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SOLID AND STRUCTURAL MECHANICS

An Integration-Reduction Scheme for Simulation of Large Systems with Local Nonlinearity and Uncertainty

Application to moving load problems in railway mechanics

SADEGH RAHROVANI

Department of Applied Mechanics
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden, 2014

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SADEGH RAHROVANI

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Department of Applied Mechanics
Chalmers University of Technology
SE-412 96 Gothenburg
Sweden
Telephone +46 (0)31 772 1000

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ABSTRACT

The focus of this research is mainly on computational efficiency, as future work is planned for reliability analysis that will be combined with design optimisation. Both these fields are known to be notoriously computationally demanding. Since the reliability-based optimization relies on fast simulations, the aim of this thesis has been on developing an efficient time-integration scheme for fast simulation of stiff FE models with local nonlinearity and uncertainty, such as railway tracks with sleepers put on a non-uniform ballast bed. Computational efficiency concerns that arise in simulation of large-scale systems are treated by using an approximation of the Jacobian matrix. This is achieved by combining the proposed exponential integrator with the developed methods for model reduction. A modal dominancy approach for modal reduction of dynamical systems is presented. Briefly stated, a quadratic dominancy metric, given in a closed form formulation, has been presented based on the modal contributions to the $\mathcal{H}_2$ norm of the frequency response function (FRF) matrix and thus related to the r.m.s prediction. However, a main issue for many of the modal dominancy metrics is to detect the non-minimality and to handle systems with multiple eigenvalues, such as a track FE model with clusters of neighbouring eigenvalues, properly. In order to treat these systems, the problem of non-uniqueness of the proposed dominancy metric is studied in detail and two different methods to circumvent this problem are proposed. Another main concern for model reduction of large-scale systems, is to be more effective and to obtain as small order approximant as possible. However, typical input-output based reduction methods usually becomes ineffective, when they are applied to systems subjected to a moving/distributed loading. In this regard, the dominancy analysis procedure is improved to be able to use the information extracted from the spectral and internal structural properties of the external excitation. Moreover, the integration algorithm's symplecticity, geometric properties and its global error are studied through Hamiltonian class examples and the efficiency and accuracy of the proposed integration-reduction scheme is investigated through a large-scale size problem that originates from a moving load problem in railway mechanics. It is demonstrated that the integrator is particularly effective in large-scale problems with local nonlinearity and/or uncertainty, when compared to the general purpose methods. Finally, the developed integration-reduction method is used in conjunction with a reliability approach using an adaptive two-stage procedure. In the first step it uses efficient methods such as First Order Reliability Form or Second Order Reliability Method, to find the approximate design point which constitutes the linearization point based on which the reduced model can be constructed. In the second step, an importance sampling technique constructed based on the obtained design point is used in conjunction with a Latin Hypercube Sampling to estimate the failure probability.

KEYWORDS: time-integration, exponential integrator, symplectic integrator, modal dominancy analysis, reliability analysis, Monte-Carlo simulations, moving load problems.

To my beloved family

PREFACE

The work presented in this thesis has been carried out between March 2011 and February 2014 at the Department of Applied Mechanics at Chalmers University of Technology. It is part of the project “Multicriterion optimization of track properties” within the Swedish National Centre of Excellence in Railway Mechanics (CHARMEC). The financial support from the centre is acknowledged.

First and formest, I would like to give my deepest gratitude to my supervisor Professor Thomas Abrahamsson for his invaluable guidance, expertise and patience.

I am particularly grateful to my co-supervisors, Professor Jens Nielson and Dr Rikard bolmsvik, who helped me during this work. I thank Dr Klas Modin for giving me valuable comments and feedbacks on the third paper in this thesis. I also thank Dr Sebastian Plica for his valuable comments in the TS14 reference group meetings.

I give my deepest gratitude to my dear parents who I owe everything in my life. My special thanks and gratitude goes to my beloved wife Fatemeh, for her endless love and support.

Last but not least, I want to thank all my dear friends and collegues for their support.

Gothenburg, February 2014

Sadegh Rahrovani

THESIS

This thesis consists of an extended summary and the following appended papers:

Paper A Rahrovani S., Vakilzadeh M. K. and Abrahamsson T., A Metric for Modal Truncation in Model Reduction Problems. Part 1: Performance and Error Analysis. Part 2: Extension to Systems with High-Dimensional Input Space, *Proceeding of IMAC XXXI*, Anaheim, (CA), USA, 2013

Paper B Rahrovani S., Vakilzadeh M. K. and Abrahamsson T., On Gramian-based Techniques for Minimal Realization of Large-Scale Mechanical Systems, *Proceeding of IMAC XXXI*, Anaheim, (CA), USA, 2013

Paper C Rahrovani S., Abrahamsson T. and Modin K., An Exponential Integrator for Fast Simulation of Large Stiff Systems with Local Nonlinearity and Uncertainty, submitted to international publication

Paper D Rahrovani S., Abrahamsson T., A New Parameter Perturbation Method Suitable for Reliability Analysis of Large Dynamic Systems, *Proceeding of ICOSSAR* 2013, New York, (NY), USA, 2013

Appended papers were prepared in collaboration with the co-authors. The auther of this thesis is responsible for the major progress of the work in papers A-D. This includes planning, theory development, numerical simulation, analysis and writing. The following publication resulted from the work carried out in the P.h.D. program and relates to paper C that has been submitted to international publication.

COMPLEMENTARY PAPERS

Paper I Rahrovani S., Abrahamsson T. and Modin K., An Efficient Exponential Integrator for Large Nonlinear Stiff Systems. Part 1: Theoretical Investigation, Part 2: Symplecticity and Global Error Analysis, *Proceeding of IMAC XXXII*, (FL), Orlando, USA, 2014

CONTENTS

PART I

Extended summary

1 MOTIVATION AND AIMS

The railway is one of society's most important transportation systems today. It is viewed as an environment friendly alternative to car and flight transportation modes. It is important to improve the performance of the railway transportation chain, by investing in higher axle loads and higher speeds, to meet the increasing demands of a new age. The use of new vehicles, capable of axle loads and higher speeds on the existing railway lines, motivates the studing and deep understanding of the dynamical behavior of track superstructure. The potential possibilities to increase the operational speed and axle loads by optimizing the track properties while being able to guarantee the safety and comfort are the driving force behind this research. Optimization of the railway sleepers, which have important roles in dynamics of the railway superstructure, is the final goal of this project.

Currently, guidelines are used in current sleeper design; two examples are the European standard [1] and an UIC leaflet [2]. Such guidelines refer to the minimum allowable capacity of the bending moment at specific locations along the sleeper. Sleeper design is presently based on a simplified analysis where the dynamic loading is accounted for via a dynamic magnification factor on the nominal static wheel load. The approach used to determine this dynamic magnification factor differs between sleeper design standards. In the analysis used in the European standard [1], the magnification factor depends on whether the train speed is below or above 200 km/h. However, the motivation for this specific splitting speed is not found in the standard or in other references. According to Buekett [3], sleeper design is based on experience rather than on theory. This may explain the differences between different standards. Another example of simplification in the design analysis is how the distribution of sleeper support is accounted for. In most cases, a uniform distribution of ballast stiffness is considered either along the complete length of the sleeper or limited to the rail seat sections. However, due to different loading condition, poor maintenance of sleeper or bad quality of ballast, a random load distribution along the sleeper-ballast interface may occur, see Figures 1 and 2. This might result in conditions such that the sleeper is fully or partially hanging over the ballast bed. The differences in result when comparing sleeper responses either calculated according to different sleeper design guidelines or measured in-field are unknown. This lack of knowledge is an obstacle considering future sleeper design, in which the design loads need to be based on both higher train speeds and higher axle loads compared to those treated by the current guidelines. Therefore, in this project, a significant effort will be put on experimental investigation into the sleeper-ballast contact pressure and the load transfer mechanism in the track structure. In-field measurement data will be used in track model calibration and validation.

Other design methods, such as probabilistic design methods, based on proper statistics of input/system stochastic parameters, could be an alternative approach. In a probability based design, the optimal design is sought for considering that the probability of failure must not exceed a specified level. The probability of failure is then found by reliability analysis. The main benefit of such indeterministic design approaches reside in the fact that they consider the effects due to uncertainty on system performance from the very begining stages of the design process. In this regard, reliability methods offer an appropriate framework in order to systematically capture the effects created due to parameter variation, on system performance. Since, by nature, the railway track properties vary significantly in space and time, a stochastic approach to the optimization will be taken.

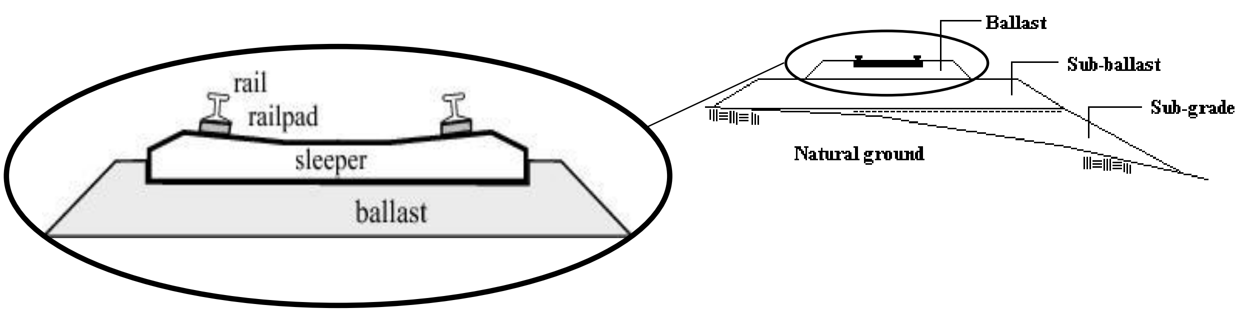

Fig. 1: The substructure mainly consists of three layers; ballast, sub-ballast and sub-grade. They rest on the subsoil, normally the natural ground.

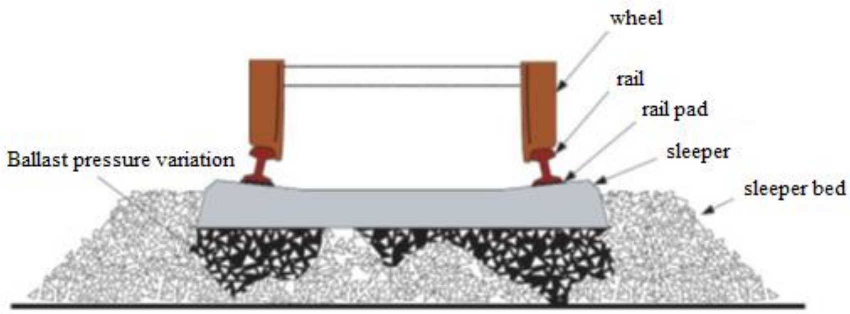

Fig. 2: Example of variation in stiffness under a sleeper due to ballast distribution change, Figure from [4]. Due to different loading condition, poor maintenance of sleeper or bad quality of ballast, a random load distribution along the sleeper-ballast interface may occur. This might result in a fully or partially hanging sleeper in which the sleeper hangs in the rails.

The focus of this research is mainly on computational efficiency, as in future work the reliability analysis will be combined with design optimisation. Both these fields are known to be notoriously computationally demanding, Figure 3. Track system models will lay the foundation for further analysis, simulation and optimization. Since the reliability-based optimization relies on fast simulations, the aim of this thesis is to develop an efficient time integration scheme for fast simulation of stiff FE models with local nonlinearity and/or uncertainty. The integration scheme, together with the developed methods for model reduction, has been used to treat large-scale systems in an efficient manner. The proposed integration-reduction scheme will be used for fast simulation of a railway track FE model which is both locally nonlinear and stochastic, in which the nonlinearities are due to a fully or partially hanging sleeper. Later on, the integration-reduction method is being used in conjunction with an adaptive reliability approch.

2 MOVING LOAD PROBLEMS IN RAILWAY MECHANICS

In this study an idealized train running on a flexible railway track structure at a constant cruising speed is considered. The vertical track dynamics is simulated in the time domain. The in-house computer program DIFF is used for simulation of the linear track model. For nonlinear and nondeterministic analysis, the DIFF code is improved to be able to model the nonlinearity and uncertainty behavior between the ballast and the hanging sleeper. The linearized complete track model consists of 1049 degrees of freedom, with and without stochastic properties used in **PAPER D** and **PAPER A**, respectively. The nonlinear model with a local nonlinearity created due to a hanging sleeper is used in **PAPER C**. The studied track model consists of 19 sleepers and 20 sleeper bays with equal length, shown in Figure 4 and 5. The loading on the track is dynamic because of the moving train, but also as a consequence of irregularities present on the rails and on the wheels. To account for the influence of a rail irregularity, an impact load is superimposed on the static wheel load at the instant when the wheel load is just above the sleeper of interest, the sleeper in bay number 10, see Figure 6.

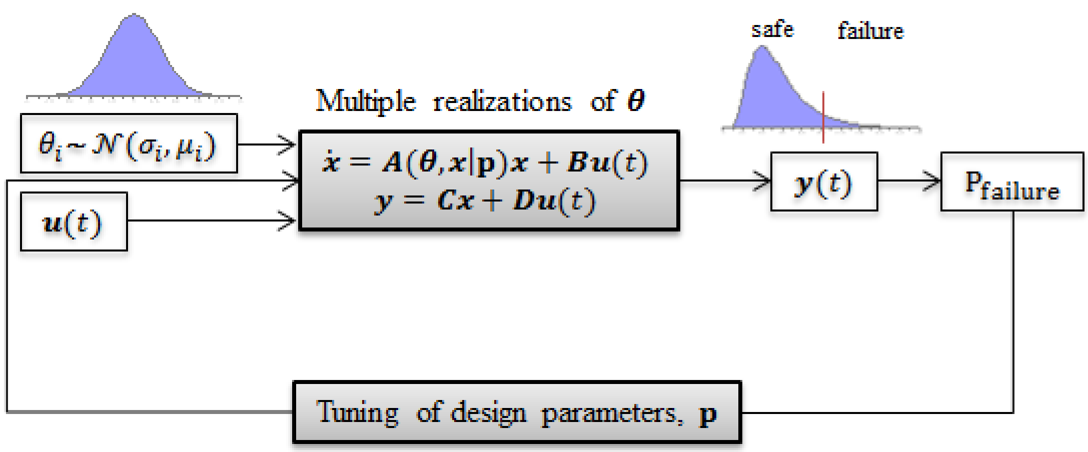

Fig. 3: Reliability-based design optimization process. The optimization loop embracing the sampling-based reliability analysis makes the design process notoriously computationally demanding.

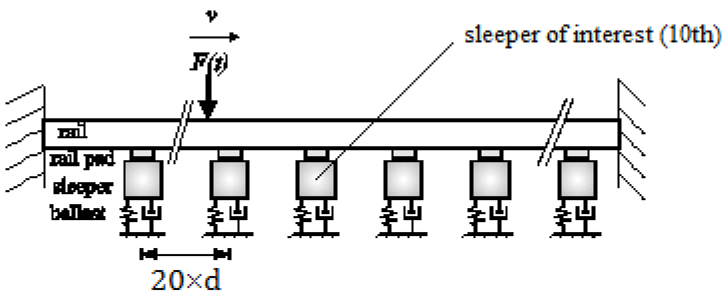

Fig. 4: The track consists of 20 sleeper bays with equal sleeper spacing. Track model is subjected to a moving axle load that is traveling the rail with speed *v* and is time varying to represent wheel/rail irregularity. Rail is long enough to minimise end condition effects.

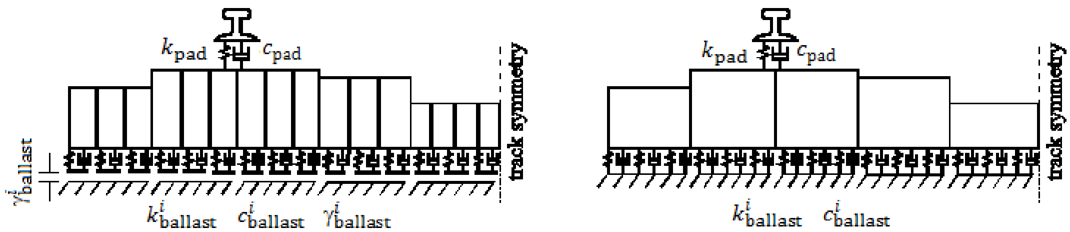

Fig. 5: In the nonlinear analysis in PAPER C, the sleeper in bay number 10 is assumed to be fully hanging before the train passage. The nonlinearity between the hanging sleeper and the ballast is modelled by gap-spring elements that produces a bilinear behaviour and can be activated during train passage (left). The remaining 18 sleepers are assumed to be connected to the ballast before and during train passage (right). The ballast is modelled with a Winkler foundation.

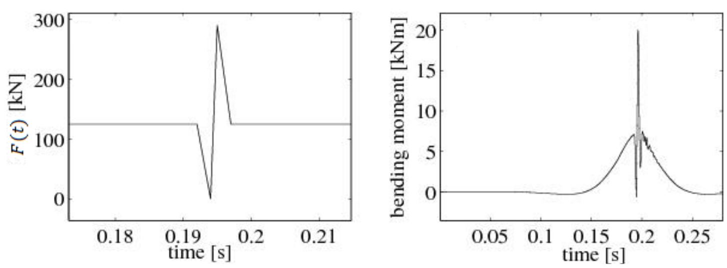

Fig. 6: The prescribed moving load time history applied to the rail at the passage of the sleeper of interest. The rapid variation is to represent influence of a large wheel-flat. The magnitude of this impact load is set to 295 kN, which according to maximum wheel-flat load allowed on Swedish tracks, 290 kN, is a marginally safe level (left). The bending moment at the railseat of sleeper number 10, computed for a linear model in nominal situation, shown in PAPER D (right).

In the nonlinear model, one sleeper in bay number 10 is considered to be fully hanging before the train passage. Nonlinear forces can be created due to the local nonlinear behaviour between this sleeper and the ballast, during the train passage when the gap between sleeper and ballast is closed because of the heavy axle load. The nonlinearity is modelled by spring-gap soil elements, each acting like a bilinear spring. Finite element discretization results in a large-scale differential equation set with 2098 states. The state-space nonlinear model of the track subjected to the external loading is simulated and moments at two critical cross sections, in the hanging sleeper, in bay number 10, are considered as outputs.

3 COMPUTATIONAL CHALLENGES

For the reliability analysis, a simple and yet very accurate method is the standard Monte Carlo (SMC) method that is a simulation method to approximate the failure probability integral. However as probabilities of failure are intended to be small in a reliability analysis this method is usually extremely time consuming, since it contain simulations of the physical system for a large number of realizations to achieve convergence. The costs drastically increase, when SMC method is used for structural reliability of large-scale nonlinear dynamical systems.

For deterministic simulation, most modern-days structural problems must rely on a large-scale FE solution that typically contains time-integration of a large ordinary differential equation (ODE) set. Simulation of large-scale FE models usually requires high computational efforts, particularly for nonlinear dynamical systems. Moreover the presence of a wide range of temporal scales in a differential equation set, that typically arises in a variety of FE models originated from large-scale structural dynamics and multi-body dynamics problems, poses more major efficiency concerns in the time-integration process [5].

PAPER C addresses this problem by developing an efficient exponential integration scheme for fast simulation of large-scale finite element models with local nonlinearity, such as railway tracks with hanging sleepers. To achieve this, simulation of the underlying linearized system is needed to be fast. In this regard, model order reduction methods techniques can be an extremely useful tool [6], particulary for large-scale problems with a limited number of inputs and outputs, which can lead to substantial savings in simulation time. An important feature of the presented integrator is the exact (or very precisely) evaluation of the part of solution that corresponds to the contribution of the underlying linearized system, which generally contains the stiff and oscillatory nature of the system. As a result, the nonlinear parts can be treated with a better stability property [7]. However, in order to obtain the exact solution of the linearized system, the exponential of the Jacobian matrix needs to be computed [8]. Additionally, to compute the solution that corresponds to the contribution of the nonlinear and stochastic terms, the presented integration scheme requires iterative simulation of the underlying linearized system. Both these two steps, however, are time consuming for systems with a large-scale Jacobian matrix. This issue is addressed in **PAPER C** and **PAPER D**, by using an approximation of the Jacobian. This is achieved by combining the proposed integration scheme with a modal dominancy procedure, modified to take the system nonlinearity and uncertainty effects into account. It is done by modifying the system input/output relationship, based on both the external input force and the so called *pseudo-forces*, created due to nonlinearity and/or uncertainty of the system, and then perfoming a modal dominancy analysis in order to identify the dominant modes of the underlying linearized system. The proposed modal ranking analysis for modal reduction of linear time invariant (LTI) systems, is presented in [9] and **PAPER A (Part I)**, in detail. A quadratic dominancy metric, given in closed form formulation, has been presented based on the modal contributions to the $\mathcal{H}_2$ norm of the frequency response function (FRF) matrix. However, a main issue for many of the modal dominancy metrics is to detect the non-minimality and to handle systems with multiple eigenvalues, properly. In order to treat these systems such as the track FE model with clusters of neighbouring eigenvalues, the problem of non-uniqueness of the proposed dominancy metric is studied in detail and two different methods to circumvent this problem are proposed in **PAPER A (Part II)** and **PAPER B**. Another main concern for model reduction of large-scale systems, is to be more effective and to obtain as small order approximant as possible. However, typical input-output based reduction methods usually becomes ineffective, when they are applied to systems subjected to a moving/distributed loading. **PAPER A (Part II)** treats this issue by incorporating the information extracted from external input spectral and internal structure, in the modal dominancy analysis. It is shown that this approach is particularly effective in problems with high-dimensional temporal correlated input, which typically arises in moving/distributed loading problems. It should also be noted that the geometric and structural properties of the presented numerical integrator are examined and the preservation of properties such as phase-plane area and energy consistency are investigated. The algorithm's symplecticity and its global error are studied through small-scale Hamiltonian examples and superiority of the method for simulation of Hamiltonian systems with close to chaotic-type behavior, have been investigated through a 2D pendulum and a 3D spring-pendulum example (**PAPER C**).

For efficiency improvement of reliability analysis, standard Monte-Carlo sampling can be replaced by Latin hypercube sampling in the importance sampling techniques [10]. **PAPER D** presents an adaptive reliability method that consists of two steps. In the first step, it uses an efficient analytical method such as the First Order Reliabilitry Method (FORM) or the Second Order Reliabilitry Method (SORM), to find the approximate design point which constitutes the linearization point based on which the reduced model can be constructed. However, since the FORM and SORM methods are known to be less accurate for highly nonlinear reliability limit state functions, a full order model can be used to increase the accuracy of simulation. This enables us to find a more accurate design point. Later on, a reduced order model is constructed based on the dominant modes of the full physical model which is linearized with respect to the obtained design point. In the second step, an importance sampling technique constructed based on the obtained design point is used in conjunction with a Latin Hypercube Sampling to estimate the failure probability. Several simulations, required for calculating the response for different system realizations, are performed through the previousely developed integration-reduction scheme.

4 DEVELOPMENTS

Briefly stated, the focus of this thesis is to develop an efficient integration-reduction scheme for fast simulation of large-scale stiff systems with local nonlinearity and uncertainty. This will be used for fast simulation of a railway track model with a local nonlinearity, created due to a patially hanging sleeper. Finally, the developed integration-reduction scheme method is used in conjunction with an adaptive reliability approach. In summary, the investigated issues in **PAPERS A-D** are as follows:

- Developing an efficient method for modal reduction of large-scale Linear Time Invariant (LTI) systems, based on accurate preservation of the input/output relationship, which is also able to treat systems with multiple eigenvalues, e.g. a track FE model with clusters of neighbaring eigenvalues ([9], **PAPER A**, **PAPER B**).

- Exploiting the information extracted from the spectral and internal structure of the external input, to treat reduction of large-scale systems subjected to a moving and/or distributed loading, e.g. a track model subjected to moving train load, in a more effectively way (**PAPER A**).
- Developing an exponential integration scheme for fast simulation of stiff systems with local nonlinearity and uncertainty, e.g. a track model with hanging sleepers (**PAPER C**). It is demonstrated that the method is very fast when the system is either weakly nonlinear or weakly stochastic (**PAPER D**).
- Improving the developed modal dominancy analysis, to be able to take the effects created due to nonlinearity and/or uncertainty of the system into account. This is used for approximation of a large-scale size Jacobian matrix, when calculating the Jacobian exponential function. It is demonstrated that the presented integration-reduction scheme is very effective when either the nonlinearity or the stochasticity are local (**PAPER C** and **PAPER D**).
- It has been demonstrated in **PAPER C** and **PAPER D** that by using a proper nominal linearized model the presented method becomes very efficient particularly in Monte-Carlo simulations, reliability analysis and model calibration. For these types several simulations are normally required for different perturbed system realizations that are mostly close to a nominal model. In such cases, the convergence of the iteration algorithm of the method is achieved very fast.
- Proposing an efficient reliability method that contains constructing the system response surface (RS) and estimating the failure probability, in an adaptive two-step algorithm. This is done by combining the presented integration-reduction scheme with importance sampling approach (**PAPER D**).
- It is also demonstrated in **PAPER C** that the integration scheme is symplectic and thus preserves physically-related structure of the original Ordinary Differential Equation (ODE) set which is beneficial in the following senses:
  (1) As geometric integration methods give an energy evolution that is accurate with accuracy that does not deteriorate as time evolves [11-12] due to numerically-induced damping. Such an energy-consistent integration method can be an appropriate candidate for calibration problems that attempt to calibrate the damping very accurately, by comparing the simulation and test measurement time response. Moreover, as shown in the spring-pendulum example of **Paper C**, energy-consistent integration schemes are useful to identify and avoid numerically-induces chaos when simulating dynamical systems with close to chaotic-type behavior.
  (2) The superiority of such geometric integrators becomes more evident when they are used either in long-term time integration or for very fast simulations, both of them need to choose the integration time-step size as large as accuracy constraints allows. The first is useful in solar system and molecular dynamics simulation. The latter arises in design optimization of nonlinear large-scale systems where fast simulation is a crucial requirement.

5 SUMMARY OF APPENDED PAPERS

5.1 Paper A

In this paper (as the continuation of [9]), a modal dominancy approach for reduction of dynamical systems is presented. Briefly stated, the reduced modal model is determined such that it minimizes the frequency response error in an $\mathcal{H}_2$ sense and is thus related to the r.m.s. prediction error. First, a performance and error analysis of the proposed modal dominancy metric is carried out, the problem of non-uniqueness of the metric for a class of systems with multiple eigenvalues is described and a method to circumvent this problem is proposed. The method is also able to use the information extracted from the spatial and spectral properties of the external excitation. It is shown that such improvement is particularly effective in problems with high-dimensional temporal correlated input, such as in distributed load and moving load problems. Performance of the method is validated for a railway track moving load problem and the results are compared with a reduction-after-balancing approach. Also, techniques to handle the efficiency issues that arise either in the eigensolution or in the dominancy metric calculations of such problems are discussed.

5.2 Paper B

In this paper, a review of Gramian-based minimal realization algorithms is presented, several comments regarding their properties are given and the ill-condition and efficiency problems that typically arise in balancing of large-scale realizations, are addressed. A new algorithm to treat non-minimal realization of very large second-order systems with dense clusters of close eigenvalues is proposed. The method benefits the effectiveness of balancing techniques in treating of non-minimal realizations in combination with the computational efficiency of modal techniques to treat large-scale problems.

5.3 Paper C

In this study, a symplectic exponential integration scheme for computing solutions of large stiff systems is introduced. It is claimed that the integrator is particularly effective in large-scale problems with localized nonlinearity when compared to general purpose methods. Computational efficiency concerns that arise in simulation of large-scale systems are treated by using an approximation of the Jacobian matrix. This is achieved by combining the proposed integration scheme with a modal truncation approach modified in order to treat the nonlinear problems. Furthermore, geometric and structural properties of the presented integrator are examined and the preservation of properties such as phase-plane area and energy consistency are investigated. The integration algorithm's symplecticity and its global error are studied through small-scale Hamiltonian examples and the efficiency and accuracy of the proposed exponential integrator are investigated through a large-scale size problem that originates from a moving load problem in railway mechanics. Moreover, the superiority of such energy-consistent integration schemes to identify and avoid numerically-induced chaos when simulating systems with close to chaotic behavior is investigated (see [13]).

5.4 Paper D

To decrease the computational efforts needed for reliability analysis of large-scale finite element (FE) models, fast simulation is a necessity. To achieve this, model reduction approach offers a systematic way to capture the main input-output behaviour of the original dynamic model with a simpler one. However to be useful for reliability analysis, the approximant must provide a good accuracy for a range of stochastic parameters variation. This problem is addressed by introducing a reduced modal basis based on a reference LTI FE model. A method is employed that uses an iterative algorithm to obtain perturbed solutions that are due to the effects of parameter variation. The main idea is to iteratively approximate the perturbed solution in the space spanned by reduced basis. This technique is used to develop an adaptive two-step reliability method. In a first step it uses efficient methods such as FORM or SORM, to find the approximate design point which constitutes the linearization point based on which the reduced model can be constructed. In the second step, an importance sampling technique constructed based on the obtained design point is used in conjunction with a Latin Hypercube Sampling to estimate the failure probability.

6 CONCLUDING REMARKS AND OUTLOOK

The proposed time-integration scheme, together with the developed methods for model reduction, has now been taken to "production" status. That means that it is ready for use in a probabilistic design setting in which thousands of realizations needs to be simulated efficiently. The developed method performance for fast simulation of linear systems with localized uncertain parameters has been validated at **PAPER D** and its efficiency to treat large-scale nonlinear systems has been investigated in **PAPER C** and [13].

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PART II

Appended papers A-D