Zonal Love and Shida numbers estimated by VLBI

H. Krásná, J. Böhm, R. Haas, H. Schuh

Abstract The deformation of the anelastic Earth as a response to external forces from the Moon and Sun is characterized with proportionality parameters, the so-called Love and Shida numbers. The increasing precision and quality of the VLBI (Very Long Baseline Interferometry) measurements allow determining those parameters. In particular, the long history of the VLBI data enables the estimation of Love and Shida numbers at the low frequencies with the longest period of a tidal wave at 18.6 years. In this study we analyze 27 years of VLBI measurements (1984.0 - 2011.0) following the recent IERS Conventions 2010. In several global solutions, we estimate the complex Love and Shida numbers of the solid Earth tides for the main long-period tidal waves. Furthermore, we determine the Love and Shida numbers of the rotational deformation due to polar motion, the so-called pole tide.

Keywords Love and Shida numbers, solid Earth tides, pole tide

1 Introduction

Deformation of the Earth due to solid Earth tides is caused by tidal forces arising from the gravitation attraction of celestial bodies surrounding the Earth. The displacement of the Earth is proportional to the tidal potential by factors which reflect the amount by which the surface of the Earth responds to the tidal forces. The proportionality numbers which link the tidal potential to the surface displacement are so-called Love (h) and Shida (l) numbers. For a basic Earth model where the Earth is considered to be spherical, non-rotating, elastic and isotropic the Love and Shida numbers are dependent on the degree n of the tidal potential V_n^t. The displacement vector Δd^t induced by the tidal potential in the local coordinate system (radial ($\hat{r}$), east($\hat{e}$), north ($\hat{n}$)) is then written as:

$$\begin{aligned} \Delta d^t = & \frac{1}{g}\sum_{n=2}^{\infty} h_n \cdot V_n^t \,\hat{r} \\ & + \frac{1}{g\cos\Phi}\sum_{n=2}^{\infty} l_n \cdot \frac{\partial V_n^t}{\partial \Lambda}\,\hat{e} \\ & + \frac{1}{g}\sum_{n=2}^{\infty} l_n \cdot \frac{\partial V_n^t}{\partial \Phi}\,\hat{n}, \end{aligned} \tag{1}$$

where Φ and Λ are geocentric coordinates of the station and g is gravitational acceleration. The recent theory of solid Earth tidal displacements is based upon the model of Wahr (1981) who considered the effects of rotation and ellipticity of the Earth. The deformation of the Earth's surface caused by lunisolar tides is based on the sum of the tidal potential with spherical harmonic degrees n and orders m, where the effective values of Love and Shida numbers additionally depend on the frequency of the tidal wave. In the long-period band the frequency dependence is mainly due to mantle anelasticity. The anelasticity model adopted in Petit and Luzum (2010) is the one from Widmer et al. (1991). The variation of the Love and Shida number across the zonal tidal band (h_{20} and l_{20}) is described by equations (2) and (3) (formula (7.4) in Petit and Luzum (2010)). Love and Shida numbers from these equations are also tabulated in the IERS Conventions 2010 and we used them as a priori values for their estimation in

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the global adjustment.

$$h_{20} = 0.5998 - 9.96 \times 10^{-4} \cdot X, \tag{2}$$

$$l_{20} = 0.0831 - 3.01 \times 10^{-4} \cdot X, \tag{3}$$

where

$$X = \left\{ \cot \frac{\alpha\pi}{2} \left[1 - \left(\frac{f_m}{f} \right)^{\alpha} \right] + i \left(\frac{f_m}{f} \right)^{\alpha} \right\}. \tag{4}$$

f is the frequency of the zonal tidal constituent, f_m is a reference frequency equivalent to a period of 200 s, and the power law index $\alpha = 15$. To ensure 1 mm accuracy by the computed displacement of the crust five tidal waves have to be taken into account (Petit and Luzum, 2010). In addition for purpose of this work, the annual tidal wave S_a was added to this group. The tidal waves are described in Table 1. The frequency-dependent correction of the displacement caused by the long-period tides follows from Mathews et al. (1997, equation (2)):

$$\delta d_f = \sqrt{\frac{5}{4\pi}} H_f \left\{ \left(\tfrac{3}{2} \sin^2 \Phi - \tfrac{1}{2} \right) (\delta h_f \cos\theta_f)\, \hat{r} + \tfrac{3}{2} \sin 2\Phi (\delta l_f \cos\theta_f)\, \hat{n} \right\}. \tag{5}$$

H_f is the amplitude of a tidal term of frequency f defined by the convention of Cartwright and Tayler (1971), θ_f is the argument for the tidal constituent with the frequency f, and δh_f and δl_f are the corrections to the Love and Shida numbers of degree two.

Similar to the deformation of the solid Earth due to the tidal potential, there is deformation of the crust Δd^c caused by variations in centrifugal potential V^c. This change of centrifugal potential arises from variations in orientation of the rotation axis, i.e. from variations in the pole position. The direct response of the crust is called the pole tide and its maximum in radial direction can reach 25 mm, with a maximum horizontal displacement of about 7 mm (Petit and Luzum, 2010). The perturbation in the centrifugal potential caused by the changes in position of the rotation axis can be written as (Wahr, 1985; Petit and Luzum, 2010):

$$V^c(\Theta, \Lambda) = -\frac{\Omega^2 r_{\oplus}^2}{2} \sin 2\Theta (m_1 \cos\Lambda + m_2 \sin\Lambda), \tag{6}$$

where $r_{\oplus}$ is the geocentric distance to the station (6378000 m), Θ and Λ geocentric co-latitude and longitude of the station. Ω is the mean angular velocity of the Earth rotation (7.292115e-5 rad/s) and m_1 with m_2 describe the time-dependent offset of the instantaneous rotation pole from the mean rotation pole.

By using the basic relation between the displacement vector and the perturbing potential (equation (1)) the final expression for the pole tide at a particular station follows as:

$$\begin{aligned} \Delta d^c = & dR^c \sin 2\Theta (m_1 \cos\Lambda + m_2 \sin\Lambda)\, \hat{r} \\ & - dT^c \cos\Theta (m_1 \sin\Lambda - m_2 \cos\Lambda)\, \hat{e} \\ & - dT^c \cos 2\Theta (m_1 \cos\Lambda + m_2 \sin\Lambda)\, \hat{n}, \end{aligned} \tag{7}$$

where dR^c and dT^c are given in [m/as] as:

$$\begin{aligned} dR^c &= h_{20} \frac{-\Omega^2 r_{\oplus}^2}{2g} \cdot \pi/180/3600, \\ dT^c &= l_{20} \frac{-\Omega^2 r_{\oplus}^2}{g} \cdot \pi/180/3600. \end{aligned} \tag{8}$$

The nominal values for the Love and Shida numbers are computed following equations (2) and (3) for the frequency appropriate to the pole tide, where we used the frequency of the Chandler wobble. The theoretical pole tide Love number is then 0.6206 and the Shida number 0.0894.

Table 1 Period and amplitude H_f of six zonal tidal waves for which the Love and Shida numbers were estimated.

Name	Period [solar days]	Cartwright-Tayler amplitude [mm]
Ω_1	6797.38 (= 18.6 yr)	27.9
S_a	365.25	−4.9
S_{sa}	182.62	−30.9
M_m	27.55	−35.2
M_f	13.66	−66.7
M'_f	13.63	−27.6

2 VLBI analysis

We used the Vienna VLBI Software VieVS (Böhm et al., 2012) to analyze 4.6 million observations from 1984.0 to 2011.0 included in 3360 24-hour sessions of the International VLBI Service for Geodesy and Astrometry (IVS; (Schuh and Behrend, 2012)). For the modeling of the theoretical time delays the IERS Conventions 2010 (Petit and Luzum, 2010) were followed, with the exception of applying a priori corrections on station coordinates due to non-tidal atmospheric loading (Petrov and Boy, 2004) which is a common procedure in the VLBI analysis. For each session the normal equation (NEQ) system was formulated including the station coordinates and velocities, source coordinates, Earth orientation parameters, zenith wet delays, tropospheric gradients, clock parameters, and the Love and Shida numbers. In the module Vie_GLOB (Krásná et al., 2013a) of VieVS a common adjustment of all sessions was carried out after local parameters (connected only to a single session) were reduced from the normal equations per session in a first step. The NEQ system of the global solution contains only the station coordinates, station velocities, source coordinates, and the Love and Shida parameters.

3 Love and Shida numbers for the long-period tides

To ensure an accuracy of 0.05 mm for the computed radial displacements of the crust in the long-period band, five tidal waves (M'_f, M_f, M_m, S_{sa}, and Ω_1) have to be taken into account (Petit and Luzum, 2010). Three solutions for the estimation of the zonal Love and Shida numbers were performed. In the first solution S1 the default parametrization was applied and Love and Shida numbers for the five main zonal tidal waves were estimated. In the second solution S2 hydrology loading corrections (provided by the NASA GSFC VLBI group (Eriksson and MacMillan; http://lacerta.gsfc.nasa.gov/hydlo)) were additionally applied a priori to the station coordinates. These corrections mainly contain annual and semi-annual signals. Solution S3 is identical to solution S2, but the Love and Shida numbers for the annual tidal wave S_a were also estimated. The real parts of the estimated complex Love and Shida numbers are listed in Tables 2 and 3. The second column of both tables contains the theoretical real part of the complex Love and Shida numbers (Mathews et al. (1997) and Petit and Luzum (2010)). Columns three, four and five list the real parts of the estimated Love and Shida numbers from solution S1, S2, and S3. In the last columns the differences between the a priori and the estimated Love and Shida numbers from solution S3, expressed as differences in amplitudes of the tidal term in millimeters are given:

$$\delta R^t_f = \sqrt{\frac{5}{4\pi}} H_f \delta h^R_f, \tag{9}$$

$$\delta T^t_f = \frac{3}{2} \sqrt{\frac{5}{4\pi}} H_f \delta l^R_f. \tag{10}$$

The real parts of the Love numbers from solution S1 show a relatively large difference of about 0.073 ± 0.019 and −0.078 ± 0.009 with respect to their theoretical values for the tidal waves Ω_1 and S_{sa}. The application of hydrology loading corrections on station coordinates (solution S2) leads to a decrease of the difference between the theoretical and estimated values of the Love number for the Ω_1 wave (0.003 ± 0.020), whereas the expected improvement of the estimated Love number of the semi-annual tide S_{sa} is small (the difference to the theoretical value is now −0.065 ± 0.009). In the third solution S3 the additional estimation of the Love number for the annual tide S_a causes another slight decrease of the difference between estimated and theoretical Love number for the semi-annual term S_{sa} (−0.055 ± 0.010). The larger formal error of the estimated Love number for the annual tide S_a is related to its small amplitude. The estimated Love number of the semi-annual tide S_{sa}, which corresponds to a 1.07 ± 0.19 mm difference in the radial amplitude of the crustal displacement with respect to the theoretical value, may reflect deficiencies in the a priori station displacement modeling of long-period origin. The larger formal error of the displacement amplitude for the Ω_1 tide is likely due to the not sufficiently long history of observations. A more detailed description of the analysis including our estimates of the imaginary parts of the Love and Shida numbers is given in Krásná et al. (2013b).

4 Love and Shida number for the pole tide

Several solutions were computed where the Love and Shida numbers for the polar motion were estimated. In these solutions the influence of a priori modeling of the mean pole and the application of hydrology loading corrections on station coordinates were investigated. The analysis of VLBI data was done according to the default parametrization with the following differences between the solutions:

- P1 - default parametrization (cubic function for mean pole (IERS Conventions 2010)),
- P2 - amplitudes of annual and semi-annual station position variations were estimated as additional parameters in the global solution and a cubic function for the mean pole was applied,
- P3 - as P2 but the mean pole was modeled by a linear approximation,
- P4 - as P2 but the mean pole was set to zero,
- P5 - as P1 but hydrology loading corrections were applied a priori on the station coordinates, ampli-

Table 2 Real parts of the complex Love numbers h_f^R for the long-period tidal waves estimated within three different solutions. $\Delta\delta R_f^t$ shows the difference in displacements when using solution S3 and values given in IERS Conventions 2010.

Name	h_f^R from (2)	h_f^R this work S1	h_f^R this work S2	h_f^R this work S3	$\Delta\delta R_f^t$ from S3 [mm]
Ω_1	0.6344	0.7071 ± 0.0188	0.6372 ± 0.0199	0.6372 ± 0.0199	0.05 ± 0.35
S_a	0.6207	-	-	0.5708 ± 0.0612	0.15 ± 0.19
S_{sa}	0.6182	0.5405 ± 0.0090	0.5531 ± 0.0094	0.5635 ± 0.0095	1.07 ± 0.19
M_m	0.6126	0.5965 ± 0.0076	0.5887 ± 0.0079	0.5905 ± 0.0079	0.49 ± 0.18
M_f	0.6109	0.6036 ± 0.0042	0.6052 ± 0.0043	0.6049 ± 0.0043	0.25 ± 0.18
M_f'	0.6109	0.6024 ± 0.0100	0.5878 ± 0.0105	0.5893 ± 0.0105	0.38 ± 0.18

Table 3 Real parts of the complex Shida numbers l_f^R for the long-period tidal waves estimated within three different solutions. $\Delta\delta T_f^t$ shows the difference in displacements when using solution S3 and values given in IERS Conventions 2010.

Name	l_f^R from (3)	l_f^R this work S1	l_f^R this work S2	l_f^R this work S3	$\Delta\delta T_f^t$ from S3 [mm]
Ω_1	0.0936	0.1147 ± 0.0044	0.1079 ± 0.0047	0.1078 ± 0.0047	0.37 ± 0.12
S_a	0.0894	-	-	0.1079 ± 0.0146	−0.09 ± 0.07
S_{sa}	0.0886	0.0955 ± 0.0021	0.0954 ± 0.0022	0.0984 ± 0.0023	−0.28 ± 0.07
M_m	0.0870	0.0851 ± 0.0018	0.0819 ± 0.0019	0.0825 ± 0.0019	0.15 ± 0.06
M_f	0.0864	0.0855 ± 0.0010	0.0865 ± 0.0010	0.0864 ± 0.0010	0.01 ± 0.06
M_f'	0.0864	0.0842 ± 0.0024	0.0771 ± 0.0025	0.0772 ± 0.0025	0.24 ± 0.07

tudes of annual station position variations were estimated as additional parameters in the global solution together with the complex Love and Shida numbers for the five main zonal tidal waves.

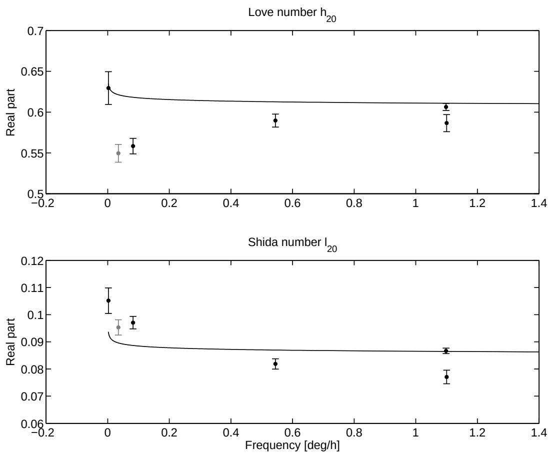

Fig. 1 Real parts of the five zonal Love and Shida numbers (black color) estimated together with the Love and Shida numbers for the pole tide (grey color) in solution P5. The solid black lines represent the theoretical values given by equations (2) and (3).

Table 4 Pole tide Love and Shida numbers.

solutions	h_2 - pole tide	l_2 - pole tide
theoretical value	0.6206	0.0894
P1	0.4638 ± 0.0092	0.1038 ± 0.0023
P2	0.5354 ± 0.0118	0.0943 ± 0.0029
P3	0.5353 ± 0.0118	0.0946 ± 0.0029
P4	0.5353 ± 0.0118	0.0956 ± 0.0029
P5	0.5495 ± 0.0109	0.0953 ± 0.0028
(Petrov, 1998)	0.65 ± 0.20	0.11 ± 0.05
(Gipson and Ma, 1998)	0.636 ± 0.025	0.087 ± 0.007

In Table 4 results of the estimated Love and Shida numbers from the five solutions are summarized. The largest difference to the theoretical value appears in solution P1. In solution P2 the determination of the remaining annual and semi-annual signals in the station coordinates (especially height) within the global adjustment brings the estimated Love number closer to its theoretical value. The Love numbers obtained from solutions P2, P3 and P4 are almost identical. This shows that the modeling of the mean pole (cubic, linear, or a total omission) does not have any influence on the Love and Shida number estimates. In solution P5 the hydrology loading corrections were applied a priori on the station coordinates and in the global adjustment the complex Love and Shida for the five zonal tidal waves (Ω_1, S_{sa}, M_m, M_f, M_f') together with the remaining annual signal in the station coordinates were estimated. The corresponding Love and Shida numbers are plotted in Figure 1. The good agreement between the estimated Love number of the semi-annual tide S_{sa} (0.558 ± 0.010) and of the pole tide (0.550 ± 0.011) is clearly visible. The vertical amplitude of the estimated harmonic annual signal at most of the stations reaches several millimeters (not shown here). This approach in solution P5 gives the best agreement between the estimated and theoretical pole tide Love number from all five solutions which were carried out. In the last two rows of Table 4 results obtained by Petrov (1998) and Gipson and Ma (1998) are shown. Petrov

(1998) used only early VLBI data covering time span of 4 years (from 1984 to 1987) for his computation. Even though his Love number estimate (0.65) lies close to the theoretical value (0.62) its large formal error of 0.20 reflects the high uncertainty of the result. Gipson and Ma (1998) included VLBI sessions from 1979 to 1996 and their estimates agree with the theoretical values within the formal errors.

5 Conclusions

Our estimate of the Love number for the semi-annual tide is 9.7% lower than the theoretical value. Similarly, the Love number of the pole tide is lower by about 11.4% than in theory. Both the a priori application of a hydrology loading model (mainly annual and semi-annual frequency content) in the analysis and the estimation of annual station positions slightly bring the estimates of zonal Love numbers closer to their theoretical values but still a significant difference remains. The empirical Shida numbers for the periods of half year and longer are always bigger than the theoretical values. A next step could be a revision of the theoretical model of solid Earth tides by re-estimating the included Earth parameters.

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