THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

IN SOLID AND STRUCTURAL MECHANICS

Nonlinear Structural Identification Using a Multi-Harmonic Frequency Response Functions

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CHALMERS UNIVERSITY OF TECHNOLOGY

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Nonlinear multi harmonic FRF lays on top of linear FRF

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ABSTRACT

In industrial structural dynamics problems, linear FE-models commonly serve to represent the global structural behavior. However, when test data are available they often show evidence of nonlinear dynamic characteristics. In that case, an initial linear model may be judged insufficient in representing the dynamics of the structure. The causes of the non-linear characteristics may be local in nature whereas the major part of the structure is satisfactorily represented by linear descriptions. Although the initial model can serve as a good foundation, all physical properties required for representing the real structure with high fidelity are most likely not included in the initial model. Therefore, a set of parameterized candidate properties controlling the nonlinear effects have to be added. The selection of the candidates is a delicate task which must be based on insight into the physical processes that control the structure at hand.

The focus of this thesis is on the selection of uncertain model parameters together with the forming of the objective function to be used for calibration. To give precise estimation of parameters in the presence of measurement noise, the objective function data have to be informative with respect to the selected parameters. Also, to get useful test data for calibration, the system stimuli need to be properly designed. A multi-harmonic stationary sinusoidal excitation is here considered since the corresponding steady-state responses at the different harmonic orders are shown to contain valuable information for the calibration process.

KEYWORDS: Nonlinear structure, inverse problem, grey box system identification, finite element model updating, identifiability, Fisher information matrix, multi-harmonic frequency response function

To my beloved family

PREFACE

The project *Nonlinear structural identification using a multi harmonic frequency response function* was carried out at the division of structural dynamics at Chalmers University of Technology. The project has been funded as a part of *Swedish National Aviation Engineering Program* (*NFFP5*).

First of all, I would like to give my deepest gratitude to my advisor, *Prof. Thomas Abrahamsson* for his constant help and guidance. I thank him because of giving me the opportunity to expand my knowledge and sharpen my research skills under his supervision. I would like to issue my sincere gratitude to my co-supervisor, Dr. Andreas Linderholt, because of his support and rewarding discussions that we had.

I am pleased to acknowledge all my colleagues and friends especially our small Iranian group “Shabneshini”.

I would like to thank my dear parents in Iran for their support and inspiration throughout my life. Finally and foremost, I would like to express my love and special and deep gratitude to my lovely soulmate, *Elham*, for her understanding and infinite patience.

Gothenburg, December 2013

Vahid Yaghoubi Nasrabadi

“Most people, if you describe a train of events to them will tell you what the result will be. There are few people, however that if you told them a result, would be able to evolve from their own inner consciousness what the steps were that led to that result. This power is what I mean when I talk of reasoning backward.“

Sherlock Holmes, A Study in Scarlet, Sir Arthur Conan Doyle, 1887

THESIS

This thesis consists of an extended summary and the following appended papers:

Paper A V. Yaghoubi, T. Abrahamsson, Automated Modal Analysis Based on Frequency Responses Function Estimates, *Conference Proceedings of the IMAC XXX*, California, USA, 2012.

Paper B V. Yaghoubi, T. Abrahamsson, An Efficient Simulation Method for Structures with Local Nonlinearity, *Conference Proceedings of IMAC XXXII*, Florida, USA, 2014.

Paper C V. Yaghoubi, Y. Chen, A. Linderholt, T. Abrahamsson, Locally Non-Linear Model Calibration Using Multi Harmonic Responses - Applied on Ecole de Lyon Non Linear Benchmark Structure, *Conference Proceedings of IMAC XXXI*, California, USA, 2013.

Paper D Y. Chen, V. Yaghoubi, A. Linderholt, T. Abrahamsson, Model Calibration of a Locally Non-Linear Structure Utilizing Multi Harmonic Response Data, *Conference Proceedings of IMAC XXXII*, Florida, USA, 2014.

CONTENTS

EXTENDED SUMMARY

1 MOTIVATION AND BACKGROUND

Although nonlinear systems have caught much attention from both academy and industry, linear finite element models are still dominating the representation of the global structural dynamics behavior of complex mechanical systems. This is due to the fact that linear models are computationally efficient and have simple input-output relationships which most often offer good insight into the models' dynamics. If significant nonlinear characteristics are found from testing of the system, a linear model may be judged being insufficient in representing the structural behavior whereby a nonlinear model has to be utilized. For many mechanical systems, the causes of nonlinear behavior are local in nature. Structural joints introducing gaps or dry friction and structural parts subjected to large rotations are examples of such. An initial linear model can then form a good foundation for a nonlinear model taken to model calibration. However, the required parameters to substantially increase the model's capability of representing the real structural behaviour are most likely not included in such initial model. Therefore, a set of candidate parameterized properties that controls the nonlinear effects have to be added. The selection of such candidates is a delicate task which solution must be based on insight into the physical processed that govern the behaviour of the system at hand.

This thesis treats different aspects of calibration of a computational non-linear model. The application target is in aeronautical engineering in which back-lash at control surface hinges are examples of local nonlinearities.

2 INVERSE PROBLEM IN LINEAR STRUCTURAL DYNAMCIS

The forward problem of structural dynamics relates to the problem of obtaining the system response with a given model and given system stimuli. As indicated in Figure 1, the inverse problem is, in contrast to the forward problem, either the problem of obtaining the system stimuli with given response and a given system model or obtaining the system model given system stimuli and response. These are the two classes of the inverse problem for which this thesis treats the second class, which is also called the system identification problem. Two types of models are common in the field of system identification:

Black box model: in black box system identification, there is no available prior knowledge about the model. The model is selected from one model class, for which the state-space model, the ARX model or the ARMAX model are commonly used classes, see Reference 18.

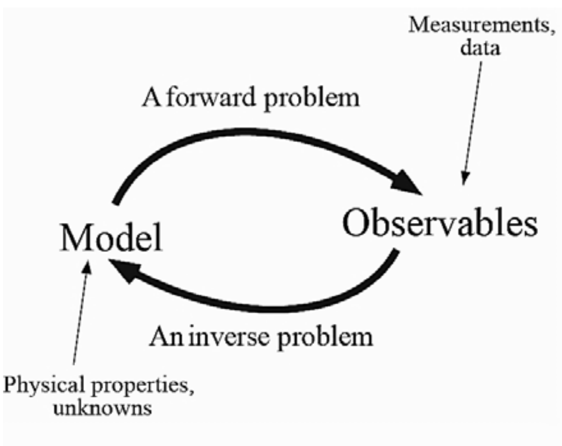


Figure 1. Schematic of forward and inverse problems.

Grey box model: in grey box modeling, although what is going on inside the system are not entirely known, a certain model based on both insight into the system and experimental data is constructed, most often based on first principles and discretization. This model does however still have a number of unknown properties that can be parameterized and estimated using system identification. Structural identification belongs to this group of identification.

In the following, the forward problem is treated because it is most often solved repeatedly in the solution of the inverse problem. The system identification problem is described in some detail and a specific eigenvector correlation metric that has been developed in the course of this work is presented.

2.1 The forward problem

For a general mechanical structure, the governing second-order equation of motion is

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{V}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{f}(t) \tag{1}$$

in which $\boldsymbol{M}$, $\boldsymbol{V}$ and $\boldsymbol{K}$ are the mass, damping and stiffness of a structure respectively and $\boldsymbol{f}$ is the force vector.

Equation (1), can be rewritten in a state-space formulation as

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \end{aligned} \tag{2}$$

in which $\boldsymbol{y}$ is the system's output. $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are state-space matrices and the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ of the dynamic equation can be obtained by the following

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{V} & -\boldsymbol{M}^{-1}\boldsymbol{K} \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1} \end{bmatrix}, \tag{3}$$

$\boldsymbol{A}$ and $\boldsymbol{B}$ are constant-valued matrices and only the system stimuli $\boldsymbol{u}$ depend on time, therefore, the system is called Linear and Time-Invariant (LTI) system. The system stimuli $\boldsymbol{u}$ consists of the nonzero force elements of $\boldsymbol{f}$ as $\boldsymbol{f} = \boldsymbol{U}\boldsymbol{u}$ with a suitable $\boldsymbol{U}$.

To obtain a numerical solution, the continuous-time ordinary differential equation (ODE) of Equation (2), needs to be discretized into a time-marching recursive algorithm with time step T. This can be made through the recursive formula

$$\boldsymbol{x}(kT + T) = \boldsymbol{A}_{\text{disc}}\boldsymbol{x}(kT) + \boldsymbol{B}_{\text{disc}}\boldsymbol{u}(kT) \tag{4}$$

in which the state at time $t=kT+T$ is obtained from data given by the previous step at $t=kT$ along with discrete-time versions of the $\boldsymbol{A}$ and $\boldsymbol{B}$ matrices. The exact coefficient matrices of the discretized form can be shown to be

$$\begin{aligned} \boldsymbol{A}_{\text{disc}} &= \mathrm{e}^{\boldsymbol{A}T} \\ \boldsymbol{B}_{\text{disc}}\boldsymbol{u} &= \boldsymbol{B}\int_{kT}^{kT+T} \mathrm{e}^{\boldsymbol{A}(kT+T-\tau)}\, \boldsymbol{u}(\tau)d\tau \end{aligned} \tag{5}$$

The integral expression for the source term $\boldsymbol{B}_{\text{disc}}\boldsymbol{u}$ can be established only approximately for a general loading $\boldsymbol{u}(t)$.

2.2 Black box system identification

As mentioned, for a linear and time-invariant (LTI) system the state-space model in discrete time can be written as

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{A}_{\text{disc}}\boldsymbol{x}_{\boldsymbol{k}} + \boldsymbol{B}_{\text{disc}}\boldsymbol{u}_{\boldsymbol{k}}, \\ \boldsymbol{y}_k &= \boldsymbol{C}\boldsymbol{x}_k + \boldsymbol{D}\boldsymbol{u}_k \end{aligned} \tag{6}$$

with the notation $\boldsymbol{x}_k = \boldsymbol{x}(kT)$ and $\boldsymbol{x}_{k+1} = \boldsymbol{x}(kT + T)$. Here $\boldsymbol{x}$ is the *n*-dimensional state vector, $\boldsymbol{A}_{\text{disc}}, \boldsymbol{A} \in \mathscr{R}^{\text{nxn}}$, $\boldsymbol{B}_{\text{disc}}, \boldsymbol{B} \in \mathscr{R}^{\text{nxm}}$, $\boldsymbol{C} \in \mathscr{R}^{\text{pxn}}$ and $\boldsymbol{D} \in \mathscr{R}^{\text{pxm}}$. The vector $\boldsymbol{u}$ is the *m*-dimensional excitation vector and $\boldsymbol{y}$ is the *p*-dimensional response vector of the system.

For LTI systems, several identification methods have been proposed to find state-space matrices from stimuli and response data given by testing. An efficient such method being the State-Space Sub-Space based method called N4SID[1]. With this method, the system identification method user only needs to define a proper model order and the methods estimate the corresponding system parameters very efficiently. However, the degree of unmodeled dynamics or the level of over-parameterization depend on the choice of the model order, therefore, a proper model order become a key problem in these methods. When the model order is sufficiently large to describe the linear dynamics adequately then, the model doesn't have unmodeled dynamics and the model can approximate the measured results reasonably well[2] without being too complex. Other state-of-the-art methods for multi-input-multi-output (MIMO) systems being the poly-reference least square complex frequency domain method called PolyMAX[3], and the Poly-Reference Complex Exponential (PRCE) method[4]. A proper model order is still an issue for these. Therefore, in recent years several approaches have been proposed to find a proper model order [5-8]. In the most recent work, Cara *et al*[9] presented a criteria based on modal contribution to find state space model order. A method based on maximum-likelihood identification to find modal parameters has been presented by El-kafafy *et al* [7].

In implementations of identification methods, the user often gets help in the model order selection by the use of pole stabilization charts or plots that shows drop of Hankel singular values[10]. For a system with well separated modes it is relatively easy to find a good model order by means of inspection of complex mode indicating functions [11] but for systems with closely spaced or highly damped modes or with nonlinear distortion, finding model order by visual inspection is very tedious and the identification results are user dependent. Thus, finding a proper model order to realize FRF data is still a challenge in this area. This thesis addresses aspects of this problem.

A multi-step procedure for obtaining a proper model order from experimental frequency response functions is proposed in this thesis. The approach commences with the identification of a high-order state-space model, called the Exhaustive Model, using the full FRF data set from testing. Then, modal states that give negligible contribution to the output, quantified by a metric associated to the observability grammian, are rejected from the exhaustive model resulting in what is called a Reference Model. The next step of rejection is based on the statistical evaluation of an ensemble of state-space models all identified from the same-size fractions of the full frequency response functions, with the same model order as the reference model but with different realizations based on a bootstrapping scheme; these models are called Bootstrapping Models. Eigensolutions of the bootstrapping models are then paired by the eigensolutions of the reference model based on newly developed Modal Observability Correlation (MOC) indices[12]. In a second reduction stage, a criterion based on the mean value and standard deviation of the maximum correlation number is used for rejection of spurious modes. The method has been implemented on several different case studies and the results [13-15], showed robustness and accuracy of this algorithm to find the proper model order.

2.3 Grey box system identification

For the representation of a complex structure with excitation and response in a wide frequency range, the need of selecting a model with many DOFs is inevitable. However, the estimation of all possible model parameters from data given by measurements may quickly become intractable. A solution to this problem is to use structural modeling techniques, which compute the model parameters based on the geometrical and mechanical properties of the structure. This is the main reason that structural identification is recognized as grey box system identification. It uses a given model structure with undetermined parameter settings.

Despite the high sophistication of structural modeling, practical applications often reveal considerable discrepancies between the model predictions and experimental results, due to three sources of errors, namely modeling errors (*e.g*, imperfect boundary conditions or improper damping modelling), parameter errors (*e.g.* inaccuracy of material stiffness) and testing errors (*e.g.* noise introduced in the measurement process). There is thus the need to improve structural models through the comparison with vibration measurements performed on the real structure (see Figure 2). This is referred to as *structural model updating*. Very often, the initial model structure is created using the *finite element method* and structural model updating is termed *finite element model updating* or *finite element model calibration* [16].

Before any updating take place, the selection of the test data to use in combination with the selection of model parameters to be involved should be examined. Test data which is used for model updating have to fulfil at least two requirements. Firstly, data should be informative with respect to the parameters used for updating. Secondly, data have to change differently for changes of different parameters, which is to say that the parameters should be *identifiable*. Should these two requirements not be fulfilled it is simply impossible to get a reliable parameter value estimation made using that test data [17].

With informative data and identifiable parameters, the system identification is usually straightforward. Several methods have been developed to numerically estimate models and from these the related modal parameters, *i.e.* eigenfrequencies, eigenvectors and modal dampings, from frequency response data. These methods often require little user interaction.

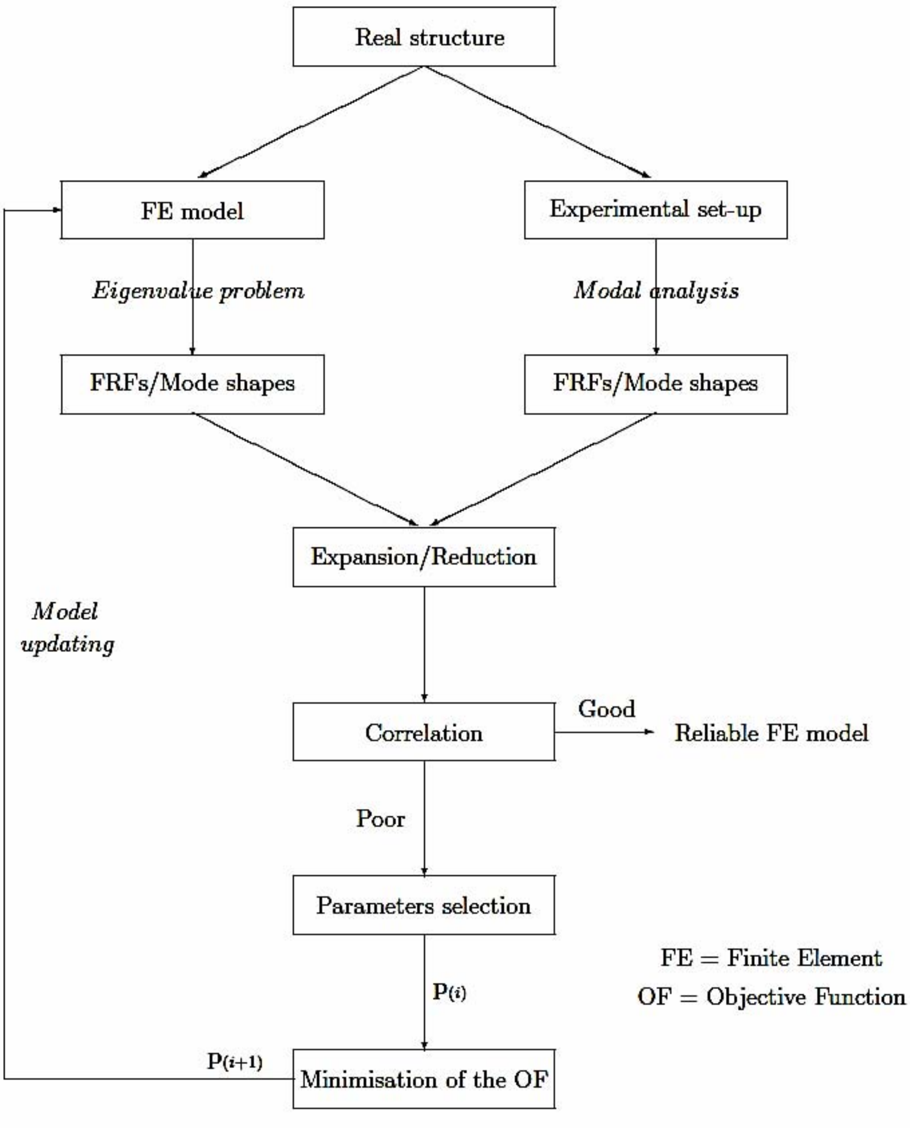


Figure 2: Linear finite element model updating algorithm

A finite element model updating procedure that uses frequency response functions (FRF) synthezised from state-space model data and relies on damping equalization is proposed by Abrahamsson and Kammer[18]. In this, the dampings of the analytical finite element model and the corresponding experimental model are set equal in an intermediate step. The damping equalization is made to avoid the mode correlation and mode-pairing problem that need to be solved in other model updating procedures. It is demonstrated that one particular use of frequency response data gives a calibration deviation metric that is smooth in the variation of model parameters and give a large radius of convergence to the global calibration minimum. The method is combined with model reduction for increased speed and employs the Levenberg-Marquardt minimizer with randomized multiple starting points in the parameter space to achieve the calibration solution. The method has been implemented in a Matlab toolbox called FEMcali.

To conclude, a good start for finite element model updating is when sensitive and informative data are available, a good parameterized finite element model as a model structure has been built and a state-space model of proper model order is identified from test data.

2.3.1 Model Correlation

Many methods for model calibration relies on eigenvector correlation. Comparison of experimental modal vectors, estimated from measured data, to eigenvectors that have been determined from finite element calculation needs a correlation metric. Such comparison is required by many updating methods to calibrate models to achieve a good match between experimentally found eigensolutions and the eigensolutions of the finite element analysis. The most well-known eigenvector correlation metric is the Modal Assurance Criterion (MAC) which is defined as a scalar constant relating the degree of consistency (linearity) between two modal vectors as follows[19],

$$MAC = \left|\Psi_i^H \Psi_j\right|^2 / \left(\Psi_i^H \Psi_i \Psi_j^H \Psi_j\right), \qquad (7)$$

Note that the vectors need not come from the same source, but need to be of equal length. For instance, the *i*th and *j*th modes may stem from data from two different tests of the same test object or from FE analysis on one hand and experimental modal analysis on the other hand.

For system with well-separated eigenvalues with many system degrees-of-freedom (DOF) represented in the eigenvectors it is normally easy to distinguish eigenvectors associated to different eigenvalues by low MAC correlation numbers. However, for eigenvectors with a sparse DOF sampling or closely spaced eigenvalues it may be hard to distinguish between vectors. Besides that problem, since MAC has no information about the eigenfrequency correspondence to the related eigenmodes in its formulation, it can sometimes indicate good correlation between high frequency modes and low frequency modes due to spatial aliasing.

Some works in the literature consider the effect of eigenfrequency in the vector correlations. Fotsch and Ewins changed the way of presenting MAC in order to consider the eigenfrequency[20], and Philips and Allemang presented a correlation metric, which they called the *pw*MAC, in which state vectors are used instead of eigenvectors[21]. By using this correlation metric, the distinction between the modes with similar sampled eigenvectors but with different eigenvalues is possible, but there is still some problem with distinguishing between the modes with close eigenvalues and similar eigenvectors but with different levels of modal contribution to the output.

To make the problem of distinguishing between eigenvectors smaller, a new correlation metric based on the observability matrix of the diagonal state-space realization is proposed by Yaghoubi and Abrahamsson[12]. We call this metric the Modal Observability Correlation or *MOC* in short. This is defined as

$$MOC_{ij} = |\{\mathcal{O}_i\}^{\mathrm{H}}\{\mathcal{O}_j\}|^2 / \max(\{\mathcal{O}_i\}^{\mathrm{H}}\{\mathcal{O}_i\},\{\mathcal{O}_j\}^{\mathrm{H}}\{\mathcal{O}_j\})^2 \qquad (8)$$

in which

$$\mathcal{O}_{\mathrm{i}} = \begin{bmatrix} \boldsymbol{C}\Psi_i T_2^{ii} \\ \boldsymbol{C}\Psi_i T_2^{ii}\lambda_i \\ \boldsymbol{C}\Psi_i T_2^{ii}\lambda_i^2 \\ \dots \\ \boldsymbol{C}\Psi_i T_2^{ii}\lambda_i^{n-1} \end{bmatrix} \tag{9}$$

is the i^{th} column of the balanced modal observability matrix. Ψ_i and λ_i is the eigenvector and eigenvalue of the of i^{th} mode respectively. $\boldsymbol{C}$ is the matrix in state-space representation (see equation (2)) and T_2^{ii} is a transformation to balance the i^{th} mode to have equal observability and controllability grammians.

3 INVERSE PROBLEM IN NONLINEAR SYSTEMS

The simulation of the forward nonlinear problem is treated in some detail, because its close relation to the corresponding inverse problem. Also, an example of a nonlinear identification is described in the following.

3.1 Simulating the forward problem

Simulating the nonlinear behavior of structures demands a lot of computational effort and efficient simulation tools are necessary. Researchers in different fields that normally involve much computation, such as system identification and system reliability, are increasingly interested in nonlinear systems which has spurred in the attempts to develop fast nonlinear simulation methods[22-25].

Nonlinearity in structures can be characterized as being either local or global. Locally nonlinear structures are structures that are mainly linear but have one or more locally nonlinear devices/properties that make the structural behavior nonlinear. Local nonlinearity in mechanical structures often stems from nonlinear structural joints and can make its response highly nonlinear. There are two major difficulties in simulating a nonlinear structure, the first one is to make accurate predictions/simulations of nonlinearity effects in a structure's response and the next one is the efficiency in simulation in order to simulate the structural behavior fast-enough for convenience.

To speed up the simulation of nonlinear structures, several methods have been proposed. Some methods are based on the model reduction of nonlinear structures[26,27], others are focused on the integration part to make it faster and more stable[22,23] while others deal with nonlinear elements based on remodeling and piecewise linearization [28].

Avitabile and O'Callahan[29] presented three efficient techniques to treat the nonlinear connection between linear parts. They called them the *Equivalent Reduced Model Technique* (ERMT), the *Modal Modification Response Technique* (MMRT), and the *Component Element Method* (CEM). In MMRT, the coefficient matrices governing the structural response should iteratively be updated using structural dynamics modification [28] in a process done in modal space, then intermediate result should be returned to physical space to check for possible change in linear response. Marione *et al*[24] applied MMRT to three different cases and it was shown that the main efficiency gain was obtained by doing model reduction of the systems. In ERMT, the well-known SEREP[30] method was used to reduce the linear system before discrete nonlinear connections were assembled to the system. Thibault *et al*[31] applied ERMT and performed case studies.

One of the well-established methods to consider the effect of structural system nonlinearity is the pseudo-force method[32]. In this method, the nonlinearity is considered as nonlinear external forces acting on an otherwise linear structure. Felippa and Park[33] used this method to treat the nonlinearity in nonlinear structural dynamics. They implemented the method in the first-order system context and used the Linear Multistep Method to discretize the equations, *i.e.* the whole response history of the system was used to find the response of the system in the next iteration. To reduce the required time for finding the response of the system in the next iteration Brusa and Nigro[34] presented a one-step method for discretizing a first-order system. They applied the method on linear systems only. Feng-Bao *et al*[35] presented an iterative pseudo-force method for second order systems to treat the non-proportional damping in the structures. They also proved the convergence of the method.

The state-space formulation, see Equation (6), is the most common first-order representation of linear systems. To find the system response, integration can be done using numerical integration schemes like the one used by the Runge-Kutta method or other time-stepping schemes based on triangular hold interpolation of the loading[36].

We propose a method to efficiently treat localized nonlinearity in a structure. The system is separated by a linearized part and a nonlinear part. The non-linear part is considered as external pseudo forces that act on the linearized system. The response of the system is obtained by iterations. Since the method is presented in linear state-space form, all linear manipulation like similarity transformations and well-established state-space model reduction can easily be exploited.

The method can be regarded as an iterative pseudo force method applied on first order system that was discretized using a linear single step method. We call the method the *Pseudo Force in State Space* (PFSS) method. To do integration, the triangular hold interpolation is used.

The governing second-order equation of motion for a non-linear mechanical structure can be written as follows (*c.f.* Equation (1))

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{V}_{\mathrm{L}}\dot{\boldsymbol{q}} + \boldsymbol{V}_{\mathrm{NL}}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{K}_{\mathrm{L}}\boldsymbol{q} + \boldsymbol{K}_{\mathrm{NL}}(\boldsymbol{q})\boldsymbol{q} = \boldsymbol{f}(t) \tag{10}$$

in which $\boldsymbol{M}$, $\boldsymbol{V}$ and $\boldsymbol{K}$ are the mass, damping and stiffness of a structure respectively. Subscript L stands for linearized part of the matrices and NL is for the state-dependent nonlinear part. The force vector is $\boldsymbol{f}$.

Without approximation all nonlinearity can be moved to the RHS side of the equation and to be treated like external force caused be nonlinearities as

$$\boldsymbol{M}_{\mathrm{L}}\ddot{\boldsymbol{q}} + \boldsymbol{V}_{\mathrm{L}}\dot{\boldsymbol{q}} + \boldsymbol{K}_{\mathrm{L}}\boldsymbol{q} = \boldsymbol{f} + \boldsymbol{f}_{\mathrm{NL}} \tag{11}$$

Here,

$$\boldsymbol{f}_{\mathrm{NL}} = -\boldsymbol{V}_{\mathrm{NL}}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{K}_{\mathrm{NL}}(\boldsymbol{q})\,\boldsymbol{q} \tag{12}$$

Equation (10), can be rewritten in state-space formulation as

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \end{aligned} \tag{13}$$

in which $\boldsymbol{y}$ is the system's output. $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are state-space matrices and the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ of the dynamic equation can be obtained by the following.

$$\begin{gathered} \boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{V}_{\mathrm{L}} & -\boldsymbol{M}^{-1}\boldsymbol{K}_{\mathrm{L}} \end{bmatrix}, \\ \boldsymbol{B} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1} \end{bmatrix}, \end{gathered} \tag{14}$$

and the non-zero elements of the external and nonlinear forces $\boldsymbol{f}$ and $\boldsymbol{f}_{\mathrm{NL}}$ define the stimulus $\boldsymbol{u}$ as

$$\boldsymbol{U}\boldsymbol{u} = \boldsymbol{f} + \boldsymbol{f}_{\mathbf{NL}}$$

with the proper Boolean distribution matrix $\boldsymbol{U}$.

To conclude, we use the following algorithm to achieve a solution to the nonlinear simulation problem:

1. Find ***K*, *V*** and ***M*** of the underlying linear system
2. Establish the state-space matrices (13)
3. Do the time discretization as in (4) to (5)
4. Find the response of the linear system at time step k ($\boldsymbol{u}_k = \boldsymbol{u}_{\mathrm{L},k} + \boldsymbol{u}_{\mathrm{NL},k}(x_k, y_k, kT)$)
    4.1. For first iteration, set the nonlinear force to zero
    4.2. Evaluate the non-linear source term
5. Check if there is any change in response of the system from previous iteration
    5.1. If YES : Update nonlinear force and go to step 4
    5.2. If NO: go for next time step and start a new iteration sequence at step 4

3.2 Grey box system identification

In contrast to linear systems, there is no general identification method that can be applied to a generic nonlinear system. Kerschen *et al* [16] recently did a literature review for nonlinear system identification in structural dynamics. Among the methods they covered, *Structural Model Updating* recently attracts a lot of attention. In Structural Model Updating, the parameters of a parameterized finite element model are calibrated to minimize the deviation to test data. The model structure is thus given, and the identification is thus grey-box.

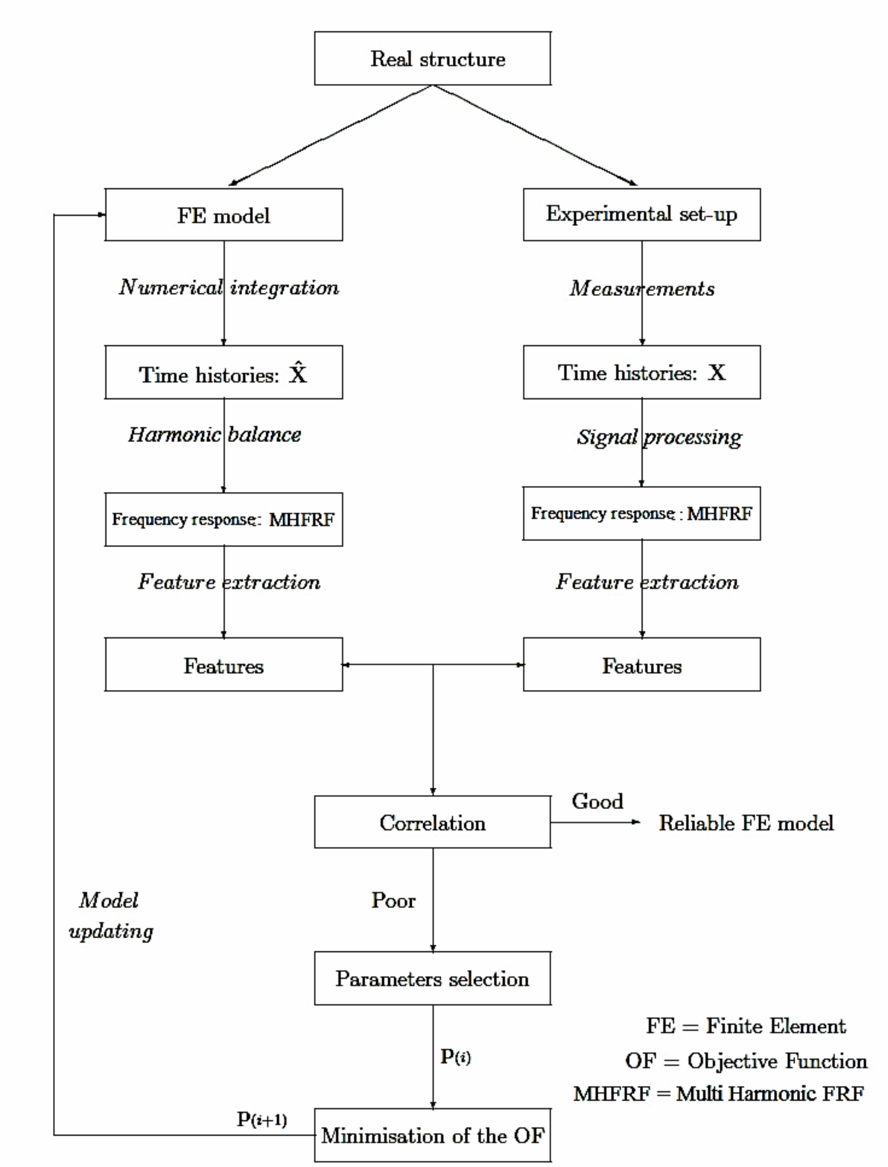


Figure 3: Nonlinear finite element model updating algorithm in frequency domain

Large deviations between test data and the corresponding analytical data stemming from a calibrated linear model may indicate that there is unmodeled nonlinearity in the structural problem. The necessary model parameters to capture the nonlinear effects and increase the model's capability of representing the real structural behavior are most likely not included in the initial linearized model. Therefore, a set of candidate parameters that models nonlinear effects, have to be added. The candidate parameters have to be chosen using insight into the observed physical effects at hand.

In nonlinear systems, processing all data that are available from test sometimes may hide or destroy information that is in the original data. Some test data simply decreases the usefulness of the overall test data; such data should be excluded before the updating is made. For a nonlinear structure (as well as for a linear), the FIM and its inverse, and thereby the data informativeness and the parameter identifiability, varies with the response data used. It is therefore important to include all available data that carries information that differentiate parameter settings. This is partly controlled by the selection of excitation and response measurement during test; that is the actuator and sensor placement together with the excitation time history. Another part is the choice of the perspective on which the test data are looked upon.

The excitation force fed into a structure during a vibrational test depends on the signal sent to the exciter but also to the interaction between the exciter and the structure itself. When nonlinearities are present, an intended pure mono-harmonic excitation therefore often becomes a multi-harmonic excitation consisting of the fundamental frequency together with sub and super harmonics of this. If the structure was linear, the responses at the sub and super harmonics would be given by its frequency response function in combination with the excitation. The deviation from linear response is therefore stemming from the nonlinearities within the structure. By using a multiple harmonic excitation, the responses at the multiple frequencies can be compared with known nonzero values stemming from a linear model. The response deviations, between test and analyses, around the linearized system's eigenfrequencies are found to contain the most valuable information for calibration of models of locally nonlinear structures. An example that follows illustrate a nonlinear calibration procedure.

3.3 Nonlinear calibration example

3.3.1 Experimental Set-Up

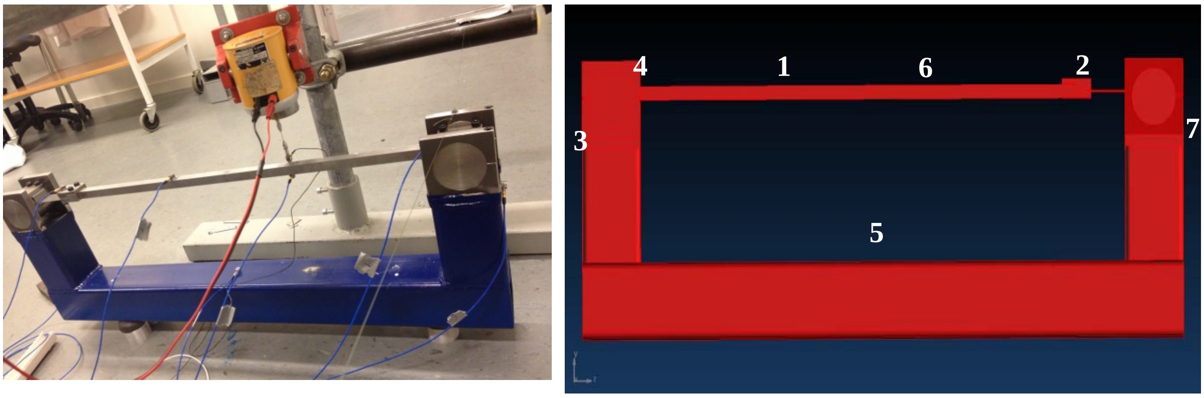
The structure studied, a replica of the Ecole de Lyon (ECL) nonlinear test-bench structure, consists of two joined cantilever beams, see Figure 4. The main part is a beam (member #1 in figure 4), which is clamped at one end and connected to a thin beam through a joint in its other end[37]. The thin cantilever beam introduces a nonlinear behavior to the structure when forced to large rotational displacement. The experiments are made for three different load levels, 1, 2 and 5 N, and with two different pretension levels for each load levels. The load is in the plane of the frame. In total 6 different cases have been evaluated.

The excitation force is designed to contain sub and super harmonics together with the fundamental sinusoidal. The magnitudes of these side harmonics are made small compared to the fundamental harmonic but large compared to the expected noise.

3.3.2 Finite Element Model

The frame is modeled by linear beam elements; CBAR within MSC Nastran[38]. The cantilever beam is modeled by 13 linear beam elements. The beam is rigidly connected (by an RBE2-element within MSC Nastran) to the frame. The membrane, responsible for the nonlinear restoring force, is modeled by 4 linear beam elements. The membrane is rigidly connected (by an RBE2-element within MSC Nastran) to the frame.

A local nonlinearity is added to the linear model. The nonlinearity within the structure originates from pretension of the thin beam and misalignment at the joint due to the weight of the main beam. The quantities are shown schematically in Figure 5.



(a) (b)

Figure 4: ECL Set-Up (a) experimental setup b) CAD model of the structure with numbers, which are accelerometers position, input is at the same place as one of the accelerometers(#1).

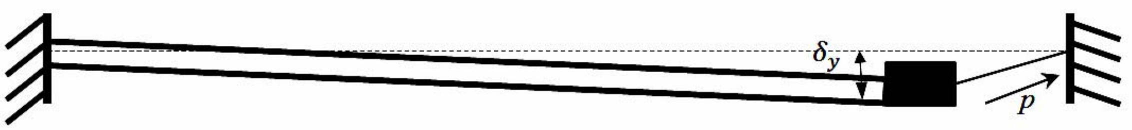


Figure 5: Modeling of nonlinearity in the setup, misalignment in the joint due to the weight of the main beam δ_y, and pretension *P*.

3.3.3 Finite Element Model Updating

A baseline linear model was calibrated using at the response due to low-level excitation damping equalization method, implemented in FEMcali toolbox. That was made in order to calibrate the density and Young's modulus of the frame, cantilever beam and the thin beam. The nonlinear parameters were also calibrated using damping equalization method but in multi-harmonic FRF procedure.

The selection of data to be used in model calibration is vital. The data selection is coupled to the model parameterization chosen. Using an over-parameterized model for calibration makes the estimated parameter values indeterminate. When there is a lack of information to discriminate between different parameters, or groups of linearly combined parameters, a re-parameterization or a modification of the data used has to be made, see Linderholt [17]. The Cramer-Rao lower bound quantifies a limit to the accuracy of parameter estimates from the information in the test data. The inverse of the Cramer-Rao lower bound is known as the Fisher information (FIM), see Udwadia [39]. The Cramer-Rao lower bound and the FIM are useful quantities to assess test data informativeness and parameter identifiability and they can be estimated a priori using a calculation model. The test data should be chosen such that the expected variances of the estimated parameters are small. Since the amount of information depends on the raw data available and the usage of them, one possibility is to process the test data differently before calibration. A tempting solution may be to simply add more test data but, as shown in this paper, the opposite could be an alternative. Disregarding low excessive data may make the remaining data better to discriminate between different parameter settings. The result of the calibration is described in Paper D of this thesis.

4 SUMMARY OF APPENDED PAPERS

4.1 Paper A

In paper A, *Automated Modal Analysis Based on Frequency Responses Function Estimates*, a multi-step procedure for obtaining a proper model order from experimental frequency response functions is presented. The approach commences with the identification of a high-order state-space model, then at each step some modes will be rejected. At the end, the most important modes based on a newly developed correlation metrics, Modal Observability Correlation (MOC), will remain.

4.2 Paper B

In paper B, *Efficient Simulation Method for Structures with Local Nonlinearity*, a method for fast and accurate simulation of nonlinear structures is presented. The method is compared, in the sense of accuracy and efficiency, with the Runge-Kutta method and the Modal Modification Response Techniques (MMRT). The latter being among the latest available efficient method for simulating structures with local nonlinearity presented in the literature. The method, called Pseudo Force in State Space (PFSS), is shown to perform well in comparison. The drawback of the method is its conditional stability, which can be circumvented by taking smaller time steps in the numerical integration.

4.3 Paper C

In paper C, *Locally Non-Linear Model Calibration Using Multi Harmonic Responses - Applied on Ecole de Lyon Nonlinear Benchmark Structure*, a finite element model of a structure developed at Ecole de Lyon (ECL) was parameterized and updated using a damping equalization method followed by a multi-harmonic FRF matching procedure. Four parameters were selected for calibration. These were the pretension in a thin beam, the Young’s modulus, and coefficients of linear, quadratic and cubic spring forces.

4.4 Paper D

In paper D, *Model Calibration of a Locally Non-Linear Structure Utilizing Multi Harmonic Response Data*, a properly parameterized finite element model of the ECL testbed structure was calibrated with measured data with the method of damping equalization and with multi-harmonics FRF data. Firstly, a linear model was calibrated in order to find calibrated values of Young’s moduli and densities of frame parts, a cantilever beam and a thin beam member. That was made using the FEMcali Toolbox for finite element model calibration. In a second step the nonlinearities, originated from misalignment in a joint and the pretension in a thin beam, were calibrated using the multi-harmonic FRF matching procedure.

5 CONCLUDING REMARK AND FUTURE WORK

In this project, a finite element model updating method based on multi-harmonic frequency response function has been proposed. The method started with calibrating the underlying linear structure. To do so, firstly, a parameterized finite element model for underlying linear structure have to be obtained and calibrated with experimental data gathered from nonlinear structure which was simulated with very small force level in order to not excite the nonlinearity. The calibration can be done by the use of the FEMcali toolbox.

An algorithm to obtain a proper state-space model order to realize measured FRF data for linear system has been proposed. The method which is based on a new correlation metric has been applied on several case studies and the results presented in paper A. More work is planned to make the model order determination more robust.

In the next step of updating the structure, nonlinearity was modelled based on our knowledge about the physical and geometrical properties of the structure. The parameterized nonlinear model was calibrated with experimental data stemmed from nonlinear structure by the use of the Levenberg-Marquardt algorithm and multi start procedure. The objective function selected for calibration minimization is the logarithmic distance between vectorized multiharmonic FRF data obtained from experimental and analytic model. Simulating the response of the nonlinear structure was one issue in this step. Therefore, an efficient simulation method has been developed and applied on several case studies and the results shown efficiency and accuracy of the method presented in paper B. The method which is called Pseudo Force in State-Space, is conditionally stable. Future work is planned to improve the stability properties of the method.

Since in model calibration, more data will not necessarily result in better parameter estimation, it is a crucial step to assure the data informativeness and parameter identifiability. The Cramer-Rao lower bound and its inverse, the Fisher information matrix, were used to assess the former and latter respectively. Only the part of data that carries rich information is used for calibrating the nonlinear model. The method applied on a nonlinear benchmark called ECL and the results were presented in papers C and D.

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