rightarrow H(\omega) = 0$. At $\omega = \omega_n$ a particular behavior occurs, the response is amplified and this phenomenon is called resonance.

2.2 Modal Analysis Method

In the previous chapter the solution of the dynamics of the single DoF system has been derived analytically, i.e. solving directly the differential equation of motion and in doing so no particular effort has been put into. When dealing with multi DoF systems solving the set of equations analytically and directly can require high computational power and time. For this reason lots of powerful methods exist to derive the dynamics in a more efficient way. One of those methods is the Modal Analysis Method. This method is based on the superposition principle, i.e. the response of a n-DoF system can be decomposed into n modal responses, each of them representing a single DoF system.

Recalling the equation of motion of a multi DoF system, equation 2.19, where [M] is the mass matrix, [C] is the damping matrix, [K] is the stiffness matrix and $\{x\}$ is the set of nodal or physical DoF.

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} \tag{2.19}$$

Equation 2.21 is the modal equation of motion, where $[\bar{M}]$ is the modal mass matrix, $[\bar{C}]$ is the modal damping matrix, $[\bar{K}]$ is the modal stiffness matrix and $\{q\}$ is the set of *modal DoF*.

$$[\bar{M}]\{\ddot{q}\} + [\bar{C}]\{\dot{q}\} + [\bar{K}]\{q\} = \{\bar{F}\} \tag{2.20}$$

As stated at the beginning the physical set of DoF can be expressed as a superposition of modal contributions:

$$\{u(t)\} = \sum_{i=1}^{N} [\phi_n]_i \{q(t)\}_i \tag{2.21}$$

where N is the total number of DoF, $[\phi_n]$ is the *natural modes* matrix of vibration of the multi-DoF system.

2.3 Eigenvalue problem

In order to perform the modal analysis, as stated previously, is necessary to calculate natural frequencies and vibration modes of the system we want to analyze. Starting, for simplicity, from the EOM of the free undamped system:

$$[M]\{\ddot{x}\} + [K]\{x\} = 0 \tag{2.22}$$

Considering the physical response as a linear combination of modes and modal response:

$$\{u(t)\} = [\phi_n]\{q(t)\} \tag{2.23}$$

and assuming the modal response to be a harmonic function of the type:

$$\{q(t)\} = Acos\omega_n t + Bsin\omega_n t \tag{2.24}$$

It is possible to combine equation 2.23 and 2.24, so that:

$$\{u(t)\} = [\phi_n](Acos\omega_n t + Bsin\omega_n t) \tag{2.25}$$

imposing $\{\lambda\} = \{\omega_n^2\}$ and substituting this equation into the EOM 2.22 gives:

$$([K] - \lambda[M])\phi_n = 0 \tag{2.26}$$

The equation 2.25 represent our *Eigenvalue Problem* and the solution is a set of of eigenvalues $\{\lambda\}$ and eigenvectors $[\phi_n]$.

From the eigenvalues is possible to derive a diagonal matrix of natural frequencies, since $\{\lambda\} = \{\omega_n^2\}$:

$$[\omega_n^2] = \begin{bmatrix} \omega_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \omega_N^2 \end{bmatrix} \tag{2.27}$$

The eigenvectors $[\phi_n]$ are the modes of vibrations and they can be represent in a modal matrix:

$$[\phi_n] = \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1r} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ \phi_{N1} & \phi_{N2} & \cdots & \phi_{Nr} \end{bmatrix} \tag{2.28}$$

where each column or eigenvector represent a vibration mode.

2.4 n-DoF System

Increasing the level of complexity of a linear dynamic system the 2-DoF system can be used to gain a better understanding about vibrations modes, natural frequencies, transfer function and other important properties related to a dynamic system.

For simplicity in this section only mass-spring systems excited by a harmonic force will be considered and the excitation frequency is considered the same for both masses.

Physical representation. In figure 2.10 is shown a 2-DoF system, made of series of masses and elastic elements or springs. This configuration is called fixed-free since at one end the system is fixed to

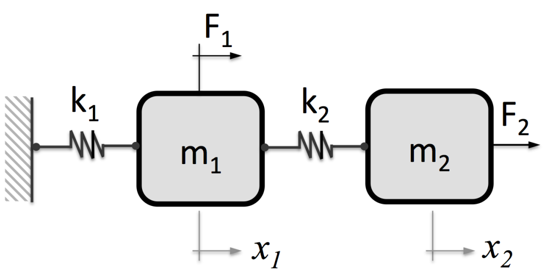

Figure 2.10: 2-DoF mass-spring system, with external force.

a rigid support, while the other end is free to move. Each mass has just one DoF, i.e. it can move only horizontally. It is clear that the dynamic of each mass is influenced by the dynamic of the other mass and this lead to a n EOM coupled to each other, equation 2.29

$$\begin{cases} m_1\ddot{x_1}(t) + (k_1 + k_2)x_1(t) - k_2x_2(t) = F_1 cos\omega(t) \\ m_2\ddot{x_2}(t) - k_2x_1(t) + (k_2 + k_3)x_2(t) = F_2 cos\omega(t) \\ \vdots \\ m_n\ddot{x_n}(t) - k_nx_{n-1}(t) + k_nx_n(t) = F_n cos\omega(t) \end{cases} \tag{2.29}$$

This system of equations can be rewritten in a matrix form, shown by equation 2.19, into equation 2.30.

$$\begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_n \end{bmatrix} \cdot \begin{Bmatrix} \ddot{x_1} \\ \ddot{x_2} \\ \vdots \\ \ddot{x_n} \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & -k_2 & \cdots & 0 \\ -k_2 & (k_2 + k_3) & \cdots & 0 \\ \vdots & \vdots & \ddots & -k_n \\ 0 & 0 & -k_n & k_n \end{bmatrix} \cdot \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} = \begin{Bmatrix} F_1 cos\omega(t) \\ F_2 cos\omega(t) \\ \vdots \\ F_n cos\omega(t) \end{Bmatrix} \tag{2.30}$$

It can be seen that the stiffness matrix [K] is not diagonal and this derive from the fact that the dynamic of mass 1 is influencing and is influenced by the dynamics of mass 2 and the other way around. In order to obtain also a diagonal stiffness matrix the modal analysis can be applied, solving first the eigen-problem and deriving the modes and natural frequencies, apply the transformation from the physical representation into the modal, solving each single-DoF system and then retrieving the physical solution from the modal using the superposition principle.

Eigen-Problem. It is possible to calculate natural frequencies and the modes matrix solving the equation $([K] - \lambda[M])\phi_n = 0$. Natural frequencies and natural modes will be computed:

$$[\omega_n^2] = \begin{bmatrix} \omega_1^2 & 0 & \cdots & 0 \\ 0 & \omega_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \omega_n^2 \end{bmatrix} \tag{2.31}$$

And the modes matrix is:

$$[\phi_n] = \left[\begin{Bmatrix} \phi_{11} \\ \phi_{21} \\ \vdots \\ \phi_{n1} \end{Bmatrix} \begin{Bmatrix} \phi_{12} \\ \phi_{22} \\ \vdots \\ \phi_{n2} \end{Bmatrix} \cdots \begin{Bmatrix} \phi_{1n} \\ \phi_{2n} \\ \vdots \\ \phi_{nn} \end{Bmatrix} \right] \tag{2.32}$$

Modal representation. In order to transform the equation 2.19 into the equation 2.21 it is necessary to apply the following transformation:

$$\begin{cases} [\bar{M}] = [\phi_n]^T[M][\phi_n] \\ [\bar{K}] = [\phi_n]^T[K][\phi_n] \\ [\bar{F}] = [\phi_n]^T[F] \end{cases} \tag{2.33}$$

Equation 2.30 is transformed into:

$$\begin{bmatrix} \bar{m}_1 & 0 & \cdots & 0 \\ 0 & \bar{m}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \bar{m}_n \end{bmatrix} \cdot \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \vdots \\ \ddot{q}_n \end{Bmatrix} + \begin{bmatrix} \bar{k}_1 & 0 & \cdots & 0 \\ 0 & \bar{k}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \bar{k}_n \end{bmatrix} \cdot \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{Bmatrix} = \begin{Bmatrix} \bar{F}_1 cos\omega(t) \\ \bar{F}_2 cos\omega(t) \\ \vdots \\ \bar{F}_n cos\omega(t) \end{Bmatrix} \tag{2.34}$$

where both the mass matrix and the stiffness matrix are diagonal, i.e. the system is made up of n uncoupled equations that can be solved separately.

Superposition principle. It is possible to calculate the n modal displacements by solving the equations 2.34. Again the solution means of two parts: a transient part due to initial conditions and a steady-state solution due to the external force, $q = q_0 + q_p$:

$$\begin{cases} q_1 = (A_1 cos\omega_{n_1} t + B_1 sin\omega_{n_1} t) + \dfrac{\bar{F}_1(cos\omega t - cos\omega_{n_1} t)}{\bar{m}_1(\omega_{n_1}^2 - \omega^2)} \\ \\ q_2 = (A_2 cos\omega_{n_2} t + B_2 sin\omega_{n_2} t) + \dfrac{\bar{F}_2(cos\omega t - cos\omega_{n_2} t)}{\bar{m}_2(\omega_{n_2}^2 - \omega^2)} \\ \vdots \\ q_N = (A_n cos\omega_{n_N} t + B_n sin\omega_{n_N} t) + \dfrac{\bar{F}_N(cos\omega t - cos\omega_{n_N} t)}{\bar{m}_N(\omega_{n_N}^2 - \omega^2)} \end{cases} \tag{2.35}$$

where A_1, A_2, B_1 ,B_2 are constants related to the initial conditions of each mass. Applying the superposition principle expressed in equation 2.21 it is possible to calculate the physical displacement, using the modes matrix, as shown in equation 2.36.

$$\begin{cases} x_1 = \eta_{1_{1st}} + \eta_{1_{2nd}} = \phi_{11}q_1 + \phi_{12}q_2 + \cdots + \phi_{1n}q_n \\ x_2 = \eta_{2_{1st}} + \eta_{2_{2nd}} = \phi_{21}q_1 + \phi_{22}q_2 + \cdots + \phi_{2n}q_n \\ \vdots \\ x_N = \eta_{n_{1st}} + \eta_{n_{2nd}} = \phi_{n1}q_1 + \phi_{n2}q_2 + \cdots + \phi_{nn}q_n \end{cases} \tag{2.36}$$

where η_{1st} is the displacement due to the first mode and η_{2nd} id the displacement due to the second mode.

Initial conditions. In order to calculate the constants A_1, A_2 ..., B_1 ,B_2 ... that depend on the physical initial condition of our system, it is necessary to follow some steps. First the physical initial conditions have to be transformed into a modal initial conditions. Assuming X_0 and V_0 respectively the physical initial displacement and velocity vectors:

$$\begin{cases} X_0 = \{x_{0_1}, x_{0_2}, \cdots, x_{0_n}\} \\ V_0 = \{\dot{x}_{0_1}, \dot{x}_{0_2}, \cdots, \dot{x}_{0_n}\} \end{cases} \tag{2.37}$$

where x_0 is respectively the initial displacement of the mass, while $\dot{x}_0$ is the initial velocity of the mass. Second it is necessary to apply a transformation from the physical domani to the modal:

$$\begin{cases} \bar{X}_0 = [\phi_n]^T[M]X_0 = \{\bar{x}_{0_1}, \bar{x}_{0_2}, \cdots, \bar{x}_{0_n}\} \\ \bar{V}_0 = [\phi_n]^T[M]V_0 = \{\bar{\dot{x}}_{0_1}, \bar{\dot{x}}_{0_2}, \cdots, \bar{\dot{x}}_{0_n}\} \end{cases} \tag{2.38}$$

where $\bar{X}_0$ and $\bar{V}_0$ are respectively the initial modal displacement and velocity. Finally it is possible to calculate the constants as shown:

$$\begin{cases} A_1 = \bar{x}_{0_1} \\ A_2 = \bar{x}_{0_2} \\ \vdots \\ A_n = \bar{x}_{0_n} \\ \\ B_1 = \bar{\dot{x}}_{0_1}/\omega_{n_1} \\ B_2 = \bar{\dot{x}}_{0_2}/\omega_{n_2} \\ \vdots \\ B_n = \bar{\dot{x}}_{0_n}/\omega_{n_n} \end{cases} \tag{2.39}$$

Dynamic Substructuring Theory

Dynamic Substructuring (DS) is based on the principle of dividing a structure into smaller sub-structures. Evaluate the dynamic behavior for each substructure and then assemble all those substructure to compute the dynamics of the initial structure. It is possible to study each substructure in the time domain using a physical or a modal representation, or it is possible to study the dynamic behavior in the frequency domain using a frequency response function (FRF). Moreover there are different possibilities to assemble the substructures, using a primal assembly where the substructures are assembled using the interface displacements i.e. the two substructure must have the same set of interface DoF. Or a dual approach that uses the interface forces, i.e. the connection forces on both sides of the interface have to be in equilibrium. Figure fig 2.11 resumes what it has been said until now.

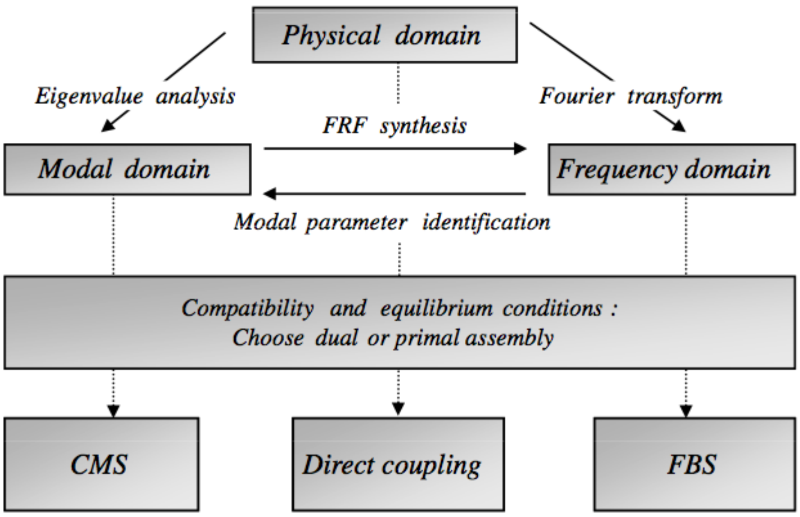

Figure 2.11: Dynamic dynamic representations (time and frequency domain) and their possible assembly methods. [1]

The Dynamic Substructuring approach has sereval advantages:

- It allows to study structures with a high number of DoF. In our case a full vehicle model can contain about 40 millions of DoF. With this approach it is easier to study such a very detailed model.
- FEM components and experimental data can be combined in order to study the dynamic of the whole structure.
- It allows model simplification since it is possible to get rid of those subsystems that don't contribute on the total dynamic behavior and moreover local dynamic behavior and its influence on the gobal behavior can be determined easily.
- Using reduction techniques within the substructuring approach allows to reduce the total number of DoF and then reducing the simulation time drastically, moreover a reduced system needs less computational power and it is easier to be handled.

2.5 Component Mode Synthesis

This chapter focuses on the Component Mode Synthesis that uses modes to represent the dynamic behavior of the system. In the time domain the CMS approach uses a certain type of modes to derive the dynamic behavior of the system by dissecting the system into small sub-system and analyzed the dynamics of each sub-system and then try to deduce the behavior of the complete structure from this information. CMS it is very efficient for large eigenvalue problems when just few modes are retained from the full set.

Two methods will be discussed: The Craig-Bampton and the Craig-Chang method. The Craig-Bampton uses fixed-interface and constraint modes, while Craig-Chang uses free-interface, rigid body and residual attachment modes. The next section will try to explain the modes used for each method, how to calculate those modes, how to compute the reduction matrix and how to reduce the original system. Finally differences between the two methods will be discussed.

2.6 The Craig-Bampton Method

The first method it will be discussed is the Craig-Bampton method. This reduction method includes two type of modes in the reduction basis: the fixed-vibrations modes and the constraints modes.

2.6.1 Fixed-Interface vibration modes.

These modes contain vibrational informations of the structure kept fixed at its boundary DoF. In order to calculate these modes it's necessary to partition the system into boundary DoF (x_b) and internal DoF (x_i).

$$\begin{bmatrix}[M_{ii}] & [M_{ib}]\\ [M_{bi}] & [M_{bb}]\end{bmatrix}\cdot\begin{Bmatrix}\ddot{x}_i\\ \ddot{x}_b\end{Bmatrix}+\begin{bmatrix}[K_{ii}] & [K_{ib}]\\ [K_{bi}] & [K_{bb}]\end{bmatrix}\cdot\begin{Bmatrix}x_i\\ x_b\end{Bmatrix}=\begin{Bmatrix}[F_i]\\ [F_b]\end{Bmatrix} \tag{2.40}$$

Then it is necessary to constrain the boundary DoF, i.e. ($\{x_b\} = 0$),

$$[M_{ii}]\{\ddot{x}_{ii}\} + [K_{ii}]\{x_{ii}\} = 0 \tag{2.41}$$

And finally solving the eigenvalue problem, described previously by equation 2.26, but related to the internal constrained DoF:

$$([K_{ii}] - \omega_i^2[M_{ii}])\{\phi_i\} = 0 \tag{2.42}$$

The result are the eigenmodes and eigenfrequencies of the system constrained at its boundary DoF. The full matrix of eigenmodes $[\Phi_i] = [\{\phi_{i_1}\}\{\phi_{i_2}\}\ldots\{\phi_{i_n}\}]$ represents the fixed-interface vibration modes.

2.6.2 Constraint Modes.

These modes are the static deformation due to a unit displacement applied to one of the boundary DoF, while the others boundary DoF are restrained and no forces are applied to the internal DoF. Thus, the constraint modes are nothing else than the static response of the structure resulting from a unit deflection imposed at the interface DoF. To calculate the constraint modes it is necessary again to divide the DoF in internal and boundary as shown in equation 2.40. Expanding the first equation in 2.40 and considering no forces applied at the internal DoF, i.e. $[F_i] = 0$, then:

$$[M_{ii}]\{\ddot{x}_i\} + [M_{ib}]\{\ddot{x}_b\} + [K_{ii}]\{x_i\} + [K_{ib}]\{x_b\} = 0 \tag{2.43}$$

Since the static response has to be calculated, it is possible to neglect the inertia forces and than the remaining terms can be rearranged in:

$$\{x_{i_{stat}}\} = -[K_{ii}]^{-1}[K_{ib}]\{x_b\} \tag{2.44}$$

where the term $-[K_{ii}]^{-1}[K_{ib}]$ is the static modes matrix. Finally it is possible to write the constraint modes matrix considering that the original set of DoF $\{x\}$ has been divided into internal and external:

$$\begin{bmatrix}\{x_i\}\\ \{x_b\}\end{bmatrix} = [\Phi_C]\{x_b\} = \begin{bmatrix}[-[K_{ii}]^{-1}[K_{ib}]]\\ [I]\end{bmatrix}\{x_b\} \tag{2.45}$$

where $[\Phi_C]$ are the constraint modes matrix.

2.6.3 Reduction Matrix.

Once the free interface modes $[\Phi_i]$ and the constraint modes $[\Phi_C]$ have been calculated, it is possible to compute the reduction matrix $[R]_{CB}$ in order to reduce the structure. Recalling equation equation 2.40, where the DoF have splitted in internal and external DoF and considering that now the internal DoF are described in terms of fixed-interface modes $[\Phi_i]$ and constraint modes $[\Phi_C]$:

$$\{x_i\} = [\Phi_i]\{\eta_i\} + [\Phi_C]\{x_b\} \tag{2.46}$$

and the reduction basis in matrix form will be:

$$\begin{bmatrix} \{x_i\} \\ \{x_b\} \end{bmatrix} = \begin{bmatrix} [\Phi_i]\{\eta_i\} + [\Phi_C]\{x_b\} \\ \{x_b\} \end{bmatrix} = \begin{bmatrix} [\Phi_i] & [\Phi_C] \\ 0 & [I] \end{bmatrix} \begin{bmatrix} \{\eta_i\} \\ \{x_b\} \end{bmatrix} = [R]_{CB} \begin{bmatrix} \{\eta_i\} \\ \{x_b\} \end{bmatrix} \tag{2.47}$$

Using the reduction matrix $[R]_{CB}$ it is possible to reduce the original mass and stiffness matrices:

$$\begin{cases} [\tilde{M}]_{CB} = [R]_{CB}^T[M][R]_{CB} \\ [\tilde{K}]_{CB} = [R]_{CB}^T[K][R]_{CB} \end{cases} \tag{2.48}$$

2.7 The Craig-Chang Method

The reduction basis is made of free-interface modes, i.e. the component is considered unconstrained at its interface DoF, rigid-body modes and residual inertia relief attachment modes.

2.7.1 Free-Interface vibration modes.

These vibration modes are simply the structure modes if the boundary or interface DoF are unconstrained. They can be computed by solving the eigen-problem for the full mass and stiffness matrix as mentioned early in this chapter. Recalling the equation 2.26:

$$([K] - \omega_f^2[M])\{\phi_f\} = 0 \tag{2.49}$$

where, $\{\phi_f\}$ is the free vibration mode linked to its eigenfrequency ω_f^2. Thus these modes contains the full vibration content of the system and since now it will be used the notation $[\Phi_f]$ to relate to the free-interface modes set.

2.7.2 Rigid-Body modes.

These modes can be considered a special type of free-interface vibration modes. They are the vibration modes of the structure if it would not be fully constrained and then it can displace without deformations, i.e. as a rigid body. Since the eigenfrequencies associated to these modes are zero, recalling equation 2.49 and setting $\omega_f^2 = 0$, then it is possible to derive:

$$[K][\Phi_r] = 0 \tag{2.50}$$

where $[\Phi_r]$ represent the set of rigid body modes.

2.7.3 Residual Inertia Relief Attachment modes.

$$[\Phi_{am}] = [G]_{am}[F] \tag{2.51}$$

2.7.4 Reduction Matrix.

Finally the physical set of DoF can be calculated as the sum of the three different contributions of free, rigid and attachment modes as shown by equation 2.52.

$$\{x_i\} = [\Phi_f]\{\eta_f\} + [\Phi_r]\{\eta_r\} + [\Phi_{am}]\{\eta_{am}\} \tag{2.52}$$

Putting together all the sets of modes it is possible to compute the reduction matrix $[R]_{CC}$ for the Craig-Chang method.

$$\{x_i\} = \begin{bmatrix} [\Phi_f] & [\Phi_r] & [\Phi_{am}] \end{bmatrix} \begin{bmatrix} \{\eta_f\} \\ \{\eta_r\} \\ \{\eta_{am}\} \end{bmatrix} = [R]_{CC} \begin{bmatrix} \{\eta_f\} \\ \{\eta_r\} \\ \{\eta_{am}\} \end{bmatrix} \tag{2.53}$$

Using the reduction matrix $[R]_{CC}$ it is possible to reduce the original mass and stiffness matrices:

$$\begin{cases} [\tilde{M}]_{CC} = [R]_{CC}^T[M][R]_{CC} \\ [\tilde{K}]_{CC} = [R]_{CC}^T[K][R]_{CC} \end{cases} \tag{2.54}$$

2.8 Differences between methods

The main difference between the Craig-Bampton method and the Craig-Chang is that the Craig-Bampton is a fixed-interface method while the Craig-Chang uses a free-interface condition. So in choosing one method rather than the other the interface condition of the component has to be analyzed. If the fixed condition more represents the real condition then the Craig-Bampton should be used, if the free condition more represents the interface condition then the Craig-Chang method should be used.

In the Craig-Bampton method if the interface changes, for example different number of connections or different number of interface DoF keeping the same interface points, than all the reduction basis has to be calculated again. Indeed both fixed-modes and contraint-modes are directly linked to the interface configuration. In the Craig-Chang free-modes and rigid-modes are not dependent from the interface configuration, only the residual attachment modes have to be recomputed.

In Craig-Bampton fixed-modes and constraint-mode are very easy to compute and moreover the original interface DoF are retained and it is easy to assembly the reduced substructure. In Craig-Chang the residual attachment modes require intensive calculation compared to the constraint modes. But an advantage to use residual attachment modes is that they may improve convergence and they account for the elasticity of the deleted free modes.

3

First Study Case

IN this first study case the reduction methods discussed in chapter 2 will be applied to a very simple structure. This model is made of a box, relatively more flexible, a frame and a rigid element that simulate the stiffness of a very stiff subsystem (for example the and engine case).The purpose of this study is to:

- study the effect of the modal truncation;
- compare Craig-Bampton and Craig-Chaing methods;
- study the influence of the connection type;
- compare the solution time for unreduced and reduced system.

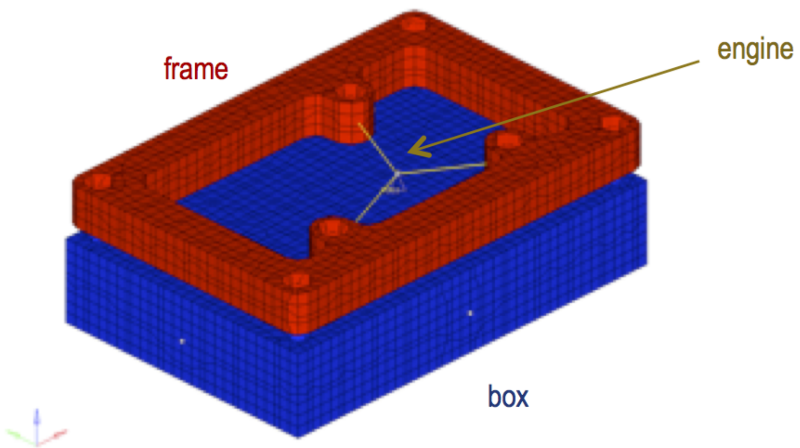

Figure 3.1: Frame-Box system, FEM model.

Table 3.1: Material Properties.

Properties	Symbol	Value
Density	ρ	2700 $[kg/m^3]$
Modulus of elasticity	E	69 [GPa]
Poisson Coeff	ν	0.3
Material Damping	η	0.06
Thickness	t	0.5 mm

The finite element model has been created in HyperWorks and has 114000 degree of freedom, figure 3.1. There are four connections between the box and the frame and three connections between the frame and the rigid element. Both frame and box have the same material properties and also the thickness is the same, as shown in table 3.1.

3.1 Simulation set-up:

Usually in NVH problems we are interested in frequencies up to 300-350 Hz, but for this specific case, since this system is very stiff it has been necessary to study the dynamic behavior up to 1000 Hz in order to include some flexible modes. As shown figure 3.2 it has been applied a load in the rigid element that simulate the engine and the response has been taken at the centre of the bottom face of the box. In this way the excitation has to travel through all the components and all the connections, so that it is possible to see the effect of both reduction type and connection type.

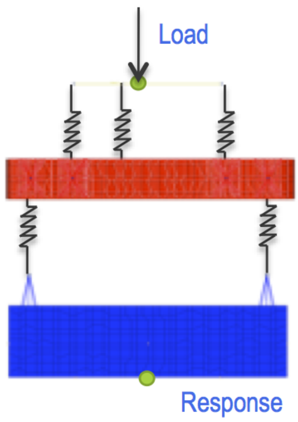

Figure 3.2: Load and Response points.

3.2 Method.

Reduction basis comparison: Firstly, the effect of including a different number of modes will be carried out. Considering that the solution is up to 1000 Hz, different cases will be studied that include a different number of modes considering different cut-off frequency for the reduction (modes up to 500Hz, up to 1000Hz, up to 1500Hz and so on), as shown in table 3.2. For this first analysis the connections have been considered rigid.

Table 3.2: Comparison between different reduction basis for a specific reduction method.

Method	Cut-off Hz	Total DoF
Full model	-	114 000
Craig-Bampton	500 Hz	120
	1500 Hz	214
	2000 Hz	279
	3000 Hz	433
	5000 Hz	736
Craig-Chang	500 Hz	196
	1500 Hz	301
	3000 Hz	526
	5000 Hz	834

Craig-Bampton and Craig-Chang comparison: Once the best reduction basis will be found out, a comparison between Craig-Bampton and Craig-Chang will be performed. Again, the connection are kept rigid.

Table 3.3: Comparison between different reduction methods.

Method	Cut-off Hz	Total DoF
Full model	-	114 000
Craig-Bampton	3000 Hz	433
Craig-Chang	3000 Hz	526

Connections comparison: Finally the influence of connections, soft and hard bushings, will be examined in order to understand if the connections affect the choice of the reduction method.

Table 3.4: Comparison between different connection types (rigid-bushing).

Connection Type	Method	Cut-off Hz	Total DoF
Bushing (K=200 Nmm, G=0.1)	Full model	-	114 000
	Craig-Bampton	3000 Hz	433
	Craig-Chang	3000 Hz	526
Bushing (K=2 Nmm, G=0.1)	Full model	-	114 000
	Craig-Bampton	3000 Hz	433
	Craig-Chang	3000 Hz	526

Solution and reduction time comparison: The reduction time for different reduction basis and methods will be compared. Moreover the solution time for the reduced models will be compared with respect the full model. It will be taken into account also the solution method, direct or modal.

Table 3.5: Reduction basis, number of DoF, solution method for different reduction methods.

Method	Modes up to (Hz)	Total DoF	Solution Method
Full model (Direct)	-	114000	direct
Full model (Modal)	1500 Hz	114000	modal
Craig-Bampton	1500 Hz	214	modal
Craig-Bampton	3000 Hz	433	modal
Craig-Chang	1500 Hz	301	modal
Craig-Chang	3000 Hz	526	modal

3.3 Results.

Reduction basis comparison: In the first graph in figure 3.3 has been plotted the direct response for the full model with FE components (back line) and the modal response for the model reduced with Craig-Bampton considering different reduction basis, i.e. keeping modes up to 500Hz, 1500Hz, 2000Hz, 3000Hz and 5000Hz for each subsystem. In the second graph in figure 3.3 the response has been discretized into 1/3 octave bands and the error between the full model solution and the reduced ones has been plotted. The error has been calculated only for the most significant curves.

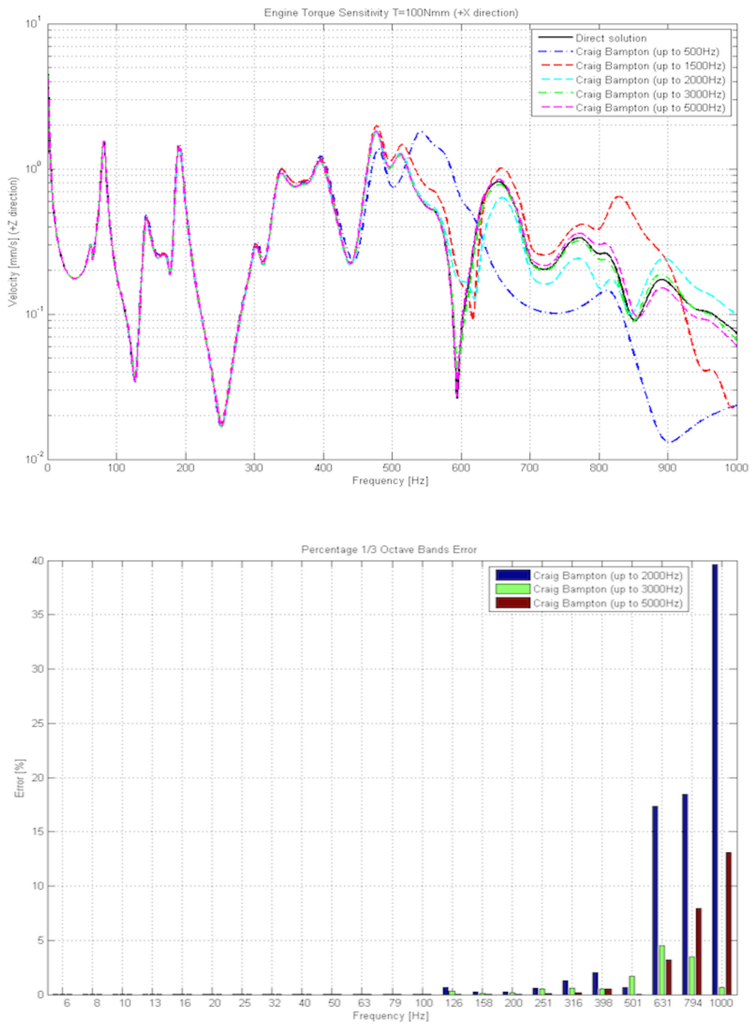

Figure 3.3: Response and Error for Craig-Bampton method.

In the first graph in figure 3.4 has been plotted the direct response of the full model and the modal response for the model reduced with the Craig-Chang method with different reduction basis, i.e. keeping modes up to 500Hz, 1500Hz, 3000Hz and 5000Hz for each subsystem. In the second graph in figure 3.4 again the response has been discretized into 1/3 octave bands and the error between the full model solution and the reduced ones has been plotted.

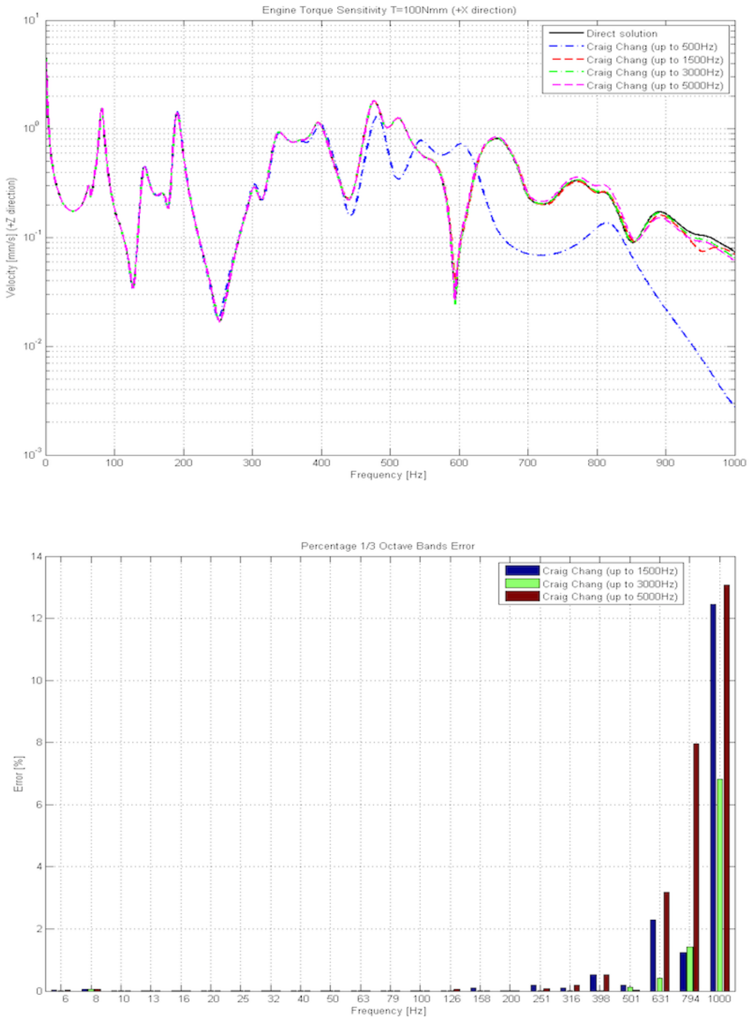

Figure 3.4: Response and Error for Craig-Chang method.

Craig-Bampton and Craig-Chang comparison: For this analysis it has been chosen the same reduction basis for both reduction methods, i.e. it has been kept modes up to 3000Hz. Moreover the subsystems are connected using rigid elements. In the first graph in figure 3.5 is shown the response for the full model, i.e. the reference response, and the response for the reduced model with Craig-Bampton and Craig-Chang. In the second graph in figure 3.5 is shown the error between the reference response and the response using Craig-Bampton (red boxes) and the error between the reference response and the response using Craig-Chang as reduction method (green boxes).

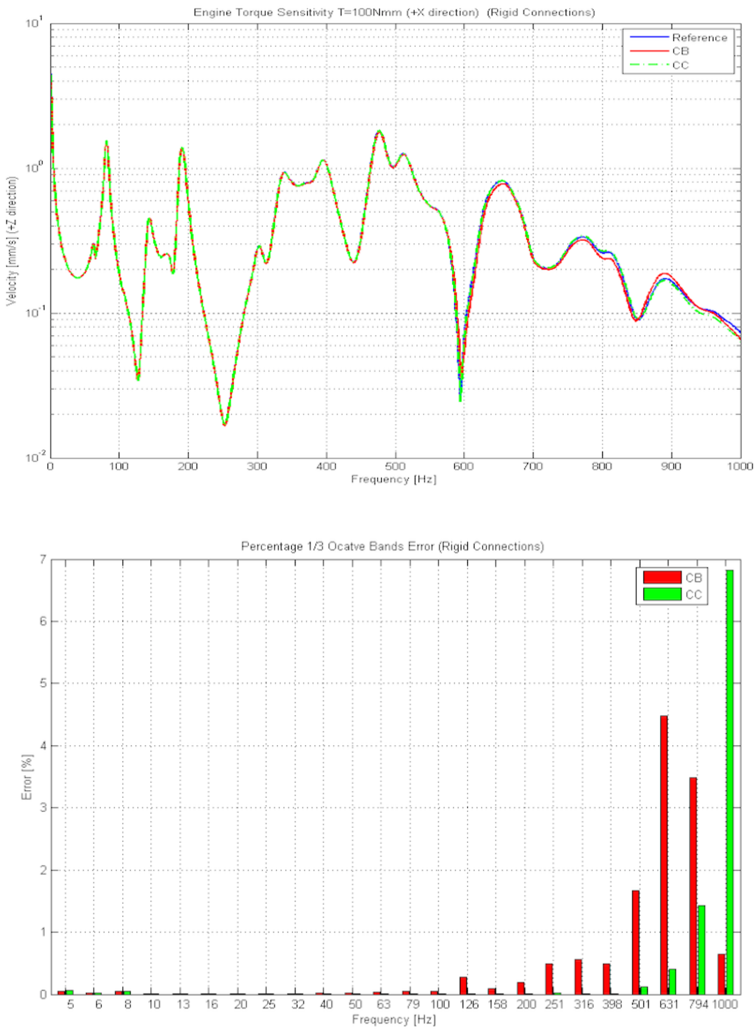

Figure 3.5: Craig-Bampton and Craig-Chang comparison, same reduction basis (up to 3000Hz).

Connections comparison: For this comparison the reduction basis for both method as been kept constant, i.e. modes up to 3000Hz has been retained. While it has been varied the connections stiffness, from 200Nmm (first graph in figure 3.6) to 2Nmm (second graph in figure 3.6). This represents an extreme case but it has been performed in order to check if a limit exist in choosing the Craig-Chang method with respect the Craig-Bampton and the other way around.

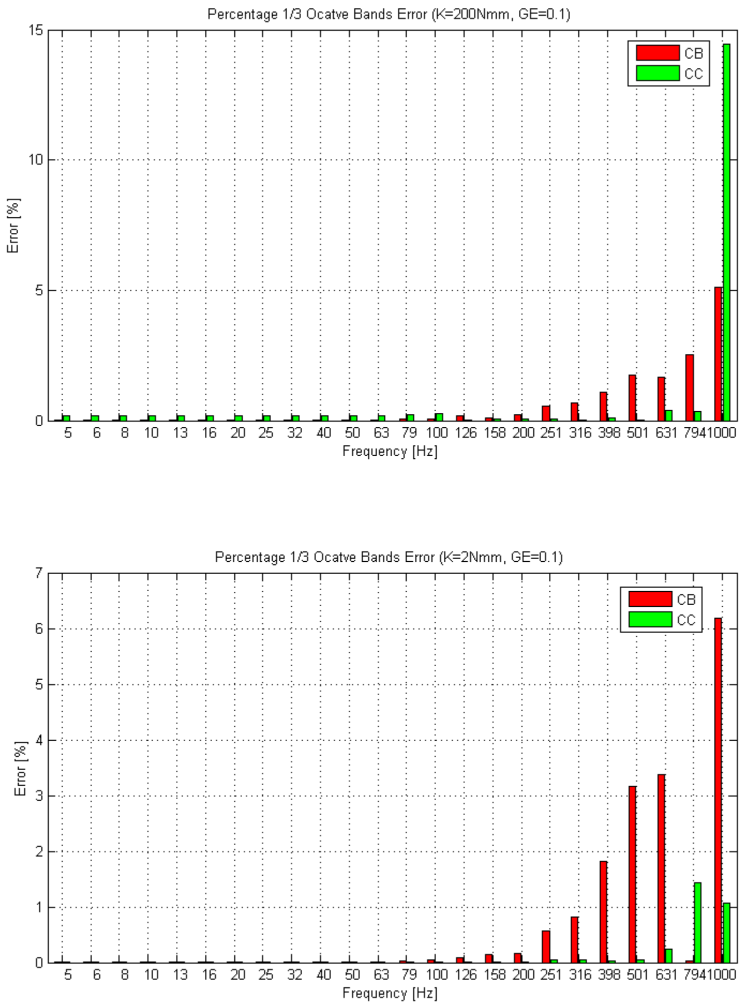

Figure 3.6: Craig-Bampton and Craig-Chang comparison, same reduction basis (up to 3000Hz), with different connections stiffness.

Solution and reduction time comparison: In table 3.6 are shown reduction times and solution times for different method as discussed above. It must be pointed out the big potential offered by the reduction method in general since it takes just 5-12 seconds to compute the response with respect 12 minutes for the full model with the modal method.

Table 3.6: Reduction time, simulation time for different reduction methods and reduction basis.

Reduction Method	Reduction Time	Solution Time	Total Time
Full model (Direct)	-	136 min	136 min
Full model (Modal)	-	12 min	12 min
Craig-Bampton (up to 1500Hz)	14 min	5 sec	14 min 5 sec
Craig-Bampton (up to 3000Hz)	17 min	12 sec	17 min 12 sec
Craig-Chang(up to 1500Hz)	12 min	6 sec	12 min 6 sec
Craig-Chang(up to 3000Hz)	12 min	8 sec	12 min 8 sec

3.4 Discussions.

At this point it is possible to make some conclusions about what has been done until now:

- the Craig-Bampton method, with fixed-interface condition, shows the best compromise when modes up to 3000Hz are retained for both sub-system;
- the Craig-Chang method, with free-interface condition, shows the best compromise already when modes up to 1500Hz are retained.
- comparing Craig-Bampton and the Craig-Chang method, with the same reduction basis, i.e. modes kept up to 3000Hz, than from figure 3.5 it is possible to see that the Craig-Chang shows better results, even if the last frequency band has an error higher than the average.
- connections properties have a little influence on the choice of the reduction method, since from what can be seen in figure 3.6 in all the frequency bands the error is less than 5%. An exception can be done for the last frequency band (1000Hz) since for this frequency range, for very low values of the connection stiffness, is advisable to use the Craig-Chang method with free- interface conditions, since the subsystems behave more free.

Considering the reduction time, the solution time, the solution error with respect the full model, it is possible to asses that the Craig-Chang reduction performs better than Craig-Bampton. In these cases it has been necessary to keep modes up to 3 times the highest response frequency since the sub-systems are quite poor from a vibrational point of view. It means it is necessary to go very high in frequency in order to find some flexible modes that contribute in enriching the reduction basis.

4

Application to the Volvo V40

THIS STUDY CASE will analyze a real vehicle structure. It has been chosen the the Volvo V40 full trim body, with its front and rear subframes as show in figure 4.1. The full finite element model has more than 13 millions of degree of freedom. The load is applied in the front subframe and the different response points are taken, from the front and rear subframes and different responses of the body.

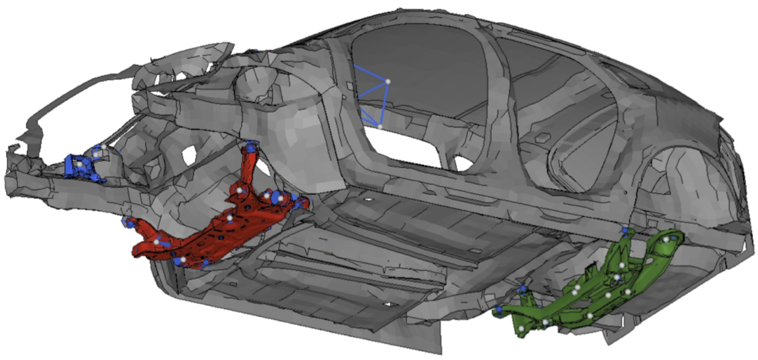

Figure 4.1: Volvo V40 trim body and subframes, FEM model.

As for the previous study case the purpose of the study is to:

- study the effect of the modal truncation;
- compare Craig-Bampton and Craig-Chaing methods;
- study the influence of the connection type;
- compare the solution time for unreduced and reduced system.

The front subframe is attached to the body through 4 connections, while the rear subframe is connected to the body through 6 connections. In the production series the front subframe is connected to the body through very stiff bushings, while the rear subframe is directly bolted to the body. In our study case, first both subframes will be considered bolted, i.e. rigidly connected to the body, and different reduction method with different reduction basis will be compared. Then both subframes will be connected to the body through bushings and differences with respect the previous cases, if any, will be highlighted.

4.1 Simulation set-up.

The frequency range of interest, as it has been said in the previous chapter, in NVH matter is up to 300-375 Hz and so the response will be computed just up to 350Hz. As shown in figure 4.2 it can be seen that a load is applied to the front subframe and the responses are taken respectively from the front subframe, from the body floor and from the rear subframe. It has been chosen this configuration because:

- since the load is applied to the front subframes, its dynamic depends barely on the dynamic of the body;
- the rear subframe dynamic strongly depends on the dynamic of the body.

In this way it is possible to understand the effect of the mutual modal truncation for the two different situations, considering also different type of connections.

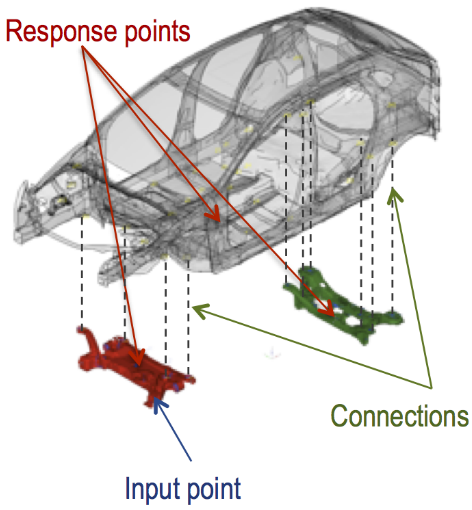

Figure 4.2: Load, response points and connections.

From a software side for the simulation has been used Hyperwork and MatLab for the post-processing. From the hardware side it has been used 10 cpu's and all the memory needed for the calculation without any constraints.

4.2 Method.

CMS reduced components: The first step was to reduced all the single components with the two different reduction methods, i.e. Craig-Bampton and Craig-Chang, considering different reduction basis, i.e. modes up to different frequencies has been calculated. In figure 4.1 it is possible to see the number of modes relative to different reduction basis for the Craig-Bampton method. In figure 4.2 the number of modes are shown for the Craig-Chang reduction method. It can be noticed that the subframes have few vibration modes with respect the body, that accounts for the most vibrational content in the full system.

Table 4.1: Number of modes for Craig-Bampton reduction.

Craig-Bampton Reduction - FIXED-interface condition			
Component	Modes up to 350 Hz	Modes up to 525 Hz	Modes up to 700 Hz
Trim Body	4362	7834	11834
Front Sub.	31	40	46
Rear Sub.	48	54	60

Table 4.2: Number of modes for Craig-Chang reduction.

Craig-Chang Reduction - FREE-interface condition			
Component	Modes up to 350 Hz	Modes up to 525 Hz	Modes up to 700 Hz
Trim Body	4429	7908	11919
Front Sub.	41	48	55
Rear Sub.	59	66	74

Effect of modal truncation: The aim of this first analysis is to understand which is the best reduction basis for each reduction method, it means to find out up to which frequency modes have to be retained in order to satisfy some constraints:

- Error between FEM full model response and CMS model response less than 5%;
- Reduction time as short as possible;
- Solution time as short as possible ;
- CMS file size as smallest as possible.

To calculate the error between the responses, first the continuous response in frequency domain has been discretized into 1/3 octave bands. For each band the mean value has been computed and then the percentage error has been calculated between each relative frequency band between the FEM full model and the CMS model.

In table 4.3 are resumed all the cases for the Craig-Bampton method that have been studied. In table 4.4 are resumed all the cases for the Craig-Chang and in table 4.5 are resumed all the cases for the Mixed reduction, i.e. some components reduced with Craig-Bampton and others with Craig-Chang. As aforementioned at the beginning of the chapter, for this analysis the subframes were considered bolted rigidly to the body.

Table 4.3: Study cases for Craig-Bampton reduction.

Craig-Bampton Study Cases - Modes kept up to Hz.					
Component	Case 1	Case 2	Case 3	Case 4	Case 5
Trim Body	350 Hz	700 Hz	350 Hz	700 Hz	525 Hz
Front Sub.	350 Hz	350 Hz	700 Hz	700 Hz	525 Hz
Rear Sub.	350 Hz	350 Hz	700 Hz	700 Hz	525 Hz

Table 4.4: Study cases for Craig-Chang reduction.

Craig-Chang Study Cases - Modes kept up to Hz.					
Component	Case 1	Case 2	Case 3	Case 4	Case 5
Trim Body	350 Hz	700 Hz	350 Hz	700 Hz	525 Hz
Front Sub.	350 Hz	350 Hz	700 Hz	700 Hz	525 Hz
Rear Sub.	350 HZ	350 Hz	700 Hz	700 Hz	525 Hz

Table 4.5: Study cases for Mixed reduction.

Mixed reduction Study Cases - Modes kept up to Hz.					
Component	Case 1	Case 2	Case 3	Case 4	Case 5
Trim Body	350 Hz (CC)	525 Hz (CC)	350 Hz (CC)	350 Hz (CB)	525 Hz (CC)
Front Sub.	350 Hz (CB)	700 Hz (CB)	700 Hz (CB)	700 Hz (CC)	525 Hz (CB)
Rear Sub.	350 Hz (CB)	700 Hz (CB)	700 Hz (CB)	700 Hz (CC)	525 Hz (CB)

Craig-Bampton and Craig-Chang comparison: Once the best case for each reduction method has been found then a comparison between each other will be performed in order to understand which reduction method performs better than the others. After a focused analysis on reduction time, solution time, cmd files size and solution error, It has been decided to compare case 4 for the Craig-Bampton method, case 5 for the Craig-Chang and case 2 for the Mixed method, as shown in table 4.6.

Table 4.6: Study cases for reduction method comparison.

Reduction Method	Component	Modes up to Hz
Craig-Bampton	Trim Body	700 Hz
	Front Sub.	700 Hz
	Rear Sub.	700 Hz
Craig-Chang	Trim Body	525 Hz
	Front Sub.	525 Hz
	Rear Sub.	525 Hz
Mixed	Trim Body	525 Hz (CC)
	Front Sub.	700 Hz (CB)
	Rear Sub.	700 Hz (CB)

Influence of Connections type: Finally the influence of the connection property on the reduction method and basis will be carried out. For this analysis the subframes were considered attached to the body using bushings. In table 4.7 the properties for the bushing used in the simulation have been reported.

Table 4.7: Bushing Property.

Connection Property	X-dir	Y-dir	Z-dir
Stiffness [N/mm]	1000	2000	1000
Damping [N/mm/s]	0.1	0.1	0.1

In tables 4.8, 4.9, 4.10 resumes all the cases studied for different reduction method and basis, as it has been done previously with rigid connections.

Table 4.8: Study cases for Craig-Bampton reduction.

Craig-Bampton Study Cases - Modes kept up to Hz.			
Component	Case 1	Case 2	Case 3
Trim Body	350 Hz	525 Hz	700 Hz
Front Sub.	350 Hz	525 Hz	700 Hz
Rear Sub.	350 Hz	525 Hz	700 Hz

Table 4.9: Study cases for Craig-Chang reduction.

Craig-Chang Study Cases - Modes kept up to Hz.			
Component	Case 1	Case 2	Case 3
Trim Body	350 Hz	525 Hz	700 Hz
Front Sub.	350 Hz	525 Hz	700 Hz
Rear Sub.	350 Hz	525 Hz	700 Hz

Table 4.10: Study cases for Mixed reduction.

Mixed reduction Study Cases - Modes kept up to Hz.		
Component	Case 1	Case 2
Trim Body	525 Hz (CC)	525 Hz (CC)
Front Sub.	700 Hz (CB)	525 Hz (CB)
Rear Sub.	700 HZ (CB)	525 Hz (CB)

Reduction and Solution time comparison: Finally a comparison between the full model solution time, direct and modal method, and the cms reduced model with Craig-Chang, with modes up to 525Hz, will be compared in order to show what actually will be gained in terms of time.

Table 4.11: Reduction basis, required memory (RAM), number of DoF and solution method.

Method	Modes up to Hz	Required Memory	Total DoF	Solution Method
Full model	-	180 Gb	13 millions	direct
Full model	525 Hz		13 millions	modal
Craig-Chang	525 Hz	800 Mb	8154	modal

4.3 Results.

CMS reduced components: In tables 4.1 and 4.2 the number of modes kept for each reduction basis have been reported, i.e. 350Hz, 525Hz, 700Hz. Here will be reported only the reduction time and the size of the CMS file for the body using different reduction methods and basis. It has been chosen to show only the results for the body since for the subframes both reduction time and file size didn't show particular behavior that should be underlined moreover no changes in terms of time and size were found in increasing the reduction basis. in figure 4.3 data are shown, it is possible to notice the increment in the file size and in the reduction time if the higher number of modes are kept. The Craig-Bampton takes a little more time to perform the reduction with respect Craig-Chang for the same reduction basis.

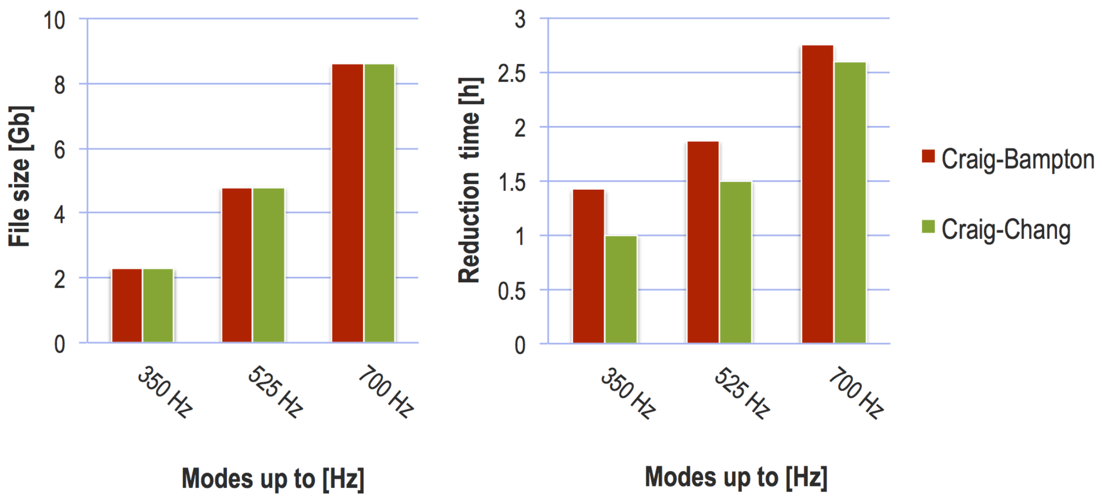

Figure 4.3: CMS file size and reduction time for Craig-Bampton and Craig-Chang, different reduction basis.

Effect of modal truncation: For this analysis only few cases of the Craig-Bampton cases will be shown. Looking at table 4.3, will be reporter the response error for case 1, 2 and 3. Case 1 includes modes up to 350Hz for all the components. In Case 2 modes up to 700Hz only for the body are retained. In Case 3 mode up to 700Hz only for the subframes. From figure 4.4 it is possible to notice that increasing the reduction basis for the body improve the solution especially at the highest frequency band. Increasing the reduction basis for the subframes instead improve the solution at the mid frequencies. The same behavior has been found if the Craig-Chang reduction is applied.

	CRAIG-BAMPTON CASE 1			CRAIG-BAMPTON CASE 2			CRAIG-BAMPTON CASE 3		
	Body Response Error [%]			Body Response Error [%]			Body Response Error [%]		
Freq. Band [Hz]	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir
100	0,33	1,94	0,03	0,32	0,50	1,06	0,71	1,80	1,22
126	1,14	5,82	1,07	1,03	4,24	1,16	0,18	2,75	0,40
158	0,48	8,26	1,64	2,87	7,46	1,80	2,71	1,36	0,12
200	5,17	3,26	1,56	8,36	4,26	1,20	15,16	0,56	2,12
251	49,32	8,51	30,11	26,65	10,25	18,11	62,30	1,03	14,13
316	19,41	10,76	24,73	1,39	2,00	1,45	30,99	10,27	21,19

	Front Subframe Response Error [%]			Front Subframe Response Error [%]			Front Subframe Response Error [%]		
Freq. Band [Hz]	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir
100	45,18	8,44	10,60	45,87	8,44	10,75	0,91	0,35	0,29
126	95,80	15,63	12,31	97,45	14,94	12,63	0,92	0,11	0,57
158	79,64	19,12	18,90	79,26	19,96	17,52	0,57	2,68	1,53
200	80,75	19,63	16,08	75,51	21,12	12,44	5,58	5,73	6,36
251	49,11	16,83	39,45	51,44	21,92	42,04	0,08	14,98	6,84
316	67,03	15,45	65,91	69,20	5,91	62,34	11,72	61,62	3,26

	Rear Subframe Response Error [%]			Rear Subframe Response Error [%]			Rear Subframe Response Error [%]		
Freq. Band [Hz]	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir
100	0,21	0,46	5,35	0,44	1,44	3,09	0,61	0,03	4,59
126	0,43	0,77	2,04	0,62	2,71	1,98	0,04	0,08	0,05
158	0,98	9,79	14,61	2,07	10,80	17,03	0,91	1,40	1,17
200	2,96	5,23	17,17	3,91	9,24	25,78	1,02	0,83	6,79
251	25,46	5,84	17,27	19,97	2,94	25,26	3,84	4,48	7,21
316	73,11	70,38	22,43	37,82	12,42	31,71	24,60	50,53	57,50

Figure 4.4: Craig-Bampton: Case 1, 2, 3 response error comparison.

Craig-Bampton and Craig-Chang comparison: Comparing the two methods Craig-Bampton and Craig-Chang, an important result has been found . The Case 4 for Craig-Bampton include modes up to 700Hz for all components, while the Case 5 for Craig-Chang just up to 525Hz. As it can be seen in figure 4.5 the Craig-Chang method performs better since for all the frequency bands the error is less than 5%. It means the free-modes, rigid-modes and attachment-modes better describe the dynamic of the full system rather then fixed-modes and constraint-modes. This is due to the presence of the attachment-modes in the Craig-Chang reduction, since those mode account for the elasticity of the truncated free-modes [2].

	CRAIG-BAMPTON CASE 4			CRAIG-CHANG CASE 5		
	Body Response Error [%]			Body Response Error [%]		
Freq. Band [Hz]	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir
100	0,06	0,27	0,26	0,10	0,01	0,37
126	0,11	0,94	0,35	0,23	0,08	0,52
158	0,03	0,41	0,18	0,31	0,02	0,06
200	1,51	0,19	0,71	0,00	0,65	0,61
251	2,63	0,33	0,50	4,16	1,68	0,92
316	3,87	0,30	0,75	0,83	0,21	0,66
	Front Subframe Response Error [%]			Front Subframe Response Error [%]		
Freq. Band [Hz]	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir
100	0,36	0,08	0,28	1,49	0,27	0,08
126	0,38	0,30	0,37	2,82	0,07	0,05
158	0,38	1,40	0,29	2,00	0,28	0,14
200	0,68	4,45	2,84	0,57	0,88	1,05
251	1,52	8,85	3,53	1,66	3,77	1,98
316	6,18	8,75	1,60	3,63	1,39	3,14
	Rear Subframe Response Error [%]			Rear Subframe Response Error [%]		
Freq. Band [Hz]	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir
100	0,41	0,94	2,33	0,17	0,01	0,47
126	0,25	1,77	0,12	0,14	0,16	0,16
158	0,16	2,52	0,98	0,21	0,20	0,09
200	0,87	2,94	2,58	0,19	0,02	0,01
251	5,61	6,53	4,41	1,90	0,26	0,24
316	2,09	10,86	6,26	2,38	1,60	0,04

Figure 4.5: Craig-Bampton Case 4 and Craig-Chang Case 5 response error comparison.

Influence of Connections type: Previously in table 4.7 the bushing property have been showed. Indeed for this part bushing have been used instead of rigid connections. Two of all the cases shown in tables 4.8, 4.9, 4.10 will be proposed here for a comparison, from table 4.9 the Case 2 with a Craig-Chang reduction (modes kept up to 525 Hz) and from 4.10 Case 1 with a Mixed reduction (body reduced with Craig-Chang up to 525Hz and subframes with Craig-Bampton up to 700Hz.

In table 4.6, it is possible to notice the the Craig-Chang reduction performs better than the mixed reduction, especially in describing the dynamic of the subframes. It means that for the mixed reduction keeping modes up to 700 Hz for the subframes is not enough.

	MIXED – CASE 1			CRAIG-CHANG CASE 2		
	Body Response Error [%]			Body Response Error [%]		
Freq. Band [Hz]	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir
100	0,93	1,10	0,69	0,01	0,10	0,04
126	0,49	0,43	0,35	0,06	0,06	0,29
158	1,22	0,21	0,24	0,36	0,01	0,00
200	0,22	0,60	0,27	0,47	0,47	0,18
251	1,79	0,88	0,45	0,66	1,17	0,50
316	0,30	0,15	0,81	1,25	0,06	0,83
	Front Subframe Response Error [%]			Front Subframe Response Error [%]		
Freq. Band [Hz]	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir
100	0,14	1,34	1,87	0,02	0,06	0,20
126	0,36	1,07	0,49	0,80	1,76	0,24
158	4,66	3,33	0,56	2,51	4,75	0,07
200	1,97	5,14	2,99	2,94	4,82	0,07
251	3,77	4,69	1,48	2,71	2,29	0,21
316	5,10	4,31	2,31	1,12	4,31	1,33
	Rear Subframe Response Error [%]			Rear Subframe Response Error [%]		
Freq. Band [Hz]	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir
100	0,69	1,07	3,12	0,00	0,13	0,75
126	0,39	1,79	2,23	0,08	0,11	0,32
158	1,97	1,22	0,18	0,52	0,20	0,30
200	4,52	0,53	1,45	0,33	0,10	0,16
251	5,53	2,39	4,14	3,37	1,29	2,87
316	3,52	11,01	13,69	1,37	3,31	10,15

Figure 4.6: Mixed Case 1 and Craig-Chang Case 2 response error comparison with bushing connections.

Reduction and Solution time comparison: In the introduction it has been stated that one of the benefit of using modal sub-structuring techniques is to reduce the total number of degree of freedom of our model and the simulation time. In table 4.11 it has been reported the number of degree of freedom of the model reduced using Craig-Chang up to 525Hz and it easy to see how powerful is this techniques since the model is reduced from 13 millions of DoF to just almost 8 thousands. Here, in table 4.12, are shown the reduction time and simulation time. Although the reduced model required time to reduce all the components, the solution time goes from 16 hours for the full model with direct method to only 29 minutes using the reduced model with Craig-Chang, hence both the advantages have been confirmed in using CMS model instead of a full FEM model.

Table 4.12: Reduction and solution time comparison.

Method	Reduction Time	Solution Time	Total Time
Full model (Direct)	-	16 hours	16 hours
Full model (Modal)	-	1 hour 30 min	1 hour 30 min
Craig-Chang Case 5	1 hour 30 min	**29 min**	2 hours

4.4 Discussions.

As for the first study case also in this case some conclusions can be made:

- the Craig-Bampton method, with fixed-interface condition, shows the best compromise when modes up to 700 Hz are retained for body and subframes, i.e. with a reduction basis factor 2, even though for some frequency bands the error is greater than 5% for the subframes;
- the Craig-Chang method, with free-interface condition, shows the best compromise already when modes up to 525 Hz are retained, i.e. with a reduction basis factor 1.5
- comparing Craig-Bampton and the Craig-Chang method, the Craig-Chang method performs better with less modes, i.e. for the same cut-off frequency or reduction basis factor.
- connections properties have a little influence on the choice of the reduction method, since even if bushings are used the Craig-Chang method still performs better.
- using reduced model means that all the components have to be reduced and it takes time, but the advantage is clear when it comes to solution time. The Craig-Chang reduced model take 1/3 of the solution time for the FEM full model, if a modal method is used to compute the response. Moreover the memory required by the CMS (reduced) model is only 800Mb with respect the 180Gb of memory required by the FEM full model.

5

Conclusions

THE AIM of this thesis is to find criteria when different reduction techniques should be used. To find out some criteria different analysis have been carried out. Firstly the effect of modal truncation has been studied, then the comparison between the two method, Craig-Bampton and Craig-Chang, has been carried out. Finally the effect of connections has been deepen.

Starting with the modal truncation problem, it is clear that when dealing in component mode synthesis techniques, that uses modes to represent the dynamic of a system, the number of modes kept are a key factor for accurate dynamic representation. As it has been said CMS methods are efficient only if few modes are kept but this is in contrast with reaching high accuracy of the description of the dynamic behavior, since a low number of modes kept leads to a pour dynamic description, and with reducing the solution time and the CMS file size. Hence a compromise has to be find.

Looking at both the study cases, the frame-box and the Volvo V40, a constant trend exist. The Craig-Chang method needs a less number of modes with respect the Craig-Bampton method to describe the dynamic of the complete system in an accurate way, hence the Craig-Chang method results to be the most efficient. Different study cases have been tried out in order to understand the reason of this outcome and it has been found out that the presence of the residual inertia relief attachment modes have a big influence in describing the dynamic of the system especially a high frequency bands since those modes account for the elasticity of the truncated modes, moreover the trim body behave more as it was in free condition without any constraints and for this reason the free-modes included in the reduction basis of the Craig-Chang method better describe this condition. If criteria should be given i suggest:

- if the Craig-Bampton method is used in reducing a FEM component then it is advisable to keep modes up to 2 times the maximum frequency the response has to be calculated, i.e. if the response is calculate up to 100 Hz then it is advisable to keep modes up to 200 Hz. It means a factor 2 has to be used for the reduction basis.

- if the Craig-Chang method is used in reducing a FEM component then it is advisable to keep modes up to 1.5 times the maximum frequency the response has to be calculated, i.e. if the response is calculate up to 100 Hz then it is advisable to keep modes up to 150 Hz. It means a factor 1.5 has to be used for the reduction basis.

Secondly, it is very common using different type of connections in assembling subsystems to the body. Combinations of bushings and bolt connections are used in the Volvo V40 and for this reason the necessity of understanding how treat different connections within different subsystem and also within the same subsystem arises when it comes to choose between free-condition (Craig-Chang) and fixed-condition (Craig-Bampton).

Looking at the results for both study cases no particular effect of the connections type has been found. It means that both reduction method perform in the same way if bolt or bushing connections are used. The only effect that has been found was when studying the frame-box system. When a very low stiffness value was used for the bushing connections the Craig-Chang method performed even better than using bolt connection and this is what we expected since with a more loosely connections the subsystem behave more as they were in free-condition and then their dynamics is better described by the free-modes. In conclusion:

- apply Craig-Chang reduction method when bolt or bushing connections are used to assembly the subsystems.

Finally a last look has been given to what actually is gained in using these reduction methods. We start this thesis with the purpose to assess how powerful is the Component Mode Synthesis approach in terms of solution time reduction and in reducing the number of DoF. After hundreds simulations and many hours spent in gathering information it is possible to say that CMS methods actually reduce the simulation time of about 1/3 in the case of the Volvo V40. This value is not constant, indeed it is a function of how many modes are kept in the reduction basis. Lower the number of modes kept lower the simulation time, but the necessity to have a high accuracy in describing the real dynamic of the complete system leads to find a compromise, as usual in engineering problems. Moreover reducing the complete model from 13 millions of DoF to just 8 thousands it is very good results since it is possible to avoid upgrading constantly the computer hardware, hence reducing cost.

6

Future Work

THIS THESIS from one side answers to some simple questions but from the other side opens up new questions. Some of those are relative to the thesis itself and other are relative to the topic in general. There is a need to validate and correlate the findings of this thesis through an experimental analysis, moreover since the CMS has shown good performance it would be advisable to include in the comparison more than only two methods. Lots of other reduction methods exist, for example the Modal Dominance or the Strain-Energy or the Balanced Gramians method that show even better results in terms of accuracy in describing the dynamic with respect the number of DoF kept. Hence this work has to be seen as an 'incipit', as a starting point within the dynamic substructuring field.

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