THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

On modelling of postglacial gravity change

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Cover:
Red dots represent stations along the Fennoscandian gravity uplift lines (Section 2.3), blue dots the NKG/AG-network (Section 2.4) and white dots superconducting gravity stations (Section 2.4). Isolines show the rate of change of gravity [μGal/year] predicted with a linear relation to modelled land uplift rates (Paper III).

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Abstract

Glacial isostatic adjustment (GIA) is the Earth's response to glacial-induced load variations on its surface. This phenomenon can today be observed in, for example, North America and Fennoscandia. Different observables, such as vertical and horizontal deformation of the crust, relative sea level change and disturbances in the gravity field, contribute different and complemental information on the phenomenon. Knowledge of the gravitational component is important for understanding the underlying geodynamical processes. Further, accurate predictions of the gravity change are important for e.g. reductions of geodetic observations to a reference epoch.

During the last decade, efforts to observe the surface gravity change in Fennoscandia have been intensified and the observational accuracy successively improved. This offers new possibilities at the same time as it puts new demands on modelling. The purpose of this thesis is to study some aspects of the modelling of GIA-induced gravity change.

We show that gravity stations close to the sea are affected by non-tidal sea level variations and that the direct attraction from the sea water constitutes a crucial contribution. Accurate modelling of the direct attraction from sea water is an intricate matter. We use different methods to model the direct attraction from GIA-induced sea level change and show that standard methods are not adequate. Further we solve the forward GIA modelling problem and show numerically how predictions of the gravity change are affected using different approximate methods. We also investigate the relation between gravity change and vertical displacement as they are predicted with the earth model depending on different sets of structural and rheological parameters. A linear relation is used as a reference. Deviations from the linear approximation are small, especially in Fennoscandia. The relation differs more between different regions included in the study than between different earth models within each region.

The thesis also includes an overview of observational efforts to determine the postglacial land uplift and gravity change in Fennoscandia, as well as a general discussion on some modelling issues that form the background for the motivation of the thesis.

Keywords: Glacial Isostatic Adjustment, postglacial rebound, gravity change, sea level change

List of appended papers

This thesis is based on the work contained in the following appended papers, referred to in the text.

I. Per-Anders Olsson, Hans-Georg Scherneck, and Jonas Ågren. *Effects on gravity from non-tidal sea level variations in the Baltic Sea.* Journal of Geodynamics, 48:151-156, 2009.

II. Per-Anders Olsson and Martin Ekman. *Crustal Loading and Gravity Change during the Greatest Storm Flood in the Baltic Sea.* Small Publications in Historical Geophysics, 19:1-10, 2009.

III. Per-Anders Olsson, Hans-Georg Scherneck and Jonas Ågren. *Modelling of the GIA-induced surface gravity change over Fennoscandia.* Journal of Geodynamics, 61:12-22, 2012.

IV. Per-Anders Olsson, Glenn Milne, Hans-Georg Scherneck and Jonas Ågren. *The relation between gravity rate of change and vertical displacement in previously glaciated areas.* Submitted to Journal of Geodynamics.

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I would like to address the greatest thanks and appreciation to my supervisors *Hans-Georg Scherneck* for constant support, valuable guidance, challenging allures and always encouraging words, and *Jonas Ågren* for excessive engagement and long and fruitful discussions - without You this work would not have been possible. In the final stage *Holger Steffen* enriched our group. I wish this had happened earlier.

Via the Training School on GIA Modelling, organized by the *COST Action ES0701* in 2009, I was first introduced to GIA modelling, on which the major part of this dissertation relies.

I would also like to thank *Lantmäteriet* for giving me the opportunity to do research and increase my knowledge within this field. I appreciate that a lot.

Finally,
Mom and Dad, thank you for everything,
Lisa, Johanna, Jakob, Ida and Märta - you are the best.
Love you all!

Contents

1 Introduction

Glacial cycles have come and gone with a period of approximately 100,000 years. The last glacial cycle started about 110,000 years ago when the climate turned colder and continental ice sheets started to grow, especially in the northern hemisphere. Around 20,000 years ago they reached their maximum, known as last glacial maximum (LGM) (Figure 1). Then warmer climate led to a relatively quick melting of the ice and some 8000 years ago the massive ice sheets in North America, Fennoscandia and Barents-Kara-Seas were gone. The amount of frozen water that was stored in the melted continental ice sheets and further mountain glaciers, if evenly distributed over the oceans, would correspond to a global sea level rise of ~ 130 meters.

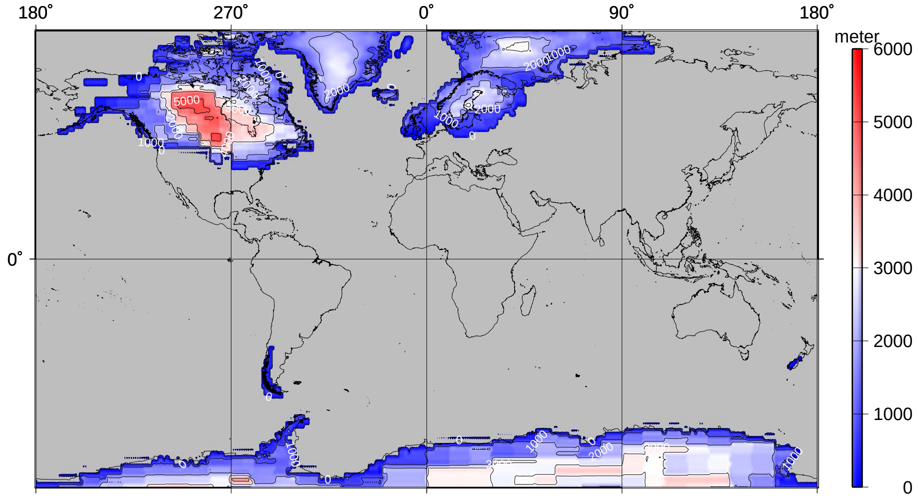

Figure 1: Thickness of continental ice sheets at last glacial maximum according to the ice model ICE-5G (Peltier, 2004).

Under the load of the ice, the Earth was deformed, the surface was depressed, and when the load disappeared the Earth started to rebound. This postglacial rebound, or glacial isostatic adjustment (GIA), is still going on and can be studied in e.g. North America and Fennoscandia.

There are two main reasons why we are interested in the GIA phenomenon: (1) we have to deal with its direct consequences, such as the ongoing deformation of the surface of the Earth, changing coastlines, and changes in the stress state of the crust that may lead to earthquakes;(2) by studying GIA we gain knowledge of the phenomenon as such, of the ice history and of the physical structure of the Earth.

1.1 Background

This dissertation is the result of a cooperation between Lantmäteriet (the Swedish mapping, cadastral and land registration authority) and Chalmers University of Technology.

Lantmäteriet is responsible for the geodetic infrastructure in Sweden. This responsibility includes establishment and maintenance of geodetic reference frames for gravity and positioning. Since Sweden is located in the centre of the Fennoscandian postglacial rebound area, the effects of postglacial rebound have to be considered when designing, maintaining and using geodetic reference frames. In the strategic plan for Lantmäteriet's geodetic activities 2011-2020 (Lantmäteriet, 2011), it is stated that:

> "In our part of the world we have land uplift as a result of the latest ice age. - - - In addition to its scientific value, the knowledge of these processes is of considerable value for the maintenance of our national reference systems. - - - We have - - - an international responsibility to provide researchers with the best possible geodetic observations - primarily GNSS observations at permanent reference stations as well as gravity changes."
>
> "We plan to - - - extend our R&D activities concerning geophysics-based models for land uplift. Further develop theories and methods for implementing deformation models in the maintenance of our reference systems."
>
> "Lantmäteriet's geodetic activities mirror a strong and genuine interest for continuing and increasing cooperation with Onsala Space Observatory."

Onsala Space Observatory, located south of Gothenburg in Sweden, is the Swedish National Facility for Radio Astronomy and an important geodetic fundamental station. It is hosted by the Institution for Earth and Space Science at Chalmers University of Technology. Besides an extensive equipment park for astronomical purposes, the observatory also conducts research related to geodesy and geodynamics and provides e.g. VLBI (very long baseline interferometry), GNSS (global navigation satellite system) and gravity observations for scientific purposes.

Gravity is, among many others, an important observable of postglacial rebound. It contributes with unique information on the underlying physics, such as the redistributions of masses within the Earth, as well as information on the location of the centre of mass (CM) of the Earth.

During the first years of the 21st century, the activity concerning absolute gravity (AG) observations of the GIA-induced gravity change in Fennoscandia increased. Various institutions contributed with observations, coordinated by the Nordic geodetic commission (NKG) (see Section 2.4). In 2006, Lantmäteriet invested in an FG5 absolute gravimeter. One of the main purposes with this investment was to contribute to, continue and ensure long time series of repeated observations of the GIA-induced absolute gravity change. About the same time Chalmers installed a superconducting gravimeter (SG) at Onsala Space Observatory, capable of measuring temporal changes in the gravitational field of the Earth at the 10^{-10} level, or 0.1 part per billion (Virtanen, 2004). With these new possibilities to observe gravity variations, a need to strengthen the knowledge related to GIA modelling and gravity change was recognized. What do we observe and why? Chalmers and Lantmäteriet decided to establish a PhD position, led by Chalmers and financed by Lantmäteriet. This dissertation is the result of that PhD position.

The mobilization of new resources for observations of the GIA-induced gravity change was not intended to be a standalone effort, but should be seen as an integrated part of a very long tradition of observations of the postglacial rebound in Fennoscandia. Scientific observations of the land uplift in Fennoscandia started with relative sea level observations in the 18^{th} century and was supplemented with repeated national levelling campaigns during the 20^{th} century . Since the 1990's, a network of permanent GNSS stations is monitoring the three-dimensional deformation of the crust. The GIA-induced gravity change was first observed with repeated relative gravity campaigns during the second half of the 20^{th} century. During the last decades they have gradually been superseded by repeated absolute gravity observations. These observational efforts form the basis for what we know about the postglacial rebound in Fennoscandia today and in Section 2 they are described in more detail.

An increasing amount of observations of different kinds of GIA observables, emphasize the question of how to relate and combine them. Relative sea level observations, by means of mareographs, measure the vertical displacement of the crust relative to the sea level; with repeated levelling campaigns the vertical displacement is measured relative to the geoid; permanent GNSS stations measure the deformation of the crust relative to a global geodetic reference frame (e.g. ITRF), and inferences of the vertical deformation of the crust from repeated observations of the surface gravity, are made relative to the centre of mass of the Earth. Another issue related to the interpretation of the observations, is how to interpolate between discrete observations, in time and space. These problems are preferably dealt with using appropriate modelling methods. In Section 3 we deepen the reasoning around these questions in general and discuss different modelling approaches.

Observations and modelling of the GIA-induced surface gravity change, in particular, are coupled to some specific questions related to the complex composition of the signal. The gravity signal is a compound of contributions from e.g. redistribution of masses within the Earth, vertical displacement of the crust, redistribution of GIA-related masses on the surface of the Earth (ice and water) and external effects. With external effects we mean redistributions of masses not related to GIA, such as e.g. sea level variations, continental water (ground water) variations and atmospheric pressure. The complex nature of the gravity signal should be recognized when observations and modelled predictions are interpreted.

1.2 Purpose and structure of the dissertation

The purpose of this dissertation is to contribute to the understanding of observed surface gravity changes, the composition of the signal, how it is accurately modelled, and how it is related to the vertical deformation of the crust. Focus is mainly on Fennoscandia although most of the conclusions are general for all previously glaciated areas with postglacial rebound. The main results are presented in four appended articles dealing with (I) the effect on gravity from nearby sea level variations, (II) a case study of the effects on gravity from an extremely large sea level change, (III) how sensitive predictions of the rate of change of gravity are to some modelling assumptions and approximations and finally (IV) the relation between the rate of change of gravity and the vertical uplift rate. A summary of the appended articles and the main conclusions are given in Section 4.

The purpose of Sections 2-3 is to put the articles in a historical and scientific context. Section 2 is a short introduction to important observational efforts to study the postglacial rebound phenomena in Fennoscandia. Section 3 deals with modelling and highlights some issues, related to modelling, that motivates this dissertation. Finally, in Section 5, we give some conclusive comments and suggestions for future work.

2 Observational efforts

In Fennoscandia, the postglacial rebound has been studied for a few hundred years starting with relative sea level observations in the 18^{th} century. The reason behind the observed relative sea level fall (or land uplift) was initially under debate but during the second half of the 19^{th} century the idea of postglacial rebound, first suggested by Jamison (1865), became generally accepted. Vening Meinesz (1934) and Haskell (1935) used uplift data from Fennoscandia to do the first estimates of the viscosity of the mantle. For a thorough review of the early history of modelling and observations of GIA see e.g. Ekman (2009). During the 20^{th} century the accuracy of land uplift observations has increased by means of longer time series as well as the introduction of other/new types of observational techniques, such as e.g. repeated levellings and during the last decades continuous GNSS observations and repeated gravity observations. Modelling of the GIA phenomenon has developed and is now able to predict GIA observables, such as the rate of land uplift or relative sea level change, on the millimeter/year level, or better.

The purpose with this section is to give an overview of important efforts to determine and study the postglacial rebound in Fennoscandia. Since the middle of the 20^{th} century, much of this work have been coordinated by the Nordic Geodetic Commission and therefore one subsection is dedicated to a short description of this association. The whole section will serve as the background for understanding where we are today. It is also the basis for the discussions in the next section concerning issues related to the choice of appropriate modelling approaches and for what we can expect from our observations of the surface gravity change. A comprehensive review of data and modelling in Fennoscandia can be found in Steffen and Wu (2011).

2.1 Empirical land uplift modelling (from geodetic observations)

For many hundred years, people in Scandinavia, inhabiting the coastlines, have been observing the postglacial land uplift in terms of relative sea level decrease. In the early 18^{th} century the Swedish astronomer and geodesist, Anders Celsius (1701-1744), started scientific investigations of the phenomenon based on sea level observations. Since the late 19^{th} century the sea level relative to the solid crust have been recorded at a large number of sea level stations around the Baltic Sea. Some 30 of these stations have continuous sea level records spanning over 100 years (Ekman, 1996) and one station, Stockholm, is spanning over 200 years and thereby constitute the world's longest sea level series (Ekman, 2003). Thorough descriptions of the historical land uplift research in this region is given by in Ekman (1991) and Ekman (2009).

During the last century repeated national levellings in the Nordic countries have been performed and thereby observations of land uplift have been extended from the coastline to the inland. Mäkinen and Saaranen (1998), for example, determine the postglacial land uplift from three precise levellings in Finland.

Figure 2 shows an, often referred to, land-uplift map constructed by Ekman (1996) from a combination of sea and lake level records and repeated levelling.

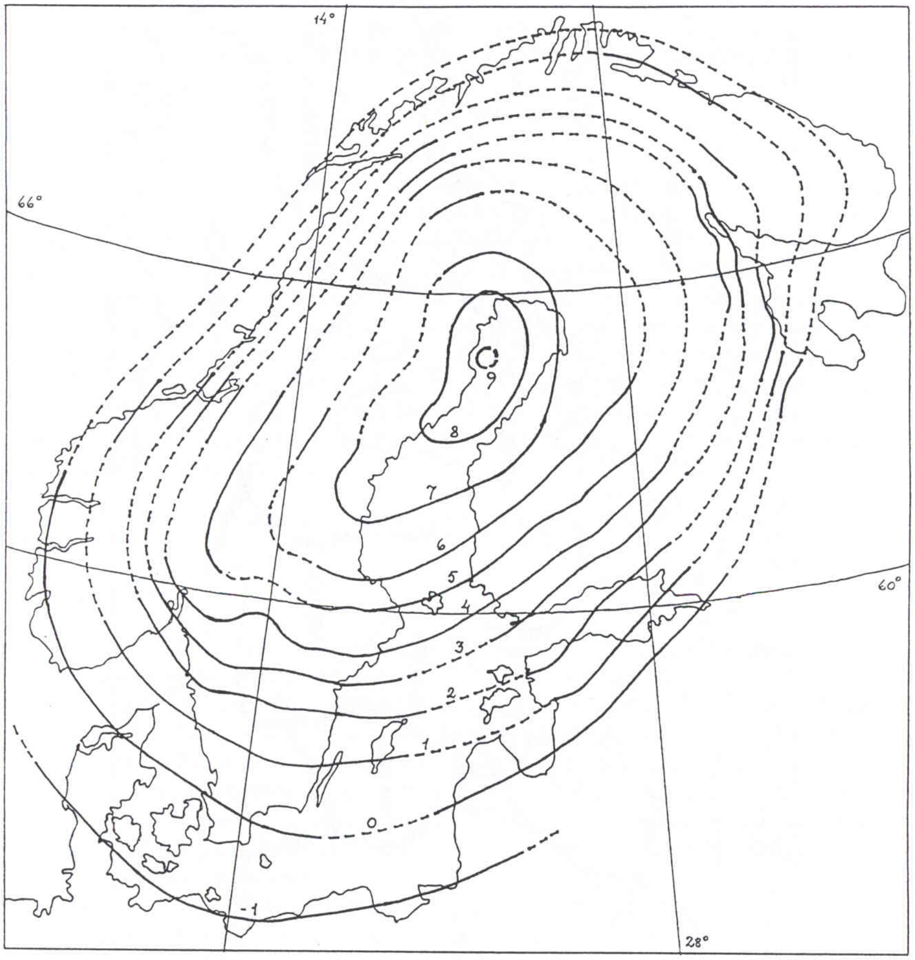

Figure 2: Empirical model of the apparent land uplift (relative to sea level) based on sea and lake level records and repeated levellings (Ekman, 1996). Dashed lines indicate interpolation, made by the author, between observations.

2.2 The Nordic Geodetic Commission

The Nordic Geodetic Commission (NKG) is an association, founded in 1953, recognized and supported by a number of organizations in the Nordic countries (Denmark, Finland, Iceland, Norway and Sweden), such as universities and national mapping authorities. The purpose of this association is to promote research, data exchange and cooperation between geodesists in the Nordic countries. Organizations from neighboring countries, such as the Baltic countries, Germany and Poland, occasionally participate in projects coordinated by NKG.

The NKG, acknowledging the importance of gravity change as a geodetic and scientific problem, delegated the responsibility to measure the phenomenon to a dedicated Working Group of Geodynamics, which has devoted its efforts in studying the effects within a broad range of questions and methods, like obtaining accurate earth tide models, agreeing on data reduction algorithms, looking into alternative kinds of instrumentation like water tube tiltmeters. The group started its work in 1967 and is still active.

The empirical land uplift model of NKG, NKG2005LU (bottom panel in Figure 8), is based on geodetic observations (Ågren and Svensson, 2007; Vestøl, 2006). In addition to sea level observations and repeated levelling, it includes GNSS observations from the BIFROST-network (see section 2.5) and also make use of a geophysical land uplift model by Lambeck et al. (1998) (see section 3.3) for extrapolation outside the observations. NKG2005LU was used in the adjustment of the new national height systems in Sweden (RH2000), Finland (N2000) and Norway (NN2000) as well as in the latest realization of the European Vertical Reference System, i.e. EVRF2007 (Sacher et al., 2008).

2.3 Fennoscandian land uplift gravity lines

A major project that was initiated and coordinated by NKG are the Fennoscandian land uplift gravity lines. They consist of four east-west high precision relative gravity profiles across the Fennoscandian postglacial rebound area, approximately following the latitudes 65°, 63°, 61° and 56°N (Figure 3) (Mäkinen et al., 1986). Measurements along the Finnish part of the 63° line started in 1966 (Kiviniemi, 1974) followed by the rest of the lines from the mid 1970s.

At the time of the establishment of the land uplift gravity lines, the geometrical land uplift was relatively well known from sea level observations and repeated levelling campaigns (section 2.1). The purpose of the gravity measurements was to determine the rate of change of gravity, $\dot{g}$, and compare this with the absolute land uplift rate, $\dot{u}$. From the ratio $\dot{g}/\dot{u}$, conclusions could then be reached concerning the underlying geodynamic processes (Kiviniemi, 1974).

From these observations Ekman and Mäkinen (1996) found the ratio between the rate of change of gravity $\dot{g}$ and the land uplift rate $\dot{u}$ to be -0.204 ± 0.058 μGal mm^{-1} (1 Gal = 0.01 m/s^2). This was later revised by Mäkinen et al. (2005), based on longer time series, to be between -0.16 and -0.20 μGal mm^{-1}. Based on this ratio and a number of approximations, for instance that the gravity change due to the mass flow in the mantle can be properly described by means of *one* planar Bouguer plate, they made the conclusion that the GIA process in Fennoscandia includes inflow of additional mantle masses.

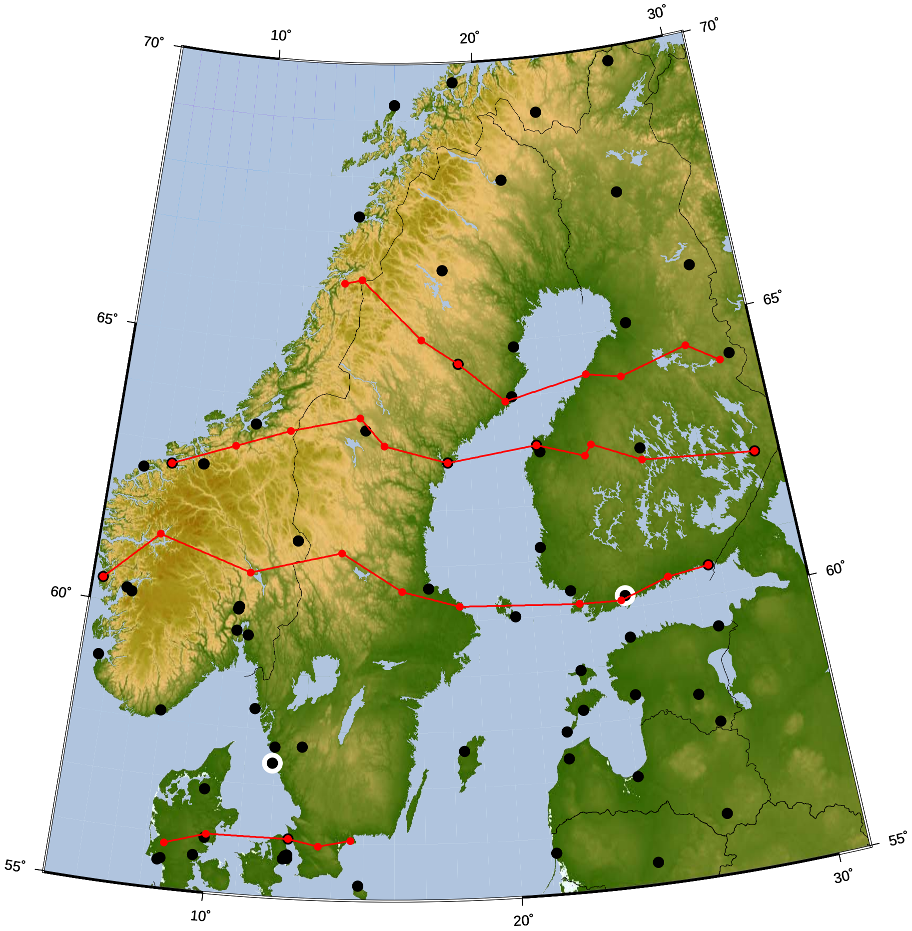

Figure 3: Red dots and lines show the Fennoscandian land uplift gravity lines (Mäkinen et al., 2005). Black dots show absolute gravity stations in the NKG/AG-network and white dots the superconducting gravimeter stations.

2.4 Nordic absolute gravity project

During the last decades the relative gravity observations have been succeeded by absolute gravity (AG) observations.

The history of absolute gravity observations in Fennoscandia (Mäkinen et al., 2010) starts with six stations observed with the Italian instrument IMGC by Istituto de Metrologia (Turino) in 1978 and two stations with the GABL instrument by USSR Academy of science in 1980.

From 1988 (-2002) the Finnish Geodetic Institute (FGI) carried out repeated measurements in Finland with a JILAg instrument and 1993 and 1995 the National Oceanic and Atmospheric Administration (NOAA), USA, and Bundesamt für Kartographie und Geodäsie (BKG), Germany, observed a number of stations with the successor instrument FG5 (free fall gravimeter manufactured by Micro-g Lacoste Inc., Colorado USA) (Niebauer et al., 1995).

In 2003 - 2008 comprehensive campaigning was carried out with an FG5 instrument by Institute für Erdmessung (IfE), Germany (Gitlein, 2009). During that time also FGI, Norwegian University of Life Sciences (UMB) and Lantmäteriet (the Swedish mapping, cadastral and land registration authority) invested in FG5 gravimeters and started with repeated AG observations. This work, hereafter referred to as the Nordic absolute gravity project, has been coordinated by NKG. The observations are performed in the so-called NKG/AG-network, consisting of a number of stations suitable for AG observations, shown in Figure 3. Most of the stations are co-located with permanent GPS-stations (see Section 2.5 below) and some of them with tide gauges.

At present observations in the NKG/AG-network are carried out repeatedly on the stations by FGI, UMB and Lantmäteriet and occasionally by IfE and BKG. The stations are intended to be reoccupied with one or a few years intervals. One long-term goal (10-30 years) of this project is a high accuracy model of the GIA-induced gravity change for Fennoscandia. Another goal is to contribute with observations to the GIA modelling research in the area. A subset of the AG stations is also planned to be used to establish new gravity reference systems in the future.

Within the area two superconducting gravimeters are installed. One at Onsala in Sweden (operated by Chalmers University of Technology) and one at Metsähovi in Finland (operated by FGI) (Virtanen, 2006). They complement the Nordic absolute gravity project with continuous observations of the gravity change providing an excellent possibility to study external effects from e.g. variations in atmospheric pressure, ocean load and continental water storage. They also serve as suitable stations for comparisons between absolute gravimeters.

2.5 BIFROST

The BIFROST (Baseline Inference for Fennoscandian Rebound) project was initiated 1993. With a network of permanent GNSS reference stations in Fennoscandia, the three-dimensional deformation of the crust is continuously measured. The network originally consisted of the Swedish (SWEPOS) and Finnish (FinnRef) national permanent GPS networks, established in 1991-1992 and 1994-1996, respectively. Later solutions also include stations in Norway (SATREF), Denmark and a selection of stations in northern Europe (Lidberg et al., 2010) (Figure 4). In Scherneck et al. (2002) a thorough description of the network and its history is given.

Solutions of the three-dimensional rates at the stations in the BIFROST network have been published in a number of articles, e.g. Johansson et al. (2002) and Lidberg et al. (2007, 2008, 2010). The results from Lidberg et al. (2010) are presented in Figure 4. Also a number of geophysical investigations, based on these BIFROST solutions, have been published, e.g. GIA modelling (see further Chapter 3.3) and inferences on the mantle viscosity profile by Milne et al. (2001, 2004) and Hill et al. (2010) and determination of a Fennoscandian strain rate field by Scherneck et al. (2010). Milne et al. (2004) also concluded that BIFROST data provide distinct constraints on mantle viscosity. As a function of depth in the mantle the resolving power varies from ~ 200 km to ~ 700 km but provide little constraint on viscosity in the bottom half of the mantle (Milne et al., 2004).

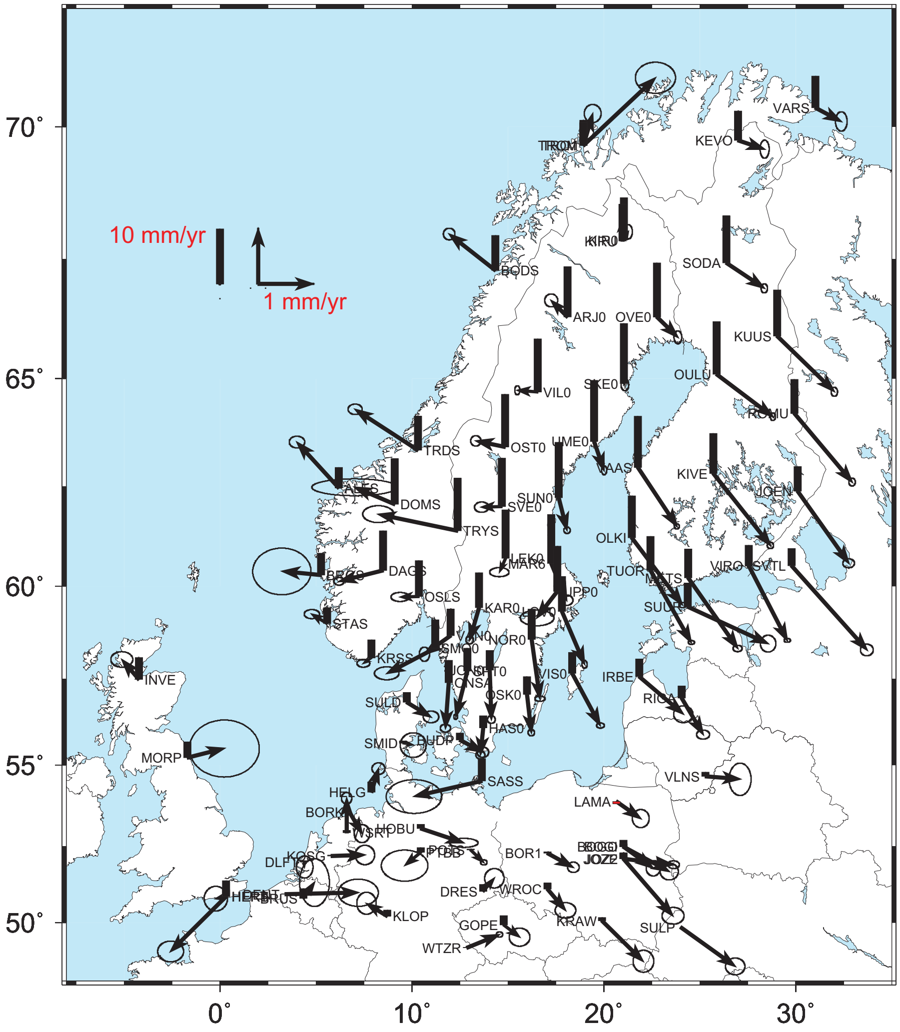

Figure 4: The BIFROST velocity field from Lidberg et al. (2010). The legend shows 10 mm/year in the vertical and 1 mm/year in horizontal components.

3 Modelling

All observations, of various kinds, of the glacial isostatic adjustment are interpreted by means of models (Figure 5). The physical prerequisites that control the GIA phenomenon, however, are too many and too complex to be understood and described in detail. We make sampled observations, associated with errors, and develop simplified models based on assumptions and approximations.

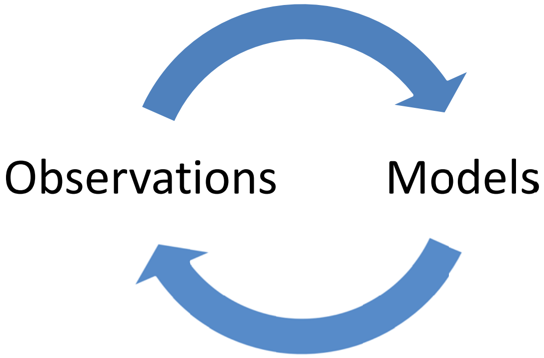

Figure 5: Observations are interpreted with models. Models are improved, revised, confirmed or rejected by observations.

In this section we discuss and give examples of some different types of models of relevance for GIA research in Fennoscandia. We make a distinction between empirical models and geophysics-based GIA models, we also emphasize some specific issues related to the GIA-induced gravity change. The purpose with this section is to substantiate and illustrate some issues discussed within the Nordic geodetic community during the last decade. These issues were important as a motivation for this PhD thesis. We end this section with a summary of prerequisites for this dissertation.

3.1 Empirical and geophysical models

Within NKG, and in this dissertation, the term *empirical model* occurs. With empirical model we mean a model whose purpose is to compile observations to get a unified picture of the same phenomenon that we observe (Figure 6). These models are typically based on carefully evaluated observations and statistical methods to filter out unwanted errors and to interpolate between the sample observations. Empirical models describe a certain observable (e.g. relative sea level change) and do not discriminate between different mechanisms that contribute to the total signal (e.g. vertical displacement of the crust, geoid change and change in ocean volume). The term empirical model sometimes refers to the predictions themselves ("Output" in Figure 6), e.g. gridded values of the land uplift rate, rather than the underlying observations and the

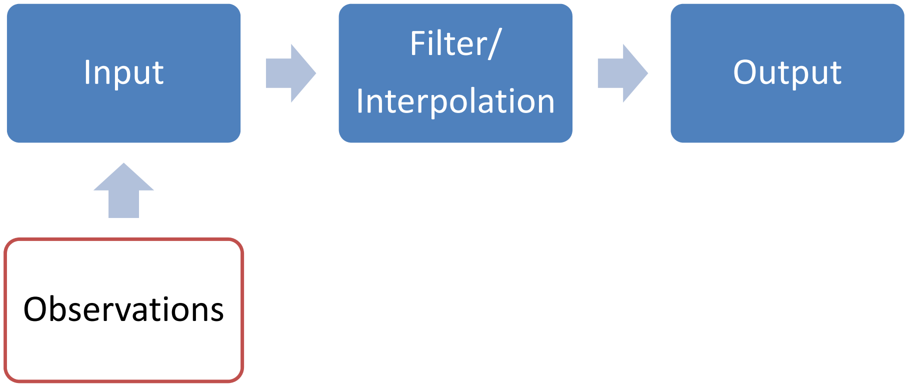

Figure 6: Empirical model.

mathematical methods used. Although empirical models do not explain the underlying mechanisms that cause the phenomenon that is observed, they are useful for practical geodetic purposes. In Fennoscandia, empirical models of the land uplift rate (Section 2.1) have typically been used to reduce observations from different time epochs to a common reference epoch.

In Figure 7 two examples of empirical models are presented. One is an estimated trend of the relative sea level change in Stockholm. From annual means of relative sea level observations (Ekman, 2003, 2009), the trend has been estimated by means of linear regression. Unwanted short-term variations have been filtered out and the estimated trend (empirical model) can be used to predict annual mean sea level in Stockholm at an optional time.

The other model in Figure 7 is more complex. It is a compilation of different types of observations of the land uplift in Fennoscandia. Repeated levelling, tide gauge observations and continuous GNSS observations have been combined and evaluated by means of least squares collocation (Vestøl, 2006). This model was developed as one important step towards an official NKG land uplift model, urged for construction of new national and regional height systems in the Nordic are (see e.g. Sacher et al., 2008; Ågren and Svensson, 2007).

The empirical models discussed above are based on observations and mathematical/statistical methods. Insights in physical conditions and constraints might indicate shortcomings of these models - or help to improve them. The top panel in Figure 8 shows the same data set as the top panel in Figure 7. Now *two* regression lines have been used to estimate the relative sea level trend, one for the years 1774-1865 (the dashed line is an extrapolation of this trend to year 2000), and one for the years 1865-2000. This approach is based on the assumption that since 1865 the local, GIA-induced relative sea level change has been complemented by a global increase of the ocean volume, induced by global warming as a result of the industrial revolution, resulting in decreased relative sea level trend in Stockholm.

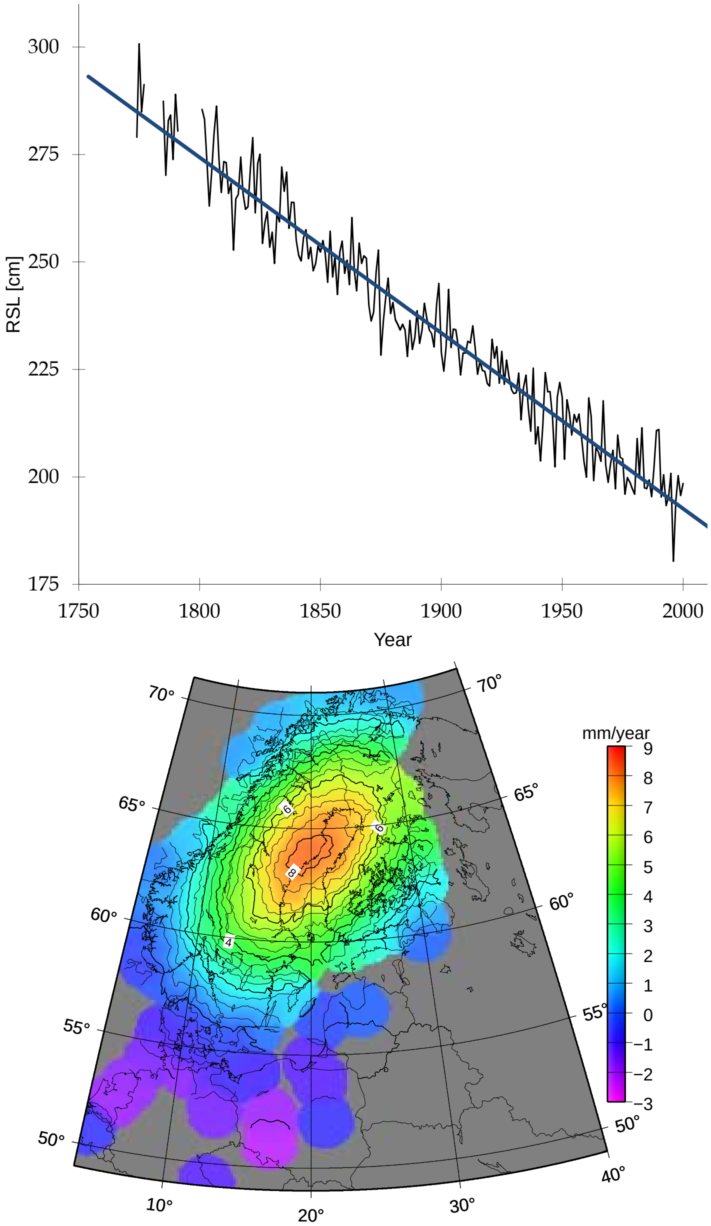

Figure 7: Two examples of empirical models; the top model is a straight line representing the relative sea level change in Stockholm (Ekman, 2003, 2009), found from linear regression of annual means of tide gauge observations; the bottom model bottom is a land uplift model, based on least squares collocation of different types of observations (Vestøl, 2006; Ågren and Svensson, 2007).

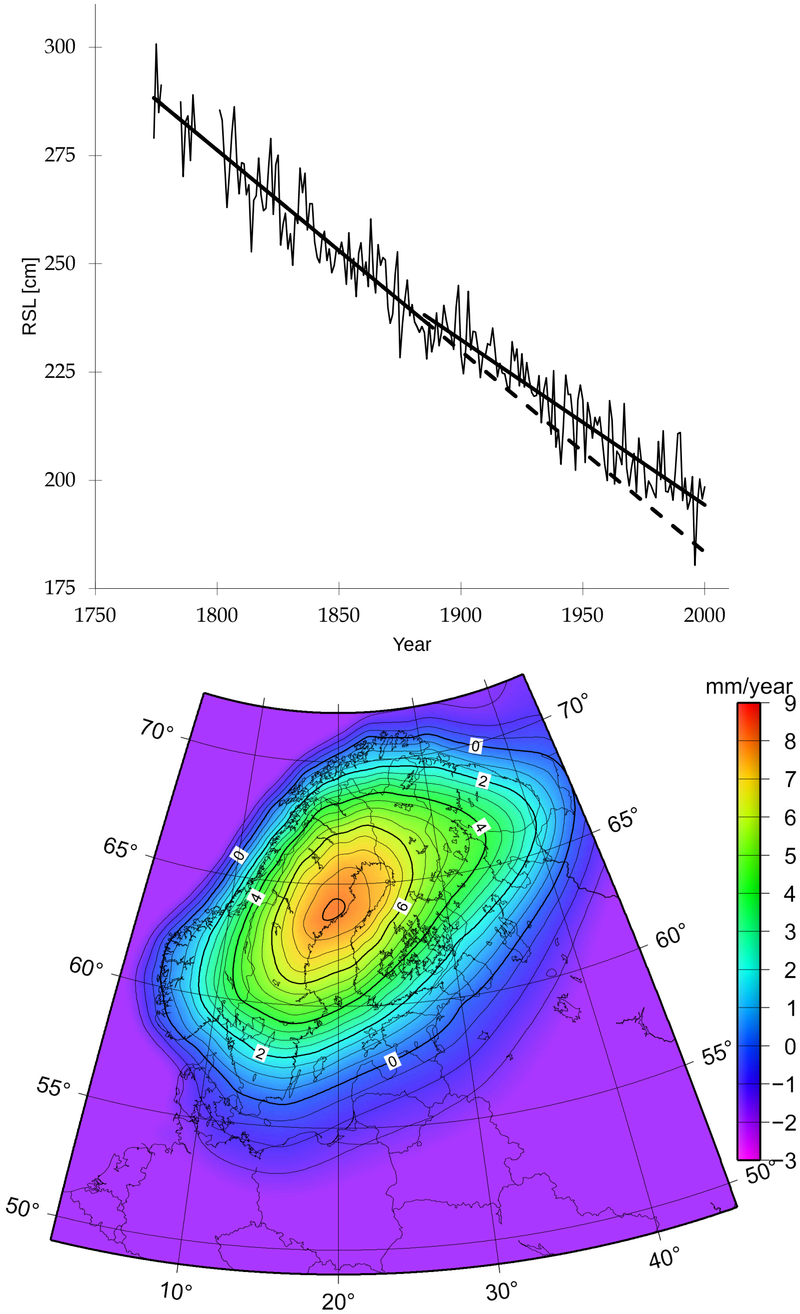

Figure 8: The same empirical models as in in Figure 7 but now refined based on insights in physical conditions and constraints (see text).

Further, the Vestøl model in the bottom panel in Figure 7 also raises some questions related to the underlying physics. Are the irregularities in the isolines physically relevant or are they caused by observational errors? How do we extrapolate outside the area covered by observations? These questions demonstrate some limitations with purely empirical models. In order to overcome these limitations Ågren and Svensson (2007) applied smoothing algorithms to the Vestøl (2006) model and combined it with a geophysics-based GIA model (see Section 3.2) of Lambeck et al. (1998) (see Section 3.3). This resulted in the official NKG land uplift model NKG2005LU shown in the bottom panel in Figure 8 (see also Section 2.2). This model can be regarded as a mix of an empirical model and different physical assumptions.

The Lambeck et al. (1998) model, mentioned above, is a *physical model* in more traditional meaning. With physical model we mean a model based on a theoretical framework of physical assumptions (Figure 9). This type of model is typically used to make predictions of observables (forward modelling), based on a number of input parameters (which values, in turn, often are based on observations). By comparing the modelled predictions with observations, conclusions can preferably be done on the presumed theoretical framework and/or input parameters. If the input parameters are sought the model can be run "backwards" (inverse modelling), i.e. from the right to the left in Figure 9. Inverse modelling often tend to have ambiguous solutions of the sought parameters, i.e. different sets of input parameters in the forward modelling can result in the same predictions of the observables. The term GIA model, in this dissertation always refer to this type of physical model.

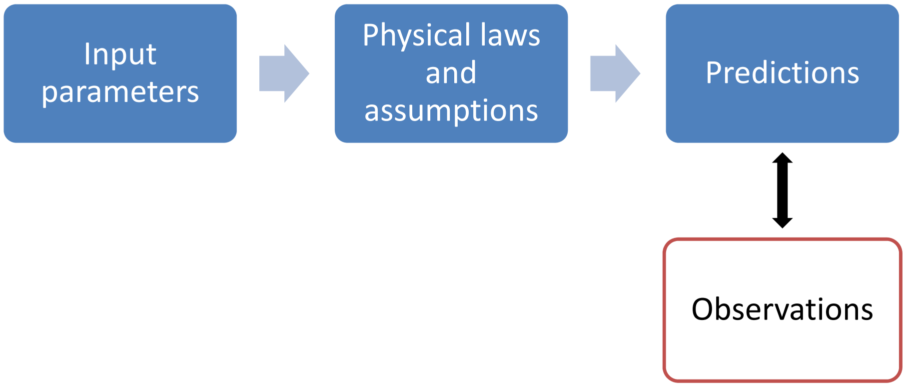

Figure 9: Physical forward modelling.

Within NKG, observational efforts and empirical modelling has been an important part of the of work. Geophysics-based GIA modelling, on the other hand, has been limited. GIA modelling efforts in Fennoscandia have often been carried out by research groups from other parts of the world. In Section 3.3 we present some important GIA modelling efforts in Fennoscandia. But first a short review of some of the most important concepts of GIA modelling.

3.2 GIA modelling concepts

With GIA model we mean a geophysics-based model (Figure 10). GIA modelling normally consists of an ice- and an earth model and a number of physically meaningful constitutive equations that control how the Earth reacts on the surface loading from the ice. The GIA model then predicts e.g. crustal deformation, gravity disturbances, sea level variations and/or disturbances of the Earth's rotational vector (Figure 10). This section is a short presentation of some fundamental concepts related to GIA modelling which are of importance for the following discussions. A state of the art report of GIA theory is given by Whitehouse (2009).

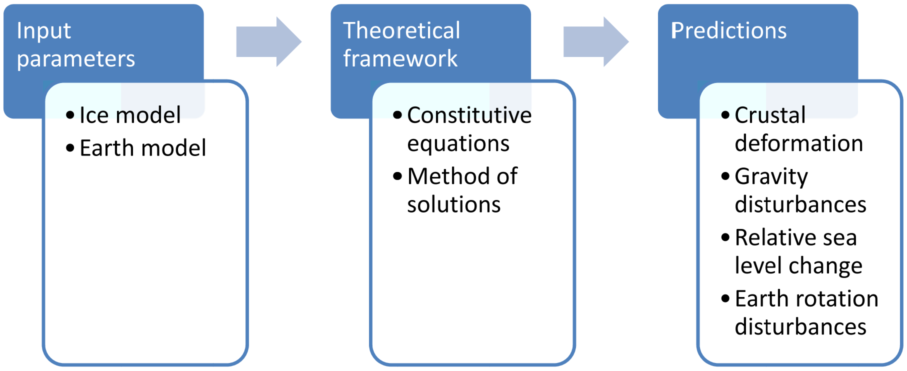

Figure 10: GIA model.

The *ice model* describes the thickness of the ice (the load) and is allowed to vary in time and space. Depending on the premises under which the ice model has been constructed it can be categorized as geophysics-based or climate driven.

Geophysics-based ice models are constructed by means of GIA modelling and are typically tuned to geological constraints like ancient sea level indicators. Examples of important geophysics-based ice models are the series of consecutive global models ICE-3G (Tushingham and Peltier, 1991), ICE-4G (Peltier, 1994, 1996) and ICE-5G (Peltier, 2004) and the regional model, explicitly tuned to observations in Fennoscandia, by Lambeck et al. (1998). The regional model is also a part of global model consisting of a compilation of regional models. Further, the ICE-5G model has a successor, ICE-6G, but at the time of this writing it has not yet been published.

Climate driven ice models are based on thermo-dynamical ice sheet models driven by paleoclimate data. The so-called Näslund ice model (SKB, 2010, Figure 11) is an example of a climate driven model for Fennoscandia.

The physical properties of the *earth model* are defined by a set of parameters, e.g. density, rigidity and viscosity, which are allowed to vary in one, two or three dimensions. A set of constitutive equations, fundamental physical laws, then governs how the earth model reacts to the surface loading. Depending on the

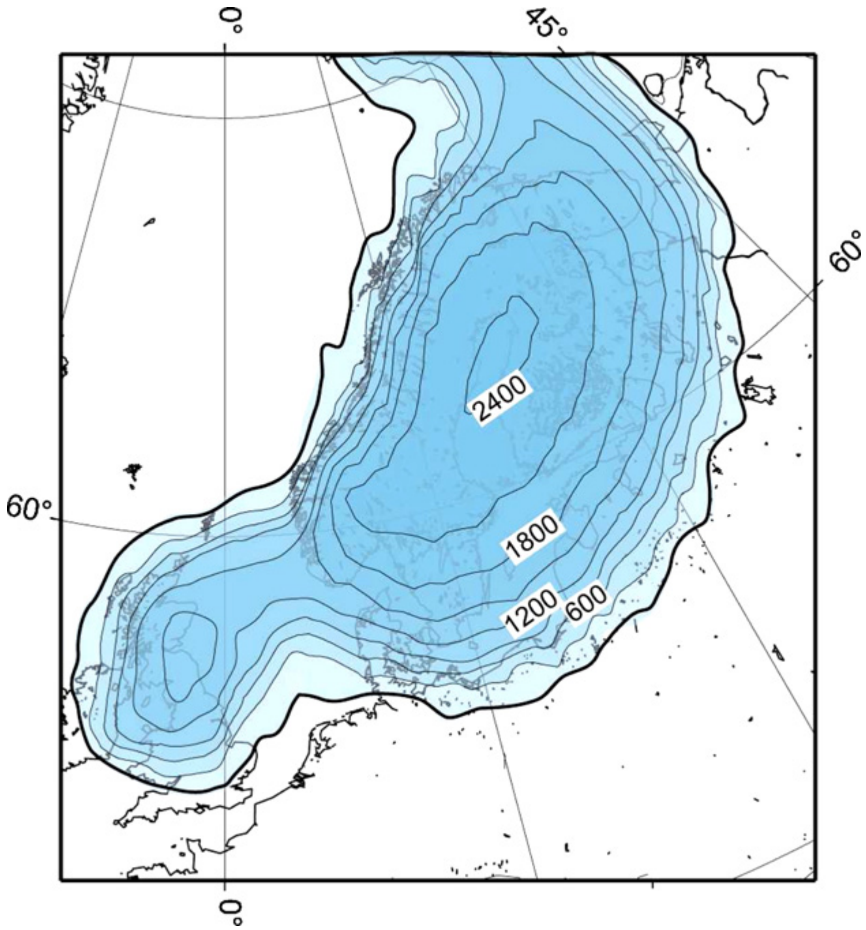

Figure 11: Ice thickness [m] in Fennoscandia at last glacial maximum from the Näslund ice model (SKB, 2010). The figure is from Steffen and Wu (2011).

type of earth model and the method of solution of the constitutive equations, GIA models can be categorized in different paradigms. The two prevailing GIA modelling paradigms today are the *normal mode method* and the *finite element method*.

The normal mode method solves the GIA problem for a one-dimensional, spherical, linearly viscoelastic earth. The physical properties are allowed to vary along the radius but are laterally homogenous. Ever since Peltier (1974) showed how the constitutive equations for a linearly viscoelastic earth, turn into the elastic problem (see e.g. Longman, 1962, 1963; Farrell, 1972) in the Laplace transform domain and can be solved there, this method has been predominant. It has been described, refined and developed in a large number of publications (see e.g. Cathles, 1975; Peltier and Andrews, 1976; Peltier, 1976; Wu, 1978; Wu and Peltier, 1982, 1983; Peltier, 1985; Sabadini and Vermeersen, 2004). The GIA models used in Paper III and Paper IV are based on the normal mode method and the preliminary reference earth model (PREM) (Dziewonski and Anderson, 1981).

The finite element method allows the physical properties to vary in two or three dimensions. Sabadini et al. (1986) first used the finite element method to study the effect of lateral variations for a flat, two-dimensional earth. Wu (2004) extended the method to a three-dimensional, spherical, self-gravitating earth. Because of limitations in the computational power, solutions with the finite element method have, compared to the normal mode method, been restricted either in spatial extend and/or grid resolution. However, since seismological results (e.g. Gregersen et al., 2002; Kozlovskaya et al., 2008; Janik et al., 2009) show that the mantle viscosity and lithospheric thickness vary laterally in Fennoscandia, the importance of three-dimensional modelling (see e.g. Schmidt, 2004) will likely increase as the computational power increases.

Irrespective of the choice of earth model and method of solution, the loading of the surface of the earth is primarily defined by the ice model. Accumulation or ablation of ice, infer redistribution of masses from the continental ice sheets to the ocean water, or the opposite. These masses are not evenly distributed over the sea but are affected by the deformation of the crust (sea floor) and disturbances in the gravitational field. The new distribution of the ocean water constitute a surface load change itself. Farrell and Clark (1976) first showed how this redistribution of loads from continental ice to ocean water should be included in the GIA modelling via the so-called sea level equation (SLE). In our GIA model (Paper III and Paper IV) we have implemented a generalized form of the sea level equation (following Mitrovica and Milne, 2003; Kendall et al., 2005) which, contrary to the original form, includes migrating shorelines. For a description of our model implementation, especially concerning SLE, see further Paper III and Olsson (2011).

The GIA modelling approaches described in this section can be used to predict different observables, such as vertical or horizontal deformation of the crust, relative sea level changes or disturbances in the gravity field. By comparing these predictions with observations, inferences can be made on the adopted ice and earth model parameters. One important contribution from GIA research is to put constraints on the viscosity profile of the Earth. In the next section we describe some important modelling efforts in Fennoscandia.

3.3 Examples of GIA modelling efforts in Fennoscandia

The observational efforts and empirical land uplift modelling, described in Section 2, are all based on various geodetic observations. The main purpose of these efforts have been to predict present-day rates, e.g. of the vertical motion of the crust or perturbation of the gravity field, for geodetic purposes. However, these observations can also be used for constraining geophysics-based GIA modelling and inferences on the ice history or earth rheology, typically the thickness of an elastic lithosphere and the viscosity of the underlying mantle. In this section we give some examples of geophysical GIA modelling efforts in Fennoscandia. For a more extensive review, see Steffen and Wu (2011).

The first attempt to determine the viscosity of the mantle from GIA observations was performed by Vening Meinesz (1934) (Ekman, 2009). He used sea level and gravity observations from the centre of the Fennoscandian uplift area, and assumed the mantle to be a homogenous, highly viscous, Newtonian fluid. He found that the viscosity, η, should be $\sim 4 \cdot 10^{21}$ Pa s. Next year, Haskell (1935), found $\eta = 10^{21}$ Pa s. This value was constrained by geological evidence of ancient relative sea change. Both of these early estimates of the mantle viscosity are remarkably close to later and resent estimates of the same.

Of the modern GIA modelling efforts, the one by Lambeck et al. (1998) has been of importance for the Nordic geodetic community. This modelling is based on the normal mode method and includes construction of a regional ice model and a vertical viscosity profile for Fennoscandia. The lithospheric thickness was inferred to be $LT \approx 75$ km, the upper mantle viscosity $\eta_{UM} \approx 0.36 \cdot 10^{21}$ Pa s and the lower mantle viscosity $\eta_{LM} \approx 8.0 \cdot 10^{21}$ Pa s. This model was explicitly constrained by geological evidence of historical shorelines in Fennoscandia. Predictions of the land uplift rate from this model were incorporated in the empirical NKG2005LU model (see Section 2.2 and 3.1) for the purpose of extrapolation outside the area of observations.

Other important modelling efforts are Milne et al. (2001, 2004); Lidberg et al. (2010) related to the BIFROST project (see Section 2.5). The ice model used in these studies is a combination of the Lambeck et al. (1998) model over Fennoscandia and ICE-3G (Tushingham and Peltier, 1991) for far field ice sheets. Via GIA modelling, the viscosity profile for Fennoscandia is inferred from GNSS observations. Milne et al. (2004) bounds the earth model parameters to $[90 < LT < 170$ km$]$, $[5 \cdot 10^{20} < \eta_{UM} < 10^{21}$ Pa s$]$ and $[5 \cdot 10^{21} < \eta_{LM} < 5 \cdot 10^{22}$ Pa s$]$. Later Lidberg et al. (2010) find the optimal parameters to be $LT = 120$ km, $\eta_{UM} = 5 \cdot 10^{20}$ Pa s and $\eta_{LM} = 5 \cdot 10^{21}$ Pa s.

Steffen et al. (2010) investigate the possibility to use data from the Gravity Recovery and Climate Experiment (GRACE) to constrain GIA modelling and to make inferences on the earth structure. They use both one-dimensional (normal mode) and three-dimensional (finite element) methods and a number of different ice models. The 1-D solutions prefer a thick lithosphere ($LT = 160$ km) which is probably due to the long wavelength characteristics of the satellite data, η_{UM} is constrained to $[2-4] \cdot 10^{20}$ Pa s and η_{LM} is 1-1.5 orders of magnitude larger. The 3-D solutions confirm the results from 1-D modelling but show that GRACE data alone poorly constrains lateral viscosity heterogeneities (cf. Wang et al., 2008).

The examples above illustrate a couple of common features, characteristic for many geophysical GIA modelling efforts. (i) The *primary* purpose of the modelling is not to make best possible predictions of the various observables, but rather to use observations to constrain inferences on the physical properties of the Earth. (ii) The observations, used as constraints, are usually of one type, or a few. These issues should be contrasted with the empirical models, which very well reproduce observations but lack of geophysical constraints. Hill et al. (2010) present an interesting approach to combine the strengths of geophysical modelling with numerous observational inputs. Using a method that is consistent with least squares collocation (Moritz, 1980) and data assimilation (Bennett, 2002) they combine geodetic data of different kinds (GNSS, tide gauges, GRACE) into an a priori geophysics-based GIA model to produce a new and updated model. With this method the resulting model is less dependent on the ice history and earth model, the individual contributions from the various data sets and their biases can be examined in a self-consistent manner and the uncertainty of the output model can be estimated.

It is also worth noting, that observed surface gravity rates have not played an essential role in GIA modelling in Fennoscandia - yet. Although, Müller et al. (2012) combine absolute gravity observations from five years of observation (Timmen et al., 2012) with GRACE data into an empirical model of the gravity rate of change. In Section 3.5 we discuss some issues related to observations and modelling of the surface gravity rate of change, but first something about the accuracy of the *implementation* of different modelling approaches.

3.4 Benchmarking

Uncertainties in the ice history and earth model limit the accuracy of GIA model predictions. But, given a certain GIA model, how much do the method of solution, the implementation of the model into code, and the numerical accuracy of the code effect on the results?

Barletta and Bordoni (2013) study the effect of different implementations of the ice model and in an acclaimed benchmark study (Spada et al., 2011), supported by European Cooperation in Science and Technology (COST) Action ES0701, seven research groups compared their implementations of GIA modelling code. Different methods, such as viscoelastic normal mode, finite elements (see Section 3.2) and spectral-finite elements (Martinec, 2000) were represented. From a number of prerequisites, the Earth's response to simple surface loading was computed and compared. During the work with the benchmarking, some misunderstandings about the theory, implementation bugs and numerical shortcomings were detected and corrected. The final results were largely consistent and it was concluded that the differences were sufficiently small such that they could be ignored.

The achieved agreement between predictions of the Earth's reaction to surface loading from Spada et al. (2011) served as the prerequisite for a second step in the benchmarking. This time the sea level equation (SLE) (see Section 3.2) was examined, i.e. how the melt water from the ice sheets is distributed over the oceans and how that, in turn, affect the GIA observables. Since a considerable part of the work behind this dissertation has been addressed to implementation of SLE, this was a golden opportunity to benchmark that work.

In a similar approach as in Spada et al. (2011) predictions were made from a set of prerequisites. Also this time the work highlighted some intricate issues related to different implementations, but the final results were largely consistent.

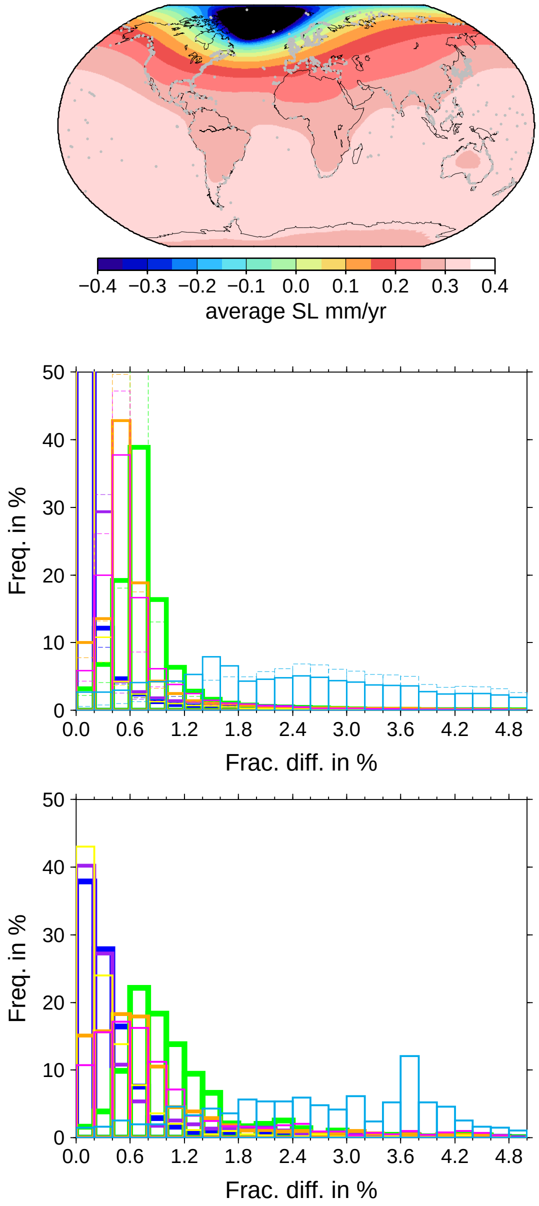

Figure 12: Unpublished results from the SLE benchmark study of relative sea level fingerprints. Top panel shows the mean from seven different SLE implementations, driven by an ice loss rate of 100 Gton/year from the Greenland ice sheet. The middle panel shows the difference (in percent) with respect to the average at grid points in a global 1×1 degree grid. Each color represents one participating code. Pink represents the code used in this thesis. The bottom panel shows the difference at a selection of tide gauge locations.

Two articles, presenting the prerequisites and the results of the SLE benchmark are, at the time of this writing, under construction. One of the two draft articles is a thorough description of the SLE benchmark study and is intended to complement the first benchmark paper (Spada et al., 2011). The other article is a shorter version, a benchmark study of predictions of the so-called sea level fingerprint (see e.g. Mitrovica et al., 2001), i.e. the spatial variation of sea level change induced by present-day melting of continental ice sheets. Seven different SLE implementations participated in the fingerprint benchmark study and Figure 12 shows some results from that. The top panel shows the mean of predicted rate of change of the relative sea level, caused by an ice loss rate of 100 Gton/year from the Greenland ice sheet. The middle panel shows the difference in percent, with respect to the average, at grid points in a global 1×1 degree grid. Each color represents one participating code. Pink represents the code used for gravity predictions in Paper III and IV. The bottom panel shows the difference at a selection of tide gauge locations.

This SLE benchmarking proved that the code used in Paper III and IV produce results that are in agreement with results based on other methods and other implementations. In our implementation of SLE, special emphasis has been put on predictions of the surface gravity change as observed by repeated absolute gravity observations. In the next section we discuss some special issues related to the GIA-induced gravity signal.

3.5 GIA-induced gravity change

The GIA-induced gravity change adds important and unique information to GIA modelling. It is sensitive to redistributions of masses within the Earth and can therefore be used to make conclusions on the underlying dynamical processes (see e.g. Ekman and Mäkinen, 1996). Another benefit is that observations of the gravity change, by nature, are done independent of any reference system definition of the centre of mass of the Earth. It therefore provides useful constraints on reference frame related uncertainties in GNSS velocities. The poor resolution of CM in GNSS data can result in uncertainties of up to 1-2 mm/year in "absolute" rates (Altamimi et al., 2007), with significant impact on geodynamic inferences (Mazzotti et al., 2011). Mazzotti et al. (2011) use repeated absolute gravity and GPS time series in North America to show that ITRF2005 (or ITRF2008) is, compared to ITRF2000, better aligned to CM and therefore more suitable for geodynamical studies.

The surface gravity change, as observed by an absolute, relative or superconducting gravimeter, consists of contributions from many different sources and might therefore be hard to interpret. For the purpose of GIA studies, the sources of surface gravity change be categorized in four main groups: (1) vertical displacement along the gravity gradient, (2) redistribution of masses within the Earth, (3) GIA-related mass redistributions on the surface of the Earth and (4) external (not GIA related) effects. When studying GIA, (1) and (2) are usually what we are searching for and (3) and (4) causing the problems.

With GIA-related mass variations we mean redistributions of masses in continental ice sheets and in the oceans (sea water). These mass movements affect the gravity value via direct, Newtonian, attraction. In areas like Fennoscandia and Laurentia, since long ice-free, ice mass variations are not an issue. However, in areas with present-day ice mass variations (PDIM) they are an important, complex and integrated part of the gravity signal (see e.g. Memin et al., 2012; Nielsen, 2013). GIA-induced relative sea level variation may add a systematic effect to (1) and (2) for locations close to the sea and is normally modelled by means of the sea level equation (see Section 3.2).

With external effects we mean mass variations, not related to GIA. They can be considered as noise in observations (Van Camp et al., 2005, 2010) and are normally not included in the modelling. External effects can be e.g. sea level variations, continental water (groundwater) variations or variations in the atmospheric pressure. Efforts have been done in order to investigate external effects for gravity observations in Fennoscandia. In Paper I, in this thesis, we investigate the sensitivity for sea level variations in the Baltic sea for stations in the NKG/AG-network (see Section 2.4) and show how the AG observations can be corrected for this effect; Gitlein and Timmen (2007) do the same for atmospheric loading and also apply this correction to AG observations in the NKG/AG-network. The effect from groundwater variations depends strongly on the local geology/hydrology and is therefore difficult to model. Investigations of the effect from local hydrology on gravity is usually conducted at geodetic fundamental stations where the gravity signal from e.g. a superconducting gravimeter can be compared to local surface- and groundwater observations and local, regional or global hydrological models (see e.g. Virtanen, 2006; Van Camp et al., 2006; Neumeyer et al., 2006; Naujoks et al., 2010).

If not corrected for, the external effects will increase the noise level in our observations and require longer time series but, as long as the noise can be considered white, not bias the inference of the rates. Van Camp et al. (2005) studied the noise spectrum for SG and AG observations, including the environmental effects, and found that for frequencies lower than one cycle per year the spectrum tends to white noise. With time series spanning 10 years or more and with a sampling rate of one observation per 1-2 years the environmental effects can thus be expected to appear as white noise.

We end this section by pointing out that the direct attraction from nearby mass variations is an intricate issue. In Paper III we show that including the direct attraction as one term in Greens function for gravity, as suggested in classical papers like Longman (1963); Farrell (1972); Peltier (1974), is not suitable for GIA modelling of surface gravity change. It is better not to include the direct attraction at all. Best is if the direct attraction is treated separately e.g. by means of analytical integration over rectangular prisms as suggested in Paper III or, as in Paper I, with numerical integration using the midpoint method and very high resolution around the point of observation.

3.6 Prerequisites for this dissertation

With reference to the background, described in Section 2 and Section 3 above, we now summarize the prerequisites for this dissertation.

A long tradition of observing the land uplift in Fennoscandia has intensified over the last few decades. New technologies, as GNSS and portable, commercially manufactured absolute gravimeters, has brought new possibilities to study the phenomena. Great efforts have been made to establish networks of permanent stations for GNSS observations (BIFROST) and repeated absolute gravity observations (NKG/AG). In two papers Wu et al. (2010) and Steffen et al. (2012) study optimal locations of GNSS and gravity observations for constraining GIA. They conclude that, except for the northwestern part of Russia, complete and adequate networks for the study of GIA parameters are provided in the Fennoscandian postglacial rebound area.

The accuracy of GIA observations increases as the time series from tide gauges, GNSS and gravity observations get longer. The BIFROST-network has continuously been collecting GNSS data since the mid of the 1990's. Lidberg et al. (2010) present vertical and horizontal displacement rates for the stations. "Realistic" 1σ uncertainties for the vertical component are around 0.2-0.4 mm/year for a majority of the stations. In the NKG/AG network, repeated observations started 1988, first sparse at a few stations, but since the beginning of the 21st century more intense. In Figure 13 the estimated observational accuracy of the gravity rate of change is plotted as function of time. These estimates are based on an a priori observational standard error of 2 μGal (Niebauer et al., 1995) and uncorrelated observation every or every second year. Empirical linear regression estimates of the standard error from a number of Swedish stations, with observational time spans 3-19 years, are also plotted. This plot is in agreement with a study by Van Camp et al. (2005) which show that with annual or semiannual AG observations we can expect a standard error of ~ 0.1 μGal/year after 15-25 years.

The Nordic geodetic community has a lot of experience concerning observations and empirical modelling. With an increasing amount of data from various observation types, that by nature are very different, the question on how to relate these observations has been highlighted and a need to increase the knowledge related to geophysics-based GIA modelling has been recognized. Earlier GIA modelling efforts in Fennoscandia has traditionally used one, or a few, types of observations to constrain inferences on earth model parameters. The use of observations of the surface gravity rate of change has been limited.

The time series get longer and the observational accuracy increase - but what do we observe? GNSS observations suffer from uncertainties related to the reference frames; the relative sea level change consists not only of vertical displacement of the crust, but also of a global change of the ocean volume and a secular change in the shape of the geoid; gravity observations are affected not only by the pure GIA signal, but also by a large variety of environmental signals.

This dissertation is an attempt to contribute to the knowledge concerning

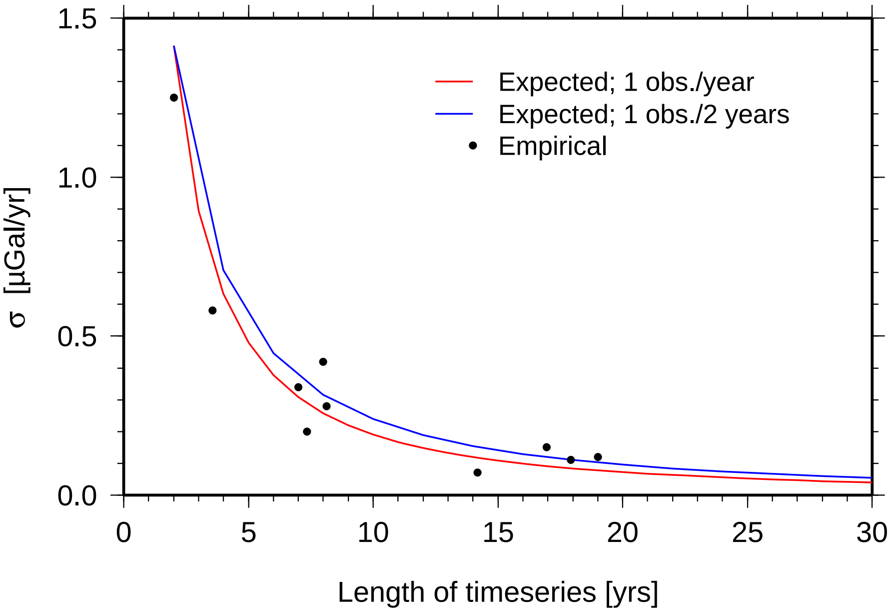

Figure 13: Empirical and expected standard error of estimated $\dot{g}$.

the issues discussed above. In four appended papers, we have studied some aspects related to the GIA-induced gravity change. A summary of the main results and conclusions of the appended papers is given in the next section.

4 Summary and conclusions

This section shortly summarizes the appended papers and presents the most important conclusions against the background given above.

4.1 Paper I: Effects on gravity from non-tidal sea level variations in the Baltic Sea.

The long-term goal with the Nordic absolute gravity project (see section 2.4) is to provide observations of the GIA-induced gravity change in Scandinavia. This is achieved by repeated absolute gravity observations (for the time being made with FG5 absolute gravimeters (Niebauer et al., 1995)) at a number of selected stations in the region (see Figure 3). The stations are intended to be reoccupied with one or a few years intervals. The observations are corrected for tidal effects and for the atmospheric pressure variations using a barometric admittance factor and the estimated absolute accuracy (standard error) is on the level of a few μGal. With this accuracy unmodelled external effects from e.g. continental water (snow, groundwater), sea level variations and three-dimensional variations in the atmospheric pressure, influence the observations and lead to increased scatter in the observed time series.

The purpose of Paper I is to investigate numerically the effects on gravity from non-tidal sea level variations in the Baltic Sea for a subset of stations in the NKG/AG-network. This is done by modelling the elastic response to the ocean load with numerical convolution of the Green's function for gravity introduced by Farrell (1972) over the ocean. Special emphasis is put on the direct attraction from the loading masses. With a dense grid representing the ocean load, a high resolution coastline and the effect of the height of the observation point above the sea surface included, an accurate prediction of the effect from the direct attraction is achieved.

Based on three different "snapshots" of the ocean load in the Baltic Sea (one theoretical uniform one meter increase of the sea level and two realistic sea levels based on tide gauge observations along the Swedish coast) the effect on twelve Swedish and one Finnish stations are predicted. The results show that the effect is significant for absolute gravity observations, easily reaching 2-3 μGal for stations with high elevation close to the coast. For one station, located only 10 meters from the coast, the effect from a one meter water increase reaches 10.9 μGal, of which 7.6 μGal comes from the direct attraction. A bounding example on how large this effect can be is presented in Paper II (Olsson and Ekman, 2009); see below.

4.2 Paper II: Crustal loading and gravity change during the greatest storm flood in the Baltic Sea.

Paper II can be seen as an extreme case of how large the effect from non-tidal sea level variations, modelled in Paper I, can be. With the methods described in Paper I, the effect on gravity and the vertical displacement of the crust was predicted for an event referred to as the greatest storm flood ever observed in the Baltic Sea.

Strong north-easterly winds redistributed the water in the southern Baltic Sea. The maximum deviation from the normal sea level occurred in the early afternoon on November 13, 1872. In the south-westernmost part of the Baltic Sea the sea level was about three meters above normal, and in the Finnish Gulf and west of Denmark (eastern North Sea), it was about one meter below normal.

This temporary redistribution of water caused an elastic vertical deformation ranging from -2.3 cm south of Sweden to 1.5 cm west of Denmark. The tilt, or gradient, of this deformation amounts to 0.2 mm km^{-1} across Denmark. (During the storm people living on the island Bornholm, close to the maximum depression of the crust, reported on an earthquake. Bornholm is located on a deformation zone in the crust and it might be suggested that the sudden depression of the crust triggered the earthquake.)

The effect on gravity was also predicted for a station located such that the effect will be close to the theoretical maximum, serving as an extreme case of how large this effect can be. Around the station the sea level was 2.6 meters above normal which induced an increase in gravity of 108 μGal, of which 104 μGal refers to the direct attraction from the water masses, 6 μGal to the vertical displacement of the crust and -2 μGal to the redistribution of masses within the Earth.

4.3 Paper III: Modelling of the GIA-induced surface gravity change over Fennoscandia.

The history of GIA observations is long and comprehensive in the Nordic countries but not so the tradition of GIA modelling. The successively increasing amount of observations of the crustal deformation from e.g. BIFROST (section 2.5) and gravity change from e.g. the Nordic absolute gravity project (section 2.4) at the same time require and enable more accurate GIA modelling.

The purpose of Paper III is to study how sensitive predictions of $\dot{g}$ in Fennoscandia are to some modelling assumptions and approximations. This is achieved by first constructing what we call the Base Model (BM), which is a normal mode GIA model for the gravity change rate based on ICE-5G (VM2) and the generalized sea level equation with migrating shorelines. Numerical predictions of $\dot{g}$ achieved with the Base Model are compared to five model variants. One of the model variants predicts $\dot{g}$ with a linear relation to $\dot{u}$; the other variants differ from the base model in how the direct attraction is treated and with simplified solutions of the sea level equation.

In the base model, Green's function for gravity is defined as in classical papers such as Longman (1963), Farrell (1972) and Peltier (1974), where it was originally derived for tidal applications. For GIA modelling in previously glaciated areas this means that the direct attraction from sea level variations is introduced with the wrong sign and also propagate inland in an improper way. The most rigorous model variant is called the full solution and here the direct attraction is computed using analytical integration over rectangular prisms. We show that predictions of $\dot{g}$ with a linear relation to $\dot{u}$ agree within 0.02 μGal yr^{-1} with the full solution over land. As the coastline is approached the direct attraction from the secular sea level change, not included in the linear model, becomes more and more important though. Solving the sea level equation with fixed coastlines results in deviations, compared to the full solution, reaching some 0.15 μGal yr^{-1} over land. The corresponding value using an eustatic solution (spatially homogenous sea level change) is ~ 0.2 μGal yr^{-1}, and over sea it exceeds 0.55 μGal yr^{-1}.

Inland, the linear model seems to be a good approximation of the rate of change of gravity (given that a good model of the uplift rate is available) but as the coastline is approached a careful treatment of the direct attraction from present-day sea level changes has to be regarded. Including the direct attraction in Green's function for gravity is not a good idea.

4.4 Paper IV: The relation between gravity rate of change and vertical displacement in previously glaciated areas.

In Scandinavia, the rate of change of gravity has traditionally been modelled as a linear relation to the vertical component of the deformation. The constant ratio between the two has been adopted from empirical studies (e.g. Ekman and Mäkinen, 1996) or from modelling efforts originally intended to enable a separation of the elastic signal from the viscous signal in areas with present day ice mass change (PDIM), like Greenland and Antarctica (e.g. Wahr et al., 1995)). In these areas, GIA models are poorly constrained by observations and the constant ratio is based on a number of assumptions and approximations. Among geodesists in the Nordic countries, the question has been raised and discussed whether this linear model is accurate enough.

The purpose of Paper IV is to study the relation between the rate of change of gravity, $\dot{g}$, and the vertical deformation, $\dot{u}$, in previously glaciated areas, like Scandinavia and Canada. Using the same modelling approach as in Paper III with a radially stratified earth model and ICE-5G ice history we predict $\dot{g}$ and $\dot{u}$ and evaluate their relation. We show numerically how the relation varies within each region, how it varies between regions, how it depends on choice of earth model and how it has varied since last glacial maximum. In a case study we investigate if local effects, such as direct attraction or high degree elastic deformation, from present-day GIA-induced relative sea level variations affect the relation.

We find that relation varies more between Canada and Fennoscandia than within each region. In Canada the ratio is ~ -0.152 μGal/mm and in Fennoscandia ~ -0.163 μGal/mm. The choice of (realistic) earth model does not significantly affect the ratio. In the spectral domain the ratio depends on the spherical harmonic degree with a lower ratio for low degrees. In Laurentia the ice load was larger compared to Fennoscandia and the GIA signal is stronger in the lower bands of the spectra which would explain the lower ratio there. The local effects from GIA-induced relative sea level variation does not significantly affect the ratio other than in extreme cases when the point of observation is located very close to the sea. Here the direct attraction from sea level variations should be recognized and treated with care.

Using the linear relation constants presented above to predict $\dot{g}$ from modelled $\dot{u}$ would give a maximum difference, compare to full modelling of $\dot{g}$, of 0.17 and 0.04 μGal/yr for Laurentia and Fennoscandia respectively. This is below the present observational accuracy of $\dot{g}$.

4.5 Complementing remark

In Paper III and IV we find that the ratio between $\dot{g}$ and $\dot{u}$ can be considered rather constant (~ -0.16 μGal/mm in Fennoscandia) not taking the direct attraction from GIA-induced sea level variations into account. It should be emphasized that these results are based on, and valid for, the modelling paradigm used, i.e. a 1D earth model with linear Maxwell rheology. Another rheological model, e.g. Burger rheology which allow for transient creep, or a laterally heterogeneous lithosphere and mantle might change these conclusions.

5 Final words and recommendations

In this dissertation we have studied some issues related to modelling of the GIA-induced surface gravity change. One of the motivating factors for the dissertation was to increase the knowledge of this independent GIA observable, particularly connected to increasing observational efforts in this regard in Fennoscandia. Below we provide some suggestions and recommendations for future work, of importance for GIA modelling, the GIA community in general and the geodetic community in Fennoscandia in particular.

- From Lantmäteriet's perspective, the most important contribution to future GIA research would be to continue the time series of observations in the GNSS- and AG-networks. These two GIA observables are independent of each other and contribute with different information on the GIA phenomenon. With improved modelling methods and increasing computational capacities, the importance of accurate observational constraints will probably increase.

- Repeated absolute gravity observations have been carried out in Fennoscandia since the early 1990's, with an increased intensity about ten years ago. These observations have so far been of limited use in terms of constraining GIA models. Many different groups have made observations at different times and a compilation and evaluation of these observations is urged. The project would also benefit from a review paper of the NKG/AG-network and related work, the purpose of the network, the infrastructure (the stations), observational efforts, observational results and related research. Gitlein (2009) treated many of these issues but limited to the work made by the Institut für Erdmessung (Leibniz Universität Hannover) during the years 2003-2008.

- Geophysics-based GIA models do not succeed to reproduce observations (empirical models) exactly. This may be due to observational errors or limitations/approximations in the GIA models. A large number of modelling efforts, based on one-dimensional earth models with linear rheology and a limited number of ice models, have been published. The greatest potential for further development towards better GIA models would be in non-linear, three-dimensional earth models as well as in new ice models (preferably climate-driven ice models which are less dependent on GIA observables that we are predicting).

- In 2013, NKG has for the first time initiated a project which aims at a geophysics-based GIA model, tuned to observations of different kinds (GNSS, gravity and sea level observations) in Fennoscandia. This work includes evaluation of earth and ice models and should be recognized and supported by the participating organizations.

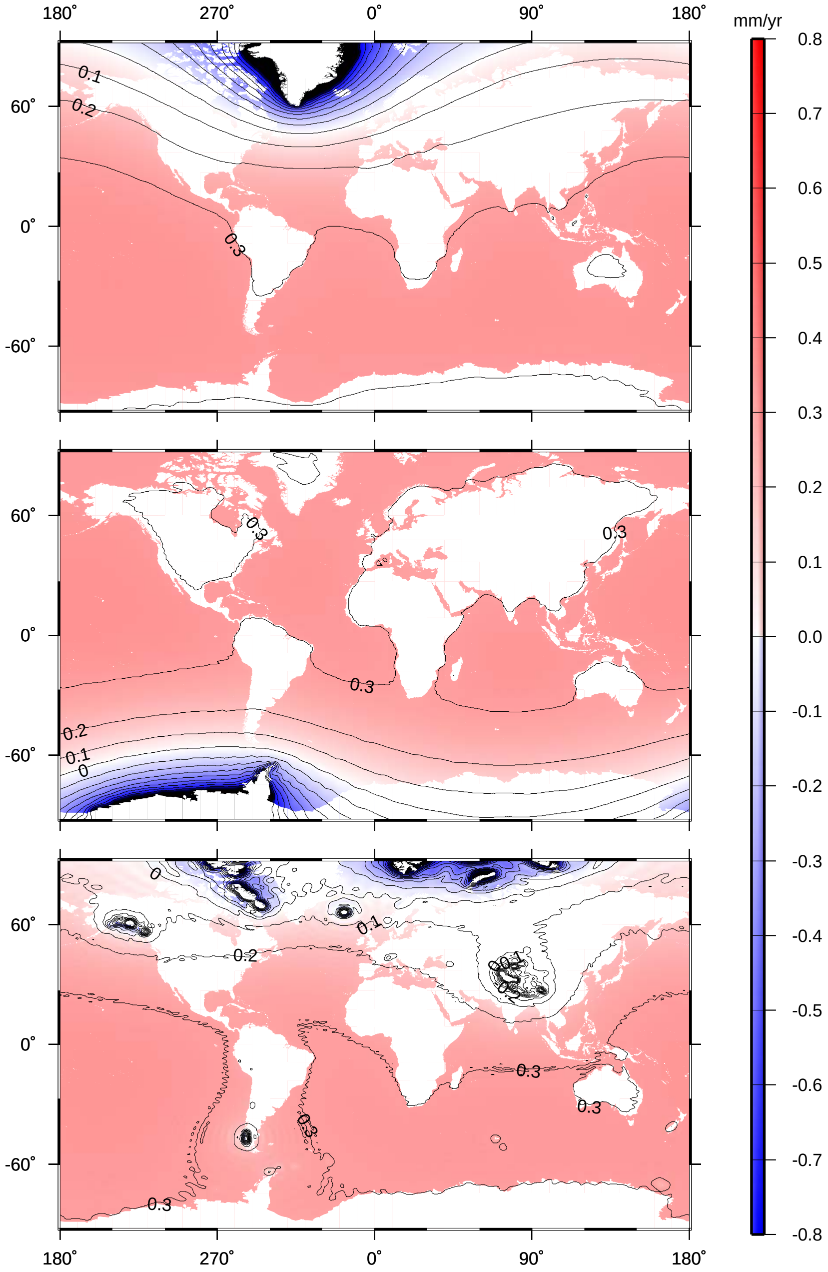

Figure 14: Expected relative sea level change from caused by a melting rate of 100 gigatons per year from (top panel) Greenland, (middle) West Antarctica, (bottom) other glaciers. Values lower than -0.8 mm/year have been blanked with black.

- The purpose of empirical models is to compile observations and filter out errors. They will continue to be important for geodetic purposes as well for constraining GIA models. Development of empirical models should therefore continue, parallel to geophysical GIA modelling efforts, as the observational databases develop.

We end this dissertation with an outlook on a possible application, not primarily for geodetic or geophysical purposes, of the theory used in this dissertation.

Lantmäteriet has been commissioned by the government to develop a national digital elevation model, based on laser scanning of the whole country, to meet the need of good elevation data for climate adaptation of the society. Another important step in the climate adaptation work is to try to understand, and predict, how sea levels will respond to potential global warming and melting of continental ice sheets. In Fennoscandia, the relative sea level change is in general negative due to the land uplift. Will this relative sea level fall be exceeded by a potential climate-induced sea level rise?

The sea level equation, as implemented in Paper III and Paper IV, is a suitable tool to predict how melt water is distributed over the ocean. An example of this given in Figure 14 which illustrate the spatial variability of the relative sea level change induced by three different melting scenarios. From the top, the three panels show the predicted rate of change of sea level from a 100 Gton/year melting rate from the Greenland ice sheet, the West Antarctica ice sheet and other continental ice sheets, respectively. Notably is that melting of the Greenland ice sheet does not affect sea level in Fennoscandia significantly (the zero isoline goes right through Sweden) and that the largest impact comes from the West Antarctica ice sheet. This kind of predictions should be regarded in future climate adaptation work.

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