DYNAMIC ANALYSIS OF PLANETARY GEAR TRAINS

Part A: **A Multibody Systems - MBS - approach to the analysis**
Part B: **Using DYMOLA to compile and solve the equations of motion**
Part C: **An illustrative numerical examples**

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COMMON ABSTRACT

This paper offers an alternative to most traditional methods to predict or simulate the fully dynamical behavior of planetary gear trains of any configuration. This novel approach is mainly utilizing two concepts that have so far been untraditional at the characterization and analysis of planetary gear train systems:
• a systems approach combined with constraint or gear mesh oriented description of the system, and
• geometrical interpretation of load proportional losses in gear meshes.

The proposed method of analysis is
• fairly general, straightforward and intuitive in the sense of multibody system analysis,
• easily implemented into other computerized analyses, and
• has a subset of procedures for very efficient analysis of planetary gear trains that operate at steady state conditions.

Any basic or compound planetary system is defined in terms of rotating elements (sun wheels, planet wheels and planet carriers) with one rotational degree of freedom each. The rotational motion of those elements is constrained by some number of permanent gear meshes. Each gear mesh is basically characterized by two radii from the theoretical gear mesh to the axes of (relative) rotation of the two specified rotating components. At non-parallel shafts is further information needed on shaft orientation. The positive sense of rotation and applied torque is essential. The concept of virtual shaft extension offers a self-contained method for definition of positive sense.

For consideration of internal inertial effects, the mass moments of inertia about the axis of rotation must be specified. Two kinds of torque losses are considered (which is reasonable at toothed constraints without speed losses):
• torque loss at each gear mesh (in a novel geometrical interpretation), and
• idling or drag torque between any combination of rotating elements as well as the stationary frame.

Part A describes the general background of dynamics of planetary systems and advises a general procedure for compiling the equation of motion for each rotating member. The set of equations obtained contains both ordinary differential equations (for torque equilibrium) and linear algebraic equations (for motion compatibility at constraints). Such a set is common in multibody dynamics and is called ***DAE*** (***D***ifferential ***A***lgebraic ***E***quations). State-of-the-art procedures have recently been developed elsewhere for solving ***DAE***. All together, Part A outlines the theoretical background for development of dedicated stand-alone programs for fully dynamic analysis of planetary gear trains.

Part B demonstrates the use of the ***Dymola*** simulation software to facilitate both the compilation and the numerical solution of dedicated equations of motion of planetary gear trains. A generic module as system component is developed for an external gear mesh or a pair of externally geared wheels on parallel shafts, where both the involved wheels may rotate relative to a rotating frame, i. e., the planet carrier. Mesh losses are considered geometrically in terms of a force pole offset. Any compound planetary gear train (containing so far external wheels on parallel shafts, only) may then be modeled interactively by dragging, dropping and interconnecting shaft ends of that module, assigning each one module two radii and one force pole offset. Inertia effects are modeled by another generic module, and drag losses may be modeled

similarly. Dymola compiles the equations of motion and evaluates the system response to any prescribed external torque or rotation.

Part C demonstrates the application of the Dymola-based methodology to a special planetary train, used as a steering gear of tracked vehicles, where the external and internal dynamic effects are supposed to be significant at transient steering commands. Even if the prime mover (engine) and the vehicle as a whole (on pneumatic wheels) are modeled in a crude way, the different modes of planetary gear train action are clearly demonstrated in agreement with common sense and experimental observations.

The quantitative influence of internal inertia as well as gear mesh losses is evaluated numerically relative to a case without both losses and inertia in the planetary gear train studied.

NOTATION

main symbol	quantity, explanation	unit
ω, w	rotational velocity	rad/s
T	torque	Nm
ΔT	drag or idling torque	Nm
v	translational velocity	m/s
Δv	constrained velocity	m/s
F	(peripheral) force	N
r, R	radius	m
Δr, f	force pole offset	m
P	power	W
J, $[J]$	mass moment(s) of inertia	kgm^2
$[C]$	constraint Jacobian matrix	[m]
$\dot{x}$, der(x)	time derivative (any variable)	any/s
{ }	column vector	
[ ]	matrix	
subscripts		
$1 \ldots i, j \ldots n$	identifier of shaft	
$1 \ldots k \ldots m < n$	identifier of constraint	
pc	special identifier of planet carrier	
superscripts		
inert	inertial action	
ext	external action	
constr	constraint action	
drag	drag action	
relat	relative (to planet carrier)	
mod	modified (due to losses)	

Part A:

1. INTRODUCTION

Characterization and analysis of planetary or epicyclic gear trains has over decades been considered to be a fairly demanding task. Many articles and even books have been devoted to that task. Such trains have a unique capability: they can split torque in definite, prescribed proportions, independent of rotational velocity, which is widely used in various automotive applications, e. g., differentials and automatic transmissions.

There are a number of traditional or established ways to describe the character and to analyze the performance of planetary gear trains, which are briefly reviewed, e. g., by Mägi [1, 2]. The analysis of performance could be subdivided into more or less complicated cases: stationary or transient operating conditions and neglecting or regarding internal power losses. Methods exist to handle all combinations of those cases.

Traditional methods to describe and analyze planetary gear trains have been ***component oriented***, where shafts with zero, one or several gear wheels constitute classical components: planet carriers, sun wheels and planet wheels, which are characterized by basic ratios, i. e., compound transmission ratios for a complete planetary system, when the planet carrier is locked. There exists, however, an alternative and versatile approach, which is influenced of the approach used in the analysis of ***M***ulti-***B***ody ***S***ystems, ***MBS***, which is ***constraint oriented***. This approach provides a fairly straightforward and general procedure for characterization and analysis of all kinds of planetary gear trains, especially when losses have to be considered, and it is very computer friendly.

Such a constraint oriented approach has so far been described, when applied to stationary operating conditions without and with losses, only, by Mägi [2, 3], where the basic approach is introduced to some detail. In the present contribution, which is a third part of that trilogy, the approach is extended to cover fully dynamic or transient operating conditions. It also describes numerical tools to integrate the arising differential equations of motion.

2. SYSTEM DEFINITION

The planetary gear train as a system is described by the following properties:

- rotating members (sun and planet wheels, planet carriers)
- constraints (gears in mesh)
- load proportional gear mesh losses (e. g., force pole offsets)
- load independent shaft losses (idling or drag torques)

At any of totally n unique rotating members, which define the independent variables of the system, there exists rotational velocities, ω, and externally acting torques, T^{ext}, identified by integer subscripts i or $j \le n$, forming the $1 \times n$ column vectors $\{\omega\}$ and $\{T\}^{ext}$. All members are assigned a positive sense, preferably the same for the whole system. If the system contains non-parallel shafts, the concept of virtual shaft extensions still allows the definition of one common positive sense for the whole system, cf. Mägi [2].

Any of totally m unique gear meshes or constraints is identified by the integer subscript $k \le m < n$, defined to act between two specific rotating members, i, j. At each constraint there acts a peripheral constraint force, F_k^{constr}, forming the $1 \times m$ column vector $\{F\}^{constr}$. To each gear mesh are related two meshing radii about the physical axes of rotation involved, i, j, denoted $r_{k,i}$ and $r_{k,j}$, respectively.

To each gear mesh there is related a quantity that defines load proportional losses due to friction, in terms of modified geometry, used at equilibrium considerations, e. g., as the force pole offset, Δr_k (at coplanar shafts), or helix angle offset, $\Delta\psi_k$ (at skew shafts).

Also, between pairs of shafts (the external non-rotating frame of reference included, subscript 0) there might exist idling or drag torques, ΔT_{ij}, from bearings, seals, etc.

3. CONSTRAINT ACTION

Constraint forces, F_k^{constr}, give rise to torque action upon two involved shafts, which is easily found by using the Lagrange multiplier approach. This might be explained by the fact that a given generalized velocity vector of a system, in terms of independent variables, $\{\dot{q}\}^{gen}$, defines the velocities at any other specified point of interest as linear combinations of the independent velocities:

$$\{\dot{q}\}^{spec} = [C]\{\dot{q}\}^{gen}, \tag{1}$$

where $[C]$ at each time instant is a matrix (in case of constraints, the Jacobian of the constraint equations).

Then the generalized force action at specified points, $\{Q\}^{spec}$, contributes to the force action along the generalized coordinates as

$$\{Q\}^{gen} = [C]^T\{Q\}^{spec}. \tag{2}$$

In the present case, $\{\dot{q}\}^{spec} = \{\Delta v\}$ is equal to the virtual difference in peripheral velocities at each gear mesh (seen from the planet carrier), if no teeth as constraints were acting. Then for each of m gear meshes or constraints with teeth, the following equation must hold:

$$\Delta v = (\omega_i - \omega_{pc})r_i \mp (\omega_j - \omega_{pc})r_j = 0, \tag{3}$$

where subscript pc defines the planet carrier involved (if any; if none, $\omega_{pc} = 0$). The upper sign is for an external gear mesh (or equivalent, as described by Mägi [2]), and the lower sign is for an internal gear mesh.

Eq. 3 applied to each gear mesh will all together implicitly define the (Jacobian) constraint or transformation matrix $[C]$:

$$\{\Delta v\} = [C]\{\omega\} = \{0\}, \tag{4}$$

where the elements of the matrix are basically all radii $r_{k,i}$ and $r_{k,j}$. Then the torque action, upon each shaft, due to constraint forces, consistent with virtual speed differences, is easily obtained as

$$\{T\}^{constr} = [C]^T\{F\}^{constr}, \tag{5}$$

where superscript T denotes transpose of the Jacobian matrix.

The Lagrange multiplier approach, as expressed in Eqs 4 and 5, eliminates the need of detailed derivation of torque equilibrium equations, which is a considerable simplification of the analysis of planetary gear trains.

4. EQUATIONS OF LOSS FREE MOTION

The equations of motion for a loss free system are constituted by torque equilibrium relationships for each unique rotating member. In the present case each member has just one degree of motion freedom: rotation about a symmetry axis. Three kinds of torque have to be considered:

- external action: $\{T\}^{ext}$,
- constraint action: $\{T\}^{constr}$, and
- inertial action: $\{T\}^{inert}$.

The external torque action may be described by any functions of time, $\{T\}^{ext} = \{T(t)\}^{ext}$.

The constraint action is a linear combination of initially unknown constraint forces, as given by Eq. 5. It should be noted that $\{F\}^{constr}$, being an internal force, could be defined positive in either sense, which motivates a $\pm$ sign in Eq. 5.

The inertial action may be described in terms of d´Alembert forces as a vector of fictitious torques

$$\{T\}^{inert} = -[J]\{\dot{\omega}\}, \tag{6}$$

where $[J]$ is a diagonal matrix, containing the mass moments of inertia about their physical axes of rotation for each member (for planet carriers the effects of non-rotating masses of planet wheels must be added).

The torque equilibrium then reads

$$\{T\}^{ext} + \{T\}^{constr} + \{T\}^{inert} = \{0\}, \tag{7}$$

which is rearranged after substitution of Eqs 5 and 6:

$$[J]\{\dot{\omega}\} + [C]^T\{F\}^{constr} = \{T(t)\}^{ext}. \tag{8}$$

Eq. 8 is not enough for solving for both $\{\omega\}$ and $\{F\}^{constr}$. To overcome that, Eq. 4 might be differentiated once and added, which yields the final set of equations of motion:

$$\begin{bmatrix} [J] & [C]^T \\ [C] & [0] \end{bmatrix} \begin{Bmatrix} \{\dot{\omega}\} \\ \{F\}^{constr} \end{Bmatrix} = \begin{Bmatrix} \{T(t)\}^{ext} \\ \{0\} \end{Bmatrix}. \tag{9}$$

Eq. 9 contains both differential and algebraic equations and belongs thus to a special category of equations: ***D***ifferential ***A***lgebraic ***E***quations, ***DAE***, for which special methods of solution have been developed, which are described in the literature on ***MBS***. The vector $\{F\}^{constr}$ has here a clear physical interpretation. In more general contexts it is often referred to as the Lagrange multiplier.

The complete solution of Eq. 9 requires additionally initial values of $\{\omega\}$ and yields then the time history of vectors $\{\omega\}$ and $\{F\}^{constr}$.

5. CONSIDERATION OF LOSSES

The consideration of internal torque losses at steady-state operating conditions has been reported in some detail by Mägi [3]. The same methodology will be used here at fully dynamic applications.

Drag or idling torque losses could be incorporated in a straightforward and systematic way by adding a drag torque vector, $\{T\}^{drag}$, to the right hand side of Eq. 9. Individual specified drag torques, ΔT_{ij}, contribute to the i-th and j-th rows of $\{T\}^{drag}$ by

$$T_i^{drag} = -T_j^{drag} = \Delta T_{ij}\text{sign}(\omega_j - \omega_i) = -\Delta T_{ij}\text{sign}(\omega_i - \omega_j). \tag{10}$$

It should be noted that ***DAE*** solvers accept influence of $\{\omega\}$ on the right hand side of Eq. 9.

Consideration of gear mesh losses is facilitated by a special approach introduced by Mägi [3]. Gear mesh losses may be given a geometrical interpretation at consideration of torque equilibrium of each shaft. At coplanar shafts tooth friction virtually moves the point of action of the peripheral component of the contact force from the theoretical pitch point towards the shaft of output of relative power, which is observed from a planet carrier that might be involved. This displacement might be called the force pole offset, denoted $\Delta r_{k,ij}$, which at external gears increases the magnitude of one and reduces of the other of the radii $r_{k,i}$ and $r_{k,j}$, respectively. E. g., at cylindrical external gears the modification is

$$r_{k,i}^{mod} = r_{k.i} + \Delta r_{k,ij}\text{sign}(P_{k,i\to j}^{relat}) \quad \& \tag{11a}$$

$$r_{k,j}^{mod} = r_{k.j} + \Delta r_{k,ij}\text{sign}(P_{k,j\to i}^{relat}), \tag{11b}$$

where the flow of power relative to the planet carrier is

$$P_{k,i\to j}^{relat} = -P_{k,j\to i}^{relat} =$$

$$= F_k^{constr} C_{k,i}(\omega_i - \omega_{pc}) = -F_k^{constr} C_{k,j}(\omega_j - \omega_{pc}), \tag{13}$$

and, further, subscript pc denotes the planet carrier that might be involved and where finally the positive sense of F_k^{constr} in Eq. 8 is defined in such a way that it applies torque in the negative sense to the shaft in question.

For intersecting and skew gears a similar modification of radii or helix angles will take place, as shown briefly by Mägi [3]. All those modifications will modify the transpose of the Jacobian matrix: $[C]^T \to [C^{mod}]^T$, which reflects the changes in the equilibrium conditions in Eq. 9.

The complete equations of motion for planetary gear trains containing internal torque losses are then

$$\begin{bmatrix} [J] & [C^{mod}]^T \\ [C] & [0] \end{bmatrix} \begin{Bmatrix} \{\dot{\omega}\} \\ \{F\}^{constr} \end{Bmatrix} = \begin{Bmatrix} \{T(t)\}^{ext} + \{T\}^{drag} \\ \{0\} \end{Bmatrix}. \tag{14}$$

The geometrical interpretation of gear mesh losses enables the use of the Lagrange multiplier approach for treatment of constraints, which normally must be loss free.

As a remark, speed losses in traction drive type planetary drives could be treated similarly. In addition to force pole offsets due to internal friction, there will also be speed pole offsets, cf. Mägi [1], which will modify the original constraint Jacobian of the system, as it appears in the lower left corner of the composite matrix in Eq. 9.

6. NUMERICAL INTEGRATION

Nowadays, solvers for ***DAE*** start to become available in about the same way as solvers for ***O***rdinary ***D***ifferential ***E***quations, ***ODE***. Until versatile ***DAE*** solvers will become readily available, the equations of motion of planetary gear trains, Eq. 13, could easily and practically be transformed to ***ODE*** form and then be solved by well established numerical methods, available as procedures for most programming languages, including the high level language ***MATLAB***.

Eq. 13 could be rewritten as

$$\begin{Bmatrix} \{\dot{\omega}\} \\ \{F\}^{constr} \end{Bmatrix} = \begin{bmatrix} [J] & [C^{mod}]^T \\ [C] & [0] \end{bmatrix}^{-1} \begin{Bmatrix} \{T(t)\}^{ext} + \{T\}^{drag} \\ \{0\} \end{Bmatrix}. \tag{14}$$

The composite system matrix to be inverted for Eq. 14 is in most cases positive definite and almost symmetrical. As far as the relative power flow doesn´t change direction, it is also constant over several time steps of numerical integration. Thus, it can be inverted relatively easily and mostly also seldom, only when relative power flow changes direction.

The following notation is used to describe the process of decomposition and solution of Eq. 14:

$$\begin{bmatrix} [J] & [C^{mod}]^T \\ [C] & [0] \end{bmatrix}^{-1} = [D] = \begin{bmatrix} [D11] & [D12] \\ [D21] & [D22] \end{bmatrix}, \tag{15}$$

where $[D11]$ is an $n \times n$ submatrix.

A set of ordinary first order differential equations in standard form is then obtained:

$$\{\dot{\omega}\} = [D11]\,(\{T(\omega,t)\}^{ext} + \{T(\omega)\}^{drag}), \tag{16}$$

where it is pointed out the possibility to let the externally acting torques be functions of not only time, but also rotational velocities. To solve this set of differential equations by ODE solvers, initial values of $\{\omega\}$ have to be specified.

Of course, the time history of peripheral constraint forces could also be evaluated in parallel, by evaluating the forces at each time step as

$$\{F\}^{constr} = [D21]\,(\{T(\omega,t)\}^{ext} + \{T(\omega)\}^{drag}). \tag{17}$$

There is, however, a numerical problem with such a procedure. Accumulating numerical integration errors will sooner or later cause the velocity vector not to fulfil the constraint conditions according to Eq. 4. This could be corrected numerically by adjusting now and then the obtained solution to meet the constraint conditions.

7. SPECIAL CONCLUSIONS

The described procedures for characterization and analysis of planetary gear trains are rather systematic and straightforward. The numerical integration of arising equations of motion could be performed efficiently by using just standard ***ODE*** solvers.

Most steps of the compilation of Eq. 13 could be automated. The only steps that are specific to the layout of a planetary gear train are the composition of the constraint matrix $[C]$ and the external torque vector $\{T\}^{ext}$. As an illustration, those items are also easily obtained for the forthcoming example in Figs 3 and 4 of Sect. 10.

Size of system:
n = 10, m = 8 (or 9, when a brake locks).

$$\{T\}^{ext} = (0 \quad T_H \quad T_{AL} \quad T_{AR} \quad T_{CL}^{x/} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)^T$$

x/ when left brake is engaged and until it locks; later zero.

$[C]$ =

$$\begin{bmatrix} -r_G & r_H & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_{AL} & 0 & 0 & 0 & -r_{BL} & 0 & 0 & 0 \\ 0 & 0 & 0 & r_{AR} & 0 & 0 & 0 & -r_{BR} & 0 & 0 \\ r_{FL}-r_{EL} & 0 & 0 & 0 & 0 & 0 & -r_{FL} & 0 & r_{EL} & 0 \\ r_{FR}-r_{ER} & 0 & 0 & 0 & 0 & 0 & 0 & -r_{FR} & 0 & r_{ER} \\ r_{DL}-r_{CL} & 0 & 0 & 0 & r_{CL} & 0 & 0 & 0 & -r_{DL} & 0 \\ r_{DR}-r_{CR} & 0 & 0 & 0 & 0 & r_{CR} & 0 & 0 & 0 & -r_{DR} \\ r_{ER}-r_{EL} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_{EL} & -r_{ER} \end{bmatrix}$$

with an additional 9th row in $[C]$ when, e. g., the left brake has locked and becomes a constraint with associated torque: $[C9] = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$.

NB All radii in $[C]$ might be replaced by corresponding numbers of teeth.

Part B:

8. INTRODUCTION OF GENERIC MODULE

Modern modeling techniques and software, e.g., ***Dymola***, described in Section 12 and by Elmqvist [5], strongly support hiearchically composed models, where generic modules can be used many times in a superordinate model, when they once are defined. In this work, a generic module for an external cylindrical gear ***mesh*** is defined, using ***Dymola*** syntax. An alternative approach is described by Otter [4], where the gear ***wheel*** is used as the main generic module. However, if the load proportional losses should be included in a general way, it is better to use the gear mesh between two wheels as the generic module, which also is the approach in this work. Further on, it has to include a carrier, in order to enable composition of planetary gear trains. In conclusion, this reasoning leads to the generic module defined in Fig. 1.

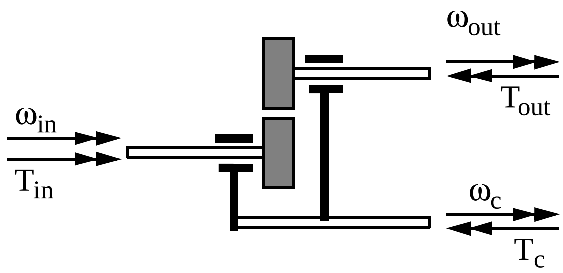

Fig. 1. Generic module for a gear mesh on a carrier with definitions of its interface quantities. T and ω are defined so that T*ω is positive in direction left to right.

Without losses, the equations for the gear mesh in Fig. 1 would read as follows using the notation speed=ω=w:

```
(win-wc)/(wout-wc)=-Rout/Rin
Tin/Rin=-Tout/Rout
Tin = Tout+Tc
```

These three equations can be referred to as compatibility equation, mesh equilibrium and global torque equilibrium, respectively.

9. LOSSES

When load proportional (dry Coulomb friction) losses in the gear mesh are to be considered, in accordance with Section 5, the three equations from Section 8 are still valid with one exception, which is that the mesh equilibrium should read:

```
Tin/(Rin+f)=-Tout/(Rout-f)
```

where `f` is the force pole offset. This offset reflects the coefficient of friction and the gear mesh geometry, cf. Mägi [1]. The magnitude of `f` is, for real gear meshes, almost constant =ΔR. However, f changes sign, dependent on the direction of the relative power flow, `PRel`:

```
PRel=(TIn/(RIn+f))*RIn*(wIn-wOut2)
```

Since f then should be switched between +ΔR and -ΔR, it can be treated as a discrete state variable. The rules for switching `f` are:

- `f` switches from **+ΔR** to -ΔR when `PRel` tends to become negative

and

- `f` switches from -ΔR to +ΔR when `PRel` tends to become positive.

The switching introduces some problems in the case of no relative power. In practice, this situation arises when the two output shafts run with the same speed. In this work, f=0 is then used. Also, a small hysteresis, expressed in relative power, is needed. Hereby, the results from such a situation are physically debatable, but numerically almost true for small losses.

The implementation of the switching is made by means of a "Petri net", see Fig. 2. A Petri net is a kind of finite state machine, with "places" (circles) and transitions (vertical bars), also used by Otter [4]. There is a place called Pos, corresponding to positive relative power, and another one called Neg, corresponding to negative ditto. The additional places, Start and Zero, are introduced in order to handle the simulation initialization and the case of no relative power, respectively.

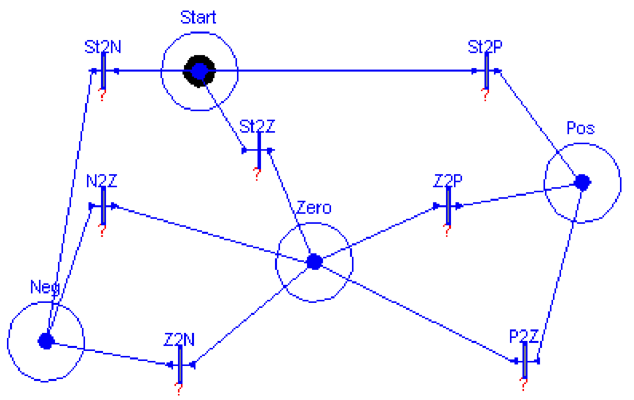

Fig. 2. Petri net used to keep track of direction of relative power flow for the single gear mesh.

In ***Dymola*** syntax, the equations for switching `f` read, in principle as:

```
P2Z.condition=PRel < 0
Z2N.condition=PRel < 0
Z2P.condition=PRel > 0
N2Z.condition=PRel > 0
when P2Z.condition or N2Z.condition then
  new(f)=if P2Z.condition then -deltaR
         else +deltaR
endwhen
```

The technique for modeling speed, torque and losses for the generic module is the same as used in Part A. A minor difference is that Part A has a global approach with no need for modularization.

10. EXAMPLE ON A GEAR TRAIN

Fig. 3 shows an example of a gear train. In Fig. 4 it is modeled by instances of the generic module. It is a steering gear used in tracked vehicles, cf. Jakobsson [6] and Thuvesen [7]. The vehicle is steered by engaging the left or right brake. The gear train is used in a larger system, Part C.

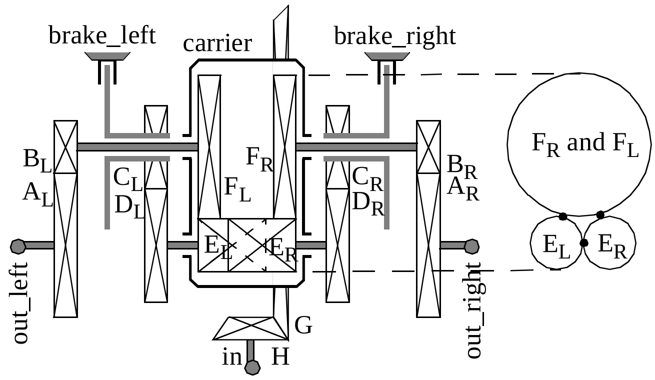

Fig. 3. Sketch of the steering gear, used as gear train example. Letters A...H denote gear wheels. Subscripts L and R denote left and right side, respectively.

Using the graphical model editor of ***Dymola***, the corresponding model of the system will look like shown in Fig. 5.

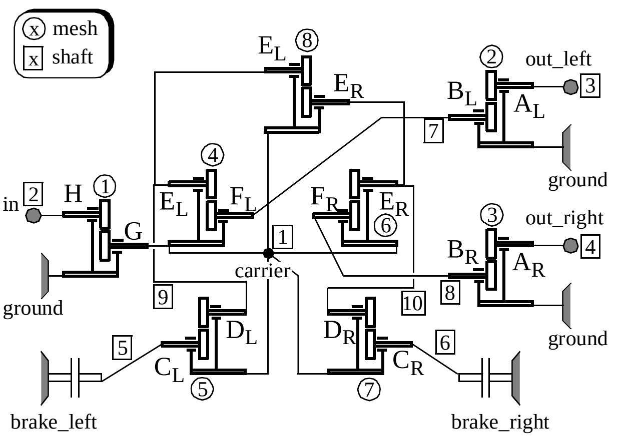

Fig. 4. Model of the gear train in Fig. 3, approximately as visible on computer screen. Also, cf. Fig. 5.

11. GENERIC EXAMPLE ON A SYSTEM

Fig. 6 shows a very simple example on a system using the gear mesh module and Fig. 7 shows corresponding simulation results. The initial conditions are zero speeds for the flywheels and an upwound spring. The flywheels follow the law of motion, `J*der(w)=`ΣT. The spring is linear, i.e., `der(T)=k*`Δw. The model includes no energy dissipation except for the load proportional losses in the gear mesh. In the simulation result one can see that the motion is damped in this way (upper diagram) and that power loss reaches zero

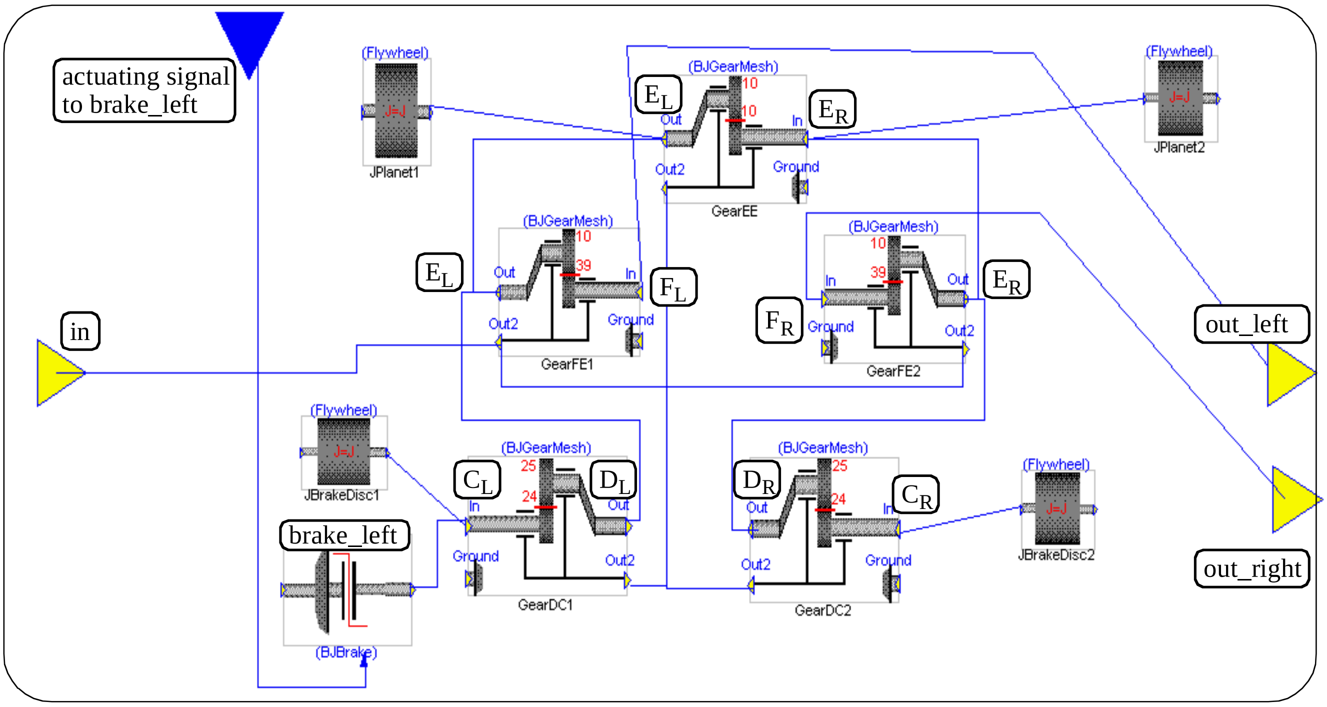

Fig. 5. The steering gear model as visible in graphical model editor on computer screen. Differences compared to Fig. 3 are: Inertias (flywheels) on each shaft are added. Neither gear meshes “G-H”, “A-B” nor brake “brake_right” are included.

every time the relative power reaches zero (lower diagram). Further, the force pole offset toggles between ΔR=+3 and ΔR=-3 governed by the direction of relative power flow.

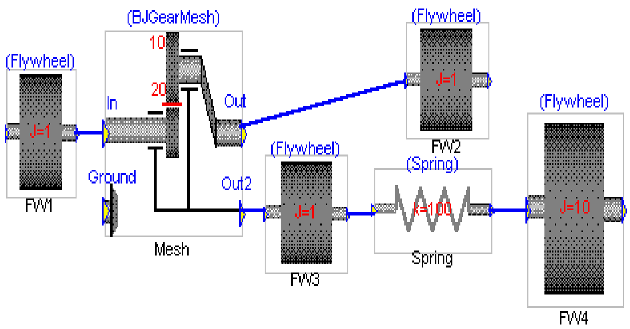

Fig. 6. Simple system as visible in graphical model editor on computer screen.

In the present example there is one constraint between the speeds of the three inertias FW1, FW2 and FW3. This occurs since they are directly interconnected via the gear mesh, which only has two motion degree of freedom. Therefore, ***Dymola*** has to differentiate the speed equations and then eliminate the constrained motion degrees of freedom. Which of the three speeds that should be eliminated is optional, but ***Dymola*** suggests speed of FW1. Then there are only four state variables left: the speeds of FW2, FW3 and FW4 and the torque of the spring. ***Dymola*** generates code for calculating the time derivatives of these four variables as functions of themselves. Then, well established and efficient ***ODE*** integration methods can be used. Also, symbolic manipulations are used so that no iteration is needed. For larger systems, the generated simulation code most often will become more efficient if ***Dymola*** is asked to generate iterative code. The system in Part C is that large that both ways are about equally efficient.

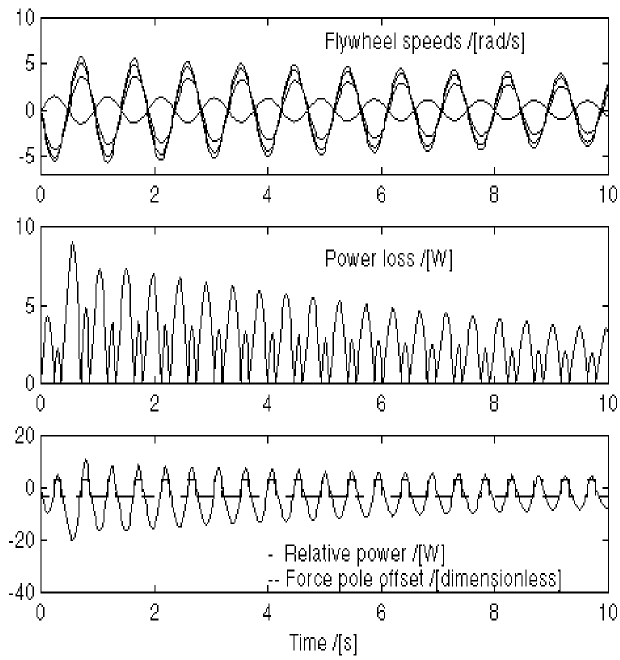

Fig. 7. Simulation results of system in Fig. 6.

A completely different way to use ***Dymola*** is to make use of its ***DAE*** integration methods. That would reduce the need for differentiation and elimination of constrained degrees of freedom. Probably, there is some diffuse limit in system complexity, where the ***DAE*** way is better. However, even in the system investigated in Part C, using the relatively complex gear train from Fig. 5, the ***ODE*** way is successful.

12. SHORT ABOUT *DYMOLA*

This section has the character of an Appendix. It describes the Software ***Dymola***, see also Elmqvist [5]. ***Dymola*** is a software for modeling and simulation of dynamic systems. One can define own "model classes" and use them in a model, see Fig. 8. ***Dymola*** has some important features:

- ***Object orientation***, which, e. g., means that the two springs in Fig. 8 are "instances" of the same model class. Hereby, the spring equation is only defined in one place, referenced to twice in the model. Parameters, such as spring stiffness **`k`**, can have different numerical values in different instances.
- ***Physical connection***, which means that a connection between two submodels differs between "across" and "flow" variables. "Across" and "flow" variables are variables that should be equal and summed to zero in a connection, respectively. E. g., in Fig. 8, the connections are the lines between the submodels. The connections generate the following equations:
  `F.w=S1.w`
  `F.w=S2.w`
  `-F.T+S1.w+S2.w=0`
  where the dot notation **`A.x`** means the variable **`x`** in submodel **`A`**.
- ***Equation orientation*** (or symbolic modeling), which means that the equations are automatically transformed to assignment form through symbolic manipulations. E.g., the flywheel equation in Fig. 8 could as well have been written **`J*der(w)-T=0`**, instead. The assignment form of a model tells how the time derivatives of the state variables are calculated knowing the state variables. In Fig. 8, the assignment form will be:
  `der(F.w)=-T`
  `der(S1.T)=S1.k*F.w`
  `der(S2.T)=S2.k*F.w`
  where **`F.w`**, **`S1.T`** and **`S2.T`** are state variables.

13. SPECIAL CONCLUSIONS

A generic module has been defined, by which general planetary gear trains can be modeled. The load proportional losses are considered. The loss model uses the force pole offset, which is automatically switched between a positive and a negative value when relative power changes sign. The situation when the relative power is zero is not covered and might be object for further development of the generic module. Also, the generic module is so far not prepared for handling internally geared wheels or wheels with non-parallel shafts.

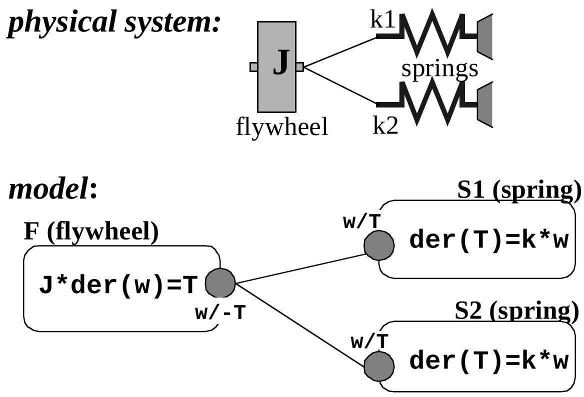

Fig. 8. Example of a *Dymola* model -- a rotating mass (flywheel) connected to rotational elasticities (flexible shafts).

Part C:

14. INTRODUCTION OF A STEERING SYSTEM AS AN APPLICATION EXAMPLE

Many planetary gear trains are used at either steady state or quasi-dynamic operating conditions, which is out of the present scope.

To demonstrate the application of the fully dynamic analysis of complex planetary gear trains, an example has here been picked from the field of vehicle dynamics. It concerns the problem of steering tracked vehicles by using a special planetary gear train. This problem was at an early stage treated by Jakobsson [6] when occurring at steady state operating conditions. Recently, it has been partially expanded to the dynamic range, as well, by Thuvesen [7].

From that example it is understood that the steering process undergoes a number of different phases:

- No brake is engaged: the gear train operates as a traditional differential that divides the propulsion effort evenly between left end right (the vehicle travels straight ahead).
- Left or right side brake is just engaged: the brake slips and applies a definite torque to the rotor, whereby the vehicle starts to change direction in a transient mode with differentiated propulsion effort on left and right hand side.
- The brake locks at some instant: a definite constant speed ratio and a differentiated torque ratio is created and maintained between the left and right hand side, which both continue to bring the vehicle asymptotically towards the limit steady state turning circle.
- Eventually, the vehicle comes to steady state turning.

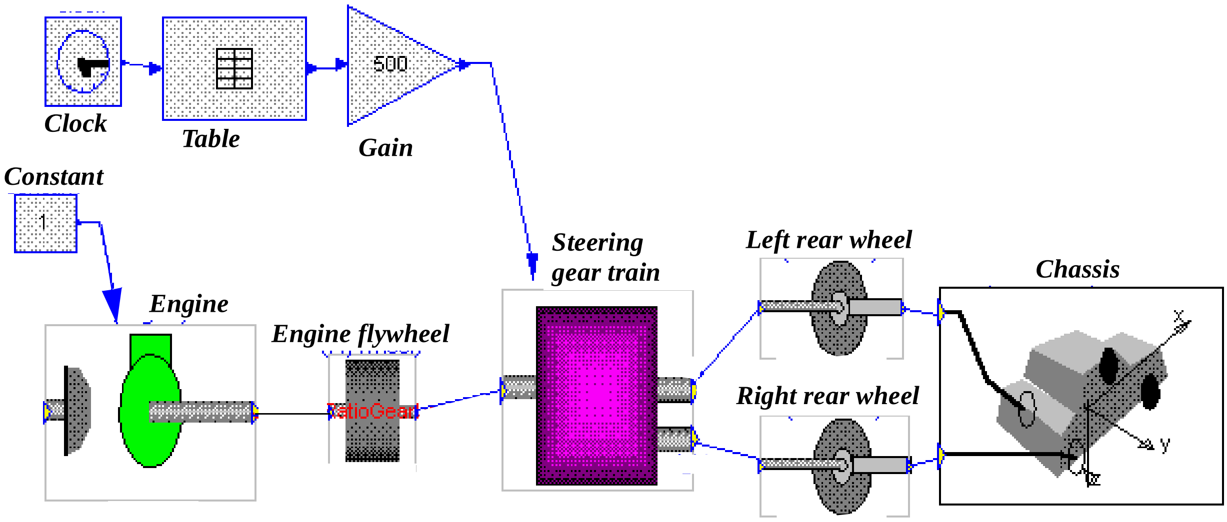

Fig. 9. Planetary steering gear system; the steering gear train is modeled in detail in Fig. 5.

Thuvesen used the ***MBS*** program package ***ADAMS*** [8] to describe and study the dynamics of the entire system, where the main emphasis was on tracks as a new program and system module. The planetary gear train was treated in a simplified way as a number of on purpose created equations that governed the motion of the planetary gear train.

In the present work ***Dymola*** is chosen as a tool for systematically composing and automatically solving the equations of motion of a planetary steering gear train according to Parts A and B.

The gear train is embedded in a larger system according to Fig. 9 to illustrate its total and integrated behavior as part of a steering system.

This system asks for the creation of some other dedicated special modules in ***Dymola*** syntax:

- the engine,
- the chassis, and
- the (driving) wheels.

15. SOME SPECIAL MODULES

The special modules needed here are intentionally modeled in a crude way without any sophistication, just to make them simple, allowing the planetary gear train module to be highlighted.

THE ENGINE MODULE:

The engine is defined in terms of a performance map, Fig. 10, displaying developed torque versus rotational velocity at various accelerator pedal positions. The maximum torque curve is intended to remind of real world internal combustion engines.

The corresponding code in ***Dymola*** syntax for the engine module is given in Tab. 1.

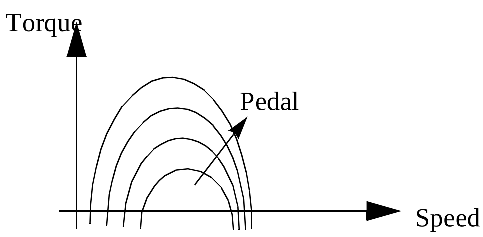

Fig. 10. Performance map of a hypothetical engine.

Tab. 1. *Dymola* code for engine module.

```
model class Base
  cut In (wIn/TIn)
  cut Out (wOut/-TOut)
end
model class (Base) Engine
local A {Defined so that TOut=0 at -50 rad/s}
parameter MaxTorque=200
parameter SpeedMaxTorque =250
  cut Pedal (Pedal)
TOut=((Pedal+0.1)/1.1)* ->
      (MaxTorque-A*(wOut-SpeedMaxTorque)**2)
0=MaxTorque-A*(-50-SpeedMaxTorque)**2 {Defines A}
wIn=0
end
```

THE CHASSIS MODULE:

The chassis is modeled as a three-wheeled vehicle with two rear driving wheels and a fixed, soft front wheel with low cornering stiffness. The chassis has three degrees of motion freedom in the horizontal plane: in longitudinal, transverse and yaw directions. It has a mass and a moment of inertia about a vertical axis. The fixed front wheel is intended to stabilize the traveling motion. At cornering, it is assumed to respond linearly to transverse slip. Its equations of motion are derived in a moving frame of reference, cf. Wong [9]. The steering action is obtained by a difference in peripheral velocities of the two rear driving wheels (cf. tracked vehicles), which transmit both longitudinal and transverse forces, while the front wheel transmits transverse force only.

The corresponding code in ***Dymola*** syntax for the chassis module is given in Tab. 2.

Tab. 2. *Dymola* code for the chassis module.

```
model class Chassis
parameter m=1000 {mass, kg}
parameter J=1000 {moment of inertia, kg*m^2}
parameter B=2 {track width, m}, L=3 {wheel base,m}
parameter WShare_r=0.75 {weight share on rear axes}
parameter RollCoeff=0.02 {N/N}
parameter AirCoeff=0.5 {N/((m/s)^2)}
parameter VxInit=0.01 {initial speed, m/s}
parameter Calphaf=1e3 {undriven front, per wheel}
constant g=9.81 {m/s^2}
output Fx,Mz,X,Y,PHI
local Resist,Vx=VxInit,Vy,Omegaz
local Fyf, Fyr, alphaf
  cut LeftShaft (VxL, VyL / FxL, FyL, NL)
  cut RightShaft1 (VxR, VyR / FxR, FyR, NR)
Fx=FxL+FxR
Mz=FxL*B/2-FxR*B/2
Resist=RollCoeff*m*g+AirCoeff*Vx*abs(Vx)
Fyr=FyL+FyR
alphaf=arctan((-Omegaz*L/2-Vy)/Vx)
Fyf=2*Calphaf*alphaf
m*(der(Vx)-Vy*Omegaz)=Fx-Resist
m*(der(Vy)+Vx*Omegaz)=Fyf+Fyr
J*der(Omegaz)=Fyf*(L/2)-Fyr*(L/2)+Mz
der(PHI)=Omegaz
der(X)=+Vx*cos(PHI)-Vy*sin(PHI)
der(Y)=+Vx*sin(PHI)+Vy*cos(PHI)
NL=WShare_r*m*g/2;   NR=WShare_r*m*g/2
VyL=Vy-Omegaz*L/2;   VyR=Vy-Omegaz*L/2
VxL=Vx+Omegaz*B/2;   VxR=Vx-Omegaz*B/2
end
```

Tab. 3. *Dymola* code for the driving wheel module.

```
model class Wheel
parameter Radius=0.3 {m}
parameter muPeak=1  {max coeff. of friction}
local k10, F
parameter slip90=0.1 {slip at 90% of muPeak}
constant pi=4*arctan(1)
output slipx, slipy
parameter J=0.5 {kg*m^2}
  cut Shaft (w/T)
  cut Hub (Vx, Vy / -Fx, -Fy, -N)
J*der(w)=T-Fx*Radius
slipx=(Radius*w-Vx)/abs(Vx)
slipy=(0       -Vy)/abs(Vx)
k10=slip90*tan(0.9*(pi/2))
F=muPeak*N*(2/pi)*
  arctan(k10*sqrt(slipx**2+slipy**2))
Fx=F*slipx/sqrt(slipx**2+slipy**2)
Fy=F*slipy/sqrt(slipx**2+slipy**2)
end
```

THE DRIVING WHEEL MODULE:

The driving wheels are modeled to give rise to frictional force in the horizontal plane as the non-linear `arctan` function of slip in a symmetrical way about the vertical axis. The module also takes into account the influence of rotational inertia about the axis of rotation.

The corresponding code in ***Dymola*** syntax for the driving wheel module is given in Tab. 3.

16. CASES INVESTIGATED

The following cases are investigated numerically by running the model in Fig. 9 at the following conditions:

- Vehicle data are kept constant as specified in the vehicle module.
- Only one type of maneuver is considered: acceleration at a constant accelerator pedal position; first driving straight ahead, then after 5 seconds, the right side brake is engaged until steady state conditions are reached.
- Four combinations of internal losses and inertia and externally applied braking torque rise time are investigated, see Tab. 4.

Tab. 4. Four cases investigated

Case name	**Gear train inertia and losses**	**Torque rise time for brake engagement**
Reference	**realistic** (about 0.1 kgm^2 per shaft and 1 % losses per gear mesh)	1 second
Simple	**zero**	as Reference case
Exaggerated	about **10 times larger** than Reference case	as Reference case
Slow	as Reference case	**5 times longer** than Reference case

The built-in speed ratio of the locked planetary gear train with one of the brakes engaged is 81/49 = 1.653. This would give rise to a theoretical turning radius of 3.06 m (at non-slipping driving wheels and track width = 2 m), which is realistic at tracked vehicles, but is extreme at wheeled vehicles.

17. NUMERICAL RESULTS

A narrow selection from the broad variety of numerical results is shown in Fig. 11 and Fig. 12, demonstrating results that are characteristic to both the whole vehicle, Fig. 11, and the planetary steering gear train, Fig. 12.

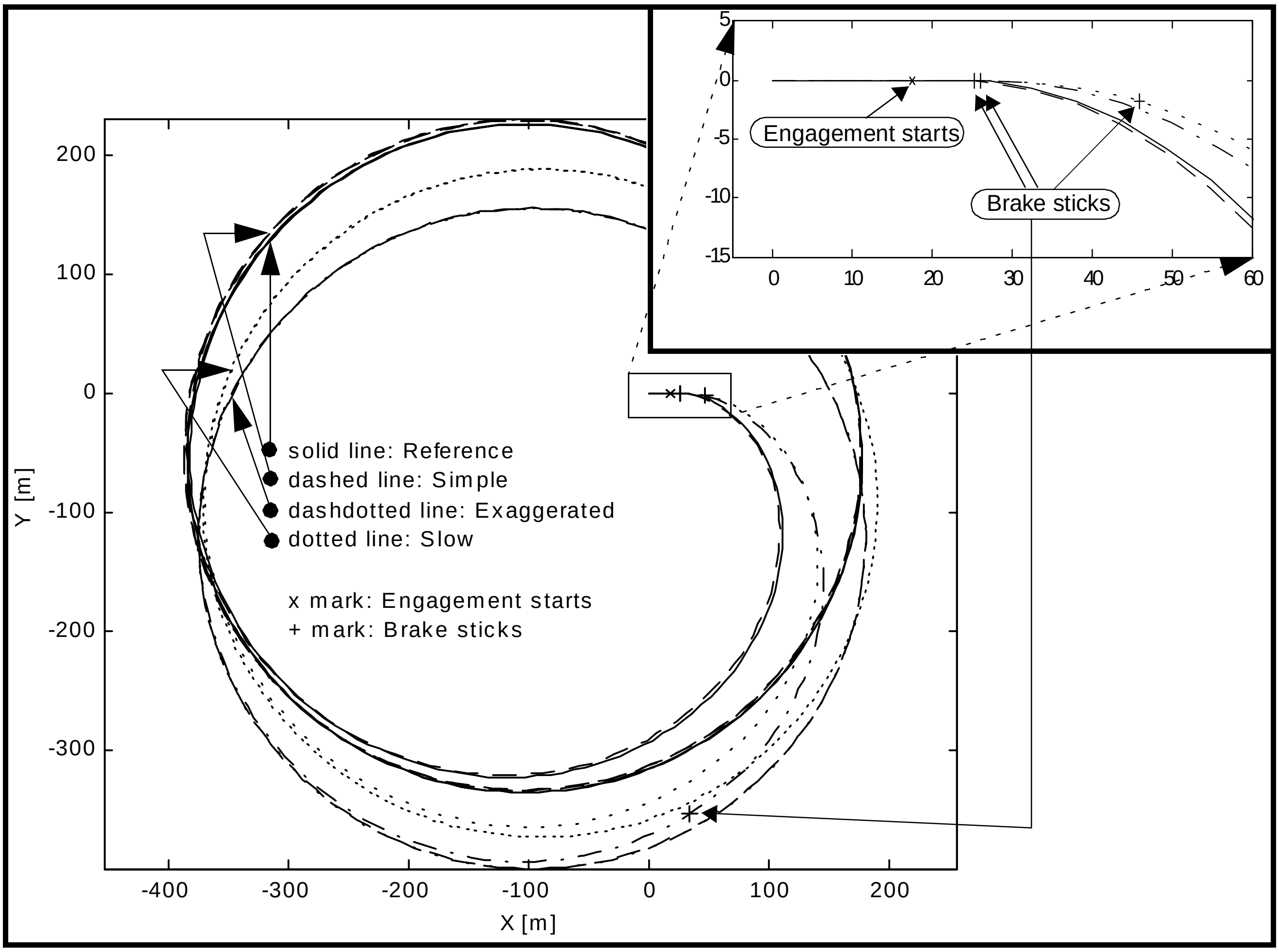

Fig. 11. Simulated vehicle trajectories for the four cases. Each trajectory covers 300 seconds.

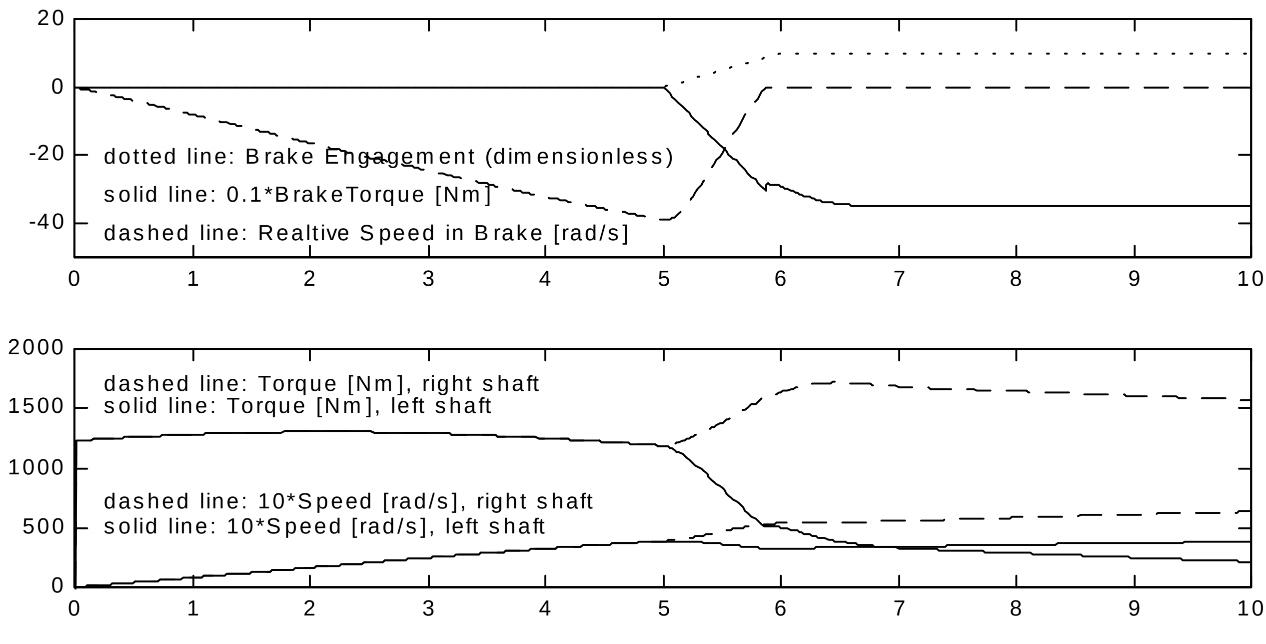

Fig. 12. Simulated transmission quantities versus time for the first 10 seconds of the Reference case.

18. SPECIAL CONCLUSIONS

This numerical evaluation of a planetary gear train operating at transient running conditions demonstrates, how the theoretical approach according to part A, and the use of ***Dymola*** for automated equation composition and integration according to Part B, are successfully applied to a combined planetary gear train and vehicle dynamics problem.

The integration of the governing differential equations of motion of the entire system is here selectively performed after automatic elimination of original constraints by using symbolic manipulations, which is an option in ***Dymola***.

Numeric results confirm and demonstrate anticipated phases of the transient steering process of vehicles. To the knowledge of the present authors, that has not been shown elsewhere for the brake slip phase, when internal tooth friction and rotational inertia are fully considered.

However, in the present case internal tooth friction and rotational inertia seem to be numerically less significant when studying the transient behavior of this particular planetary gear train.

Concerning the problem of transient steering of vehicles, the time history of the application of the braking torque is significant and its influence can be studied in detail by using the described and proposed methods.

GENERAL CONCLUSIONS

The present approach to the characterization and analysis of planetary gear trains, in particular regarding the full dynamics of them, is based upon two untraditional concepts:

- constraint or gear mesh based definition of the planetary system, and
- geometric interpretation of gear mesh losses.

Those two concepts together enable the use of the concept of Lagrange multiplier, which eliminates the need of detailed and often cumbersome study of component equilibrium.

Alternative methods have been mentioned for the integration of the governing ordinary differential equations:

- simultaneous solution of differential algebraic equations, ***DAE***, as frequently practised in ***MBS*** software,
- elimination of dependent state variables by symbolic manipulations, and
- integration of state variables disregarding constraints,

the two last mentioned methods needing ***ODE*** solvers, only.

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