Thesis for the degree of Doctor of Philosophy

Wireless Sensor Network Positioning Techniques

Mohammad Reza Gholami

CHALMERS

Department of Signals and Systems
Chalmers University of Technology

Gothenburg, Sweden 2013

Mohammad Reza Gholami
Wireless Sensor Network Positioning Techniques.

ISBN 978-91-7385-899-1
Department of Signals and Systems
Chalmers University of Technology
Ny serie Nr 3580
ISSN 0346-718X

Department of Signals and Systems
Chalmers University of Technology
SE-412 96 Gothenburg, Sweden
Telephone: + 46 (0)31-772 1792
Fax: + 46 (0)31-772 1748
Email: moreza@chalmers.se

This thesis has been prepared using LaTeX.

Printed by Chalmers Reproservice,
Gothenburg, Sweden, October 2013.

To my beloved parents, Somayeh, and Arman

Abstract

Position information is one of the key requirements for wireless sensor networks (WSNs). Since GPS receivers have drawbacks when used in low-power sensor nodes, mainly due to latency in position recovery and limited access to GPS satellites in indoor-type scenarios, extracting the position information by means of the network itself, so-called positioning or localization, has been considered an effective solution for the positioning problem. The goal of this thesis is to design and develop approaches, algorithms, and benchmarks for the positioning problem for WSNs. The contributions of the thesis, which appear in five appended papers, are as follows.

Paper A develops an eavesdropping technique for positioning multiple target nodes in a cooperative wireless sensor network in the presence of unknown turn-around times. That is, a number of reference or target nodes (secondary nodes) can listen to both signals transmitted by the target and an active (primary) node. The maximum likelihood estimator (MLE) and a theoretical lower bound as well as a suboptimal efficient linear estimator are derived for the problem. Numerical results confirm a considerable improvement for the proposed technique compared to conventional approaches, especially for low signal-to-noise ratios. Paper B studies a self-positioning problem based on TDOA measurements in the presence of unknown target node clock skew. Since the optimal MLE poses a difficult global optimization problem, two suboptimal estimators followed by a fine-tuning approach are investigated in this paper. Numerical results show that the suboptimal estimators asymptotically attain the Cramér-Rao lower bound. Paper C investigates the single target node localization problem based on received signal strength measurements in the presence of unknown channel parameters. Using approximations, the problem is rendered to a low complex problem and a simple technique is employed to solve the problem. The proposed technique shows a good trade-off between accuracy and complexity compared to the existing approaches. Paper D studies the possibility of upper bounding the position error for range-based positioning algorithms in wireless sensor networks. It is argued that in certain situations when the measured distances between sensor nodes have positive errors, the target node is confined to a closed bounded convex set, which can be derived from the measurements. In particular, the upper bounds are formulated as nonconvex optimization problems, and relaxation techniques are employed to approximately solve the nonconvex problems. Simulation results show that the proposed bounds are reasonably tight in many situations, especially for non-line-of-sight conditions. Finally, Paper E deals with identifying the feasible sets in cooperative positioning and proposes an iterative technique to cooperatively outer-approximate the feasible sets containing the locations of the target nodes. Simulation results show that the proposed technique converges after a small number of iterations.

Keywords: Wireless sensor network, cooperative positioning, maximum likelihood estimator, nonlinear and linear least squares, projection onto convex sets, outer-approximations, Cramér-Rao lower bound, semidefinite programming, clock offset, clock skew.

List of Publications

Included papers

This thesis is based on the following papers:

[A] M. R. Gholami, S. Gezici, and E. G. Ström, "Improved position estimation using hybrid TW-TOA and TDOA in cooperative networks," *IEEE Trans. Signal Process.*, vol. 60, no. 7, pp. 3770–3785, Jul. 2012.

[B] M. R. Gholami, S. Gezici, and E. G. Ström, "TDOA-based positioning in the presence of unknown clock skew," *IEEE Trans. Commun.*, vol. 61, no. 6, pp. 2522–2534, Jun. 2013.

[C] M. R. Gholami, R. M. Vaghefi, and E. G. Ström, "RSS-based sensor localization in the presence of unknown channel parameters," *IEEE Trans. Signal Process.*, vol. 61, no. 15, pp. 3752–3759, Aug. 2013.

[D] M. R. Gholami, E. G. Ström, H. Wymeersch, and M. Rydström, "Upper bounds on position error of a single location estimate in wireless sensor networks," submitted to *Signal Processing,* Sep. 2013.

[E] M. R. Gholami, H. Wymeersch, S. Gezici, and E. G. Ström, "Distributed bounding of feasible sets in cooperative wireless network positioning," *IEEE Commun. Lett.*, 2013, DOI: 10.1109/LCOMM.2013.070113.130905.

Acknowledgments

None of us got to where we are alone. Whether the assistance we received was obvious or subtle, acknowledging someone's help is a big part of understanding the importance of saying thank you.

—Harvey Mackay

I would sincerely like to thank all who supported, encouraged, and helped me in all my falling and rising until I accomplished the PhD degree. First and foremost, I would like to express my sincere gratitude to my advisor Prof. Erik G. Ström for the kind support of my PhD research, for his patience, for his motivation, and for fruitful discussions during the last five years. Fortunately, I had ample opportunities to learn many subtle leadership skills from you and to see how to manage difficult circumstances. I would like to cordially thank you for allowing me enough freedom during my PhD studies and believing in me as I defined and completed research problems.

In addition to my advisor, I would like to warmly thank Dr. Mats Rydström and Dr. Henk Wymeersch for their kind support, advice, and friendly help. I would like to sincerely thank Dr. S. Gezici, Prof. Y. Censor, Dr. L. Tetruashvili, and Prof. M. Viberg for many fruitful discussions and comments on different research problems. Finally, I would like to thank our colleagues at Newcom++, an EU FP7 Network of Excellence, for collaborating on measuring the practical data.

During my PhD studies, I had a great opportunity to visit the Adaptive Systems Laboratory at UCLA, hosted by Prof. Ali H. Sayed, to work on distributed estimation over networks. I would like to sincerely thank Prof. Sayed and his great PhD students for their warm hospitality, wonderful discussions during challenging weekly meetings, and friendly support.

Throughout my life, I have been so lucky to have great friends. If you were not in my life, I could not imagine how meaningless and tedious life could be. I would like to take this opportunity to thank all my former and current friends. In particular, I would like to thank all former and current PhD students, Associate Professors, and Professors in the division of Communication Systems, Information Theory, and Antennas. I would also like to thank all my Iranian friends in Chalmers and especially at the department of Signals and Systems. I will never forget pleasant discussions with you, especially in the department's kitchen. Finally, I would like to thank the administration staff at the department of Signals and Systems, who helped me a lot throughout this process with particular thanks to Agneta, Lars, and Natasha.

I owe my deepest gratitude to my family for their utmost support, encouragement, constant love, and unbelievable compassion. I am thoroughly grateful to my parents, Nasrollah and Zahra, for all the sacrifices they have endured to support their son. Your overwhelming support, commitment, dedication, and unconditional love cannot be expressed in words. I would like to express my heartfelt gratitude to my lovely sister, Hanieh, and my lovely brother, Hamid Reza, for their constant love, concern, and support, which are inestimable assets in my life.

Last but not least, my sincere thanks goes to my wife, Somayeh, for her understanding, love, and compassion during the past few years. Your unbelievable support and encouragement were in the end what made this dissertation possible. You have been, always, my pillar, my joy, and my guiding light. Finally, I would like to welcome my son, Arman, to this world. You have enlightened my life and brought a huge amount of pleasure, happiness, and blessing to our families. It has been quite a joy to write the thesis while looking at your smile and hearing your little giggles. I wish you a great and successful life.

This work has been supported by the Swedish Research Council (VR) under Grant 2007-6363. The research visit at UCLA was partially supported by Ericsson's Research Foundation (Reference: FOSTIFT-12:021 and FOSTIFT-13:031).

Mohammad Reza Gholami
Gothenburg, 2013.

Acronyms

AOA	Angle-of-arrival
CRLB	Cramér-Rao lower bound
CDF	Cumulative distribution function
CEP	Circular error probable
CFP	Convex feasibility problem
DCP	Difference of convex functions programming
FROB	Frobenius metric
GDOP	Geometric dilution of precision
LOS	Line-of-sight
MLE	Maximum likelihood estimator
MM	Majorization minimization
NB	Norm of bias
NLS	Nonlinear least squares
NLOS	Non-line-of-sight
OA	Outer-approximation
PDF	Probability density function
POCS	Projection onto convex sets
QCQP	Quadratically constrained quadratic programming
RF	Radio frequency
RSS	Received signal strength
RMSE	Root-mean-square-error
SNR	Signal-to-noise ratio
SDP	Semidefinite programming
TOA	Time-of-arrival
TDOA	Time-difference-of-arrival
TW-TOA	Two-way TOA
UWB	Ultra-wideband
WSN	Wireless sensor network
WNLS	Weighted nonlinear least squares

Contents

Chapter 1

Introduction

Recent advances in Micro-Electro-Mechanical System (MEMS) technology have enabled the use of tiny devices such as sensor nodes in large distributed wireless sensor networks (WSNs). WSNs have vastly found applications in, e.g., intelligent transportation systems, habitat monitoring, tracking targets, agro-technology, battlefield surveillance, and intruder detection [1, 2], just to name a few. The data gathered by sensor nodes are often meaningless unless the position information of the sensor nodes is available. For example, for location-aware services, the geographic location of a mobile phone is needed in order to provide a service. In fact, in location-based services, mobile customers are able to request and receive information, e.g., maps, emergency response, and demographic data collection, based on their geographic locations. The position information, with desired accuracy, can also be used in designing efficient routing protocols in WSNs. Information about the locations of sensor nodes can be provided by a satellite-based positioning system, e.g., global positioning system (GPS), or it can be extracted from the network itself. In general, satellite-based positioning systems have acceptable level of accuracy in outdoor scenarios in which signals from satellites can be received with good signal-to-noise ratios without blockage (line-of-sight (LOS) conditions). Table 1.1 shows two satellite-based positioning systems with applications in outdoor scenarios [3].

Due to drawbacks of using GPS receivers in low-power sensor nodes, limited access to satellites in indoor scenarios, e.g., inside tunnels or across high-rise buildings [4], and latency in position recovery, a tremendous effort has been devoted to designing low-cost and efficient algorithms to recover the location information from the network using some promising technology such as Ultra-wideband (UWB) [5]. The position information is extracted relative to local coordinates of some infrastructure devices (landmarks). In some scenarios to offer seamless positioning, e.g., tracking a target moving from an outdoor scenario to an indoor one, it may be necessary to integrate global coordinates with local coordinates.

The required positioning accuracy depends on applications, which can be in the order of centimetres to hundreds of meters. As an example, Table 1.2 shows the required ac-

Table 1.1: Two satellite-based positioning systems [3].

Systems	Technique	Accuracy	Application	Drawbacks
GPS	TDOA	10-20 m	Earth scale coverage	LOS scenarios, expensive infrastructure
Galileo	TDOA	1-5 m	Earth scale coverage	LOS scenarios, expensive infrastructure

Table 1.2: Positioning accuracy required for different applications [4].

Application	Accuracy	Example
Emergency	medium-high	emergency call
Navigation	high	directions
Information	medium	advertisement
Tracking	low	vehicle tracking

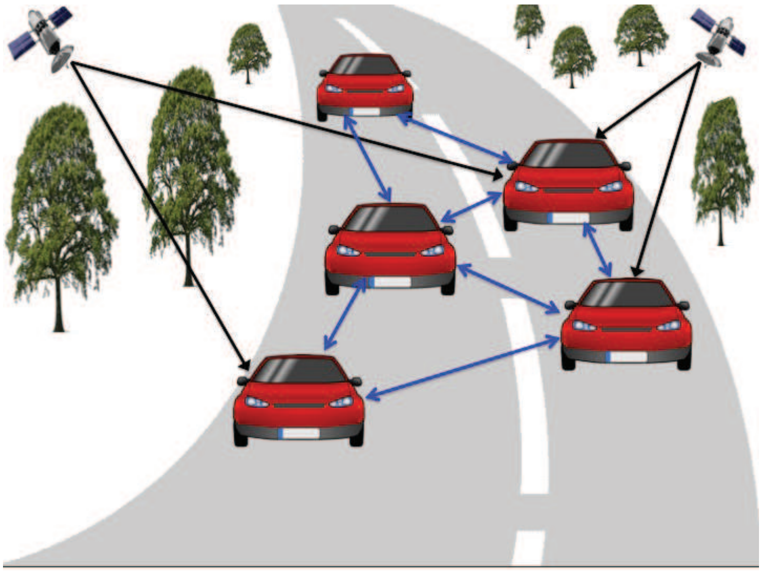

Figure 1.1: An example of a dynamic network. Vehicles can estimate their locations using a hybrid technique. Some nodes get their information from GPS, and through a collaborative technique other vehicles (with no access to satellites) can find their positions.

curacy for different applications [4]. In general, the required accuracy for network-based positioning (remote positioning) and device-based location recovery (self-positioning) can also be different. For example, for a 911 emergency call in the U.S, the accuracy for remote positioning and self-positioning should not be worse that 300 and 150 meters, respectively, in 95% of emergency calls.

1.1 Positioning of nodes

As mentioned before, the GPS receiver has some drawbacks when being used in low-power sensor nodes; hence, extracting the position information by means of the network itself, also called localization, can be alternatively employed in WSNs [6–10]. To that aim, it is commonly assumed that there are a number of fixed sensors called reference or anchor nodes whose positions are initially known using GPS receivers or manual settings [11]. The locations of reference nodes are used to localize a number of target nodes at unknown locations. In this thesis, the terms “reference nodes” and ”target nodes” will be used consistently. We assume that sensor nodes are able to make some type of measurements, which carry information about the location of target nodes. Sensor nodes in WSNs can

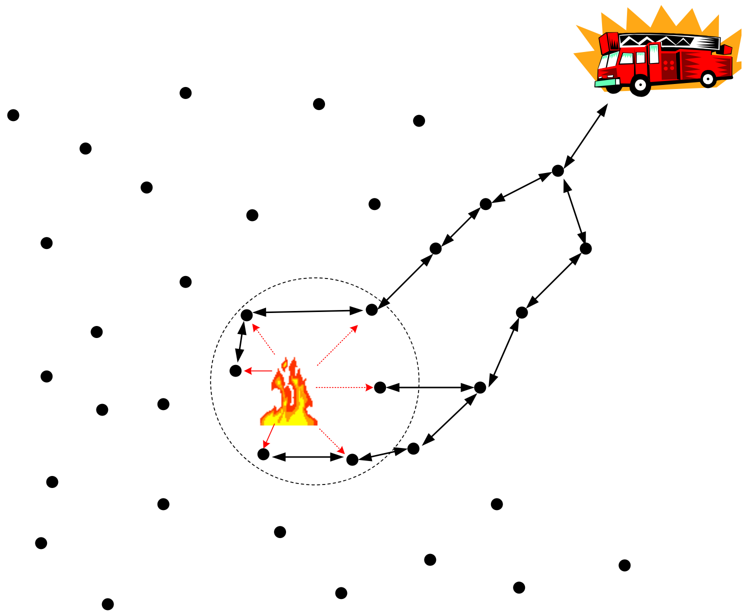

Figure 1.2: An example of the application of a WSN (with static sensor nodes). Solid dots denote wireless sensors. Sensors inside the dashed circle detect a fire event and then report an alarm to a central unit for further processing.

be either stationary or moving. Some sensors can be either transmitters or receivers, or both. To study the positioning problem in a WSN, we need suitable models for the measurements taken between sensor nodes. It is commonly assumed that measurements are made between target and reference nodes, but in some situations measurements between target nodes are also available.

The positioning problem can be categorized into two groups based on the type of interaction between target nodes: cooperative and noncooperative. In a cooperative positioning system, measurements between target nodes as well as measurements between target nodes and reference nodes are used for position recovery. Whereas, in noncooperative systems, only the latter type of measurements (between target and reference nodes) are used. In scenarios where there are a limited number of reference nodes, a collaborative technique between target nodes effectively improves the accuracy of the position estimate [12]. Fig. 1.1 and Fig. 1.2 show two applications in which the position information plays an important role for the network to function as intended. In a traffic safety application (Fig. 1.1), the position of vehicles can be properly integrated with other parameters, e.g., speed, to decrease the risk of collisions between vehicles. If there is direct access to enough satellites, the position information can be extracted from GPS signals. If satellite signals are blocked, e.g., inside tunnels, a number of vehicles are unable to find their locations from GPS signals. Using a cooperative technique, different vehicles can find their locations. In addition, the collaboration can be beneficial even when all nodes have access to satellites. In the second example (Fig. 1.2), a few sensors inside the dashed circle detect a fire event and then report this alarm to a central unit for further processing. Suppose that the short-range sensors can find a route to the central unit using a routing protocol. In the central unit, before any decision about the alarm has been made, the positions of the sensors detecting the fire event need to be known. In these two examples, we notice two types of positioning algorithms that can be used for practical applications. Namely, in Fig. 1.1, a distributed positioning algorithm due to, e.g., the dynamic nature of the network, is more suitable than the centralized positioning approach whereas in Fig. 1.2, a centralized positioning algorithm can be implemented in the processing center.

In summary, the positioning problem can be defined as follows:

Positioning problem: Given the position of reference nodes, e.g., from GPS signals, and some types of measurements between different nodes, estimate the location of target nodes as accurately as possible.

Similar to any estimation problem, positioning algorithms commonly aim at positioning target nodes as accurately as possible. The performance of a positioning algorithm depends on the type of measurement taken, reference node selection, measurement errors, geometry of networks, and other factors. Moreover, practical impairments, e.g., modeling or round-off errors, which might be unknown can considerably affect the performance of positioning algorithms. During the last decade, a large number of positioning approaches have been proposed in the literature. Thus, as it can be imagined, it is not always straightforward to compare all the different approaches based on a single criterion. Hence, to evaluate various positioning methods, we use a number of metrics, e.g., complexity or accuracy. The positioning algorithms are generally formulated as difficult nonconvex optimization problems. A number of suboptimal approaches can be used to approximately solve the problem.

In this thesis, we review the positioning problem and investigate a number of techniques to solve the problem. We further study a number of practical parameters that can affect the accuracy of the position estimate. In this study, we consider both cooperative and noncooperative scenarios in developing algorithms.

1.2 Thesis outline

This thesis investigates a number of techniques to improve positioning accuracy and also proposes some low-complex algorithms to recover the position information from the measurements, mainly from time-of-arrival-based estimates. The positioning problem is also explored in terms of a geometric interpretation and two geometric solutions to the positioning problem are introduced. To present the contributions of this thesis that appear in the appended papers, we first give a brief and general overview of the positioning problem and then introduce results of this thesis.

This thesis is organized as follows: In Chapter 2, we study different data models commonly used in the positioning literature. In Chapter 3, we briefly review a few positioning algorithms. Since most positioning algorithms are formulated as difficult nonconvex optimization problems, useful techniques for approximately solving nonconvex problems are introduced in Chapter 4. In Chapter 5, a number of metrics for evaluating positioning algorithms are discussed. The contributions of this thesis are presented in Chapter 6. This thesis is based on five papers, i.e., Papers A, B, C, D, and E, that are included in the thesis.

Chapter 2

Positioning problem

In this section, we review the positioning problem as an estimation problem in a wireless network consisting of a number of reference nodes, at known locations, and a number of targets, at unknown locations. The goal of a positioning system is to estimate the location of the target nodes as accurately as possible. The sensor nodes (references or targets) are assumed to be able to interact with each other and collect some types of measurements, carrying information about the location of sensor nodes. Concretely, a poistioning algorithm receives the location of reference nodes and measurements (and some *a priori* knowledge about target locations, if available) and locates the targets with respect to a (local or global) coordinate system.

In this thesis, we mainly focus on the positioning problem when no prior knowledge about the position of target nodes is available. In fact, we consider the location of the target nodes as unknown deterministic parameters.

In this chapter, we briefly review the positioning problem and study different measurement models commonly used in the positioning literature. We also discuss some practical imperfections in measuring the data between nodes. Finally, we review different classes of positioning algorithms.

2.1 The problem statement

Positioning problems are usually defined for a 2- or 3-dimensional network. In this thesis, we mainly focus on 2-dimensional networks, but the generalization to 3-dimensional networks, in most cases, is straightforward. In particular, we consider a network with $N+M$ sensor nodes distributed over a geographical area. Suppose that the locations of M target nodes at $\boldsymbol{x}_i = [x_{i1}\ x_{i2}]^T \in \mathbb{R}^2$, $i = 1, ..., M$, are unknown *a priori*. During the positioning process, it is assumed that the locations of N reference nodes $\boldsymbol{a}_j = [a_{j1}\ a_{j2}]^T \in \mathbb{R}^2$, $j = M+1, ..., N+M$ are known in advance. Note that we consider a static scenario, in which the position of target and reference nodes do not change with time. Every target node can communicate with nearby sensor nodes and also with other targets. To define the connectivity between sensor nodes, we consider two sets:

$$\mathcal{A}_i \triangleq \{j \mid \text{reference node } j \text{ can communicate with target } i\} \tag{2.1}$$

and

$$\mathcal{B}_i \triangleq \{j \mid i \neq j,\ \text{target } j \text{ can communicate with target } i\} \tag{2.2}$$

as the sets of indices of all reference and target nodes that can communicate with target i, respectively. For a noncooperative network, we simply set $\mathcal{B}_i = \emptyset, \ \forall i$.

As previously mentioned, to localize target nodes, we assume that sensor nodes can measure some type of information about the locations of target nodes. Formally, we define the following model for the measurement between nodes i and j collected at node i as

$$m_{ij} \triangleq \begin{cases} f(\boldsymbol{x}_i, \boldsymbol{a}_j) + \epsilon_{ij}, & j \in \mathcal{A}_i, \\ f(\boldsymbol{x}_i, \boldsymbol{x}_j) + \epsilon_{ij}, & j \in \mathcal{B}_i, \end{cases} \tag{2.3}$$

where $f(\boldsymbol{\alpha}, \boldsymbol{\gamma})$ is a deterministic function that defines a type of noiseless measurement between two sensors at positions $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$, and ϵ_{ij} is the measurement error. The function $f(\boldsymbol{\alpha}, \boldsymbol{\gamma})$ may have different shapes based on the positions $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$. For instance, for distance measurements, it is the ℓ_2-norm of difference between $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$, i.e., $f(\boldsymbol{\alpha}, \boldsymbol{\gamma}) = d(\boldsymbol{\alpha}, \boldsymbol{\gamma}) \triangleq \|\boldsymbol{\alpha} - \boldsymbol{\gamma}\|$. The measurement errors ϵ_{ij} may have any probability density function (PDF). Measurements can be collected at reference nodes, target nodes, or both reference and target nodes. As an example, Fig. 2.1 shows a cooperative network consisting of four reference nodes and two target nodes. The connectivity between different nodes and noiseless measurement (actual distances) is depicted in the figure.

Note that the model considered in (2.3), which describes a wide range of measurement models used in the positioning literature, might not hold true for a practical application. It may happen that the measurement error or its statistic also depends on the location of unknown sensor nodes. In addition, the measurement errors may affect the measurement in a nonadditive manner or even in a nonlinear fashion. It is also possible that additional unknown (nuisance) parameters appear in the model. In this section, we first review the simplest case and then briefly study the effect of some practical impairments on the measurement models.

Concretely, *the positioning problem is to find the position of M target nodes based on the position of N reference nodes at known locations, measurements in* (2.3), *and possibly some prior knowledge about the locations of target nodes.*

2.2 Measurement models

The positioning information, in general, can be extracted from the signal transmitted from one node to another. While it is possible to directly recover the position information from the received waveforms, a popular technique is to first extract intermediate estimates such as distance estimates and then to find the location from the intermediate estimates. The accuracy of the position estimate, therefore, depends on the measurement between different nodes. We will later discuss that the geometry of the network also has a significant effect on the accuracy of the position estimate. In the positioning literature, different types of measurements have been considered such as received signal strength (RSS), angle-of-arrival (AOA), time-of-arrival (TOA), or time-difference-of-arrival (TDOA). In this thesis, we only focus on distance estimates and power measurements between sensor nodes. For a discussion on other measurements, see, e.g., [6, 13].

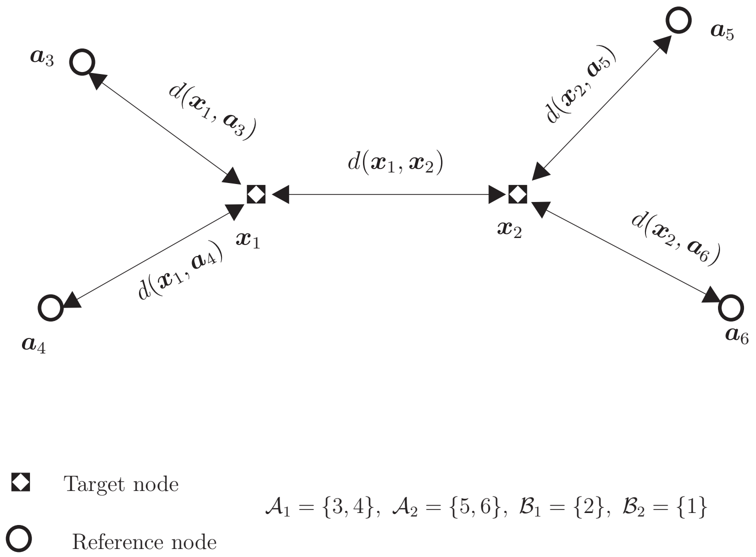

Figure 2.1: A cooperative network with two targets and four reference nodes.

2.2.1 Received signal strength

In the wireless communication literature, the log-normal model is commonly considered as a generalization of the free-space propagation model. To express shadowing effects (large-scale fading), we add a random variable in the model. Namely, the received power from transmitter i at receiver j, in dB, can be expressed as [8, 14]

$$P_{ij} = P_{0i}(d_{0i}) - 10\beta_{ij} \log\left(\frac{d(\boldsymbol{x}_i, \boldsymbol{x}_j)}{d_{0i}}\right) + n_{ij}, \tag{2.4}$$

where the base of the logarithm is 10; $P_{0i}(d_{0i})$ denotes the power at distance d_{0i} (reference power); $d(\boldsymbol{b}, \boldsymbol{c}) \triangleq \|\boldsymbol{b} - \boldsymbol{c}\|$, as before, is the Euclidean distance between $\boldsymbol{b}$ and $\boldsymbol{c}$; β_{ij} is a path-loss exponent for the link between node i and node j that is often between 2 and 6 [14], and n_{ij} is modeled as a zero-mean Gaussian random variable with variance σ_{ij}^2 [15], i.e., $n_{ij} \sim \mathcal{N}(0, \sigma_{ij}^2)$. The reference power $P_{0i}(d_{0i})$ is often calculated *a priori* or calibrated during the measurement process. To find the reference power, the reference point with distance d_{0i} from the transmitter should be in the far field region [13].

As can be seen, the model is nonlinearly dependent on the position of the transmitter i. In this model, $f(\boldsymbol{x}_i, \boldsymbol{x}_j) = P_{0i}(d_{0i}) - 10\beta_{ij} \log\left(\frac{\|\boldsymbol{x}_j - \boldsymbol{x}_i\|}{d_{0i}}\right)$. For convenience, we define $d_{ij} \triangleq \|\boldsymbol{x}_i - \boldsymbol{x}_j\|$. For a homogeneous environment, different links experience the same path-loss exponent, i.e., $\beta_{ij} = \beta$. Assuming known parameters P_{0i} and $\beta_{ij} = \beta$, the distance between node i and j can be optimally estimated (in the maximum likelihood sense) as

$$\hat{d}_{ij} = d_{0i} 10^{\frac{P_{0i}(d_{0i}) - P_{ij}}{10\beta}}. \tag{2.5}$$

It can be shown that the distance estimate obtained in (2.5) is biased. An unbiased estimate for the distance can be derived as [14]

$$\hat{d}_{ij}^u = d_{0i} 10^{\frac{P_{0i}(d_{0i}) - P_{ij}}{10\beta}} e^{-\frac{10\beta}{\sigma_{ij} \ln 10}}. \tag{2.6}$$

The Cramér-Rao lower bound (CRLB) for the variance of any unbiased distance estimator based on RSS measurements can be obtained as [8]

$$\mathbb{E}\big(\hat{d}_{ij} - \mathbb{E}(\hat{d}_{ij})\big)^2 \geq \Big(\frac{\sigma_{ij} d_{ij} \ln 10}{10\beta}\Big)^2. \tag{2.7}$$

It is observed that the distance estimate accuracy deteriorates with the distance between two nodes as well as the standard deviation σ_{ij} of measurement noise in (2.4). One interesting observation is that, as the path-loss exponent β increases, the accuracy of distance estimate improves. The reason is that the average power becomes more sensitive to distance for the larger path-loss exponents [8]. It is also observed that the transmission power has no effect on the accuracy-bound CRLB.

2.2.2 Time-of-arrival

The distance between sensor nodes can be computed from the RSS measurement between nodes, Eq. (2.5) or (2.6). Another way to estimate the distance is to measure time-of-arrival (TOA) of a signal transmitted by a node. TOA measurements provide good estimates of the distance between sensor nodes in some scenarios, e.g., in line-of-sight (LOS) conditions with high signal-to-noise ratios. To calculate the distance between two sensor nodes based on the time the signal spends traveling from one node to another node, we need a time-synchronized network that can be achieved by using a number of techniques [16–20]. The TOA estimate is commonly obtained by employing correlator or matched filter receivers [21–23]. In this thesis, we review three strategies to compute the TOA measurements: one-way TOA, two-way TOA (TW-TOA), and time-difference-of-arrival (TDOA) measurements. We first assume perfect synchronization between different nodes, and later we study the effects of clock imperfection on distance estimates.

One-way time-of-arrival

Suppose sensor nodes are synchronized with a reference clock and consider LOS conditions. Let us assume that a known signal $s_i(t)$ is transmitted by node i at time $t = 0$ and the signal is received at node j with delay τ_{ij}. We express the received signal at node j as

$$r_{ij}(t) = \alpha_{ij} s_i(t - \tau_{ij}) + v_{ij}(t), \tag{2.8}$$

where α_{ij} is the channel gain, deterministic or random, and $v_{ij}(t)$ is assumed to be zero-mean Gaussian random variable with spectral density of $N_0/2$. The unknown delay τ_{ij} can be estimated by, e.g., a maximum likelihood estimator (MLE).

Let us define the effective bandwidth B_e as

$$B_e \triangleq \left(\frac{1}{E_{s_i}} \int_{-\infty}^{\infty} f^2 |S_i(f)|^2 df\right)^{\frac{1}{2}}, \tag{2.9}$$

where E_{s_i} and $S_i(f)$ represent, respectively, the energy and the Fourier transform of the transmitted signal $s_i(t)$. Considering the effective bandwidth B_e defined in (2.9), the CRLB for the TOA estimate is computed as [8]

$$\sqrt{\mathbb{E}\big(\hat{\tau}_{ij} - \mathbb{E}(\hat{\tau}_{ij})\big)^2} \geq \frac{1}{2\sqrt{2}\pi\sqrt{\mathrm{SNR}}B_e}, \tag{2.10}$$

where the signal-to-noise ratio, SNR, is defined as SNR $\triangleq \alpha_{ij}^2 E_{s_i}/N_0$.

It is seen that increasing the SNR or effective bandwidth improves the performance of TOA estimation. For example, in UWB systems in which a large bandwidth is used, the distance can be accurately estimated using TOA.

An estimate of the delay in a synchronized network can be expressed as

$$\hat{\tau}_{ij} = \frac{\|\boldsymbol{x}_i - \boldsymbol{x}_j\|}{c} + n_{ij}, \tag{2.11}$$

where n_{ij} is often assumed to be a zero-mean Gaussian random variable, i.e., $n_{ij} \sim \mathcal{N}(0, \sigma_{ij}^2)$ [6, 22, 24] and c denotes the speed of propagation. Note that the MLE estimate asymptotically, i.e., for large number of samples, has Gaussian distribution (under some regularity conditions) [25]; hence, the Gaussian noise assumption n_{ij} for MLE is a valid assumption. The distance estimate is, then, obtained by

$$\hat{d}_{ij} \triangleq c\hat{\tau}_{ij} = \|\boldsymbol{x}_i - \boldsymbol{x}_j\| + c\, n_{ij}. \tag{2.12}$$

Two-way time-of-arrival

In a two-way TOA (TW-TOA) approach, the distance between two nodes is computed using the round-trip delay estimate without the need for a common time reference. In this method, a sensor node i sends a signal to a node j and waits for a response from it. Node j, then, replies with an acknowledgment after a turn-around time T_j^{ar}. The propagation delay can be computed using TOA estimates in each node separately. That is, every node individually estimates the TOA of the received signal and then an estimate of the delay between two nodes is obtained from TOAs (two-step estimation). Another approach is to directly estimate the delay between two nodes from both signals received at both nodes (direct approach). If there is correlation between two links, i.e., the channels from node i to node j and vice versa, the latter approach is expected to provide more accurate estimate than the former technique. Here, we only assume that an estimate of the delay based on TW-TOA is available, regardless of the approach. Let us assume that an estimate of the distance using TW-TOA can be expressed as

$$\hat{d}_{ij} = \|\boldsymbol{x}_i - \boldsymbol{x}_j\| + c\frac{T_j^{\mathrm{ar}}}{2} + c\xi_{ij}, \tag{2.13}$$

where ξ_{ij} denotes the TW-TOA estimation error. It is seen that the T_j^{ar} should be known or at least accurately estimated. Since the TOAs are separately estimated in two nodes, the measurement noise ξ_{ij} can be expressed as $\xi_{ij} = (n_{ij} + n_{ji})/2$, where n_{ij} and n_{ji} are corresponding TOA estimation errors at node i and node j for the signals transmitted from node j and i, respectively. As we will see later, the TW-TOA can be affected by another parameter, the so-called clock skew (for affine clock model). The drawback of this method is that we need to send two signals for every range measurement, compared to the TOA approach, which only needs one transmission.

Time-difference-of-arrival

Suppose two sensor nodes at positions $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ receive a signal transmitted by a target node. The TDOA of the signal transmitted by the target node can be estimated directly from waveform observations in two different nodes (assuming their receivers are synchronized) or it can be indirectly computed from two TOA estimates in each sensor node (synchronization between the transmitter of one node with the receiver of another node is required). In this thesis, we consider estimating TDOA using the difference between TOAs computed in each node. Instead of measuring the absolute distance between two nodes, which is the case for TOA and TW-TOA, we can measure the distance differences between a target node and two synchronized reference nodes using TDOA estimates. This method is used by the GPS receiver, where a receiver at an unknown position measures the TDOA of received signals from two synchronized satellites. For example, the TDOA between target node $\boldsymbol{x}_i$ and synchronized sensor nodes at positions $\boldsymbol{x}_j$ and $\boldsymbol{x}_k$ can be computed as follows. First, note that the TOA estimation error of the signal transmitted by node i at node j can be expressed as

$$\hat{t}_{ij} = T_{0i} + \frac{\|\boldsymbol{x}_i - \boldsymbol{x}_j\|}{c} + n_{ij}, \tag{2.14}$$

where T_{0i} denotes the time in which node i starts sending its signal, which may be unknown. Thus, the TDOA estimate can be obtained as

$$\hat{\tau}^i_{jk} = \hat{t}_{ij} - \hat{t}_{ik} = \frac{\|\boldsymbol{x}_i - \boldsymbol{x}_j\|}{c} - \frac{\|\boldsymbol{x}_i - \boldsymbol{x}_k\|}{c} + n_{ij} - n_{ik}. \tag{2.15}$$

Therefore, an estimation of the distance differences between nodes j and k to node i can be written as

$$\hat{d}^i_{jk} \triangleq c\,\hat{\tau}^i_{jk} = \|\boldsymbol{x}_i - \boldsymbol{x}_j\| - \|\boldsymbol{x}_i - \boldsymbol{x}_k\| + c(n_{ij} - n_{ik}). \tag{2.16}$$

As can be seen from (2.16), this technique creates correlation between measurements: e.g., $\hat{\tau}^i_{jk}$ and $\hat{\tau}^i_{lk}$ are correlated through n_{ik}.

It is also possible to use hybrid measurements for positioning. A number of hybrid schemes have been studied in the literature, e.g., TOA/AOA [26], TDOA/AOA [27], TDOA/TW-TOA [28, 29], and TOA(TDOA)/RSS [30, 31].

In addition to the measurement errors, there are other sources of errors, which affect the TOA-based estimates. Generally, the main sources of error in time-based ranging are [22]; *propagation effects*, *clock imperfections*, and *interference*. The propagation effects include multipath fading, direct-path delay, and direct-path blockage. Imperfect synchronization between nodes causes range estimates to have large errors [18–20]. Finally, the interference from other signals using the same frequency band (or neighboring band) will deteriorate the range estimate. In the next section, we review only some of these practical impairments.

2.2.3 Practical considerations

In this section, we take into account a few practical effects on measurements. In particular, we briefly study the effects of direct-path blockage and clock imperfections on range estimates. Note that it may be difficult to identify all practical degradations, especially in indoor scenarios. The details of practical imperfections are beyond the scope of this thesis, and we refer the reader to, e.g., [3, 22] and references therein.

Non-line-of-sight errors

In wireless channels, the signal traveling between sensor nodes is mainly affected by, e.g., objects, people, and walls, and hence the channel is multipath. It may happen that the direct path is blocked, e.g., in indoor scenarios, and the distance measured between sensor nodes will be affected by NLOS errors. The distance estimate by sensor nodes can be expressed as

$$\hat{d}_{ij} = d(\boldsymbol{x}_i, \boldsymbol{x}_j) + b_{ij} + \tilde{n}_{ij}, \tag{2.17}$$

where b_{ij} and $\tilde{n}_{ij}$ denote the NLOS error and measurement error, respectively. A number of distributions have been considered to model NLOS conditions, e.g., an exponential distribution or a uniform distribution [32]. The Gaussian distribution with large positive mean has also been considered to model the NLOS condition [32, 33]. In some scenarios the NLOS can be a large positive value that makes the estimated distance greater than the actual distance. The statistics of NLOS errors may be unknown in practical scenarios, and so designing algorithms to relay on the statistics of measurement errors may not be straightforward. In general, the positioning problem for NLOS conditions is a challenging problem and still is an ongoing research topic.

Clock imperfections

The distance between two sensors corresponds to the traveling time of a signal from one node to another node. The time of flight depends on a number of parameters, such as the propagation speed, channel behaviors, or response time (turn-around time) in one node (for two-way ranging). Moreover, the delay is computed with respect to local time in sensor nodes. That is, the arrival time in a sensor node is measured with respect to its local clock. Hence, any deviation from reference (global) clock in sensor nodes can remarkably affect the accuracy of the range estimate. For many applications in WSNs, e.g., transmission scheduling, data fusion, power management, and tracking, a high level of accuracy is required for the network to function as intended. In the past few years, a huge amount of research has been devoted to design efficient and robust synchronization algorithms, see, e.g., [34–36] and references therein.

In the synchronization literature, it is common to model the behavior of a local clock of a sensor node by a polynomial function of the reference clock. The most widely used model in the literature is an affine model expressed as [34]

$$C_i(t) \triangleq \theta_{i0} + w_i t + \epsilon_i(t), \tag{2.18}$$

where $C_i(t)$ is the local clock of node i at ideal (reference) time t, θ_{i0} is the clock offset (phase difference), w_i denotes the clock skew, and $\epsilon_i(t)$ denotes a random noise process, which can also be considered as a model mismatch error. For a short time period, we can assume that $\epsilon_i(t)$ is fixed, hence, during sufficiently short time periods, we can absorb $\epsilon_i(t)$ into θ_{i0}. Fig. 2.2 illustrates the relationship between the local clock of node i and a reference clock. As noted in the figure, perfect synchronization corresponds to $\theta_{0i} = 0$ and $w_i = 1$. The clock skew w_i shows the slope of the local clock. For example, in the figure, the local time varies faster than the ideal time, i.e., $w_i > 1$. The affine model for the clock is a common model and has been justified in the literature, e.g., see [16, 35, 36] and

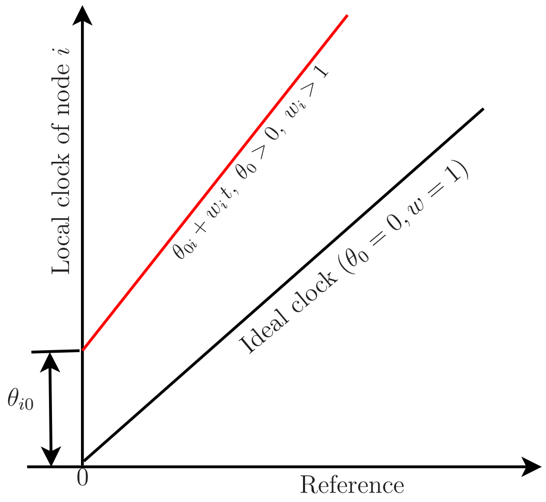

Figure 2.2: Local clock versus real clock for node i.

references therein. It can be observed that the local clock of every node can be expressed based on the local clock of another node, through an affine relationship.

Note that another well-known model for describing the behavior of a local clock is given by a quadratic function. The quadratic model is more suitable for studying the synchronization problem for long-term scenarios.

We now investigate the effect of clock parameters on TOA-based measurements. For simplicity, we assume that there is a target at unknown position $\boldsymbol{x}$.

Time-of-arrival: Let us consider Fig. 2.3 illustrating the kth round of TOA measured at reference node i for a signal transmitted by a target. Namely, the target node sends its signal at real time T_0^k, which is unknown to the reference node i. After a delay corresponding to the distance between the target and the reference node i, the signal arrives at reference node i at $C_i(T_0^k + d(\boldsymbol{a}_i, \boldsymbol{x})/c)$ (corresponding to the real time $T_0^k + d(\boldsymbol{a}_i, \boldsymbol{x})/c$). The TOA measurement for the signal transmitted from the target node at reference node i for the kth measurement can be written as [37, 38]

$$C_i(T_0^k + d(\boldsymbol{a}_i, \boldsymbol{x})/c) = \theta_{0i} + w_i \left(T_0^k + \frac{d(\boldsymbol{a}_i, \boldsymbol{x})}{c} \right) + \tilde{n}_i^k, \quad k = 1, \ldots, K, \quad i = 1, \ldots, N, \tag{2.19}$$

where $\tilde{n}_i^k$ is the TOA estimation error at reference node i for the signal transmitted from the target node at time T_0^k, and K is the number of TOA measurements (messages) for every link between a reference node and the target node.

Note that if time stamping is performed in the MAC layer, a model including fixed and random delays with no measurement noise can be considered. Such a model has been extensively studied in the synchronization literature, e.g., in [36] and references therein. The estimation error is often modeled by a zero-mean Gaussian random variable with variance σ_i^2/c^2; i.e., $\tilde{n}_i^k \sim \mathcal{N}(0, \sigma_i^2/c^2)$ [8, 24]. Note that we assume that θ_{0i} and w_i are fixed unknown parameters for $k = 1, \ldots, K$. From Eq. (2.19), it is observed that unknown parameters in the estimation process are $\boldsymbol{x}$, $\{\theta_{0i}, w_i\}_{i=1}^N$, and $\{T_0^k\}_{k=1}^K$.

A similar expression can be obtained when the target node collects signals transmitted

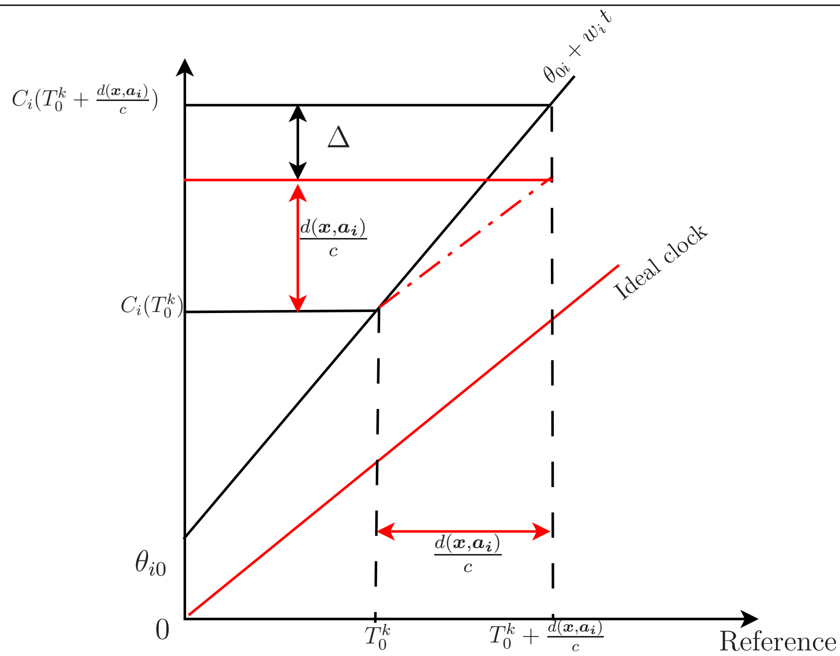

Figure 2.3: TOA estimation of signal transmitted by reference node i at a target node.

by a reference node. Namely, the TOA estimation in the target node is given by

$$C(T_{0,i}^k + d(\boldsymbol{a}_i, \boldsymbol{x}_i)/c) = \theta_0 + w(T_{0,i}^k + d(\boldsymbol{a}_i, \boldsymbol{x})/c) + \bar{n}_i^k, \quad k = 1, \ldots, K, \quad i = 1, \ldots, N, \tag{2.20}$$

where $\bar{n}_i^k$ is the TOA estimation error in the target node, $T_{0,i}^k$ is the time instant that reference node i transmits its signal, and θ_0 and w are, respectively, the clock offset and clock skew. It is observed that the unknown parameters are θ_0, w, $\boldsymbol{x}$, and $\{\{T_{0,i}^k\}_{i=1}^N\}_{k=1}^K$.

From (2.19) and (2.20), it is noted that different sets of unknown parameters appear for remote and self positioning scenarios for asynchronous networks.

Time-difference-of-arrival: Let us consider a self-positioning scenario, i.e., the measurement model in (2.20). The model indicates that in order to obtain an estimate of the distance between the target node and a reference node, parameters θ_0, w, and $T_{0,i}^k$ (as nuisance parameters) should also be estimated. Let us assume that reference nodes are synchronized, e.g., using GPS signals, and assume that $T_{0,i}^k = \cdots = T_{0,N}^k = T_0^k$, meaning reference nodes transmit their signals simultaneously. We, then, subtract TOA measurements of the signals sent from reference nodes i and j, and form a TDOA measurement as follows:

$$\Delta t_{i,j}^k = t_i^k - t_j^k = w\left(\frac{d(\boldsymbol{a}_i, \boldsymbol{x})}{c} - \frac{d(\boldsymbol{a}_j, \boldsymbol{x})}{c}\right) + \bar{n}_i^k - \bar{n}_j^k, \quad i \neq j = 1, \ldots, N. \tag{2.21}$$

As observed from (2.21), the clock offset θ_0 and $T_{0,i}^k = T_0^k$ are canceled out, but the clock skew affects the TDOA measurements and it should be considered when estimating the target node position. If the clock skew is very close to one, its impact on TDOA measurements is negligible.

Two-way TOA: As mentioned previously, in the TOA approach the unknown clock offset can considerably affect the distance estimate, see Eq.(2.20), for self-positioning.

One way to get rid of the clock offset for distance estimates is to use two-way TOA. A TW-TOA measurement between the target node and the ith reference node is carried out as follows: (a) the target sends a message to the reference node at (global) time $t_{i,1}^k$, (b) the message arrives at the reference node at time $t_{i,2}^k$, (c) the reference node sends a return message at time $t_{i,3}^k$, and (d) the return message arrives at the target node at time $t_{i,4}^k$. Clearly, $t_{i,2}^k - t_{i,1}^k = t_{i,4}^k - t_{i,3}^k = d(\boldsymbol{x}, \boldsymbol{a}_i)/c$. Moreover, $t_{i,3}^k = t_{i,2}^k + T_i^{\text{ar}}$, where T_i^{ar} is the turn-around time in the ith reference node, which is assumed to be fixed during the positioning process. The TW-TOA measurement is computed at the target node as

$$z_i^k \triangleq \frac{1}{2}\left[C(t_{4,i}^k) - C(t_{1,i}^k) + n_i^k\right] = w\frac{d(\boldsymbol{x}, \boldsymbol{a}_i)}{c} + w\frac{T_i^{\text{ar}}}{2} + \frac{n_i^k}{2}, \quad k = 1, 2, \ldots, K, \tag{2.22}$$

where n_i^k is the TW-TOA measurement error, modeled as $n_i^k \sim \mathcal{N}(0, 2\sigma_i^2)$, and K is the number of the TW-TOA measurements during the positioning process. Note that we assumed that the variance of TOA estimation errors in the reference node i and target are the same and equal to σ_i^2.

The unknown parameter T_i^{ar} either might be extremely small and can be neglected [20] or it needs to be estimated. One way to deal with the unknown parameter T_i^{ar} is to jointly estimate it along with the location of the target node [39, 40]. It can also be estimated by reference node i using a loopback test and is sent back to the target node [38, 41, 42].

In Fig. 2.3, Δ shows the effect of an imperfect clock skew on TOA measurements. Similar behaviors hold for TW-TOA scenarios.

For recent advances in positioning problems for asynchronous networks, we refer the reader to, e.g., [43, 44].

2.3 Algorithms classifications

The positioning problem can be studied in different aspects. Practical constraints often pose limited freedom in designing algorithms. For example, due to limited source of energy in WSNs, it is necessary to design low-complexity algorithms. In addition, due to uncertainties in some parameters, such as clock parameters or the location of the reference nodes, the designed algorithms should be robust and reliable in practical applications. One important degradation parameter is NLOS conditions, which significantly affect the performance of the positioning algorithm. Positioning algorithms (problems) can be classified into different groups. Here, we categorize the positioning algorithms into different well-known classes, as discussed below.

2.3.1 Centralized vs distributed approaches

In order to estimate the locations of target nodes, the measurements collected in different nodes can be processed in two different ways: centralized or distributed. In a centralized processing approach, there is a fusion center, which collects all measurements from different nodes across the network and estimates the positions of target nodes, optimally or suboptimally. In a distributed processing approach, however, there is no central unit and sensor nodes work cooperatively to find the location of unknown targets. That is, nodes have local interactions with their immediate neighbors to solve the positioning problem. Implementing a centralized algorithm in a distributed manner, in general, may be challenging. Techniques from optimizations, e.g., dual and primal decomposition methods, or from graphical models, e.g., factor graph based approaches, are commonly used to obtain distributed algorithms. The optimality conditions in which the distributed algorithms attain the centralized solution might not be satisfied in some scenarios. Moreover, an important question in designing a distributed algorithm is how fast the algorithm convergences to the solution. Distributed processing usually contains two steps: broadcasting the estimate and combining the local estimates in every local node. It has been argued that the most costly part is the broadcasting step. As a rule of thumb, the energy required to transmit one bit of information over 100 meters is approximately equivalent to the energy consumption of executing 3 million instructions [45]. Therefore, the number of broadcasts, generally, determines how complex an algorithm is.

2.3.2 Cooperative vs noncooperative techniques

The traditional positioning problem is defined over a network consisting of a number of reference nodes connected to a single target or multiple targets. It is known that as the number of reference nodes increases, the accuracy of the position estimate considerably improves. In situations in which there is limited access to reference nodes, the cooperation technique can improve the accuracy of the position estimate. That is, a target node is allowed to connect other target nodes and collect some type of measurements. In fact, in a cooperative approach, both measurements between a target and reference nodes and between the target and a number of other targets are available. Whereas, in noncooperative scenarios only measurements between a target and reference nodes are available.

2.3.3 Self-positioning vs remote positioning

Extracting the position information of a target node can be carried out in the target node (self-positioning) or it can be estimated in reference nodes and sent back to the target node (remote positioning). For example, in positioning recovery via GPS, every target node receives GPS signals and finds its position, whereas in target tracking the location information is extracted in reference nodes. In terms of algorithm developments, we may need to optimize different parameters for self-positioning and remote positioning as discussed earlier when we reviewed the effects of clock imperfections.

2.3.4 Classification based on measurements

It is also possible to classify different algorithms based on the measurements taken by sensor nodes. The common positioning approaches use a single type of measurements such as RSS, TOA, or AOA or a combination of measurements. *Finger-printing* based approaches have also been considered for some applications. In general, the different techniques can be summarized as [6, 22]:

- **RSS** is simple to implement and not sensitive to timing errors. It requires an accurate model of the RSS-distance dependency. However, range estimation using RSS is not accurate compared to, e.g., TOA-based approaches;

- **AOA** is strongly affected by NLOS conditions. The accuracy depends on RF bandwidth and SNR.

- **TOA/TDOA** is an accurate technique that suffers from NLOS conditions. For perfectly synchronized networks, the accuracy depends on RF bandwidth and SNR.
- **Mapping method (finger printing-based)** is robust against NLOS conditions. Its need to construct an extensive database and then to search through it make the approach complicated.

Chapter 3

Positioning algorithms

During the last few years, a huge number of positioning algorithms, based on different criteria, have been proposed in the literature [6–9, 11, 12, 46]. The traditional positioning system consists of a number of reference nodes at known locations, a number of target nodes at unknown locations that need to be localized, and a processing center that gathers the measurements made in different nodes and implements a positioning algorithm. Recently, there has been a huge interest in distributed algorithms to estimate the location of target nodes in a decentralized manner. That is, nodes interact with their immediate neighbors and finally find their estimates in a distributed fashion.

As mentioned in the previous section, the position information can be directly extracted from the signal traveling between sensor nodes, but from a practical perspective (mainly complexity) it is preferred to first estimate certain parameters, e.g., distance between sensor nodes, and then to extract the position information from the intermediate estimates (a suboptimal approach) [8].

Fig. 3.1 shows a high-level implementation of a positioning algorithm. Reference nodes (or even unknown target nodes) receive waveform signals from a target node and generate some type of intermediate estimates, e.g., TOA. A positioning algorithm is consequently applied to estimate the positions of unknown targets. Positioning algorithms can be either centralized or distributed; therefore, the measurements need to be sent to a center (centralized) or to be locally processed (distributed).

During the last decade, various positioning algorithms have been proposed for WSNs. If models of the measurements are known, classical approaches, e.g., least squares or maximum likelihood, can be applied to solve the positioning problem. A well-known approach, the so-called *multilateration* technique, has traditionally been used for position information recovery [13]. In some scenarios due to, e.g., complexity constraints, a coarse estimate is required and a number of low-complexity algorithms can be used to provide a coarse estimate. For example, the *centroid* algorithm, which relies only on connectivity information, computes the average of reference nodes' locations connected to a target as a coarse estimate of the target location [47, 48]. A number of heuristic approaches have been proposed in the literature, which can provide good coarse estimates in different scenarios. For details of such approaches, we refer the reader to, e.g., [13]. Another class of algorithms are based on *finger printing* approaches, especially for RSS measurements, which can be used in small networks. In this technique, first, in the offline phase a database (map) for the network is constructed for each region– represented by a point– of the network, including an ID and RSS measurements corresponding to that region. Then

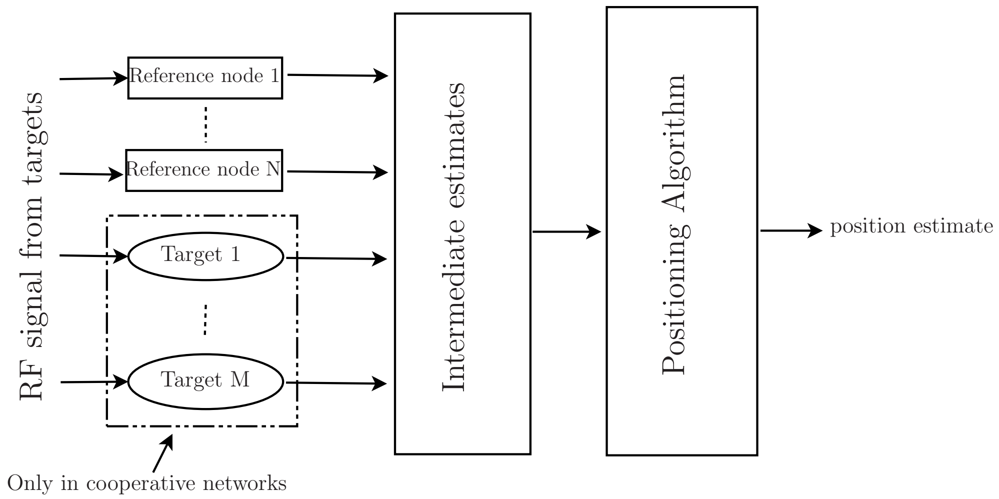

Figure 3.1: A high-level description of positioning (two-step positioning).

in the online phase, we estimate the location of the target as a point in the data set in which the new measurements have the best match with offline measurements corresponding to that point [49]. For large networks, this technique has drawbacks when it comes to implementation, due to cost and memory constraints. Moreover, the network geometry should be known in advance, which may not be possible for some applications, e.g., for dynamic networks.

In this thesis, we briefly review a number of positioning algorithms with a focus on range-based methods. In particular, we study the MLE and least squares algorithms and two approaches based on a geometric interpretation of the positioning problem.

The positioning problem described in Section 2.1 can be generally formulated as an optimization problem

$$\begin{aligned} \underset{\boldsymbol{X}\in\mathbb{R}^{d\times M}}{\text{minimize}} \quad & \ell(\boldsymbol{X}, \boldsymbol{m}) \\ \text{subject to} \quad & \boldsymbol{X} \in \mathcal{D}, \end{aligned} \tag{3.1}$$

where $\boldsymbol{m} \triangleq \{\{m_{ij}\}_{j\in\mathcal{A}_1\cup\mathcal{B}_1} \;\cdots\; \{m_{Mj}\}_{j\in\mathcal{A}_M\cup\mathcal{B}_M}\}$, $\boldsymbol{X} = [\boldsymbol{x}_1 \cdots \boldsymbol{x}_M]^T \in \mathbb{R}^{d\times M}$, $\mathcal{D}$ is a set containing the locations of targets (a feasible set), and $\ell(\boldsymbol{X}, \boldsymbol{m})$ denotes a loss function. The loss function may be originated from, e.g., an statistical criterion or a geometric interpretation.

The problem in (3.1) is in general a centralized optimization problem and the distributed version of (3.1) may require a suitable decomposition technique.

3.1 Statistical estimators

In this section, we review the maximum likelihood estimator (MLE) and the least squares approximation (both nonlinear and linear) for the positioning problem.

3.1.1 Maximum likelihood estimator

Suppose that measurement errors are independent and identically distributed (i.i.d.). Let the joint PDF of the measurements in (2.3) be $\Pi_{i=1}^{M}\Pi_{j\in\mathcal{A}_i\cup\mathcal{B}_i}p_i(m_{ij};\boldsymbol{X})$. The joint MLE estimate of the locations of M targets can be obtained as follows [50, 51]:

$$\hat{\boldsymbol{X}} = \underset{\boldsymbol{X}\in\mathbb{R}^{d\times M}}{\arg\max}\sum_{i=1}^{M}\sum_{j\in\mathcal{A}_i\cup\mathcal{B}_i}\log p_i(m_{ij};\boldsymbol{X}). \tag{3.2}$$

In general, the optimization problem in (3.2) is nonconvex and difficult to solve. The MLE is, under some regularity conditions, asymptotically unbiased and efficient [25, 50]; hence, for the positioning problem the MLE tends, on average, to be true positions for high SNRs, a large number of observations, or a large number of reference nodes [52].

As an example, the MLE for range-based measurements using the TOA estimates for synchronized networks and zero-mean Gaussian measurement errors, can be expressed as

$$\hat{\boldsymbol{X}} = \underset{\boldsymbol{X}\in\mathbb{R}^{d\times M}}{\arg\min}\sum_{i=1}^{M}\left(\sum_{j\in\mathcal{B}_i}\frac{1}{\sigma_{ij}^2}\left(\hat{d}_{ij}-\|\boldsymbol{x}_i-\boldsymbol{x}_j\|\right)^2+\sum_{j\in\mathcal{A}_i}\frac{1}{\sigma_{ij}^2}\left(\hat{d}_{ij}-\|\boldsymbol{x}_i-\boldsymbol{a}_j\|\right)^2\right). \tag{3.3}$$

where σ_{ij}^2 is the variance of the Gaussian measurement errors. The problem (3.3), in general, is difficult to solve. For more complex examples of positioning problems, see, e.g., [53–55].

It is noted that in the localization problem there might be additional nuisance parameters, e.g., unknown clock parameters in range-based techniques or unknown transmission power and path-loss exponent, which need to be taken into account in the estimation process. The nuisance parameters can be removed from the models, e.g., by manipulating the measurements, or they can be estimated along with the location of the target nodes. For some examples in different scenarios, see, e.g., [42, 53–56].

As mentioned, the MLE is asymptotically efficient and unbiased; i.e., it attains the CRLB. To compute the MLE, it is necessary to know the distribution of measurements; however, in practice, it is difficult to obtain *a priori* knowledge of the full statistics of measurement errors.

3.1.2 Nonlinear least squares

Let us define the residual errors for measurements in (2.3) as $e_r^a(i,j) \triangleq m_{ij}-f(\boldsymbol{x}_i,\boldsymbol{a}_j),\ j\in\mathcal{A}_i$ and $e_r^t(i,j) \triangleq m_{ij}-f(\boldsymbol{x}_i,\boldsymbol{x}_j),\ j\in\mathcal{B}_i$. The least squares approximation commonly used in the positioning literature tries to find an estimate $\hat{\boldsymbol{X}}$ of $\boldsymbol{X}$ based on minimizing the squares of residuals $e_r^a(i,j)$ and $e_r^t(i,j)$. Namely, a nonlinear least squares algorithm for positioning is formulated as [57]:

$$\hat{\boldsymbol{X}} = \underset{\boldsymbol{X}\in\mathbb{R}^{d\times M}}{\arg\min}\sum_{i=1}^{M}\left(\sum_{j\in\mathcal{A}_i}\left(e_r^a(i,j)\right)^2+\sum_{j\in\mathcal{B}_i}\left(e_r^t(i,j)\right)^2\right). \tag{3.4}$$

When the variances of measurement errors are available, the NLS can be formulated as a weighted nonlinear least squares (WNLS) technique

$$\hat{\boldsymbol{X}} = \underset{\boldsymbol{X}\in\mathbb{R}^{d\times M}}{\arg\min}\sum_{i=1}^{M}\left(\sum_{j\in\mathcal{A}_i}\left(\frac{e_r^a(i,j)}{\sigma_{ij}}\right)^2+\sum_{j\in\mathcal{B}_i}\left(\frac{e_r^t(i,j)}{\sigma_{ij}}\right)^2\right), \tag{3.5}$$

where σ_{ij}^2 is the variance of ϵ_{ij}, see (2.3).

In general, the WNLS solution coincides with the ML estimate if the measurement errors are i.i.d. Gaussian. As an example, the WNLS for range-based positioning is similar to (3.3).

An alternative approach in the positioning literature is to apply the squared-range NLS squares (SR-NLS) [58–61] for the range-based positioning. Then

$$\hat{\boldsymbol{X}} = \arg\min_{\boldsymbol{X}\in\mathbb{R}^{d\times M}} \sum_{i=1}^{M} \left(\sum_{j\in\mathcal{A}_i} \left(\hat{d}_{ij}^2 - \|\boldsymbol{x}_i - \boldsymbol{a}_j\|^2\right)^2 + \sum_{j\in\mathcal{B}_i} \left(\hat{d}_{ij}^2 - \|\boldsymbol{x}_i - \boldsymbol{x}_j\|^2\right)^2 \right). \tag{3.6}$$

Note that the SR-NLS approach is suboptimal in the maximum likelihood sense, but it can be shown that a global solution to (3.6) can be obtained efficiently for noncooperative scenarios [60].

There are a number of techniques to approximately solve the nonconvex MLE or NLS, e.g., via semidefinite relaxation, second-order cone programming, or sum of squares approaches. For details of these approaches, consult, e.g., [58, 62, 63].

The MLE (NLS or SR-NLS) problem is a centralized problem; hence, it is needed to transfer all measurements to a fusion center for processing. In situations where there is no central processing unit, the positioning problem needs to be solved in a distributed manner. Since the problem, in general, is nonconvex and coupled, the distributed implementation can be challenging. The techniques based on decomposition or consensus [64–68] can be used to solve the problem. A fast technique based on alternating direction method of multipliers [69] has been recently proposed to solve the problem [70]. Another approach is to resort to a graphical framework and solve the problem using, e.g., factor graph-based approaches [12].

3.1.3 Linear least squares

The measurement models in the positioning problem are, in general, nonlinear, based on the location of targets, and the positioning problems are often formulated as nonlinear and nonconvex problems, which can be difficult to solve. One technique to solve the nonconvex problem is approximating the problem by a convex problem, e.g., adopting suitable relaxations, and obtaining a good coarse estimate of the location. The estimate can be further improved using a refining approach. An efficient method that often gives good coarse estimates, which asymptotically attains the CRLB at high SNR, is to linearize the measurements based on the position of the target nodes and then to employ the linear least squares (LLS) criterion. A number of studies have been devoted to derive linear estimators in the positioning literature, see, e.g., [41, 42, 53, 55, 71–73]. To form a linear least squares problem, we need to find a signal model that is linear in unknown parameters [71]. If there are nuisance parameters such as unknown clock parameters in the TOA-based approaches or unknown channel parameters in the RSS-based technique, the unknown vector may also contain nuisance parameters. It may also be possible to remove the nuisance parameters, using suitable transformations, and derive a linear model based on unknown locations. To explain the technique, we consider the measurements between a target node and reference nodes connected to the target. The technique is based on nonlinear processing of measurements, which allows us to express the modified measurements by means of a linear model based on unknown parameters as follows. In order to use this technique properly, it is necessary to have at least three nodes (four nodes in three-dimensional networks), noncolinear, with known locations connected to a target node. Let the measurement error be small compared to the true distances, and assume that the distance measurement between target i and reference node j is given by

$$\hat{d}_{i,j} = d(\boldsymbol{x}_i, \boldsymbol{a}_j) + \epsilon_{ij}, \qquad j \in \mathcal{A}_i, \tag{3.7}$$

where ϵ_{ij} is measurement noise with variance σ_{ij}^2. Here, there is no particular assumption about noise statistics except it is zero mean. If ϵ_{ij} has nonzero mean, we first need to subtract the mean from both sides of (3.7). Knowing the variance of the noise helps to obtain a more accurate estimate.

Squaring both sides of (3.7) and rearranging terms yields:

$$\tilde{d}_{ij} \triangleq \hat{d}_{ij}^2 - \|\boldsymbol{a}_j\|^2 = [-2\boldsymbol{a}_j{}^T \ 1]\boldsymbol{\psi}_i + 2d(\boldsymbol{x}_i, \boldsymbol{a}_j)\epsilon_{ij} + \epsilon_{ij}^2, \qquad j \in \mathcal{A}_i, \tag{3.8}$$

where $\boldsymbol{\psi}_i = \left[\boldsymbol{x}_i^T \ \|\boldsymbol{x}_i\|^2\right]^T$. As it is observed, a semi-linear model based on the location of the target nodes, $\boldsymbol{x}_i$, and square norm of the location, $\|\boldsymbol{x}_i\|^2$, is obtained. We consider the model (3.8) to be semilinear because the statistics of noise also depend on the location of the target node. There are two techniques to find an estimate of the location $\boldsymbol{x}_i$: eliminating the quadratic term and estimating the quadratic term along with the location. In the elimination approach, the common quadratic term is eliminated by subtracting two modified measurements, say $\tilde{d}_{ij} - \tilde{d}_{ik}$, $j \neq k$, and then a new set of measurements is formed based on the unknown location $\boldsymbol{x}_i$. In the second approach, both $\boldsymbol{x}_i$ and $\|\boldsymbol{x}_i\|^2$ are jointly estimated. It has been shown that both techniques (in the least squares scenes) yield the same solution [71]. If the statistics of the measurement errors are known, a more accurate estimate is obtained based on the weighted least squares approach. Here, we study a joint estimation of the location and the squared norm of the location. We further assume that the measurement noise ϵ_{ij} is small; hence,

$$\tilde{d}_{ij} \simeq [-2\boldsymbol{a}_j{}^T \ 1]\boldsymbol{\psi}_i + 2d(\boldsymbol{x}_i, \boldsymbol{a}_j)\epsilon_{ij}, \quad j \in \mathcal{A}_i. \tag{3.9}$$

Now a set of linear equations can be written as

$$\boldsymbol{d}_i = \boldsymbol{A}_i\boldsymbol{\psi}_i + \boldsymbol{\nu}_i, \tag{3.10}$$

where

$$\boldsymbol{d}_i \triangleq \left[\tilde{d}_{ij_1} \ \tilde{d}_{ij_2} \ \ldots \ \tilde{d}_{ij_k}\right]^T, \tag{3.11a}$$

$$\boldsymbol{A}_i \triangleq \begin{bmatrix} -2\boldsymbol{a}_{j_1}^T & 1 \\ \vdots & \vdots \\ -2\boldsymbol{a}_{j_k}^T & 1 \end{bmatrix} \tag{3.11b}$$

$$\boldsymbol{\nu}_i \triangleq \left[2d(\boldsymbol{x}_i, \boldsymbol{a}_{j_1})\,\epsilon_{ij_1} \ \ldots \ 2d(\boldsymbol{x}_i, \boldsymbol{a}_{j_k})\,\epsilon_{ij_k}\right]^T, \tag{3.11c}$$

where $\mathcal{A}_i = \{j_1, \ldots, j_k\}$, and $k = |\mathcal{A}_i|$ is the cardinality of set $\mathcal{A}_i$.

If the matrix $\boldsymbol{A}_i$ is full rank, then using the (unconstrained) least squares criterion, the unknown parameter $\boldsymbol{\psi}_i$ can be estimated as [50]

$$\hat{\boldsymbol{\psi}}_i = (\boldsymbol{A}_i^T \boldsymbol{C}_{\boldsymbol{\nu}_i}^{-1} \boldsymbol{A}_i)^{-1} \boldsymbol{A}_i^T \boldsymbol{C}_{\boldsymbol{\nu}_i}^{-1} \boldsymbol{d}_i, \tag{3.12}$$

where the weighting matrix $\boldsymbol{C}_{\boldsymbol{\nu}_i}$, for i. i. d. measurement errors, is given by [74]

$$\boldsymbol{C}_{\boldsymbol{\nu}_i} = \operatorname{diag}\left(4d^2(\boldsymbol{x}_i, \boldsymbol{a}_{j_1})\sigma_{ij_1}^2, \ldots, 4d^2(\boldsymbol{x}_i, \boldsymbol{a}_{j_k})\sigma_{ij_k}^2\right). \tag{3.13}$$

The covariance matrix of $\hat{\boldsymbol{\psi}}_i$ can be computed as [50]

$$\operatorname{cov}(\hat{\boldsymbol{\psi}}_i) = \left(\boldsymbol{A}_i^T \boldsymbol{C}_{\boldsymbol{\nu}_i}^{-1} \boldsymbol{A}_i\right)^{-1}. \tag{3.14}$$

To compute the weighting matrix $\boldsymbol{C}_{\boldsymbol{\nu}_i}$, the real distances between known nodes and the target i are required. Since in practice the real distances are not available, we instead use the measured distances in (3.13). The linear estimator derived in the positioning literature is suboptimal [73]; hence, we can employ a number of techniques to improve the estimate, e.g., correction techniques [72, 75] or a refining technique based on the Taylor series expansion [55]. Here, we review the correction technique and refer the reader to [41, 55] for details about refining the estimate using the Taylor series expansion.

Let us express each element of (3.12) as

$$\begin{aligned} [\hat{\boldsymbol{\psi}}_i]_1 &= x_{i,1} + e_1, \\ [\hat{\boldsymbol{\psi}}_i]_2 &= x_{i,2} + e_2, \\ [\hat{\boldsymbol{\psi}}_i]_3 &= \|\boldsymbol{x}_i\|^2 + e_3, \end{aligned} \tag{3.15}$$

where $\boldsymbol{\epsilon} = [e_1\ e_2\ e_3]^T$ is the error of estimation $\boldsymbol{\epsilon} = \hat{\boldsymbol{\psi}}_\ell - \boldsymbol{\psi}_\ell$, and $\boldsymbol{x}_i = [x_{i,1}\ x_{i,2}]^T$. We assume that the error of estimation, $\boldsymbol{\epsilon}$, is considerably small. Let us square both sides of the first two elements of (3.15) to obtain

$$\begin{aligned} [\hat{\boldsymbol{\psi}}_i]_1^2 &\simeq x_{i,1}^2 + 2x_{i,1}e_1, \\ [\hat{\boldsymbol{\psi}}_i]_2^2 &\simeq x_{i,2}^2 + 2x_{i,2}e_2, \end{aligned} \tag{3.16}$$

where we have neglected terms of the form e_i^2. We then express the squared terms in (3.16) and combine with $[\hat{\boldsymbol{\psi}}_i]_3$ as

$$\boldsymbol{b}_i = \boldsymbol{B}\boldsymbol{\phi}_i + \boldsymbol{\zeta}_i, \tag{3.17}$$

where

$$\begin{aligned} \boldsymbol{b}_i &\triangleq \left[[\hat{\boldsymbol{\psi}}_i]_1^2\ [\hat{\boldsymbol{\psi}}_i]_2^2\ [\hat{\boldsymbol{\psi}}_i]_3\right]^T, \\ \boldsymbol{\zeta}_i &\triangleq \left[2x_{i,1}e_1\ 2x_{i,2}e_2\ e_3\right]^T \\ \boldsymbol{\phi}_i &\triangleq \left[x_{i,1}^2\ x_{i,2}^2\ \right]^T, \\ \boldsymbol{B} &\triangleq \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}. \end{aligned} \tag{3.18}$$

The least squares approximation of $\boldsymbol{\phi}_i$ is obtained from (3.17) as

$$\hat{\boldsymbol{\phi}}_i = (\boldsymbol{B}^T \boldsymbol{C}_{\boldsymbol{\zeta}_i}^{-1} \boldsymbol{B})^{-1} \boldsymbol{B}^T \boldsymbol{C}_{\boldsymbol{\zeta}_i}^{-1} \boldsymbol{b}_i, \tag{3.19}$$

where the covariance matrix $\boldsymbol{C}_{\boldsymbol{\zeta}}^{-1}$ can be computed as

$$\boldsymbol{C}_{\boldsymbol{\zeta}_i} = \mathbb{E}\{(\boldsymbol{b}_i - \boldsymbol{B}\boldsymbol{\phi}_i)(\boldsymbol{b}_i - \boldsymbol{B}\boldsymbol{\phi}_i)^T\} = \boldsymbol{\Lambda}_i \operatorname{cov}(\hat{\boldsymbol{\psi}}_i)\boldsymbol{\Lambda}_i, \tag{3.20}$$

where $\boldsymbol{\Lambda}_i = \operatorname{diag}(2x_{i,1}, 2x_{i,2}, 1)$.

To compute matrix $\boldsymbol{\Lambda}_i$, we use the estimate in (3.12). The covariance matrix of $\hat{\boldsymbol{\phi}}_i$ is given by

$$\operatorname{cov}(\hat{\boldsymbol{\phi}}_i) = (\boldsymbol{B}^T \boldsymbol{C}_{\boldsymbol{\zeta}_i}^{-1} \boldsymbol{B})^{-1}. \tag{3.21}$$

Finally, the target position can be obtained as follows:

$$\tilde{x}_{i,j} = \operatorname{sgn}([\hat{\boldsymbol{\psi}}_i]_j)\sqrt{\left|[\hat{\boldsymbol{\phi}}_i]_j\right|}, \qquad j = 1, 2, \tag{3.22}$$

where $\operatorname{sgn}(\cdot)$ denotes the signum function defined as

$$\operatorname{sgn}(t) = \begin{cases} 1, & \text{if } t \geq 0 \\ -1, & \text{otherwise.} \end{cases} \tag{3.23}$$

An estimate of the covariance matrix of the estimator in (3.22) can be obtained by a linearization technique using the first-order Taylor series expansion. We refer the reader to [72] for details about the approach.

It is noted that the least squares solution to (3.10) is obtained by solving the following nonconvex problem:

$$\begin{aligned} &\underset{\boldsymbol{x}_i}{\text{minimize}}\, \|\boldsymbol{d}_i - \boldsymbol{A}\boldsymbol{\psi}_i\|^2 \\ &\text{subject to} \quad \boldsymbol{\psi}_i = \left[\boldsymbol{x}_i^T \; \|\boldsymbol{x}_i\|^2\right]^T. \end{aligned} \tag{3.24}$$

We will see later how to solve such a nonconvex quadratic programming problem using, e.g., a relaxation technique. We also study an efficient approach (in terms of complexity) to solve such a problem.

3.2 Geometric estimators

Another approach to formulate the positioning problem is to consider a geometric interpretation of the measurements taken between nodes. In this section, we review the concept of the geometric approach for solving a positioning problem. Let us consider the distance estimate $\hat{d}_{ij} = d(\boldsymbol{x}_i, \boldsymbol{x}_j) + v_{ij},\ j \in \mathcal{A}_i \cup \mathcal{B}_i$ between node i and node j, where v_{ij} denotes the estimation error. In the absence of measurement errors, i.e., $\hat{d}_{ij} = d(\boldsymbol{x}_i, \boldsymbol{x}_j)$, it is clear that target i, at position $\boldsymbol{x}_i$, lies in the intersection of a number of circles with radii $\hat{d}_{ij}$ and centers $\boldsymbol{x}_j,\ j \in \mathcal{A}_i \cup \mathcal{B}_i$ ($\boldsymbol{x}_j$ is known in advance if $j \in \mathcal{A}_i$ and is unknown if $j \in \mathcal{B}_i$).

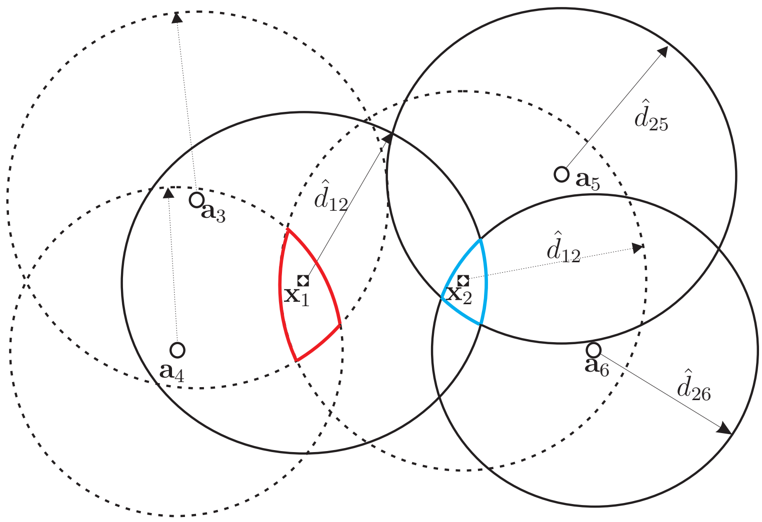

Figure 3.2: For distance measurements with nonnegative errors, target nodes 1 and 2 for the network shown in Fig. 2.1 lie in closed bounded sets.

Suppose that the measurement errors are nonnegative, $v_{ij} \geq 0$. Let us define the disc $\mathcal{D}_{ij}$ centered at $\boldsymbol{x}_j$ with radius $\hat{d}_{ij}$ as

$$\mathcal{D}_{ij} \triangleq \left\{\boldsymbol{x} \in \mathbb{R}^2 \mid \|\boldsymbol{x} - \boldsymbol{x}_j\| \leq \hat{d}_{ij}\right\}, \quad j \in \mathcal{A}_i \cup \mathcal{B}_i. \tag{3.25}$$

We define an estimate of $\boldsymbol{x}_i$ as a point in the intersection $\mathcal{D}_i$ of the discs $\mathcal{D}_{ij}$, that is

$$\hat{\boldsymbol{x}}_i \in \mathcal{D}_i \triangleq \bigcap_{j \in \mathcal{A}_i \cup \mathcal{B}_i} \mathcal{D}_{ij}, \quad \text{for } i = 1, 2, \ldots, M. \tag{3.26}$$

In fact, every point in the intersection area can be, potentially, an estimate of the target location. Although not all feasible points are optimal, e.g., in the mean square sense, this technique provides a robust approach for the positioning problem, especially for large NLOS errors, since the intersection is not affected considerably with larger positive errors as long as there are a few measurements with small errors. The positioning problem, then, can be transformed to the following problem:

$$\text{find } \hat{\boldsymbol{X}} = [\hat{\boldsymbol{x}}_1 \cdots \hat{\boldsymbol{x}}_M] \text{ such that } \hat{\boldsymbol{x}}_i \in \mathcal{D}_i, \; i = 1, \ldots, M. \tag{3.27}$$

As an example, Fig. 3.2 illustrates the intersections including target nodes one and two for the network shown in Fig. 2.1. For noncooperative scenarios, the problem (3.27) is equivalent to the well-known *convex feasibility* problem (CFP), which aims to obtain a point in the intersection of a number of convex sets. For cooperative networks, the problem (3.27), however, is different from the traditional CFP since the intersection including the target nodes is dependent on $\hat{\mathbf{X}}$ [39, 76, 77].

To improve the accuracy of the estimate in the geometric approach, we can decrease the intersection area in which the target nodes are most probably located, as considered in some recent studies [77, 78]. That is, instead of a disc defined in (3.25), we consider a ring or a circle for the possible location of the target nodes as

$$\mathcal{R}_{ij} = \left\{\boldsymbol{x} \in \mathbb{R}^2 \mid \hat{d}_{ij} - \epsilon_l \leq \|\boldsymbol{x}_i - \boldsymbol{x}_j\| \leq \hat{d}_{ij} + \epsilon_u\right\}, \quad j \in \mathcal{A}_i \cup \mathcal{B}_i, \tag{3.28}$$

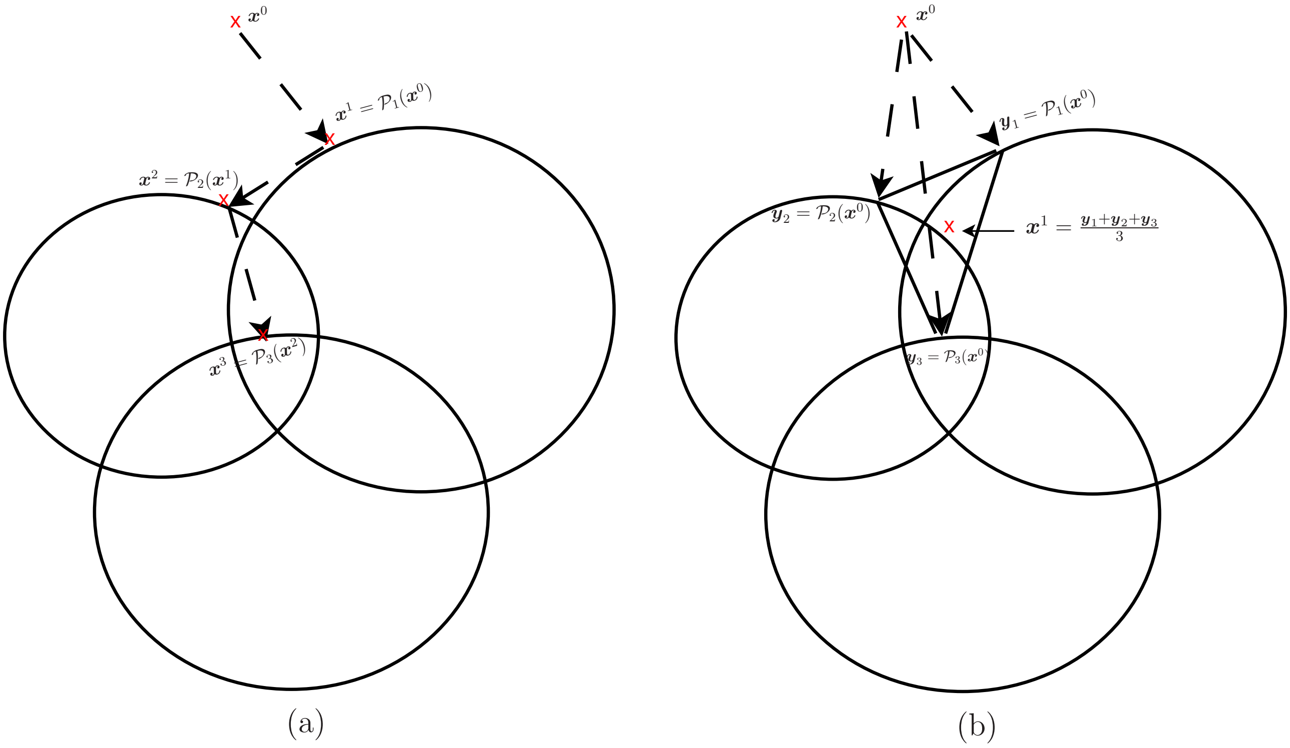

Figure 3.3: An example of sequential and parallel projections. Here, $\mathcal{P}_j$ is the projection onto the jth disc. Red crosses denote how the estimates are updated. (a) sequential projection (b) parallel projection.

where $\epsilon_l \geq 0$, $\epsilon_u \geq 0$, and the control parameter $\epsilon_l + \epsilon_u$ determines the width of the ring that can be connected to the distribution of noise (if available). We then define an estimate of the location of the target node as the following *convex-concave* feasibility problem:

$$\tilde{\boldsymbol{x}}_i \in \mathcal{R}_i \triangleq \bigcap_{j \in \mathcal{A}_i \cup \mathcal{B}_i} \mathcal{R}_{ij}, \quad \text{for } i = 1, 2, \ldots, M. \tag{3.29}$$

In fact, for zero-mean Gaussian measurement errors, it is more likely to find a target close to the boundary of the disc defined in (3.25). However, the problem defined in (3.29) is not convex and may not be easy to solve.

In the sequel, we consider the intersection of a number of discs and investigate two approaches for solving the problem, namely, one based on *projection onto convex sets* (POCS) and the second based on *outer-approximation* (OA).

3.2.1 Projections onto convex sets

The POCS technique was first introduced to solve the convex feasibility problem [79, 80]. POCS has then been applied for different problems in various fields, e.g., in image restoration problems [81–83]. There are generally two variants of projection techniques: sequential and parallel. In a sequential approach, first a set is selected among different sets, according to a rule, then the current point (estimate) is projected onto that set. Whereas, in a parallel projection approach, first a current point is projected onto different sets and then a convex combination of all projected points generates a new estimate. Fig. 3.3 shows an example of sequential and parallel projection techniques for solving a CFP consisting of three balls. The details of projection techniques are beyond the scope of this thesis, and we refer the reader to, e.g., [80].

In the positioning literature, the authors in [84, 85], for the first time, applied a POCS approach to solve a positioning problem for noncooperative scenarios. To improve the convergence of POCS, a technique was introduced in [86]. The POCS approach, then, was applied to different problems in different scenarios, e.g., [48, 87, 88].

The convergence of POCS has been extensively studied in the literature [80, 85, 89]. It has been shown that for the consistent case, i.e., with a nonempty intersection, the POCS estimate converges to a point in the intersection. For the inconsistent case, using suitable relaxation parameters, POCS converges to a point that minimizes the *sum of squared distances to the convex sets* (here a number of discs). The performance of POCS evaluated through practical data confirms theoretical claims [90, 91].

There are a few recent studies based on projection techniques for cooperative positioning. The authors in [78] proposed a technique based on projection onto the boundary of the disc determining the location of target nodes. The technique provides good estimates if suitable initial points close enough to the optimal solutions are available. Another technique based on a sequential projection approach was proposed in [77], which is a robust technique for NLOS scenarios. A new method based on parallel projection, which is amendable for distributed implementation, has been recently proposed in [76]. The authors also provide a convergence proof for the proposed algorithm.

In general, the projection-based approaches for the positioning problem are simple with medium accuracy. These techniques can provide good coarse estimates, which can be used in a refining algorithm yielding an accurate estimate.

3.2.2 Bounding the feasible set

As mentioned in the previous section, for positive measurement errors, the intersection of discs is nonempty and the location of the target node is definitely found there. Note that it is also possible to have a nonempty intersection for mixed positive and negative measurement errors, but the intersection no longer contains the location of the target node. The positive error assumption can be fulfilled in some scenarios. For example, measurement errors based on TW-TOA tend to be positive in practical situations, even for LOS conditions (as clarified in recent work on localization based on practical UWB measurements [92, 93]). In fact, for TW-TOA measurements, even if the threshold of detecting the first peak for the direct path (based on a correlation technique) is carefully adjusted, the peak corresponding to the time of arrival rarely happens before the true arrival time. For more details of this phenomenon, see [92].

The intersection in general may have any convex shape and every point in the intersection can potentially be an estimate of the target position. POCS gives one point as an estimate. In contrast to POCS, the OA approach tries to approximate the feasible set by a suitable shape and then one point inside of it is taken as an estimate, e.g., the middle of the approximated set. The main problem is how the intersection can be accurately approximated. Generally speaking, two kinds of approximations, i.e., inner-approximation and outer-approximation, have been extensively studied in the literature. In the inner-approximation family, finding the maximum-volume ellipsoid contained in an intersection of ellipsoids or the maximum-volume ellipsoid contained in a polyhedron given as a set of linear equalities are tractable problems [94]. A number of outer-approximation problems are also known to be tractable, such as the minimum-volume ellipsoid containing a polyhedron and the minimum-volume ellipsoid containing a union of ellipsoids [94]. Note that the minimum-volume ellipsoid enclosing the intersection of a number of ellipsoids seems to be an intractable problem. For noncooperative positioning problems, a number of researchers proposed to approximate the intersection by convex regions such as polytopes, ellipsoids, or discs [91, 95–97].

For cooperative networks, identifying the feasible sets is challenging since the intersection containing the location of a target may also depend on the locations of other targets at unknown positions. A few studies are available in the literature to approximate the intersection by a convex shape. The authors in [39, 77] use a heuristic approach to approximate the intersection including a target node by a disc. The approach is implemented in a distributed manner, which allows one to incorporate the technique with a distributed positioning algorithm to obtain a more accurate estimate. In Paper E, a technique based on ellipsoid outer-approximation is proposed that performs better than the one introduced in [77].

Note that if the intersection is empty, the OA approach is not directly applicable. In some scenarios, it is possible to manually modify the measurements such that the intersection becomes nonempty and contains the location of target nodes. Recent work investigates such a technique in more detail [98].

Chapter 4

Techniques for approximately solving nonconvex problems

The positioning problem, generally, involves solving a (difficult) nonconvex problem. The cooperative positioning problem poses further difficulties. In this chapter, we study a number of techniques to approximately solve the nonconvex problem. The study here is a brief review of the literature, and for more details we refer the reader to suitable references.

4.1 Relaxation techniques

We consider the quadratically constrained quadratic programming (QCQP), which is used to formulate a wide range of applications. Moreover, many other types of optimization problems can be cast as a QCQP.

Let us consider a QCQP as

$$\begin{aligned} &\underset{\boldsymbol{x}\in\mathbb{R}^n}{\text{minimize}}\ \boldsymbol{x}^T\boldsymbol{A}_0\boldsymbol{x} + 2\boldsymbol{b}_0^T\boldsymbol{x} + c_0 \\ &\text{subject to}\ \boldsymbol{x}^T\boldsymbol{A}_i\boldsymbol{x} + 2\boldsymbol{b}_i^T\boldsymbol{x} + c_i \le 0, \quad i = 1,\dots,N \end{aligned} \tag{4.1}$$

for $\boldsymbol{A}_i \in \mathbb{S}^n$, with $\mathbb{S}^n$ denoting the set of n by n symmetric matrices, $\boldsymbol{b}_i \in \mathbb{R}^n$, and $c_i \in \mathbb{R}$. The constraints in the QCQP problem includes the equality constraints, since an equality constraint can be expressed based on two inequalities. When all symmetric matrices $\boldsymbol{A}_i,\ i = 0,\dots,N$ are positive (semi)definite, the problem (4.1) is convex, which can be solved efficiently using, e.g., the interior point method. Here, we assume that at least one $\boldsymbol{A}_i$ is not positive (semi)definite. Except in rare cases, which will be discussed later, the nonconvex QCQP is difficult to solve. The nonconvex QCQP problem is, generally, claimed to be an NP hard problem, but there is no rigorous proof for the claim in the literature.

We consider two techniques to (approximately) solve the problem. In fact, we first change the problem to a convex problem, which can be solved efficiently. We introduce a fact for quadratic function, which will be used later. For a proof of the claim in Lemma 1, see, e.g., [94, 99].

LEMMA 1 *A quadratic function* $f(\boldsymbol{x}) = \boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x} + 2\boldsymbol{b}^T\boldsymbol{x} + c$, *with symmetric* n *by* n *matrix* $\boldsymbol{A}$ *is always nonnegative for all* $\boldsymbol{x} \in \mathbb{R}^n$ *if and only if*

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{b} \\ \boldsymbol{b}^T & c \end{bmatrix} \succeq 0. \tag{4.2}$$

4.1.1 Semidefinite relaxations

For nonconvex QCQP in (4.1), we can employ a relaxation technique and obtain a semidefinite programming problem (SDP) as follows. Let us rewrite the problem in (4.1) as [94]

$$\begin{aligned} &\underset{\boldsymbol{x}\in\mathbb{R}^n}{\text{minimize}} \ \operatorname{tr}(\boldsymbol{A}_0\boldsymbol{x}\boldsymbol{x}^T) + 2\boldsymbol{b}_0^T\boldsymbol{x} + c_0 \\ &\text{subject to } \operatorname{tr}(\boldsymbol{A}_i\boldsymbol{x}\boldsymbol{x}^T) + 2\boldsymbol{b}_i^T\boldsymbol{x} + c_i \le 0, \qquad i = 1, \ldots, N. \end{aligned} \tag{4.3}$$

Now, by replacing $\boldsymbol{Z} = \boldsymbol{x}\boldsymbol{x}^T$ and then relaxing it as $\boldsymbol{Z} \succeq \boldsymbol{x}\boldsymbol{x}^T$, we obtain an SDP as

$$\begin{aligned} &\underset{\boldsymbol{x}\in\mathbb{R}^n,\ \boldsymbol{Z}\in\mathbb{S}^n}{\text{minimize}} \ \operatorname{tr}(\boldsymbol{A}_0\boldsymbol{Z}) + 2\boldsymbol{b}_0^T\boldsymbol{x} + c_0 \\ &\text{subject to } \operatorname{tr}(\boldsymbol{A}_i\boldsymbol{Z}) + 2\boldsymbol{b}_i^T\boldsymbol{x} + c_i \le 0, \qquad i = 1, \ldots, N \\ &\qquad\qquad \begin{bmatrix} \boldsymbol{Z} & \boldsymbol{x} \\ \boldsymbol{x}^T & 1 \end{bmatrix} \succeq 0. \end{aligned} \tag{4.4}$$

For details of the relaxation technique, see, e.g., [66, 99]. Using the Schur complement [94], we expressed the constraint $\boldsymbol{Z} \succeq \boldsymbol{x}\boldsymbol{x}^T$ as a linear matrix inequality in (4.4). By adopting the relaxation in (4.4), we expand the feasible set therefore, the objective function in (4.4) is minimized over a larger set than in (4.1), meaning the optimal value in (4.4) gives a lower bound on the optimal value in (4.1). If the optimal solution $\boldsymbol{Z}^*$ is rank-1, the optimal solution of the problem in (4.4) is at hand. For example, we can use Cholesky factorization of $\boldsymbol{Z}^*$ and obtain the optimal solution. Otherwise, as it happens in most cases, we need to extract a rank-1 approximation from a higher rank matrix [66]. Among different approaches, we discuss two techniques based on singular value decomposition and randomization.

SVD approach: Let us decompose the rank-r ($r > 1$) matrix $\boldsymbol{Z}^*$ using the eigen-decomposition as

$$\boldsymbol{Z}^* = \sum_{i=1}^{r} \lambda_i \boldsymbol{y}_i \boldsymbol{y}_i^T, \tag{4.5}$$

where $\lambda_1 \geq \cdots \geq \lambda_r > 0$ are eigenvalues of the semidefinite matrix $\boldsymbol{Z}^*$ and $\boldsymbol{y}_1, ..., \boldsymbol{y}_r$ are corresponding eigenvectors. The best rank-1 approximation in the least two-norm sense is given by $\tilde{\boldsymbol{Z}} = \lambda_1\boldsymbol{y}_1\boldsymbol{y}_1^T$. Thus, we can get $\hat{\boldsymbol{x}} = \sqrt{\lambda_1}\,\boldsymbol{y}_1$ as an estimate of the target position.

Randomization: Here, we study another technique based on randomization, which often provides good solutions. That is, we first generate enough samples $\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_K$ from a multivariate Gaussian random vector, $\boldsymbol{\xi}$, with zero mean vector and covariance matrix $\boldsymbol{Z}^*$, i.e., $\boldsymbol{\xi} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{Z}^*)$. Therefore, on average, the sample vector $\boldsymbol{\xi}_k$ is the optimal solution to (4.3). We choose samples that satisfy the constraint $\operatorname{tr}(\boldsymbol{A}_i\boldsymbol{\xi}_\ell\boldsymbol{\xi}_\ell^T) + 2\boldsymbol{b}_i^T\boldsymbol{\xi}_\ell + c_i \le 0,\ i =$

$1, \ldots, N$, say samples $\boldsymbol{\xi}_{i_1}, \ldots, \boldsymbol{\xi}_{i_K}$ (we can also project samples that lie outside the feasible set onto the feasible set). We then evaluate the objective function in (4.3) for samples $\boldsymbol{\xi}_{i_1}, \ldots, \boldsymbol{\xi}_{i_K}$ and pick the one that minimizes the objective function, say, sample jth, $\boldsymbol{\xi}_j$. Then a near optimal rank-1 solution is derived as $\boldsymbol{x}^* = \boldsymbol{\xi}_j$ and the near optimal value of the cost function is computed as $\operatorname{tr}(\boldsymbol{A}_0\boldsymbol{x}^*\boldsymbol{x}^{*T}) + 2\boldsymbol{b}_0^T\boldsymbol{x}^* + c_0$.

For details about rank-1 approximation in an SDP relaxation approach, see, e.g., [66, 99–101] and references therein.

4.1.2 Lagrangian relaxations

Another approach to solve the QCQP problem (4.1) is to use the Lagrangian relaxation approach. Let us define the dual function as

$$f_d(\boldsymbol{x}, \boldsymbol{\lambda}) \triangleq \boldsymbol{x}^T\boldsymbol{A}_\lambda\boldsymbol{x} + 2\boldsymbol{b}_\lambda^T\boldsymbol{x} + c_\lambda, \tag{4.6}$$

where

$$\begin{aligned}
\boldsymbol{\lambda} &\triangleq [\lambda_1 \cdots \lambda_N]^T, \quad \lambda_i \geq 0 \\
\boldsymbol{A}_\lambda &\triangleq \boldsymbol{A}_0 + \sum_{i=1}^N \lambda_i\boldsymbol{A}_i \\
\boldsymbol{b}_\lambda &\triangleq \boldsymbol{b}_0 + \sum_{i=1}^N \lambda_i\boldsymbol{b}_i \\
c_\lambda &\triangleq c_0 + \sum_{i=1}^N \lambda_i c_i.
\end{aligned} \tag{4.7}$$

Assume that there is an $\alpha \in \mathbb{R}$ such that

$$f_d(\boldsymbol{x}, \boldsymbol{\lambda}) - \alpha \geq 0, \quad \forall \boldsymbol{x} \in \mathbb{R}^n,\ \lambda_i \geq 0. \tag{4.8}$$

In other words, α is a lower bound on the optimal value of the QCQP problem (4.1). Using the result of Lemma 1, the inequality in (4.8) holds true if and only if

$$\begin{bmatrix} \boldsymbol{A}_\lambda & \boldsymbol{b}_\lambda \\ \boldsymbol{b}_\lambda & c_\lambda - \alpha \end{bmatrix} \succeq 0. \tag{4.9}$$

In order to obtain a tight lower bound on the optimal value of the QCQP problem (4.1), we solve the following SDP problem:

$$\begin{aligned}
&\underset{\boldsymbol{\lambda} \in \mathbb{R}_+^N,\ \alpha \in \mathbb{R}}{\text{maximize}} \ \alpha \\
&\text{subject to} \begin{bmatrix} \boldsymbol{A}_\lambda & \boldsymbol{b}_\lambda \\ \boldsymbol{b}_\lambda & c_\lambda - \alpha \end{bmatrix} \succeq 0.
\end{aligned} \tag{4.10}$$

The SDP approaches in (4.4) and (4.10) provide lower bounds on the optimal value of the QCQP problem (4.1). The problems (4.4) and (4.10) are dual of each other and hence, the lower bounds are the same assuming a constraint qualification holds true [94, 102].

4.1.3 General trust region subproblem

There is a specific nonconvex QCQP in which the relaxation approach provides the optimal solution, which is a QCQP with a single constraint. In this section, we consider a QCQP with a single equality constraint and review a simple technique to solve the problem with complexity lower than an SDP-based approach. Let us consider a QCQP problem with one equality constraint as

$$\begin{aligned} &\underset{\boldsymbol{x}\in\mathbb{R}^n}{\text{minimize}} \;\; \boldsymbol{x}^T\boldsymbol{A}_0\boldsymbol{x} + 2\boldsymbol{b}_0^T\boldsymbol{x} + c_0 \\ &\text{subject to } \boldsymbol{x}^T\boldsymbol{A}_1\boldsymbol{x} + 2\boldsymbol{b}_1^T\boldsymbol{x} + c_1 = 0. \end{aligned} \tag{4.11}$$

The problem in (4.11) minimizes a quadratic function over a quadratic constraint. This type of problem is called a generalized trust region subproblem [103]. It is known that the general trust region subproblem has no duality gap and the optimal solution can be extracted from the dual solution [103–105]. A necessary and sufficient condition for $\boldsymbol{x}^*$ to be optimal in (9) is that [104]

$$\begin{aligned} &(\boldsymbol{A}_0 + \gamma\boldsymbol{A}_1)\boldsymbol{x}^* + (\boldsymbol{b}_0 + \gamma\boldsymbol{b}_1) = 0, \\ &(\boldsymbol{x}^*)^T\boldsymbol{A}_1\boldsymbol{x}^* + 2\boldsymbol{b}_1^T\boldsymbol{x}^* + c_1 = 0, \\ &(\boldsymbol{A}_0 + \gamma\boldsymbol{A}_1) \succ 0. \end{aligned} \tag{4.12}$$

Under conditions considered in (4.12), the solution to the problem of (4.11) is given by

$$\boldsymbol{x}(\gamma) = -(\boldsymbol{A}_0 + \gamma\boldsymbol{A}_1)^{-1}(\boldsymbol{b}_0 + \gamma\boldsymbol{b}_1). \tag{4.13}$$

In such a situation, to find γ, we replace (4.13) into constraint $\boldsymbol{x}^T\boldsymbol{A}_1\boldsymbol{x} + 2\boldsymbol{b}_1^T\boldsymbol{x} + c_1 = 0$, i.e.,

$$\phi(\gamma) = \boldsymbol{x}^T(\gamma)\boldsymbol{A}_1\boldsymbol{x}(\gamma) + 2\boldsymbol{b}_1^T\boldsymbol{x}(\gamma) + c_1 = 0, \quad \gamma \in \mathcal{I}, \tag{4.14}$$

where the interval $\mathcal{I}$ consists of all γ such that $\boldsymbol{A}_0 + \gamma\boldsymbol{A}_1 \succeq 0$. From (4.14) and (4.13), and considering (4.12), a simple iterative approach can be employed to solve the general trust region subproblem. For details about the approach, see, e.g., [60, 106].

4.2 Majorization minimization approach

For approximately solving a nonconvex problem, one technique is to find a bound on the nonconvex function and then optimize the bound. The approach lies in a well-known class of optimization, so-called majorization minimization (MM) [107]. In the MM approach, the optimization is performed over a surrogate function of the original function, which might be difficult to optimize. The well-known expectation-maximization (EM) algorithm is a special type of the MM technique. The surrogate function can have any shape, but here we focus on a convex surrogate function (or concave surrogate function for a maximization problem).

Definition: consider a function $g(\boldsymbol{x}, \boldsymbol{y})$, $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$. We say the real function $g(\boldsymbol{x}, \boldsymbol{y})$ majorizes a real function $f(\boldsymbol{x})$ at $\boldsymbol{y}$ if

$$\begin{aligned} &f(\boldsymbol{x}) \le g(\boldsymbol{x}, \boldsymbol{y}), \;\; \forall \boldsymbol{x} \in \mathbb{R}^n, \\ &f(\boldsymbol{x}) = g(\boldsymbol{x}, \boldsymbol{x}). \end{aligned} \tag{4.15}$$

Suppose that the minimum of function $g(\boldsymbol{x}, \boldsymbol{x}^k)$ happens at $\boldsymbol{x} = \boldsymbol{x}^{k+1}$. From the property of the surrogate function, we can write

$$\begin{aligned} f(\boldsymbol{x}^{k+1}) &\leq g(\boldsymbol{x}^{k+1}, \boldsymbol{x}^k) \\ &\overset{(a)}{\leq} g(\boldsymbol{x}^k, \boldsymbol{x}^k) = f(\boldsymbol{x}^k), \end{aligned} \tag{4.16}$$

where (a) follows from the fact that $\boldsymbol{x}^{k+1}$ is the minimizer of $g(\boldsymbol{x}, \boldsymbol{x}^k)$, i.e., $g(\boldsymbol{x}^{k+1}, \boldsymbol{x}^k) \leq g(\boldsymbol{x}, \boldsymbol{x}^k),\ \forall \boldsymbol{x}$.

If this procedure continues, decreasing sequences $f(\boldsymbol{x}^0) \geq f(\boldsymbol{x}^1) \geq \cdots \geq f(\boldsymbol{x}^k)$ can be obtained.

The MM technique has been considered in a few studies in the positioning literature, see, e.g., [108, 109].

4.3 Difference of convex functions programming

Let us consider a class of problem in the form of difference of convex functions programming (DCP) as follows [110]:

$$\begin{aligned} &\underset{\boldsymbol{x} \in \mathbb{R}^n}{\text{minimize}}\ \ f_0(\boldsymbol{x}) - g_0(\boldsymbol{x}) \\ &\text{subject to } f_i(\boldsymbol{x}) - g_i(\boldsymbol{x}) \leq 0,\ \ i = 1, \ldots, M, \end{aligned} \tag{4.17}$$

where $f_i(\boldsymbol{x})$ and $g_i(\boldsymbol{x})$ are both smooth convex functions for $i = 1, \ldots, M$. We consider a sequential approach to solve (4.17). That is, we first approximate the concave function $(-g_i(\boldsymbol{x}))$ with a convex one by an affine approximation. In fact, the approach for solving the DCP can be considered as a special case of the MM technique. Let us consider a point $\boldsymbol{x}^j$ in the domain of the problem in (4.17), linearize the concave function around $\boldsymbol{x}^j$ and write the optimization problem in (4.17) as

$$\begin{aligned} &\underset{\boldsymbol{x} \in \mathbb{R}^n}{\text{minimize}}\ \ f_0(\boldsymbol{x}) - g_0(\boldsymbol{x}^j) - \bigtriangledown g_0(\boldsymbol{x}^j)^T(\boldsymbol{x} - \boldsymbol{x}^j) \\ &\text{subject to }\ \ f_i(\boldsymbol{x}) - g_i(\boldsymbol{x}^j) - \bigtriangledown g_i(\boldsymbol{x}^j)^T(\boldsymbol{x} - \boldsymbol{x}^j) \leq 0. \end{aligned} \tag{4.18}$$

The convex problem in (4.18) can now be solved efficiently. Denote the solution of (4.18) as $\boldsymbol{x}^{j+1}$. Next, we further improve the solution by convexifying (4.17) for the new point $\boldsymbol{x}^{j+1}$ similar to the procedure employed for $\boldsymbol{x}^j$. This sequential programming procedure, called concave-convex programming (CCCP), continues for a number of iterations. The convergence of the CCCP to a stationary point has been shown in the literature, e.g., [110, 111] and references therein. Fig. 4.1 illustrates an example of the unconstrained CCCP approach for a DCP.

For an application of the CCCP approach to positioning problems, see [112].

4.4 Other techniques

In the literature, a number of techniques have been proposed to solve nonconvex problems. The details of such techniques are beyond the scope of this thesis, and we cite only some well-known approaches.

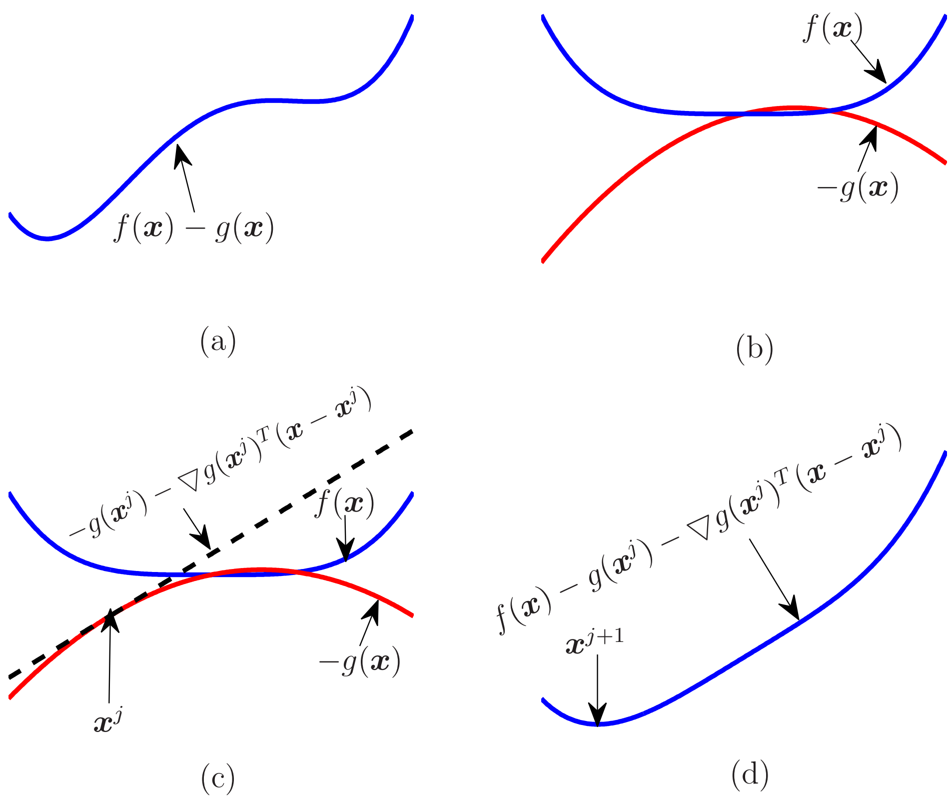

Figure 4.1: An example of difference of convex programming. (a) original function as the difference of two convex functions, (b) separating the difference of convex functions of (a) into a convex and a concave function, (c) linearizing the concave function $-g(\boldsymbol{x})$ around $\boldsymbol{x}^j$, and (d) a convex approximation of the function in (a) around $\boldsymbol{x}^j$. The minimum of the approximated function appears in $\boldsymbol{x}^{j+1}$. The same procedures are repeated for the new point at $\boldsymbol{x}^{j+1}$.

In some problems, a nonconvex problem can be transformed to a convex programming by adopting a suitable transformation [113]. For example, a class of nonconvex problems, so-called *geometric programming* (GP) with vast applications in different fields, can be efficiently solved, and the global solution can be obtained with polynomial time complexity. For details about the GP, see, e.g., [114, 115].

For some nonconvex polynomial programming problems, the problem can be approximately solved by an approach called the *sum of squares* (SOS) [116, 117]. In the SOS technique, the polynomial function is transformed to a convex quadratic function using some relaxation techniques and the resulting problem can be efficiently solved. Such a technique has also been applied to the positioning problem [58]. The SDP approach for solving the QCQP, discussed earlier, can be considered as a special case of the SOS approach.

In addition to the techniques proposed to approximately solve nonconvex problems, a technique based on *branch and bound* can be used to solve the nonconvex problem. It provides a provable upper and lower bound on the optimal solution [118]. The approach generally has slow convergence speed, but in some scenarios the algorithm converges with reasonable complexities. For details about the approach see, e.g., [119].

Chapter 5

Performance evaluation

A positioning algorithm provides an estimate of the target location, with respect to a coordinate system, based on the measurements made between sensor nodes. It is common in the literature to evaluate the position estimate through a statistical metric, e.g., a lower bound on the mean-square-error, or a metric based on the geometry of the networks, e.g., the Geometric Dilution of Precision (GDOP) metric [6, 14]. The position estimate can also be evaluated for the worst-case position error. The worst-case position error can also be used to efficiently design location-based services for different applications.

The sensor network may consist of a few or many sensor nodes [6]. Thus, one way to assess algorithms is to consider whether an algorithm designed for a small network can be extended to a large network, often referred to as scalability. For example, in centralized processing, a proposed algorithm for a small network can be extended to a large network with more complexity, while for the distributed version, answering this question is not straightforward.

In this section, we study a number of metrics for evaluating the performance of a positioning algorithm. Some metrics have been studied in detail in [37], and here we briefly review different parameters. As discussed in [37], in the literature there is no unique criterion to compare and evaluate various approaches (to the best of our knowledge).

5.1 Metrics on position errors

A positioning algorithm takes measurements made in sensor nodes, and the location of the reference nodes, and provides an estimate of the target position. In this thesis, we consider the target location as an unknown deterministic vector, i.e., there is no *a priori* information about the location. If some *a priori* knowledge about the target position is available, we may need to modify the discussion in this section such that the prior information is properly used in the derivation of performance metrics.

5.1.1 Position errors

Suppose a positioning algorithm provides estimates of the location of M targets as $\hat{\boldsymbol{x}}_i,\ i = 1, \ldots, M$. The error for the position estimate is defined as follows:

$$\boldsymbol{e}_i \triangleq \hat{\boldsymbol{x}}_i - \boldsymbol{x}_i, \quad i = 1, \ldots, M, \tag{5.1}$$

where $\boldsymbol{x}_i$ is the true location of the ith target. Due to randomness in measurement errors or network deployment, the vector $\boldsymbol{e}_i$ is a random vector. There are different ways to study the performance of a positioning algorithm through the error $\boldsymbol{e}_i$. In this thesis, we define the *position error* as the Euclidean norm of the error $\boldsymbol{e}_i$, i.e., $\|\boldsymbol{e}_i\|$, and study different functions of the position error.

We say a position algorithm is unbiased if $\mathbb{E}(\boldsymbol{e}_i) = 0$, that is, the position estimate on average tends to the true position. It is noted that in practice the actual position error can not be computed since the true location is unknown in advance (although unbiasedness can help to find the true location).

5.1.2 The Cramér-Rao lower bound

As mentioned in the previous section, the position error is a random variable. One way to assess the accuracy of the position estimation is to establish a lower bound on the mean-squared error $\mathbb{E}\|\boldsymbol{e}_i\|^2$. There are different lower bounds in the literature, which can be used to evaluate a positioning algorithm, such as the Ziv-Zakai bound [120]. Among different lower bounds, the Cramér-Rao lower bound (CRLB) is by far the simplest bound to compute. The CLRB can be computed if the PDF of the measurements is known and satisfies some regularity conditions [50]. The CRLB is a lower bound on the variance of any unbiased estimator. If the estimator is biased, it is not necessarily bounded by the CRLB and other bounds should be considered [121, 122].

Suppose that the PDF of measurements in (2.3) is given by $p_i(\{m_{ij}\}_{j\in\mathcal{A}_i\cup\mathcal{B}_i};\boldsymbol{\theta}),\ i = 1,\ldots,M$. Assuming i.i.d. measurements, the PDF of the complete set of measurements is given by the product of each PDF, i.e.,

$$p(\boldsymbol{m};\boldsymbol{\theta}) = \prod_{i=1}^{M} p_i(\{m_{i,j}\}_{j\in\mathcal{A}_i\cup\mathcal{B}_i};\boldsymbol{\theta}), \tag{5.2}$$

where $\boldsymbol{m} \triangleq \{\{m_{ij}\}_{j\in\mathcal{A}_1\cup\mathcal{B}_1}\ \cdots\ \{m_{Mj}\}_{j\in\mathcal{A}_M\cup\mathcal{B}_M}\}$ and $\boldsymbol{\theta} \triangleq [\boldsymbol{x}_1^T\ \cdots\ \boldsymbol{x}_M^T]^T$.

Hence, the Fisher Information Matrix (FIM) can be computed as [50]

$$\boldsymbol{F}(\boldsymbol{\theta}) \triangleq \mathbb{E}\{[\triangledown_{\boldsymbol{\theta}} \ln p(\boldsymbol{m};\boldsymbol{\theta})][\triangledown_{\boldsymbol{\theta}} \ln p(\boldsymbol{m};\boldsymbol{\theta})]^T\}, \tag{5.3}$$

where $\triangledown_{\boldsymbol{\theta}}$ denotes partial derivatives with respect to vector $\boldsymbol{\theta}$.

For the FIM computed in (5.3), we can find a lower bound on the variance of any unbiased estimator $\hat{\boldsymbol{\theta}}$ as [50]

$$\mathrm{var}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \geq \mathrm{Tr}\{\boldsymbol{F}(\boldsymbol{\theta})^{-1}\}. \tag{5.4}$$

As an example, the CRLB for the position estimate based on the RSS measurements for single target node at position $\boldsymbol{x} = \boldsymbol{x}_1$ (assuming known path-loss and transmission power) can be computed as

$$\mathbb{E}\|\hat{\boldsymbol{x}} - \boldsymbol{x}\|^2 \geq \frac{F_{11} + F_{22}}{F_{11}F_{22} - F_{12}^2}, \tag{5.5}$$

where

$$F_{ii} = \left(\frac{10\beta}{\sigma_{\text{dB}}^2 \ln 10}\right)^2 \sum_{j\in\mathcal{A}_1} \frac{(x_i - a_{j,i})^2}{d(\boldsymbol{x}, \boldsymbol{a}_j)^4}, \quad i = 1, 2$$

$$F_{12} = \left(\frac{10\beta}{\sigma_{\text{dB}}^2 \ln 10}\right)^2 \sum_{j\in\mathcal{A}_1} \frac{(x_1 - a_{j,1})(x_2 - a_{j,2})}{d(\boldsymbol{x}, \boldsymbol{a}_j)^4}, \tag{5.6}$$

where σ_{dB}^2 is the variance of shadowing, which is assumed to be the same for different links for simplicity.

The CRLB determines a lower bound on the variance of any unbiased estimator and can be used as a benchmark for evaluating the position algorithms. To compute the CRLB, we often need to know the true positions. In evaluating the positioning algorithm, we may be interested in studying the behavior of the positioning error or a function of positioning errors, not necessary the average.

Note that if there is *a priori* information about the locations of unknown targets, the CRLB may not be a bound. One possibility for deriving a lower bound, when there is *a priori* information, is the Bayesian CRLB [123], which considers the prior distribution about the locations of targets as well as the distribution of the measurement errors.

5.1.3 Cumulative distribution function

One way to evaluate the performance of an algorithm is to investigate the PDF or CDF of the position error $\|\boldsymbol{e}_i\|$. Suppose a positioning algorithm generates K position estimates for every target node, i.e., $\hat{\boldsymbol{x}}_i^k,\ k = 1, \ldots, K$, as the kth estimate of the location of target i. Defining the position error for the kth estimate of the location of target node i as $\|\boldsymbol{e}_i^k\|$, the empirical CDF of the position error can be calculated as

$$\text{CDF}(\gamma) = \Pr(\text{position error} \leq \gamma) = \frac{\sum_{k=1}^{K}\sum_{m=1}^{M} I(\|\boldsymbol{e}_i^k\| - \gamma)}{KM}, \tag{5.7}$$

where the function $I(t)$ is defined as

$$I(t) = \begin{cases} 1, & \text{if } t \geq 0, \\ 0, & \text{otherwise.} \end{cases} \tag{5.8}$$

The CDF gives more insight into the performance of positioning algorithms than, for instance, root-mean-square-error, which gives one value. For example, two different algorithms may have relatively different performances for different error intervals. One algorithm might be superior for small errors while the other may perform better for medium errors. Let us consider Fig. 5.1, which shows the CDF of the position error for Algorithms 1 and 2. It is observed that different algorithms have different behaviors. For example, 40% of the time, the position error produced by Algorithm 2 does not exceed 16 cm, while Algorithm 1 yields estimates with position errors no larger than 20 cm. As another interpretation, let us fix the maximum position error to be 10 cm. Algorithm 1 satisfies this constraint 20% of the time, whereas Algorithm 2 provides estimates with position errors less than 10 cm 30% of the time.

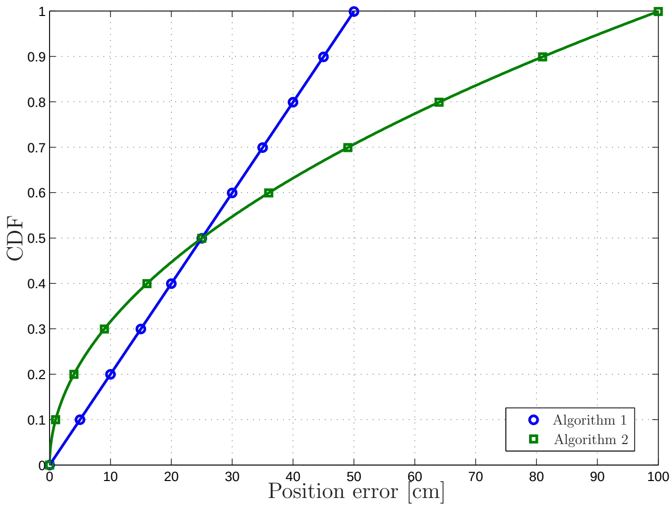

Figure 5.1: Comparison between two CDFs of the position error.

5.1.4 The worst-case position error

In this section, the realization of an error is discussed, not its statistical properties (PDF, CDF, or MSE). In some applications, it may be necessary to know the worst-case position error on position estimates. If there are K estimates available for every target location, we can define the maximum position error for target node i as follows [124]:

$$\text{Max-Error}_i \triangleq \max_{k=1,\ldots,K} \|\hat{\boldsymbol{x}}_i^k - \boldsymbol{x}_i\|, \quad i = 1, 2, \ldots, M. \tag{5.9}$$

It is clear that to find the worst-case position error we need to know the true location, which is a drawback in practical applications. Instead of evaluating the position error, we try to find an upper bound on the position error. Namely, we assume that the target node i belongs to a closed bounded set, say $\mathcal{B}_i$, and determine the maximum position error with respect to the feasible set $\mathcal{B}_i$. In fact, using this technique, we can find an upper bound on the position error of every single position estimate. We define an upper bound on a single position error $\hat{\boldsymbol{x}}_i$ as follows:

$$\|\boldsymbol{e}_i^k\| \leq \max_{\boldsymbol{x}_i \in \mathcal{B}_i} \|\hat{\boldsymbol{x}}_i^k - \boldsymbol{x}_i\|. \tag{5.10}$$

In general, determining the feasible set $\mathcal{B}_i$ is not easy; however, there are situations in which a feasible set containing the location of the target node can be quantified from the measurements, e.g., if all measurement errors are positive. Note that the tightness of the bound depends on the feasible set and also on the accuracy of the estimate.

In general, the upper bound in (5.10) is difficult to compute since the corresponding optimization problem is a difficult nonconvex problem, especially for 3-dimensional networks. We can, however, approximately solve the problem using, e.g., a relaxation technique.

As an example, consider Fig. 5.2, illustrating an estimate, $\hat{\boldsymbol{x}}_i$, of the location of a target at unknown location $\boldsymbol{x}_i$ using three reference nodes. When all distance estimates have positive errors, the intersection of the three discs centered at the location of the reference nodes with radii $\hat{d}_i$, $i = 1, 2, 3$, determines a closed bounded set containing the location

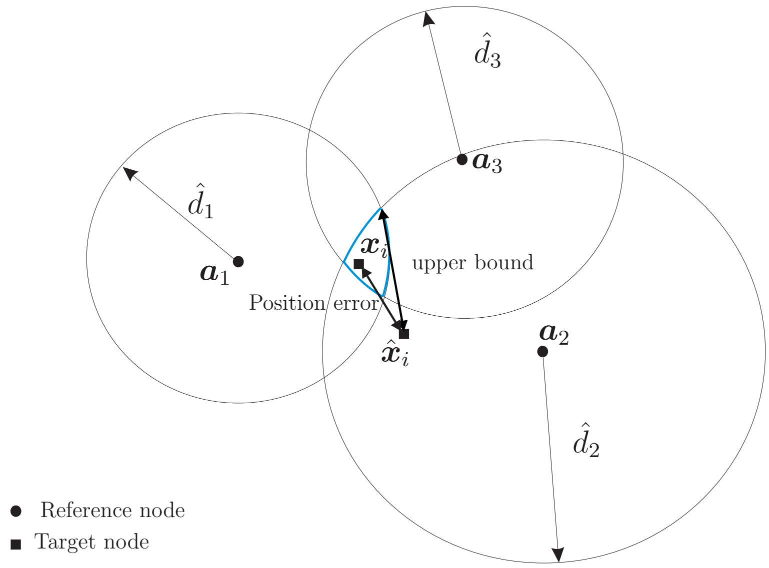

Figure 5.2: The position error and an upper bound on the position error for an estimate $\hat{\boldsymbol{x}}_i$ of a target node at location $\boldsymbol{x}_i$ for a network consisting of three reference nodes. In this figure, it is assumed that the measurement errors are positive.

of the target node. Now, an upper bound on the position error can be computed with respect to the feasible set.

5.1.5 The circular error probable (CEP)

Let us consider an estimate of the location of target i as $\hat{\boldsymbol{x}}_i$ with the mean of the estimate as $\mathbb{E}\,\hat{\boldsymbol{x}}_i$. The CEP is defined as the radius of a disc centered at $\mathbb{E}\,\hat{\boldsymbol{x}}_i$ that contains half of the realizations of the location estimates [125]. If the estimator is unbiased, the CEP gives a measure of the estimator uncertainty [13]. To provide some insight, the geometry of the CEP is shown in Fig. 5.3. It simply indicates that with probability 0.5, the distance between an estimate $\hat{\boldsymbol{x}}_i$ and the true location $\boldsymbol{x}_i$ is less than $\|\boldsymbol{x}_i - \mathbb{E}\,\hat{\boldsymbol{x}}_i\| +$ CEP.

Suppose that the PDF of the estimate $\hat{\boldsymbol{x}}_i$ is denoted by $q(\boldsymbol{x})$. Then, the CEP can be obtained by solving the following equation [13, 125]:

$$0.5 = \int_S q(\boldsymbol{y}) d\boldsymbol{y}, \tag{5.11}$$

where S is defined as

$$S \triangleq \{\boldsymbol{x} : \|\boldsymbol{x} - \mathbb{E}\,\hat{\boldsymbol{x}}_i\| \leq \text{CEP}\}. \tag{5.12}$$

In general, there is no closed-form solution for the integral in (5.11) and we need to solve it numerically. A common approximation for the solution in (5.11) is given by [13, Ch. 3]

$$\text{CEP} \simeq 0.75\sqrt{\text{Tr}\{\mathbb{E}(\hat{\boldsymbol{x}}_i - \mathbb{E}\,\hat{\boldsymbol{x}}_i)(\hat{\boldsymbol{x}}_i - \mathbb{E}\,\hat{\boldsymbol{x}}_i)^T\}}. \tag{5.13}$$

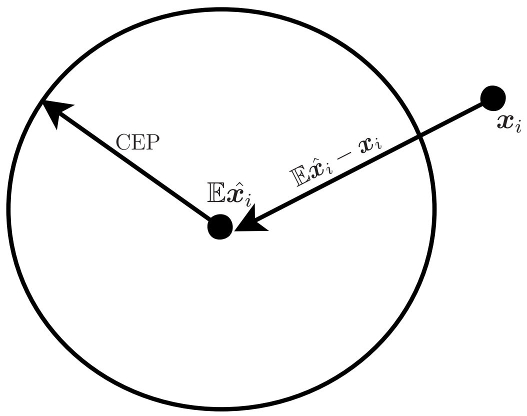

Figure 5.3: The geometry of CEP metric.

5.1.6 Geometric dilution of precision

To evaluate the effect of the geometry on the position estimate, the GDOP measure is often used in the positioning literature. GDOP generally shows how much the distance estimate errors are magnified through the geometry of the network [13]. Consider an estimate of the location of target i as $\hat{\boldsymbol{x}}_i$ and assume that the variance of the measurement errors is σ^2. The GDOP is defined as [6]

$$\mathrm{GDOP} \triangleq \sqrt{\frac{\mathbb{E}(\hat{\boldsymbol{x}}_i - \mathbb{E}\hat{\boldsymbol{x}}_i)^T(\hat{\boldsymbol{x}}_i - \mathbb{E}\hat{\boldsymbol{x}}_i)}{\sigma^2}}. \tag{5.14}$$

A large GDOP indicates that the network deployment is not appropriate for achieving a high degree of accuracy in the positioning context. GDOP can be written in terms of CEP as [13]

$$\mathrm{GDOP} \simeq 0.75\sigma\mathrm{CEP}. \tag{5.15}$$

5.1.7 Frobenius metric (FROB)

Suppose that the distance $\tilde{d}_{ij}$ is the distance between a located target i, i.e., $\hat{\boldsymbol{x}}_i$, and node j, i.e., $\tilde{d}_{ij} = \|\hat{\boldsymbol{x}}_i - \boldsymbol{a}_j\|,\ j \in \mathcal{A}_i$ and $\tilde{d}_{ij} = \|\hat{\boldsymbol{x}}_i - \hat{\boldsymbol{x}}_j\|,\ j \in \mathcal{B}_i$. The FROB, which has been considered as a method for evaluating positioning algorithms in the literature, is defined as [126]

$$\mathrm{FROB} = \sqrt{\frac{1}{\sum_{i=1}^{M} |\mathcal{A}_i \cup \mathcal{B}_i|} \sum_{i=1}^{M} \sum_{j \in \mathcal{A}_i \cup \mathcal{B}_i} (\tilde{d}_{ij} - d_{ij})^2}, \tag{5.16}$$

where $|\mathcal{X}|$ denotes the cardinality of set $\mathcal{X}$, and d_{ij} is the actual distance between target i and node j.

In all accuracy metrics described above except the upper bound on position error in (5.10), we need to know the position of the target to compute performance metrics. For instance, to compute the CDF of the position error, we should subtract the target's estimated position from true locations of target nodes. In simulation scenarios, the true location is available, making the computation of different metrics straightforward. However, in practice such prior knowledge of the target location is not available in advance. In fact, the geometry of the network is not initially known. In such a scenario, the accuracy metric should be defined regardless of the geometry of the network, i.e., independent of the location of targets. In [127] a metric based on average distance error (ADE), which can be considered as an accuracy measure, was defined as

$$\text{ADE} = \frac{1}{\sum_{i=1}^{M} |\mathcal{A}_i \cup \mathcal{B}_i|} \sum_{i=1}^{M} \sum_{j \in \mathcal{A}_i \cup \mathcal{B}_i} (\hat{d}_{ij} - \tilde{d}_{ij}), \tag{5.17}$$

where $\tilde{d}_{ij}$ is the distance between node i and j after positioning, and $\hat{d}_{ij}$ is the observed (measured) distance.

5.2 Other metrics

Different metrics defined in the previous section are, generally, defined based on position errors. In this section, we consider two other important metrics in evaluating position algorithms: cost and coverage.

Cost metrics: As is clear from the name of the metric, the cost metric determines the cost of implementing an algorithm. To study the cost metric, the following items are commonly considered [6]:

- **Reference-to-node ratio** is defined as the number of reference nodes divided by the number of sensor nodes in a network. It is commonly used to investigate the trade-off on accuracy of algorithms. For instance, it determines how the accuracy of an algorithm changes if the percentage of reference nodes decreases [6];

- **Communication overhead** is defined as the number of packets transmitted or the actual power consumed to reach the positioning goal;

- **Power consumption** determines the lifetime of a sensor node. Power consumption is a combination of the power required to perform local processing, e.g., the task of a sensor node for the distributed processing, and the power used to send and receive packets;

- **Algorithm complexity** determines the computational complexity in time and space for implementing an algorithm;

- **Convergence time** is defined based on both the time required to gather measurements and the time needed for a positioning algorithm to converge.

Coverage metrics: The coverage metric is the percentage of target nodes in a WSN that can be positioned, regardless of accuracy. The geometry and the node density have the most effect on coverage results. For a target to be positioned successfully, there should be enough reference nodes around it and sufficient measurements taken by sensor nodes. Density can be determined as the minimum number of neighbors required for target nodes to be positioned considering a certain level of accuracy [6]. If the density of the deployment is low, it is possible that a number of nodes cannot be positioned, due to lacking enough reference nodes around a target node. In this case, cooperation between target nodes can remedy the problem and then improve the coverage metric. Increasing the density also improves the coverage metric, but this might not be an option due to increased message collisions and energy consumption. In addition to node density, the reference node placement has a great impact on positioning error. As mentioned previously, the effect of the geometry of reference nodes is studied through the GDOP metric [6, 14]. GDOP analysis shows that if target nodes are located inside the convex hull of the reference nodes, they can be localized with lower error.

It is also common to define some composite metrics as opposed to independent metrics. For example, the cost metric defined in [128] is one such composite metric, merging accuracy and complexity in one performance measure.

In conclusion, although there is no unique way to compare different positioning algorithms, various positioning approaches can be evaluated based on a number of metrics, e.g., the performance measures considered in this chapter. A comprehensive assessment of an algorithm may require that a hardware implementation of the algorithm is tested in a real-world scenario.

Chapter 6

Conclusions and future work

6.1 Thesis contributions

In this thesis, the main effort has been devoted to designing new positioning algorithms and evaluating position estimates for both cooperative and noncooperative scenarios. In general, the thesis evaluates the positioning estimates based on statistical and geometric approaches. Moreover, some practical imperfections (as nuisance parameters) are involved in developing position algorithms. The contributions of the thesis, five appended papers, can be, generally, categorized into two groups: statistical algorithms and geometric approaches.

6.1.1 Statistical algorithms

The thesis formulates the positioning problem of different scenarios as estimation problems and employs an asymptotically optimal algorithm, i.e., MLE, to solve the problem (Paper A, B, and C). Since the MLE in general poses difficult global optimization problems, a number of suboptimal techniques are introduced, which show a good trade-off between complexity and accuracy. Concretely, the contributions of this class of papers are to

- take a number of practical impairments into account to develop positioning algorithms;
- introduce a new idea based on eavesdropping signals in silent nodes to decrease the delay and power consumption;
- derive the optimal estimator (MLE) and CRLB as benchmarks for different scenarios;
- propose suboptimal linear estimators, which are asymptotically optimal, for different scenarios;
- introduce a number of suboptimal techniques, which are asymptotically optimal, based on convex optimization approaches.

6.1.2 Geometric approaches

Papers D and E consider a geometric interpretation for cooperative and noncooperative positioning problems and conclude that the location of target nodes can be confined into a number of closed bounded convex sets derived from distance estimates if the measurement errors tend to be positive. The feasible sets can then be used to design a constrained positioning algorithm and to obtain an upper bound on the realization of a position estimate. In particular, the main contributions of Papers D and E can be summarized as

- introducing the idea of bounding the location of target nodes to a number of convex feasible sets for range estimates with positive errors;
- formulating upper bounds on position errors for noncooperative scenarios as nonconvex problems and then approximately solving the problems using convex relaxation techniques;
- identifying the feasible sets in cooperative positioning using a distributed approach that relays on ellipsoid outer-approximation of the feasible set.

6.2 Summaries of papers

The main contributions of this thesis are found in five appended papers. In this section, we review the contributions of each paper in more detail.

6.2.1 Paper A

M. R. Gholami, S. Gezici, and E. G. Ström, "Improved position estimation using hybrid TW-TOA and TDOA in cooperative networks," *IEEE Trans. Signal Process.*, vol. 60, no. 7, pp. 3770–3785, Jul. 2012.

Motivations– In general, the accuracy of the position estimate can be improved by increasing the number of reference nodes connected to target nodes. In situations in which there is limited access to reference nodes, the cooperation technique can be employed to improve the accuracy of the estimate. That is, target nodes will connect to their neighbors and take some sort of measurements. As the number of sensors (reference or target) connected to target nodes increases, the number of packets exchanged between different nodes increases. This implies an increased delay in the localization process. There might also be some constraints, e.g., complexity or power consumption, and an increase in active nodes may face limitations. The question is how to improve accuracy without increasing active nodes.

Contributions– To decrease the number of packets exchanged over the network during the positioning process and to improve the accuracy of the estimate, this paper proposes a new positioning scenario in cooperative networks. The main idea is to eavesdrop packets exchanged between a target and other active nodes in a number of reference or target nodes that remain silent during the measurement process for the target. Namely, the silent nodes (secondary reference or target nodes) listen to both the signal transmitted by a reference node (primary reference node) and the reply signal by a target node. Concretely, primary reference nodes and secondary (reference or target) nodes measure

TW-TOA and TDOA, respectively. All measurements collected in target and reference nodes are used to estimate the location of target nodes. In summary, the paper

- models the positioning problem in a cooperative network using an eavesdropping technique;
- derives a lower bound (CRLB) on the performance of the optimal unbiased estimator for the corresponding estimation problem;
- derives the optimal estimator (MLE), which poses a difficult nonconvex problem;
- proposes a simple (suboptimal) linear estimator to solve the positioning problem.

Numerical studies show that the eavesdropping technique can improve the accuracy of the estimate, especially for low SNRs.

6.2.2 Paper B

M. R. Gholami, S. Gezici, and E. G. Ström, "TDOA-based positioning in the presence of unknown clock skew," *IEEE Trans. Commun.,* vol. 61, no. 6, pp. 2522–2534, Jun. 2013.

Motivations– In ranging based on TOA, the time instant that a reference node starts transmitting a signal is usually unknown to the target node receiving the signal. Another approach is to use the TDOA approach, which is commonly used in practice, e.g., in GPS receivers. That is, two synchronized reference nodes transmit the signal at the same time, and a target node separately estimates the TOAs for signals received from reference nodes and then subtracts two TOAs to remove the common time offset. While the TDOA approach can remove the common time offset, it can be affected by local imperfections in the target node. A common model for describing the behavior of a local clock is to employ an affine model, which contains two main parameters: clock offset and clock skew. (For an ideal clock, the clock offset is equal to zero and clock skew is one.) The accuracy of TDOA-based positioning can be considerably affected if the clock skew deviates considerably from one. For small deviations from ideal clock skew, the performance of positioning for very large networks can also be affected in high SNR regions.

Contributions– This paper studies a self-positioning problem based on TDOA measurements in the presence of unknown clock skew in a target node. In fact, the clock of the target node is modeled by an affine function, and it is shown that the positioning problem is affected by the clock skew. The problem is formulated as an estimation problem in which the clock skew, as a nuisance parameter, needs to be estimated along with the location of the target node. The paper mainly

- investigates the CRLB and the MLE for the problem;
- proposes two suboptimal estimators based on linear least squares and semidefinite programming;
- introduces a refining step based on a linearization technique using the first-order Taylor expansion.

The numerical results show that the proposed suboptimal techniques asymptotically attain the CRLB.

6.2.3 Paper C

M. R. Gholami, R. M. Vaghefi, and E. G. Ström, "RSS-based sensor localization in the presence of unknown channel parameters," *IEEE Trans. Signal Process.,* vol. 61, no. 15, pp. 3752–3759, Aug. 2013

Motivations– RSS-based position recovery is a simple and attractive technique for practical scenarios, e.g., in emergency call applications. The log-normal model is commonly considered for modeling the received power of signals transmitted by a target node. In the log-normal model, the received power is a deterministic function of the reference power and path-loss exponent (which we call channel parameters) and the distance plus a log-normally distributed shadow-fading term. In practice, it may be difficult to know the exact values of the channel parameters. The positioning accuracy is, therefore, affected by unknown channel parameters.

Contributions– This paper investigates the single target localization problem based on RSS-measurements in the presence of unknown channel parameters. Using suitable approximations, the problem is rendered to a low-complexity problem, a general trust region subproblem, and a simple technique is employed to solve the problem. The paper studies different scenarios, i.e., unknown transmission power, unknown path-loss exponent, or both unknown transmission power and unknown path-loss exponent. In some scenarios, two-step estimators are employed to solve the positioning problem. In summary, the paper

- introduces a linearization technique and formulates the positioning problem in the presence of unknown channel parameters as nonconvex QCQPs;
- employs a simple iterative technique to solve the problem.

The proposed technique shows a good trade-off between accuracy and complexity compared to the existing approaches.

6.2.4 Paper D

M. R. Gholami, E. G. Ström, H. Wymeersch, and M. Rydström, "Upper bounds on position error of a single location estimate in wireless sensor networks," submitted to *Signal Processing,* Sep. 2013.

Motivations– Identifying reasonable bounds on the position error is often of great value for different applications. For instance, a lower bound on the mean-square position error is a common metric to benchmark the accuracy of the position estimate. There exists a number of such lower bounds for the positioning algorithms in the literature. For example the CRLB, which gives a lower bound on the variance of any unbiased estimator, can be computed if the PDF of the measurement error is known and satisfies some regularity conditions. In addition to a lower bound on the position error, in some applications it may be useful to know the worst-case behavior of the position error. Such knowledge may be useful not only for the evaluation of different services provided by WSNs but also for design and resource management. Similarly, in evaluating the worst-case position error, we may be interested in assessing the quality of a single point estimate.

Contributions– This paper studies the possibility of upper bounding the position error for range-based positioning algorithms in wireless sensor networks. In this study, it is argued that, in certain situations, when the measured distances between sensor nodes have positive errors, e.g., in non-line-of-sight (NLOS) conditions, the target node is confined to a closed bounded convex set (a feasible set), which can be derived from the measurements. Then, two classes of geometric upper bounds are formulated with respect to the feasible set. If an estimate is available, either feasible or infeasible, the position error can be upper bounded as the maximum distance between the estimate and any point in the feasible set (the first bound). Alternatively, if an estimate given by a positioning algorithm is always feasible, the maximum length of the feasible set is an upper bound on position error (the second bound). These bounds are formulated as nonconvex optimization problems. To progress, convex relaxations techniques are employed to approximately solve the nonconvex problems. In summary, the main contributions of this study are

- introducing the concept of an instantaneous upper bound for a single point position estimate when the distance measurements have positive errors, e.g., in NLOS conditions;
- proposing an upper bound on the position error based on a convex relaxation technique when an estimate of the target position is available (feasible or infeasible);
- proposing three upper bounds for an estimator always giving a feasible point as an estimate (e.g., the POCS estimate) based on the idea of the maximum length of the feasible set or a relaxed feasible set including the target node.

Simulation results show that the proposed bounds are reasonably tight in many situations, especially for NLOS conditions.

6.2.5 Paper E

M. R. Gholami, H. Wymeersch, S. Gezici, and E. G. Ström, "Distributed bounding of feasible sets in cooperative wireless network positioning," *IEEE Commun. Lett.,* 2013, doi: 10.1109/LCOMM.2013.070113.130905.

Motivations– Locations of target nodes in cooperative wireless sensor networks can be confined to a number of feasible sets in certain situations, e.g., when the estimated distances between sensors are larger than the actual distances. Quantifying feasible sets is often challenging in cooperative positioning since the intersections involving the location of the target nodes depend on the location of the target nodes.

Contributions– This study proposes an iterative technique to cooperatively outer-approximate the feasible sets containing the locations of the target nodes. That is, first an ellipsoid outer-approximation of a feasible set including a target node location is obtained. Then, the ellipsoid is extended with the measured distances between sensor nodes, resulting in larger ellipsoids. The larger ellipsoids are used to determine the intersections containing other targets. In summary, the paper

- argues how to confine the targets locations into a number of feasible sets resulted from distance estimates with positive measurements errors;

- introduces an iterative technique, amendable to distributed implementation, to (outer-)approximate the intersection involving the locations of targets by a number of ellipsoids.

Simulation results show that the proposed technique converges after a small number of iterations. The feasible sets, then, can be used as constraints to improve the performance of a positioning algorithm.

6.3 Future work

In this thesis, a number of approaches have been proposed to improve the accuracy of the position estimate. Both statistical and geometric approaches have been investigated for different scenarios. In addition, a number of suboptimal approaches have been studied, which can provide good coarse estimates. The effect of clock imperfections has also been studied in some detail. The problem has, mainly, been formulated as a centralized optimization problem. One direction in future studies is to design efficient distributed algorithms, e.g., based on dual decomposition techniques. Another important challenge for the positioning problem is the consideration of NLOS errors, especially for cooperative scenarios. In this thesis, a technique was investigated to upper bound the position error for a single estimate. The results show good performance in some scenarios, whereas it is necessary to improve the tightness of the bound for some other scenarios. One possible open problem for future studies is to investigate techniques to improve the tightness of the bound. In this thesis, cooperative positioning was studied to some extent when the locations of reference nodes are exactly known. One open problem in studying cooperative positioning is to consider uncertainties in the location of reference nodes and to design robust algorithms.

6.4 Related contributions

Other related publications by the author, which are not included in this thesis, are listed below.

Book chapters

[BC1] D. Dardari, M. Dio, A. Emmanuele, D. Fontanella, S. Gezici, M. R. Gholami, M. Kieffer, E. Lagunas, J. Louveaux, A. Mallat, M. Nájar, M. Navarro, M. Nicoli, L. Reggiani, M. Rydström, E. G. Ström, L. Vandendorpe, and F. Zanier, *Innovative Signal Processing Techniques for Wireless Positioning, in Satellite and Terrestrial Radio Positioning Techniques - A signal processing perspective*, edited by D. Dardari, E. Falletti, and M. Luise, Elsevier, pp. 207–315, 2012.

[BC2] C. Pau, A. Conti, D. Dardari, N. Decarli, E. Falletti, C. Fernádez-Prades, M. R. Gholami, M. Nájar, E. Lagunas, M. Pini, M. Rydström, F. Sottile, and E. G. Ström, *Casting Signal Processing to Real-World Data*, edited by D. Dardari, E. Falletti, and M. Luise, Elsevier, pp. 383–415, 2012.

Journals

[J1] M. R. Gholami, H. Wymeersch, E. G. Ström, and M. Rydström, “Wireless network positioning as a convex feasibility problem,” *EURASIP Journal on Wireless Communications and Networking* 2011, 2011:161.

[J2] R. M. Vagehfi, M. R. Gholami, R. M. Buehrer, and E. G. Ström, “Cooperative received signal strength-based sensor localization with unknown transmit powers,” *IEEE Trans. Signal Process.*, vol. 61, pp. 1389–1403, 2013.

[J3] M. R. Gholami, S. Gezici, and E. G. Ström, “A concave-convex procedure for TDOA based positioning,” *IEEE Commun. Lett.*, vol. 17, no. 4, pp. 765–768, 2013.

[J4] M. R. Gholami, L. Tetruashvili, E. G. Ström, and Y. Censor, “Cooperative wireless sensor network positioning via implicit convex feasibility,” accepted for publication, *IEEE Trans. Signal Process.*, 2013.

[J5] M. R. Gholami, S. Gezici, and E. G. Ström, “TW-TOA based positioning in the presence of clock imperfections,” to be submitted to *IEEE Trans. Wireless Commun.*, 2013.

[J6] M. R. Gholami, E. G. Ström, H. Wymeersch, and S. Gezici, “Upper bounds on position error of a single location estimate in wireless sensor networks,” submitted to *EURASIP J. Advances in Signal Processing (issue on Digital Signal Processing for Localization)*, May 2013.

[J7] M. R. Gholami, E. G. Ström, and A. H. Sayed, “Distributed estimation over cooperative networks with missing data,” to be submitted to *IEEE Trans. Signal Process.*

[J8] W. Sun, E. G. Ström, F. Brännström, and M. R. Gholami, “Random broadcast based distributed consensus clock synchronization for mobile networks,” to be submitted to *IEEE Trans. Commun.*

Conferences

[C1] M. R. Gholami, E. G. Ström, and A. H. Sayed, “Distributed estimation over cooperative networks with missing data,” accepted for publication, *IEEE GlobalSIP*, 2013.

[C2] W. Sun, M. R. Gholami, E. G. Ström, and F. Brännström “Consensus based distributed clock synchronization for mobile Ad Hoc networks,” accepted for publication, *IEEE Globecom Workshop - International Workshop on Device-to-Device (D2D) Communication With and Without Infrastructure*, 2013.

[C3] M. R. Gholami, S. Gezici, and E. G. Ström, “Range based sensor node localization in the presence of unknown clock skews,” in Proc. *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vancouver, Canada, 2013.

[C4] P. Oguz-Ekim, J. Gomes, P. Oliveira, M. R. Gholami, and E. G. Ström, “TW-TOA based cooperative sensor network localization with unknown turn-around time," in Proc. *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vancouver, Canada, 2013.

[C5] M. R. Gholami, H. Wymeersch, E. G. Ström, and M. Rydström, "Robust distributed positioning algorithms for cooperative networks," in Proc. *IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, pp.156–160, 2011.

[C6] M. R. Gholami, S. Gezici, E. G. Ström, and M. Rydström, "Positioning algorithms for cooperative networks in the presence of an unknown turn-around time," in Proc. *IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, pp. 166–170, 2011.

[C7] M. R. Gholami, S. Gezici, E. G. Ström, and M. Rydström, "Hybrid TW-TOA/TDOA positioning algorithms for cooperative wireless networks," in Proc. *IEEE International Conference on Communication (ICC)*, 2011.

[C8] M. R. Gholami, S. Gezici, E. G. Ström, and M. Rydström, "Positioning algorithms for cooperative networks in the presence of an unknown turn-around time," in Proc. *IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, pp. 166–170, 2011.

[C9] R. M. Vaghefi, M. R. Gholami, and E. G. Ström, "RSS-based sensor localization with unknown transmit power," in Proc. *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Prague, pp. 2480–2483, 2011.

[C10] M. R. Gholami, E. G. Ström, F. Sottile, D. Dardari, A. Conti, S. Gezici, M. Rydström, and M. Spirito, "Static positioning using UWB range measurements," in Proc. *Future Network & Mobile Summit*, Florence, Italy, June 16–18, 2010.

[C11] M. R. Gholami, S. Gezici, M. Rydström, and E. G. Ström, "A distributed positioning algorithm for cooperative active and passive sensors," in Proc. *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, pp. 1711–1716, 2010.

[C12] M. R. Gholami, M. Rydström, and E. G. Ström, "Positioning of node using plane projection onto convex sets," in Proc. *IEEE Wireless Communication and Networking Conference (WCNC)*, 2010.

[C13] R. M. Vaghefi, M. R. Gholami, and E. G. Ström, "Bearing-only target localization with uncertainties in observer position," in Proc. *IEEE International Workshop on Advances in Positioning and Location-Enabled Communications (in conjunction with PIMRC'10)*, pp. 238–242, 2010.

[C14] M. R. Gholami, E. G. Ström, and M. Rydström, "Indoor sensor node positioning using UWB range measurements," in Proc. *European Signal Processing Conference (Eusipco)*, pp. 1943–1947, 2009.

Bibliography

[1] D. Puccinelli and M. Haenggi, "Wireless sensor networks: applications and challenges of ubiquitous sensing," *IEEE Circuits and Systems Magazine*, vol. 5, no. 3, pp. 19–31, 2005.

[2] C.-Y. Chong and S. P. Kumar, "Sensor networks: evolution, opportunities, and challenges," *Proceedings of the IEEE*, vol. 91, no. 8, pp. 1247–1256, 2003.

[3] D. Dardari, M. Luise, and E. Falletti, *Satellite and Terrestrial Radio Positioning Techniques: A Signal Processing Perspective*, Academic Press, 2011.

[4] M. Hazas, J. Scott, and J. Krumm, "Location-aware computing comes of age," *Computer*, vol. 37, no. 2, pp. 95–97, 2004.

[5] Z. Sahinoglu, S. Gezici, and I. Guvenc, *Ultra-Wideband Positioning Systems: Theoretical Limits, Ranging Algorithms, and Protocols*, New York: Cambridge University Press, 2008.

[6] G. Mao and B. Fidan, *Localization Algorithms and Strategies for Wireless Sensor Networks*, Hershey, New York: Information Science Reference, 2009.

[7] A. H. Sayed, A. Tarighat, and N. Khajehnouri, "Network-based wireless location: challenges faced in developing techniques for accurate wireless location information," *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 24–40, Jul. 2005.

[8] S. Gezici, "A survey on wireless position estimation," *Wireless Personal Communications*, vol. 44, no. 3, pp. 263–282, Feb. 2008.

[9] S. Gezici, Z. Tian, G. B. Giannakis, H. Kobayashi, A. F. Molisch, H. V. Poor, and Z. Sahinoglu, "Localization via ultra-wideband radios: A look at positioning aspects for future sensor networks," *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 70–84, Jul. 2005.

[10] R. Huang and G. V. Zaruba, "Beacon deployment for sensor network localization," in *IEEE Wireless Communications and Networking Conference*, Mar. 2007, pp. 3188–3193.

[11] N. Bulusu, J. Heidemann, and D. Estrin, "GPS-less low-cost outdoor localization for very small devices," *IEEE Personal Commun.*, vol. 7, no. 5, pp. 28–34, Oct. 2000.

[12] H. Wymeersch, J. Lien, and M. Z. Win, "Cooperative localization in wireless networks," *Proc. IEEE*, vol. 97, no. 2, pp. 427–450, Feb. 2009.

[13] D. Munoz, F. B. Lara, C. Vargas, and R. Enriquez-Caldera, *Position Location Techniques and Applications*, Academic Press, 2009.

[14] N. Patwari, "Location estimation in sensor networks," PhD dissertation, University of Michigan, Ann Arbor, 2005.

[15] A. F. Molisch, *Wireless Communications*, Piscataway, NJ: IEEE Press/Wiley, 2005.

[16] I. Sari, E. Serpedin, K. Noh, Q. Chaudhari, and B. Suter, "On the joint synchronization of clock offset and skew in RBS-protocol," *IEEE Trans. Commun.*, vol. 56, no. 5, pp. 700–703, May 2008.

[17] Q. M. Chaudhari, E. Serpedin, and K. Qaraqe, "Some improved and generalized estimation schemes for clock synchronization of listening nodes in wireless sensor networks," *IEEE Trans. Commun.*, vol. 58, no. 1, pp. 63–67, Jan. 2010.

[18] J. Zheng and Y. Wu, "Joint time synchronization and localization of an unknown node in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1309–1320, Mar. 2010.

[19] N. Khajehnouri and A. H. Sayed, "A distributed broadcasting time-synchronization scheme for wireless sensor networks," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing*, vol. 5, Mar. 2005, pp. v/1053 – v/1056.

[20] Q. Chaudhari, E. Serpedin, and J. Kim, "Energy-efficient estimation of clock offset for inactive nodes in wireless sensor network," *IEEE Trans. Inf. Theory*, vol. 56, no. 1, pp. 582–596, Jan. 2010.

[21] A. A. D'Amico, U. Mengali, and L. Taponecco, "Energy-based TOA estimation," *IEEE Trans. Wireless Commun.*, vol. 7, no. 3, pp. 838–847, Mar. 2008.

[22] D. Dardari, A. Conti, U. Ferner, A. Giorgetti, and M. Z. Win, "Ranging with Ultrawide bandwidth signals in multipath environments," *Proc. IEEE*, vol. 97, no. 2, pp. 404–426, Feb. 2009.

[23] A. A. D'Amico, U. Mengali, and L. Taponecco, "TOA estimation with the IEEE 802.15.4a standard," *IEEE Trans. Wireless Commun.*, vol. 9, no. 7, pp. 2238–2247, Jul. 2010.

[24] N. Patwari, J. Ash, S. Kyperountas, A. O. Hero, and N. C. Correal, "Locating the nodes: Cooperative localization in wireless sensor network," *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 54–69, Jul. 2005.

[25] L. Wasserman, *All of Statistics: A Concise Course in Statistical Inference*, Springer, 2004.

[26] S. Venkatraman and J. Caffery Jr, "Hybrid TOA/AOA techniques for mobile location in non-line-of-sight environments," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, vol. 1, Mar. 2004, pp. 274–278.

[27] L. Cong and W. Zhuang, "Hybrid TDOA/AOA mobile user location for wideband CDMA cellular systems," *IEEE Trans. Wireless Commun.*, vol. 1, no. 3, pp. 439–447, Jul. 2002.

[28] S. Gezici and Z. Sahinoglu, "Enhanced position estimation via node cooperation," in *Proc. IEEE International Conference on Communications (ICC)*, Cape Town, South Africa, May 23–27, 2010.

[29] M. R. Gholami, S. Gezici, E. G. Ström, and M. Rydström, "A distributed positioning algorithm for cooperative active and passive sensors," in *Proc. IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, Sep. 2010.

[30] A. Catovic and Z. Sahinoglu, "The Cramér-Rao bounds of hybrid TOA/RSS and TDOA/RSS location estimation schemes," *IEEE Commun. Lett.*, vol. 8, no. 10, pp. 626–628, Oct. 2004.

[31] J. Huang and Q. Wan, "Comments on 'the Cramér-Rao bounds of hybrid TOA/RSS and TDOA/RSS location estimation schemes'," *IEEE Commun. Lett.*, vol. 11, no. 11, pp. 848–849, Nov. 2007.

[32] P.-C. Chen, "A non-line-of-sight error mitigation algorithm in location estimation," in *Proc. IEEE Wireless Communications and Networking Conference*, 1999, pp. 316–320 vol. 1.

[33] C. Liang and R. Piche, "Mobile tracking and parameter learning in unknown non-line-of-sight conditions," in *Proc. 13th International Conference on Information Fusion*, Edinburg, U.K., Jul. 26–29, 2010.

[34] E. Serpedin and Q. M. Chaudhari, *Synchronization in wireless sensor networks: Parameter estimation, performance benchmarks and protocols*, New York: NY, USA: Cambridge University Press, 2009.

[35] N. M. Freris, S. R. Graham, and P. R. Kumar, "Fundamental limits on synchronizing clocks over networks," *IEEE Trans. Autom. Control*, vol. 56, no. 6, pp. 1352–1364, Jun. 2011.

[36] Y.-C. Wu, Q. Chaudhari, and E. Serpedin, "Clock synchronization of wireless sensor networks," *IEEE Signal Process. Mag.*, vol. 28, no. 1, pp. 124–138, Jan. 2011.

[37] M. R. Gholami, "Positioning algorithms for wireless sensor networks," Licentiate thesis, Chalmers University of Technology, Mar. 2011, [Online]. Available: http://publications.lib.chalmers.se/records/fulltext/138669.pdf.

[38] Z. Sahinoglu, "Improving range accuracy of IEEE 802.15.4a radios in the presence of clock frequency offsets," *IEEE Commun. Lett.*, vol. 15, no. 2, pp. 244–246, Feb. 2011.

[39] M. R. Gholami, H. Wymeersch, E. G. Ström, and M. Rydström, "Robust distributed positioning algorithms for cooperative networks," in *IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, San Francisco, USA, 2011, pp. 156–160.

[40] M. R. Gholami, S. Gezici, and E. G. Ström, "Range based sensor node localization in the presence of unknown clock skews," in *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Vancouver, Canada, 2013.

[41] M. R. Gholami, S. Gezici, E. G. Ström, and M. Rydström, "Hybrid TW-TOA/TDOA positioning algorithms for cooperative wireless networks," in *Proc. IEEE International Conference on Communications (ICC)*, Kyoto, Japan, Jun. 2011.

[42] ——, "Positioning algorithms for cooperative networks in the presence of an unknown turn-around time," in *IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, San Francisco, Jun. 26–29 2011, pp. 166–170.

[43] S. P. Chepuri, R. T. Rajan, G. Leus, and A. D. Even, "Joint clock synchronization and ranging: Asymmetrical time-stamping and passive listening," *IEEE Signal Process. Lett.*, vol. 20, no. 1, pp. 51–54 2013.

[44] Y. Wang, X. Ma, and G. Leus, "Robust time-based localization for asynchronous networks," *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4397–4410, Sep. 2011.

[45] G. J. Pottie and W. J. Kaiser, "Wireless integrated network sensors," *Communications of the ACM*, vol. 43, no. 5, pp. 51–58, 2000.

[46] R. L. Moses, D. Krishnamurthy, and R. M. Patterson, "A self-localization method for wireless sensor networks," *EURASIP J. Adv. Sig. Pr.*, no. 4, pp. 348–358, Mar. 2003.

[47] K. Yedavalli and B. Krishnmachari, "Sequence-based localization in wireless sensor networks," *IEEE Trans. Mobile Comput.*, vol. 7, no. 1, pp. 1–14, Jan. 2008.

[48] M. R. Gholami, M. Rydström, and E. G. Ström, "Positioning of node using plane projection onto convex sets," in *Proc. IEEE Wireless Communication and Networking Conference (WCNC)*, Apr. 2010.

[49] M. Bshara, U. Orguner, F. Gustafsson, and L. Van Biesen, "Fingerprinting localization in wireless networks based on received-signal-strength measurements: A case study on WIMAX networks," *IEEE Trans. Veh. Technol.*, vol. 59, no. 1, pp. 283–294, 2010.

[50] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation theory*, Englewood Cliffs, NJ: Prentice-Hall, 1993.

[51] X. Sheng and Y. Hu, "Maximum likelihood multiple-source localization using acoustic energy measurements with wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 53, no. 1, pp. 44–53, Jan. 2005.

[52] R. M. Vaghefi, M. R. Gholami, and E. G. Ström, "Bearing-only target localization with uncertainties in observer position," in *Proc. IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, pp. 238–242.

[53] R. Vaghefi, M. Gholami, R. Buehrer, and E. Ström, "Cooperative received signal strength-based sensor localization with unknown transmit powers," *IEEE Trans. Signal Process.*, vol. 61, no. 6, pp. 1389–1403, Mar. 2013.

[54] M. R. Gholami, S. Gezici, and E. G. Ström, "TDOA based positioning in the presence of unknown clock skew," *IEEE Trans. Commun.*, vol. 61, no. 6, pp. 2522–2534, Jul. 2013.

[55] ——, "Improved position estimation using hybrid TW-TOA and TDOA in cooperative networks," *IEEE Trans. Signal Process.*, vol. 60, no. 7, pp. 3770–3785, Jul. 2012.

[56] R. M. Vaghefi, M. R. Gholami, and E. G. Ström, "RSS-based sensor localization with unknown transmit power," in *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, May 2011, pp. 2480–2483.

[57] G. Destino and G. Abreu, "Reformulating the least-square source localization problem with contracted distances," in *Proc. Asilomar*, 2009, pp. 307–311.

[58] J. Nie, "Sum of squares method for sensor network localization," *Comput. Optim. Appl.*, vol. 43, pp. 151–179, 2009.

[59] A. Beck, M. Teboulle, and Z. Chikishev, "Iterative minimization schemes for solving the single source localization problem," *AIAM Journal on Optimization*, no. 3, pp. 1397–1416, 2008.

[60] A. Beck, P. Stoica, and J. Li, "Exact and approximate solutions of source localization problems," *IEEE Trans. Signal Process.*, vol. 56, no. 5, pp. 1770–1778, May 2008.

[61] E. G. Larsson and D. Danev, "Accuracy comparison of LS and squared-range LS for source localization," *IEEE Trans. Signal Process.*, vol. 58, no. 2, pp. 916–923, 2010.

[62] P. Biswas, T.-C. Lian, T.-C. Wang, and Y. Ye, "Semidefinite programming based algorithms for sensor network localization," *ACM Trans. Sens. Netw.*, vol. 2, no. 2, pp. 188–220, 2006.

[63] S. Srirangarajan, A. Tewfik, and Z.-Q. Luo, "Distributed sensor network localization using SOCP relaxation," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 4886–4895, Dec. 2008.

[64] D. P. Bertsekas, *Nonlinear Programming*, 2nd ed, Athena Scientific, 1999.

[65] M. Chiang, S. H. Low, A. R. Calderbank, and J. C. Doyle, "Layering as optimization decomposition: A mathematical theory of network architectures," *Proc. IEEE*, vol. 95, no. 1, pp. 255–312, 2007.

[66] D. P. Palomar and Y. C. Eldar, *Convex Optimization in Signal Processing and Communications*, Cambridge University Press, 2010.

[67] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[68] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and distributed computation*, Old Tappan, NJ (USA): Prentice Hall Inc., 1989.

[69] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends® in Machine Learning*, vol. 3, no. 1, pp. 1–122, 2011.

[70] T. Erseghe, "A distributed and scalable processing method based upon ADMM," *IEEE Signal Process. Lett.*, vol. 19, no. 9, pp. 563–566, 2012.

[71] C. Meesookho, U. Mitra, and S. Narayanan, "On energy-based acoustic source localization for sensor networks," *IEEE Trans. Signal Process.*, vol. 56, no. 1, pp. 365–377, Jan. 2008.

[72] M. Sun and K. C. Ho, "Successive and asymptotically efficient localization of sensor nodes in closed-form," *IEEE Trans. Signal Process.*, vol. 57, no. 11, pp. 4522–4537, Nov. 2009.

[73] S. Gezici, I. Guvenc, and Z. Sahinoglu, "On the performance of linear least-squares estimation in wireless positioning systems," in *Proc. IEEE International Conference on Communications (ICC)*, May 2008, pp. 4203–4208.

[74] K. C. Ho, X. Lu, and L. Kovavisaruch, "Source localization using TDOA and FDOA measurements in the presence of receiver location errors: analysis and solution," *IEEE Trans. Signal Process.*, vol. 55, no. 2, pp. 684–696, Feb. 2007.

[75] X. Wang, Z. Wang, and B. O'Dea, "A TOA-based location algorithm reducing the errors due to non-line-of-sight (NLOS) propagation," *IEEE Trans. Veh. Technol.*, vol. 52, no. 1, pp. 112–116, Jan. 2003.

[76] M. R. Gholami, L. Tetruashvili, E. G. Ström, and Y. Censor, "Cooperative wireless sensor network positioning via implicit convex feasibility," to appear in *IEEE Trans. Signal Process.*, 2013.

[77] M. R. Gholami, H. Wymeersch, E. G. Ström, and M. Rydström, "Wireless network positioning as a convex feasibility problem," *EURASIP Journal on Wireless Communications and Networking 2011*, 2011:161.

[78] T. Jia and R. Buehrer, "A set-theoretic approach to collaborative position location for wireless networks," *IEEE Trans. Mobile Comput.*, pp. 1264–1275, 2011.

[79] L. G. Gubin, B. Polyak, and E. Raik, "The method of projection for finding the common point of convex sets," *USSR Computational Mathematics and Mathematical Physics*, vol. 7, pp. 1–24, 1967.

[80] Y. Censor and S. A. Zenios, *Parallel Optimization: Theory, Algorithms, and Applications*, New York: Oxford Unversity Press, 1997.

[81] P. Oskoui-Fard and H. Stark, "Tomographic image reconstruction using the theory of convex projections," *IEEE Transactions on Medical Imaging*, vol. 7, no. 1, pp. 45–58, Mar. 1988.

[82] G. T. Herman and L. B. Meyer, "Algebraic reconstruction techniques can be made computationally efficient [positron emission tomography application]," *IEEE Transactions on Medical Imaging*, vol. 12, no. 3, pp. 600–609, Sep. 1993.

[83] D. C. Youla and H. Webb, "Image restoration by the method of convex projections: Part 1: theory," *IEEE Transactions on Medical Imaging*, vol. 1, no. 2, pp. 81–94, Oct. 1982.

[84] A. O. Hero and D. Blatt, "Sensor network source localization via projection onto convex sets (POCS)," in *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, vol. 3, Philadelphia, USA, Mar. 2005, pp. 689–692.

[85] D. Blatt and A. O. Hero, "Energy-based sensor network source localization via projection onto convex sets," *IEEE Trans. Signal Process.*, vol. 54, no. 9, pp. 3614–3619, Sep. 2006.

[86] ——, "APOCS: a rapidly convergent source localization algorithm for sensor networks," in *IEEE/SP Workshop on Statistical Signal Processing*, Jul. 2005, pp. 1214–1219.

[87] J. Wang and P. A. Regalia, "Sensor network localization via boundary projections," in *Proc. 43rd Annual Conference on Information Sciences and Systems*, Mar. 2009, pp. 224–229.

[88] M. Rydström, E. G. Ström, and A. Svensson, "Robust sensor network positioning based on projection onto circular and hyperbolic convex sets," in *IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, pp.1–5, 2006.

[89] Y. Censor, A. R. D. Pierro, and M. Zaknoon, "Steered sequential projections for the inconsistent convex feasibility problem," *Nonlinear analysis: theory, method, and application, series A*, vol. 59, pp. 385–405, 2004.

[90] M. R. Gholami, E. G. Ström, and M. Rydström, "Indoor sensor node positioning using UWB range measurements," in *Proc. 17th European Signal Processing Conference (Eusipco)*, 2009, pp. 1943–1947.

[91] M. R. Gholami, E. G. Ström, F. Sottile, D. Dardari, S. Gezici, M. Rydström, M. A. Spirito, and A. Conti, "Static positioning using UWB range measurements," in *Proc. Future Network and Mobile Summit*, Jun. 2010.

[92] H. Wymeersch, S. Maranó, W. M. Gifford, and M. Z. Win, "A machine learning approach to ranging error mitigation for UWB localization," *IEEE Trans. Commun.*, vol. 60, no. 6, pp. 1719–1728, Jun. 2012.

[93] S. Marano, W. M. Gifford, H. Wymeersch, and M. Z. Win, "Nonparametric obstruction detection for UWB localization," in *Proc. IEEE Global Communication Conference*, Dec. 2009, pp. 1–6.

[94] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[95] J. Zhang and S. Sastary, "Distributed position estimation for sensor networks," in *Proc. 15th Triennial World Congress*, Barcelona, Spain, 2002.

[96] L. Doherty, K. S. J. Pister, and L. E. Ghaoui, "Convex position estimation in wireless sensor networks," in *INFOCOM*, vol. 3, 2001, pp. 1655–1663.

[97] A. Beck, "On the convexity of a class of quadratic mappings and its application to the problem of finding the smallest ball enclosing a given intersection of balls," *Journal of Global Optimization*, vol. 39, pp. 113–126, Sep. 2007.

[98] M. R. Gholami, E. G. Ström, H. Wymeersch, and S. Gezici, "Upper bounds on position error of a single location estimate in wireless sensor networks," submitted to *EURASIP J. Advances in Signal Processing (special issue on Digital Signal Processing for Localization)*, 2013.

[99] A. Ben-Tal and A. Nemirovski, "Lectures on modern convex optimization," 2012, [Online]. Available: http://www2.isye.gatech.edu/~nemirovs/Lect_ModConvOpt.pdf.

[100] A. Nemirovski, C. Roos, and T. Terlaky, "On maximization of quadratic form over intersection of ellipsoids with common center," *Mathematical Programming*, vol. 86, pp. 463–473, 1999.

[101] Z.-Q. Luo, W.-K. Ma, A. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, May 2010.

[102] M. S. Bazaraa, H. D. Sherali, and C. M. Shetty, *Nonlinear Programming Theory and Algorithms*, 3rd ed, New Jersey: John Wiley & Sons, 2006.

[103] J. J. More, "Generalization of the trust region problem," *Optimization Methods and Software*, vol. 2, pp. 189–209, 1993.

[104] D. Sorensen, "Newton's method with a model trust region modification," *SIAM J. Numer. Anal.*, vol. 19, no. 2, pp. 409–426, 1982.

[105] A. Beck and Y. C. Eldar, "Strong duality in nonconvex quadratic optimization with two quadratic constraints," *SIAM J. on Optimi.*, vol. 17, no. 3, pp. 844–860, Sep. 2006.

[106] C. Fortin and H. Wolkowicz, "The trust region subproblem and semidefinite programming," *Optim. Methods Softw.*, vol. 19, no. 1, pp. 41–67, 2004.

[107] D. R. Hunter and K. Lange, "A tutorial on MM algorithms," *The American Statistician*, vol. 58, no. 1, pp. 30–37, 2004.

[108] P. Oguz-Ekim, J. P. Gomes, J. Xavier, and P. Oliveira, "Robust localization of nodes and time-recursive tracking in sensor networks using noisy range measurements," *IEEE Trans. Signal Process.*, vol. 59, no. 8, pp. 3930–3942, 2011.

[109] J. A. Costa, N. Patwari, and A. O. Hero, "Distributed weighted-multidimensional scaling for node localization in sensor networks," *ACM Transactions on Sensor Network*, vol. 2, no. 1, pp. 39–64, 2006.

[110] R. Horst and N. V. Thoai, "DC programming: Overview," *Journal of Optimization Theory and Applications*, vol. 103, pp. 1–43, 1999.

[111] T. D. Quoc and M. Diehl, "Sequential convex programming methods for solving nonlinear optimization problems with DC constraints," 2011, [Online]. Available: arXiv:1107.5841v1.

[112] M. R. Gholami, S. Gezici, and E. G. Ström, "A concave-convex procedure for TDOA based positioning," *IEEE Commun. Lett.*, vol. 17, no. 4, pp. 765–768, 2013.

[113] M. Chiang, "Nonconvex optimization of communication systems," *Advances in Mechanics and Mathematics*, vol. 3, pp. 137–196, 2008.

[114] S. Boyd, S.-J. Kim, L. Vandenberghe, and A. Hassibi, "A tutorial on geometric programming," *Optimization and Engineering*, vol. 8, no. 1, pp. 67–127, 2007.

[115] M. Chiang, *Geometric Programming for Communication Systems*, Now Publishers Inc, 2005.

[116] P. A. Parrilo, "Semidefinite programming relaxations for semialgebraic problems," *Mathematical Programming*, vol. 96, no. 2, pp. 293–320, 2003.

[117] ——, "Structured semidefinite programs and semialgebraic geometry methods in robustness and optimization," PhD dissertation, Citeseer, 2000.

[118] S. Boyd and J. Mattingley, "Branch and bound methods," 2007, [Online]. Available: http://www.stanford.edu/class/ee364b/notes/bb_notes.pdf.

[119] E. L. Lawler and D. E. Wood, "Branch-and-bound methods: A survey," *Operations Research*, vol. 14, no. 4, pp. 699–719, 1966.

[120] V. M. Chiriac and A. M. Haimovich, "ZivZakai lower bound on target localization estimation in MIMO radar systems," in *IEEE Radar Conference*, 2010, pp. 678–683.

[121] Y. C. Eldar, "Uniformly improving the Cramér-Rao bound and maximum-likelihood estimation," *IEEE Trans. Signal Process.*, vol. 54, no. 8, pp. 2943–2956, Aug. 2006.

[122] Z. Ben-Haim and Y. C. Eldar, "On the constrained Cramér-Rao bound with a singular Fisher information matrix," *IEEE Signal Process. Lett.*, vol. 16, no. 6, pp. 453–456, Jun. 2009.

[123] H. L. V. Trees, *Detection, Estimation, and Modulation Theory*, New York: John Wiley & Sons, 2004.

[124] S. S. Slijepcevic, S. Megerian, and M. Potkonjak, "Location errors in wireless embedded sensor networks: sources, models, and effects on applications," *Sigmobile Mobile Computing and Communications Review*, vol. 6, pp. 67–78, Jul. 2002.

[125] D. J. Torrieri, “Statistical theory of passive location systems,” *IEEE Transactions on Aerospace and Electronic Systems*, no. 2, pp. 183–198, 1984.

[126] A. Efrat, D. Forrester, A. Iyer, S. G. Kobourov, C. Erten, and O. Kilic, “Force-directed approaches to sensor localization,” *ACM Transaction on Sensor Network*, vol. 7, pp. 1–27, Oct. 2010.

[127] L. Girod, “A self-calibrating system of distributed acoustic arrays,” PhD dissertation, University of California Los Angeles, USA, 2005.

[128] A. A. Ahmed, H. Shi, and Y. Shang, “SHARP: A new approach to relative localization in wireless sensor networks,” in *Proc. IEEE International Conference on Distributed Computing Systems Workshops*, Jun. 2005, pp. 892–898.