

# CHALMERS



## Crest Factor Reduction Using Peak Strainer

*Master of Science thesis in the Communication Engineering Programme*

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## **Abstract**

Multiple-Input Multiple-Output systems are getting pushed as a solution to better utilize the frequency spectrum and to achieve higher throughput and energy efficiency. By shaping beams and direct them to the intended receiver, less energy is wasted and picked up as interference in unwanted directions. By obtaining stronger signal to noise ratio, and therefore the potential to use higher modulation schemes, there is a need to reduce the amount of error vector magnitude that the transmitter introduces, in order to increase throughput.

This thesis presents a novel solution, namely peak strainer, to achieve crest factor reduction, which improves transmitter efficiency without any error vectors and unwanted spectrum broadening that conventional time domain clipping creates. The peak strainer applies the method of phase shifting on the individual input signals into each antenna element in a phased array, such that a lower peak to average power ratio is achieved. By making sure that the orthogonality of each beams is kept, there is no added error vector or spectral broadening in the signal.

## **Keywords**

Adjacent Channel Leakage Ratio, Complementary Cumulative Distribution Function, Crest Factor Reduction, Error Vector Magnitude, Long Term Evolution, Orthogonal Frequency Division Multiplexing, Peak to Average Power Ratio, Peak Strainer, Multiple-Input Multiple-Output

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Simon Andersson, Zixin Zhang, Gothenburg

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# 1

## Introduction

**M**OVING FROM single antenna systems towards massive multiple-input multiple-output (MIMO) systems creates new problems and challenges that need to be solved, in order to reap all the promised benefits. Techniques and methods that were appropriate for previous systems may need to be revisited and revised to fit the next generation of cellular systems. The focus of this thesis is to ease the demand for the transmitter amplifiers in a multi-antenna base station. This will be done by signal processing rather than changing existing hardware.

### 1.1 Aim and objective

Today's cellular systems use modulation schemes where the composite signal varies greatly in amplitude. With a small probability, the maximum amplitude of the signal can be several orders of magnitude larger than the average [1]. In general, a power amplifier generally has a lower efficiency when linearly amplifying a signal which varies greatly in amplitude [2]. The thesis will explain how peak strainer can be used to create a more power efficient base station.

One way to achieve higher efficiency is to lower the peak to average power ratio (PAPR), also called crest factor reduction (CFR), of the signals going through each antenna element. A technology that is used today is called clipping [3]. Clipping introduces an error in the transmitted signal which makes the receiver experience an increased error vector magnitude (EVM), a difference of the received signal from the ideal one, which in the worst case can result in an erroneous decoded symbol. Clipping also widens the signal's spectrum by creating sharp edges in the signal. These sharp corners can be smoothed out at a cost of increased EVM. The spectrum broadening causes adjacent channel leakage (ACL) which leaks energy outside the system's initial frequency channel. Clipping can also leak unwanted energy inside the system's own frequency band that will also contribute to EVM [3].

In this thesis, a better method called peak strainer is introduced, that reduces the PAPR of the signal in a multi-user MIMO (MU-MIMO) system without any degradation of the received signal, while retaining the spectral shape of the initial signal. Therefore, peak strainer introduces no EVM and no extra adjacent channel leakage ratio (ACLR).

## 1.2 Outline

In chapter 2, the background theory of the relevant topics are described. An overview and explanation of how peak strainer works, along with the underlying assumptions are given in chapter 3. Chapter 4 presents the simulation results and compares them with other known methods. Chapter 5 summarizes the report with the most important conclusions.



# 2

## Theory

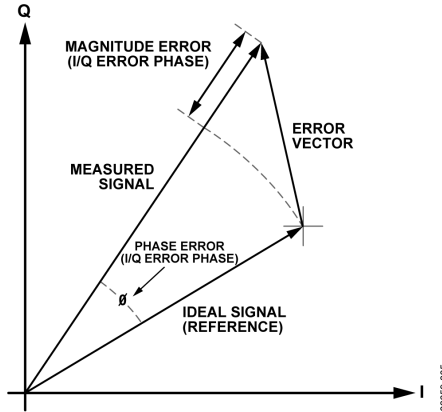
**T**HIS CHAPTER will explain the necessary theory to understand the problem that peak strainer solves. Since this thesis does not focus on hardware, this chapter briefly explains the behaviour of power amplifiers while concepts and ideas of multi-antenna systems are covered in more depth.

### 2.1 Quality parameters

A special section of the parameters used to measure the quality of a CFR solution are given, so that the results in chapter 4 are more easily understood.

#### 2.1.1 Error vector magnitude

The unit EVM is a metric of how well a received signal resembles the ideal signal [4]. EVM is introduced by different sources from different parts in a communication system. Some reasons for EVM are imperfections in hardware and crest factor reduction (CFR) using clipping [5]. EVM will result in a transmitted signal with a delta from the ideal as in figure 2.1.



**Figure 2.1:** Schematic illustration of EVM [6].

The definition of an error vector is the vector between the ideal transmitted point versus the measured point [4]. The average of the power of the error vector gives the error vector magnitude, which is usually expressed in dB by formula 2.1 or percentage by formula 2.2.

$$EVM(dB) = 10 \times \log_{10} \left( \frac{P_{error}}{P_{reference}} \right) \quad (2.1)$$

$$EVM(\%) = \sqrt{\frac{P_{error}}{P_{reference}}} \times 100\% \quad (2.2)$$

It is easy to see that EVM will limit the use of standard higher order modulations, whose constellation plots are more sensitive to errors. The table below gives some examples. That is why there are requirements for how much EVM a communication system can tolerate for a specific modulation to guarantee a reliable transmission. [7].

EVM Constellation	
Modulation	EVM(%)
QPSK	17.5%
16 QAM	12.5%
64 QAM	8%

### 2.1.2 Adjacent channel interference

A communication system is always limited by how much spectrum is allowed to use [8]. Different systems have to co-exist in the same medium and therefore it is important that each communication system is only transmitting within its allocated spectrum.

However, non-ideal hardware and non-linearities will create adjacent channel leakage, which causes power to leak outside the system's own channel; this is the so called out-of-band distortion [9]. Therefore, requirements on how much power is allowed to be leaked into an adjacent channel have been specified by 3GPP, as a ratio of power in the main channel versus power leaked into the adjacent channel [10]. The same reasons can also put unwanted power within the system's own channel which is called in-band distortion; this distortion will be seen as EVM in the receiver [9].

### 2.1.3 Complementary cumulative distribution function

The complementary cumulative distribution function (CCDF) in this report is used to show the probability for an OFDM symbol to be above a certain PAPR. It is calculated by taking several OFDM symbols and calculating their PAPR and how often this level occurs. CCDF gives a reasonable measurement of how well a CFR works by looking at a specific probability, e.g  $10^{-3}$ , and seeing the difference between the non-CFR OFDM symbols versus CFR OFDM symbols.

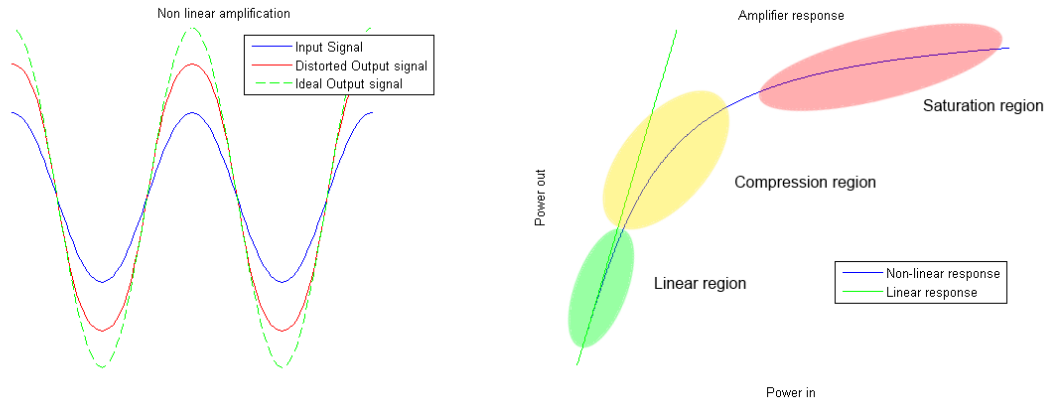
## 2.2 Power amplifier - non-linearities

The focus of this thesis is in software, but a quick introduction to amplifiers is given to provide an understanding of the importance of a signal's PAPR and its effect on amplifier efficiency.

An ideal amplifier should take any input signal and proportionally increase the amplitude as an output signal. This means that the output is just a scaled version of the input. However, in reality, the output signal will never be a perfect scaling version of the input, since amplifiers have certain characteristics that will affect the output signal [11]. Figure 2.2 shows how this is mathematically represented [12] while Figure 2.3 shows the behaviour of an ideal amplifier versus a real amplifier.

$$a \cos(\omega t) \quad \triangle \quad A \cos(\omega t) + B \cos^2(\omega t) + \dots$$

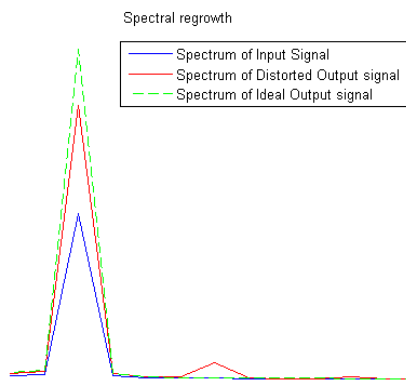
**Figure 2.2:** Mathematical description of a non-linear power amplifier.



(a) Effect of a non-linear amplification of a signal in time domain (b) Typical amplification response of a non-ideal versus an ideal amplifier.

**Figure 2.3:** A non-ideal amplifier’s output signal will not be a perfect scaled version of the input signal.

As is shown in Figure 2.3 (b), the amplifier will have a near linear response region initially, but the behaviour starts to deviate from the ideal linear with higher input power, this behaviour is seen in the compression region. Eventually, this compression area will lead into the saturation region where the amplifier gives the same output no matter what the input voltage is [13]. This non-linearity will create spectral broadening, which means that the output signal will contain more frequencies, with regards to the input signal [14]. This is shown in Figure 2.4 where the frequency components of Figure 2.3 (a) are plotted. Out of band distortion creates problems in a radio frequency (RF) environment and leads to ACLR. In band distortion leads to EVM in the received signal.



**Figure 2.4:** The non-ideal amplification leads to spectral regrowth, which can be seen in this frequency spectrum plot. This, then, becomes out of band and in band distortion.

One way to achieve linearity in an amplifier is to reduce the input power to make

the signal vary within the linear region. The drawback of this is that the efficiency of the amplifier is at its lowest in the linear region and at its highest close to the saturation point [15]. To get closer to the saturation point, a digital pre-distortion (DPD) can be used before the amplifier, to linearize the response [16]. A simple way to describe a DPD is that it has the inverse distortion characteristics of the non-linear amplifier, so that the product of the two becomes linear. The complexity of the DPD increases when the input signal gets closer to the saturation region, this is why normally the input signal has to be scaled down even when using a DPD.

When a signal with high PAPR needs to be amplified, more backing off is needed to make sure that the high peaks, even though they are improbable, do not reach the saturation region. Thus, on average the signal will be amplified with low efficiency to adhere to linear amplification of higher peaks. Lowering the PAPR of the signal makes it possible to use the amplifier more efficiently since the average input power can be moved closer to the compression region with small probability of high peaks crossing the saturation region.

## 2.3 Previous CFR methods

The definition of a signal's crest factor is,

$$C = \frac{|x|_{peak}}{x_{rms}} \quad (2.3)$$

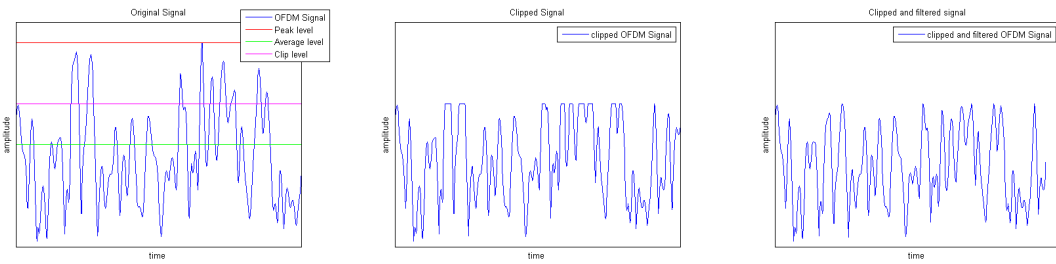
This metric gives a sense of how much a signal varies in amplitude. To put this in terms of power, the so called peak to average power ratio (PAPR) is defined as:

$$PAPR = \frac{|x|_{peak}^2}{x_{rms}^2} = C^2 \quad (2.4)$$

As previously explained, a power amplifier can achieve better efficiency with a signal that has a lower PAPR.

### 2.3.1 CFR using clipping

A common way to lower the PAPR of a signal in today's systems is to use CFR with Clipping [17]. It works by clipping a signal if it goes above a certain threshold and then filter this sharp edge to band limit the signal. This cut will introduce two problems, EVM and ACLR. The EVM comes from that the transmitted signal will no longer be the ideal one and it will result in an error vector at the receiver. The sharp edges created from the clipping are bad from a spectrum point of view, since they contain an infinite amount of frequencies and will increase the ACLR. Figure 2.5 and 2.6 show how clipping affects the signal in time and frequency respectively.

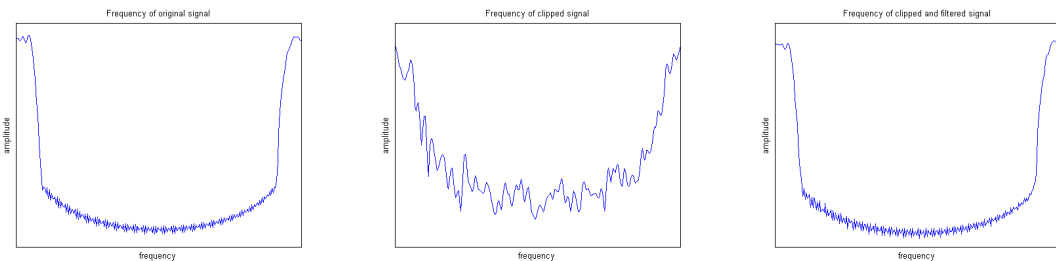


(a) The original signal

(b) Clipped signal

(c) Clipped &amp; filtered signal

**Figure 2.5:** The signal above the threshold is being clipped which creates a sharp edge. By filtering, it retains a band limited shape.



(a) The Original signal in frequency

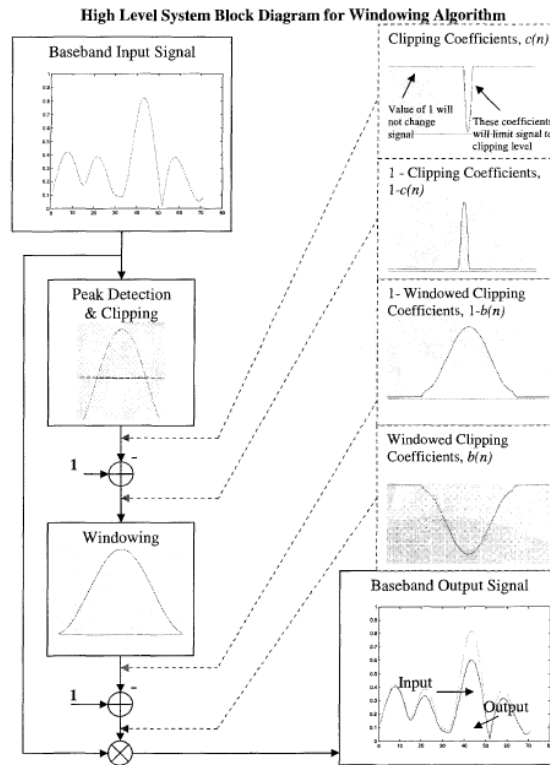
(b) Clipped signal in frequency

(c) Clipped &amp; filtered signal in frequency

**Figure 2.6:** The initial signal is band limited in frequency but a hard clip creates spectral broadening, which can be reduced by filtering around the clipped areas.

To remove the sharp edges a smoothing of the clipping is done. This will affect the signal over a sector around the clipped area which will lead to even more EVM, but the signal will retain a band limited shape which will give a better ACLR. So, there is a trade off between EVM and ACLR.

The following picture explains the concepts in more detail.



**Figure 2.7:** The steps taken to reduce the crest factor of a signal by using clipping and windowing [17].

### 2.3.2 CFR using partial transmit sequence

This CFR with partial transmit sequence (PTS) technique is only applicable for an OFDM modulation system. Since the improbable high peaks in an OFDM system come from constructive interference from the individual subcarriers, it is possible to remove these peaks by phase shifting groups of subcarriers or individual subcarriers to rid the OFDM symbols of their high peaks. However, this will create problems in the receiver of how to equalise these shifts. Therefore, information of the phase shifts will have to be sent to the receiver at the same time. The benefit from doing this instead of clipping is that potentially no EVM is introduced and no extra ACLR is added [18].

## 2.4 Beamforming

By having several antennas in a uniformly spaced array, it is possible to direct the radiation pattern towards the receiver, a technique known as beamforming. This technique has several applications, such as sonar, radar, acoustic measurements or in communications. Sonar and radar can use beamforming to focus on one area by using several small elements and therefore use less energy and filter out noise sources from other angles. By using microphones in an array setup it is possible for auto mobile engineers to more accurately pin-point where noise is coming from and to try to suppress this. In communication, it is possible to use beamforming to let several users communicate at the same time, by utilizing spatial multiplexing. Signals can be steered in such a way that only the intended user elements receive the signal without interference [19].

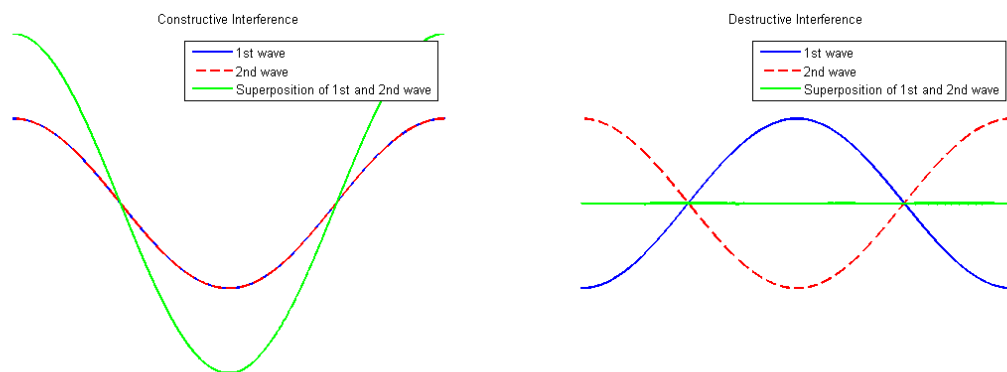
This subsection will show the essential physical properties that makes beamforming possible and how this can be used to steer signals.

### 2.4.1 Superposition of waves

Electromagnetic waves combine linearly, which means that superposition apply when they interfere with each other [20]. There are two types of interference, constructive and destructive. If the sum of the two waves has a higher amplitude than each of them individually, the interference is constructive. On the other hand, if the amplitude of the sum is lower than the individual waves, the interference is destructive. An example is given in Figures 2.4.1 (a) and 2.4.1 (b) below using equations 2.5 and 2.6.

$$\cos(2\pi t) + \cos(2\pi t), \text{ where } t = [0, 0.0001, \dots, 1] \quad (2.5)$$

$$\cos(2\pi t) + \cos(2\pi t + \pi), \text{ where } t = [0, 0.0001, \dots, 1] \quad (2.6)$$



(a) Constructive interference

(b) Destructive interference

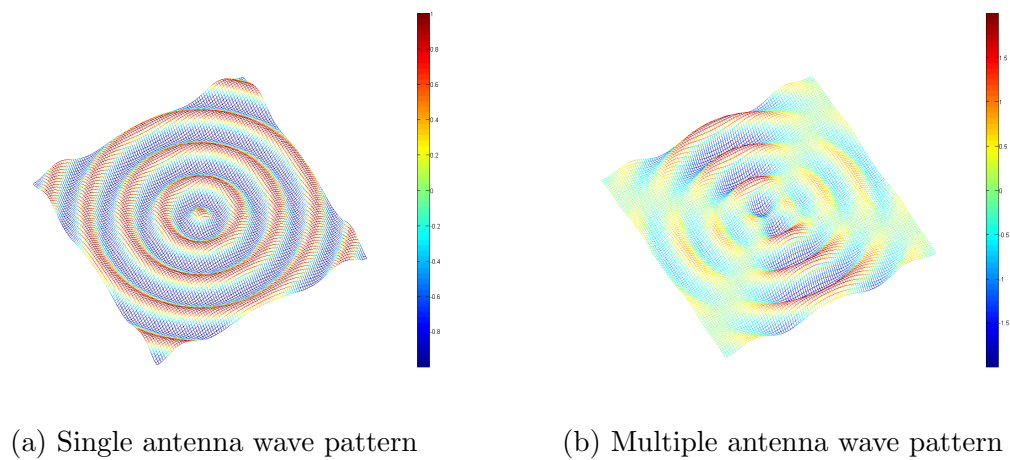
**Figure 2.8:** Completely constructive vs completely destructive interference



The above figures demonstrate the two extremes, fully constructive and destructive superposition. This behaviour can then be used in a phased array to steer beams in different directions.

### 2.4.2 Phased array

A phased array consists of two or more antennas spaced closely to each other relative to the wavelength of the signal. Figures 2.9 (a) and 2.9 (b) below show how electromagnetic waves propagate for a single isotropic antenna versus a phased array consisting of two isotropic antennas.

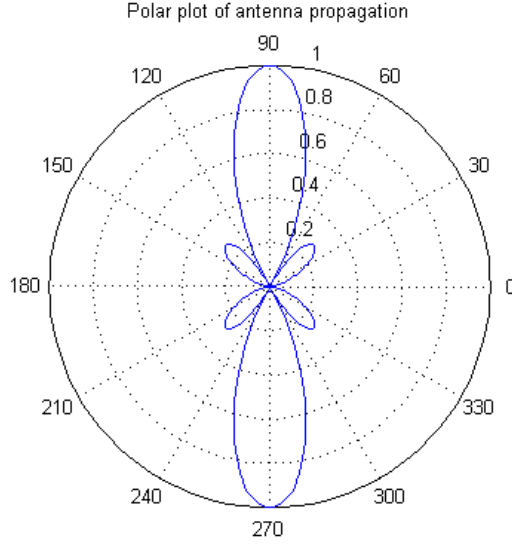


**Figure 2.9:** Wave pattern for single and multiple antennas.

As is shown in the pictures, the radiation pattern is not uniformly distributed when using multiple antennas. The deep red and blue points represent the amplitude maxima, which can be calculated by using formula 2.7:

$$|\cos(d \times \lambda \times 2\pi + \varphi)| = 1 \text{ where } d \text{ is the distance from transmitter, } \lambda \text{ is wavelength and } \varphi \text{ is phase.} \quad (2.7)$$

When a point in space has an in-phase maxima from both waveforms, that point will experience a stronger signal. From formula 2.7, one sees that the distance, which corresponds to a maxima, depends on the phase of the signal. Looking at Figure 2.9 (a), a change of phase translates into moving the position of maxima. By using this fact and looking at Figure 2.9 (b), it is then possible to change the angle at which constructive interference occurs in the multiple antenna case. This property is being used by phased arrays to achieve beamforming.



**Figure 2.10:** A 2D wave pattern which shows mainlobes (90 and 270 degrees), sidelobes (40, 140, 220 and 320 degrees) and nulls (0, 60, 120, 180, 240 and 300 degrees).

In Figure 2.10, a two dimensional plot of a phased array radiation pattern is shown using four antennas with half a wavelength spacing. It shows the three different characteristics that a phased array pattern can have: mainlobe, sidelobe and null [21]. The mainlobes are the directions where most of the energy is transmitted to, sidelobes are areas that have much less energy than the mainlobes and nulls are where no energy propagates. In order to get no interference when transmitting multiple beams from one antenna array, the nulls must appear where other beams have their mainlobes. This is also known as orthogonal beamforming [22]. The theory and math behind this technique are explained in the following subsection.

### 2.4.3 Antenna spacing

The antenna elements for both the transmitter  $N_t$  and the receiver  $N_r$  are placed on uniform linear arrays with normalized lengths  $L_t$  and  $L_r$ . The normalized spacing of the antenna arrays are defined as  $t = L_t/N_t$  and  $r = L_r/N_r$  respectively. The length of the phased arrays are calculated as

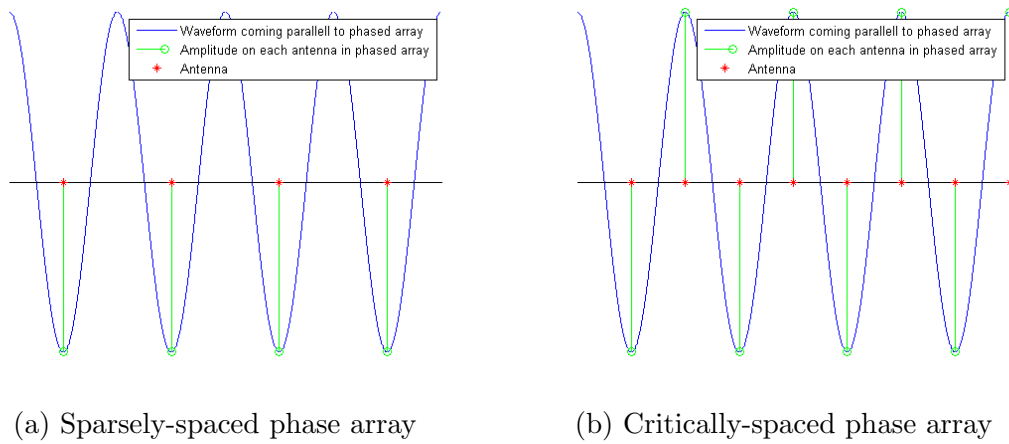
$$L_r = \Delta_r \times N_r \text{ and } L_t = \Delta_t \times N_t \quad (2.8)$$

where  $N_t$  and  $N_r$  are the numbers of the  $T_x$  and  $R_x$  antennas respectively,  $\Delta_t$  and  $\Delta_r$  are the antenna spacings in unit of wavelength.

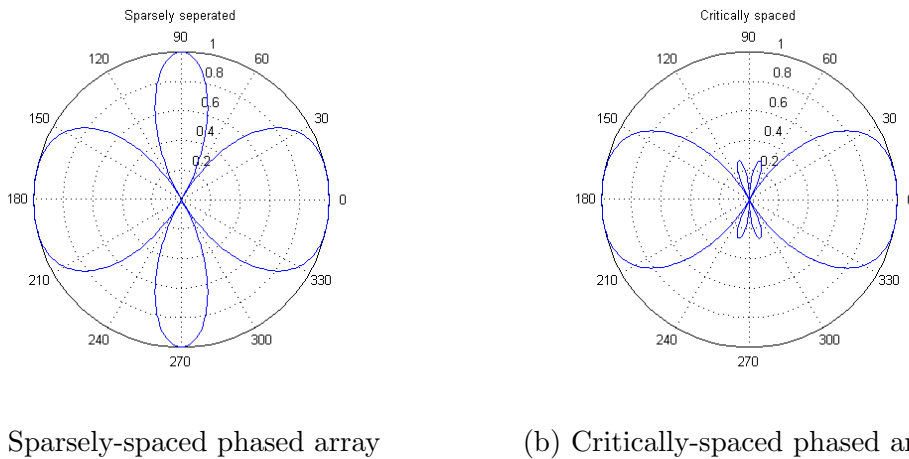
The mainlobes, which retain orthogonality from each other, of the beams are at angles with

$$\cos(\varphi) = \left(\frac{k}{L_r} - 1\right) \bmod \frac{1}{\Delta_r}, \text{ where } k = [0, 1, \dots, N_r - 1] \quad (2.9)$$

For  $\Delta_r = 0.5$ , which is the critically-spaced case,  $\frac{1}{\Delta_r} = 2$  and  $\frac{k}{L_r}$  is from 0 to 2, therefore there will be a unique solution of  $\cos(\varphi) = \frac{k}{L_r}$  [23]. In the sparsely-spaced case, that is  $\frac{1}{\Delta_r} < 2$ , there will be multiple solutions of  $\cos(\varphi) = \frac{k}{L_r}$  for some values of  $k$ . In the densely-spaced cases,  $\frac{1}{\Delta_r} > 2$ , for  $L_r < k < N_r - L_r$ , there will be no solutions of  $\cos(\varphi) = \frac{k}{L_r}$  at all. Only in the critically-spaced case, there is an one-to-one mapping corresponding to a physical direction.



**Figure 2.11:** Waveform coming parallel to phase array

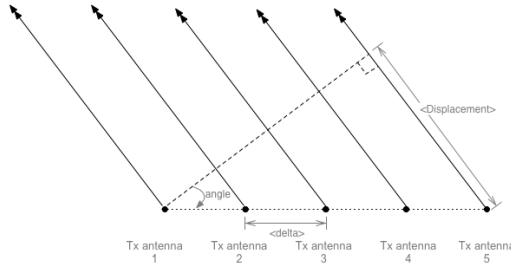


**Figure 2.12:** Polar plot of how the sparsely and critical spaced phased array react to a parallel waveform towards the antenna array

To give a more intuitive view, the phased array could be considered as the sampling points of a waveform. As is shown in Figure 2.11, the waveform is coming parallel to the phased array and the antennas are experiencing the amplitudes accordingly. At one time instance, when the phased array is critically-spaced with regards to wavelength, the array could distinguish the parallel direction of the waveform. However, in the sparsely-spaced case, all the antennas experience the same amplitude offset. This behaviour also happens when the waveform is coming completely vertical to the phased array. So the sparsely-spaced array can not distinguish these two different possibilities and therefore treats them as the same. As a result, the sparsely-spaced array can not detect the direction as precise as the critically-spaced array. Figure 2.12 shows that a sparsely-spaced array treats a vertical and a horizontal waveform the same since it can not differentiate between them.

#### 2.4.4 Beam steering

As was explained in previous subsections, by changing the phase of the antenna elements, it is possible to change the angle of the mainlobe. This section explains how the phase offset for each element is calculated from a given angle.



**Figure 2.13:** Transmitter phase array with line-of-sight path, receiver is infinitely far away. The signals from the transmit antennas arrive almost in parallel at the receiving antennas.

The quantity

$$(i - 1)\Delta_r\lambda_c \cos(\varphi_i) \quad (2.10)$$

is the displacement of transmit antenna  $i$  from transmit antenna 1 in the direction of the line-of-sight.  $\Delta_r$  is the antenna spacing,  $\lambda_c$  is the wavelength of the transmit signal. By giving each antenna a certain phase shift  $\varphi_i$  which is determined by the  $Displacement_i$ , the beam could be steered towards the desired direction.

# 3

## Peak Strainer

**S**INCE THE THESIS had restrictions both in time and resources, the simulations were done fully in software, namely in MATLAB. In this section, the simulation parameters will be described as well as the motivation behind the chosen parameters.

### 3.1 System parameters

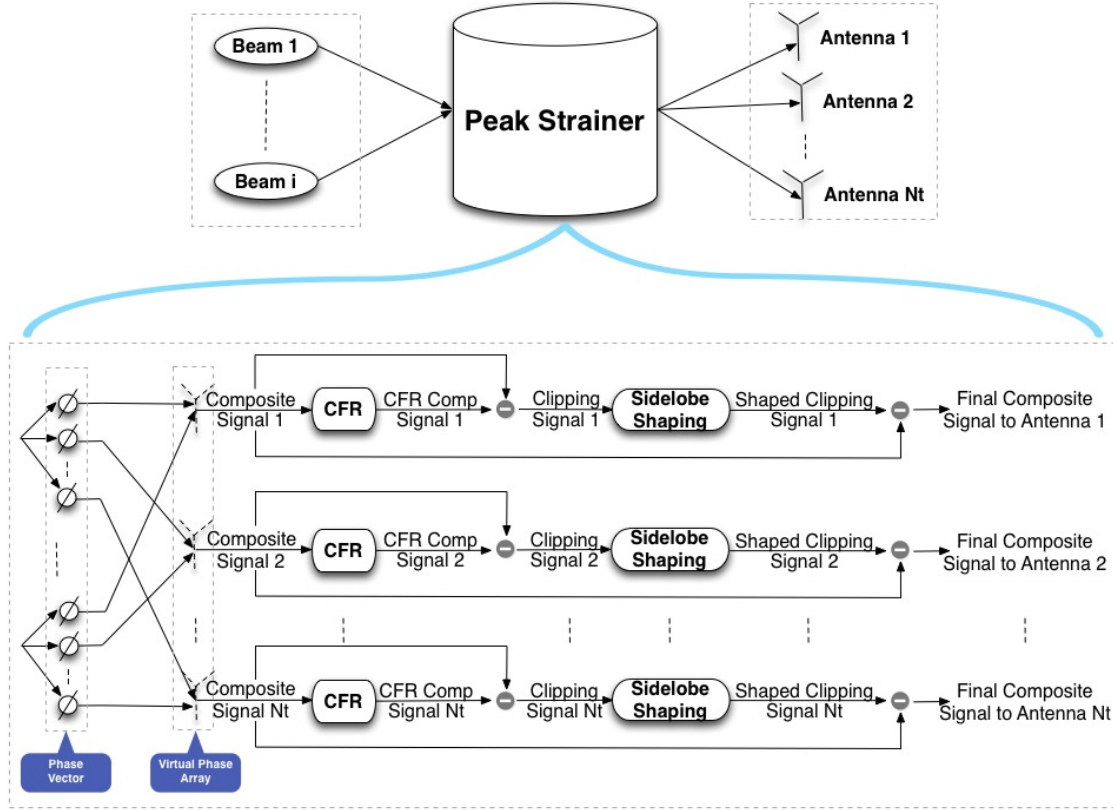
In this system, omnidirectional antennas are used to simplify calculations, while no fading, noise or path loss are being considered. The motivation for this is that peak strainer does not depend on the antenna type, while fading, noise and path loss are irrelevant in the measurement of any possible improvement. The antennas are uniformly spaced half a wavelength to optimally achieve beamforming.

Since orthogonal frequency division multiplexing (OFDM) is what is used in long term evolution (LTE), it was chosen as the baseband signal type in our simulations [18]. 64-QAM was chosen since it is the highest modulation that LTE supports, and the most sensitive to EVM [24].

A technique called massive MIMO refers to very large multiple-input multiple-output system. The amount of antennas in a massive MIMO base station could be around 10 per user [25]. Since this technology is getting more focus for the coming generation of communication systems, the amount of antennas modeled will be in this range.

### 3.2 System design

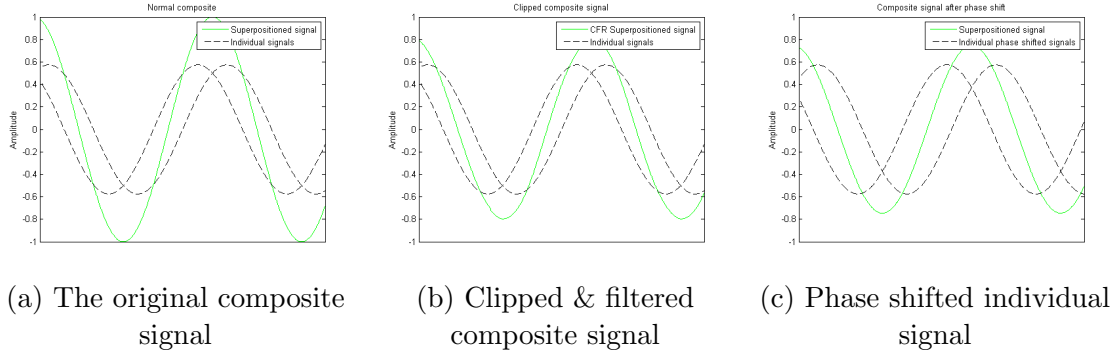
This section describes the core of the idea behind the proposed peak strainer, starting with PAPR reduction and proceeding with compensation for loss of orthogonality between beams. Figure 3.1 is a simple overview of how peak strainer operates.



**Figure 3.1:** An overview of how peak strainer affects a normal phased array. It starts by changing the peak strainer phase vector for the PAPR reduction at the phase array. Then removes the sidelobe power in other beam directions.

### 3.2.1 CFR by phase shift

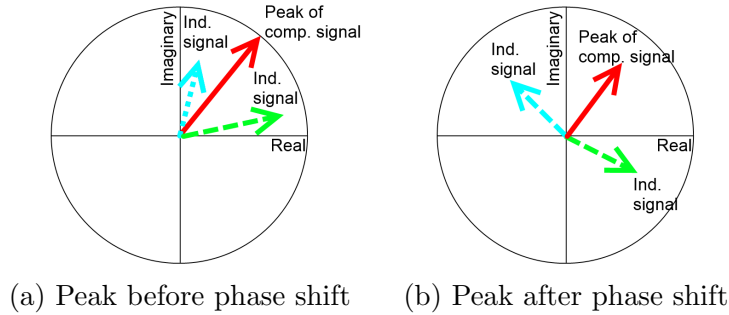
To reduce the PAPR on each antenna element, a way to lower the peaks needs to be developed. For a multi-user system, there will be more than one signal going into each antenna. High peaks will appear if all or some of the signals are constructively added up. The basic idea of the CFR in peak strainer is to phase shift the incoming signals in such a way that those signals will not add up constructively above a certain threshold. How phase can change the amplitude of a composite signal and how it compares to clipping is shown in Figures 3.2 (a), (b) and (c).



**Figure 3.2:** A composite signal can be lowered in amplitude by either CFR using clipping or phase shifting the individual signals

The advantage over clipping is that by phase shifting, the spectral characteristics of a signal is kept, whereas by clipping, a filtering has to be done in order to lower the created out of band frequencies.

The signals coming into each antenna have a standard PAPR around 12dB [18]. To lower the PAPR on each antenna element to a desired threshold, the algorithm finds the individual signals that contribute to the highest peak. Then, phase shift them in the opposite direction of the peak, with an increasing strength to achieve quicker convergence. As is shown in Figure 3.3, the red arrow is the peak of the composite signal in one antenna element, the blue and green arrows are the peaks of individual signals that contribute to the high peak. Phase shifting the blue and green ones away from the high peak will result in a lower peak of the composite signal. After this, check the PAPR of the new resulting signal, if the PAPR is lower than the threshold then this CFR part is done, otherwise, find the new highest peak and repeat the above steps until the PAPR has been lowered to the threshold.



**Figure 3.3:** Phase shift method in CFR unit

### 3.2.2 Sidelobe shaping

Since the composite signals going into each antenna have been phase shifted, the phased array vector is no longer the original ideal one for beamforming. That means that the

perfect orthogonality coming from formulas 2.9 and 2.10 has been distorted; thus, the new scheme will put power from one beam's sidelobe into another beam's mainlobe as interference. This interference will deteriorate the received signal, thereby leading to increased EVM.

To solve this problem and have a resulting system with no EVM, we need to make sure that the CFR using phase shift will not affect the mainlobe directions. Therefore, a sidelobe shaping unit is introduced. After the CFR unit, a new composite signal with lower PAPR for each antenna has been generated. By comparing this new signal with the original composite signal, a 'clipping signal' on each antenna can be calculated as in formula 3.1:

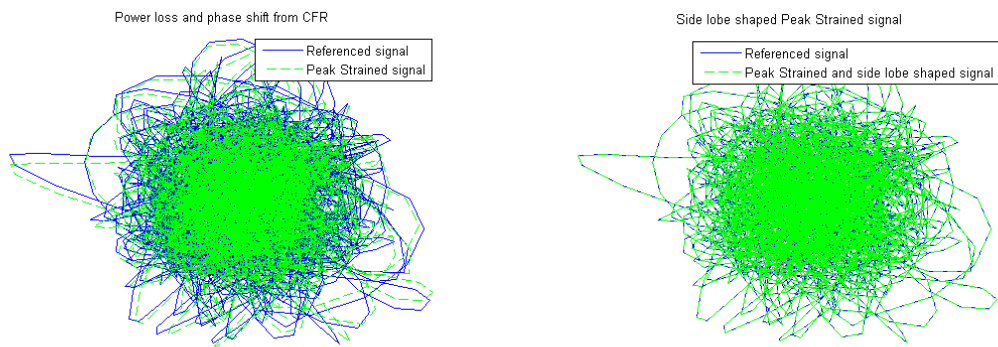
$$\begin{aligned} \text{OrigCompSignal}(n) - \text{CFRCompSignal}(n) &= \text{ClippingSignal}(n) \\ \text{where } n &= [1, 2, \dots, N_t] \end{aligned} \quad (3.1)$$

This clipping signal is the reason why one beam's sidelobe leaks into another beam's mainlobe. A way to not disturb the mainbeam directions with this clipping signal has to be included. By calculating how the clipping signal is received in all the orthogonal angles, and setting the power in the angles that have mainlobes in them to be zero, and then calculate the signal back to the transmitter, this new clipping signal no longer destroys the orthogonality in the beam mainlobe directions. However, it does mean that the output power could be increased in other un-known directions. By using these new clean clipping signals as in equation 3.2, the final composite signals will be ready to send to each antenna element.

$$\begin{aligned} \text{OrigCompSignal}(n) - \text{CleanClippingSignal}(n) &= \text{FinalCompSignal}(n) \\ \text{where } n &= [1, 2, \dots, N_t] \end{aligned} \quad (3.2)$$

The sidelobe shaping could be seen as an angle domain filter. The signals that goes to unwanted directions have been filtered, just like a normal filter that filters out unwanted frequencies. Peak strainer removes peaks on each antenna element just like in clipping, which introduces a power loss. To guarantee no EVM in the mainlobe directions, more power has to be distributed among the antenna elements to compensate the power loss. It is important to note that this will change the PAPR after the CFR, and raise the average power level (RMS) in an unknown way at the antenna elements. But as is seen by the results it is still bounded. This sidelobe shaping can be seen as zero forcing (ZF) for the clipping signal [26]. Figure 3.4 shows how the receiver sees the interferenced signal vs the ideal, and how the sidelobe shaping removes any interference.





(a) Received OFDM signal with interference vs Original OFDM signal

(b) Compensated OFDM signal vs Original OFDM signal

**Figure 3.4:** Sidelobe shaping completely removes any EVM in the received signal since there is no interference between different beams.

# 4

## Results and Evaluation

**T**HIS CHAPTER shows how well peak strainer performs under different conditions and with different parameters. A discussion and evaluation are followed after each result.

### 4.1 Calculation of PAPR

The PAPR of the signal after the CFR unit is the one that has been specified by using formula 4.1. However, since the sidelobe shaping will introduce an RMS increase, the whole power level is increased which pushes both the mean and maximum power up. If the standard formula for measuring PAPR (4.1) is used one could be misled, since the increase in RMS is not explicitly shown. For instance, it can not be considered a gain to have a PAPR reduction of 5dB while the resulting RMS increase is 8dB. Therefore, the actual PAPR gain of the PS should be calculated using formula 4.2 to take into account the RMS increase that PS introduces.

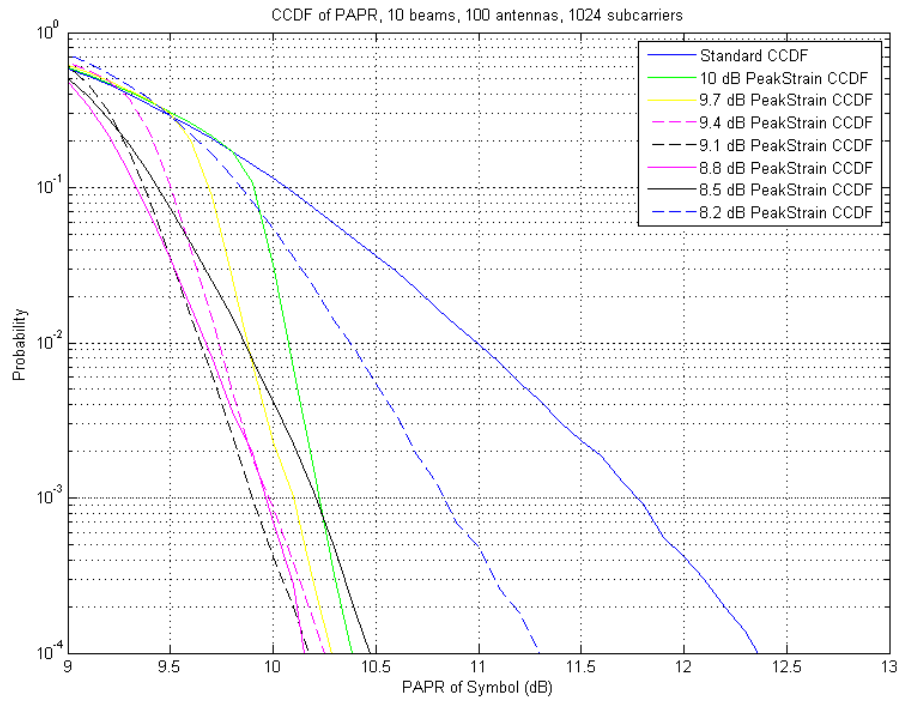
$$PAPR(dB)_{classical} = 20 \times \log_{10}(\max(\text{abs}(\text{OutputSignal}))/\text{rms}(\text{OutputSignal})) \quad (4.1)$$

$$PAPR(dB)_{proposed} = 20 \times \log_{10}(\max(\text{abs}(\text{OutputSignal}))/\text{rms}(\text{InputSignal})) \quad (4.2)$$

### 4.2 Find optimal CFR level

Peak strainer is given a CFR threshold where it will reduce the PAPR to at each antenna element. However, the stronger the reduction, the more compensation has to be done in order for the receiver to get a correct signal, which leads to an increased RMS power. This increase will raise the RMS power at each antenna element accordingly, which

also raises the peaks, and this will reduce the PAPR gains. A simple explanation, the more peak strainer reduces peaks, the more the RMS power increases which will raise the power over the whole signal. Therefore, there is an optimal level to be found with regards to the threshold and actual PAPR reduction. Since more beams give more degrees of freedom at each antenna element, this optimal point should depend on the number of beams.



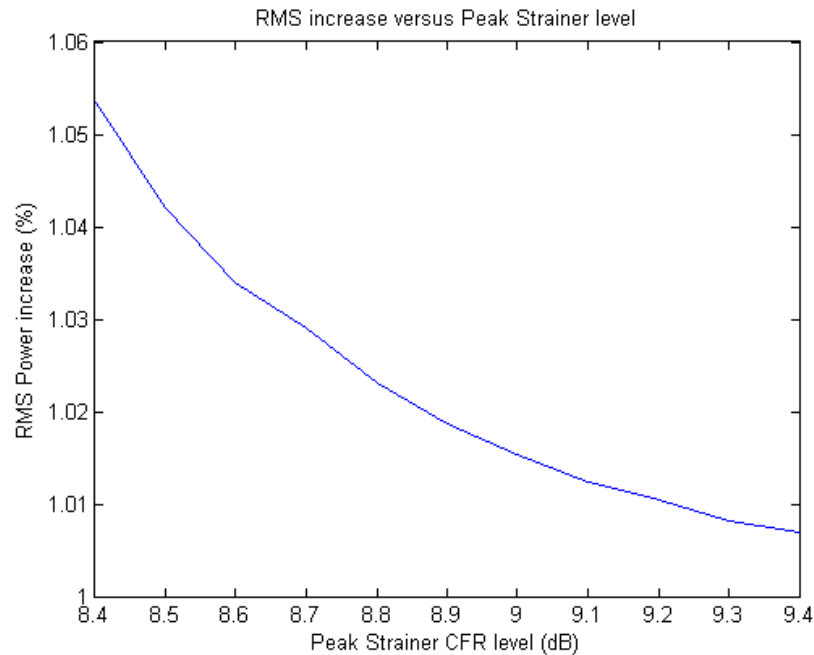
**Figure 4.1:** The CCDF of PAPR at each antenna element without peak strainer and with different levels of peak strainer. Simulations using 10 beams and 100 antennas.

Peak Strainer Gain	
Peak strainer level	PAPR gain at probability $10^{-3}$
10dB	1.5dB
9.7dB	1.6dB
9.4dB	1.75dB
9.1dB	1.9dB
8.8dB	1.75dB
8.5dB	1.5dB
8.2dB	1.0dB

There is a clear gain by using peak strainer; it can lower the PAPR of 10 beams by almost 2dB with no degradation of the received signal and by only a cost of complexity. The results show that the optimal CFR level is at 9.1dB for 10 beams with 100 transmit antennas. It is also worth noting that the gains decrease rapidly once the CFR level goes below the optimal, which means that the RMS power increases faster than the gains.

### 4.3 Increase in RMS power

The compensation to remove EVM that is done by peak strainer raises the average power. Depending on amplifier type, an increase in RMS power could be associated with a higher energy cost. Figure 4.2 shows how the RMS increase depend on the CFR level using the same parameters as in the previous section.

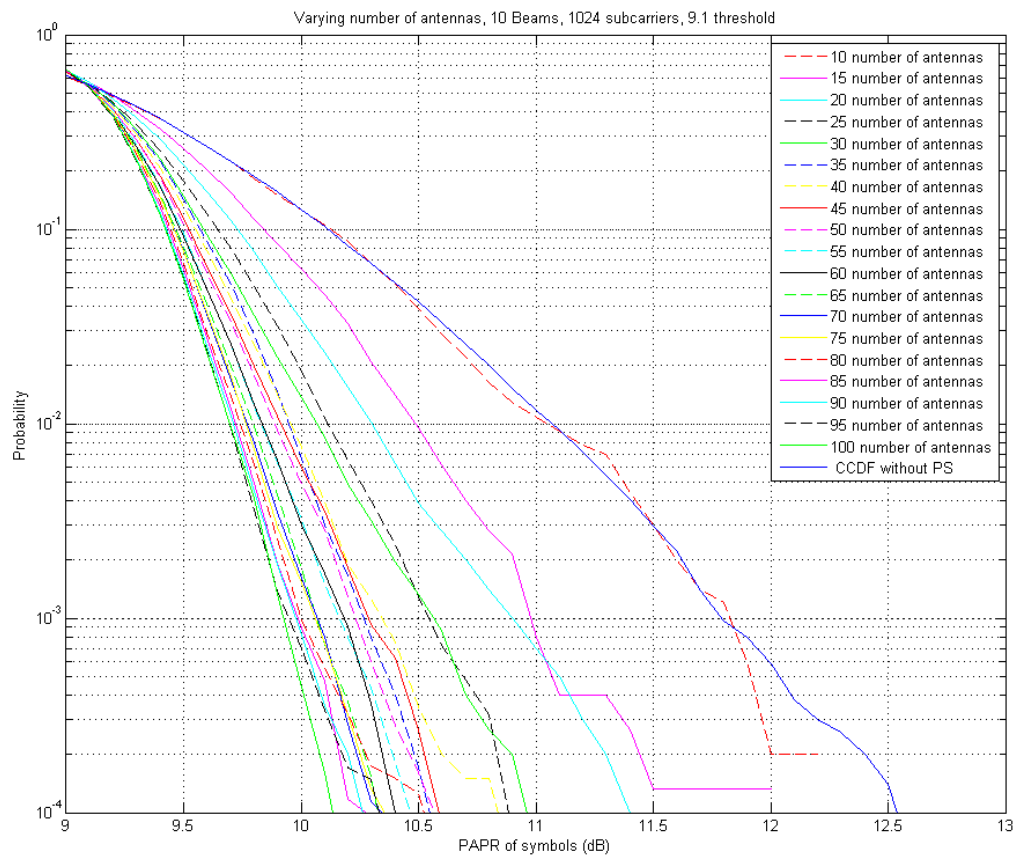


**Figure 4.2:** The RMS power increase at each antenna element, depending on the CFR level.

The RMS increase has an exponential response to the CFR level, which is why the gains in the previous section are getting increasingly worse with heavier CFR once below the optimum CFR level. This is what limits peak strainer from lowering PAPR further, the heavier peak strainer pushes, the higher the RMS increases and the PAPR gain decreases.

#### 4.4 Antenna influence

Due to the fact that peak strainer only works in a multi-antenna system, it is of interest to see how it responds to an increasing number of antennas. In Figure 4.3, the resulting CCDF by simulating peak strainer using 10 beams and with different number of antennas is shown.



**Figure 4.3:** The difference in CCDF with varying number of antennas.

Performance for varying number of antennas	
Antennas	PAPR gain at probability $10^{-3}$
10	0.0dB
15	0.6dB
20	1.1dB
25	1.2dB
30	1.2dB
35	1.5dB
40	1.5dB
45	1.6dB
50	1.7dB
55	1.6dB
60	1.6dB
65	1.7dB
70	1.8dB
75	1.8dB
80	1.9dB
85	1.8dB
90	1.8dB
95	1.9dB
100	1.8dB

As the table shows, in order to get 1dB PAPR reduction at least 20 antennas are needed. To double this gain, about 4 times as many antennas are needed, which shows that the PAPR reduction gain scales logarithmically with the number of antennas.

## 4.5 Subcarrier influence

Since the CFR in peak strainer works by smoothening out the peaks using phase shifts, it is possible to accidentally create new peaks that are above the threshold while lowering a peak. Therefore, it could be beneficial for the peak strainer to use signals with less subcarriers, since it is less likely that new peaks will appear when the signal that is being phase shifted contains less samples. Figure 4.4 shows the CCDF of the PAPR of each antenna element with varying number of subcarriers after peak strainer at the same CFR level.

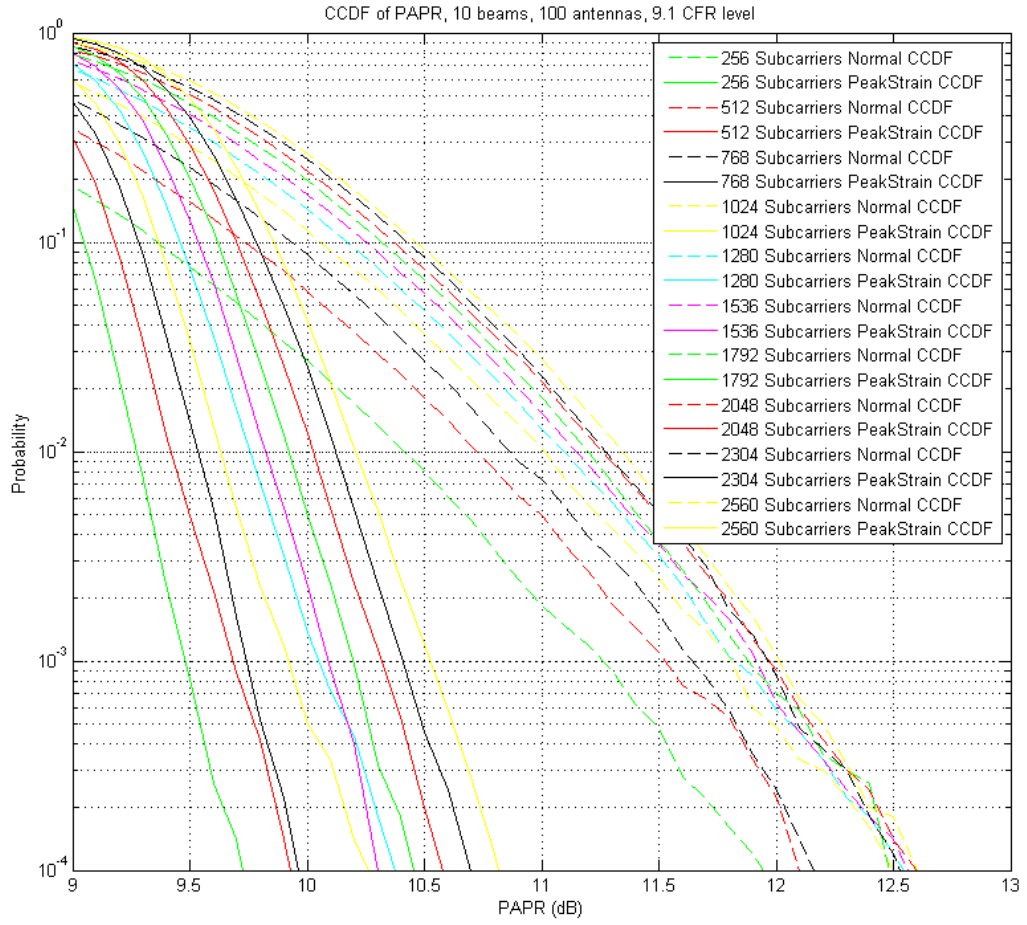


Figure 4.4: PS response to varying number of subcarriers at the same CFR level.

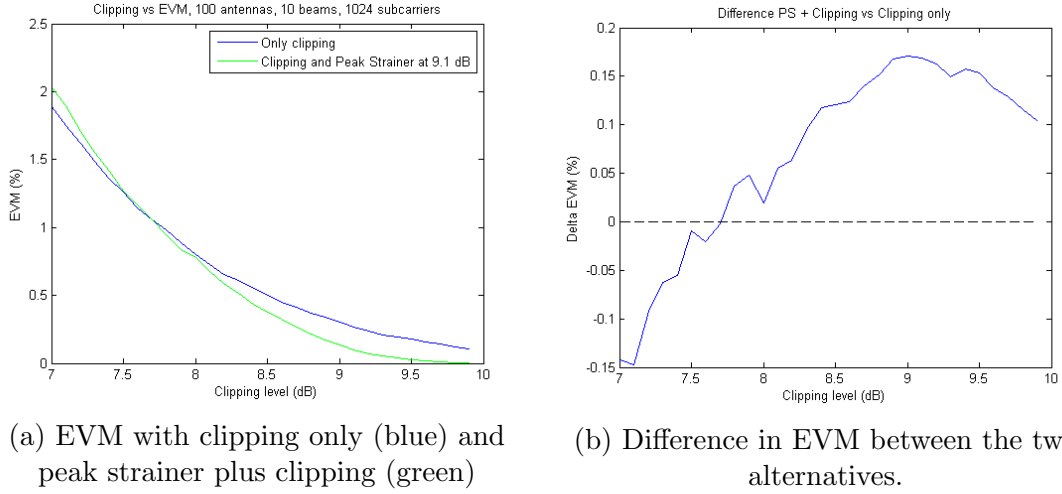


Performance for varying number of subcarriers	
Subcarriers	PAPR gain at probability $10^{-3}$
256	1.8dB
512	1.8dB
768	1.9dB
1024	1.9dB
1280	1.8dB
1536	1.8dB
1792	1.7dB
2048	1.7dB
2304	1.6dB
2560	1.5dB

The table below shows the PAPR gain at probability  $10^{-3}$  with CFR level at 9.1dB. The result shows that lowering the amount of subcarriers gives a slightly lower gain compared to the maximum of 1.9dB. However, the CFR level was chosen from the optimal point found by using 1024 subcarriers, so there might be room to investigate this further and see if more gains could be obtained. Especially since the higher subcarriers drastically decreases in gain which would suggest that there are improvements to be made. Due to time limitations, this further analysis will not be done in this thesis.

## 4.6 Clipping comparison

This subsection is of most interest since it compares the previous method of doing CFR by clipping, against the proposed peak strainer. It is also of value to see if these two methods can be combined, where peak strainer is first used to lower the PAPR initially and then later add clipping on each antenna element to further reduce the PAPR.



**Figure 4.5:** Peak straining and clipping

We can observe that up to CFR 7.7dB, it is beneficial to use both peak strainer and clipping, but after this, it actually gives less EVM to let CFR only be done by clipping. If the modulation that is used requires a very low EVM to function, such as 1024-QAM, where the total EVM requirement is 1.2%, it is beneficial to use peak strainer plus clipping compared to only using clipping [27].

The reason for this behaviour most likely comes from the way peak strainer smoothens out the highest peak, but at the same time, also potentially creates new peaks due to the phase shifts. Although these peaks are below the threshold for the CFR using peak strainer, the clipping algorithm will reduce the PAPR further and has to clip these new lower peaks, which is harder to do than clipping one large peak.

However, for the time being, peak strainer does not give a huge gain for existing systems, and as been seen, can actually perform worse than by just using clipping. In both of these simulations the ACLR was under the required levels for LTE [28].

# 5

## Conclusion

**T**HIS chapter summarizes the main findings of this thesis and suggests potential future research.

### 5.1 Summary of results

In the beginning of this thesis, we did not consider how the performance of CFR using clipping will change by adding more antennas. All numbers that were used as reference EVM with standard clipping were in relation to a single antenna system. According to a previous study, clipping at 8.8dB for a single antenna system gives 3.5% EVM [29]. However, for a multiple antenna system with 100 antennas at the transmitter, clipping introduces only 0.4% EVM at the same CFR level. Therefore, the performance of CFR using clipping is actually improving with the number of antennas. This happens since the EVM on each antenna element that is introduced by clipping, is averaged out through all antennas; at the receiver side, the final EVM will decrease hugely compared to the EVM on each transmit antenna individually. As was seen in section 4.6, with heavy CFR in a large antenna array, the use of peak strainer plus clipping performs worse than by only using clipping.

Peak strainer is beneficial for systems that have very high modulation schemes since they require very low EVM. An alternative way to use peak strainer to get more benefit, is for each beam to use different frequencies. In this way, sidelobe shaping is unnecessary, since sidelobes from another beams will not affect the mainlobe because they lie in different frequencies. This will lead to less RMS increase and a higher PAPR gain.

## 5.2 Future research

As discussed in section 4.4, it could be interesting to further analyze how different number of subcarriers affect the system, when using the optimal CFR level for each amount of subcarriers used. As section 4.4 pointed out, it seems reasonable that a small number of subcarriers leads to further gains, but what will happen in single carrier systems? This thesis focused entirely on multi-carrier systems using OFDM modulation, which leaves open other types of modulation, such as single-carrier FDMA, together with peak strainer for analysis.

With OFDM it is possible to do CFR using PTS; therefore, it would be interesting to see how peak strainer and PTS are combined together. Since PTS makes the individual OFDM symbols have small PAPR, peak strainer could then make the superposition of several symbols smaller to further lower the resulting PAPR.

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