Reversing Quantum Trajectories with Analog Feedback

G. de Lange, 1 D. Ristè, 1 M. J. Tiggelman, 1 C. Eichler, 2 L. Tornberg, 3 G. Johansson, 3 A. Wallraff, 2 R. N. Schouten, 1 and L. DiCarlo 1

1 Kavli Institute of Nanoscience, Delft University of Technology, P.O. Box 5046, 2600 GA Delft, Netherlands
2 Department of Physics, ETH Zürich, CH-8093 Zürich, Switzerland
3 Department of Microtechnology and Nanoscience, MC2, Chalmers University of Technology, SE-412 96 Gothenburg, Sweden

(Received 22 November 2013; published 24 February 2014)

We demonstrate the active suppression of transmon qubit dephasing induced by dispersive measurement, using parametric amplification and analog feedback. By real-time processing of the homodyne record, the feedback controller reverses the stochastic quantum phase kick imparted by the measurement on the qubit. The feedback operation matches a model of quantum trajectories with a measurement efficiency \( \eta \approx 0.5 \), consistent with the result obtained by postselection. We overcome the bandwidth limitations of the amplification chain by numerically optimizing the signal processing in the feedback loop and provide a theoretical model explaining the optimization result.

DOI: 10.1103/PhysRevLett.112.080501
PACS numbers: 03.67.Lx, 42.50.Dv, 42.50.Pq, 85.25.-j

In a quantum measurement, information gain is accompanied by backaction, altering superposition states of the observed system [1]. Tunable strength measurements have been devised to balance the tradeoff between information gain and backaction. These can be realized, for example, by controlling the interaction of the observed qubit with an ancillary qubit, followed by strong measurement of the ancilla [2–4]. Depending on the choice of ancilla measurement basis, the observed qubit either acquires a stochastic phase kick, or is partially projected towards one of the basis states, in a direction that is determined by the measurement result. Similarly, a cavity mode can serve as an ancilla, with the measurement basis set by the detected field quadrature [5], and a continuous range of measurement results and associated kickbacks [5, 6].

For an efficient measurement [1], the correlation between the stochastic evolution of the system, also known as quantum trajectory, and the measurement record of the ancilla can be exploited to undo any unwanted backaction [7, 8] or to reverse the measurement altogether [9]. Probabilistic reversal of measurement backaction has been pursued with superconducting [10], photonic [11], and ionic systems [12]. Deterministic reversal, requiring feedback control, has only been demonstrated with ions [13]. Recent improvements in quantum coherence in circuit quantum electrodynamics (cQED) [14] have allowed first demonstrations of feedback control with superconducting qubits. Digital feedback, based on fully projective measurement, enabled on-demand qubit state initialization [15, 16], deterministic teleportation [17], and generation of deterministic entanglement by parity measurement [18]. Analog feedback, instead, is required to counteract the continuous range of measurement kickbacks in a qubit-cavity system. A first implementation of analog feedback relied on continuous monitoring of a driven qubit to stabilize Rabi oscillations [19].

In this Letter, we demonstrate the real-time reversal of measurement-induced qubit dephasing in cQED, using phase-sensitive parametric amplification [20] and analog feedback control, as proposed in Ref. [21]. The recovery of coherence by feedback is quantitatively consistent with a measurement efficiency \( \eta \approx 0.5 \) for the homodyne detection chain, closely matching the result obtained by open-loop postselection. Furthermore, we demonstrate a numerical procedure that finds the optimal weight function for the homodyne signal integration, circumventing the inefficiency arising from the finite detection bandwidth.

We study measurement-induced dephasing of a transmon qubit (transition frequency \( \omega_Q/2\pi = 5.430 \) GHz) coupled to the fundamental mode of a 3D cavity (frequency \( f_r = 6.5433 \) GHz, linewidth \( \kappa/2\pi = 1.4 \) MHz). The qubit-cavity Hamiltonian in the presence of a measurement drive at frequency \( f_m \) and valid in the dispersive regime of our experiment is [22]

\[
H = (\Delta_r - \chi Z)a^\dagger a - \omega_Q Z/2 + \epsilon_m(t)a + \epsilon_m(t)a^\dagger,
\]

in a frame rotating at \( f_m \), with \( \Delta_r/2\pi = f_r - f_m, a (a^\dagger) \) the photon annihilation (creation) operator, and \( Z \) the qubit Pauli \( z \) operator. Above, we have grouped terms to highlight the dependence of the cavity resonance on the qubit state. The transmitted signal is sent to a Josephson parametric amplifier (JPA) operated in phase-sensitive mode [20, 23]. The homodyne signal obtained by demodulation is recorded for postprocessing purposes and also sampled by a feedback controller implementing real-time phase correction (discussed further below) [Fig. 1(a)]. We choose for \( f_m \) the average of the cavity frequencies for the qubit in \( |0\rangle \) (\( f_r \) ) and \( |1\rangle \) (\( f_r + \chi/\pi \), with \( \chi/\pi = -3.2 \) MHz) [Fig. 1(b)].

Applying a measurement pulse entangles the qubit with the cavity field [6, 24]. If the measurement record is
Inherent to the applied measurement, the pulse envelope sequence to reduce the dephasing from mechanisms not an echo sequence [Fig. 1(c), preferred over a Ramsey reduced, where \( r_{\text{off}} \), the qubit coherence at the end of the echo sequence for \( \tilde{c}_m = n \), to the open-loop coherence \( r_{\text{ol}} \). According to theory [26], \( r_{\text{ol}} = r_{\text{off}} \exp \left[ \int_0^t \Gamma_d(\tau) d\tau \right] \), with instantaneous measurement-induced dephasing rate \( \Gamma_d(t) = 2\Re(\delta_0(t) \delta_{\text{opt}}(t)) \), as 

\[
\rho_{01}(V_{\text{int}}) = r_{\text{off}} \exp \left[ (\eta - 1) \int \Gamma_d(\tau) d\tau + i\phi \right],
\]

where \( \phi = c \text{V}_{\text{int}} + \bar{\phi} \), with \( c \propto \epsilon_m \) and \( \bar{\phi} \) the deterministic ac-Stark phase shift [26]. Here, \( \eta \) is the quantum efficiency, modeled as losses in the readout chain leading up to the JPA.

In particular, for \( \phi = 0 \), the averaged homodyne response is equal and opposite for the qubit in \( |0\rangle \) and \( |1\rangle \), \( \langle V_j \rangle_0 = -\langle V_j \rangle_1 \), whereas for \( \phi = \pi/2 \), \( \langle V_j \rangle_0 = \langle V_j \rangle_1 \) [Figs. 1(d), S2] [25]. The measurement reduces \( r_{\text{off}} \), the qubit coherence at the end of the echo sequence for \( \tilde{c}_m = n \), to the open-loop coherence \( r_{\text{ol}} \). According to theory [26], \( r_{\text{ol}} = r_{\text{off}} \exp \left[ \int_0^t \Gamma_d(\tau) d\tau \right] \), with instantaneous measurement-induced dephasing rate \( \Gamma_d(t) = 2\Re(\delta_0(t) \delta_{\text{opt}}(t)) \), as 

\[
\rho_{01}(V_{\text{int}}) = r_{\text{off}} \exp \left[ (\eta - 1) \int \Gamma_d(\tau) d\tau + i\phi \right],
\]

where \( \phi = c \text{V}_{\text{int}} + \bar{\phi} \), with \( c \propto \epsilon_m \) and \( \bar{\phi} \) the deterministic ac-Stark phase shift [26]. Here, \( \eta \) is the quantum efficiency, modeled as losses in the readout chain leading up to the JPA.

In our experiment, the zero-average envelope of the measurement pulse, which makes \( \int w(t) dt = 0 \), is chosen to suppress the infiltration of excess low-frequency noise in \( V_{\text{int}} \) [30]. Furthermore, the integration window extends 6.5/\( \kappa \) = 0.75 \( \mu s \) past the end of the applied measurement pulse [Fig. 2(a)] in order to capture the total field emitted by the cavity as it returns to the vacuum state [21]. Binning the tomography results \( M_I \) on \( V_{\text{int}} \) reveals the stochastic phase \( \delta \phi \) induced by the measurement [Figs. 2(b)–2(d)] [5]. Rather than relying on the weight function predicted by theory, we numerically optimize \( w = w_{\text{opt}} \) to maximize the conditioned coherence \( r_{\text{cond}} = \sum C(V_{\text{int}}) |r(V_{\text{int}})| \), with \( r \) the absolute coherence and \( C \) the fraction of counts for the bin centered at \( V_{\text{int}} \) [25]. From the conditioned coherence, we place a lower bound on \( \eta \), absorbing signal losses after the JPA and classical processing of \( V_j \) in an overall measurement efficiency \( \tilde{\eta} \) in Eq. (1). We find quantitative agreement with the data for \( \tilde{\eta} = 0.50 \) [Figs. 2(c)–2(d)].
The optimal choice for the feedback gain ($c_{\text{fb}}$) in feedback control. In real time, the controller samples the measurement-induced kickback by employing analog tomospheric prerotation ($\phi$). The homodyne record $V_Q^\text{int}$ is acquired for a total duration of 1.25 $\mu$s from the start of the measurement pulse. Light (dark) trace: single (average) record. (b) Measurement scheme. (c) Conditional state tomography (left) and corresponding fraction of counts $C$ (right) in open-loop operation. Solid (dashed) curves: data (model with $\tilde{\eta} = 0.50$). The tomography outcomes $M_i$ are binned on $V_{\text{int}} = \sum_n w[n]V_Q[n]$, where $V_Q$ is sampled every 10 ns. The weight function $w = w_{\text{opt}}$ is obtained by numerical optimization using the records $V_Q$ (see also Fig. 4. (d) Stochastic qubit phase $\delta\phi$ (dots) and absolute coherence $r$ (squares), binned on $V_{\text{int}}$ and model for $\delta\phi$ with $\tilde{\eta} = 0.50$ (solid) and 1 (dashed line). In closed-loop operation (e), corresponding to $c_{\text{fb}} = -10$ in Fig. 3(a)], $V_Q$ is fed to the feedback controller, which calculates $V_{\text{int}}$ using $w_{\text{opt}}$ and translates it into $\delta\phi$, setting the phase of $R_{x,\phi}$. (f) Measured distribution of $\delta\phi$ (grey scale) produced by $M_Q$ and refocusing by analog feedback. This refocusing increases the unconditioned coherence from $r_{\text{ad}} = 0.40$ (black arrow) to $r_{\text{ad}} = 0.56$ (pink arrow). Dashed circle: maximum $r = r_{\text{off}}$ that would be obtained with $\tilde{\eta} = 1$.

Moving beyond postselection, we now set off to cancel the measurement-induced kickback by employing analog feedback control. In real time, the controller samples $V_Q$, calculates $V_{\text{int}}$ using $w_{\text{opt}}$, and adjusts the phase of the tomographic prerotation $R_{x,\phi}$ by $\delta\phi = c_{\text{fb}}V_{\text{int}}$ (Figs. S4, S5 [25]). The optimal choice for the feedback gain ($c_{\text{fb}} = c_{\text{opt}}$) removes all the azimuthal phase dependence on $V_{\text{int}}$.

![FIG. 2 (color online). Conditional qubit tomography and cancellation of measurement-induced dephasing by analog feedback. (a) The measurement $M_Q$ is performed with a pulse at $f_m$ with amplitude $\tilde{e}_m = 0.4$ V and 500 ns length (dashed trace). The homodyne record $V_Q$ is acquired for a total duration of 1.25 $\mu$s from the start of the measurement pulse. Light (dark) trace: single (average) record. (b) Measurement scheme. (c) Conditional state tomography (left) and corresponding fraction of counts $C$ (right) in open-loop operation. Solid (dashed) curves: data (model with $\tilde{\eta} = 0.50$). The tomography outcomes $M_i$ are binned on $V_{\text{int}} = \sum_n w[n]V_Q[n]$, where $V_Q$ is sampled every 10 ns. The weight function $w = w_{\text{opt}}$ is obtained by numerical optimization using the records $V_Q$ (see also Fig. 4. (d) Stochastic qubit phase $\delta\phi$ (dots) and absolute coherence $r$ (squares), binned on $V_{\text{int}}$ and model for $\delta\phi$ with $\tilde{\eta} = 0.50$ (solid) and 1 (dashed line). In closed-loop operation (e), corresponding to $c_{\text{fb}} = -10$ in Fig. 3(a)], $V_Q$ is fed to the feedback controller, which calculates $V_{\text{int}}$ using $w_{\text{opt}}$ and translates it into $\delta\phi$, setting the phase of $R_{x,\phi}$. (f) Measured distribution of $\delta\phi$ (grey scale) produced by $M_Q$ and refocusing by analog feedback. This refocusing increases the unconditioned coherence from $r_{\text{ad}} = 0.40$ (black arrow) to $r_{\text{ad}} = 0.56$ (pink arrow). Dashed circle: maximum $r = r_{\text{off}}$ that would be obtained with $\tilde{\eta} = 1$.

![FIG. 3 (color online). Extraction of measurement efficiency from the extent of coherence recovery. (a) Coherence versus feedback gain $c_{\text{fb}}$ for $\tilde{e}_m = 0.2$–0.7 V, with $w_{\text{opt}}$ optimized at $\tilde{e}_m = 0.4$ V. Top left: average homodyne voltage $\langle V_Q \rangle$ for the same range of $\tilde{e}_m$. The maximum coherence $r_{\text{cl}}$ corresponds to the optimum feedback gain $c_{\text{opt}}$ (lower inset), directly proportional to $\tilde{e}_m$. The horizontal dashed line indicates the coherence $r_{\text{off}}$ for no measurement drive ($\tilde{e}_m = 0$). Error bars are the standard deviations of eight repetitions. (b) Contour plot of the measurement efficiency $\tilde{\eta}$, with curves at 0.1 steps. For each $\tilde{e}_m$, $r_{\text{cl}}$ is obtained by a quadratic fit of $r$ around the maximum and $r_{\text{ad}}$ is the measured average for $c_{\text{fb}} = 0$ in (a). The best fit of Eq. (2) (orange dashed line) to the data yields $\tilde{\eta} = 0.49 \pm 0.01$. [Figs. 2(e)–2(f)]. Crucially, $r_{\text{con}}$ is unaffected, demonstrating that feedback does not introduce additional errors. To fully quantify the performance of the active coherence recovery, we repeat the experiment in Fig. 2(b) for various measurement-drive amplitudes $\tilde{e}_m$ and feedback gains $c_{\text{fb}}$ [Fig. 3(a)]. Whereas the variance of $V_{\text{int}}$ is independent of $\tilde{e}_m$, as expected, the phase dependence $\partial \phi / \partial V_{\text{int}}$ grows linearly with $\tilde{e}_m$ [27,29], requiring the optimum $c_{\text{opt}} \propto \tilde{e}_m$ [Fig. 3(a) inset]. Following from Eq. (1), the measured $r_{\text{cl}}$ (corresponding to $c_{\text{fb}} = 0$), $r_{\text{off}}$ ($\tilde{e}_m = 0$) and $r_{\text{cl}}$ ($c_{\text{fb}} = c_{\text{opt}}$) are related by

$$r_{\text{ad}}/r_{\text{cl}} = (r_{\text{cl}}/r_{\text{off}})^{\tilde{\eta}}.$$  

(2)

We obtain the best-fit $\tilde{\eta} = 0.49 \pm 0.01$ [Fig. 3(b)].
To understand how the JPA response impacts $w_{\text{opt}}$, we apply the recent mode-matching theory of Ref. [31]. This theory predicts the optimum weight function $w_{\text{mm}} \propto \langle b_{\text{out}}^\dagger(t)Z \rangle$, with $b_{\text{out}}^\dagger(t)$ the operator for the outgoing field after amplification by the JPA [25]. As shown in the Supplemental Material [25], $w_{\text{mm}} \propto F^{-1}[(\alpha_{0,\Delta}^* - \alpha_{\Delta}^*)/2G_{\text{in,}\Delta}]$, where $\alpha_{i,\Delta} = \langle a_{\Delta}^i \rangle$ for the qubit in $|i\rangle$, with $a_{\Delta}$ the Fourier component of the intracavity field at detuning $\Delta$ from the pump, $G_{\text{in,}\Delta}$ the $\Delta$-dependent small-signal gain, and $F$ the Fourier transform. Interestingly, $w_{\text{mm}}$ coincides with the expected $\langle V_j \rangle$ for the qubit in $|0\rangle$, corresponding to the quadrature demultiplexed by the JPA for $\phi = \pi/2$. We find a good agreement between the predicted $w_{\text{mm}}$ and the experimental $w_{\text{opt}}$ [Fig. 4(b)].

In conclusion, we demonstrated the suppression of measurement-induced dephasing of a transmon qubit using parametric amplification and analog feedback. Optimal real-time processing of the homodyne signal makes the recovery of coherence independent of detection bandwidth and equal to the maximum achievable with the quantum efficiency $\eta$. We estimate that applying the same feedback scheme to the cavity-assisted parity measurement [29,32] in the same conditions as Ref. [18] would improve concurrence from the measured 34% to 42%.

Improving quantum efficiency will be essential to fully undo measurement kickback and for protocols, such as qubit-state stabilization [33,34] and continuous-time error correction [35], requiring near-perfect correlation between measurement record and kickback. Alternatively, analog feedback schemes that rely on qubit projection can tolerate a lower efficiency, since estimation of the quantum state improves with the measurement strength. Similarly to the first implementations of digital feedback in the solid state [15–18], which reached high fidelity in spite of moderate efficiencies, analog feedback using projective measurement offers the capability to create and stabilize entanglement [36,37] with the current state of the art.

We thank C. A. Watson for experimental assistance, W. F. Kindel and K. W. Lehnert for the parametric amplifier, and A. F. Kockum and M. Dukalski for helpful discussions. We acknowledge funding from the Dutch Organization for Fundamental Research on Matter (FOM), the Netherlands Organization for Scientific Research (NWO, VIDI scheme), and the EU FP7 integrated projects SOLID and SCALEQIT. G. d. L. and D. R. contributed equally to this work.

---

[30] With a square envelope, instead, we obtain \( \eta = 0.39 \) using Eq. (3).