Fractional Fokker-Planck Equation vs Tsallis' Statistical Mechanics

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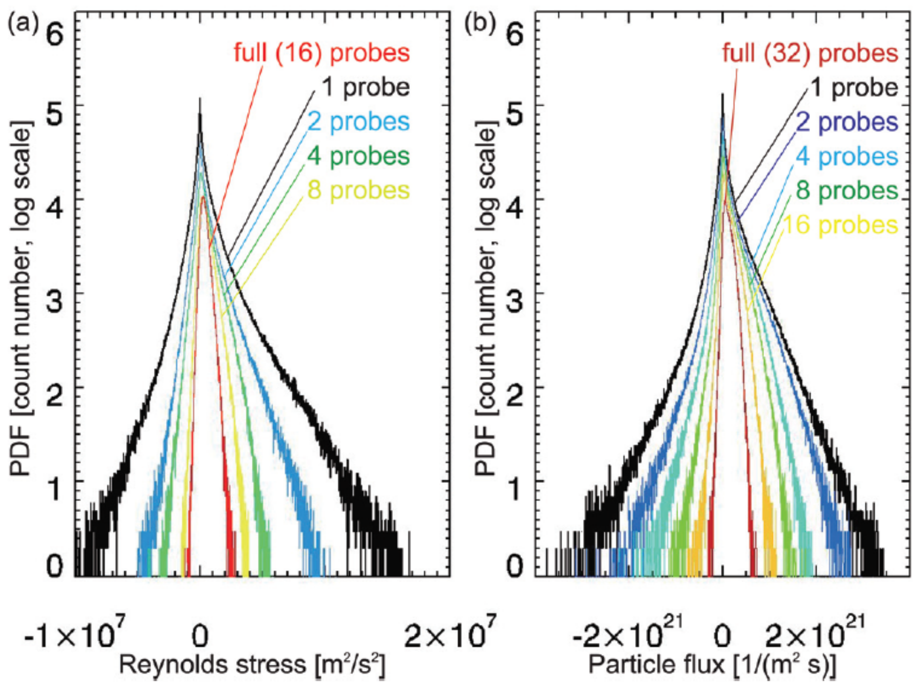
Non-Gaussian statistical properties and Fractional FPE

$$P(Z) \propto \exp(-\xi \langle \Pi \rangle^{X})$$



Nagashima et al (2011)

The FFPE (Moradi et al 2011, 2012)

$$\frac{\partial F}{\partial t} + \vec{v}\frac{\partial F}{\partial \vec{r}} + \frac{F}{m}\frac{\partial F}{\partial \vec{v}} = \upsilon \frac{\partial}{\partial \vec{v}}(\vec{v}F) + D\frac{\partial^{\alpha} F}{\partial |\vec{v}|^{\alpha}}$$

Consider the simplified Eq. in 1D

$$\frac{\partial F}{\partial t} = \upsilon \frac{\partial}{\partial \vec{v}}(\vec{v}F) + D\frac{\partial^{\alpha} F}{\partial |\vec{v}|^{\alpha}}$$

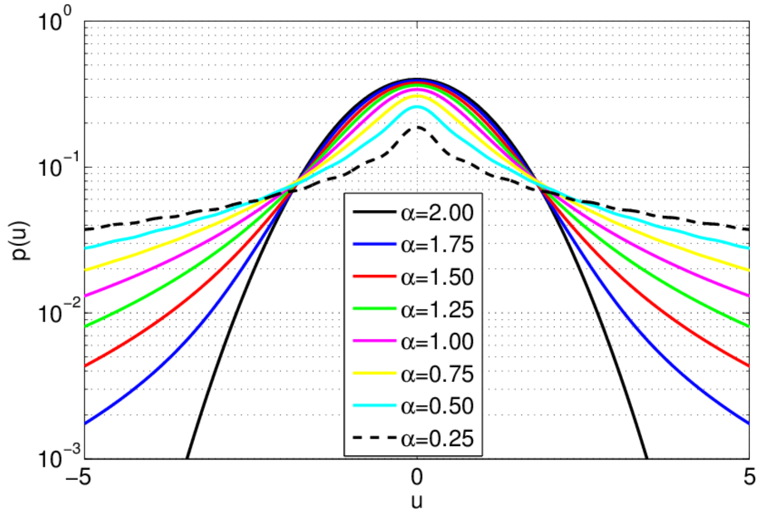
With solution

$$F(v) = \frac{F_0}{2\pi}\int_{-\infty}^{\infty} dq \exp\left(-\frac{D}{\upsilon\alpha}|q|^{\alpha} + iqv\right)$$

Tarasov 2006

PDFs with fits

Cauchy-Lorentz distribution



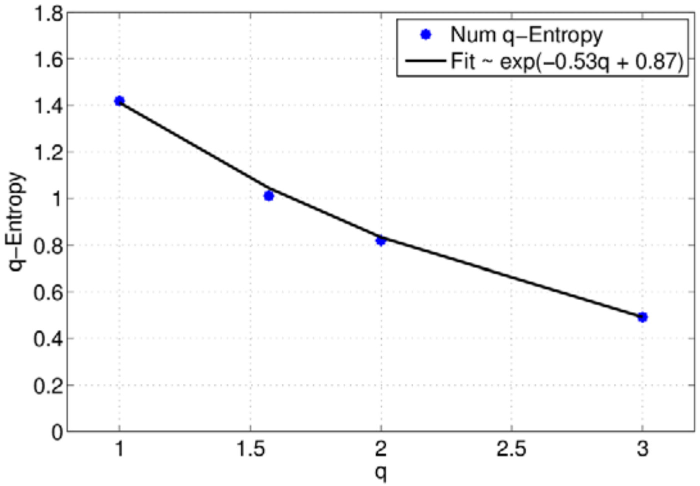
$$F(v) = \frac{a}{(1+bv^2)^{1/(q-1)}} \quad q = \frac{\alpha+1}{\alpha}$$

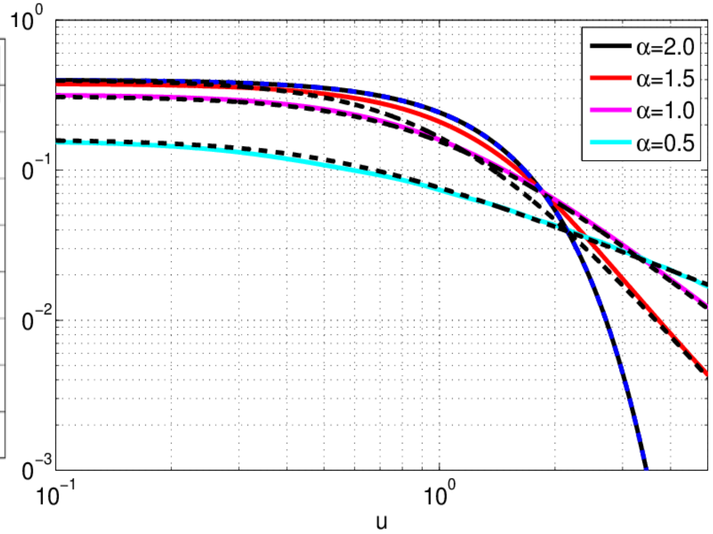
$$q > \frac{5}{3} \ \text{ and } \ \alpha < \frac{3}{2}$$

It is claimed that for q>5/3 ->Levy distribution

Non-Extensive system

$$S_q = \frac{1-\int dv F^q(v)}{q-1}$$





Conclusions, Future work and References

- The numerical solutions indicate that the PDFs from the simplified FFPE follows the predicted Cauchy-Lorentz distributions.
- Remaining work includes looking the properties of the non-extensive entropy, q-expectations etc.
- Relation to NL system or stochastic DEs. Stochastic DEs have direct relation to PDEs ala Feynman-Kac.
- All suggestions and ideas are most welcome.

- S. Moradi and J. Anderson, A non-local gyrokinetic model of ion-temperature-gradient modes, Phys. Plasmas 19, 082307 (2012).
- S. Moradi, J. Anderson and B. Weyssow, Non-local linear drift wave transport model, Phys. of Plasmas 18, 062106 (2011)