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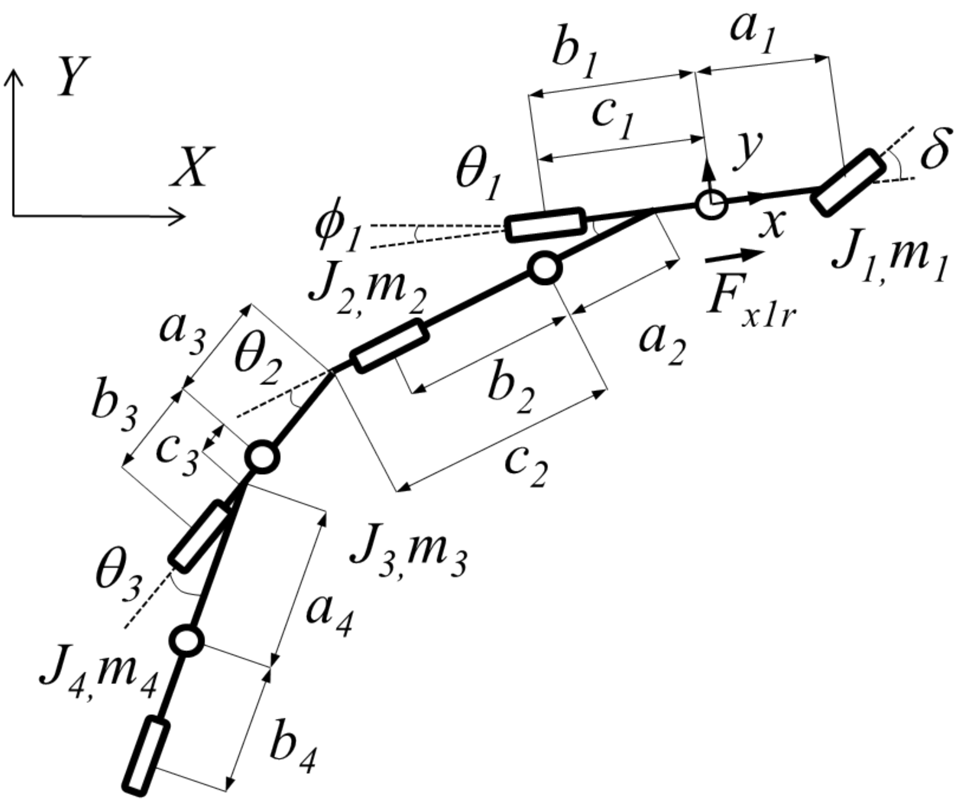

Single-track models of an A-double heavy vehicle combination

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Cover:
Illustration of single track model parameters

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ABSTRACT

This report presents single-track models representing an A-double vehicle combination. The equations of motion are expressed relative the center of mass for the first vehicle unit and are derived using Lagrangian formalism. The symbolic manipulations of the equations have been done using the software Mathematica [1].

The model derived in this report is intended to be valid for studies of vehicle cornering in high speed with moderate lateral and longitudinal acceleration levels. In order to keep the model complexity as low as possible, the derived equations of motion are simplified. The steering angle and the articulation angles of the towed vehicle units are assumed to be small. All products of the steering angle, the yaw rate of the first unit and the articulation angles and their time derivatives are set to zero. Further, the representation of longitudinal vehicle velocity leads to different level of model complexity. Firstly, the longitudinal velocity is assumed to be constant. Together with the use of a linear lateral tyre model this results in a fully linear vehicle model, consisting of eight states and one input. Secondly, the longitudinal velocity is allowed to be varying and is represented as a model state variable. Also here a linear lateral tyre model is used. The resulting vehicle model becomes non-linear, consisting of nine states and six inputs. Further simplifications of the longitudinal dynamics is carried out by assuming e.g. slowly varying longitudinal velocity and longitudinal forces only on the towing unit. This results in a non-linear model consisting of nine states and two inputs. This model is to be compared with the fully linear model.

Realistic linear tyre parameters are found by tuning the derived linear single-track model with results from a high fidelity vehicle model developed at Volvo Group Trucks Technology (VGTT). The tuning was made using the software Matlab and a built-in Genetic Algorithm(GA) [2], where the summarized squared difference in yaw rate gain for vehicle units was minimized.

The performance of the derived vehicle model, including the difference in treatment of the longitudinal velocity, is evaluated in a comparison with the high fidelity model developed at VGTT. The open-loop analysis carried out in the comparison are step steer response analysis, single-sine steer response analysis, braking in a turn analysis (ISO14794) [6].

Keywords: Commercial vehicles, Heavy trucks, Long vehicle combinations, Single track model

Contents

1 Introduction

This report presents single-track models of an A-double vehicle combination. The vehicle combination which is illustrated in Fig. 1.1 consists of four units; tractor, semi-trailer, dolly and semi-trailer. The total length of the vehicle is 32[m] and the total mass are set to 80[t]. The designated A-double combination is longer than what is currently allowed in traffic in Europe. It is one out of several vehicle combinations denoted as prospective modular combinations described in [3]. The prospective modular combinations have the potential to increase the productivity, but also holds an extended complexity when it comes to vehicle dynamics.

A single-track model is a common way to represent the basic vehicle cornering response. Its relative simplicity facilitates the derivation of the model as well as the understanding of lateral vehicle dynamics and lateral stability. The derivation of a single unit single-track model can be found extensively in literature [7], [9], [5]. Also, single-track models representing articulated vehicles are found in [9], [5]. The model follows on an assumption that the effects of all tyres on one axle are combined into one single virtual tyre. Phenomena which are not captured with this type of model are according to [7] e.g. large deviations from Ackerman geometry within an axle and varying axle cornering stiffness due to lateral load shift and different axle propulsion/braking left/right. The effects of the latter can however be handled by using axle sub-models. Other important phenomena, more specific for heavy-duty trucks, which are not captured by a single track model are connected to the lack of frame torsion flexibility and the cabin with its suspension.

The model derived in this report is intended to be valid for studies of vehicle cornering in high speed with moderate lateral and longitudinal acceleration levels. This means that the vehicle velocity typically varies in the range 30-80[km/h] and the absolute level of the longitudinal and the lateral acceleration is below 3[m/s^2] and 1.5[m/s^2] respectively. Linear lateral tyre properties without effects of combined slip are assumed. Important vehicle measures that needs to be captured by the derived model are the Performance Based Characteristics (PBC) for long heavy vehicle combinations [8] which are related to high speed maneuvering.

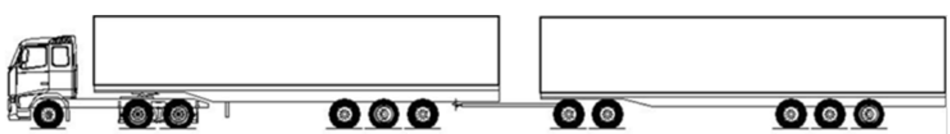

Figure 1.1: *Illustration of an A-double vehicle combination*

2 Vehicle Model

This chapter is organized as follows. First the Lagrangian equations are introduced and transformation to quasi-coordinates are carried out. Secondly, the kinetic energy as well as generalized forces are derived. Finally, simplified equations of motion are presented.

2.1 Lagrange's equations of motion

The Lagrangian equations defined as

$$\frac{d}{dt} \cdot \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \tag{2.1}$$

$i = 1, ..., N$ are the generalized coordinates are according to Boström [4] a system of differential equations. The generalized coordinates q_i are the dependent variables, T is the kinetic energy, V is the potential energy and Q_i are the generalized forces. The main benefit of using Lagrangian formalism when deriving the equations of motion for the given vehicle representation is that coupling forces between the vehicle units are excluded and the number of equations are correspondingly fewer. The chosen generalized coordinates are

$$q = [X,\ Y,\ \phi_1,\ \theta_1,\ \theta_2,\ \theta_3] \tag{2.2}$$

where X and Y are the global coordinates of the center of mass for the first vehicle unit, ϕ_1 is the heading angle of the first unit and θ_i are the articulation angles of the towed units. The parameters are illustrated in Fig. 2.1.

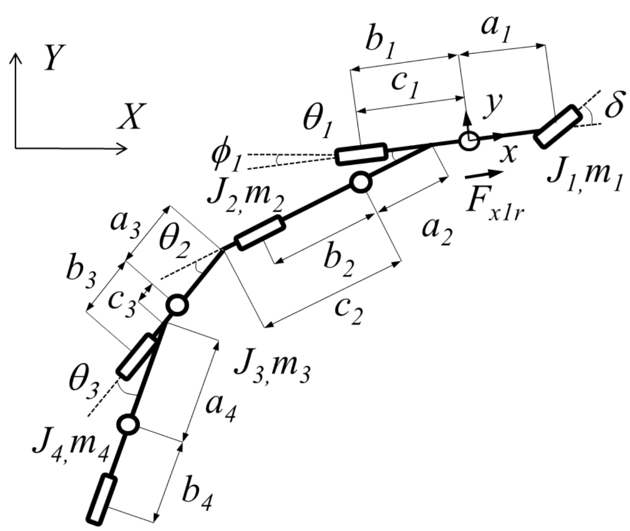

Figure 2.1: *Parameters of a transient one-track model of an A-double combination*

To improve the usefulness of the forthcoming vehicle model, the velocity at the center of mass of the first unit is transformed into a local coordinate system. The longitudinal and lateral velocities, denoted u and v, expressed relative the local coordinate frame are not derivatives of generalized coordinates but may be regarded as derivatives of quasi-coordinates and can therefore be introduced in the Lagrange equations. The relationship between the velocities expressed in the global frame and relative the local coordinate frame are

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \cos(\phi_1) & \sin(\phi_1) \\ -\sin(\phi_1) & \cos(\phi_1) \end{bmatrix} \cdot \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} \tag{2.3}$$

In order to derive the equations of motion in the direction of the quasi-coordinates, the method described in [5], [9] is followed. The transformation from generalized coordinates to quasi-coordinates is done using (2.4).

$$\begin{aligned}
\frac{\partial T}{\partial \dot{X}} &= \frac{\partial T}{\partial u} \cdot \frac{\partial u}{\partial \dot{X}} + \frac{\partial T}{\partial v} \cdot \frac{\partial v}{\partial \dot{X}} = A \cdot \cos\phi_1 - B \cdot \sin\phi_1 \\
\frac{\partial T}{\partial \dot{Y}} &= \frac{\partial T}{\partial u} \cdot \frac{\partial u}{\partial \dot{Y}} + \frac{\partial T}{\partial v} \cdot \frac{\partial v}{\partial \dot{Y}} = A \cdot \sin\phi_1 + B \cdot \cos\phi_1 \\
\frac{\partial T}{\partial X} &= \frac{\partial T}{\partial Y} = 0 \\
\frac{\partial T}{\partial \phi_1} &= \frac{\partial T}{\partial u} \cdot v - \frac{\partial T}{\partial v} \cdot u
\end{aligned} \tag{2.4}$$

Performing the derivatives with respect to time as stated in (2.1) the equations of motions can be written

$$\begin{bmatrix} \cos(\phi_1) & -\sin(\phi_1) \\ \sin(\phi_1) & \cos(\phi_1) \end{bmatrix} \cdot \begin{bmatrix} \dot{A} - B \cdot \dot{\phi}_1 \\ \dot{B} + A \cdot \dot{\phi}_1 \end{bmatrix} = \begin{bmatrix} Q_X \\ Q_Y \end{bmatrix} \tag{2.5}$$

Pre-multiplying (2.5) with the inverse of the yaw rotation matrix and introducing the generalized forces Q_x and Q_y defined in the local frame it follows that the equations of motion in the quasi-coordinates direction can be written as

$$\begin{bmatrix} \dot{A} - B \cdot \dot{\phi}_1 \\ \dot{B} + A \cdot \dot{\phi}_1 \end{bmatrix} = \begin{bmatrix} Q_x \\ Q_y \end{bmatrix} \tag{2.6}$$

2.1.1 Kinetic energy

The kinetic energy of the system is the sum of the rotational and translational energy for all units.

$$T = \sum_{i=1}^{4} \frac{1}{2} \cdot m_i \cdot \bar{v}_i^2 + \frac{1}{2} \cdot J_i \cdot \dot{\phi}_i^2, \quad i = 1, ..., 4 \text{ are the vehicle units} \tag{2.7}$$

In (2.7) m_i are the masses, J_i are the moment of inertia about the z-axis. The potential energy for the system is zero.

In order to calculate the total kinetic energy, the translational velocity vectors $\bar{v}_i$ and the rotational velocity vectors $\dot{\phi}_i$ at the center of mass for each vehicle unit $i = 1, ..., 4$ are needed. Using the global position vectors $\bar{p}_i$ (2.8) as starting point, the global translational velocities are derived in (2.9).

$$\begin{aligned}
\bar{p}_1 &= [X, Y] \\
\bar{p}_2 &= [X - c_1 \cdot \cos\phi_1 - a_2 \cdot \cos\phi_2, \\
&\quad Y - c_1 \cdot \sin\phi_1 - a_2 \cdot \sin\phi_2] \\
\bar{p}_3 &= [X - c_1 \cdot \cos\phi_1 - a_2 \cdot \cos\phi_2 - c_2 \cdot \cos\phi_2 - a_3 \cdot \cos\phi_3, \\
&\quad Y - c_1 \cdot \sin\phi_1 - a_2 \cdot \sin\phi_2 - c_2 \cdot \sin\phi_2 - a_3 \cdot \sin\phi_3] \\
\bar{p}_4 &= [X - c_1 \cdot \cos\phi_1 - a_2 \cdot \cos\phi_2 - c_2 \cdot \cos\phi_2 - a_3 \cdot \cos\phi_3 - c_3 \cdot \cos\phi_3 - a_4 \cdot \cos\phi_4, \\
&\quad Y - c_1 \cdot \sin\phi_1 - a_2 \cdot \sin\phi_2 - c_2 \cdot \sin\phi_2 - a_3 \cdot \sin\phi_3 - c_3 \cdot \sin\phi_3 - a_4 \cdot \sin\phi_4]
\end{aligned} \tag{2.8}$$

$$\begin{aligned}
\bar{v}_1 &= \frac{d}{dt}\bar{p}_1 = [\dot{X}, \dot{Y}] \\
\bar{v}_2 &= \frac{d}{dt}\bar{p}_2 \\
&= [\dot{X} + c_1 \cdot \sin\phi_1 \cdot \dot{\phi}_1 + a_2 \cdot \sin\phi_2 \cdot \dot{\phi}_2, \\
&\quad \dot{Y} - c_1 \cdot \cos\phi_1 \cdot \dot{\phi}_1 - a_2 \cdot \cos\phi_2 \cdot \dot{\phi}_2] \\
\bar{v}_3 &= \frac{d}{dt}\bar{p}_3 \\
&= [\dot{X} + c_1 \cdot \sin\phi_1 \cdot \dot{\phi}_1 + a_2 \cdot \sin\phi_2 \cdot \dot{\phi}_2 + c_2 \cdot \sin\phi_2 \cdot \dot{\phi}_2 + a_3 \cdot \sin\phi_3 \cdot \dot{\phi}_3, \\
&\quad \dot{Y} - c_1 \cdot \cos\phi_1 \cdot \dot{\phi}_1 - a_2 \cdot \cos\phi_2 \cdot \dot{\phi}_2 - c_2 \cdot \cos\phi_2 \cdot \dot{\phi}_2 - a_3 \cdot \cos\phi_3 \cdot \dot{\phi}_3] \\
\bar{v}_4 &= \frac{d}{dt}\bar{p}_4 \\
&= [\dot{X} + c_1 \cdot \sin\phi_1 \cdot \dot{\phi}_1 + a_2 \cdot \sin\phi_2 \cdot \dot{\phi}_2 + c_2 \cdot \sin\phi_2 \cdot \dot{\phi}_2 + a_3 \cdot \sin\phi_3 \cdot \dot{\phi}_3 + c_3 \cdot \sin\phi_3 \cdot \dot{\phi}_3 + a_4 \cdot \sin\phi_4 \cdot \dot{\phi}_4, \\
&\quad \dot{Y} - c_1 \cdot \cos\phi_1 \cdot \dot{\phi}_1 - a_2 \cdot \cos\phi_2 \cdot \dot{\phi}_2 - c_2 \cdot \cos\phi_2 \cdot \dot{\phi}_2 - a_3 \cdot \cos\phi_3 \cdot \dot{\phi}_3 - c_3 \cdot \cos\phi_3 \cdot \dot{\phi}_3 - a_4 \cdot \cos\phi_4 \cdot \dot{\phi}_4]
\end{aligned} \tag{2.9}$$

The rotational velocity vectors of the different units are given as $\dot{\phi}_1$, $\dot{\phi}_2$, $\dot{\phi}_3$ and $\dot{\phi}_4$. For the studied vehicle representation the kinetic energy is given as

$$
\begin{aligned}
T = & \frac{1}{2} \cdot \left(J_1 \cdot \dot{\phi_1}^2 + J_2 \cdot \dot{\phi_2}^2 + J_3 \cdot \dot{\phi_3}^2 + J_4 \cdot \dot{\phi_4}^2\right. \\
& + m_1 \left(\dot{u}^2 + \dot{v}^2\right) \\
& + m_2 \cdot \left(\sin\phi_1 \cdot u + \cos\phi_1 \left(v - c_1 \cdot \dot{\phi_1} - a_2 \cdot \cos\phi_2 \cdot \dot{\phi_2}\right)^2 + \left(\cos\phi_1 \cdot u + \sin\phi_1 \left(-v + c_1 \cdot \dot{\phi_1} + a_2 \cdot \sin\phi_2 \cdot \dot{\phi_2}\right)^2\right)\right) \\
& + m_3 \cdot \left(\left(-\sin\phi_1 \cdot u + \cos\phi_1 \left(-v + c_1\dot{\phi_1}\right) + (a_2 + c_2) \cdot \cos\phi_2 \cdot \dot{\phi_2} + a_3 \cdot \cos\phi_3 \cdot \dot{\phi_3}\right)^2 + \left(\cos\phi_1 \cdot u\right.\right. \\
& \left.\left. + \sin\phi_1 \left(-v + c_1 \cdot \dot{\phi_1}\right) + (a_2 + c_2) \cdot \sin\phi_2 \cdot \dot{\phi_2} + a_3 \cdot \sin\phi_3 \cdot \dot{\phi_3}\right)^2\right) \\
& + m_4 \cdot \left(\left(-\sin\phi_1 \cdot u + \cos\phi_1 \cdot \left(-v + c_1 \cdot \dot{\phi_1}\right) + (a_2 + c_2) \cdot \cos\phi_2 \cdot \dot{\phi_2} + (a_3 + c_3) \cdot \cos\phi_3 \cdot \dot{\phi_3}\right.\right. \\
& \left. + a_4 \cdot \cos\phi_4\right)^2 + \left(\cos\phi_1 \cdot u + \sin\phi_1 \cdot \left(-v + c_1 \cdot \dot{\phi_1}\right) + (a_2 + c_2) \cdot \sin\phi_2 \cdot \dot{\phi_2} + (a_3 + c_3) \cdot \sin\phi_3 \cdot \dot{\phi_3}\right. \\
& \left.\left. + a_4 \cdot \sin\phi_4 \cdot \dot{\phi_4}\right)^2\right)\Big)
\end{aligned}
\tag{2.10}
$$

2.1.2 Generalized forces

According to [4], the generalized forces can be calculated using

$$
Q_i = \sum_{k=1}^{5} F_k \cdot \frac{\partial p_k}{\partial q_i} \tag{2.11}
$$

$k = 1, \ldots, 5$ are the axle positions

$i = 1, \ldots, 6$ are the generalized coordinates

where F_k are the tyre forces, p_k are the position vectors for the forces and q_i are the generalized coordinates.

The tyre forces expressed in the global frame are

$$
\begin{aligned}
F_{1f} &= \left[F_{x1f} \cdot \cos(\phi_1 + \delta) - F_{y1f} \cdot \sin(\phi_1 + \delta),\ F_{x1f} \cdot \sin(\phi_1 + \delta) + F_{y1f} \cdot \cos(\phi_1 + \delta)\right] \\
F_{1r} &= \left[F_{x1r} \cdot \cos(\phi_1) - F_{y1r} \cdot \sin(\phi_1),\ F_{x1r} \cdot \sin(\phi_1) + F_{y1r} \cdot \cos(\phi_1)\right] \\
F_{2r} &= \left[F_{x2r} \cdot \cos(\phi_2) - F_{y2r} \cdot \sin(\phi_2),\ F_{x2r} \cdot \sin(\phi_2) + F_{y2r} \cdot \cos(\phi_2)\right] \\
F_{3r} &= \left[F_{x3r} \cdot \cos(\phi_3) - F_{y3r} \cdot \sin(\phi_3),\ F_{x3r} \cdot \sin(\phi_3) + F_{y3r} \cdot \cos(\phi_3)\right] \\
F_{4r} &= \left[F_{x4r} \cdot \cos(\phi_4) - F_{y4r} \cdot \sin(\phi_4),\ F_{x4r} \cdot \sin(\phi_4) + F_{y4r} \cdot \cos(\phi_4)\right]
\end{aligned}
\tag{2.12}
$$

The positions of the tyres are expressed in global coordinates and can be written as

$$
\begin{aligned}
p_{1f} &= \left[X + a_1 \cdot \cos\phi_1, Y + a_1 \cdot \sin\phi_1\right] \\
p_{1r} &= \left[X - b_1 \cdot \cos\phi_1, Y - b_1 \cdot \sin\phi_1\right] \\
p_{2r} &= [X - c_1 \cdot \cos\phi_1 - (a_2 + b_2) \cdot \cos(\phi_1 + \theta_1), \\
&\qquad Y - c_1 \cdot \sin\phi_1 - (a_2 + b_2) \cdot \sin(\phi_1 + \theta_1)] \\
p_{3r} &= [X - c_1 \cdot \cos\phi_1 - (a_2 + c_2) \cdot \cos(\phi_1 + \theta_1) - (a_3 + b_3) \cdot \cos(\phi_1 + \theta_1 + \theta_2), \\
&\qquad Y - c_1 \cdot \sin\phi_1 - (a_2 + c_2) \cdot \sin(\phi_1 + \theta_1) - (a_3 + b_3) \cdot \sin(\phi_1 + \theta_1 + \theta_2)] \\
p_{4r} &= [X - c_1 \cdot \cos\phi_1 - (a_2 + c_2) \cdot \cos(\phi_1 + \theta_1) - (a_3 + c_3) \cdot \cos(\phi_1 + \theta_1 + \theta_2) - (a_4 + b_4) \cdot \cos(\phi_1 + \theta_1 + \theta_2 + \theta_3), \\
&\qquad Y - c_1 \cdot \sin\phi_1 - (a_2 + c_2) \cdot \sin(\phi_1 + \theta_1) - (a_3 + c_3) \cdot \sin(\phi_1 + \theta_1 + \theta_2) - (a_4 + b_4) \cdot \sin(\phi_1 + \theta_1 + \theta_2 + \theta_3)]
\end{aligned}
\tag{2.13}
$$

The generalized forces which are needed for the Lagrange equations are derived using (2.11) where the tyre forces are expressed in (2.12) and the tyre positions in (2.13). The transformation to quasi-coordinates are carried out in (2.15). All longitudinal resistance forces such as wind- and rolling resistance are neglected.

$$
\begin{aligned}
Q_X =&F_{x1r} \cdot \cos\phi_1 + F_{x1f} \cdot \cos(\delta + \phi_1) + F_{x2r} \cdot \cos(\phi_1 + \theta_1) + F_{x3r} \cdot \cos(\phi_1 + \theta_1 + \theta_2) \\
&+ F_{x4r} \cdot \cos(\phi_1 + \theta_1 + \theta_2 + \theta_3) - F_{y1r} \cdot \sin\phi_1 - F_{y1f} \cdot \sin(\delta + \phi_1) - F_{y2r} \cdot \sin(\phi_1 + \theta_1) \\
&- F_{y3r} \cdot \sin(\phi_1 + \theta_1 + \theta_2) - F_{y4r} \cdot \sin(\phi_1 + \theta_1 + \theta_2 + \theta_3) \\
Q_Y =&F_{x1r} \cdot \sin\phi_1 + F_{x1f} \cdot \sin(\delta + \phi_1) + F_{x2r} \cdot \sin(\phi_1 + \theta_1) + F_{x3r} \cdot \sin(\phi_1 + \theta_1 + \theta_2) \\
&+ F_{x4r} \cdot \sin(\phi_1 + \theta_1 + \theta_2 + \theta_3) + F_{y1r} \cdot \cos\phi_1 + F_{y1f} \cdot \cos(\delta + \phi_1) + F_{y2r} \cdot \cos(\phi_1 + \theta_1) \\
&+ F_{y3r} \cdot \cos(\phi_1 + \theta_1 + \theta_2) + F_{y4r} \cdot \cos(\phi_1 + \theta_1 + \theta_2 + \theta_3) \\
Q_{\phi_1} =&- b_1 \cdot F_{y1r} - (a_2 + b_2) \cdot F_{y2r} - (a_3 + b_3) \cdot F_{y3r} - (a_4 + b_4) \cdot F_{y4r} - c_1 \cdot F_{y2r} \cdot \cos\theta_1 - (a_2 + c_2) \cdot F_{y3r} \cdot \cos\theta_2 \\
&+ a_1 \cdot \Big(F_{y1f} \cdot \cos\delta + F_{x1f} \cdot \sin\delta\Big) - (a_3 + c_3) \cdot \big(F_{y4r} \cdot \cos\theta_3 + F_{x4r} \cdot \sin\theta_3\big) - (a_2 + c_2) \cdot \big(F_{y4r} \cdot \cos(\theta_2 + \theta_3) \\
&+ F_{x3r} \cdot \sin\theta_2 + F_{x4r} \cdot \sin(\theta_2 + \theta_3) - c_1 \cdot \big(F_{y3r} \cdot \cos(\theta_1 + \theta_2) + F_{y4r} \cdot \cos(\theta_1 + \theta_2 + \theta_3) + F_{x2r} \cdot \sin\theta_1 \\
&+ F_{x3r} \cdot \sin(\theta_1 + \theta_2) + F_{x4r} \sin(\theta_1 + \theta_2 + \theta_3) \\
Q_{\theta_1} =&- (a_2 + b_2) \cdot F_{y2r} - (a_3 + b_3) \cdot F_{y3r} - (a_4 + b_4) \cdot F_{y4r} - (a_3 + c_3) \cdot F_{y4r} \cos\theta_3 - (a_3 + c_3) \cdot F_{x4r} \cdot \sin\theta_3 \\
&- (a_2 + c_2) \cdot \sin\theta_2 \cdot \big(F_{x3r} + F_{x4r} \cdot \cos\theta_3 - F_{y4r} \cdot \sin\theta_3\big) \\
Q_{\theta_2} =&- (a_3 + b_3) \cdot F_{y3r} - (a_4 + b_4) \cdot F_{y4r} - (a_3 + c_3) \cdot \big(F_{y4r} \cdot \cos\theta_3 + F_{x4r} \cdot \sin\theta_3\big) \\
Q_{\theta_3} =&- (a_4 + b_4) \cdot F_{y4r}
\end{aligned}
\tag{2.14}
$$

$$
\begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = \begin{bmatrix} \cos(\phi_1) & \sin(\phi_1) \\ -\sin(\phi_1) & \cos(\phi_1) \end{bmatrix} \cdot \begin{bmatrix} Q_X \\ Q_Y \end{bmatrix}
\tag{2.15}
$$

$$
\begin{aligned}
Q_x =&F_{x1r} + F_{x1f} \cdot \cos\delta + F_{x2r} \cdot \cos\theta_1 + F_{x3r} \cdot \cos(\theta_1 + \theta_2) + F_{x4r} \cdot \cos(\theta_1 + \theta_2 + \theta_3) - F_{y1f} \cdot \sin\delta \\
&- F_{y2r} \cdot \sin\theta_1 - F_{y3r} \cdot \sin(\theta_1 + \theta_2) - F_{y4r} \cdot \sin(\theta_1 + \theta_2 + \theta_3) \\
Q_y =&F_{x1f} \cdot \sin\delta + F_{x2r} \cdot \sin\theta_1 + F_{x3r} \cdot \sin(\theta_1 + \theta_2) + F_{x4r} \cdot \sin(\theta_1 + \theta_2 + \theta_3) + F_{y1r} + F_{y1f} \cdot \cos\delta \\
&+ F_{y2r} \cdot \cos\theta_1 + F_{y3r} \cdot \cos(\theta_1 + \theta_2) + F_{y4r} \cdot \cos(\theta_1 + \theta_2 + \theta_3)
\end{aligned}
\tag{2.16}
$$

The generalized forces are simplified using an assumption of small angles i.e $\sin(\alpha) \to \alpha$ and $\cos(\alpha) \to 1$. The assumptions are made for all articulation angles and the steering angle. The resulting simplified generalized forces can be written

$$
Q_{x,simp} = F_{x1f} + F_{x1r} + F_{x2r} + F_{x3r} + F_{x4r} - F_{y1f} \cdot \delta - F_{y2r} \cdot \theta_1 + F_{y3r} \cdot (-\theta_1 - \theta_2) + F_{y4r} \cdot (-\theta_1 - \theta_2 - \theta_3) \tag{2.17a}
$$

$$
Q_{y,simp} = F_{y1f} + F_{y1r} + F_{y2r} + F_{y3r} + F_{y4r} + F_{x1f} \cdot \delta + F_{x2r} \cdot \theta_1 + F_{x3r} \cdot (\theta_1 + \theta_2) + F_{x4r} \cdot (\theta_1 + \theta_2 + \theta_3) \tag{2.17b}
$$

$$
\begin{aligned}
Q_{\phi_1,simp} =& a_1 \cdot F_{y1f} - b_1 \cdot F_{y1r} + (-a_2 - b_2 - c_1) \cdot F_{y2r} + (-a_2 - a_3 - b_3 - c_1 - c_2) \cdot F_{y3r} \\
&+ (-a_2 - a_3 - a_4 - b_4 - c_1 - c_2 - c_3) \cdot F_{y4r} + a_1 \cdot F_{x1f} \cdot \delta - c_1 \cdot F_{x2r} \cdot \theta_1 + F_{x3r} \\
&\cdot (-c_1\theta_1 + (-a_2 - c_1 - c_2) \cdot \theta_2) + F_{x4r} \cdot (-c_1\theta_1 + (-a_2 - c_1 - c_2) \cdot \theta_2 + (-a_2 - a_3 - c_1 - c_2 - c_3) \cdot \theta_3)
\end{aligned}
\tag{2.17c}
$$

$$
\begin{aligned}
Q_{\theta_1,simp} =& (-a_2 - b_2) \cdot F_{y2r} + (-a_2 - a_3 - b_3 - c_2) \cdot F_{y3r} + (-a_2 - a_3 - a_4 - b_4 - c_2 - c_3) \\
&\cdot F_{y4r} + (-a_2 - c_2) \cdot F_{x3r} \cdot \theta_2 + F_{x4r} \cdot ((-a_2 - c_2) \cdot \theta_2 + (-a_2 - a_3 - c_2 - c_3) \cdot \theta_3)
\end{aligned}
\tag{2.17d}
$$

$$
Q_{\theta_2,simp} = (-a_3 - b_3) \cdot F_{y3r} + (-a_3 - a_4 - b_4 - c_3) \cdot F_{y4r} + (-a_3 - c_3) \cdot F_{x4r} \cdot \theta_3 \tag{2.17e}
$$

$$
Q_{\theta_3,simp} = -(a_4 + b_4) \cdot F_{y4r} \tag{2.17f}
$$

2.1.3 Simplified equations of motion

The equations are simplified using an assumption of small steering and articulation angles. The simplifications are in general valid when considering vehicle cornering in high speed and moderate lateral acceleration levels. Also, all products of the steering angle δ, and the states $[\dot{\phi}_1,\ \theta_1,\ \theta_2,\ \theta_3]$ and their time derivatives are set to zero. These simplifications follows the assumption of small steering angles and large cornering radius together with limited vehicle speed. Note that the equations are not fully linearized in the vehicle states since the terms including $v \cdot \dot{\phi}_1, u \cdot \dot{\phi}_1, \dot{u} \cdot \theta_1, \dot{u} \cdot \theta_2$ and $\dot{u} \cdot \theta_3$ are still present.

The simplified equations of motion expressed relative the center of mass for the first unit can then be found as (2.18a)-(2.18f).

$$(m_1 + m_2 + m_3 + m_4) \cdot \left(\dot{u} - v \cdot \dot{\phi}_1\right) = Q_x \tag{2.18a}$$

$$\begin{aligned}&(m_1+m_2+m_3+m_4)\cdot\left(u\cdot\dot{\phi}_1+\dot{v}\right)-((a_3+c_2)\cdot m_3+(a_3+a_4+c_2+c_3)\cdot m_4+a_2\cdot(m_2+m_3+m_4)+c_1\cdot(m_2+m_3+m_4))\cdot\ddot{\phi}_1\\&\quad-((a_3+c_2)\cdot m_3+(a_3+a_4+c_2+c_3)\cdot m_4+a_2\cdot(m_2+m_3+m_4))\cdot\ddot{\theta}_1+(-a_3\cdot m_3-(a_3+a_4+c_3)\cdot m_4)\cdot\ddot{\theta}_2-a_4\cdot m_4\cdot\ddot{\theta}_3=Q_y\end{aligned} \tag{2.18b}$$

$$\begin{aligned}&(-(a_2+c_1)\cdot m_2-(a_2+a_3+c_1+c_2)\cdot m_3-(a_2+a_3+a_4+c_1+c_2+c_3)\cdot m_4)\cdot u\cdot\dot{\phi}_1\\&\quad+(a_2\cdot m_2+(a_2+a_3+c_2)\cdot m_3+(a_2+a_3+a_4+c_2+c_3)\cdot m_4)\cdot\theta_1\cdot\dot{u}+(a_3\cdot m_3+(a_3+a_4+c_3)\cdot m_4)\cdot\theta_2\cdot\dot{u}\\&\quad+a_4\cdot m_4\cdot\theta_3\cdot\dot{u}-(a_2+c_1)\cdot m_2\cdot\dot{v}-(a_2+a_3+c_1+c_2)\cdot m_3\cdot\dot{v}-(a_2+a_3+a_4+c_1+c_2+c_3)\cdot m_4\cdot\dot{v}\\&\quad+\left(J_1+J_2+J_3+J_4+(a_2+c_1)^2\cdot m_2+(a_2+a_3+c_1+c_2)^2\cdot m_3+(a_2+a_3+a_4+c_1+c_2+c_3)^2\cdot m_4\right)\cdot\ddot{\phi}_1+(J_2+J_3+J_4\\&\quad+a_2\cdot(a_2+c_1)\cdot m_2+(a_2+a_3+c_2)\cdot(a_2+a_3+c_1+c_2)\cdot m_3+(a_2+a_3+a_4+c_2+c_3)\cdot(a_2+a_3+a_4+c_1+c_2+c_3)\cdot m_4)\\&\quad\cdot\ddot{\theta}_1+(J_3+J_4+a_3\cdot(a_2+a_3+c_1+c_2)\cdot m_3+(a_3+a_4+c_3)\cdot(a_2+a_3+a_4+c_1+c_2+c_3)\cdot m_4)\\&\quad\cdot\ddot{\theta}_2+(J_4+a_4\cdot(a_2+a_3+a_4+c_1+c_2+c_3)\cdot m_4)\cdot\ddot{\theta}_3=Q_{\phi_1}\end{aligned} \tag{2.18c}$$

$$\begin{aligned}&(-a_2\cdot m_2-(a_2+a_3+c_2)\cdot m_3-(a_2+a_3+a_4+c_2+c_3)\cdot m_4)\cdot u\cdot\dot{\phi}_1\\&\quad+(a_2\cdot m_2+(a_2+a_3+c_2)\cdot m_3+(a_2+a_3+a_4+c_2+c_3)\cdot m_4)\cdot\theta_1\cdot\dot{u}+(a_3\cdot m_3+(a_3+a_4+c_3)\cdot m_4)\cdot\theta_2\cdot\dot{u}+a_4\\&\quad\cdot m_4\cdot\theta_3\cdot\dot{u}-a_2\cdot m_2\cdot\dot{v}-(a_2+a_3+c_2)\cdot m_3\cdot\dot{v}-(a_2+a_3+a_4+c_2+c_3)\cdot m_4\cdot\dot{v}+(J_2+J_3+J_4+a_2\cdot(a_2+c_1)\\&\quad\cdot m_2+(a_2+a_3+c_2)\cdot(a_2+a_3+c_1+c_2)\cdot m_3+(a_2+a_3+a_4+c_2+c_3)\cdot(a_2+a_3+a_4+c_1+c_2+c_3)\cdot m_4)\\&\quad\cdot\ddot{\phi}_1+\left(J_2+J_3+J_4+a_2^2\cdot m_2+(a_2+a_3+c_2)^2\cdot m_3+(a_2+a_3+a_4+c_2+c_3)^2\cdot m_4\right)\\&\quad\cdot\ddot{\theta}_1+(J_3+J_4+a_3\cdot(a_2+a_3+c_2)\cdot m_3+(a_3+a_4+c_3)\cdot(a_2+a_3+a_4+c_2+c_3)\cdot m_4)\\&\quad\cdot\ddot{\theta}_2+(J_4+a_4\cdot(a_2+a_3+a_4+c_2+c_3)\cdot m_4)\cdot\ddot{\theta}_3=Q_{\theta_1}\end{aligned} \tag{2.18d}$$

$$\begin{aligned}&(-a_3\cdot m_3-(a_3+a_4+c_3)\cdot m_4)\cdot u\cdot\dot{\phi}_1+(a_3\cdot m_3+(a_3+a_4+c_3)\cdot m_4)\cdot\theta_1\cdot\dot{u}+(a_3\cdot m_3+(a_3+a_4+c_3)\cdot m_4)\cdot\theta_2\cdot\dot{u}+a_4\cdot m_4\cdot\theta_3\\&\quad\cdot\dot{u}-a_3\cdot m_3\cdot\dot{v}-(a_3+a_4+c_3)\cdot m_4\cdot\dot{v}+(J_3+J_4+a_3\cdot(a_2+a_3+c_1+c_2)\cdot m_3+(a_3+a_4+c_3)\cdot(a_2+a_3+a_4+c_1+c_2+c_3)\cdot m_4)\\&\quad\cdot\ddot{\phi}_1+(J_3+J_4+a_3\cdot(a_2+a_3+c_2)\cdot m_3+(a_3+a_4+c_3)\cdot(a_2+a_3+a_4+c_2+c_3)\cdot m_4)\cdot\ddot{\theta}_1\\&\quad+\left(J_3+J_4+a_3^2\cdot m_3+(a_3+a_4+c_3)^2\cdot m_4\right)\cdot\ddot{\theta}_2+(J_4+a_4\cdot(a_3+a_4+c_3)\cdot m_4)\cdot\ddot{\theta}_3=Q_{\theta_2}\end{aligned} \tag{2.18e}$$

$$\begin{aligned}&(a_4\cdot m_4)\cdot\left(-u\cdot\dot{\phi}_1+\theta_1\cdot\dot{u}+\theta_2\cdot\dot{u}+\theta_3\cdot\dot{u}-\dot{v}\right)+(J_4+a_4\cdot(a_2+a_3+a_4+c_1+c_2+c_3)\cdot m_4)\cdot\ddot{\phi}_1\\&\quad+(J_4+a_4\cdot(a_2+a_3+a_4+c_2+c_3)\cdot m_4)\cdot\ddot{\theta}_1+(J_4+a_4\cdot(a_3+a_4+c_3)\cdot m_4)\cdot\ddot{\theta}_2+\left(J_4+a_4^2\cdot m_4\right)\cdot\ddot{\theta}_3=Q_{\theta_3}\end{aligned} \tag{2.18f}$$

2.2 Lateral tyre model

Firstly, we neglect the effects on the tyre characteristics assigned to re-distribution of the normal load between the axles. Assuming no combined slip and linear tyre properties the lateral tyre forces can be stated as

$$F_{yi} = C_{\alpha i} \cdot \alpha_i = C_{\alpha i} \cdot \left(\tan^{-1}(\frac{v_{yi}}{v_{xi}}) \right), \quad i = 1, ..., 5 \text{ are the axle positions} \tag{2.19}$$

where v_{xi} and v_{yi} are velocities at the axles expressed in wheel coordinates. The velocity vectors for the axles expressed in the global coordinate frame are derived by taking the time derivative of the position vectors defined in (2.10). Transformation of the velocities to local tyre coordinate systems are done by multiplication with the yaw rotation matrices defined in (2.20).

$$\begin{aligned} R_i &= \begin{bmatrix} \sin\psi_i & \cos\psi_i \\ -\sin\psi_i & \cos\psi_i \end{bmatrix}, \quad i = 1, ..., 5 \text{ are the axle positions} \\ \psi_i &= [(\delta + \phi_1), \phi_1, \phi_2, \phi_3, \phi_4] \end{aligned} \tag{2.20}$$

Further, the local velocities are linearized using an assumption of small articulation angles and steering angle. Also, all products including the articulation angles, v and $\dot{\phi}_1$ are set to zero. The final expression of the linear tyre forces are given as

$$\begin{aligned} F_{y1f} &= \left(C_{1f} \cdot (u \cdot \delta - a_1 \cdot \dot{\phi}_1 - \dot{y})\right)/u \\ F_{y1r} &= \left(C_{1r} \cdot (b_1 \cdot \dot{\phi}_1 - \dot{v})\right)/u \\ F_{y2r} &= \left(C_{2r} \cdot \left(u \cdot \theta_1 + (a_2 + b_2 + c_1) \cdot \dot{\phi}_1 + (a_2 + b_2) \cdot \dot{\theta}_1 - \dot{v}\right)\right)/u \\ F_{y3r} &= \left(C_{3r} \cdot \left(u\,(\theta_1 + \theta_2) + (a_2 + a_3 + b_3 + c_1 + c_2) \cdot \dot{\phi}_1 + (a_2 + a_3 + b_3 + c_2) \cdot \dot{\theta}_1 \right.\right. \\ &\quad \left.\left. + (a_3 + b_3) \cdot \dot{\theta}_2 - \dot{v}\right)\right)/u \\ F_{y4r} &= (1/u) \cdot C_{4r} \cdot (u\,(\theta_1 + \theta_2 + \theta_3) + (a_2 + a_3 + a_4 + b_4 + c_1 + c_2 + c_3) \cdot \dot{\phi}_1 + (a_2 + a_3 + a_4 + b_4 \\ &\quad + c_2 + c_3) \cdot \dot{\theta}_1 + (a_3 + a_4 + b_4 + c_3) \cdot \dot{\theta}_2 + (a_4 + b_4) \cdot \dot{\theta}_3 - \dot{v}) \end{aligned} \tag{2.21}$$

2.3 Linear time invariant state-space model for constant vehicle speed

In this section a linear model representing the vehicle cornering behavior at high speed and moderate acceleration levels is presented. As a starting point we use the equations of motion (2.18a)- (2.18f), the simplified generalized forces (2.17a)- (2.17f) and the linear lateral tyre forces (2.21).

The first unit's longitudinal speed u is assumed to be constant and is further named v_x. All terms including the time derivative of u are consequently zero. Assume that each axle longitudinal force is moderate and the articulation and steering angles are small. In (2.17a), the terms including longitudinal forces then becomes much smaller than the terms including lateral forces. Hence we approximate the terms with longitudinal forces to zero in (2.17a)-(2.17f). (Note that this is even exact if all propulsion and braking is done with F_{x1r}). Then the equations of longitudinal motion (2.17a) and (2.18a) becomes decoupled. The equations (2.17a) and (2.18a) can be neglected while solving the motion of the vehicle combination and then used to calculate how a certain linear combination of the axle longitudinal forces has to vary during the maneuver in order to keep v_x constant. A model of propulsion and braking which has a certain distribution between axles can then be used to calculate the longitudinal force on each axle. Eq. (2.17a) and (2.18a) will not be considered more in Section 2.3.

Finally, the state vector $z = \left[v,\ \dot{\phi}_1,\ \theta_1,\ \dot{\theta}_1,\ \theta_2,\ \dot{\theta}_2,\ \theta_3,\ \dot{\theta}_3\right]^T$ and three new equations which describes the relationship between the articulation angles and their time derivative are introduced. The remaining five equations of motion and the additional three equations are linear in the state vector variables and can be written in a linear time invariant state-space form (2.22). The linear system includes eight states and one input. The model matrices and parameters are found in Eq. (2.22)- (2.26) and Appendix A- B.

$$
\begin{aligned}
\dot{z}_{lin} &= M^{-1} \cdot (A \cdot z_{lin} + B \cdot u_{lin}) \\
y_{lin} &= C \cdot z_{lin} + D \cdot u_{lin} \\
z_{lin} &= \left[v,\ \dot{\phi}_1,\ \theta_1,\ \dot{\theta}_1,\ \theta_2,\ \dot{\theta}_2,\ \theta_3,\ \dot{\theta}_3\right]^T \\
u_{lin} &= \delta
\end{aligned}
\tag{2.22}
$$

$$
M = \begin{bmatrix}
M_{11} & M_{12} & 0 & M_{14} & 0 & M_{16} & 0 & M_{18} \\
M_{21} & M_{22} & 0 & M_{24} & 0 & M_{26} & 0 & M_{28} \\
M_{31} & M_{32} & 0 & M_{34} & 0 & M_{36} & 0 & M_{38} \\
M_{41} & M_{42} & 0 & M_{44} & 0 & M_{46} & 0 & M_{48} \\
M_{51} & M_{52} & 0 & M_{54} & 0 & M_{56} & 0 & M_{58} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\tag{2.23}
$$

$$
A = \begin{bmatrix}
A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} \\
A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} & A_{28} \\
A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & A_{37} & A_{38} \\
A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & A_{47} & A_{48} \\
A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} & A_{57} & A_{58} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\tag{2.24}
$$

$$
B = \begin{bmatrix}
C_{1f} \\
a_1 \cdot C_{1f} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\tag{2.25}
$$

$$
C = I \tag{2.26}
$$

$$
D = 0 \tag{2.27}
$$

2.4 Non-linear model for variable vehicle speed

In this section the aim is to derive a model for vehicle cornering at high speed and moderate acceleration levels which includes variable longitudinal vehicle speed. The equations of motion (2.18a)- (2.18f), the simplified generalized forces (2.17a)-(2.17f) and the linear lateral tyre forces (2.21) are used as a starting point.

The vehicle speed u are considered as a model state and the equation of motion in the longitudinal direction cannot be neglected as in the case of the linear time invariant model. The state vector $z_{nl,1} = \left[v,\ \dot{\phi}_1,\ \theta_1,\ \dot{\theta}_1,\ \theta_2,\ \dot{\theta}_2,\ \theta_3,\ \dot{\theta}_3,\ u\right]^T$ and three equations describing the relationship between the the articulation angles and their time derivative are introduced. Utilizing the six equations of motion and the additional three equations, the non-linear model can be written as

$$\begin{aligned} \dot{z}_{nl,1} &= f\left(z_{nl,1}, u_{nl,1}\right) \\ z_{nl,1} &= \left[v,\ \dot{\phi}_1,\ \theta_1,\ \dot{\theta}_1,\ \theta_2,\ \dot{\theta}_2,\ \theta_3,\ \dot{\theta}_3,\ u\right]^T \\ u_{nl,1} &= \left[\delta,\ F_{x1f},\ F_{x1r},\ F_{x2r},\ F_{x3r},\ F_{x4r}\right] \end{aligned} \tag{2.28}$$

No longitudinal tyre slip model are introduced and effects from combined slip are neglected. The model input are the longitudinal forces for all axles. Normally, a propulsion force is only applied on the rear axle of the towing unit. However, braking forces are normally applied on all axles of all units. The explicit expressions of the matrices in (2.28) is not given in this report.

In an attempt to find an even simpler model that still includes the main effects of variable vehicle speed on the vehicle cornering, further simplifications are made. If the longitudinal velocity varies slower than the lateral forces varies, the influence of the longitudinal acceleration on the vehicle cornering can be assumed to be small since this influence appear as $\dot{u} \cdot \theta_1, \dot{u} \cdot \theta_2$ and $\dot{u} \cdot \theta_3$ and both $\dot{u}, \theta_1, \theta_2$ and θ_3 are small. This means that all terms including the time derivative of u are set to zero in all equations of motion apart from the equation describing the longitudinal motion. Further, as in Section 2.3, we assume that each axle longitudinal force is moderate and the articulation and steering angles are small. We approximate the terms including longitudinal forces to zero in (2.17b)-(2.17f). Then the equations of longitudinal motion (2.17a) and (2.18a) becomes decoupled and can be neglected while solving the motion of the vehicle combination. Equation (2.17a) can be used to calculate the sum of longitudinal forces for the axles in order to keep the desired longitudinal vehicle speed. Here the distribution is simplified to a single force acting on the rear axle of the towing unit. This is normally exact for propulsion but an approximation for braking. The final model are very similar to the linear model described in Section 2.3. The differences are that the vehicle speed is considered as a state variable and that the equation of motion in the longitudinal direction is used. The equation of motion in the longitudinal direction is however simplified. The state vector $z_{nl,2} = \left[v,\ \dot{\phi}_1,\ \theta_1,\ \dot{\theta}_1,\ \theta_2,\ \dot{\theta}_2,\ \theta_3,\ \dot{\theta}_3,\ u\right]^T$ and equations describing the relationship between the articulation angles and their time derivative are introduced. The model matrix components and parameters are found in Eq. (2.29)- (2.32) and Appendix A- B.

$$\begin{aligned} \dot{z}_{nl,2} &= M_{nl,2}^{-1} \cdot \left(A\left(u\left(t\right)\right)_{nl,2} \cdot z_{nl,2} + B_{nl,2} \cdot u_{nl,2}\right) \\ z_{nl,2} &= \left[v,\ \dot{\phi}_1,\ \theta_1,\ \dot{\theta}_1,\ \theta_2,\ \dot{\theta}_2,\ \theta_3,\ \dot{\theta}_3\ ,\ u\right]^T \\ u_{nl,2} &= \left[\delta,\ F_{x1r}\right] \end{aligned} \tag{2.29}$$

$$M_{nl,2} = \begin{bmatrix} M_{11} & M_{12} & 0 & M_{14} & 0 & M_{16} & 0 & M_{18} & 0 \\ M_{21} & M_{22} & 0 & M_{24} & 0 & M_{26} & 0 & M_{28} & 0 \\ M_{31} & M_{32} & 0 & M_{34} & 0 & M_{36} & 0 & M_{38} & 0 \\ M_{41} & M_{42} & 0 & M_{44} & 0 & M_{46} & 0 & M_{48} & 0 \\ M_{51} & M_{52} & 0 & M_{54} & 0 & M_{56} & 0 & M_{58} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.30}$$

$$A\left(u\left(t\right)\right)_{nl,2} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} & 0 \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} & A_{28} & 0 \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & A_{37} & A_{38} & 0 \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & A_{47} & A_{48} & 0 \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} & A_{57} & A_{58} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.31}$$

$$B_{nl,2} = \begin{bmatrix} C_{1f} & 0 \\ a_1 \cdot C_{1f} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_1+m_2+m_3+m_4} \end{bmatrix} \tag{2.32}$$

3 Linear tyre cornering stiffness

The linear lateral tyre model considered in this report is described in Section 2.2. The cornering stiffness values for the different axles of the vehicle model have been determined by correlating results from the linear single-track model derived in Section 2.3 and a high fidelity vehicle model. The high fidelity model is a two-track model developed at Volvo GTT in the Matlab/Simulink. The model includes detailed sub-models of the vehicle chassis (including frame compliance), cab suspensions, steering system, powertrain, and brakes. The Magic formula tyre model [9] with combined slip, dynamic relaxation, tyre normal load dependency, and rolling resistance is used as tyre model for all tyres in the vehicle combination. The longitudinal and lateral motion control is performed by a velocity interface for coordinating engine propulsion and braking of each wheel in the combination.

The correlation was formulated as an optimization problem and genetic algorithm optimization from Matlab optimization toolbox has been used to find a solution. The objective function formulated according to (3.1) aimed to minimize the difference in yaw rate gain in the frequency interval 0-2[Hz] at the constant vehicle speed 70[km/h]. The yaw rate gains of the high fidelity model $(r/\delta)_{i,hf}$, where $i = 1,...,4$ represents the vehicle units, were determined using pseudo-random steering input analysis. The yaw rate gains of the linear single track model $(r/\delta)_{i,st}$, $i = 1,...,4$, were determined using the built-in Matlab function *bode*.

$$\min_x \sqrt{\sum_j^N \left(\left((r/\delta)_{1,hf,j} - (r/\delta)_{1,st,j}\right)^2 + \left((r/\delta)_{2,hf,j} - (r/\delta)_{2,st,j}\right)^2 + \left((r/\delta)_{3,hf,j} - (r/\delta)_{3,st,j}\right)^2 + \left((r/\delta)_{4,hf,j} - (r/\delta)_{4,st,j}\right)^2 \right)} \tag{3.1}$$

subject to

$$x^{LB} \leq x \leq x^{UB} \tag{3.2}$$

$j = 1,...,N$ are the number of samples

The optimization variables x_k where $k = 1,...,5$ are the cornering stiffness values of the different axles. The lower and upper bound of the optimization variables were set to $x^{LB} = 1e5$[N/rad] and $x^{UB} = 5e6$[N/rad]. In Fig. 3.1- 3.2, the yaw rate gains of the high fidelity vehicle model are compared to the single-track model including the final cornering stiffness values. The final cornering stiffness values can be found in Table B.1.

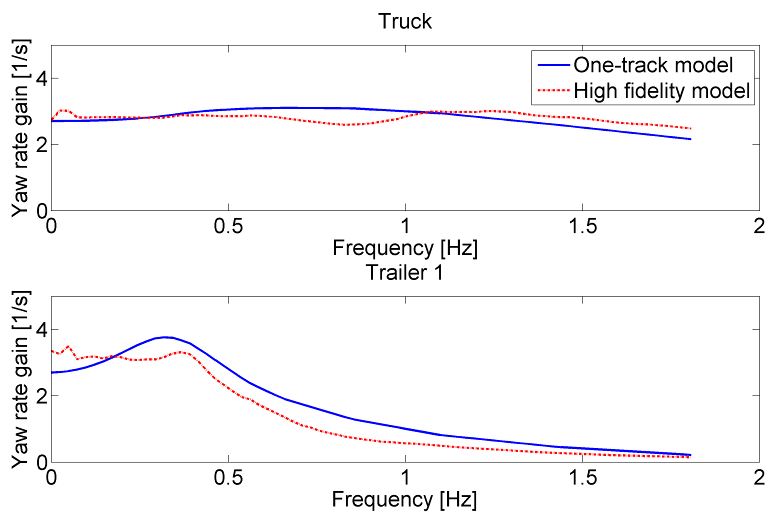

Figure 3.1: *Comparison of yaw rate gains calculated at constant speed 70[km/h] using a high fidelity model and a linear single-track model. The top panel shows the result for the tractor unit and the bottom panel shows the result for the 1st semi-trailer.*

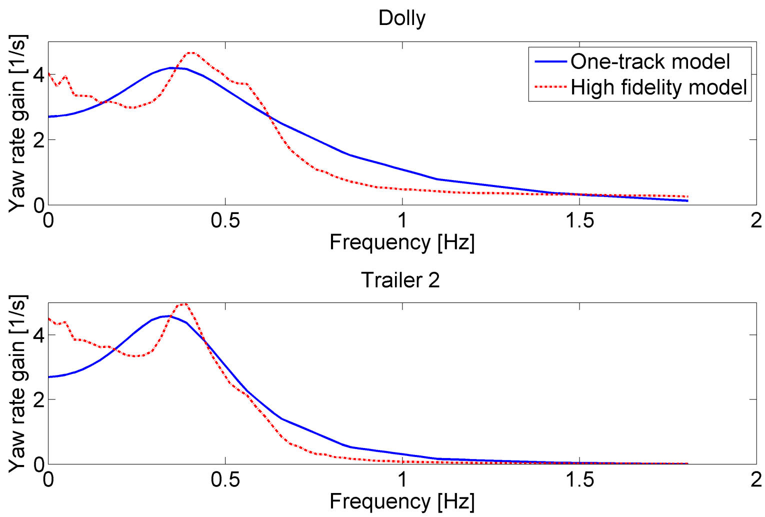

Figure 3.2: *Comparison of yaw rate gains calculated at constant speed 70[km/h] using a high fidelity model and a linear single-track model. The top panel shows the result for the dolly unit and the bottom panel shows the result for the 2nd semi-trailer.*

4 Simulation

In this chapter open-loop simulations have been carried out and the results are compared. The models used in the comparison are the two non-linear models described in 2.4 and the high fidelity model developed at VGTT. The non-linear models are separated by the notation non-linear model 1 and non-linear model 2, as in Section 2.4. For both single-track models an air resistance force equivalent to $F_x = -\frac{1}{2} \cdot c_d \cdot \rho \cdot A \cdot v_x^2$ have been implemented. Here, c_d is the drag coefficient, ρ is the air density, A is the front area of the truck and v_x is the vehicle speed. The simulations are carried out assuming level ground. The performed open-loop analysis are step steer response analysis, single-sine steer response analysis and braking in a turn analysis (ISO14794) [6]. The two first analysis, Section 4.1- 4.2, are carried out at constant vehicle speed which mean that the two single track models are identical to the linear model introduced in 2.3. In the third analysis the vehicle speed is varying and the behavior of the single track models differs.

4.1 Step steer analysis

In the considered analysis, the vehicle combination performs a step steer input with front wheel steering angle δ=1[deg] at 80[km/h] on dry road.

In Fig. 4.1 the longitudinal velocity and acceleration of the first vehicle unit are shown. The vehicle speed is constant and consequently the longitudinal acceleration is zero for all vehicle models. Fig. 4.2 shows the lateral acceleration of the front and last axle of the vehicle combination. The transient behavior that occurs for the front axle in the high fidelity model when the step is initiated at t = 7[s], is not fully captured by any of the single track-models. The constant acceleration level that is reached after the initial transient also differs slightly. It is also noted that for the last axle of semi trailer 2, there is a time delay of the peak acceleration of approximately 0.35[s]. Fig. 4.3 shows the yaw rate of the front and last axle of the vehicle combination. As in the case of the lateral acceleration, there is a time delay of the peak value of approximately 0.35[s] for the last axle. In Fig. 4.4 the global X and Y paths are shown for the first and last axle of the combination. For both axles is can be seen that for a given longitudinal displacement, the high fidelity model has a higher value of the lateral displacement than the single-track models.

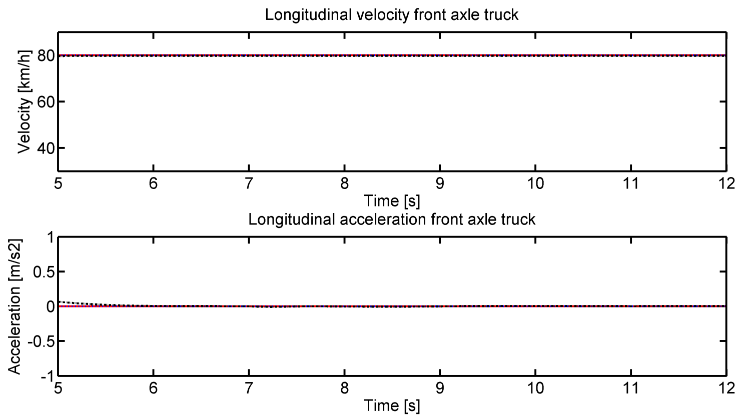

Figure 4.1: *Step steer analysis. Vehicle longitudinal speed (top) and acceleration (bottom). The non-linear model 1 (blue solid line) and the non-linear model 2 (red dashed line) are compared to the high fidelity model (dashed black line)*

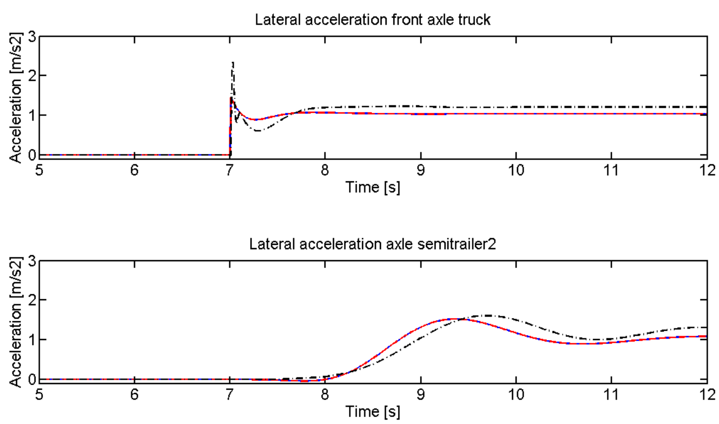

Figure 4.2: *Step steer analysis. Lateral acceleration of the truck front axle (top) and last axle of 2nd semi-trailer (bottom). The non-linear model 1 (blue solid line) and the non-linear model 2 (red dashed line) are compared to the high fidelity model (dashed black line)*

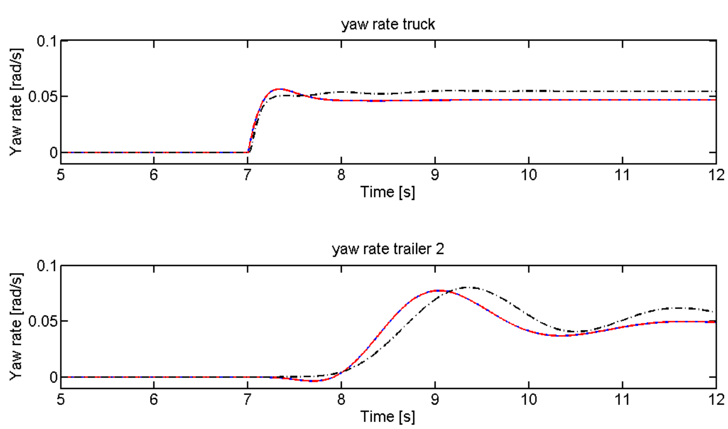

Figure 4.3: *Step steer analysis. Yaw rate of the truck unit (top) and last axle of 2nd semi-trailer (bottom). The non-linear model 1 (blue solid line) and the non-linear model 2 (red dashed line) are compared to the high fidelity model (dashed black line)*

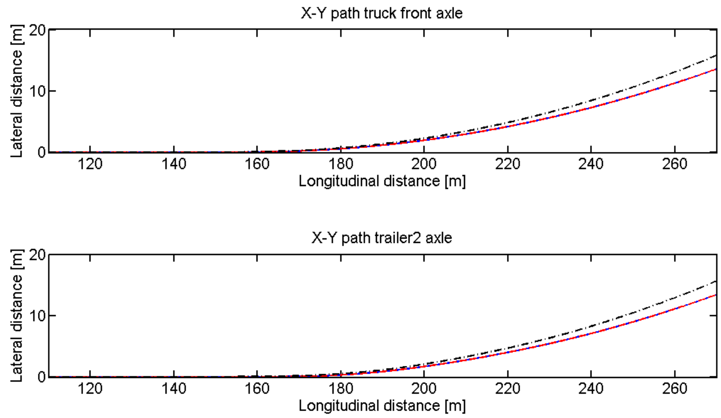

Figure 4.4: *Step steer analysis. Global X and Y paths of the truck front axle (top) and last axle of 2nd semi-trailer (bottom). The non-linear model 1 (blue solid line) and the non-linear model 2 (red dashed line) are compared to the high fidelity model (dashed black line)*

4.2 Single-sine steer analysis

In the considered analysis, the vehicle combination performs a single lane change at 80[km/h] on dry road. The front wheel steering angle input corresponds to a sinusoidal input with the amplitude 3[deg] and frequency 0.4[Hz].

In Fig. 4.5 the longitudinal velocity and acceleration of the first vehicle unit are shown. The vehicle speed is constant and consequently the longitudinal acceleration is zero for all vehicle models. Fig. 4.6 shows the lateral acceleration of the front and last axle of the vehicle combination. It is noted that there is a time delay of the peak acceleration of approximately 0.35[s] for both the front axle of the truck and the last axle of trailer 2. Fig. 4.7 shows the yaw rate of the front and last axle of the vehicle combination. As in the case of the lateral acceleration, there is a time delay of the peak values. The time delay is increasing for the last axle of semi-trailer 2. In Fig. 4.8 the global X and Y paths are shown for the first and last axle of the combination. For the single-track models, the heading angle of the first unit returns to the initial zero value after the sine input. This is not the case for the high fidelity model.

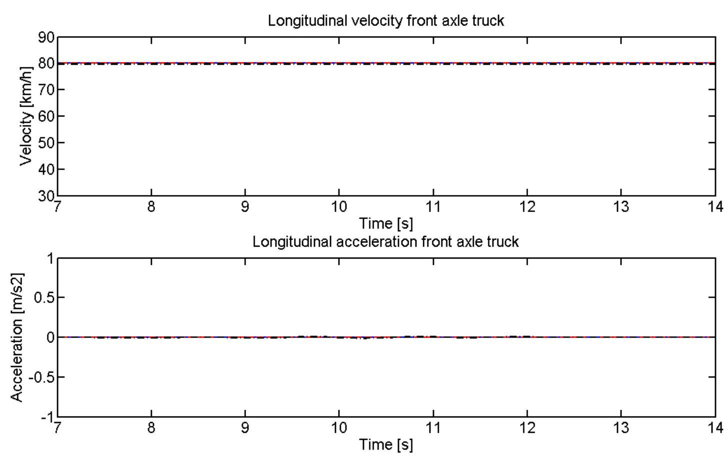

Figure 4.5: *Single-sine analysis. Vehicle longitudinal speed (top) and acceleration (bottom). The non-linear model 1 (blue solid line) and the non-linear model 2 (red dashed line) are compared to the high fidelity model (dashed black line)*

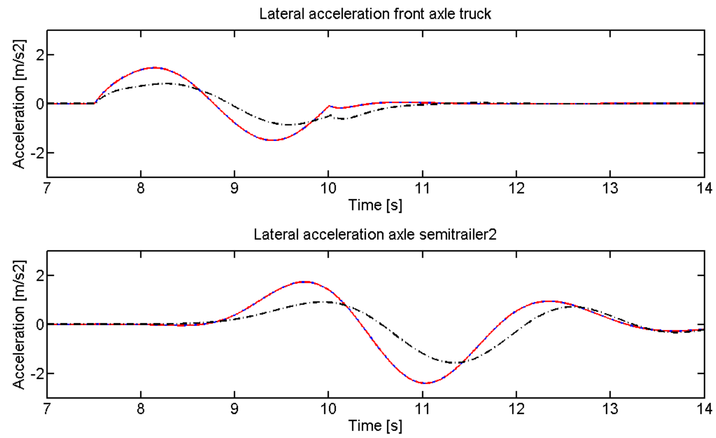

Figure 4.6: *Single-sine analysis. Lateral acceleration of the truck front axle (top) and last axle of 2nd semi-trailer (bottom). The non-linear model 1 (blue solid line) and the non-linear model 2 (red dashed line) are compared to the high fidelity model (dashed black line)*

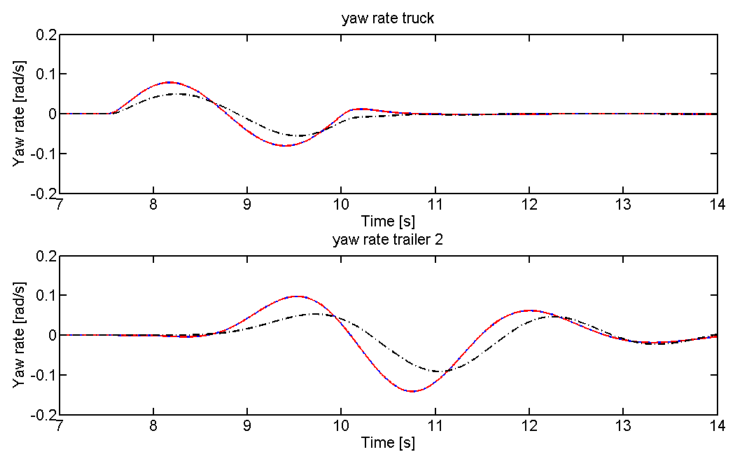

Figure 4.7: *Single-sine analysis. Yaw rate of the truck unit (top) and last axle of 2nd semi-trailer (bottom). The non-linear model 1 (blue solid line) and the non-linear model 2 (red dashed line) are compared to the high fidelity model (dashed black line)*

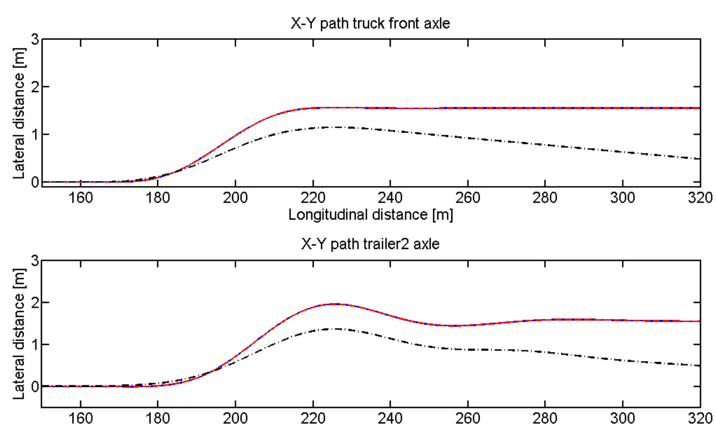

Figure 4.8: *Single-sine analysis. Global X and Y paths of the truck front axle (top) and last axle of 2nd semi-trailer (bottom). The non-linear model 1 (blue solid line) and the non-linear model 2 (red dashed line) are compared to the high fidelity model (dashed black line)*

4.3 Braking in a turn analysis

In the considered analysis, the vehicle combination is starting in a steady-state condition with the driving radius 100[m], lateral acceleration 3[m/s^2] and the vehicle speed 63[km/h]. The vehicle steady-state condition is followed by deceleration to standstill with constant front wheel steering angle.

In Fig. 4.9- 4.10 the lateral acceleration and the yaw rate of the front axle of the vehicle combination are shown. At initiation of the deceleration at the time t = 54[s], a transient behavior can seen in the high fidelity model which is not noted in non-linear model 1. However, in non-linear model 2 a small transient can be noted.

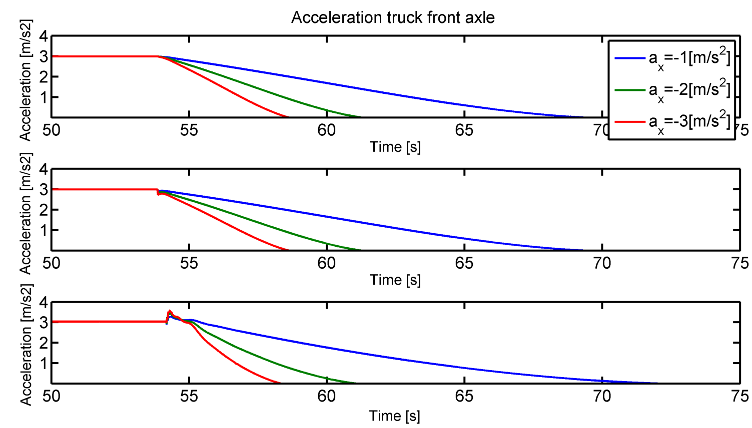

Figure 4.9: *Braking in a turn analysis. Lateral acceleration of the truck front axle. The non-linear model 1 (top), the non-linear model 2 (middle) and the high fidelity model (bottom). Three deceleration levels have been used where a_x = -1[m/s^2] (blue solid line), a_x = -2[m/s^2] (green dashed line), a_x = -3[m/s^2] (red dotted line).*

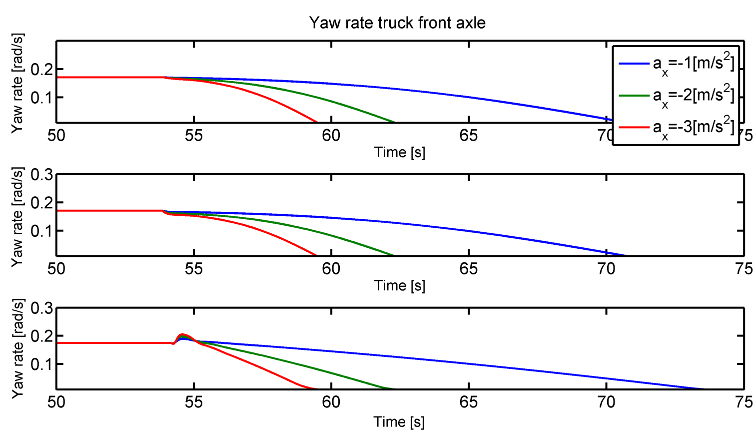

Figure 4.10: *Braking in a turn analysis. Yaw rate of the truck unit. The non-linear model 1 (top), the non-linear model 2 (middle) and the high fidelity model (bottom). Three deceleration levels have been used where a_x = -1[m/s^2] (blue solid line), a_x = -2[m/s^2] (green dashed line), a_x = -3[m/s^2] (red dotted line).*

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A LTI vehicle model matrix components

$$M_{11} = m_1 + m_2 + m_3 + m_4$$

$$M_{12} = -a_3 \cdot m_3 - c_2 \cdot m_3 - a_3 \cdot m_4 - a_4 \cdot m_4 - c_2 \cdot m_4 - c_3 \cdot m_4 - a_2 \cdot (m_2 + m_3 + m_4) - c_1 \cdot (m_2 + m_3 + m_4)$$

$$M_{14} = -c_2 \cdot m_3 - a_4 \cdot m_4 - c_2 \cdot m_4 - c_3 \cdot m_4 - a_3 \cdot (m_3 + m_4) - a_2 \cdot (m_2 + m_3 + m_4)$$

$$M_{16} = -(a_4 + c_3) \cdot m_4 - a_3 \cdot (m_3 + m_4)$$

$$M_{18} = -a_4 \cdot m_4$$

$$M_{21} = -(a_3 + c_2) \cdot m_3 - (a_3 + a_4 + c_2 + c_3) \cdot m_4 - a_2 \cdot (m_2 + m_3 + m_4) - c_1 \cdot (m_2 + m_3 + m_4)$$

$$M_{22} = J_1 + J_2 + J_3 + J_4 + a_2^2 \cdot m_2 + 2 \cdot a_2 \cdot c_1 \cdot m_2 + c_1^2 \cdot m_2 + a_2^2 \cdot m_3 + 2 \cdot a_2 \cdot a_3 \cdot m_3 + a_3^2 \cdot m_3 + 2 \cdot a_2 \cdot c_1 \cdot m_3 + 2 \cdot a_3 \cdot c_1 \cdot m_3 + c_1^2 \cdot m_3 + 2 \cdot a_2 \cdot c_2 \cdot m_3 + 2 \cdot a_3 \cdot c_2 \cdot m_3 + 2 \cdot c_1 \cdot c_2 \cdot m_3 + c_2^2 \cdot m_3 + a_2^2 \cdot m_4 + 2 \cdot a_2 \cdot a_3 \cdot m_4 + a_3^2 \cdot m_4 + 2 \cdot a_2 \cdot a_4 \cdot m_4 + 2 \cdot a_3 \cdot a_4 \cdot m_4 + a_4^2 \cdot m_4 + 2 \cdot a_2 \cdot c_1 \cdot m_4 + 2 \cdot a_3 \cdot c_1 \cdot m_4 + 2 \cdot a_4 \cdot c_1 \cdot m_4 + c_1^2 \cdot m_4 + 2 \cdot a_2 \cdot c_2 \cdot m_4 + 2 \cdot a_3 \cdot c_2 \cdot m_4 + 2 \cdot a_4 \cdot c_2 \cdot m_4 + 2 \cdot c_1 \cdot c_2 \cdot m_4 + c_2^2 \cdot m_4 + 2 \cdot a_2 \cdot c_3 \cdot m_4 + 2 \cdot a_3 \cdot c_3 \cdot m_4 + 2 \cdot a_4 \cdot c_3 \cdot m_4 + 2 \cdot c_1 \cdot c_3 \cdot m_4 + 2 \cdot c_2 \cdot c_3 \cdot m_4 + c_3^2 \cdot m_4$$

$$M_{24} = J_2 + J_3 + J_4 + a_2^2 \cdot m_2 + a_2 \cdot c_1 \cdot m_2 + a_2^2 \cdot m_3 + 2 \cdot a_2 \cdot a_3 \cdot m_3 + a_3^2 \cdot m_3 + a_2 \cdot c_1 \cdot m_3 + a_3 \cdot c_1 \cdot m_3 + 2 \cdot a_2 \cdot c_2 \cdot m_3 + 2 \cdot a_3 \cdot c_2 \cdot m_3 + c_1 \cdot c_2 \cdot m_3 + c_2^2 \cdot m_3 + a_2^2 \cdot m_4 + 2 \cdot a_2 \cdot a_3 \cdot m_4 + a_3^2 \cdot m_4 + 2 \cdot a_2 \cdot a_4 \cdot m_4 + 2 \cdot a_3 \cdot a_4 \cdot m_4 + a_4^2 \cdot m_4 + a_2 \cdot c_1 \cdot m_4 + a_3 \cdot c_1 \cdot m_4 + a_4 \cdot c_1 \cdot m_4 + 2 \cdot a_2 \cdot c_2 \cdot m_4 + 2 \cdot a_3 \cdot c_2 \cdot m_4 + 2 \cdot a_4 \cdot c_2 \cdot m_4 + c_1 \cdot c_2 \cdot m_4 + c_2^2 \cdot m_4 + 2 \cdot a_2 \cdot c_3 \cdot m_4 + 2 \cdot a_3 \cdot c_3 \cdot m_4 + 2 \cdot a_4 \cdot c_3 \cdot m_4 + c_1 \cdot c_3 \cdot m_4 + 2 \cdot c_2 \cdot c_3 \cdot m_4 + c_3^2 \cdot m_4$$

$$M_{26} = J_3 + J_4 + a_2 \cdot a_3 \cdot m_3 + a_3^2 \cdot m_3 + a_3 \cdot c_1 \cdot m_3 + a_3 \cdot c_2 \cdot m_3 + a_2 \cdot a_3 \cdot m_4 + a_3^2 \cdot m_4 + a_2 \cdot a_4 \cdot m_4 + 2 \cdot a_3 \cdot a_4 \cdot m_4 + a_4^2 \cdot m_4 + a_3 \cdot c_1 \cdot m_4 + a_4 \cdot c_1 \cdot m_4 + a_3 \cdot c_2 \cdot m_4 + a_4 \cdot c_2 \cdot m_4 + a_2 \cdot c_3 \cdot m_4 + 2 \cdot a_3 \cdot c_3 \cdot m_4 + 2 \cdot a_4 \cdot c_3 \cdot m_4 + c_1 \cdot c_3 \cdot m_4 + c_2 \cdot c_3 \cdot m_4 + c_3^2 \cdot m_4$$

$$M_{28} = J_4 + a_4 \cdot (a_2 + a_3 + a_4 + c_1 + c_2 + c_3) \cdot m_4$$

$$M_{31} = -(a_3 + c_2) \cdot m_3 - (a_3 + a_4 + c_2 + c_3) \cdot m_4 - a_2 \cdot (m_2 + m_3 + m_4)$$

$$M_{32} = J_2 + J_3 + J_4 + a_2^2 \cdot m_2 + a_2 \cdot c_1 \cdot m_2 + a_2^2 \cdot m_3 + 2 \cdot a_2 \cdot a_3 \cdot m_3 + a_3^2 \cdot m_3 + a_2 \cdot c_1 \cdot m_3 + a_3 \cdot c_1 \cdot m_3 + 2 \cdot a_2 \cdot c_2 \cdot m_3 + 2 \cdot a_3 \cdot c_2 \cdot m_3 + c_1 \cdot c_2 \cdot m_3 + c_2^2 \cdot m_3 + a_2^2 \cdot m_4 + 2 \cdot a_2 \cdot a_3 \cdot m_4 + a_3^2 \cdot m_4 + 2 \cdot a_2 \cdot a_4 \cdot m_4 + 2 \cdot a_3 \cdot a_4 \cdot m_4 + a_4^2 \cdot m_4 + a_2 \cdot c_1 \cdot m_4 + a_3 \cdot c_1 \cdot m_4 + a_4 \cdot c_1 \cdot m_4 + 2 \cdot a_2 \cdot c_2 \cdot m_4 + 2 \cdot a_3 \cdot c_2 \cdot m_4 + 2 \cdot a_4 \cdot c_2 \cdot m_4 + c_1 \cdot c_2 \cdot m_4 + c_2^2 \cdot m_4 + 2 \cdot a_2 \cdot c_3 \cdot m_4 + 2 \cdot a_3 \cdot c_3 \cdot m_4 + 2 \cdot a_4 \cdot c_3 \cdot m_4 + c_1 \cdot c_3 \cdot m_4 + 2 \cdot c_2 \cdot c_3 \cdot m_4 + c_3^2 \cdot m_4$$

$$M_{34} = J_2 + J_3 + J_4 + a_2^2 \cdot m_2 + a_2^2 \cdot m_3 + 2 \cdot a_2 \cdot a_3 \cdot m_3 + a_3^2 \cdot m_3 + 2 \cdot a_2 \cdot c_2 \cdot m_3 + 2 \cdot a_3 \cdot c_2 \cdot m_3 + c_2^2 \cdot m_3 + a_2^2 \cdot m_4 + 2 \cdot a_2 \cdot a_3 \cdot m_4 + a_3^2 \cdot m_4 + 2 \cdot a_2 \cdot a_4 \cdot m_4 + 2 \cdot a_3 \cdot a_4 \cdot m_4 + a_4^2 \cdot m_4 + 2 \cdot a_2 \cdot c_2 \cdot m_4 + 2 \cdot a_3 \cdot c_2 \cdot m_4 + 2 \cdot a_4 \cdot c_2 \cdot m_4 + c_2^2 \cdot m_4 + 2 \cdot a_2 \cdot c_3 \cdot m_4 + 2 \cdot a_3 \cdot c_3 \cdot m_4 + 2 \cdot a_4 \cdot c_3 \cdot m_4 + 2 \cdot c_2 \cdot c_3 \cdot m_4 + c_3^2 \cdot m_4$$

$$M_{36} = J_3 + J_4 + a_2 \cdot a_3 \cdot m_3 + a_3^2 \cdot m_3 + a_3 \cdot c_2 \cdot m_3 + a_2 \cdot a_3 \cdot m_4 + a_3^2 \cdot m_4 + a_2 \cdot a_4 \cdot m_4 + 2 \cdot a_3 \cdot a_4 \cdot m_4 + a_4^2 \cdot m_4 + a_3 \cdot c_2 \cdot m_4 + a_4 \cdot c_2 \cdot m_4 + a_2 \cdot c_3 \cdot m_4 + 2 \cdot a_3 \cdot c_3 \cdot m_4 + 2 \cdot a_4 \cdot c_3 \cdot m_4 + c_2 \cdot c_3 \cdot m_4 + c_3^2 \cdot m_4$$

$$M_{38} = J_4 + a_4 \cdot (a_2 + a_3 + a_4 + c_2 + c_3) \cdot m_4$$

$$M_{41} = -a_3 \cdot m_3 - (a_3 + a_4 + c_3) \cdot m_4$$

$$M_{42} = J_3 + J_4 + a_2 \cdot a_3 \cdot m_3 + a_3^2 \cdot m_3 + a_3 \cdot c_1 \cdot m_3 + a_3 \cdot c_2 \cdot m_3 + a_2 \cdot a_3 \cdot m_4 + a_3^2 \cdot m_4 + a_2 \cdot a_4 \cdot m_4 + 2 \cdot a_3 \cdot a_4 \cdot m_4 + a_4^2 \cdot m_4 + a_3 \cdot c_1 \cdot m_4 + a_4 \cdot c_1 \cdot m_4 + a_3 \cdot c_2 \cdot m_4 + a_4 \cdot c_2 \cdot m_4 + a_2 \cdot c_3 \cdot m_4 + 2 \cdot a_3 \cdot c_3 \cdot m_4 + 2 \cdot a_4 \cdot c_3 \cdot m_4 + c_1 \cdot c_3 \cdot m_4 + c_2 \cdot c_3 \cdot m_4 + c_3^2 \cdot m_4 M_{44}$$

$$= J_3 + J_4 + a_2 \cdot a_3 \cdot m_3 + a_3^2 \cdot m_3 + a_3 \cdot c_2 \cdot m_3 + a_2 \cdot a_3 \cdot m_4 + a_3^2 \cdot m_4 + a_2 \cdot a_4 \cdot m_4 + 2 \cdot a_3 \cdot a_4 \cdot m_4 + a_4^2 \cdot m_4 + a_3 \cdot c_2 \cdot m_4 + a_4 \cdot c_2 \cdot m_4 + a_2 \cdot c_3 \cdot m_4 + 2 \cdot a_3 \cdot c_3 \cdot m_4 + 2 \cdot a_4 \cdot c_3 \cdot m_4 + c_2 \cdot c_3 \cdot m_4 + c_3^2 \cdot m_4$$

$$M_{46} = J_3 + J_4 + a_3^2 \cdot m_3 + a_3^2 \cdot m_4 + 2 \cdot a_3 \cdot a_4 \cdot m_4 + a_4^2 \cdot m_4 + 2 \cdot a_3 \cdot c_3 \cdot m_4 + 2 \cdot a_4 \cdot c_3 \cdot m_4 + c_3^2 \cdot m_4$$

$$M_{48} = J_4 + a_4 \cdot (a_3 + a_4 + c_3) \cdot m_4$$

$$M_{51} = -a_4 \cdot m_4$$

$$M_{52} = J_4 + a_4 \cdot (a_2 + a_3 + a_4 + c_1 + c_2 + c_3) \cdot m_4$$
$$M_{54} = J_4 + a_4 \cdot (a_2 + a_3 + a_4 + c_2 + c_3) \cdot m_4$$
$$M_{56} = J_4 + a_4 \cdot (a_3 + a_4 + c_3) \cdot m_4$$
$$M_{58} = J_4 + a_4^2 \cdot m_4$$
$$A_{11} = -\left(\left(C_{1f} + C_{1r} + C_{2r} + C_{3r} + C_{4r}\right)/v_x\right)$$
$$A_{12} = \left(-a_1 \cdot C_{1f} + b_1 \cdot C_{1r} + a_2 \cdot C_{2r} + b_2 \cdot C_{2r} + c_1 \cdot C_{2r} + a_2 \cdot C_{3r} + a_3 \cdot C_{3r} + b_3 \cdot C_{3r} + c_1 \cdot C_{3r} + c_2 \cdot C_{3r} + a_2 \cdot C_{4r} + a_3 \cdot C_{4r} + a_4 \cdot C_{4r} + b_4 \cdot C_{4r} + c_1 \cdot C_{4r} + c_2 \cdot C_{4r} + c_3 \cdot C_{4r} - m_1 \cdot v_x^2 - m_2 \cdot v_x^2 - m_3 \cdot v_x^2 - m_4 \cdot v_x^2\right)/v_x$$
$$A_{13} = C_{2r} + C_{3r} + C_{4r}$$
$$A_{14} = (b_2 \cdot C_{2r} + a_3 \cdot C_{3r} + b_3 \cdot C_{3r} + c_2 \cdot C_{3r} + a_3 \cdot C_{4r} + a_4 \cdot C_{4r} + b_4 \cdot C_{4r} + c_2 \cdot C_{4r} + c_3 \cdot C_{4r} + a_2 \cdot (C_{2r} + C_{3r} + C_{4r}))/v_x$$
$$A_{15} = C_{3r} + C_{4r}$$
$$A_{16} = (b_3 \cdot C_{3r} + (a_4 + b_4 + c_3) \cdot C_{4r} + a_3 \cdot (C_{3r} + C_{4r}))/v_x$$
$$A_{17} = C_{4r}$$
$$A_{18} = ((a_4 + b_4) \cdot C_{4r})/v_x$$
$$A_{21} = \left(-a_1 \cdot C_{1f} + b_1 \cdot C_{1r} + a_2 \cdot C_{2r} + b_2 \cdot C_{2r} + c_1 \cdot C_{2r} + a_2 \cdot C_{3r} + a_3 \cdot C_{3r} + b_3 \cdot C_{3r} + c_1 \cdot C_{3r} + c_2 \cdot C_{3r} + a_2 \cdot C_{4r} + a_3 \cdot C_{4r} + a_4 \cdot C_{4r} + b_4 \cdot C_{4r} + c_1 \cdot C_{4r} + c_2 \cdot C_{4r} + c_3 \cdot C_{4r}\right)/v_x$$
$$A_{22} = -\left(\left(a_1^2 \cdot C_{1f} + b_1^2 \cdot C_{1r} + a_2^2 \cdot C_{2r} + 2 \cdot a_2 \cdot b_2 \cdot C_{2r} + b_2^2 \cdot C_{2r} + 2 \cdot a_2 \cdot c_1 \cdot C_{2r} + 2 \cdot b_2 \cdot c_1 \cdot C_{2r} + c_1^2 \cdot C_{2r} + a_2^2 \cdot C_{3r} + 2 \cdot a_2 \cdot a_3 \cdot C_{3r} + a_3^2 \cdot C_{3r} + 2 \cdot a_2 \cdot b_3 \cdot C_{3r} + 2 \cdot a_3 \cdot b_3 \cdot C_{3r} + b_3^2 \cdot C_{3r} + 2 \cdot a_2 \cdot c_1 \cdot C_{3r} + 2 \cdot a_3 \cdot c_1 \cdot C_{3r} + 2 \cdot b_3 \cdot c_1 \cdot C_{3r} + c_1^2 \cdot C_{3r} + 2 \cdot a_2 \cdot c_2 \cdot C_{3r} + 2 \cdot a_3 \cdot c_2 \cdot C_{3r} + 2 \cdot b_3 \cdot c_2 \cdot C_{3r} + 2 \cdot c_1 \cdot c_2 \cdot C_{3r} + c_2^2 \cdot C_{3r} + a_2^2 \cdot C_{4r} + 2 \cdot a_2 \cdot a_3 \cdot C_{4r} + a_3^2 \cdot C_{4r} + 2 \cdot a_2 \cdot a_4 \cdot C_{4r} + 2 \cdot a_3 \cdot a_4 \cdot C_{4r} + a_4^2 \cdot C_{4r} + 2 \cdot a_2 \cdot b_4 \cdot C_{4r} + 2 \cdot a_3 \cdot b_4 \cdot C_{4r} + 2 \cdot a_4 \cdot b_4 \cdot C_{4r} + b_4^2 \cdot C_{4r} + 2 \cdot a_2 \cdot c_1 \cdot C_{4r} + 2 \cdot a_3 \cdot c_1 \cdot C_{4r} + 2 \cdot a_4 \cdot c_1 \cdot C_{4r} + 2 \cdot b_4 \cdot c_1 \cdot C_{4r} + c_1^2 \cdot C_{4r} + 2 \cdot a_2 \cdot c_2 \cdot C_{4r} + 2 \cdot a_3 \cdot c_2 \cdot C_{4r} + 2 \cdot a_4 \cdot c_2 \cdot C_{4r} + 2 \cdot b_4 \cdot c_2 \cdot C_{4r} + 2 \cdot c_1 \cdot c_2 \cdot C_{4r} + c_2^2 \cdot C_{4r} + 2 \cdot a_2 \cdot c_3 \cdot C_{4r} + 2 \cdot a_3 \cdot c_3 \cdot C_{4r} + 2 \cdot a_4 \cdot c_3 \cdot C_{4r} + 2 \cdot b_4 \cdot c_3 \cdot C_{4r} + 2 \cdot c_1 \cdot c_3 \cdot C_{4r} + 2 \cdot c_2 \cdot c_3 \cdot C_{4r} + c_3^2 \cdot C_{4r} - a_2 \cdot m_2 \cdot v_x^2 - c_1 \cdot m_2 \cdot v_x^2 - a_2 \cdot m_3 \cdot v_x^2 - a_3 \cdot m_3 \cdot v_x^2 - c_1 \cdot m_3 \cdot v_x^2 - c_2 \cdot m_3 \cdot v_x^2 - a_2 \cdot m_4 \cdot v_x^2 - a_3 \cdot m_4 \cdot v_x^2 - a_4 \cdot m_4 \cdot v_x^2 - c_1 \cdot m_4 \cdot v_x^2 - c_2 \cdot m_4 \cdot v_x^2 - c_3 \cdot m_4 \cdot v_x^2\right)/v_x\right)$$
$$A_{23} = -b_2 \cdot C_{2r} - c_1 \cdot C_{2r} - a_3 \cdot C_{3r} - b_3 \cdot C_{3r} - c_1 \cdot C_{3r} - c_2 \cdot C_{3r} - a_3 \cdot C_{4r} - a_4 \cdot C_{4r} - b_4 \cdot C_{4r} - c_1 \cdot C_{4r} - c_2 \cdot C_{4r} - c_3 \cdot C_{4r} - a_2 \cdot (C_{2r} + C_{3r} + C_{4r})$$
$$A_{24} = -\left(\left(b_2^2 \cdot C_{2r} + b_2 \cdot c_1 \cdot C_{2r} + a_3^2 \cdot C_{3r} + 2 \cdot a_3 \cdot b_3 \cdot C_{3r} + b_3^2 \cdot C_{3r} + a_3 \cdot c_1 \cdot C_{3r} + b_3 \cdot c_1 \cdot C_{3r} + 2 \cdot a_3 \cdot c_2 \cdot C_{3r} + 2 \cdot b_3 \cdot c_2 \cdot C_{3r} + c_1 \cdot c_2 \cdot C_{3r} + c_2^2 \cdot C_{3r} + a_3^2 \cdot C_{4r} + 2 \cdot a_3 \cdot a_4 \cdot C_{4r} + a_4^2 \cdot C_{4r} + 2 \cdot a_3 \cdot b_4 \cdot C_{4r} + 2 \cdot a_4 \cdot b_4 \cdot C_{4r} + b_4^2 \cdot C_{4r} + a_3 \cdot c_1 \cdot C_{4r} + a_4 \cdot c_1 \cdot C_{4r} + b_4 \cdot c_1 \cdot C_{4r} + 2 \cdot a_3 \cdot c_2 \cdot C_{4r} + 2 \cdot a_4 \cdot c_2 \cdot C_{4r} + 2 \cdot b_4 \cdot c_2 \cdot C_{4r} + c_1 \cdot c_2 \cdot C_{4r} + c_2^2 \cdot C_{4r} + 2 \cdot a_3 \cdot c_3 \cdot C_{4r} + 2 \cdot a_4 \cdot c_3 \cdot C_{4r} + 2 \cdot b_4 \cdot c_3 \cdot C_{4r} + c_1 \cdot c_3 \cdot C_{4r} + 2 \cdot c_2 \cdot c_3 \cdot C_{4r} + c_3^2 \cdot C_{4r} + a_2^2 \cdot (C_{2r} + C_{3r} + C_{4r}) + a_2 \cdot (2 \cdot b_2 \cdot C_{2r} + c_1 \cdot (C_{2r} + C_{3r} + C_{4r}) + 2 \cdot (b_3 \cdot C_{3r} + c_2 \cdot C_{3r} + a_4 \cdot C_{4r} + b_4 \cdot C_{4r} + c_2 \cdot C_{4r} + c_3 \cdot C_{4r} + a_3 \cdot (C_{3r} + C_{4r})))\right)/v_x\right)$$
$$A_{25} = -b_3 \cdot C_{3r} - c_1 \cdot C_{3r} - c_2 \cdot C_{3r} - a_4 \cdot C_{4r} - b_4 \cdot C_{4r} - c_1 \cdot C_{4r} - c_2 \cdot C_{4r} - c_3 \cdot C_{4r} - a_2 \cdot (C_{3r} + C_{4r}) - a_3 \cdot (C_{3r} + C_{4r})$$
$$A_{26} = -\left(\left(b_3^2 \cdot C_{3r} + b_3 \cdot c_1 \cdot C_{3r} + b_3 \cdot c_2 \cdot C_{3r} + a_4^2 \cdot C_{4r} + 2 \cdot a_4 \cdot b_4 \cdot C_{4r} + b_4^2 \cdot C_{4r} + a_4 \cdot c_1 \cdot C_{4r} + b_4 \cdot c_1 \cdot C_{4r} + a_4 \cdot c_2 \cdot C_{4r} + b_4 \cdot c_2 \cdot C_{4r} + 2 \cdot a_4 \cdot c_3 \cdot C_{4r} + 2 \cdot b_4 \cdot c_3 \cdot C_{4r} + c_1 \cdot c_3 \cdot C_{4r} + c_2 \cdot c_3 \cdot C_{4r} + c_3^2 \cdot C_{4r} + a_3^2 \cdot (C_{3r} + C_{4r}) + a_2 \cdot (b_3 \cdot C_{3r} + (a_4 + b_4 + c_3) \cdot C_{4r} + a_3 \cdot (C_{3r} + C_{4r})) + a_3 \cdot (2 \cdot b_3 \cdot C_{3r} + c_2 \cdot C_{3r} + 2 \cdot a_4 \cdot C_{4r} + 2 \cdot b_4 \cdot C_{4r} + c_2 \cdot C_{4r} + 2 \cdot c_3 \cdot C_{4r} + c_1 \cdot (C_{3r} + C_{4r}))\right)/v_x\right)$$
$$A_{27} = -(a_2 + a_3 + a_4 + b_4 + c_1 + c_2 + c_3) \cdot C_{4r}$$

$$
\begin{aligned}
A_{28} &= -\left(\left((a_4 + b_4) \cdot (a_2 + a_3 + a_4 + b_4 + c_1 + c_2 + c_3) \cdot C_{4r}\right)/v_x\right)\\
A_{31} &= \left(b_2 \cdot C_{2r} + (a_3 + b_3 + c_2) \cdot C_{3r} + (a_3 + a_4 + b_4 + c_2 + c_3) \cdot C_{4r} + a_2 \cdot (C_{2r} + C_{3r} + C_{4r})\right)/v_x\\
A_{32} &= -\Big(\Big(b_2^2 \cdot C_{2r} + b_2 \cdot c_1 \cdot C_{2r} + (a_3 + b_3)^2 \cdot C_{3r} + a_3 \cdot c_1 \cdot C_{3r} + b_3 \cdot c_1 \cdot C_{3r} + 2 \cdot a_3 \cdot c_2 \cdot C_{3r} + 2 \cdot b_3 \cdot c_2 \cdot C_{3r} + c_1 \cdot c_2 \cdot C_{3r} + c_2^2\\
&\quad \cdot C_{3r} + a_3^2 \cdot C_{4r} + 2 \cdot a_3 \cdot a_4 \cdot C_{4r} + a_4^2 \cdot C_{4r} + 2 \cdot a_3 \cdot b_4 \cdot C_{4r} + 2 \cdot a_4 \cdot b_4 \cdot C_{4r} + b_4^2 \cdot C_{4r} + a_3 \cdot c_1 \cdot C_{4r} + a_4 \cdot c_1 \cdot C_{4r} + b_4 \cdot c_1\\
&\quad \cdot C_{4r} + 2 \cdot a_3 \cdot c_2 \cdot C_{4r} + 2 \cdot a_4 \cdot c_2 \cdot C_{4r} + 2 \cdot b_4 \cdot c_2 \cdot C_{4r} + c_1 \cdot c_2 \cdot C_{4r} + c_2^2 \cdot C_{4r} + 2 \cdot a_3 \cdot c_3 \cdot C_{4r} + 2 \cdot a_4 \cdot c_3 \cdot C_{4r} + 2 \cdot b_4 \cdot c_3\\
&\quad \cdot C_{4r} + c_1 \cdot c_3 \cdot C_{4r} + 2 \cdot c_2 \cdot c_3 \cdot C_{4r} + c_3^2 \cdot C_{4r} + a_2^2 \cdot (C_{2r} + C_{3r} + C_{4r}) - ((a_3 + c_2) \cdot m_3 + (a_3 + a_4 + c_2 + c_3) \cdot m_4) \cdot v_x^2 + a_2\\
&\quad \cdot \Big(2 \cdot b_2 \cdot C_{2r} + 2 \cdot (a_3 + b_3 + c_2) \cdot C_{3r} + 2 \cdot (a_3 + a_4 + b_4 + c_2 + c_3) \cdot C_{4r} + c_1 \cdot (C_{2r} + C_{3r} + C_{4r}) - (m_2 + m_3 + m_4) \cdot v_x^2\Big)\Big)/v_x\Big)\\
A_{33} &= -b_2 \cdot C_{2r} - a_3 \cdot C_{3r} - b_3 \cdot C_{3r} - c_2 \cdot C_{3r} - a_3 \cdot C_{4r} - a_4 \cdot C_{4r} - b_4 \cdot C_{4r} - c_2 \cdot C_{4r} - c_3 \cdot C_{4r} - a_2 \cdot (C_{2r} + C_{3r} + C_{4r})\\
A_{34} &= -\Big(\Big(b_2^2 \cdot C_{2r} + a_3^2 \cdot C_{3r} + 2 \cdot a_3 \cdot b_3 \cdot C_{3r} + b_3^2 \cdot C_{3r} + 2 \cdot a_3 \cdot c_2 \cdot C_{3r} + 2 \cdot b_3 \cdot c_2 \cdot C_{3r} + c_2^2 \cdot C_{3r} + a_3^2 \cdot C_{4r} + 2 \cdot a_3 \cdot a_4 \cdot C_{4r} + a_4^2 \cdot C_{4r} + 2 \cdot a_3 \cdot b_4 \cdot C_{4r}\\
&\quad + 2 \cdot a_4 \cdot b_4 \cdot C_{4r} + b_4^2 \cdot C_{4r} + 2 \cdot a_3 \cdot c_2 \cdot C_{4r} + 2 \cdot a_4 \cdot c_2 \cdot C_{4r} + 2 \cdot b_4 \cdot c_2 \cdot C_{4r} + c_2^2 \cdot C_{4r} + 2 \cdot a_3 \cdot c_3 \cdot C_{4r} + 2 \cdot a_4 \cdot c_3 \cdot C_{4r} + 2 \cdot b_4 \cdot c_3 \cdot C_{4r} + 2 \cdot c_2 \cdot c_3\\
&\quad \cdot C_{4r} + c_3^2 \cdot C_{4r} + a_2^2 \cdot (C_{2r} + C_{3r} + C_{4r}) + 2 \cdot a_2 \cdot (b_2 \cdot C_{2r} + b_3 \cdot C_{3r} + c_2 \cdot C_{3r} + a_4 \cdot C_{4r} + b_4 \cdot C_{4r} + c_2 \cdot C_{4r} + c_3 \cdot C_{4r} + a_3 \cdot (C_{3r} + C_{4r}))\Big)/v_x\Big)\\
A_{35} &= -b_3 \cdot C_{3r} - c_2 \cdot C_{3r} - a_4 \cdot C_{4r} - b_4 \cdot C_{4r} - c_2 \cdot C_{4r} - c_3 \cdot C_{4r} - a_2 \cdot (C_{3r} + C_{4r}) - a_3 \cdot (C_{3r} + C_{4r})\\
A_{36} &= -\Big(\Big(b_3^2 \cdot C_{3r} + b_3 \cdot c_2 \cdot C_{3r} + a_4^2 \cdot C_{4r} + 2 \cdot a_4 \cdot b_4 \cdot C_{4r} + b_4^2 \cdot C_{4r} + a_4 \cdot c_2 \cdot C_{4r} + b_4 \cdot c_2 \cdot C_{4r} + 2 \cdot a_4 \cdot c_3 \cdot C_{4r} + 2 \cdot b_4 \cdot c_3 \cdot C_{4r} + c_2 \cdot c_3 \cdot C_{4r} + c_3^2 \cdot C_{4r}\\
&\quad + a_3^2 \cdot (C_{3r} + C_{4r}) + a_2 \cdot (b_3 \cdot C_{3r} + (a_4 + b_4 + c_3) \cdot C_{4r} + a_3 \cdot (C_{3r} + C_{4r})) + a_3 \cdot (2 \cdot b_3 \cdot C_{3r} + 2 \cdot (a_4 + b_4 + c_3) \cdot C_{4r} + c_2 \cdot (C_{3r} + C_{4r}))\Big)/v_x\Big)\\
A_{37} &= -(a_2 + a_3 + a_4 + b_4 + c_2 + c_3) \cdot C_{4r}\\
A_{38} &= -\left(\left((a_4 + b_4) \cdot (a_2 + a_3 + a_4 + b_4 + c_2 + c_3) \cdot C_{4r}\right)/v_x\right)\\
A_{41} &= \left((a_3 + b_3) \cdot C_{3r} + (a_3 + a_4 + b_4 + c_3) \cdot C_{4r}\right)/v_x\\
A_{42} &= -\Big(\Big(b_3^2 \cdot C_{3r} + a_2 \cdot (a_3 + b_3) \cdot C_{3r} + b_3 \cdot c_1 \cdot C_{3r} + b_3 \cdot c_2 \cdot C_{3r} + (a_4 + b_4)^2 \cdot C_{4r} + a_4 \cdot c_1 \cdot C_{4r} + b_4 \cdot c_1 \cdot C_{4r} + a_4 \cdot c_2 \cdot C_{4r} + b_4 \cdot c_2\\
&\quad \cdot C_{4r} + 2 \cdot a_4 \cdot c_3 \cdot C_{4r} + 2 \cdot b_4 \cdot c_3 \cdot C_{4r} + c_1 \cdot c_3 \cdot C_{4r} + c_2 \cdot c_3 \cdot C_{4r} + c_3^2 \cdot C_{4r} + a_2 \cdot (a_3 + a_4 + b_4 + c_3) \cdot C_{4r} + a_3^2 \cdot (C_{3r} + C_{4r})\\
&\quad - (a_4 + c_3) \cdot m_4 \cdot v_x^2 + a_3 \cdot \Big((2 \cdot b_3 + c_1 + c_2) \cdot C_{3r} + 2 \cdot (a_4 + b_4) \cdot C_{4r} + (c_1 + c_2 + 2 \cdot c_3) \cdot C_{4r} - (m_3 + m_4) \cdot v_x^2\Big)\Big)/v_x\Big)\\
A_{43} &= -b_3 \cdot C_{3r} - (a_4 + b_4 + c_3) \cdot C_{4r} - a_3 \cdot (C_{3r} + C_{4r})\\
A_{44} &= -\Big(\Big(b_3^2 \cdot C_{3r} + b_3 \cdot c_2 \cdot C_{3r} + a_4^2 \cdot C_{4r} + 2 \cdot a_4 \cdot b_4 \cdot C_{4r} + b_4^2 \cdot C_{4r} + a_4 \cdot c_2 \cdot C_{4r} + b_4 \cdot c_2 \cdot C_{4r} + 2 \cdot a_4 \cdot c_3 \cdot C_{4r} + 2 \cdot b_4 \cdot c_3 \cdot C_{4r} + c_2 \cdot c_3 \cdot C_{4r} + c_3^2 \cdot C_{4r}\\
&\quad + a_3^2 \cdot (C_{3r} + C_{4r}) + a_2 \cdot (b_3 \cdot C_{3r} + (a_4 + b_4 + c_3) \cdot C_{4r} + a_3 \cdot (C_{3r} + C_{4r})) + a_3 \cdot (2 \cdot b_3 \cdot C_{3r} + 2 \cdot (a_4 + b_4 + c_3) \cdot C_{4r} + c_2 \cdot (C_{3r} + C_{4r}))\Big)/v_x\Big)\\
A_{45} &= -b_3 \cdot C_{3r} - (a_4 + b_4 + c_3) \cdot C_{4r} - a_3 \cdot (C_{3r} + C_{4r})\\
A_{46} &= -\Big(\Big(b_3^2 \cdot C_{3r} + (a_4 + b_4 + c_3)^2 \cdot C_{4r} + a_3^2 \cdot (C_{3r} + C_{4r}) + 2 \cdot a_3 \cdot (b_3 \cdot C_{3r} + (a_4 + b_4 + c_3) \cdot C_{4r})\Big)/v_x\Big)\\
A_{47} &= -(a_3 + a_4 + b_4 + c_3) \cdot C_{4r}\\
A_{48} &= -\left(\left((a_4 + b_4) \cdot (a_3 + a_4 + b_4 + c_3) \cdot C_{4r}\right)/v_x\right)\\
A_{51} &= \left((a_4 + b_4) \cdot C_{4r}\right)/v_x\\
A_{52} &= -\Big(\Big((a_4 + b_4) \cdot (a_2 + a_3 + a_4 + b_4 + c_1 + c_2 + c_3) \cdot C_{4r} - a_4 \cdot m_4 \cdot v_x^2\Big)/v_x\Big)\\
A_{53} &= -(a_4 + b_4) \cdot C_{4r}\\
A_{54} &= -\left(\left((a_4 + b_4) \cdot (a_2 + a_3 + a_4 + b_4 + c_2 + c_3) \cdot C_{4r}\right)/v_x\right)\\
A_{55} &= -(a_4 + b_4) \cdot C_{4r}\\
A_{56} &= -\left(\left((a_4 + b_4) \cdot (a_3 + a_4 + b_4 + c_3) \cdot C_{4r}\right)/v_x\right)\\
A_{57} &= -(a_4 + b_4) \cdot C_{4r}\\
A_{58} &= -\Big(\Big((a_4 + b_4)^2 \cdot C_{4r}\Big)/v_x\Big)
\end{aligned}
\tag{A1}
$$

B Model parameters

Table B.1: Vehicle model parameters

Parameter	Symbol	Value	Unit
Mass, unit 1	m_1	9841	[kg]
Mass, unit 2	m_2	33601	[kg]
Mass, unit 3	m_3	2700	[kg]
Mass, unit 4	m_4	33801	[kg]
Inertia z-axis, unit 1	J_1	20e3	[kgm^2]
Inertia z-axis, unit 2	J_2	543e3	[kgm^2]
Inertia z-axis, unit 3	J_3	2e3	[kgm^2]
Inertia z-axis, unit 4	J_4	546e3	[kgm^2]
Distance from COG to front axle, unit 1	a_1	1.45	[m]
Distance from COG to connection point front, unit 2	a_2	4.43	[m]
Distance from COG to connection point front, unit 3	a_3	4.55	[m]
Distance from COG to connection point front, unit 4	a_4	4.65	[m]
Distance from COG to rear axle, unit 1	b_1	2.23	[m]
Distance from COG to rear axle, unit 2	b_2	3.27	[m]
Distance from COG to rear axle, unit 3	b_3	0.65	[m]
Distance from COG to rear axle, unit 4	b_4	3.05	[m]
Distance from COG to connection point rear, unit 1	c_1	1.95	[m]
Distance from COG to connection point rear, unit 2	c_2	5.97	[m]
Distance from COG to connection point rear, unit 3	c_3	0.	[m]
Front axle cornering stiffness, unit 1	C_{1f}	4.07e5	[N/rad]
Rear axle cornering stiffness, unit 1	C_{1r}	2.07e6	[N/rad]
Rear axle cornering stiffness, unit 2	C_{2r}	1.24e6	[N/rad]
Rear axle cornering stiffness, unit 3	C_{3r}	1.17e6	[N/rad]
Rear axle cornering stiffness, unit 4	C_{4r}	1.42e6	[N/rad]