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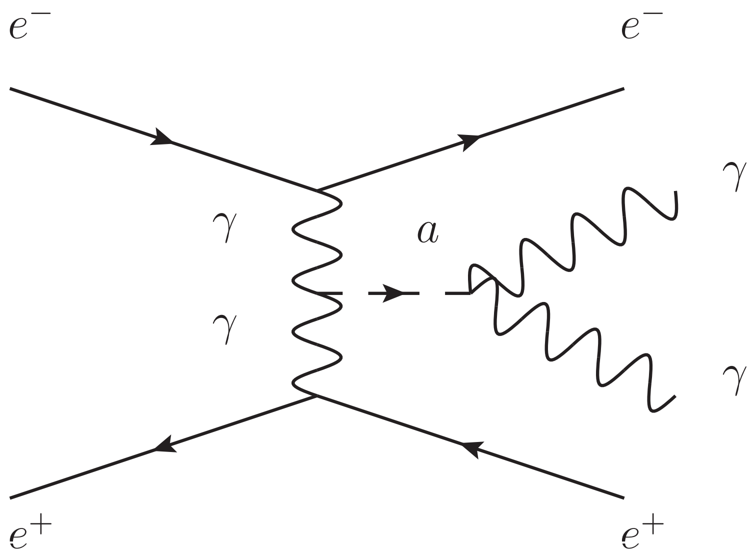

LHC, the Higgs particle and physics beyond the Standard Models

Simulation of an additional scalar particle a's decay.

Bachelor of Science Thesis for the Engineering Physics program

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We would like to commit a special thanks to our supervisor, Gabriele Ferretti, without whom this project would not be what it is. He has led us to the right path when we strayed too far off course, shown us light when all we saw was darkness and supported us when we most needed it.

Abstract

This thesis explores a possible addition of a scalar boson to the Standard Model. Apart from a quadratic coupling to the Higgs boson, it couples to the photon and the gluon. To fully be able to explore this new boson, it is necessary to get acquainted with some of the vast background theory in form of quantum field theory. This involves the most fundamental ideas of relativistic quantum mechanics, the Lagrangian formulation, cross section, decay rate, calculations of scatteringamplitude, Feynman diagrams, the Feynman rules and the Higgs mechanism. To analyse the particle, it was necessary to use computer aid in form of FeynRules, a package to Mathematica, for retrieve the Feynman rules for the particle, and MadGraph 5 for numerical calculations of decay rate and cross section. This was used to find limits to coupling constants with in the Lagrangian to concur with experimental findings.

Sammanfattning

Denna rapport undersöker en möjlig extra skalärboson till standardmodellen. Förutom kopplingen med higgsbosonen kopplar den även till fotonen och gluonen. För att kunna undersöka denna nya skalärboson så är det nödvändigt att förklara det mest relevanta av kvantfältsteorin. Detta omfattar grundläggande kunskaper i relativistisk kvantmekanik, lagrangefunktioner, tvärsnitt, sönderfallsbredd, spridningsamplitud, feynmandiagram och higgsmekanismen. För att analysera partikeln, behövs stöd av datorsimuleringar där FeynRules tar fram feynmanreglerna och MadGraph 5 gör numeriska beräkningar för att ta fram bredd och tvärsnitt. Dessa används för att ta fram gränser för kopplingskonstanterna i lagrangiefunktionen, för att partikeln ska kunna ge experimentiella fynd.

Contents

1 Introduction

Scientists have always tried to explain the universe using simple models. As the understanding increases new more precise models improve or replace the older ones. Today the current understanding of the universe involves theories on both cosmic scale and down to subatomic scale. At the subatomic scale, the ruling theory is the Standard Model. The Standard Model is a theory that contains all the fundamental particles and their interactions with each other. But the Standard Model is not complete, for example it does not explain gravity or dark matter, that we know for a fact exist. Most recently we found the famous Higgs boson which completed the picture of the standard model. Even so, there may be other hidden particles which could further improve the Standard Model and our understanding of the universe.

This thesis aims to study an addition of a new particle, a scalar boson, to the Standard Model as a new possible decay product of the Higgs boson. To do this it was necessary to dive into the vast background theory of the Standard Model, quantum field theory and Feynman diagrams. However both these theories are quite extensive and therefore only the most important parts needed to understand this thesis are included. As well as interacting with the Higgs boson our particle also interact with the photon and the gluon. For instance this would allow a decay from the Higgs boson to four photons.

To find limits for the parameters of the added particle, different scattering processes will be simulated using computers, such that the theory can concur with experimental data and possibly cause new physics.

In the second chapter some background theory is explained. It starts with going over relativistic quantum mechanics which provides a basic for the quantum field theory needed in this project. After that it follows with the Lagrangian, a function used to describe the dynamics of a system, and its applications in quantum field theory. This is followed by two subchapters on the width and cross section, which are numerical values to describe the decays and collisions of the particles in the Standard Model. To calculate these it is needed to use something named the scattering amplitude. How to calculate these are covered in the following subchapter. The theory chapter ends with a description of the Higgs mechanism, which gives mass to the other particles in the Standard Model.

The third chapter covers some theory needed for our model particle. It consists of an derivation of the width for the Higgs decay to our model particle and values for our coupling constant.

The fourth chapter goes through how the simulations are done including how to implement our scalar boson in the simulation programs.

The fifth chapter shows some results from our simulations. It starts off showing how the cross section depends on whether our model boson is a scalar or a pseudo-scalar. It continues with some simulations related to the LEP (Large Electron-Positron Collider) experiment, and ends with showing how the decays from the model particle depends on the coupling constants.

The sixth and last chapter discusses the decay rate from Higgs to our model as well as the branching ratio for our model's decays to the photon and gluon. The report ends with a conclusion of the limits of our coupling constants.

2 Theory

To be able to add particles to the Standard Model, it is necessary to know its background theory. This theory is quantum field theory, which is a relativistic version of quantum mechanics. This section will treat the most important aspects of quantum field theory necessary to understand this thesis, but since it is a complex theory this is far from a complete description. After a short glossary, some brief explanations on notation and units used in this thesis will be conducted. Secondly a short run through of basic relativistic quantum mechanics, followed by a section on the Lagrangian, a way to describe all the physical interactions within a theory. The two coming sections treat the golden rules for decay rate and cross section. After that is a section which explains how to calculate the scattering amplitude found in the golden rules, Feynman diagrams and Feynman rules. The last section is on the Higgs mechanism, the field that gives mass to the particles in the Standard Model.

2.1 Glossary

This is a simple list of important words in the theory section with short explanation.

- Anticommutator: Is a mathematical construct defined as $\{\hat{A},\hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A}$.
- Boson: A type of particle with an integer spin.
- Commutator: Is a mathematical construct defined as $[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$.
- Fermion: A type of particle with a half integer spin. They follow the Pauli exclusion principal that states that two fermions can not occupy the same state.
- Metric: A metric is a function giving a measurement of length to a set, in special relativity it can be seen as rank two tensor.
- Minkowski space: Allso known as space-time. Minkowski space is a geometry of 4 dimensions, 3 space dimensions and 1 time dimension, with a metric tensor $\eta^{\mu\nu} = diag\{[1,-1,-1,-1]\}$.
- Parity: A quantity p such that $\psi(t,-x) = (-1)^p\psi(t,x)$.
- Pseudoscalar field: A quantum field of spin zero and negative parity (p=1).
- Quantum field: An operator describing the creation and annihilation one type of particle over space time.
- Scalar field: A quantum field of spin zero and even parity (p=0).
- Scale of New Physics: Energies and mass above the reach of the Standard Model. It is often denoted as Λ.
- Tensor: A geometric object, represented with an array of numbers, transforming analogous to a vector under coordinate transformations.

2.2 Notation and units

The theory behind the Standard Model is quantum field theory. Quantum field theory is the merge of quantum mechanics and special relativity. This means in order to understand the mathematics in this thesis it is necessary to be familiar with the notation of these two areas of physics.

In quantum mechanics it is sometimes convenient to use Dirac-notation. In Dirac-notation a state is written with $\langle\psi|$, a so called, "bra" or $|\psi\rangle$ a "ket". The inner product of two states is $\langle\psi|\phi\rangle$. If an operator $\hat{A}$ is applied to the ket the inner product is $\langle\psi|\hat{A}|\phi\rangle$. If the same operator is applied to the bra $\langle\psi|\hat{A}^\dagger|\phi\rangle$, the $\dagger$ takes the hermitian conjugate of the operator. More on Dirac notation may be found in standard textbooks on quantum mechanics. [1]

Another useful notation is the covariance tensor notation from relativity. A vector could be written as its components in a particular base, for instance normally a vector is written $\vec{A} = (A_1, A_2, ..., A_n)$, but in tensor notation the vector is written as one general component of the vector with an index like A_i. If one index is written twice in an factor it is assumed that it is being summed over. For instance a scalar product of two vectors in Euklidian space is written as $A_i B_i$. A tensor is an object which transform similar to a vector under coordinate transformations, but is represented with multiple index. When working with 4-vectors in Minkowski space it is common to use Greek letters as index which range from 0 to 3 where 0 is the time coordinate. When Latin letters are used as index they refer to the space coordinates and range from 1 to 3. The Lorentz invariant metric tensor will be $(+,-,-,-)$ and denoted with $\eta_{\mu\nu}$. In Minkowski space it is critical to make difference of if the index is up or down. It is possible to change an index position using $x^\mu = \eta^{\mu\nu}x_\nu$. We will assume some familiarity with working with tensor notation and for further references, more on tensors can be found in most textbooks on special and general relativity.[2]

This thesis will be working in natural units. This means that the speed of light and the Planck's reduced constant is set to one, $c = \hbar = 1$. All dimension full quantities will be measured in terms of the energy unit GeV. For instance the unit of length is GeV^{-1}.[3] The only exception is cross section for which the standard in particle physics is to use $\text{b} = 10^{-28}\text{m}^2$ ("barn"). The reason to choose natural units is that many identities becomes simpler, an example of this is the squared energy relation from special relativity. Normally it is written as

$$E^2 = p^2c^2 + m^2c^4.\text{[2]} \tag{1}$$

In natural units however it is just

$$E^2 = p^2 + m^2. \tag{2}$$

Since there will be many integrations it is assumed that

$$\int d^4x f(x) = \int_{\mathbb{R}^{3,1}} d^4\mathbf{x} f(\mathbf{x}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,x,y,z)\,dt\,dx\,dy\,dz \tag{3}$$

where $\mathbb{R}^{3,1}$ is Minkowski space and $f(x)$ is an arbitrary integrable function of space time coordinates to some complex value.

2.3 Relativistic Quantum Mechanics

In classical quantum mechanics the dynamics of a system is determined by applying a combination of Noether's theorem and Wigner's theorem. Noether's theorem states that for any differentiable symmetry of the action, exists an invariant physical quantity [4], while Wigner's theorem states that for any symmetry transformation of an Hilbert space, exists an unitary or antiunitary operator. The relation between the conserved quantity's operator $\hat{A}$ and the associated symmetry's unitary operator $\hat{U}$ is

$$\hat{U}(\alpha) = \exp{(i\hat{A}\alpha)}. \tag{4}$$

Under the assumption that a quantum mechanical system is time translation invariant and the associated conserved quantity is energy, the Schrodinger equation

$$i\frac{\partial \psi}{\partial t} = \hat{H}\psi \tag{5}$$

can be derived, where ψ is the time dependent state of the system. However for convenience it is better to work within the Heisenberg picture where the operators change over time, rather than the state. In the relativistic case, of course both Noether's theorem and Wigner's theorem still apply. The change is the symmetries of the action S. The time invariance in classical quantum mechanics is an effect of Galileo invariance. Relativistic mechanics on the other hand is Lorentz invariant, therefore the action S of relativistic quantum mechanics must be invariant under transformations of the Lorentz group. This means that

$$\begin{aligned} x^\mu &\to \Lambda^{\mu\nu} x_\nu \\ S &\to S, \end{aligned} \tag{6}$$

where x^μ is the position in Minkowski space in one initial frame, and $\Lambda^{\mu\nu}$ is the Lorentz transformation between two initial frames.

It took quite a long time to connect the quantum theories with special relativity. One approach was to use the energy momentum relation [2] for one particle

$$E^2 = p^2 + m^2, \tag{7}$$

where m is the rest mass of the particle [2]. Inserting the quantum mechanical operators for the energy $E = i\frac{\partial}{\partial t}$ and momentum $p = -i\nabla$, the Klein-Gordon equation is obtained

$$(\frac{\partial^2}{\partial^2 t} - \nabla^2 + m^2)\psi = 0. \tag{8}$$

Unfortunately this equation only describes spinless bosons like the pion or the Higgs boson. By factorising equation (7) it is possible to derive the Dirac equation describing relativistic fermions, like electrons and muons. The energy momentum relation written with covariant tensor notation is [5]

$$p^\mu p_\mu - m^2 = 0. \tag{9}$$

In the zero momentum frame for the particle the conjugate rule can be applied

$$(p^0)^2 - m^2 = (p^0 - m)(p^0 + m) = 0. \tag{10}$$

If the particle is observed in other frames it is necessary to assume the vectors β^μ and γ^μ such that [5]

$$p^\mu p_\mu - m^2 = (\beta^\mu p_\mu + m)(\gamma^\mu p_\mu - m). \tag{11}$$

Expanding the product yields

$$p^\mu p_\mu - m^2 = \beta^\mu \gamma^\nu p_\mu p_\nu + m(\gamma^\mu - \beta^\mu)p_\mu - m^2. \tag{12}$$

Since the lowest order of the momentum on the left side is 2, $\gamma^\mu = \beta^\mu$ must be true so that the second term on the right hand vanishes [5]. The second order momentum term is

$$\gamma^\mu \gamma^\nu p_\mu p_\nu$$

and must satisfy

$$p^\mu p_\mu = \gamma^\mu \gamma^\nu p_\mu p_\nu.$$

This will only be so if

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \tag{13}$$

where $\{,\}$ is the anticommutator and $\eta^{\mu\nu}$ is the Minkowski metric. γ^μ will only satisfy equation (13) if they are 4×4 matrices. It may be noted that these matrices do not act on Lorentz vectors and thus the left side and the right side of the equation can be satisfied if we allow the elements of the metric to be 4 by 4 matrices. [5] These 4 matrices has the form of

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{14}$$

and

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \tag{15}$$

where σ^i is the Pauli matrices. Using this in (11) we will now get

$$(\gamma^\mu p_\mu + m)(\gamma^\mu p_\mu - m) = 0. \tag{16}$$

The quantum mechanical operator for the four momentum is $p_\mu = i\partial_\mu$ and when applying ψ it can be found that

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0. \tag{17}$$

Using slash notation the equation becomes

$$\not{\partial}\psi - m\psi = 0 \tag{18}$$

An important notice is that the field ψ in both the Dirac equation and Klein-Gordon equation must not be confused with the state in the Schrodinger equation, but rather operator fields acting on the vacuumstate $|0\rangle$.

However an alternative approach to relativistic quantum mechanics was discovered by R.P. Feynman. The former approach was to set up equations for the local behaviour of a system, analogous to Newtonian and Hamiltonian mechanics. But the new way was by setting up equation for the global behaviour, using methods from functional analysis like in Lagrangain analytic mechanics. Feynman discovered that the amplitude of a system could be obtained by evaluating functional integrals of the form

$$\int \mathcal{D}\phi(x)e^{i\int d^4x\mathcal{L}(\partial_\mu\phi,\phi)} \tag{19}$$

where $\phi(x)$ is a quantum field and $\mathcal{L}$ is the Lagrangian for the system, or rather the Lagrangian density [6]. Lagrangians are further explained in the following section.

2.4 Lagrangian

It is easy to describe the dynamics of a system using a function called the Lagrangian L. In many classical systems, the Lagrangian is a function of generalised coordinates q_i and their velocities $\partial q_i/\partial t$. These parameters are in turn functions of time. This is because from a classical point of view, time is an independent variable. However in quantum field theory it is not and instead it is necessary to work with relativistic coordinates (x, y, z, t), and the independent variables are chosen otherwise [5].

In classical mechanics the Lagrangian is simply $L = T - V$, where T is the kinetic energy of the system and V is potential energy. The Lagrangian is then used to describe the dynamics. An example is the conservation of momentum $p_i = \frac{\partial L}{\partial \dot{q}_i}$. As long as L is q_i invariant under the symmetry $q_i \to qi + a_i$, this can be written as $\dot{p}_i = \frac{\partial L}{\partial q_i} = 0$ [7].

Every theory in physics is described by an action S, which links a real number to the trajectory of the dynamics of the system. The most important aspect of the action is the action principle, which states that the action is stationary to first order [8]. This can mathematically be written as

$$\delta S = 0. \tag{20}$$

The action S is related to the Lagrangian L, through $S = \int Ldt$. Since working with the Lagrangian is easier than working directly with the action, it would be efficient to have an equation that transforms the action principle to the Lagrangian domain. We will do this by looking at a small change in the path $\vec{r}(t)$ to $\vec{r}(t) + \vec{\epsilon}(t)$. To make the formula simpler we will assume a rectangular movement, limiting us to one coordinate $x(t)$. Of course this could also be done with freer directions. When looking at the small change in the action

$$S[x(t) + \epsilon(t)] - S[x(t)] = \int (L(x(t) + \epsilon(t), \dot{x}(t) + \dot{\epsilon}(t))) - L(x(t), \dot{x}(t)))dt. \tag{21}$$

ϵ is chosen to be infinitely small, so that only factors linear to ϵ remains. This gives us $(L(x(t) + \epsilon(t), \dot{x}(t) + \dot{\epsilon}(t))) = L(x(t), \dot{x}(t)) + \epsilon(t)\frac{\partial L}{\partial x}(t) + \dot{\epsilon}(t)\frac{\partial L}{\partial \dot{x}}(t)$. Putting this in equation (21) grants

$$S[x(t) + \epsilon(t)] - S[x(t)] = \int \left(\epsilon(t)\frac{\partial L}{dx}(t) + \dot{\epsilon}(t)\frac{\partial L}{\partial \dot{x}}(t)\right) dt. \tag{22}$$

Since $\epsilon = 0$ at the endpoints it is possible to use partial integration to obtain, in accordance with the action principle, that

$$S[x(t) + \epsilon(t)] - S[x(t)] = \int \epsilon(t)(\frac{\partial L}{dx}(t) - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}}(t))dt = 0. \tag{23}$$

From this we find the Euler-Lagrangian equation

$$\frac{\partial L}{\partial x}(t) - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}}(t) = 0. \tag{24}$$

When working in quantum field theory, instead of looking at the traditional Lagrangian L it is more convenient to work with the Lagrangian density $\mathcal{L}$. Due to this the Lagrangian density is often called the Lagrangian.

Since equations of motions should be local, we use $L = \int \mathcal{L} d^3x$, where $\mathcal{L}$ will be used to denote the Lagrangian density. Using this notation we get the action

$$S = \int \mathcal{L} d^4x. \tag{25}$$

Working in a relativistic system it is important that the action of the theory is Lorentz invariant. We find immediately that d^4x is Lorentz invariant since $\bar{x}^\mu = \Lambda^\mu_\nu x^\nu$ and $det\Lambda = 1$ is equivalent with

$$d^4\bar{x} = |det\Lambda| d^4x = d^4x. \tag{26}$$

Since d^4x is clearly Lorentz invariant, for the action in equation (25) to be Lorentz invariant, it is needed that $\bar{\mathcal{L}}(\bar{x}) = \mathcal{L}(x)$. The Lagrangian density has to be a Lorentz scalar.

When describing the standard model we describe each particle as a field. We will get a huge Lagrangian density containing all the information about the fields' kinetic contributions and how the fields interact with each other. To make sure that it is Lorentz invariant, it will consist of linear combinations of scalars related to the fields.

For instance, when working with a fermion field ψ, ψ is not Lorentz invariant. However by using the dirac conjugate of ψ, $\bar{\psi} = \psi^\dagger\gamma^0$, we find the scalar $\bar{\psi}\psi$ which is Lorentz invariant. In a similar approach another example of a scalar related to the fermion field would be $\bar{\psi}\gamma^\mu\partial_\mu\psi$. The linear coefficients for the Lagrangian will be coupling constants which can be measured experimentally [6].

Otherwise there are some other factors that we need to take in regard when creting the Lagrangian. Since the action is dimensionless, equation (25) says that the dimension for the Lagrangian actually is $(\text{GeV})^4$. It is important that all terms in Lagrangian densities have such a dimension.

Most coupling constants are dimensionless, but they can also be given in $(\text{GeV})^{-n}$. If a coupling constant has a unit of $(\text{GeV})^{-n}$, that means that it will be inversely proportional to Λ^n, where Λ is something along the "Scale of New Physics" [9].

This means that Λ corresponds to the mass of a particle above the reach of the current model [10]. There is no sensible argument to set an upper limit to Λ, except the Planck mass, which is nature's maximum allowed point mass at 10^{19}GeV. But as $\Lambda \to \infty$ the coupling fades away. Λ also needs to be substantially higher than the electroweak scale, $\Lambda \gg m_t$, where m_t is the mass of the top quark, the heaviest particle in the Standard Model. Therefore Λ will often be in the scale of TeV.

2.5 Decay rate

Particles have a certain mean lifetime τ, which depends on how easy it can decay into other particles. By taking $\frac{1}{\tau} = \Gamma_T$ where $\Gamma_T = \sum_i^n \Gamma_i$, and Γ_i is the probability of the particle to decay per unit time into something else, usually two or three other particles.

The fraction of width and the total particles created is called the branching ratio and is defined as

$$BR(i) = \frac{\Gamma_i}{\Gamma_T}. \tag{27}$$

The way to calculate decay-rate is best described in Griffiths [5] by using Fermi's golden rule:

$$\Gamma = \frac{S}{2m_1} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 - p_2 - p_3 \ldots - p_n) \prod_{j=2}^{n} 2\pi\delta(p_j^2 - m_j^2)\theta(p_j^0)\frac{d^4 p_j}{(2\pi)^4}, \tag{28}$$

where S is a product of statistical factors $\frac{1}{j!}$ from each group of identical particle, j is the number of identical particles in the final state. n is the total number of particles in the interaction. m_i and p_i are the particle mass and four-momentum. For us S is either 1 for two different outgoing particles or $\frac{1}{2}$ for two identical ones. $\mathcal{M}$ is usually a matrix and holds the dynamics of the system which will be futher explained in section 2.7

We further get when we are in the restframe of one particle going into two particles described in Griffiths:

$$\Gamma = \frac{S|\mathbf{p}|}{8\pi m_1^2}|\mathcal{M}|^2, \tag{29}$$

where $\mathbf{p}$ is the three-momentum of one of the outgoing particles.

2.6 Cross section

Cross section is best described as the measurement of the likelihood of a certain interaction between two particles in to some other particles. It is independent on the luminosity which is useful trait of comparison. As for the decay rate, the cross section formula is given in a very similar way:

$$\sigma = \frac{S}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2}} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 \ldots - p_n) \prod_{j=3}^{n} 2\pi\delta(p_j^2 - m_j^2)\theta(p_j^0)\frac{d^4 p_j}{(2\pi)^4}. \tag{30}$$

From this we can express cross sections differential which is useful because $\mathcal{M}$ is dependent of the direction and amplitude of the outgoing particles' momentum $\mathbf{p}$ so the integral is not trivial.

$$d\sigma = \frac{S|\mathcal{M}|^2}{64\pi^2(E_1 + E_2)^2}\frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}d\Omega, \tag{31}$$

where $E_{1,2}$ is equal to $p^0_{1,2}$, $|\mathbf{p}_f|$ is the magnitude of the momentum either outgoing momentum particle , $|\mathbf{p}_i|$ is the magnitude of the momentum for either incoming particle and $d\Omega = \sin(\theta)d\theta d\phi$.

2.7 Calculating the scattering amplitude

The most important factor in the golden rules for decay rate and cross-section is the scattering amplitude. This is the connection to the Lagrangian of the theory and holds the information of the dynamics of the system. The processes of calculating it are based mostly on the so called Feynman rules, which are described later, but to retrieve these from a Lagrangian could be quite extensive which motivates the use of computer programs like FeynRules. This section will cover the basic principal of one way to calculate the scattering amplitude and show some simple examples. Later the Feynman rules will be explained.

2.7.1 Theoretical foundation for calculation of the scattering amplitude

In the interaction picture we assume an interaction Lagrangian $\mathcal{L}_{int}$ for a quantum field theory and we seek the scattering amplitude for a process with initial state $|i\rangle$ at time $t = -\infty$ and a final state $|f\rangle$ at time $t = \infty$. In our quantum mechanical theory we have an operator $\hat{M}$ that transforms the initial state to the final state. Therefore the scattering amplitude becomes

$$S = \langle f|\hat{M}|i\rangle. \tag{32}$$

Since $\hat{M}$ evolves the system over time it must be the time evolution operator, thus we get

$$S = \langle f|\hat{U}(-\infty, \infty)|i\rangle. \tag{33}$$

The time evolution operator is connected through Noether's and Wigner's theorems to the systems Hamiltonian as [1]

$$\hat{U}(-\infty, \infty) = T \exp -i \int_{-\infty}^{\infty} \hat{H}(t)\, dt. \tag{34}$$

Since the Hamiltonian is dependent on time we can't assume that the Hamiltonian at time t commutes with it self at time $t' \neq t$, therefore the time order operator is necessary. The standard way to proceed is to write down the series expansion of the time ordered exponential [1] , which yields

$$\hat{U}(-\infty, \infty) = 1 - i \int_{-\infty}^{\infty} \hat{H}(t)\, dt - \frac{1}{2} \int_{-\infty}^{\infty} \hat{H}(t) \int_{-\infty}^{t} \hat{H}(t')\, dt' dt + \cdots \tag{35}$$

The relation between the interaction Lagrangian and the Hamiltonian is

$$\hat{H} = -\int d^3x \mathcal{L}_{int}, \tag{36}$$

[1] Deriving the series expansion of the time ordered exponential in equation (34) is quite extensive and will therefore be left out of this thesis.

which gives the final formula for calculating the scattering amplitude as

$$S = \langle f|1 + i \int_{-\infty}^{\infty} \int d^3x \mathcal{L}_{int} dt - \frac{1}{2} \int_{-\infty}^{\infty} \int d^3x \mathcal{L}_{int}(t) \int_{-\infty}^{t} \int d^3x' \mathcal{L}_{int}(t')\, dt' dt + \cdots |i\rangle \tag{37}$$

The quantity $\mathcal{M}$ is defined as

$$S = i(2\pi)^4 \delta^4(k_{\text{in}} - k_{\text{out}})\mathcal{M} \tag{38}$$

where k_{in} and k_{out} is the total momentum in the initial state respective the final state. [6]

In most cases it is impossible to evaluate the whole sum, in which case it is imperative to use perturbation theory. In this case we assume that only a few of the first terms are large enough to have an significance for the result.

This is just the general formula, for more specific cases it is required to add a theory for creation and annihilation of particles. To do so it is necessary to introduce the creation and annihilation operators. These operators are completely analogous to the creation and annihilation operators of the harmonic oscillator and could be treated as such. The creation operator $a(k)^\dagger$ is defined by

$$|k\rangle = \sqrt{2E_k} a^\dagger(k)|0\rangle. \tag{39}$$

That is if it operates on the vacuum, it creates a particle with momentum k and energy E_k. The annihilation operator that is the Hermitian conjugate to the creation operator removes one particle with momentum k. In Dirac notation that is

$$\frac{1}{\sqrt{2E_k}} a(k)|k'\rangle = (2\pi)^3 \delta^3(k - k')|0\rangle. \tag{40}$$

An important feature is that an annihilation operator can not remove particles from the vacuum. This is mathematically written as

$$\hat{a}|0\rangle = 0, \tag{41}$$

where $|k\rangle$ is the state of a particle with momentum k and $|0\rangle$ is the vacuum state, the state without any particles. The commutation rules for our two operators are as follows

$$[a(q_1), a(q_2)] = 0 \tag{42}$$
$$\left[a^\dagger(q_1), a^\dagger(q_2)\right] = 0 \tag{43}$$
$$\left[a(q_1), a^\dagger(q_2)\right] = (2\pi)^3 \delta(q_1 - q_2) \tag{44}$$

for bosons [6]. Fermions has to obey the Pauli principal and thus they follow the anticommutation relations

$$\{a(q_1), a(q_2)\} = 0 \tag{45}$$
$$\left\{a^\dagger(q_1), a^\dagger(q_2)\right\} = 0 \tag{46}$$
$$\left\{a(q_1), a^\dagger(q_2)\right\} = (2\pi)^3 \delta(q_1 - q_2). \tag{47}$$

A quantum field can be seen as the Fourier transform of the corresponding creation and annihilation operators. A spin zero bosonic field can be written as [6]

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3\sqrt{2E_k}} \left(a_k e^{-ix^\mu k_\mu} + a_k^\dagger e^{ix^\mu k_\mu} \right). \tag{48}$$

The quantisation of the fermionic and spin one bosonic fields is done in similar fashion, but is not necessary for the following theory.

2.7.2 Scattering of free particles

Consider the case of non-interacting free particles. In this case the Lagrangian is $\mathcal{L}_{int} = 0$. This results in that the time evolution operator is the identity operator $\hat{I}$. Assuming that there is n particles with momentum $p_1, p_2, \cdots, p_n$ in the initial state and m particles with momentum $k_1, k_2, \cdots, k_m$ in the final state. The scattering amplitude is then

$$S = \langle k_1, k_2, \cdots, k_m | p_1, p_2, \cdots, p_n \rangle. \tag{49}$$

Using equation (39) the scattering amplitude can be written as

$$\begin{aligned} S =& \sqrt{2E_{k_1} 2E_{k_2} \cdots 2E_{k_m} 2E_{p_1} \cdots E_{p_n}} \\ & \langle 0 | a(k_1) a(k_2) \cdots a(k_m) a^\dagger(p_1) a^\dagger(p_2) \cdots a^\dagger(p_n) | 0 \rangle. \end{aligned} \tag{50}$$

By switching place of $a(k_m)$ and $a^\dagger(p_1)$ and using commutation rule (44) the right hand side in equation (50) is

$$\begin{aligned} S =& \sqrt{2E_{k_1} 2E_{k_2} \cdots 2E_{k_m} 2E_{p_1} \cdots E_{p_n}} \\ & \big(\langle 0 | a(k_1) a(k_2) \cdots a(k_{m-1}) a^\dagger(p_1) a(k_m) a^\dagger(p_2) \cdots a^\dagger(p_n) | 0 \rangle + \\ & \langle 0 | a(k_1) a(k_2) \cdots a(k_{m-1}) a^\dagger(p_2) \cdots a^\dagger(p_n) | 0 \rangle (2\pi)^3 \delta^3(k_m - p_1) \big). \end{aligned} \tag{51}$$

Letting $a^\dagger(p_1)$ wander to the left in this manner this yields that

$$\begin{aligned} S =& \sqrt{2E_{k_1} 2E_{k_2} \cdots 2E_{k_m} 2E_{p_1} \cdots E_{p_n}} \\ & \big(\langle 0 | a^\dagger(p_1) a(k_1) a(k_2) \cdots a(k_m) a^\dagger(p_2) \cdots a^\dagger(p_n) | 0 \rangle + \\ & + \langle 0 | a(k_2) \cdots a(k_m) a^\dagger(p_2) \cdots a^\dagger(p_n) | 0 \rangle (2\pi)^3 \delta^3(k_1 - p_1) + \\ & \sum_{j=2}^{m-1} \langle 0 | a(k_1) \cdots a(k_{j-1}) a(k_{j+1}) \cdots a(k_m) a^\dagger(p_2) \cdots a^\dagger(p_n) | 0 \rangle (2\pi)^3 \delta^3(k_j - p_1) + \\ & + \langle 0 | a(k_1) \cdots a(k_{m-1}) a^\dagger(p_2) \cdots a^\dagger(p_n) | 0 \rangle (2\pi)^3 \delta^3(k_m - p_1) \big). \end{aligned} \tag{52}$$

In the first term there is an creation operator directly applied to the vacuum state in the bra. This is equivalent to annihilation of the vacuum as in rule (41). Therefore the first term vanishes.

This process could be performed for each of the other creation operators. If so, it is easy to see that only the case $n = m$ will yield an non zero result. This could be verified by looking at the other two cases. If $n > m$ there will be creation operators left after all annihilation operators are consumed, this means that there will be one creation operator directly applied to the vacuum bra and all the terms yields zero. The other case $n < m$ is analog in the aspect that there will be only annihilation operators left free to be applied to the vacuum ket. As a result the number of particles in the free case must be conserved.

For the actually scattering amplitude there will be $n!$ terms of the form

$$\prod_{(i,j)\in R} (2\pi)^3 \delta^3(k_j - p_i) \tag{53}$$

where $R = \{(1, P_1(n)), (2, P_2(n)), \cdots, (n, P_n(n))\}$ in which $P_i(n)$ is a permutation of the numbers $i = 1, \cdots, n$.

2.7.3 Scattering of a simple model

To calculate the scattering of the process where one scalar boson decays to two other bosons with the interaction Lagrangian

$$\mathcal{L}_{int} = \lambda \phi_1(x) \phi_2(x)^2 \tag{54}$$

where λ is just a constant, $\phi_1(x)$ and $\phi_2(x)$ are the quantum fields for the particles defined as in equation (48). The initial state of the process is one particle with momentum p corresponding to $\phi_1(x)$. This state will be denoted $|^{\phi_1}p\rangle$. The final state is two particles with momentum k_1 and k_2 corresponding to $\phi_2(x)$, these states are denoted $|^{\phi_2}k_1,^{\phi_2}k_2\rangle$. To calculate the scattering amplitude in equation (37), it is necessary to only do a first order perturbation. Therefore the scattering matrix is approximated as

$$S \approx \langle^{\phi_2}k_1,^{\phi_2}k_2|1 + i\int d^4\mathcal{L}_{int}|^{\phi_1}p\rangle. \tag{55}$$

The first term correspond to the case of free particles. It is shown in the previous section that the number of particles must be conserved. This is clearly not the case, since there are two outgoing particles but only one incoming particle. Therefore this term gives no contribution to the final scattering amplitude. In most cases the free particle case is ignored since it is not interesting when looking at the interactions. The scattering amplitude must then be, after inserting the interaction Lagrangian,

$$S \approx \langle^{\phi_2}k_1,^{\phi_2}k_2|i\int d^4x \lambda\phi_1(x)\phi_2(x)^2|^{\phi_1}p\rangle. \tag{56}$$

Replacing the fields with their definitions as in equation (48) this becomes

$$\begin{aligned} S \approx & i\lambda\langle^{\phi_2}k_1,^{\phi_2}k_2|\int d^4x \\ & \left(\int \frac{d^3k_a}{(2\pi)^3\sqrt{2E_{k_a}}}\left(a_1(k_a)e^{-ix^\mu k_{a\mu}} + a_1^\dagger(k_a)e^{ix^\mu k_{a\mu}}\right)\right. \\ & \int \frac{d^3k_b}{(2\pi)^3\sqrt{2E_{k_b}}}\left(a_2(k_b)e^{-ix^\mu k_{b\mu}} + a_2^\dagger(k_b)e^{ix^\mu k_{b\mu}}\right) \\ & \left.\int \frac{d^3k_c}{(2\pi)^3\sqrt{2E_{k_c}}}\left(a_2(k_c)e^{-ix^\mu k_{c\mu}} + a_2^\dagger(k_c)e^{ix^\mu k_{c\mu}}\right)\right)|^{\phi_1}p\rangle. \end{aligned} \tag{57}$$

The only operators in this expression are the creation and annihilation operators. The best way to proceed is to put them all in to a new operator defined as

$$\hat{Q} = (a_1(k_a)e^{-ix^\mu k_{a\mu}} + a_1^\dagger(k_a)e^{ix^\mu k_{a\mu}}) \tag{58}$$
$$(a_2(k_b)e^{-ix^\mu k_{b\mu}} + a_2^\dagger(k_b)e^{ix^\mu k_{b\mu}}) \tag{59}$$
$$(a_2(k_c)e^{-ix^\mu k_{c\mu}} + a_2^\dagger(k_c)e^{ix^\mu k_{c\mu}}). \tag{60}$$

.

The scattering amplitude can now be written as

$$S \approx i\lambda \int \frac{d^4x d^3k_a d^3k_b d^3k_c}{(2\pi)^9\sqrt{2E_{k_a}2E_{k_b}2E_{k_c}}} \langle^{\phi_2}k_1,^{\phi_2}k_2|\hat{Q}|^{\phi_1}p\rangle. \tag{61}$$

The last factor in the integrand, expands to eight terms. However there has to be an equal number of creation and annihilation operators acting on the vacuum. Thus only one of the terms survives and $\hat{Q}$ yields

$$\begin{aligned}\langle^{\phi_2}k_1,^{\phi_2}k_2|\hat{Q}|^{\phi_1}p\rangle &= \sqrt{2E_{k_1}2E_{k_2}2E_p}\\ &\langle 0|a_2(k_1)a_2(k_2)a_1(k_a)a_2^\dagger(k_b)a_2^\dagger(k_c)a_1^\dagger(p)|0\rangle e^{-ix^\mu(k_{a\mu}-k_{b\mu}-k_{c\mu})}.\end{aligned} \tag{62}$$

The annihilation operator $a_1(k_a)$ commutes with all the creation operators except $a_1^\dagger(p)$. This is because there are two different types of particles. When $a_1(k_a)$ swaps place with $a_1^\dagger(p)$ there will be an annihilation operator acting on the vacuum so the only surviving term is the commutator. This yields

$$\begin{aligned}\langle^{\phi_2}k_1,^{\phi_2}k_2|\hat{Q}|^{\phi_1}p\rangle &= \sqrt{2E_{k_1}2E_{k_2}2E_p}\\ &\langle 0|a_2(k_1)a_2(k_2)a_2^\dagger(k_b)a_2^\dagger(k_c)|0\rangle\\ &(2\pi)^3\delta^3(k_a-p)e^{-ix^\mu(k_{a\mu}-k_{b\mu}-k_{c\mu})}.\end{aligned} \tag{63}$$

The remaining creation and annihilation operators could be paired up in two different ways. By using their commutation relations the final expression is

$$\begin{aligned}\langle^{\phi_2}k_1,^{\phi_2}k_2|\hat{Q}|^{\phi_1}p\rangle &= \sqrt{2E_{k_1}2E_{k_2}2E_p}(2\pi)^9 e^{-ix^\mu(k_{a\mu}-k_{b\mu}-k_{c\mu})}\\ &\delta^3(k_a-p)(\delta^3(k_1-k_b)\delta^3(k_2-k_c)+\delta^3(k_1-k_c)\delta^3(k_2-k_b)).\end{aligned} \tag{64}$$

The sign of the terms is the same since the process only contains bosons, in the case of fermions the sign would be different because of their anti-commutation rules.

The scattering amplitude is therefore

$$\begin{aligned}S \approx& i\lambda \int \frac{d^4x d^3k_a d^3k_b d^3k_c}{(2\pi)^9\sqrt{2E_{k_a}2E_{k_b}2E_{k_c}}}\sqrt{2E_{k_1}2E_{k_2}2E_p}(2\pi)^9 e^{-ix^\mu(k_{a\mu}-k_{b\mu}-k_{c\mu})}\\ &\delta^3(k_a-p)(\delta^3(k_1-k_b)\delta^3(k_2-k_c)+\delta^3(k_1-k_c)\delta^3(k_2-k_b)).\end{aligned} \tag{65}$$

Consider the integration over the momentum spaces one at a time. The first momentum integration over k_a has a Dirac-function, $\delta^3(k_a-p)$ this has the effect of "swaping" every k_a with p. For the other two integrals there are two terms of Dirac-functions. Therefore the amplitude is now

$$S \approx -i\lambda \left(\int d^4x e^{-ix^\mu(p_\mu - k_{1\mu} - k_{2\mu})} + \int d^4x e^{-ix^\mu(p_\mu - k_{2\mu} - k_{1\mu})} \right). \tag{66}$$

Since addition is commutative the two integrals are the same. The four dimensional Fourier transform of the Dirac-function is 1 and the integral has the form of the inverse Fourier transform except for a constant $\frac{1}{(2\pi)^4}$. Thus the scattering amplitude of this process is

$$S \approx -i32\pi^4\lambda\delta^4(p_\mu - k_{1\mu} - k_{2\mu}). \tag{67}$$

The Dirac function agrees with the conservation of momentum. Since this is an observed property in all processes in nature it is convention to not include the factor $(2\pi)^4\delta^4(p_\mu - k_{1\mu} - k_{2\mu})$ in the final scattering amplitude. Thus the final amplitude is

$$i\mathcal{M} = -2i\lambda. \tag{68}$$

It is also possible to do this calculation using path integrals, like equation 19. However this require functional analysis which is not a requirement of this thesis.

2.7.4 Feynman diagrams and the Feynman rules

It is fairly obvious that the method to calculate the scattering amplitude presented earlier is complicated. In many cases it is necessary to use higher order terms in the series expansion of the time ordered exponential and this complicates things.

There are however simpler methods to get analytical expressions for the scattering amplitude using Feynman diagrams and the Feynman rules.

A Feynman diagram is a graphical representation of the reactions where every particle is represented with an line in two dimensions and every interaction is represented with a node where three lines converge[2]. A diagram is read from left to right with all in going particles to the left and all outgoing particles to the right.

In figure 1 a simple example over a Feynman diagram representing the coulomb interaction between an electron and a positron. Different types of particles have different styles of lines. The photons are represented with a squiggly line. Gluons is represented with a curly line. Fermions have a strait line with an arrow point to the right, while antifermions have an arrow pointing to the left. Scalar bosons may have different styles of lines in this thesis a dashed line with an arrow is used but others may use a strait line without an arrow.

The Feynman rules tells how to transform a Feynman diagram to the scattering amplitude. These are as follows

1. All ingoing and outgoing particle is assigned momentum.

[2] This is not always true, but in most of those cases it just shows that there are more fundamental interaction at a higher energy. However there are a case in the Standard Model with four gluons which is a fundamental interaction [5]

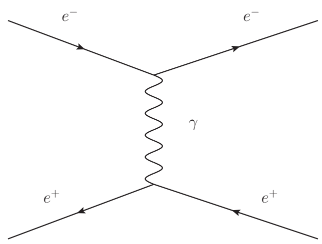

Figure 1: Simple electron positron coulomb interaction diagram.

2. All vertices's assigned with an coupling constant, these are derived from the interaction Lagrangian using first order perturbation theory.

3. The internal lines are assigned propagators derived from the kinetic terms of the Lagrangian using the least action principal.

4. Integrate over all the internal particles momentum.

5. The remaining delta function enforcing conservation of momentum is ignored.

These rules is taken from Griffiths Introduction to Elementary Particle Physics [5].

Consider the interaction in the previous section. It is described by the very simple diagram in figure 2. The first naive thought would be that the scattering

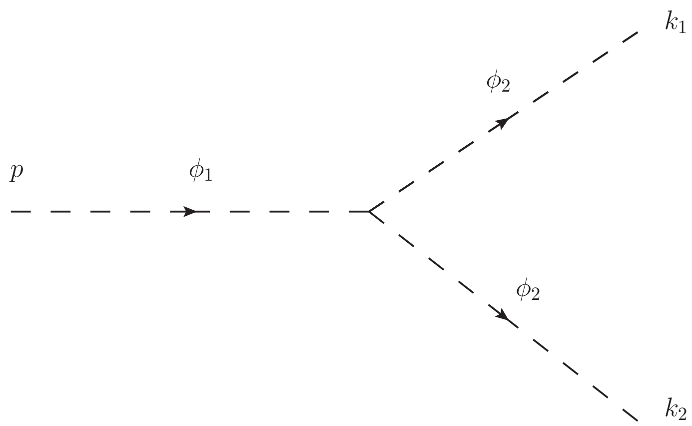

Figure 2: A single vertex Feynman diagram describing the first order interaction calculated earlier. This is used to find the vertex coupling used to evaluating the diagrams in figure 3.

amplitude is the same as the one calculated earlier $\mathcal{M} = -2i\lambda$. However the real amplitude is just half of it $\mathcal{M} = -i\lambda$. This is because the outgoing particles can switch place and thus resulting in a second diagram, and since there where no difference between the out going particles this was taken in to account earlier. Since this is a diagram with just one vertex and no internal particles, the amplitude of this diagram must be the coupling constant for interaction of this type.

To further demonstrate the power of Feynman diagrams take a look of the interaction with two incoming ϕ_2 bosons and two outgoing ϕ_2 bosons. What is

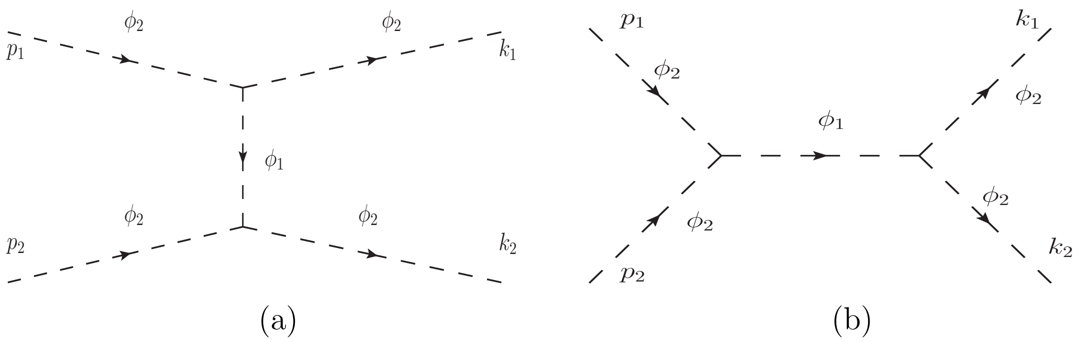

Figure 3: The two possible lowest order non free interaction between two ϕ_2 scalar bosons, with the interaction Lagrangian described in equation (54). Diagram (a) corresponds to two ϕ_2 bosons exchange one ϕ_1 boson, diagram (b) corresponds to two ϕ_2 bosons collides and produce one ϕ_1 boson that then decays to two ϕ_2 bosons.

the amplitude of the lowest order non free interaction? In this case there are two diagrams to calculate (see figure 3).

However, the principal for calculating each diagram is the same, therefore only calculation of diagram (a) will be demonstrated. The first step of assigning all in going and outgoing particles momentum is already done. The second step is assigning all vertices's coupling constants. Since they are of the same type as the interaction in figure 2 the coupling constants are $-i\lambda$, we get therefor a factor $-\lambda^2$ in the scattering amplitude. The third step is to find the propagator for the internal line. A propagator for a particle is a Greens function from its equation of motion. Since the ϕ_1 particle is a scalar boson, the Klein-Gordon equation (8) describes its motion. A Greens function for the Klein-Gordon equation can be found by solving the equation

$$(\partial^\mu \partial_\mu + m^2)\Delta(x) = \delta^4(x) \tag{69}$$

where $\Delta(x)$ is the propagator in Minkovski space and m is the mass of the particle. However it is only necessary for further calculations to solve the equation in momentum space. The equation in momentum space is given by a four dimensional Fourier transform, and thus the equation becomes

$$(-k^\mu k_\mu + m^2)\Delta(k) = 1. \tag{70}$$

From this the propagator is

$$\Delta(k) = \frac{-1}{k^\mu k_\mu - m^2} \tag{71}$$

The fourth step is to add a Dirac function for each vertex, and then integrate over the momentum of the internal particle. Let q be the four-momentum of the internal particle then the expression to evaluate is

$$\begin{aligned} S &= \lambda^2 \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^\mu q_\mu - m^2} (2\pi)^4 \delta(p_1 - q - k_1)(2\pi)^4 \delta(p_2 + q - k_2) \\ &= \frac{\lambda^2}{(k_2 - p_2)^2 - m^2} (2\pi)^4 \delta(p_1 + p_2 - k_1 - k_2). \end{aligned} \tag{72}$$

And finally by removing the remaining $(2\pi)^4\delta(p_1+p_2-k_1-k_2)$ and multiply with i the amplitude is obtained as

$$\mathcal{M} = i\frac{\lambda^2}{(k_2-p_2)^2-m^2}. \tag{73}$$

Of course this is for the diagram (a), for diagram (b) the result is similar but the denominator changes to $(k_1+k_2)^2-m^2$.

2.8 The Higgs mechanism

The Higgs boson was a highly anticipated prediction to the standard model. It is the excitation of the Higgs field. The properties of the Higgs field are that it is responsible for the mass of all the massive particles in the Standard Model.[3] We can visualise it as that every massive particle is floating on top of the Higgs field like rubble in the water. In the same simile the Higgs Bosons would be the waves forming when throwing a rock in the water. [11]

Using quantum mechanic terms the Higgs mechanism actually derives from something called a spontaneous symmetry breaking. This occurs when the ground state of the system does not share the full symmetry of the theory. An important consequence of spontaneous symmetry breaking is that there are excitations whose energy goes to zero in the long wavelength limit. These excitations do not create the Higgs boson but massless theoretical bosons named the Goldstone Bosons.

To understand the Higgs mechanism we will look at the relativistic field theory for a complex scalar field ϕ. In this theory we are working with the Lagrangian

$$\mathcal{L} = \partial_\mu\bar{\phi}\partial^\mu\phi - V(\phi) = \partial_\mu\bar{\phi}\partial^\mu\phi - m^2\bar{\phi}\phi - \frac{1}{2}\lambda(\bar{\phi}\phi)^2, \tag{74}$$

where m is the mass and λ is the self-interaction coupling constant of the field. This model has $U(1)$-symmetry since it is invariant under a phase-change,

$$\phi(x) \to \phi(x)e^{i\alpha}. \tag{75}$$

As long as $m^2 > 0$, this model describes a self-interacting scalar field for particles with the mass m. However, if $m^2 < 0$ then $\phi = 0$ is a maximum of the potential V. If we set the minimum for V to $\phi_0 = v$, we can rewrite

$$V(v) = \frac{1}{2}\lambda v^4 + m^2v^2. \tag{76}$$

We can then minimise the potential by using its derivative

$$V'(v) = 2\lambda v^3 + 2m^2v = 0. \tag{77}$$

Using this we find that the minimum is given by $v^2 = -\frac{m^2}{\lambda}$. The field's minimum now lies on the circle $|\phi|^2 = v^2$. In the ground state we expect ϕ to be nonzero with a magnitude close to v.

The potential $V(\phi)$, which describes an unstable equilibrium, is often called the sombrero potential. This can be seen graphically in figure 4.

[3]This is not to be confused with the mass shown on a bathroom scale since that is mostly relativistic effects of coupling quarks within protons and neutrons.

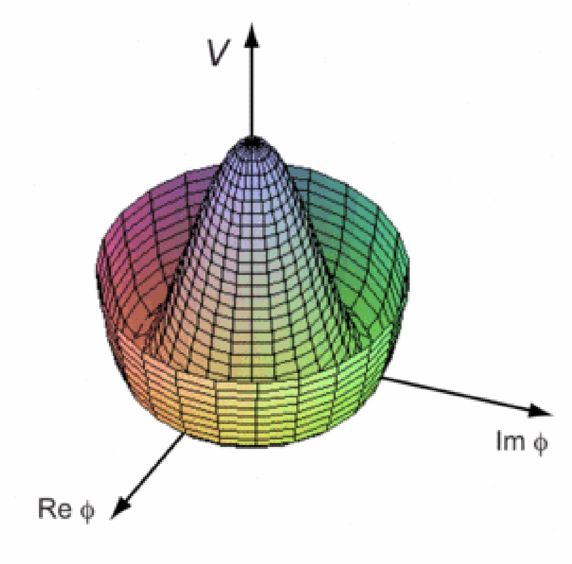

Figure 4: The famous sombrero potential [11]. The field's minima lies on the circle $|\phi|^2 = \frac{v^2}{2}$. In the ground state it is expected that ϕ is nonzero with a magnitude close to $\frac{v}{\sqrt{2}}$.

From this we can observe the Goldstone bosons. By choosing a particular minimum where ϕ is real and positive, and expand it with

$$\phi = (v + \psi_1 + i\psi_2), \tag{78}$$

it is given that

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu \psi_1)^2 + (\partial_\mu \psi_2)^2] - V(\psi_1, \psi_2), \tag{79}$$

where

$$V(\psi_1, \psi_2) = -\frac{1}{2}\lambda v^4 + \frac{1}{2}\lambda v^2 \psi_1^2 + 2\lambda v \psi_1(\psi_1^2 + \psi_2^2) + \frac{1}{2}\lambda(\psi_1^2 + \psi_2^2). \tag{80}$$

By construction there is no linear term. Using canonical quantisation we find that this model describes two kinds of particle, the ψ_1 of mass $\sqrt{\lambda} v$ and the massless ψ_2 with cubic and quartic couplings. We find that ψ_2, corresponding to a variation of the phase angle, is the Goldstone boson. In the same way the massive ψ_1 is a perfect candidate for the Higgs boson. In this theory ψ_2 is physical and should be kept. However if we couple ϕ to electromagnetism, ψ_2 can be removed by a gauge transform. We will ignore it from now on and consider the physics of the $h(x)$ field only.

If we define the Higgs boson $\psi_1 = h(x)$ we can from equation (78) rewrite the original Higgs field as

$$\phi(x) = (h(x) + v). \tag{81}$$

The alternative Higgs field $h(x)$ will preserve the couplings of the $\phi(x)$ field, but the shift v will appear in the couplings giving masses to the gauge bosons through the gauge couplings and giving masses to the leptons through the Yukawa couplings.

Gauge couplings are couplings where all particles are interacting with the massless theoretical Goldstone bosons B and $W_i, i = 1, 2, 3$ While the Yukawa couplings are different coupling constants which are used in the interaction between a scalar field and a Dirac field, the aptly named Yukawa interaction.

The Yukawa interaction term is

$$\mathcal{L}_{Yukawa} = -g\bar{\Psi}\phi\Psi, \tag{82}$$

where g is a coupling constant and Ψ is a fermion field [12]. By expanding using equation (81) and that v is a constant we get

$$\mathcal{L}_{Yukawa} = -g\bar{\Psi}(h(x) + v)\Psi = -g\bar{\Psi}h(x)\Psi - gv\bar{\Psi}\Psi. \tag{83}$$

The term $-gv\bar{\Psi}\Psi$ is quadratic and similar to a mass term $-m^2\bar{\phi}\phi$. The Yukawa coupling will give the fermion a mass contribution equals to $\sqrt{gv}$.

3 The Scalar Boson a

The main purpose of this project has been to expand the current Standard Model with an extra particle, a scalar boson. The point of this scalar boson would be to explain some of the Higgs boson's decay to 4 photons. When looking further in the subject [13], we have found that if the particle decays into photons it would also probably decay into gluons as well.

The interaction Lagrangian for a particle like that, in correlation with the previously discussed theories, would be

$$\begin{aligned}\mathcal{L}_{int} =& \lambda_{photon} \cos\theta_{photon} aF^{\mu\nu}F_{\mu\nu} + \lambda_{photon} \sin\theta_{photon} aF^{\mu\nu}\tilde{F}_{\mu\nu} \\ &+ \lambda_{gluon} \cos\theta_{gluon} aG^{\mu\nu i}G_{\mu\nu i} + \lambda_{gluon} \sin\theta_{gluon} aG^{\mu\nu i}\tilde{G}_{\mu\nu i} \\ &- \mu a^2\bar{\phi}\phi,\end{aligned} \tag{84}$$

where μ is a unitless coupling constant, λ_{photon} och λ_{gluon} are coupling constants with units GeV^{-1}. The field $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ corresponds to swapping the electric field and magnetic field. The two angles θ_{photon} and θ_{gluon} decides whether the field is a scalar field or a pseudoscalar field, and all linear combinations of those.

To find the mass of the particle a we can expand the interaction part with the Higgs field ϕ. We know that $\phi = (h + v)$, which leads to

$$\mathcal{L}_{a-h} = -\mu a^2\bar{\phi}\phi = -\mu a^2(h + v)^2 = -\mu a^2h^2 - 2\mu a^2hv - \mu a^2v^2. \tag{85}$$

We see that the last part of the expansion is similar to a mass term, as example $-m_a^2a^2$ where the mass $m_a = \sqrt{\mu}v$.

3.1 Width of Higgs decay to two a

The interaction of the a and the Higgs boson result in a Feynman diagram for a Higgs decaying in to two a as shown in figure 5.

This diagram arises from the term $-\mu a^2\bar{\phi}\phi$ in the Lagrangian. Expanding the factor $\bar{\phi}\phi$ as in equation (85), and the second of these terms result in the diagram.

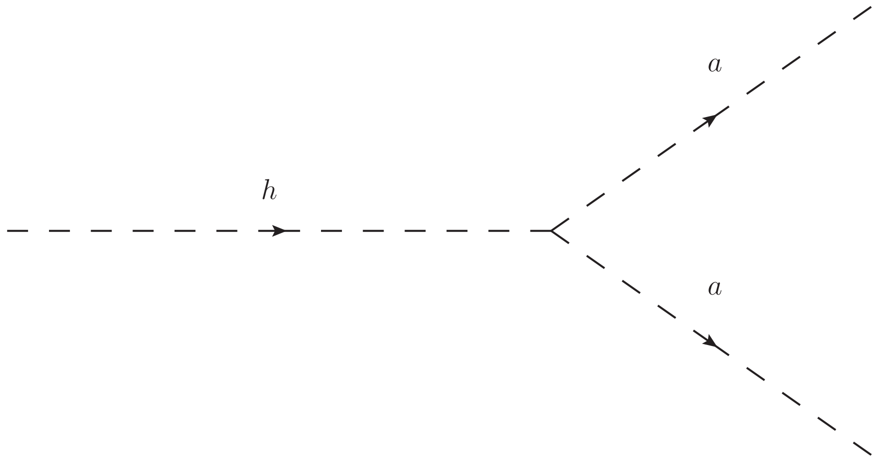

Figure 5: The first order Feynman diagram describing a Higgs boson decaying to two a.

This interaction is exactly the same as in the example calculation of the scattering matrix in section 2.7.3. In this case λ is $2\mu v$ and thus the scattering amplitude is $\mathcal{M} = -2\mu v$. To calculate the width we use the formula (28). The parameter S is $\frac{1}{2}$ since we have two identical particles in the final state. Therefore the width becomes the integral

$$\Gamma = \frac{1}{4m_h}\int 4\mu^2 v^2 (2\pi)^4 \delta^4(p-k_1-k_2)(2\pi)^2\delta(k_1^2-\mu v^2)\delta(k_2^2-\mu v^2)\theta(k_1^0)\theta(k_2^0)\frac{d^4k_1 d^4k_2}{(2\pi)^8} \tag{86}$$

where m_h is the mass of the Higgs boson, p is the 4-momentum of the incoming Higgsboson, k_1, k_2 is the 4-momentum of the outgoing particles and $\theta(x)$ is the Heaviside function. The simplest way to solve this integral is to take the integration over the time components of the outgoing momentum. Due to the Heaviside function it is only necessary to take the time integration lower limit to 0. Another thing to notice is that we can write $k_1^2 = (k_1^0)^2 - \vec{k}_1^2$ and $k_2^2 = (k_2^0)^2 - \vec{k}_2^2$ so we get the integral

$$\begin{aligned}\Gamma =& \frac{\mu^2 v^2}{(2\pi)^2 m_h}\int d^3\vec{k}_1 d^3\vec{k}_2 \int_0^\infty dk_1^0 \int_0^\infty dk_2^0 \\ & \underbrace{\delta^4(p-k_1-k_2)}_{\text{Dirac function 1}}\underbrace{\delta((k_1^0)^2-\vec{k}_1^2-\mu v^2)}_{\text{Dirac function 2}}\underbrace{\delta((k_2^0)^2-\vec{k}_2^2-\mu v^2)}_{\text{Dirac function 3}}.\end{aligned} \tag{87}$$

Dirac function 1 is factorised to one time dependent factor and one space dependent factor. The second Dirac functions will yield an factor $\frac{1}{2\sqrt{\vec{k_1}^2+\mu v^2}}$ and "replace" k_1^0 with $\sqrt{\vec{k_1}^2+\mu v^2}$ in the remaining integrand after integration over k_1^0. The integration over k_2^0 is analogues to the one over k_1^0. After reducing the integral to integration over space the width is

$$\Gamma = \frac{\mu^2 v^2}{(2\pi)^2 m_h}\int d^3\vec{k}_1 d^3\vec{k}_2 \frac{\delta(p^0-\sqrt{\vec{k}_1^2+\mu v^2}-\sqrt{\vec{k}_2^2+\mu v^2})\delta^3(\vec{p}-\vec{k}_1-\vec{k}_2)}{2\sqrt{\vec{k}_1^2+\mu v^2}2\sqrt{\vec{k}_2^2+\mu v^2}}. \tag{88}$$

Since Γ must be an Lorentz invariant quantity the calculation can be performed in the initial frame of incoming Higgs boson. Due to that $\vec{p} = 0$. Continuing with integrating over $\vec{k}_2$. Since we have choose $\vec{p} = 0$ the factor $\delta^3(\vec{p} - \vec{k}_1 - \vec{k}_2)$ under integration is equivalent with swapping all $\vec{k}_2$ with $-\vec{k}_1$ in the remaining expression. What remains is

$$\Gamma = \frac{\mu^2 v^2}{4(2\pi)^2 m_h} \int d^3\vec{k}_1 \frac{\delta(p^0 - 2\sqrt{\vec{k}_1^2 + \mu v^2})}{\vec{k}_1^2 + \mu v^2}. \tag{89}$$

Since the only occurrence of the integration variable $\vec{k}_1$ is in square the integrand clearly is spherical symmetric. Therefor it is beneficial to switch to spherical coordinates. The Jacobian determinant for spherical coordinates is $r^2 \cos\theta$ where r is the distance, θ is the angle between the vector and z-axis. The Jacobian is the only angular dependence in the integrand and then the angular integration results in a factor 4π, the remaining integral is

$$\Gamma = \frac{\mu^2 v^2}{4\pi m_h} \int_0^\infty dr \frac{\delta(p^0 - 2\sqrt{r^2 + \mu v^2})}{r^2 + \mu v^2} r^2. \tag{90}$$

A second variable substitution, $q = \sqrt{r^2 + \mu v^2}$, $dq = \frac{r}{\sqrt{r^2 + \mu v^2}} dr$. The lower integration limit is $\sqrt{\mu} v$, so the integral is

$$\Gamma = \frac{\mu^2 v^2}{4\pi m_h} \int_{\sqrt{\mu} v}^\infty \frac{\delta(p^0 - 2q)\sqrt{q^2 - \mu v^2}}{4q} dq. \tag{91}$$

If p^0 is less than $2\sqrt{\mu} v$ then the width is zero. Since the integration is in the zero momentum frame the time component of the Higgs bosons momentum is the mass of the Higgs. This can be interpreted as a condition for the mass of of a. For this decay to happen the mass of a must be less than half the Higgs mass. The mass of a is $\sqrt{\mu} v$. If $p^0 > 2\sqrt{\mu} v$ the width is

$$\Gamma = \frac{\mu^2 v^2}{4\pi m_h^2} \sqrt{\frac{m_h^2}{4} - \mu v^2} = \frac{\mu^2 v^2}{8\pi m_h} \sqrt{1 - 4\frac{m_a^2}{m_h^2}}. \tag{92}$$

This is the same as the width given in [13], after obvious changes in notations.

3.2 Coupling constants λ_{photon} and λ_{gluon}

When attempting to find a connection between our coupling constants it is of interest to check if they are related to the New Physics Constant Λ, a concept which is briefly explained at the end of section 2.4. To do that it is important to check if any of our coupling constants μ, λ_{photon} and λ_{gluon} has a dimension. This would happen since our added Lagrangian should have the dimension GeV^4. The first part is the Higgs coupling from equation (84)

$$\mathcal{L}_{h-a} = -\mu a^2 \bar{\phi}\phi. \tag{93}$$

Since both our model particle and Higgs are scalar fields they each have dimensions (GeV) [5]. Because of that $a^2\bar{\phi}\phi$ has the dimension (GeV)^4. The coupling constant μ is dimensionless and there is no need to invoke any New Physics scale.

The second part is the coupling to photons

$$\mathcal{L}_{a-\gamma} = \lambda_{photon} \cos\theta_{photon} a F^{\mu\nu} F_{\mu\nu} + \lambda_{photon} \sin\theta_{photon} a F^{\mu\nu} \tilde{F}_{\mu\nu}. \tag{94}$$

This case differs from the first since the electromagnetic field tensors $F^{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ are rank-2 tensors and therefore have the dimension $(\text{GeV})^2$ [5]. Since $aF^{\mu\nu}F_{\mu\nu}$ and $aF^{\mu\nu}\tilde{F}_{\mu\nu}$ have the dimension $(\text{GeV})^5$, λ_{photon} needs to have a dimension of (GeV^{-1}). This is equivalent to that $\lambda_{photon} \propto \Lambda^{-1}$. It would also be reasonable that $\lambda_{photon} \propto \alpha$, where α is the electromagnetic force coupling constant. By looking at the similar axion [14], λ_{photon} could be chosen such that

$$\lambda_{photon} = \frac{\alpha}{4\pi\Lambda}. \tag{95}$$

Similar arguments could be made for the gluon coupling

$$\mathcal{L}_{a-g} = \lambda_{gluon} \cos\theta_{gluon} a G^{\mu\nu i} G_{\mu\nu i} + \lambda_{gluon} \sin\theta_{gluon} a G^{\mu\nu i} \tilde{G}_{\mu\nu i} \tag{96}$$

and thus λ_{gluon} is also $\propto \Lambda^{-1}$. Since this coupling is related to quantum chromodynamics instead of quantum electrodynamics like in the previous case, we let $\lambda_{gluon} \propto \alpha_s$, where α_s is the strong force coupling constant. λ_{gluon} can then similarly be chosen

$$\lambda_{gluon} = \frac{\alpha_s}{4\pi\Lambda}. \tag{97}$$

While it is not sure that it is the same New Physics constant Λ in both cases for simplicity it can be assumed that it is. Our model's Lagrangian can now be written

$$\begin{aligned}\mathcal{L}_{int} =& \frac{\alpha}{4\pi\Lambda} \cos\theta_{photon} a F^{\mu\nu} F_{\mu\nu} + \frac{\alpha}{4\pi\Lambda} \sin\theta_{photon} a F^{\mu\nu} \tilde{F}_{\mu\nu} \\ &+ \frac{\alpha_s}{4\pi\Lambda} \cos\theta_{gluon} a G^{\mu\nu i} G_{\mu\nu i} + \frac{\alpha_s}{4\pi\Lambda} \sin\theta_{gluon} a G^{\mu\nu i} \tilde{G}_{\mu\nu i} \\ &- \mu a^2 \bar{\phi}\phi.\end{aligned} \tag{98}$$

4 Simulation method

As seen in the theory section, it is possible to calculate cross section and width for some scattering processes analytically. Unfortunately in most cases this is not an option which motivates the aid of computer simulations. To fully explore different scattering processes involving the imagined particle a introduced earlier, such simulations is performed. These following section will introduce the computer programs used, explain the implementation of the Lagrangian, how to run simulations and the actual simulations.

4.1 Simulation tools

To do the calculations of cross section and width by hand is hard work, and most of the times impossible. This is why we used computer programs to explore our model. The simulation chain goes through three main programs. First the Feynman rules for the theory is calculated from the Lagrangian using a Mathematica package called FeynRules. The rules are then imported in to the simulation program MadGraph that set up Monte Carlo simulations for the desired process. A more detailed description of these tools follows.

4.2 FeynRules

FeynRules is a package for the Computer Algebraic System (CAS) suit Mathematica, capable of calculate the Feynman rules for a quantum field theory described by a Lagrangian. This is done by setting up the Lagrangian and its quantum fields in a file that then can be loaded in Mathematica. Mathematica can then use its algebraic kernels to derive the vertex coupling constants. It can also generate a folder containing python code with the derived Feynman rules, which can later be used as a model in MadGraph.

4.3 MadGraph

The main simulation program is MadGraph 5. MadGraph 5 is best described by the MadGraph wiki:n "MadGraph5_aMC@NLO is a framework that aims at providing all the elements necessary for SM and BSM phenomenology, such as the computations of cross sections, the generation of hard events and their matching with event generators, and the use of a variety of tools relevant to event manipulation and analysis." ("SM" in the quote stands for the "Standard Model" and "BSM" stands for "Beyond the Standard Model".)[15] MadGraph can load different particle models created with tools like FeynRules. These models could be used to set up simulations of scattering processes commonly found with in high energy particle colliders like the LHC. By interfacing to programs like MadAnalysis it provides a huge set of tool for statistical analysis of the resulting data. The program uses a Monte Carlo simulation to explore the scattering processes. Monte Carlo is a type of numerical approximation method to find an probability distribution by repeated random tries.[16]

4.4 Simulating the particle a

The particle described in section 3, was simulated using the programs introduced in the previous section. To do this the Lagrangian had to be implemented in FeynRules from which a model folder for MadGraph could be generated.

4.4.1 Implementing the model in FeynRules

An implementation of the standard model lagrangian for FeynRules was handed to the authors from their supervisor, complete with addition of the Higgs boson. See program in appendix A. To this an implementation of the interaction Lagrangian in equation (84) was added.

First the scalar field had to be added. This was done by the addition of the following code just below the declaration of the Higgs filed.

```
S[2] == {
        ClassName -> F,
        SelfConjugate -> True,
  Mass -> {Mf, Internal},
        Width -> {Wf, Internal},
  PropagatorLabel -> "f",
  PropagatorType -> Straight,
  PropagatorArrow -> Forward,
        TeXParticleName -> "\\Psi",
        TeXClassName -> "\\Psi",
        FullName -> "f" }
```

The reason that the particle is not given the name "a" is that this name is already take by the photon.

The constants "Mf" and "Wf" corresponds to the mass and width of the particle, and is declared in the section for internal parameters.

```
Mf == {
  ParameterType -> Internal,
  Value -> Sqrt[muF v v]
        },

Wf == {
  ParameterType -> Internal,
    Value -> 1.4334 LBD^(-2)
        },
```

The interaction Lagrangian was created with the code

```
Lp := lbd Cos[thetaF] F FS[A, mu, nu] FS[A, mu, nu] + 1/2 lbd Sin[
    thetaF] F FS[A,mu,nu] Eps[mu,nu,eta,xi] FS[A,eta,xi] + lbd2 Cos[
    thetaG] F FS[G, mu, nu, b] FS[G, mu, nu, b] + 1/2 lbd2 Sin[thetaG]
    F FS[G, mu, nu, b] Eps[mu, nu, xi, eta] FS[G, xi, eta, b] - muF (F
    F) (Phibar.Phi);
```

and was then added to the Standard Model Lagrangian LSM.

The constants "lbd", "lbd2", "muF", "thetaF", "thetaG" and "LBD" where defined with the following lines of code in the section for External parameters.

```
   lbd == {
        ParameterType -> Internal,
        Value -> \[Alpha]EW/(4 Pi LBD),
        InteractionOrder -> {QED, 1},
        Description -> "Photon Interaction Parameter" },

   lbd2 == {
        ParameterType -> Internal,
        Value -> \[Alpha]S/(4 Pi LBD),
        InteractionOrder -> {QED, 1},
        Description -> "Gluon Interaction Parameter" },

    muF == {
        ParameterType -> External,
```

```
        Value -> 0.0106779,
        InteractionOrder -> {QED, 2},
        TeX -> \[Mu],
        Description -> "Coefficient of the quadratic piece of the F potential"},

    thetaF == {
        ParameterType -> External,
        Value -> (0),
        InteractionOrder -> {QED, 1},
        Description -> "Photon Interaction Parameter" },

   thetaG == {
        ParameterType -> External,
        Value -> (0),
        InteractionOrder -> {QED, 1},
        Description -> "Gluon Interaction Parameter" },

    LBD == {
        ParameterType -> External,
        Value -> (5000),
        InteractionOrder -> {QED, 1},
        Description -> "New Physics constant" }
```

Finally the ".fr" file was loaded in FeynRules, from which the Feynman rules could be derived. By using the command

```
WriteUFO[LSM]
```

in the Mathematica console, an UFO folder containing python scripts with all the coupling constants was generated which is the MadGraph model.

4.4.2 Calculating cross section and width with MadGraph

The UFO folder generated by FeynRules was moved to the model folder in the MadGraph folder. The model could then be loaded by using the command

```
import model modelname
```

where "modelname" is replaced with the folder name of the UFO folder. The screen output of this command contains amongst other thing a list of all the particles names with in the model of choice. Now it is possible for MadGraph to calculate both cross section and width for a scattering process. For example if two protons collide and generate one Higgs boson which then decay through the boson a to four photons the commands to give to MadGraph is (remember that the boson a is called "F" with in this model since the name "a" is preserved for the photon).

```
generate p p > h , ( h > F F , F > a a)
output garfield
```

The last row generate a simulation program in Fortran for the particular process named "garfield".[4] To run the simulation the command "launch garfield" is used. Before the simulation is started MadGraph asks what simulation tools to use, for instance the program PYTHIA may be chosen. Then it let changes to parameters to be done. When the simulation is running it is possible to follow its progress by opening the "html" files with in the "garfield" folder. Afterwards the simulation release a bunch of data file that can be loaded in to MadAnalysis 5[5] for deeper statistical analysis, if MadAnalysis 4 is installed with MadGraph some plots generate automatically.

4.4.3 Scattering processes of interest

There are two diagrams that have been of interest in this thesis. The first describe the decay of one Higgs boson to four photons through a and is shown in figure 6. Of course to get the cross section for this event to happen in LHC, production cross section of one Higgs from two protons must be taken in to account. This process is of interest since it can help determine limits for the mass of the particle a. It can also shine light on the question of if a is a scalar or a pseudoscalar. The second

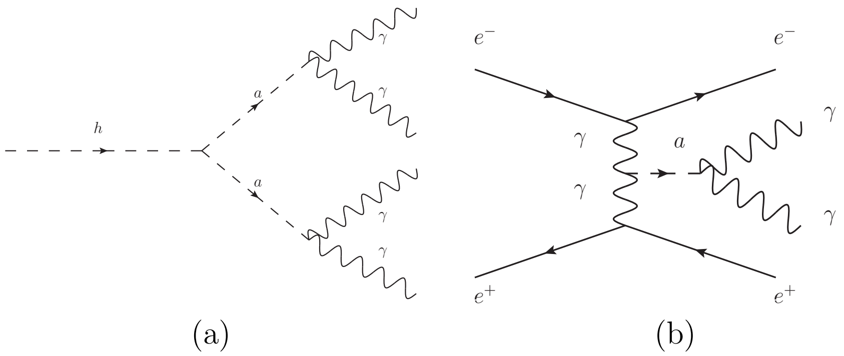

Figure 6: These two Feynman diagrams describes the processes of interest. Diagram (a) describes the total Higgs decay to four photons through the imagined particle a. This is one of the processes of interest for the simulations and has connections to possible decays in LHC. Since cross section is of interest in the case of particle accelerators, in most simulations the production of Higgs from to protons is added to the diagram. Diagram (b) describes a scattering process of an electron and a positron with the emission of two photons through one a. The interesting properties is that only the photon interaction term in the Lagrangian gives a contribution to this process cross section.

diagram of interest describes how an electron and positron interact and out comes one electron one positron and two photons, like in figure 6. This process could have occurred in LEP, but by assuming that no detection with reasonable certainty was done this could give an upper limit for λ_{photon}.

[4] This lasagna eating cat has of course nothing to do with protons and Higgs bosons but it is a good name for an example. In reality the name should be chosen so it is clear what process is simulated and what model is loaded.

[5] This program was newer used due to dependency problems under installation, however MadAnalysis 4 was used instead.

5 Simulation results

This section includes the results of the different simulation. The first simulation if the boson a is a scalar, a pseudo scalar or a combination of both. There after an upper limit is determined for λ_{photon} using condition from LEP. Last a lower limit for λ_{photon} is found by using the prompt decay limit.

5.1 The dependance of θ_{photon} for $\sigma(pp \to h \to aa \to \gamma\gamma\gamma\gamma)$

In our model the parameter θ changes the proportions in polar coordinates of aFF and $aF\tilde{F}$ from equation (84)

$$\begin{aligned}\mathcal{L}_{int} =&\lambda_{photon} \cos\theta_{photon} aF^{\mu\nu}F_{\mu\nu} + \lambda_{photon} \sin\theta_{photon} aF^{\mu\nu}\tilde{F}_{\mu\nu} \\ =&\lambda_{photon} \cos\theta_{photon} aF^{\mu\nu}F_{\mu\nu} + \lambda_{photon} \sin\theta_{photon} aF_{\mu\nu}\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}\end{aligned} \tag{99}$$

where θ_{photon} sets the proportions of scalar and pseudo scalar part of a's interaction Lagranigan with the photon. The most common theories about extra scalar particles, the coupling to the photon is usually either $aF^{\mu\nu}F_{\mu\nu}$ or $aF^{\mu\nu}\tilde{F}_{\mu\nu}$ to conserve charge parity. Nothing in QCD theory restrains charge parity (CP) from breaking, but experimentally CP is almost conserved symmetry. This simulation aimed to check if cross section was dependent on θ_{photon} in the process $pp \to h \to aa \to \gamma\gamma\gamma\gamma$ and as the result is presented in figure 7 where cross section is clearly not dependant on this parameter θ_{photon}. The simulation in MadGraph goes not from $0 \to 2\pi$ but instead $0 \to \frac{\pi}{2}$, because the the change in sign of $\sin\theta_{photon}$ does not matter. And lastly it is worth nothing that this model is only interesting if either $aF^{\mu\nu}F_{\mu\nu}$ or $aF^{\mu\nu}\tilde{F}_{\mu\nu}$ is very dominant, since no large CP breaking is observed in nature.

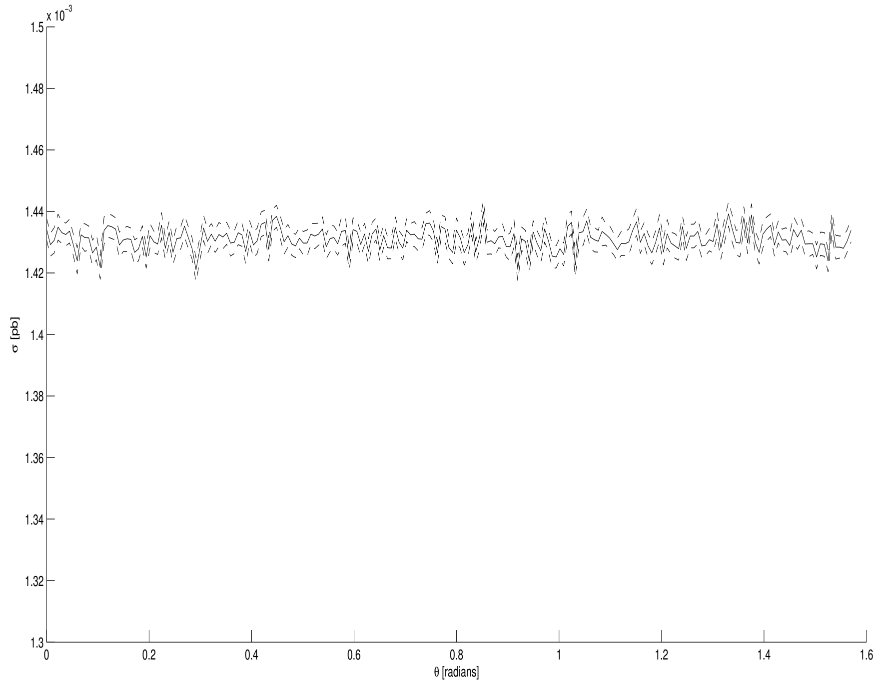

Figure 7: A plot how the cross section $\sigma(pp \to h \to aa \to \gamma\gamma\gamma\gamma)$ depends on changing the proportions between $aF^{\mu\nu}F_{\mu\nu}$ and $aF^{\mu\nu}\tilde{F}_{\mu\nu}$,through varying θ_{photon} from 0 to $\frac{\pi}{2}$. The cross section seems independant of θ_{photon}.

5.2 The scalar boson a at the LEP experiment

Our hypothetical extra particle a that couples to Higgs, photons and gluons, could be visible in experiments prior to LHC if its mass is low enough. Particle a could hypothetically been detected at LEP (large electron collider) in the $e^+ + e^- \to e^+ + e^- + \gamma + \gamma$ channel if a's branching ratio for the process in figure 8 is big enough.

Even thought it is hard to find an analytical expression for the cross section in this case, it is possible to show that it must be proportional to λ^2_{photon}. Looking at the Feynman diagram in figure 8, it can be seen that their is two vertices connecting to the particle a. These two vertices results in a factor λ_f each in the scattering amplitude. The propagator for the boson a is the Klein-Gordon propagator in equation 71. There is however a problem with that propagator, and that is there are poles on the real axis. To handle this problem a term $i\Gamma_a m_a$ is added to the denominator, where Γ_a is the total width of a. This gives

$$\mathcal{M} \propto \frac{\lambda^2_{photon}}{(k_1 + k_2)^2 - m_a^2 + i\Gamma_a m_a} \tag{100}$$

where k_1 and k_2 are the momentum of the two outgoing photons. In the cross section formula (31) the amplitude is squared, therefore

$$|\mathcal{M}|^2 \propto \frac{\lambda^4_{photon}}{((k_1 + k_2)^2 - m_a^2)^2 + \Gamma_a^2 m_a^2} \tag{101}$$

Since Γ_a has to be small, the narrow width approximation can be applied. This means that the propagator can be approximated with a Dirac function.[17] The amplitude is then proportional to

$$|\mathcal{M}|^2 \propto \frac{\pi \lambda^4_{photon}}{\Gamma_a m_a} \delta((k_1 - k_2)^2 - m_a^2). \tag{102}$$

From equation (95) and (97), it is possible to see that $\Gamma_a \propto \lambda^2_{photon}$ and therefore

$$|\mathcal{M}|^2 \propto \lambda^2_{photon}. \tag{103}$$

Since this is not dependent of phase space the cross section goes like

$$\sigma \propto \lambda^2_{photon}. \tag{104}$$

The coupling constant λ_{photon} have already an upper limit set by $\lambda_{photon} = \frac{\alpha}{4\pi\Lambda}$ where $\Lambda \gg m_h = 125.5$GeV. A simulation with $\Lambda = 1000$GeV gave a branching ratio lower than $10^{-10}\%$, which makes it impossible to detect a.

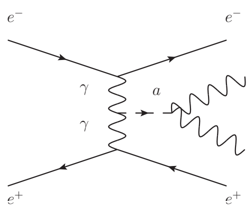

Figure 8: Feynmandiagram of electron positron interation, that is a important product of our model.

5.3 Decay Rate from the boson a

It is important that the New Physics constant Λ is at TeV scale. It can't be much smaller because then it would interfere with the scale of the Standard Model, and if it's too large the decay rates will be too small. To be observed at a modern detector is important that the decay is prompt, which means that it happens within 0.1mm from the creation of a [18]. Since

$$l = c\tau = \frac{c\hbar}{\Gamma}, \tag{105}$$

this is equal to that $\Gamma > 2 \cdot 10^{-12}$GeV.

To find out how the decay rates depends on Λ and to find an upper limit for it, the processes $a \to \gamma\gamma$ and $a \to gg$ has been generated with varying Λ. This is plotted in figure 9. Due to a very low branching ratio to photons, there is a huge difference in scale on the two cross sections. Because of that logarithmic scale has been used to plot this. This simulation gives us a lower limit for λ_{photon} at $3.6 \cdot 10^{-8}\text{GeV}^{-1}$. In accordance with what is stated about the coupling constants in section 3.2, this corresponds to an upper limit for Λ at around 16TeV . Otherwise the decay rate will be too small for the decay to be observed.

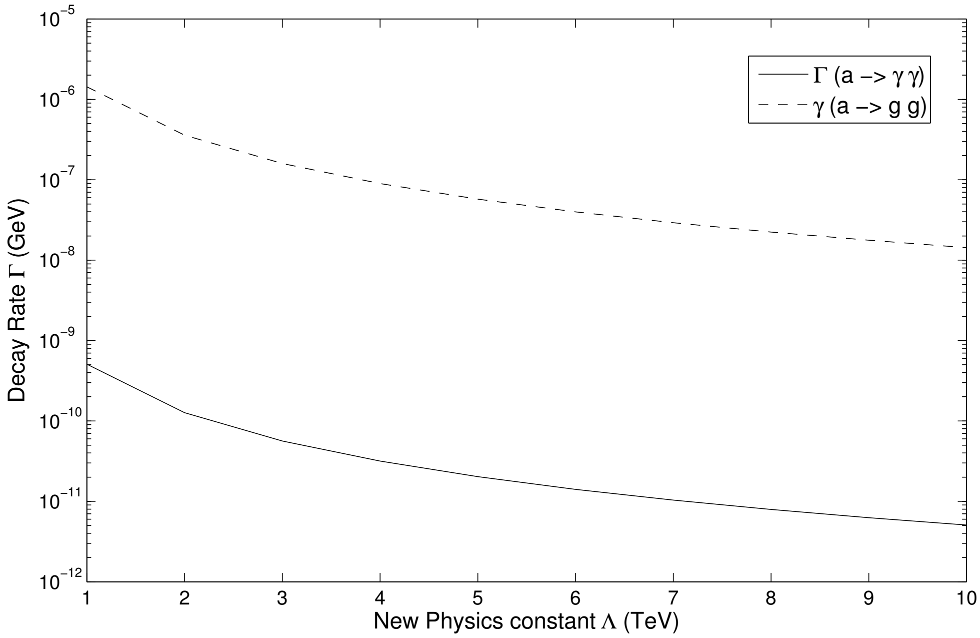

Figure 9: The decay ratios dependant of the New Physics constant Λ, logarithmic scale is used to be able to see both curves. The higher decay rate is the gluon decay, this is because of that $\alpha_s > \alpha$. For the decays to be measured in a detector it is important that the decay rates are above $2 \cdot 10^{-12}$GeV, which means that we need $\Lambda < 16$TeV.

6 Discussion

This section discusses some of the result from both the mathematical derivations and the simulations. It starts with an analysis of the decay rate from the Higgs particle and the estimate of the corresponding coupling constant μ. It follows with an estimate of the value and dependance of the branching ratio, both mathematically derived and using data from the simulations. It ends with a concluding part on the limits of our coupling constants.

6.1 Analysis of the Decay Rate from Higgs to two a

Experiments at the LHC have found that the total width of the Higgs Boson, Γ_T, is below 17.4MeV [19]. The theoretical total width Γ_T of the Higgs in the current Standard Model is around 4.1MeV, which gives us a range of about 13MeV for the $h \to aa$ decay. However 17.4MeV is a high estimate and it is unsure if our particle would be the only Higgs decay product not yet found.

We start by looking closer at the equation (92). The mass of Higgs m_h is experimentally measured to 125.5GeV and its vacuum expectation value v is set to 246GeV. Since both of these are known constants we find that the decay rate $\Gamma(h \to aa)$ only depends on the coupling constant μ. By varying μ it is possible to find a decay rate, which does not have too much of an impact to the current theoretical total width. Simultaneously we need too take account that the mass of our particle does not become to high. Due to conservation of energy it is needed that our particle has a rest mass $m_a < m_h/2$.

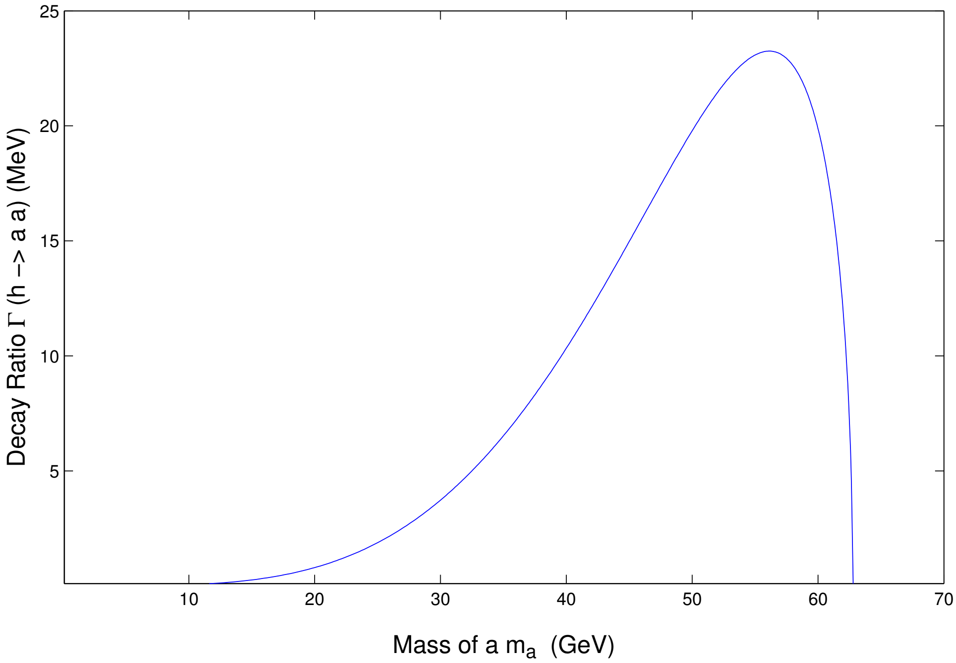

Figure 10: A plot of how the decay ratio $\Gamma(h \to aa)$ from equation (92) varies in connection with the mass of a $m_a = \sqrt{\mu}v$. There's an interestingly steep slope at approximately $m_h/2$, however it is improbable for that since the range of the slope is very small, and there is no dynamic reason for our mass to be in this area. Instead we have chosen a value of μ at the more gradual slope.

In figure 10 we can see how the decay ratio $\Gamma(h \to aa)$ depends on the mass $m_a = \sqrt{\mu}v$. Due to fact that there is an especially steep slope at $m_a \approx m_h/2$, it may seem interesting to find a mass close to that. However that is improbable since the range of the slope is very small. The steepness is merely a coincidence from the square root factor, and there is no dynamic reason for our mass to be in this area. Instead a mass at the left part of figure 10 has been estimated. The prompt decay limit from section 5.3 gives us a lower limit for μ at $3.228 \cdot 10^{-7}$ and m_a at 139MeV. The upper limit is given by our limited range at 13MeV, for a μ at 0.03 and m_a at 42.6GeV.

The μ used in most simulations has been chosen to 0.0106779. This corresponds to $m_a = 25.4$GeV and a decay rate of 2MeV which is quite large but within our range. This seems like an acceptable value for m_a, for instance in [20] a mass between 10GeV and 50GeV is used.

6.2 The Branching Ratio and dependence of New Physics constant Λ

For instance in [21] it is suggested that $Br(a \to \gamma\gamma)$ is very low. This is something that is also implied by looking at the decay rates from figure 9. Figure 9 also suggests that the branching ratio may be independent of the New Physics Constant Λ. This section gives both an analytically approach and looks at the simulations in figure 9, to investigate the branching ratios of the scalar boson a's decays.

6.2.1 Mathematical analysis

Using the modified Lagrangian from equation (98) might seem to induce a Λ dependence, however this might not be the case. For example the process $\Gamma(h \to 2a \to 4\gamma)$ could be written as

$$\Gamma(h \to \gamma\gamma\gamma\gamma) = \Gamma(h \to aa) BR(a \to 2\gamma)^2, \tag{106}$$

where the branching ratio is equal to

$$BR(a \to 2\gamma) = \frac{\Gamma(a \to 2\gamma)}{\Gamma_{tot}} = \frac{\Gamma(a \to 2\gamma)}{\Gamma(a \to 2\gamma) + \Gamma(a \to 2g)}. \tag{107}$$

The gluon field tensor $G^{\alpha\beta i}$ and the electromagnetic field $F^{\alpha\beta}$ only differs by a loop coupling factor $ig_s[\mathcal{A}_\alpha, \mathcal{A}_\beta]$ [5], which is neglectable. Other than that, while there are eight different final states for the gluon decay, there is only one type of photon. If we take the final states in account, we will get that

$$BR(a \to 2\gamma) = \frac{\lambda_{photon}^2}{\lambda_{photon}^2 + 8\lambda_{gluon}^2}. \tag{108}$$

If we now use $\lambda_f = \frac{\alpha}{4\pi\Lambda}$ och $\lambda_f = \frac{\alpha_s}{4\pi\Lambda}$ from section 3.2, we get that

$$BR(a \to 2\gamma) = \frac{\alpha^2}{\alpha^2 + 8\alpha_s^2}. \tag{109}$$

Note that here $\theta_{photon} = \theta_{gluon} = 0$ so that the field is scalar. Using the same numerical values as FeynRules, $\alpha = \frac{1}{127.9}$ and $\alpha_s = 0.1148$, leads to that

$$BR(a \to 2\gamma) = 5.52 \cdot 10^{-4}, \tag{110}$$

whereas

$$BR(a \to 2g) = \frac{8\alpha_s^2}{\alpha^2 + 8\alpha_s^2} = \frac{1}{1 + (\frac{\alpha}{\alpha_s})^2} \approx 1. \tag{111}$$

This shows not only that the branching ratio for photons is really small. It also shows that the branching ratio is independent of Λ. Because of equation (106) the decay rate $\Gamma(h \to 2a \to 4\gamma)$ should also be independent of Λ.

6.2.2 Comparations with Simulations

It would be of interest to compare these analytically produced results with results from simulations. Using the data from figure 9 we can plot figure 11. This data confirms that the branching ratio is indeed independant of Λ and also gives us numerical values to compare with the purely analytical ones. The branching ratios from the simulations are $BR(a \to \gamma\gamma) = 3.64 \cdot 10^{-4}$ and $BR(a \to gg) = 0.99964$.

These differs slightly from our theory. This is because the α_s used earlier is the one calculated for the Z boson and should be normalised to our boson a. Since gluons are hard to detect, due to the many gluon-consisting jets that emerges in colliders, the high branching ratio to gluons supports that the scalar boson a will be hard to find.

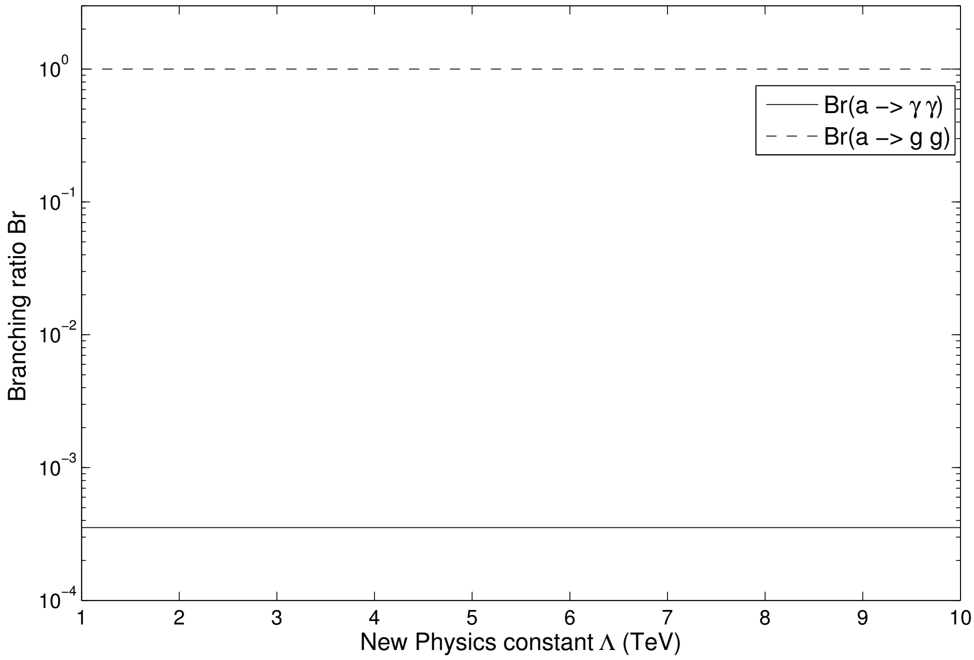

Figure 11: The decay ratios dependant of the New Physics constant Λ, logarithmic scale is used to be able to see both curves. It is noted that $BR(a \to \gamma\gamma) = 3.54 \cdot 10^{-3}$ and $BR(a \to gg) = 0.99964$.

6.3 Conclusion

The purpose of this thesis was to add a scalar boson to the Standard Model Lagrangian and find limits to its couplings to the Higgs boson, the photon and the gluon. The Higgs coupling μ was limited to an interval between $3.228 \cdot 10^{-7}$ and 0.003 otherwise the contribution to the total Higgs decayrate would be too large. The corresponding mass for a is from this limited to an interval between 139MeV and 42.6GeV. The coupling constants for the photon and the gluon was found to be related through the New Physics constant as in

$$\lambda_{photon} = \frac{\alpha}{4\pi\Lambda} \tag{112}$$

$$\lambda_{gluon} = \frac{\alpha_s}{4\pi\Lambda}. \tag{113}$$

This means that the branching ratios for the $a \to \gamma\gamma$ and $a \to gg$ is not dependent of the New Physics constant. The branching ratio of the decay to two gluons is much larger than the branching ratio for decay to two photons.

The parameter Λ has been shown to be in the scale of New Physics, which is $\Lambda \gg m_t$, where m_t is the mass of the top quark, the heaviest particle in the Standard Model. It has been verified by simulation, that the particle could not been observed within earlier experiments such as the LEP because of the scale of Λ. If $a \to \gamma\gamma$ is ever going to be observed Λ has to be lower that 16000GeV.

The question, if the decay rate is affected by whether a is a scalar or a pseudo scalar boson, resulted in that there is no difference. Even when parity symmetry is broken, there where no measurable changes whatsoever.

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A FeynRules implementation of the standard model

The following code is the implementation of the standard model provided by the supervisor. It contains an full implementation of the current understanding of the standard model including the Higgs field.

```
(*****************************************************************************)
(******                                                                 *******)
(*****************************************************************************)

M$ModelName = "Standard Model plus";

M$Information = {Authors -> {" "},
                 Version -> "1.0",
                 Date -> "2013-05-08",
                 Institutions -> {" "},
                 Emails -> {" "}};

(******* Index definitions ********)

IndexRange[ Index[Generation] ] = Range[3]

IndexRange[ Index[Colour] ] = NoUnfold[Range[3]]

IndexRange[ Index[Gluon] ] = NoUnfold[Range[8]]

IndexRange[ Index[SU2W] ] = Unfold[Range[3]]


IndexStyle[Colour, i]

IndexStyle[Generation, f]

IndexStyle[Gluon ,a]

IndexStyle[SU2W ,k]


(******* Gauge parameters (for FeynArts) ********)

GaugeXi[ V[1] ] = GaugeXi[A];
GaugeXi[ V[2] ] = GaugeXi[Z];
GaugeXi[ V[3] ] = GaugeXi[W];
GaugeXi[ V[4] ] = GaugeXi[G];
GaugeXi[ S[1] ] = 1;
GaugeXi[ S[2] ] = GaugeXi[Z];
GaugeXi[ S[3] ] = GaugeXi[W];

(***** Setting for interaction order (as e.g. used by MadGraph 5) ******)

M$InteractionOrderHierarchy = {
      {QCD, 1},
      {QED, 2}
     };

(*************** The loop coefficient ******)

sert[x_] := 1+ 7/30 x + 2/21 x^2 + 26/525 x^3;
```

```
serw[xw_, xt_] := 1 + xw * 66/235 +xw^2 * 228/1645 + xw^3 * 696/8225 +
              xw^4 * 5248/90475 +xw^5 * 1280/29939+ xw^6 * 54528/1646645-
              xt * 56/705 - xt^2 * 32/987;

serp[x_] := 1 + x/3 + x^2 * 8/45 + x^3 * 4/35;


(*************** Parameters ************)

M$Parameters = {

  (* External parameters *)

  \[Alpha]EWM1== {
        ParameterType -> External,
        BlockName -> SMINPUTS,
        ParameterName -> aEWM1,
        InteractionOrder -> {QED, -2},
        Value -> 127.9,
        Description -> "Inverse of the electroweak coupling constant"},

  Gf == {
        ParameterType -> External,
        BlockName -> SMINPUTS,
        TeX -> Subscript[G, f],
        InteractionOrder -> {QED, 2},
        Value -> 1.16637 * 10^(-5),
        Description -> "Fermi constant"},

  \[Alpha]S == {
        ParameterType -> External,
        BlockName -> SMINPUTS,
        TeX -> Subscript[\[Alpha], s],
        ParameterName -> aS,
        InteractionOrder -> {QCD, 2},
        Value -> 0.1184,
        Description -> "Strong coupling constant at the Z pole."},

  ymdo == {
        ParameterType -> External,
        BlockName -> YUKAWA,
        Value -> 5.04*10^(-3),
        OrderBlock -> {1},
      Description -> "Down Yukawa mass"},

  ymup == {
        ParameterType -> External,
        BlockName -> YUKAWA,
        Value -> 2.55*10^(-3),
        OrderBlock -> {2},
        Description -> "Up Yukawa mass"},

  yms == {
        ParameterType -> External,
        BlockName -> YUKAWA,
```

```
          Value -> 0.101,
        OrderBlock -> {3},
        Description -> "Strange Yukawa mass"},

  ymc == {
          ParameterType -> External,
          BlockName -> YUKAWA,
          Value -> 1.27,
          OrderBlock -> {4},
          Description -> "Charm Yukawa mass"},

 ymb == {
          ParameterType -> External,
          BlockName -> YUKAWA,
          Value -> 4.7,
          OrderBlock -> {5},
          Description -> "Bottom Yukawa mass"},

  ymt == {
          ParameterType -> External,
          BlockName -> YUKAWA,
          Value -> 172.0,
          OrderBlock -> {6},
          Description -> "Top Yukawa mass"},

  yme == {
          ParameterType -> External,
          BlockName -> YUKAWA,
          Value ->  5.11*10^(-4),
          OrderBlock -> {11},
          Description -> "Electron Yukawa mass"},

  ymm == {
          ParameterType -> External,
          BlockName -> YUKAWA,
          Value -> 0.10566,
          OrderBlock -> {13},
          Description -> "Muon Yukawa mass"},

  ymtau == {
          ParameterType -> External,
          BlockName -> YUKAWA,
          Value -> 1.777,
          OrderBlock -> {15},
          Description -> "Tau Yukawa mass"},

   cabi == {
          TeX -> Subscript[\[Theta], c],
          ParameterType -> External,
          BlockName -> CKMBLOCK,
          Value -> 0.227736,
          Description -> "Cabibbo angle"},

   (* Internal Parameters *)

  \[Alpha]EW == {
```

```
            ParameterType -> Internal,
            Value -> 1/\[Alpha]EWM1,
            TeX -> Subscript[\[Alpha], EW],
            ParameterName -> aEW,
            InteractionOrder -> {QED, 2},
            Description -> "Electroweak coupling contant"},

  MW == {
            ParameterType -> Internal,
            Value -> Sqrt[MZ^2/2+Sqrt[MZ^4/4-Pi/Sqrt[2]*\[Alpha]EW/Gf*MZ
    ^2]],
            TeX  -> Subscript[M, W],
            Description -> "W mass"},

   sw2 == {
            ParameterType -> Internal,
            Value -> 1-(MW/MZ)^2,
            Description -> "Squared Sin of the Weinberg angle"},

    ee == {
            TeX -> e,
            ParameterType -> Internal,
            Value -> Sqrt[4 Pi \[Alpha]EW],
            InteractionOrder -> {QED, 1},
            Description -> "Electric coupling constant"},

    cw == {
            TeX -> Subscript[c, w],
            ParameterType -> Internal,
            Value -> Sqrt[1 - sw2],
            Description -> "Cos of the Weinberg angle"},

    sw == {
            TeX -> Subscript[s, w],
            ParameterType -> Internal,
            Value -> Sqrt[sw2],
            Description -> "Sin of the Weinberg angle"},

    gw == {
            TeX -> Subscript[g, w],
            ParameterType -> Internal,
            Value -> ee / sw,
            InteractionOrder -> {QED, 1},
            Description -> "Weak coupling constant"},

    g1 == {
            TeX -> Subscript[g, 1],
            ParameterType -> Internal,
            Value -> ee / cw,
            InteractionOrder -> {QED, 1},
            Description -> "U(1)Y coupling constant"},

    gs == {
            TeX -> Subscript[g, s],
            ParameterType -> Internal,
            Value -> Sqrt[4 Pi \[Alpha]S],
```

```
        InteractionOrder -> {QCD, 1},
        ParameterName -> G,
        Description -> "Strong coupling constant"},

    v == {
        ParameterType -> Internal,
        Value -> 2*MW*sw/ee,
        InteractionOrder -> {QED, -1},
        Description -> "Higgs VEV"},

    \[Lambda] == {
        ParameterType -> Internal,
        Value -> MH^2/(2*v^2),
        InteractionOrder -> {QED, 2},
        ParameterName -> lam,
        Description -> "Higgs quartic coupling"},

    muH == {
        ParameterType -> Internal,
        Value -> Sqrt[v^2 \[Lambda]],
        TeX -> \[Mu],
        Description -> "Coefficient of the quadratic piece of the Higgs
  potential"},

    yl == {
        TeX -> Superscript[y, l],
        Indices -> {Index[Generation]},
        AllowSummation -> True,
        ParameterType -> Internal,
        Value -> {yl[1] -> Sqrt[2] yme / v, yl[2] -> Sqrt[2] ymm / v,
 yl[3] -> Sqrt[2] ymtau / v},
        ParameterName -> {yl[1] -> ye, yl[2] -> ym, yl[3] -> ytau},
        InteractionOrder -> {QED, 1},
        ComplexParameter -> False,
        Description -> "Lepton Yukawa coupling"},

    yu == {
        TeX -> Superscript[y, u],
        Indices -> {Index[Generation]},
        AllowSummation -> True,
        ParameterType -> Internal,
        Value -> {yu[1] -> Sqrt[2] ymup / v, yu[2] -> Sqrt[2] ymc / v,
 yu[3] -> Sqrt[2] ymt / v},
        ParameterName -> {yu[1] -> yup, yu[2] -> yc, yu[3] -> yt},
        InteractionOrder -> {QED, 1},
        ComplexParameter -> False,
        Description -> "U-quark Yukawa coupling"},

    yd == {
        TeX -> Superscript[y, d],
        Indices -> {Index[Generation]},
        AllowSummation -> True,
        ParameterType -> Internal,
        Value -> {yd[1] -> Sqrt[2] ymdo / v, yd[2] -> Sqrt[2] yms / v,
 yd[3] -> Sqrt[2] ymb / v},
        ParameterName -> {yd[1] -> ydo, yd[2] -> ys, yd[3] -> yb},
```

```
        InteractionOrder -> {QED, 1},
        ComplexParameter -> False,
        Description -> "D-quark Yukawa coupling"},

(* N. B. : only Cabibbo mixing! *)
  CKM == {
       Indices -> {Index[Generation], Index[Generation]},
       TensorClass -> CKM,
       Unitary -> True,
       Value -> {CKM[1,1] -> Cos[cabi],
                 CKM[1,2] -> Sin[cabi],
                 CKM[1,3] -> 0,
                 CKM[2,1] -> -Sin[cabi],
                 CKM[2,2] -> Cos[cabi],
                 CKM[2,3] -> 0,
                 CKM[3,1] -> 0,
                 CKM[3,2] -> 0,
                 CKM[3,3] -> 1},
       Description -> "CKM-Matrix"},

AH == {TeX -> Subscript[A, H],
       ParameterType -> Internal,
       InteractionOrder -> {HIW, 1},
       Value -> ee^2/4/Pi/(Pi*v)*(47/18)*serw[(MH/2/MW)^2, (MH/2/MT)
    ^2],
       Description -> "One loop coupling HAA"},

GH == {TeX -> Subscript[G, H],
       ParameterType -> Internal,
       InteractionOrder -> {HIG, 1},
       Value -> -gs^2/(4Pi(3Pi v)) sert[(MH/2/MT)^2],
       Description -> "One loop coupling HGG"}
}

(************* Gauge Groups *****************)

M$GaugeGroups = {

  U1Y == {
        Abelian -> True,
        GaugeBoson -> B,
        Charge -> Y,
        CouplingConstant -> g1},

  SU2L == {
        Abelian -> False,
        GaugeBoson -> Wi,
        StructureConstant -> Eps,
        CouplingConstant -> gw},

  SU3C == {
        Abelian -> False,
        GaugeBoson -> G,
        StructureConstant -> f,
        SymmetricTensor -> dSUN,
        Representations -> {T, Colour},
```

```
        CouplingConstant -> gs}
}

(********* Particle Classes **********)

M$ClassesDescription = {

(********** Fermions ************)
  (* Leptons (neutrino): I_3 = +1/2, Q = 0 *)
  F[1] == {
        ClassName -> vl,
        ClassMembers -> {ve,vm,vt},
        FlavorIndex -> Generation,
  SelfConjugate -> False,
  Indices -> {Index[Generation]},
        Mass -> 0.00000000001,
        Width -> 0,
  QuantumNumbers -> {LeptonNumber -> 1},
  PropagatorLabel -> {"v", "ve", "vm", "vt"} ,
  PropagatorType -> S,
  PropagatorArrow -> Forward,
        PDG -> {12,14,16},
        FullName -> {"Electron-neutrino", "Mu-neutrino", "Tau-neutrino"
   } },

  (* Leptons (electron): I_3 = -1/2, Q = -1 *)
  F[2] == {
        ClassName -> l,
        ClassMembers -> {e, m, tt},
        FlavorIndex -> Generation,
  SelfConjugate -> False,
  Indices -> {Index[Generation]},
  Mass -> {Ml, {Me, 5.11 * 10^(-4)}, {MM, 0.10566}, {MTA, 1.777}},
        Width -> 0,
  QuantumNumbers -> {Q -> -1, LeptonNumber -> 1},
  PropagatorLabel -> {"l", "e", "m", "tt"},
  PropagatorType -> Straight,
        ParticleName -> {"e-", "m-", "tt-"},
        AntiParticleName -> {"e+", "m+", "tt+"},
  PropagatorArrow -> Forward,
        PDG -> {11, 13, 15},
        FullName -> {"Electron", "Muon", "Tau"} },

  (* Quarks (u): I_3 = +1/2, Q = +2/3 *)
  F[3] == {
        ClassMembers -> {u, c, t},
        ClassName -> uq,
        FlavorIndex -> Generation,
  SelfConjugate -> False,
  Indices -> {Index[Generation], Index[Colour]},
  Mass -> {Mu, {MU, 2.55*10^(-3)}, {MC, 1.42}, {MT, 172}},
        Width -> {0, 0, {WT, 1.50833649}},
  QuantumNumbers -> {Q -> 2/3},
  PropagatorLabel -> {"uq", "u", "c", "t"},
  PropagatorType -> Straight,
  PropagatorArrow -> Forward,
```

```
        PDG -> {2, 4, 6},
        FullName -> {"u-quark", "c-quark", "t-quark"}},

  (* Quarks (d): I_3 = -1/2, Q = -1/3 *)
  F[4] == {
        ClassMembers -> {d, s, b},
        ClassName -> dq,
        FlavorIndex -> Generation,
  SelfConjugate -> False,
  Indices -> {Index[Generation], Index[Colour]},
  Mass -> {Md, {MD, 5.04*10^(-3)}, {MS, 0.101}, {MB, 4.7}},
        Width -> 0,
  QuantumNumbers -> {Q -> -1/3},
  PropagatorLabel -> {"dq", "d", "s", "b"},
  PropagatorType -> Straight,
  PropagatorArrow -> Forward,
        PDG -> {1,3,5},
        FullName -> {"d-quark", "s-quark", "b-quark"} },

(*********** Gauge Bosons **************)
  (* Gauge bosons: Q = 0 *)
  V[1] == {
        ClassName -> A,
  SelfConjugate -> True,
  Indices -> {},
  Mass -> 0,
        Width -> 0,
  PropagatorLabel -> "a",
  PropagatorType -> W,
  PropagatorArrow -> None,
        PDG -> 22,
        FullName -> "Photon" },

  V[2] == {
        ClassName -> Z,
  SelfConjugate -> True,
  Indices -> {},
  Mass -> {MZ, 91.1876},
        Width -> {WZ, 2.4952},
  PropagatorLabel -> "Z",
  PropagatorType -> Sine,
  PropagatorArrow -> None,
        PDG -> 23,
        FullName -> "Z" },

  (* Gauge bosons: Q = -1 *)
  V[3] == {
        ClassName -> W,
  SelfConjugate -> False,
  Indices -> {},
  Mass -> {MW, Internal},
        Width -> {WW, 2.085},
  QuantumNumbers -> {Q -> 1},
  PropagatorLabel -> "W",
  PropagatorType -> Sine,
  PropagatorArrow -> Forward,
```

```
    ParticleName ->"W+",
    AntiParticleName ->"W-",
            PDG -> 24,
            FullName -> "W" },

  V[4] == {
            ClassName -> G,
    SelfConjugate -> True,
    Indices -> {Index[Gluon]},
    Mass -> 0,
            Width -> 0,
            PropagatorLabel -> G,
    PropagatorType -> C,
    PropagatorArrow -> None,
            PDG -> 21,
            FullName -> "G" },

  V[5] == {
            ClassName -> Wi,
            Unphysical -> True,
            Definitions -> {Wi[mu_, 1] -> (W[mu] + Wbar[mu])/Sqrt[2],
                            Wi[mu_, 2] -> (Wbar[mu] - W[mu])/Sqrt[2]/I,
                            Wi[mu_, 3] -> cw Z[mu] + sw A[mu]},
            SelfConjugate -> True,
            Indices -> {Index[SU2W]},
            FlavorIndex -> SU2W,
            Mass -> 0,
            PDG -> {1,2,3}},

  V[6] == {
            ClassName -> B,
    SelfConjugate -> True,
            Definitions -> {B[mu_] -> -sw Z[mu] + cw A[mu]},
    Indices -> {},
    Mass -> 0,
            Unphysical -> True},


(*********** Scalar Fields *********)
    (* physical Higgs: Q = 0 *)
    S[1] == {
            ClassName -> H,
    SelfConjugate -> True,
    Mass -> {MH, 125.5},
            Width -> {WH, 0.00414},
    PropagatorLabel -> "H",
    PropagatorType -> D,
    PropagatorArrow -> None,
            PDG -> 25,
            TeXParticleName -> "\\phi",
            TeXClassName -> "\\phi",
            FullName -> "H" }
}

(*
    ********************************************************************************
```

```
    *)

(* SM Lagrangian *)

(******************* Gauge F^2 Lagrangian terms
    ************************)
(*Sign convention from Lagrangian in between Eq. (A.9) and Eq. (A.10)
    of Peskin & Schroeder.*)

 LGauge = -1/4 (del[Wi[nu, i1], mu] - del[Wi[mu, i1], nu] + gw Eps[i1,
    i2, i3] Wi[mu, i2] Wi[nu, i3])*
           (del[Wi[nu, i1], mu] - del[Wi[mu, i1], nu] + gw Eps[i1, i4,
    i5] Wi[mu, i4] Wi[nu, i5]) -

  1/4 (del[B[nu], mu] - del[B[mu], nu])^2 -

  1/4 (del[G[nu, a1], mu] - del[G[mu, a1], nu] + gs f[a1, a2, a3] G[mu,
     a2] G[nu, a3])*
     (del[G[nu, a1], mu] - del[G[mu, a1], nu] + gs f[a1, a4, a5] G[mu,
    a4] G[nu, a5]);


(******************** Fermion Lagrangian terms***********************
    *)
(*Sign convention from Lagrangian in between Eq. (A.9) and Eq. (A.10)
    of Peskin & Schroeder.*)

 LFermions = Module[{Lkin, LQCD, LEWleft, LEWright},

    Lkin = I uqbar.Ga[mu].del[uq, mu] +
      I dqbar.Ga[mu].del[dq, mu] +
      I lbar.Ga[mu].del[l, mu] +
      I vlbar.Ga[mu].del[vl, mu];

    LQCD = gs (uqbar.Ga[mu].T[a].uq +
      dqbar.Ga[mu].T[a].dq)G[mu, a];

    LBright =
      -2ee/cw B[mu]/2 lbar.Ga[mu].ProjP.l +          (*Y_lR=-2*)
      4ee/3/cw B[mu]/2 uqbar.Ga[mu].ProjP.uq -       (*Y_uR=4/3*)
      2ee/3/cw B[mu]/2 dqbar.Ga[mu].ProjP.dq;        (*Y_dR=-2/3*)

    LBleft =
      -ee/cw B[mu]/2 vlbar.Ga[mu].ProjM.vl -         (*Y_LL=-1*)
      ee/cw B[mu]/2 lbar.Ga[mu].ProjM.l  +           (*Y_LL=-1*)
      ee/3/cw B[mu]/2 uqbar.Ga[mu].ProjM.uq +        (*Y_QL=1/3*)
      ee/3/cw B[mu]/2 dqbar.Ga[mu].ProjM.dq ;        (*Y_QL=1/3*)

    LWleft = ee/sw/2(
  vlbar.Ga[mu].ProjM.vl Wi[mu, 3] -                  (*sigma3 = ( 1   0
     )*)
      lbar.Ga[mu].ProjM.l Wi[mu, 3] +                (*         ( 0  -1
     )*)

      Sqrt[2] vlbar.Ga[mu].ProjM.l W[mu] +
      Sqrt[2] lbar.Ga[mu].ProjM.vl Wbar[mu]+
```

```
      uqbar.Ga[mu].ProjM.uq Wi[mu, 3] -                (*sigma3 = ( 1    0
    )*)
      dqbar.Ga[mu].ProjM.dq Wi[mu, 3] +                (*           ( 0   -1
    )*)

      Sqrt[2] uqbar.Ga[mu].ProjM.CKM.dq W[mu] +
      Sqrt[2] dqbar.Ga[mu].ProjM.HC[CKM].uq Wbar[mu]
      );

   Lkin + LQCD + LBright + LBleft + LWleft];

(******************* Higgs Lagrangian terms**************************
  *)

 Phi :=  {0, (v + H)/Sqrt[2]};
 Phibar := {0, (v + H)/Sqrt[2]};

 LHiggs := Block[{PMVec, WVec, Dc, Dcbar, Vphi},

    PMVec = Table[PauliSigma[i], {i, 3}];
    Wvec[mu_] := {Wi[mu, 1], Wi[mu, 2], Wi[mu, 3]};

  (*Y_phi=1*)
    Dc[f_, mu_] := I del[f, mu] + ee/cw B[mu]/2 f + ee/sw/2 (Wvec[mu].
   PMVec).f;
    Dcbar[f_, mu_] := -I del[f, mu] + ee/cw B[mu]/2 f + ee/sw/2 f.(Wvec
   [mu].PMVec);

    Vphi[Phi_, Phibar_] := -muH^2 Phibar.Phi + \[Lambda] (Phibar.Phi)
   ^2;

    (Dcbar[Phibar, mu]).Dc[Phi, mu] - Vphi[Phi, Phibar]];


(************** Yukawa Lagrangian*********************)

LYuk :=   Module[{s,r,n,m,i},
                 -
       yd[n]              dqbar[s,n,i].ProjP[s,r].dq[r,n,i] (v+H)/Sqrt
   [2] -
       yu[n]              uqbar[s,n,i].ProjP[s,r].uq[r,n,i] (v+H)/Sqrt
   [2] -
       yl[n]               lbar[s,n].ProjP[s,r].l[r,n]       (v+H)/Sqrt
   [2]
            ];

LYukawa := LYuk + HC[LYuk];


(************ One loop Higgs couplings ****************)

LCPeven := -1/4 GH FS[G, mu, nu, b] FS[G, mu, nu, b] H - 1/4 AH FS[A,
   mu, nu] FS[A, mu, nu] H;

```

```
(*********Total SM Lagrangian*******)
LSM := LGauge + LHiggs + LFermions + LYukawa + LCPeven;
```

SMplus.fr

B Swedish Summary

1 Introduktion

Kandidatarbets speciella utformning har varit ett annorlunda sätt att lära oss fysik. Förutom att fokusera på att lägga till en extra skalärboson i standardmodellen har vi på egen hand läst om partikelfysik. Utöver det har programvara använts för att göra simuleringar för att mer praktiskt förstå ekvationerna.

Kandidatrapporten fokus har varit att lägga till en ny skalär boson till standard modellen som kopplar mot higgs bosonen. För att kunna göra detta har vi beskrivit det viktigaste i den bakomliggande teorin, bland annat kvantfältsteori, feynmandiagram och standardmodellen. Förutom kopplingen med higgsbosonen har vi lagt till kopplingen med fotonen och glounen för att kunna studera den processen när higgsbosonen sönderfaller till fyra fotoner.

När vi väl lärt oss lägga in nya partiklar och kopplingar mot annat visade det sig vara ganska enkelt. Det vi jobbat med har varit att bestämma kopplingskonstanter för att processerna ska kunna upptäckas i LHC experimentet samt hålla oss inom sådana gränser så att partikeln inte redan borde vara upptäckt.

2 Teori

Den teori som detta arbete bygger på är kvantfältsteori. Detta är en förening av kvantmekanik och speciell relativitetsteori. Det som används för att beskriva en kvantfältsteori kallas Lagrangefunktion. Lagrangefunktionen har sitt ursprung i analytisk mekanik men har via vägintegralsformalismen av kvantmekaniken blivit centralt sättet att beskriva partikeldynamiken inom kvantfältsteori. Lagrangianfunktionen kan ses som skillnaden mellan kinetisk energi och potentiell energi $L = T - V$. Eftersom att det inte görs skillnad på tid och rum i kvantfältsteori så är det egentligen vanligare att använda Lagrangian densiteten så att $L = \int d^3\mathcal{L}$ där $\mathcal{L}$ är Lagrangiandensiteten. För enkelhets skull, när man jobbar i kvantfältsteori så brukar man kalla lagrangiandensiteten för lagrangianfunktion.

För att kunna undersöka partiklar har det varit nödvändigt att titta på begreppen bredd och tvärsnitt. Bredd beskriver hur sannolikt det är att en partikel sönderfaller i andra, medan tvärsnitt är ett mått på sannolikheten att två eller flera partiklar kolliderar. Bredden går att få fram med hjälp av Fermis gyllene regel då

$$\Gamma = \frac{S}{2m_1} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 - p_2 - p_3 \ldots - p_n) \prod_{j=2}^{n} 2\pi\delta(p_j^2 - m_j^2)\theta(p_j^0)\frac{d^4 p_j}{(2\pi)^4}, \quad (1)$$

där S är en produkt av statistisk faktorer $\frac{1}{j!}$ från varje grupp av identiska partiklar, j är antalet identiska partiklar i sluttillståndet, n är antalet partiklar i interaktionen och m_i och p_i är partiklarnas massa och fyrdimensionella rörelsemängd. i vårt fall så är S antingen 1 för två olika utgående partiklar eller $\frac{1}{2}$ för två identiska utgående partiklar.

Ytterligare får vi att i vilosystemet, för en partikel som sönderfaller till två partiklar, är sönderfallsbredden

$$\Gamma = \frac{S|\mathbf{p}|}{8\pi m_1^2}|\mathcal{M}|^2, \tag{2}$$

där $\mathbf{p}$ är en av de utgående partiklarnas tredimensionella röremängd.

En liknande gyllene regel finns för tvärsnittet och kan tecknas

$$\sigma = \frac{S}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2}} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 \ldots - p_n) \prod_{j=3}^{n} 2\pi\delta(p_j^2 - m_j^2)\theta(p_j^0)\frac{d^4 p_j}{(2\pi)^4}. \tag{3}$$

Utifrån detta kan vi uttrycka en tvärsnittsdel, vilket är användbart eftersom $\mathcal{M}$ är riktningsberoende och amplituden av den utgående partikelns rörelsemängd $\mathbf{p}$ inte är trivial. Denna kan tecknas

$$d\sigma = \frac{S|\mathcal{M}|^2}{64\pi^2(E_1 + E_2)^2}\frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}d\Omega \tag{4}$$

där $E_{1,2} = p^0_{1,2}$, $|\mathbf{p}_i$ och $|\mathbf{p}_f$ är amplituden av momentet för den ingående respektive den utgående partikeln, och $d\Omega = \sin(\theta)d\theta d\phi$.

Spridningsamplituden $\mathcal{M}$ kommer ur beräkningen av spridningsmatrisen S som kommer av

$$S = \langle f|\hat{U}(-\infty, \infty)|i\rangle$$

där $|i\rangle$ är systemets initialtillstånd, $\langle f|$ är systemets sluttillstånd. Detta kan beräknas på flera sätt men lättast är via de så kallade feynmanreglerna och feynmandiagram. Feynmandiagram är ett simpelt sätt att grafiskt representera en spridningsprocess, feynmanreglerna talar om hur en sådan figur kan transformeras till spridningsamplituden.

Den partikel som introduceras i detta kandidatarbete får sin massa på grund av Higgsmekanismen. Higgsmekanismen introducerades till Standard Modellen (SM) som ett sätt att få bort symmetribrott i Proca Lagrangianen. Higgsmekanismen ger massa till de massiva partiklarna i SM genom ett spontant symmetribrott. Det går att se det som att massiva partiklar bildar vågor i Higgsfältet likt vågorna som bildas då sten släpps i vatten.

3 Modell

Huvudsyftet för detta arbetet har varit att utveckla en skalärboson, kallad a. Denna partikeln förklarar och fungerar som en mellanhand för higgs sönderfall till fyra fotoner. På grund av likheterna mellan fotonen och gluonen är det också rimligt att en sådan partikel även ska sönderfalla till gluoner. För att skalärbosonen skulle interagera med higgs, gluonen och fotonen tecknades lagrangefunktionen

$$
\begin{aligned}
\mathcal{L}_{int} =& \lambda_{photon} \cos\theta_{photon} a F^{\mu\nu} F_{\mu\nu} + \lambda_{photon} \sin\theta_{photon} a F^{\mu\nu} \tilde{F}_{\mu\nu} \\
&+ \lambda_{gluon} \cos\theta_{gluon} a G^{\mu\nu i} G_{\mu\nu i} + \lambda_{gluon} \sin\theta_{gluon} a G^{\mu\nu i} \tilde{G}_{\mu\nu i} \\
&- \mu a^2 \bar{\phi}\phi,
\end{aligned} \tag{5}
$$

där μ är en enhetslös kopplingskonstant och λ_{photon} och λ_{gluon} är kopplingskonstanter med enheten $(GeV)^{-1}$. De två vinklarna θ_{photon} och θ_{gluon} avgör om fälten är skalärfält eller pseudoskalära fält, eller linjärkombinationer av dessa. Partikelns a massa kommer från higgskopplingen $-\mu a^2\bar{\phi}\phi$. Genom att utnyttja att higgsfältet $\phi = (h + v)$ kan man se att higgskopplingen

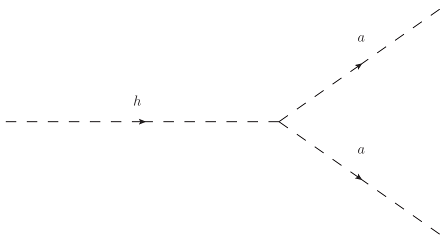
$$
\mathcal{L}_{a-h} = -\mu a^2\bar{\phi}\phi = -\mu a^2(h+v)^2 = -\mu a^2 h^2 - 2mua^2hv - \mu a^2 v^2. \tag{6}
$$

Den sista termen här påminner om en kinetisk mass term $\mathcal{L}_{kin} = -m_a^2 a^2$ där massan är $m_a = \sqrt{\mu}v$.

Denna interaktion mellan higgs och a kan ses i feynmandiagrammet i figur 1. Analytiskt kan bredden för denna process härledas via ekvation 1 till

$$
\Gamma(h \to aa) = \frac{\mu^2 v^2}{8\pi m_h}\sqrt{1 - 4\frac{m_a^2}{m_h^2}}, \tag{7}
$$

där m_h är higgsbosonens massa.



Figur 1: Första ordningens feynmandiagram för en higgs boson som sönderfaller till två a.

Eftersom λ_{photon} och λ_{gluon} har enheten $(GeV)^{-1}$ måste de vara inverst proportionella mot en nyfysikskonstant Λ. Man kan argumentera för att de skulle vara

$$
\lambda_{photon} = \frac{\alpha}{4\pi\Lambda}, \tag{8}
$$

respektive

$$
\lambda_{gluon} = \frac{\alpha_s}{4\pi\Lambda}. \tag{9}
$$

Här är α den elektromagnetiska kopplingkonstanten och α_s det starka fältets kopplingskonstant. Detta ger oss en lagrangefunktion som kan skrivas

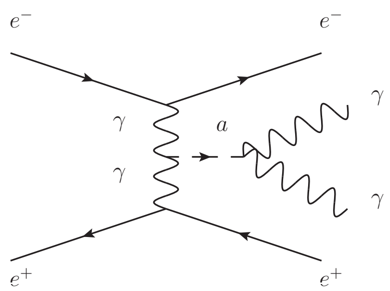
$$
\begin{aligned}
\mathcal{L}_{int} = & \frac{\alpha}{4\pi\Lambda}\cos\theta_{photon} a F^{\mu\nu}F_{\mu\nu} + \frac{\alpha}{4\pi\Lambda}\sin\theta_{photon} a F^{\mu\nu}\tilde{F}_{\mu\nu} \\
& + \frac{\alpha_s}{4\pi\Lambda}\cos\theta_{gluon} a G^{\mu\nu i}G_{\mu\nu i} + \frac{\alpha_s}{4\pi\Lambda}\sin\theta_{gluon} a G^{\mu\nu i}\tilde{G}_{\mu\nu i} \\
& - \mu a^2\bar{\phi}\phi,
\end{aligned} \tag{10}
$$

4 Simuleringar

Simuleringar har genomförts med programmen FeynRules och MadGraph 5. Vi har kunnat sätta upp reglerna för vår modell i en .fr-fil, vilken FeynRules kan konvertera till en modellkatalog. MadGraph 5 läser in modellkatalogen och kan utifrån Monte-Carlo-metoden numeriskt uppskatta tvärsnitt och bredd för en önskad partikelinteraktion.

En intressant frågeställning är huruvida det spelar roll om partikeln a är en skalärboson eller psudoskalärboson. Dvs om $a(-x) = a(x)$ eller $a(-x) = -a(x)$. Detta testades genom att svepa parametren θ_{photon} över intervallet $[0, \frac{\pi}{2}]$ och sedan låta MadGraph 5 beräkna tvärsnittet för processen $p + p \to h \to a + a \to \gamma + \gamma + \gamma + \gamma$. Det visade sig att tvärsnittet var oberoende huruvida var en skalär, pseudoskalär eller en linjärkombination av dessa.

Teoretiskt skulle partikeln a kunnat producerats i LEP (Large Electron Positron Collider), genom processen i figur 2. Våra simuleringar visar att tvärsnittet för en sådan process med vår partikel, skulle blivit alldeles för lågt jämfört med andra liknande processer.

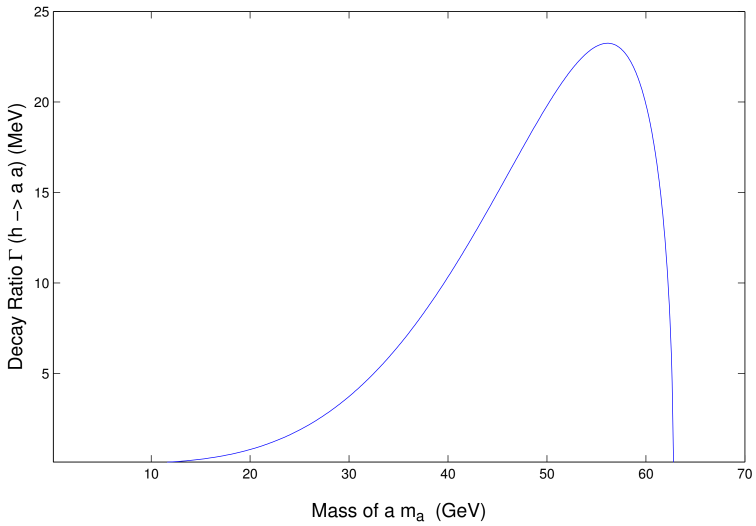


Figur 2: Första ordningens feynmandiagram för en process som skulle kunna ske på LEP, vilken involverar skalärbosonen a.

Även sönderfallen $a \to \gamma\gamma$ och $a \to gg$ har simulerats. Eftersom sönderfallet måste ske inom $0.1mm$ från skapelsen för att kunna observeras, krävs det att bredden $\Gamma > 2 \cdot 10^{-12}$. För att fotonsönderfallet ska kunna synas får vi en undre gräns för λ_{photon} på $3.6 \cdot 10^{-8}\text{GeV}^{-1}$ och en övre gräns för Λ på 16TeV.

5 Diskussion

Higgsbosonens totala bredd Γ_T är experimentellt framtagen till att vara under 17.4MeV. I den nuvarande standardmodellen har higgsbosonen en teoretisk total bredd på 4.1MeV. Detta ger oss ett område på ungefär 13 MeV för $\Gamma(h \to aa)$. Tittar vi på ekvation (7) ser vi att, eftersom $m_h = 125.5$ och $v = 246$ är kända konstanter, bredden endast beror på μ och således på vår partikels massa. Hur denna funktion ser ut kan vi se i figur 3. För att vårat sönderfall ska vara detekterbart måste vi ha en bredd på $2 \cdot 10^{-12}$. Detta motsvarar en minimal partikelmassa på $m_a = 139$MeV. Den övre gränsen fås från det begränsade området, vilket motsvarar en maximal partikelmassa på $m_a = 42.6$GeV.



Figur 3: En plot över hur sönderfallsbredden $\Gamma(h \to aa)$ från ekvation (7), varierar i samband med skalärbosonens massa $m_a = \sqrt{\mu} v$. Det är en interessant brant sluttning vid halva higgsmassan, $m_h/2$. Trots detta är det osannolikt eftersom området är väldigt smalt och det finns ingen dynamisk anledning för att våran massa ska ligga i det området.

I de flesta simuleringarna har vi valt $\mu = 0.0106779$, vilket motsvarar en massa $m_a = 25.4$GeV och $\Gamma(h \to aa) = 2$MeV. Tittar vi närmare på lagrangefunktionen i ekvation (10), ser vi att förgreningsförhållandena $Br(a \to \gamma\gamma)$ och $Br(a \to gg)$ är oberoende av nyfysikskonstanten Λ. Dessa kan matematiskt skrivas

$$BR(a \to 2\gamma) = \frac{\alpha^2}{\alpha^2 + 8\alpha_s^2}, \tag{11}$$

och

$$BR(a \to 2g) = \frac{8\alpha_s^2}{\alpha^2 + 8\alpha_s^2}. \tag{12}$$

Eftersom α_s är partikelberoende och vi inte känner till $\alpha_s(a)$ så har simuleringar i MadGraph och FeynRules genomförts för att fastställa dessa värden numeriskt. Vi får då $BR(a \to \gamma\gamma) = 3.54 * 10^-3$ och $BR(a \to gg) = 0.99964$.

Avslutningsvis har vi gränserna $139\text{MeV} \leq m_a \leq 42.6\text{GeV}$ och $m_h \ll \Lambda \leq 16\text{TeV}$.