

## The global dynamic behaviour of steel and composite bridges for high-speed railway lines

*Master of Science Thesis in the Master's Programme Structural Engineering and  
Building Technology*

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CHALMERS UNIVERSITY OF TECHNOLOGY  
Göteborg, Sweden 2013  
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Examensarbete / Institutionen för bygg- och miljöteknik,  
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A beam subjected to a moving point load.

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## ABSTRACT

Investigations of the possibilities of future high-speed railway lines have been made by the Swedish government considering a way to connect major cities of Sweden with the rest of Europe. Due to geometric requirements the ratio bridge per meter railway is substantial in high-speed railway lines. The design of high-speed railway bridges is often governed by the dynamic requirements. Usually these kinds of bridges are made of concrete. However, it is of interest to investigate the possibility to design high-speed railway bridges using steel and composite structures. Due to the low weight, these kinds of structures have a short erection time with small material consumption and fewer transports to the construction site. These factors are favourable concerning environmental aspects as well as disturbance of existing infrastructure.

This thesis investigates the demands of steel and composite bridges used in high-speed railway lines, concerning dynamic performance.

A parametric study concerning the dynamic behaviour of simply supported beams is conducted. The beams are subjected to the load model High Speed Load Model A (HSLM-A) from Eurocode and the vertical acceleration is studied.

The results of the parametric study are shown as contour plots where the vertical acceleration is a function of the flexural rigidity and mass of the beam for each investigated beam length. The contour plots indicate favourable intervals in mass of a beam concerning the dynamic response. A case study of two 50 meter simply supported bridge concepts is conducted in order to investigate the applicability of the results of the parametric study in a design situation.

The case study implies that the parametric study can be used to predict the global behaviour of bridges. However, it is important that the assumptions in the FE- model of the parametric study reflect the behaviour of the structure at hand. It can be advantageous to design high-speed railway bridges based on the dynamic behaviour. Methodologies for design and assessment of bridges for high-speed railway lines are proposed.

Key words: Dynamic behaviour, high-speed railway lines, lightweight bridges, global vertical accelerations, steel, composite

Det globala dynamiska beteendet för stål- och samverkansbroar i  
höghastighetsjärnvägslinjer

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## SAMMANFATTNING

Sveriges regering har utfört utredningar angående möjligheten till en framtida höghastighetsjärnväg som skulle binda ihop Sveriges största städer. På grund av geometriska krav är förhållandet bro per meter järnväg påtaglig i sådana järnvägsnät. Dynamiska faktorer är ofta dimensionerande för denna typ av broar. Normalt utförs sådana broar i betong men det är av intresse att undersöka möjligheterna av att använda stål- och samverkanskonstruktioner för denna typ av broar. Dessa konstruktioner är fördelaktiga då den lägre vikten ger möjlighet till färre transporter och kortare uppföringstid, vilket ger mindre påverkan av befintlig infrastruktur samt är fördelaktigt ur miljösynpunkt.

Detta arbete undersöker, med avseende på globala vertikala accelerationer, vilka krav som ställs på stål- och samverkanskonstruktioner för broar i höghastighetsjärnväg.

En parameterstudie som undersöker det dynamiska beteendet, med avseende på vertikala accelerationer, för fritt upplagda balkar är utförd. Balkarna är belastade av lastmodellen High Speed Load Model A (HSLM-A) enligt Eurocode. Resultaten från parameterstudien redovisas i nivåkurvor där vertikal acceleration presenteras som en funktion av balkens massa och böjstyvhet för varje undersökt längd. Gynnsamma massintervall, med avseende på balkens dynamiska beteende, har identifierats. För att undersöka tillämpbarheten av resultaten från parameterstudien i en designsituation har en fallstudie utförts. Fallstudien behandlar två 50 meter långa fritt upplagda broar.

Resultaten av fallstudien antyder att parameterstudien kan användas för att förutsäga det globala dynamiska beteendet av brokonstruktioner. Det är dock viktigt att beräkningsmodellen i en parameterstudie återspeglar beteendet hos den behandlade strukturen. Arbetet påvisar svårigheten av att förutse den dynamiska responsen av brokonstruktioner utsatta för höghastighetståg och genom detta vinningen av en utformning baserad på det dynamiska beteendet. Metodik för utformning och klassning av broar för höghastighetjärnväg föreslås.

Nyckelord: Dynamiskt beteende, höghastighetsjärnväg, lätta brokonstruktioner, stålkonstruktioner, samverkanskonstruktioner globala vertikala accelerationer,

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## Preface

This Master Thesis was written during the spring of 2013 as the final part of the Master's Programme Structural Engineering and Building Technology at Chalmers University of Technology. The work was carried out in collaboration between Chalmers University of Technology and WSP Bridge and Hydraulic design (WSP Bro och Vattenbyggnad) in Gothenburg.

The thesis would never have been completed if it was not for the support we have gotten from friends and families. We would especially like to thank the following people, in no particular order, for their contribution to our work.

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Anders Pagoldh Johansson & Carl Westerlund  
June 2013

# Notations

## Abbreviations

2-DOF	Two degrees of freedom
ERRI	European Railway Research Institute
FE	Finite Elements
HSLM	High Speed Load Model
IC	Initial conditions
MAC	Modal assurance correlation
MDOF	Multiple degrees of freedom
SDOF	Single degree of freedom
SLS	Serviceability limit state

## Roman upper case letters

$A$	Cross sectional area	$[m^2]$
$A_1, A_2$	Amplitude constants	$[-]$
$C_1, C_2$	Integration constants	$[-]$
$\mathbb{C}$	Damping matrix	$[Ns/m]$
$D$	Outer diameter	$[m]$
$E$	Youngs Modulus	$[GPa]$
$EI$	Flexural rigidity	$[Nm^2]$
$\mathbb{F}$	Load matrix	$[N]$
$I$	Second moment of area	$[m^4]$
$I$	Impulse load	$[Ns]$
$\mathbb{K}$	Stiffness matrix	$[N/m]$
$L$	Length	$[m]$
$\mathbb{M}$	Mass matrix	$[kg]$
$N$	Arbitrary degrees of freedom	$[-]$
$P$	Applied point load	$[N]$
$T_n$	Period time	$[s]$
$V$	Displacement of a beam	$[m]$

## Roman lower case letters

$c$	Velocity	$[m/s]$
$c$	Damping coefficient	$[Ns/m]$
$c_{cr}$	Critical damping coefficient	$[Ns/m]$

$f_1$	First natural frequency	[Hz]
$h$	Time step	[s]
$k$	Spring stiffness	[N/m]
$l$	Length	[m]
$m$	Mass	[kg]
$n_0$	Natural frequency of a bridge	[Hz]
$p$	Applied force	[N]
$t$	Time	[s]
$t$	Thickness	[m]
$u$	Displacement	[m]
$\mathbf{u}$	Displacement vector	[m]
$\dot{u}$	Velocity	[m/s]
$\dot{\mathbf{u}}$	Velocity vector	[m/s]
$\ddot{u}$	Acceleration	[m/s <sup>2</sup> ]
$\ddot{\mathbf{u}}$	Acceleration	[m/s <sup>2</sup> ]
$\bar{u}$	Average displacement	[m]
$v$	Displacement of a beam	[m]
$v_{res}$	Resonance speed	[m/s]
$v_0$	Initial deformation	[m]
$z$	Base displacement	[m]
$\dot{z}$	Base velocity	[m/s]
$p$	Applied load	[N/m]

### **Greek upper case letters**

$\hat{\Phi}$	Eigenvectors	[-]
$\Omega$	Frequency of the applied load	[rad/s]

### **Greek lower case letters**

$\delta(x - ct)$	Dirac delta function	[-]
$\vartheta$	Logarithmic decrement of damping	[-]
$\hat{\eta}$	Displacement in principle coordinates	[m]
$\hat{\dot{\eta}}$	Velocity in principle coordinates	[m/s]
$\hat{\ddot{\eta}}$	Acceleration in principle coordinates	[m/s <sup>2</sup> ]
$\lambda$	Eigenvalue	[-]
$\lambda$	Excitation wavelength	[m]

$\xi$	Damping ratio	[-]
$\rho$	Density	[kg/m <sup>3</sup> ]
$\omega$	Circular frequency of load	[rad/s]
$\omega_d$	Damped natural frequency	[rad/s]
$\omega_n$	Natural frequency	[rad/s]
$\omega_r$	Natural frequency of a beam	[rad/s]
$\omega^*$	Damped frequency	[rad/s]
$\omega_b$	Circular frequency of damping	[rad/s]

# 1 Introduction

By developing the public transportation system there is much to gain in today's society. For example better connections between major urban areas would result in better work opportunities and increased trade. A developed transportation system also results in improved possibilities for commuters to choose a way of transportation that minimises the environmental effects. Investigations of the possibilities of future high-speed railway lines have been made by the Swedish government considering a way to connect major cities of Sweden with the rest of Europe (Johansson, Anderssson et al. 2010). The development of high-speed railway traffic in Sweden is logical, concerning the development of train traffic speed during the last two centuries, see Figure 1. In the figure the two dotted lines indicate prognosis for the future design speeds of train traffic, one considering the development from the 1820's and one the development since the ending of the Second World War (Wiberg 2009).

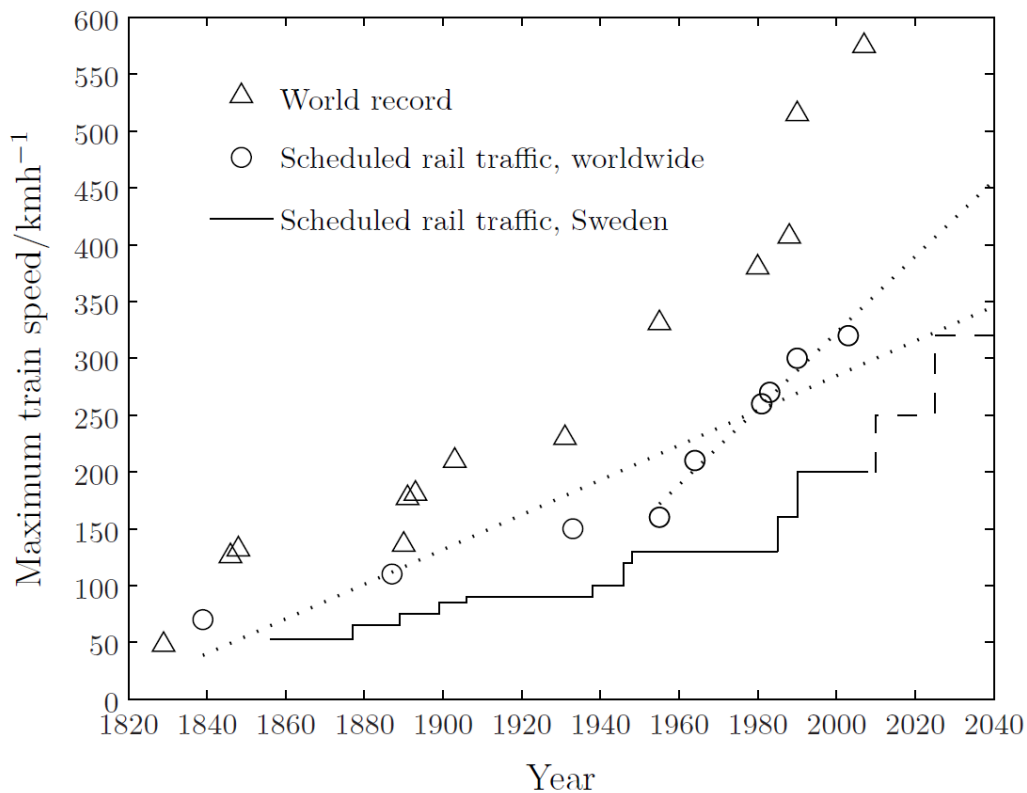


Figure 1 Development of train traffic speeds during the last two centuries. The dashed line indicates the Botnia line and the proposed future high speed line design speed in Sweden (Wiberg 2009).

In the design of high speed railways the geometric requirements from Eurocode concerning curve radius, 35000 m, and vertical inclination, 1.8%, have the effect that the ratio bridge per meter railway is substantial in high speed railway lines. The dynamic effect on these bridges is an important design factor and according to Eurocode a dynamic analysis has to be performed for railway lines where the design speed exceeds 200 km/h (Mellier 2010).

Concrete bridges with ballasted tracks are the most common structural design in high speed railway lines (Marioni 2006). However, in high traffic areas it might be

advantageous to build bridges in steel due to the low weight and the possibilities for quick erection. Further on, the usage of prefabricated parts reduces the disturbance of the existing infrastructure. Structures built using lighter materials result in fewer transports to the construction site; which is advantageous both in urban areas and inaccessible sites, particularly concerning environmental aspects.<sup>1</sup>

Previous research performed at the Royal University of Technology (KTH) shows that bridges for use in high-speed railway lines should be made of concrete. Furthermore, it is concluded that steel and composite bridges with span lengths in the range of 8-50 m seem to be inappropriate, regarding the dynamic behaviour, to be used in railway lines for high-speed traffic (Johansson, Andersson et al. 2010).

Within the industry it is of interest to study the suitability of lightweight bridges more closely. Further on in this thesis steel and composite bridges are considered to be lightweight when used in high-speed railway lines. Usually bridges are designed regarding the static loads and then the dynamic performance is checked. However, the dynamic behaviour often governs the design and therefore it is of interest to investigate the possibility of design with regard to the dynamic response of high-speed railway bridges.<sup>2</sup>

## 1.1 Aim and objective

The thesis aims at investigating the demands of steel and composite bridges for high-speed railway lines concerning global vertical accelerations.

A secondary aim is to evaluate the dynamic behaviour of simply supported beams. The results are used to develop a design methodology that can be used in conceptual design of steel- and composite bridges in high-speed railway lines.

Another secondary aim is to investigate the applicability of the design methodology via a case study.

## 1.2 Method

The work in this master thesis will be performed in three parts, a literature study, a parametric study and a case study. The methods used for the parametric- and case study are presented below.

A simplified finite element-model (FE-model) is developed in the commercial software MATLAB. This is done in order to investigate the dynamic behaviour of simply supported beams subjected to high speed trains. From each given combination of considered flexural rigidity and mass, the global vertical acceleration of the beam is calculated. In order to investigate the influence of the beam length, three different lengths are considered. In the study all beams are subjected to the High Speed Load Model-A (HSLM-A) according to Eurocode 1. The results from the parametric study are presented in contour plots where the resulting accelerations from all treated combination of properties can be seen.

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<sup>1</sup> Interview with Peter Nilsson and Roland Olsson 4/3-2013

<sup>2</sup> Interview with Peter Nilsson and Roland Olsson 4/3-2013

The case study is done in order to show how the results from the parametric study can be used in a design situation. The case study is performed in the commercial software BRIGADE/Plus where dynamic analyses are made using the load model of the parametric study.

## **1.3 Limitations**

Limitations for each part of the thesis are presented in the following sections:

### **1.3.1 Parametric Study**

- Euler Bernoulli beam theory is used.
- Simply supported beams of 20, 35 and 50 meters are analysed.
- The rails are considered as geometrically perfect.
- The trains are considered as moving point loads.
- The load model HSLM-A from Eurocode 1 will be applied.
- The load is applied centric on the beam and only bending modes will be considered.
- The parametric study is only valid for steel and composite structures.
- The analysis covers a speed range of 150 – 350 km/h and will only consider the global vertical accelerations.

### **1.3.2 Case Study**

- Only a 50 meter simply supported bridge is considered.
- The case study will only consider steel and composite bridges
- Only enclosed three dimensional structures will be considered.
- The bridge deck utilises a double tracked slab-track-system.
- Only vertical accelerations will be considered.
- No interaction between supports and the surrounding ground is considered.
- The rails are considered to be geometrically perfect.
- The load model HSLM-A will be considered in speeds between 150 and 350 km/h.

## **1.4 Common bridges used for high-speed railways**

The most preferred static scheme for high-speed railway bridges is a simply supported bridge with continuous rails resting on sleepers and ballast. The ballast enables some small movements between the rail and the bridge deck, which is necessary for continuous rails. This solution is the most commonly used for high-speed railway bridges in countries such as Italy, Taiwan, China and Japan. However there are countries, for example Spain and France, that utilize other methods (Marioni 2006).

Marioni categorizes the bridges used for high-speed railways depending on the type of crossing. For bridges crossing roads and small rivers the most common type is the simply supported beam with spans of 25-35 meters. Normally the bridge structure consists of pre-stressed concrete box girders which can be either single or double. In Italy 90% of the total bridge length consists of bridges from this category (Marioni

2006). Likewise, Sobrino (2010) claims that almost all of the bridges in the high-speed railway line in Spain consists of cast in-situ concrete bridges.

Another common solution is reinforced concrete arches supporting concrete beams. This is commonly used if the railway line crosses deep valleys, where it is impractical to place columns along the span, since it makes it possible to cross deep valleys in just one span (Marioni 2006).

When crossing rivers it is often of interest to minimize the distance between the river and the tracks, since the gradient of the railway needs to be small and low height enables shorter approach bridges. Therefore these structural designs differ from the case where the railway line crosses deep valleys. Common for bridges in this category are that they have a low structural height beneath the rail. Examples of bridge types are:

- a. Steel or concrete arches with lower railway and hangers for spans up to 100 meters.
- b. Steel trusses with lowered railway for spans up to 150 meters.
- c. Concrete or steel cable stayed bridges for spans over 150 meters.
- d. Suspension bridges for very long spans. (Marioni 2006)

### 1.4.1 Innovative steel bridges for high-speed railways

In this part of the thesis some of the more unique steel bridges used for high-speed railway traffic is presented. The selection is made by the authors and reflects their opinion of innovative solutions. Three steel bridges from Spain, Riudellots high speed line bridge, The Llinars bridge and the Sant Boi bridge, and two steel bridges from China, The Nanjing Dashengguan Bridge and The Zhengzhou yellow river road-cum-railway bridge, are shown.

#### The Riudellots high speed line bridge

The Riudellots high speed line bridge is a steel truss bridge where each span consists of two offset trusses with variable height, Figure 2. The bridge has two spans, each 53 m long with a large skew (34°). The design is made mainly with the appearance and the serviceability limit state (SLS) requirements in mind. The head designer José Romo claims that the bridge has a light impression compared with “the feeling of heaviness typical of the railways bridges”. The bridge was built onsite and launched into place in one piece (Romo 2010).



Figure 2 The Riudellots high speed line bridge (Skachkova 2012)



### The Llinars bridge and the Sant Boi bridge

The Llinars bridge and the Sant Boi bridge, which both cross roads, are quite similar when it comes to the structural design. The structural design consists of two curved steel members above a composite steel-concrete deck with ballast. The bridges are innovative for the Spanish high speed railway line since there are only three steel bridges in use on the line. Although the bridges look like suspension bridges both of them behave as continuous beams with variable height. The longest span is 75 m (Llinars bridge) and 63 m (Sant Boi) (Sobrino 2010).



Figure 3 The Llinars Bridge (Sobrino 2010)

The structural design was chosen to obtain a small vertical clearance between the rail and the crossing road (5.5 m). Several alternatives were investigated before the final design was chosen. The height of the deck is only 2.15 meter in the final design (Sobrino 2010).



Figure 4 The Sant Boi Bridge (Sobrino 2010)

The performance of the bridges satisfies the specifications of Eurocode 1, for ballasted tracks, with a maximum acceleration lower than  $3.5 \text{ m/s}^2$ . Considering the flexural frequency the Llinars bridge has a first natural frequency of 1.28 Hz and the Sant Boi 1.33 Hz (Sobrino 2010).

## The Nanjing Dashengguan Bridge

The Nanjing Dashengguan Bridge is a part of the Beijing-Shanghai High-Speed Railway line which has a design speed of 300 km/h. However, during the analysis of the bridge it was shown that the bridge met the requirements for speeds up to 350 km/h. The total length of the bridge is 9.273 km and the steel part of the bridge is 1608 m long (Gao, Yi et al. 2010).

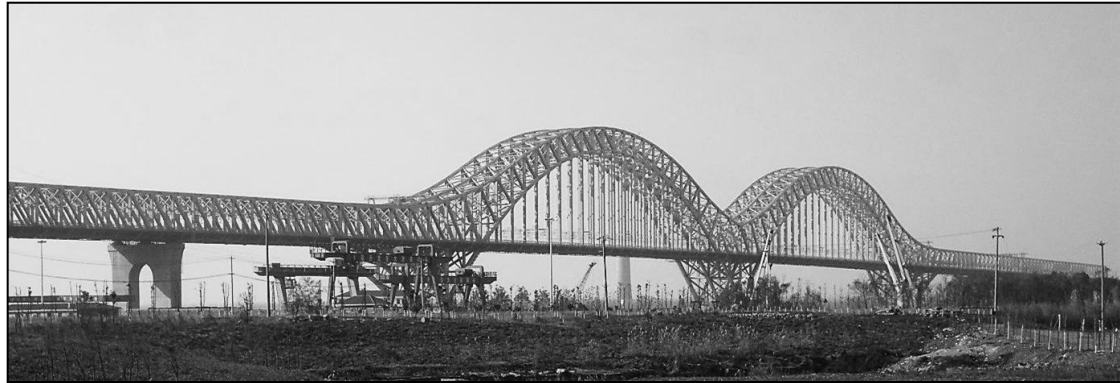


Figure 5 The Nanjing Dashengguan Bridge (东京村子 2011)

The bridge can be divided in two units, where the first unit consists of a continuous steel truss with 84 m long spans and the second unit consists of continuous arch steel trusses which connect with the first unit. The long spans and the heavy live load from the six tracks would result in very big structural members if a conventional truss had been used for the longest span. The structural design of the arches is chosen with manufacturing and erection in mind (Gao, Yi et al. 2010).

Due to high requirements of stiffness and straightness of the rail, an orthotropic steel bridge deck is used. Composite action is formed by welding the deck to the bottom chord of the truss. The rails are placed on a ballast trough consisting of a reinforced concrete structure that is 15 cm thick and 1.235 m high without any expansion joints (Gao, Yi et al. 2010).

## The Zhengzhou yellow river road-cum-railway bridge

The Zhengzhou yellow river road-cum-railway bridge is a 9177 m long cable stayed bridge; in this report the main part which is 1684 m is presented. The main bridge has one part with a continuous steel-concrete truss, which is suspended by A-shaped pylons by suspension cables, and the span in this section is 168 m. The pylons are made of steel and are 37 m high. Each pylon is connected to the truss with 5 pairs of cables. The second part consists of a continuous steel-concrete truss which spans 120 m. A six lane highway with the maximum speed of 100 km/h is on top of the truss. A twin-track railway with a maximum speed of 350 km/h is situated inside the truss (Gao 2012).



Figure 6 The main bridge of the Zhengzhou yellow river road-cum-railway bridge (Gao 2012).

The truss is designed as a parallelogram with horizontal top and bottom but inclined sides, shown in Figure 7. The structure is iterated from easier designs in order to make the structure more effective. The ballasted railway track rests on a precast concrete slab connected to a steel orthotropic deck (Gao 2012).

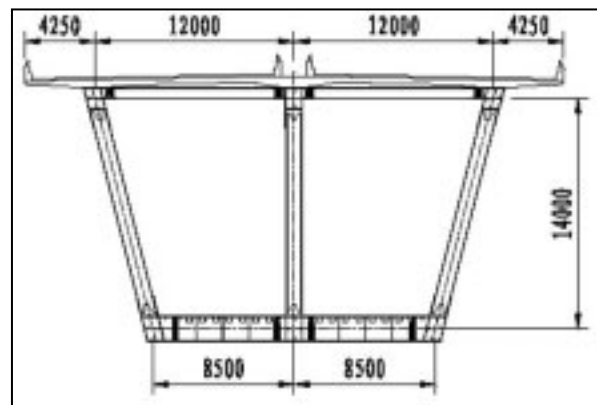


Figure 7 The truss is designed as a parallelogram (Gao 2012).

## 1.5 Previous research

Wiberg (2009) performed measurements of the dynamic response of a newly built railway bridge in Stockholm, Sweden; called the New Årsta Railway Bridge. An analysis was made of the bridge using a FE model and Euler-Bernoulli 3D beam elements. After running the FE model with parameters taken from static studies a substantial part of the author's work was to identify the relevant parameters for the dynamic model. The FE model was optimised by keeping the relevant parameters to a minimum which could be done since they were identified in the report. The implementation of the optimisation process in MATLAB was presented. Two algorithms were used in the process, the Yate's algorithm and the Nelder-Mead algorithm. In Wiberg's thesis, The Yate algorithm is used in big factorial experiments to speed up the process by simplifications while the Nelder-Mead algorithm is used by the *fminsearch* solver in MATLAB. By using the algorithms the predictions of the load effect in both static and dynamic studies were improved. It was shown that the bridge structure could be simplified with beam theory and the method of equivalent modulus of elasticity while still obtaining reliable results for simplified global analysis. The optimized FE model was used in a dynamic study of the New Årsta Railway Bridge exposed to high speed railway traffic. Calculations of the vertical

acceleration of the bridge deck showed that the results were below allowed code values from Eurocode. Multispan continuous concrete bridges may therefore be suitable for high speed railway bridges.

Kaliyaperumal, Imam et al. (2011) performed a case study of a railway bridge in Stockholm, Sweden. The aim of the study was to investigate different kinds of modelling techniques in the FE software ABAQUS and see how well the results coincided with measured data collected from studies where a train travelled across the bridge at three different velocities. The bridge was modelled as a one span beam, a continuous bridge with two spans and a six span continuous bridge (the whole bridge). In all cases two different sets of boundary conditions were used, simply supported and fixed, and both shell elements and beam elements were used to see how they capture the behaviour of the dynamic response of a bridge structure. It was also investigated whether secondary parts of the bridge structure, such as bracings, influence the dynamic behaviour of the bridge. By looking on the eigenfrequencies it was shown that secondary elements influence the response and therefore should be a part of the model. Two types of dynamic analyses were performed, implicit damping and modal damping. The results showed that the implicit damping model gave reliable results and that the model was computationally effective. Further on the authors showed that for the single span bridge model, the beam elements could not capture out-of-plane deformations of the main girders of the bridge. For the case where a three span bridge was investigated it could be shown that a time effective way to get acceptable results was to model the span of interest with shell elements and the rest of the spans with beam elements. When looking on the model of the six span bridge (full bridge) it could be seen that good results were obtained by using shell elements for the spans of interest and beam elements for the remaining spans.

## 2 Theory of structural dynamics

The dynamic behaviour of structures is a complex phenomenon and to achieve a better understanding of the work performed further on in the report some basics of structural dynamics will be explained in this chapter. The derivations in Sections 2.1 - 2.4 as well as Sections 2.6 and 0 are in accordance with Craig Jr and Kurdila (2006).

Single degree of freedom (SDOF) systems shows the general dynamic behaviour in a straight forward manner and is presented below.

Multiple degree of freedom (MDOF) systems and continuous systems are also presented. The theories of these systems are important in order to understand modelling of more complex dynamic structures such as railway bridges.

Important theories used for solving complex dynamic problems are also presented.

### 2.1 Undamped SDOF system

The most fundamental system considered will be the undamped SDOF system. Consider the SDOF in Figure 8 below. The system is assumed to be in rest at  $t = 0$  i.e.  $u(0) = \dot{u}(0) = 0$

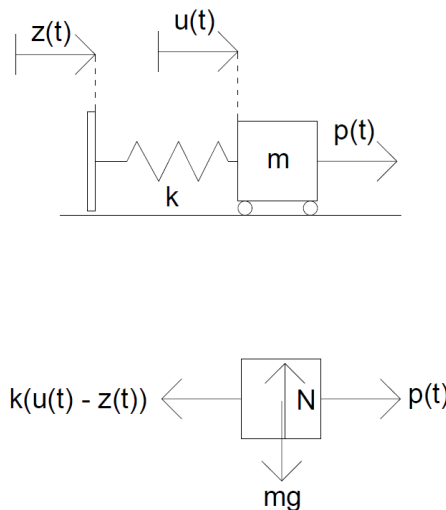


Figure 8 The undamped SDOF system, and the free body diagram of the same system.

The Equation of motion can now be expressed as:

$$m\ddot{u}(t) + ku(t) = kz(t) + p(t) \quad (1)$$

Where	$m$	Mass of the moving body	[kg]
	$k$	Spring stiffness	[N/m]
	$p(t)$	Applied load	[N]
	$u(t)$	Displacement	[m]

$\ddot{u}(t)$	Acceleration	[m/s <sup>2</sup> ]
$z(t)$	Base displacement	[m]

By introduction of the natural frequency  $\omega_n = \sqrt{\frac{k}{m}}$ , Equation (1) can be simplified as:

$$\ddot{u}(t) + \omega_n^2 u(t) = \omega_n^2 z(t) + \frac{p(t)}{m} \quad (2)$$

If the base can be assumed to be firm ( $z = 0$ ) the expression can be written as:

$$\ddot{u}(t) + \omega_n^2 u(t) = \frac{p(t)}{m} \quad (3)$$

The Differential Equation (DE) above has the solution:

$$u(t) = u_c(t) + u_p(t) \quad (4)$$

$u_c(t)$  is the complementary solution and  $u_p(t)$  is the particular solution. The latter term depends on the applied load. The complementary solution to this kind of DE can generally be written as:

$$u_c(t) = A_1 \cos \omega_n t + A_2 \sin \omega_n t \quad (5)$$

$A_1$  and  $A_2$  can be determined by the use of initial conditions (IC) and the presence of a particular solution.

### 2.1.1 Particular solution for a harmonic applied force

For the case with a harmonic applied force,  $p(t) = p_0 \cos \Omega t$ , the particular solution will be:

$$u_p(t) = \frac{p_0 \cos \Omega t}{m(\omega_n^2 - \Omega^2)} \quad (6)$$

Where	$p_0$	Amplitude of load	[N]
	$\omega_n$	Natural frequency	[rad/s]
	$\Omega$	Frequency of applied load	[rad/s]
	$m$	Mass	[kg]
	$u_p(t)$	Particular solution	[m]

When  $\Omega \rightarrow \omega_n$  the system is in resonance, which means that the displacement approach infinity.

The complete solution is solved by inserting IC and the solution of Equation (6) into Equation (4) to solve  $A_1$  and  $A_2$ .

### 2.1.2 Influence of an impulse load

In the case of an impulse load the duration of the load is much less than the time for one period of the system. The system will be derived in the following manner:

First, the impulse can be defined as:

$$I = \int_0^{t_d} p(t) dt \quad (7)$$

Where	$I$	Impulse load	[Ns]
	$p(t)$	Applied load	[N]
	$t_d$	Time of impulse	[s]

$t_d \ll T_n$ ;  $T_n$  being the period time of the system.

The equation of motion with the applied impulse becomes:

$$m\ddot{u} + ku = \begin{cases} p(t) & 0 < t \leq t_d \\ 0 & t_d < t \end{cases} \quad (8)$$

Integration with respect to time and using the ICs specified in Section 2.1 the following equation is obtained:

$$m\dot{u}(t_d) + k\bar{u}t_d = I \quad (9)$$

Where  $\bar{u}$  is the average displacement during the time  $0 < t \leq t_d$ . For a short impulse, i.e.  $t_d \rightarrow 0$ , the expression can be simplified as

$$m\dot{u}(0^+) = I \quad (10)$$

Hence, the impulse load has the effect of a given initial velocity, rather than a force. The initial velocity can be seen as:

$$\dot{u}(0^+) \approx \dot{u}(0) = \frac{I}{m} \quad (11)$$

Finally, this can be used to solve Equation 4, where  $u_p(t) = 0$ .

## 2.2 Damped SDOF system

In Section 2.1 an undamped SDOF was considered. In order to investigate damping of bridges, a linearly viscous damped SDOF system is considered. The equation of motion for the system can be derived in the same manner as for the undamped SDOF. Consider the system in Figure 9.

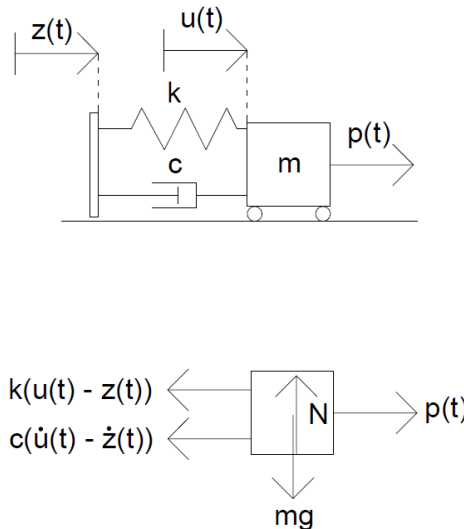


Figure 9 A damped SDOF system, and the free body diagram of the same system.

The equation of motion can be expressed as:

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = c\dot{z}(t) + kz(t) + p(t) \quad (12)$$

Where	$m$	Mass of the body	[kg]
	$c$	Damping coefficient	[Ns/m]
	$k$	Spring stiffness	[N/m]
	$u(t)$	Displacement	[m]
	$\dot{u}(t)$	Velocity	[m/s]
	$\ddot{u}(t)$	Acceleration	[m/s <sup>2</sup> ]
	$z(t)$	Base displacement	[m]
	$\dot{z}(t)$	Base velocity	[m/s]

The expression is simplified by introducing the damping ratio  $\xi = \frac{c}{c_{cr}}$  where  $c_{cr} = 2\sqrt{km}$  is the critical damping of the system.

If the damping quotient  $\xi$  is inserted in the equation of motion the following expression is obtained:

$$\ddot{u}(t) + 2\xi\omega_n \dot{u}(t) + \omega_n^2 u(t) = 2\omega_n \dot{z}(t) + \omega_n^2 z(t) + \frac{p(t)}{m} \quad (13)$$



By assuming that the base is firm,  $z(t) = 0$  the equation of motion becomes:

$$\ddot{u}(t) + 2\xi\omega_n \dot{u}(t) + \omega_n^2 u(t) = \frac{p(t)}{m} \quad (14)$$

This equation has a similar solution as the undamped case:

$$u(t) = u_c(t) + u_p(t) \quad (15)$$

### 2.2.1 Complementary solution

The complementary solution can be obtained from the equation below:

$$\ddot{u}(t) + 2\xi\omega_n \dot{u}(t) + \omega_n^2 u(t) = 0 \quad (16)$$

If the ansatz  $u_c(t) = C e^{st}$  is inserted in the equation above we get:

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad (17)$$

Where  $s$  can be solved as:

$$s_{1,2} = \omega_n(-\xi \pm \sqrt{\xi^2 - 1}) \quad (18)$$

For the solution of Equation 18 above, three important cases can be identified. These depend on the value of  $\xi$  and are called under-damped ( $\xi < 1$ ), over-damped ( $\xi > 1$ ) and critically damped ( $\xi = 1$ ) systems. When applying an initial displacement, the under-damped case the oscillation will proceed with decaying amplitude until it finally ends. For the over-damped case the motion will end without any oscillation. Critical damping defines the limit between an under- and an over-damped system, see Figure 10.

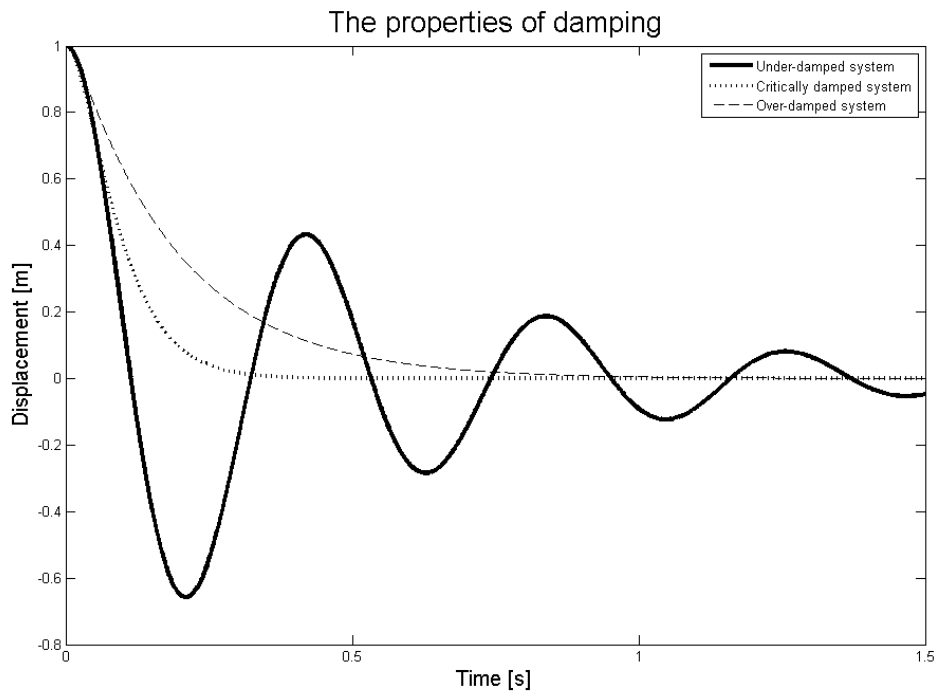


Figure 10 The difference between an over, under and critically damped system.

### Complementary solution for an under-damped system

First the under-damped case is investigated:

$$\xi < 1 \text{ gives } s_{1,2} = -\xi\omega_n \pm i\omega_d$$

Where the damped natural frequencies can be expressed as

$$\omega_d = \omega_n \sqrt{1 - \xi^2} \quad (19)$$

Insert  $s_{1,2}$  into the characteristic equation and the following complementary solution can be obtained using Eulers formula:

$$\begin{aligned} u_c(t) &= C_1 e^{ts_1} + C_2 e^{ts_2} = \\ &= e^{-\xi\omega_n t} (A_1 \cos \omega_d t + A_2 \sin \omega_d t) \end{aligned} \quad (20)$$

### Complementary solution for an over-damped system

If the damping factor  $c$  is greater than the  $c_{cr}$  the system is over-damped. The over-damped case can be solved as follows:

$$\xi > 1 \text{ gives } s_{1,2} \text{ as two negative real roots}$$

Therefore a variable  $\omega^*$  is defined as:

$$\omega^* = \omega_n \sqrt{\xi^2 - 1} \quad (21)$$

The complementary solution for this case is:

$$u_c(t) = e^{-\xi\omega_n t}(C_1 \cosh\omega^*t + C_2 \sinh\omega^*t) \quad (22)$$

### Complementary solution for a critically damped system

If the damping factor  $c$  is equal to  $c_{cr}$ , the system is critically damped:

$$\xi = 1 \text{ gives } s = -\omega_n$$

For a critically damped system the complementary solution will be:

$$u_c(t) = (C_1 + C_2 t)e^{-\omega_n t} \quad (23)$$

### 2.2.2 Particular solution for a harmonic applied load

For the case with a harmonic applied force,  $p(t) = p_0 \cos\Omega t$ , the particular solution will be as follows:

$$u_p(t) = U \cos(\Omega t - \alpha) \quad (24)$$

Where the phase shift  $\alpha$  is obtained from:

$$\tan \alpha = \frac{2\xi r}{1 - r^2} \quad (25)$$

Where

$$r = \frac{\Omega}{\omega_n} \quad (26)$$

The amplitude  $U$  is expressed as:

$$U = \frac{p_0}{k\sqrt{(1 - r^2)^2 + (2\xi r)^2}} \quad (27)$$

In this case  $\Omega \rightarrow \omega_n$  does not lead to resonance. However, this situation resembles resonance and is usually referred to as resonance.

The system can be solved by inserting the IC's to Equation (15).

## 2.3 Damped MDOF system

The damped SDOF system contains enough information to understand the dynamic behaviour of many systems. However, in order to capture the behaviour of more complex structures a system with multiple degrees of freedom needs to be considered. Here, the equation of motion consists of the stiffness matrix  $\mathbb{K}$ , the damping matrix  $\mathbb{C}$  and the mass matrix  $\mathbb{M}$ . The equation of motion will be derived for a two degrees of freedom system, see Figure 11. The procedure will be the same for an arbitrary  $N$  degree of freedom system.

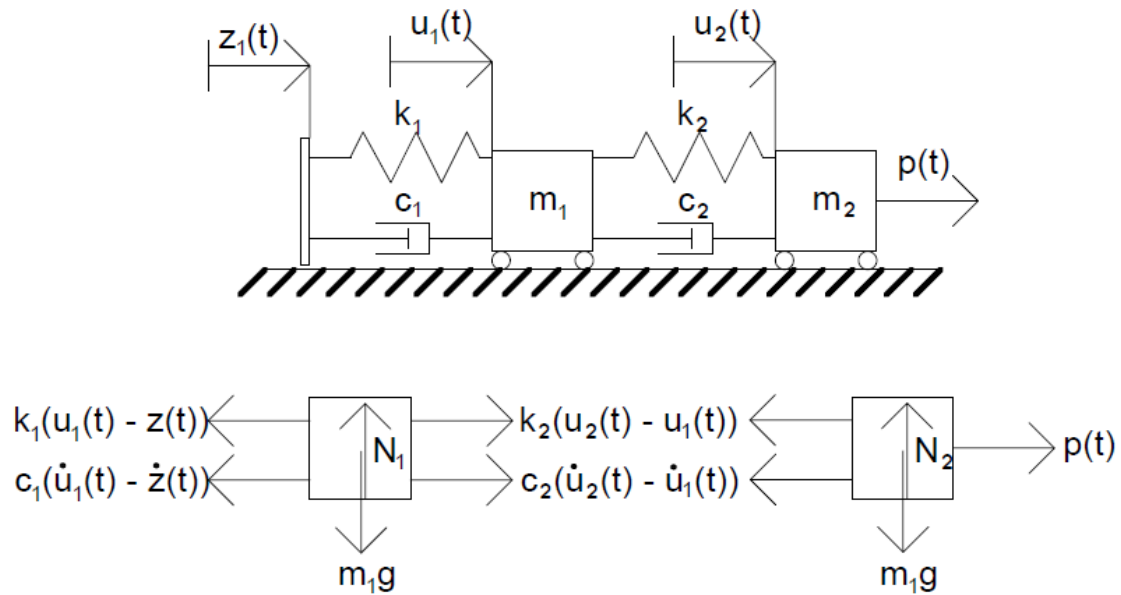


Figure 11 The two degree of freedom system considered.

In order to derive the mass matrix  $\mathbb{M}$ , the stiffness matrix  $\mathbb{K}$  and the damping matrix  $\mathbb{M}$  the first step is to use Newton's second law on each body. The base is assumed to be firm, hence  $z(t) = 0$ .

$$\begin{aligned} \Sigma F_1 &= m_1 \ddot{u}_1 = \\ &= -k_1 u_1(t) + k_2(u_2(t) - u_1(t)) - c_1 \dot{u}_1(t) + c_2(\dot{u}_2(t) - \dot{u}_1(t)) \end{aligned} \quad (28)$$

$$\Sigma F_2 = m_2 \ddot{u}_2 = -k_2(u_2(t) - u_1(t)) - c_2(\dot{u}_2 - \dot{u}_1) + p(t) \quad (29)$$

By rearranging the variables the equations can be written as:

$$\begin{aligned} \Sigma F_1 &= m_1 \ddot{u}_1 + k_1 u_1(t) - k_2((u_2(t) - u_1(t)) + c_1 \dot{u}_1(t) \\ &\quad - c_2(\dot{u}_2(t) - \dot{u}_1(t)) = 0 \end{aligned} \quad (30)$$

$$\Sigma F_2 = m_2 \ddot{u}_2 + k_2(u_2(t) - u_1(t)) + c_2(\dot{u}_2 - \dot{u}_1) = p(t) \quad (31)$$

The equation system formed by Equation (30) and (31) can be expressed in matrix form resulting in the following matrix system:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ p(t) \end{bmatrix} \quad (32)$$

which can be written as:

$$M\ddot{u} + C\dot{u} + Ku = p \quad (33)$$

The eigenfrequencies for the system is obtained by solving the eigenvalue problem  $K - \omega^2 M = 0$  as follows:

$$\left| \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} - \omega_i^2 \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \right| = 0 \quad (34)$$

The number of eigenfrequencies equals the number of degrees of freedom, in this case two.

## 2.4 Analytic solution of a simply supported beam

Continuous systems, as a beam has an infinite number of degrees of freedom. In this thesis the Euler-Bernoulli beam theory will be adapted.

Consider the Euler-Bernoulli beam in Figure 12.

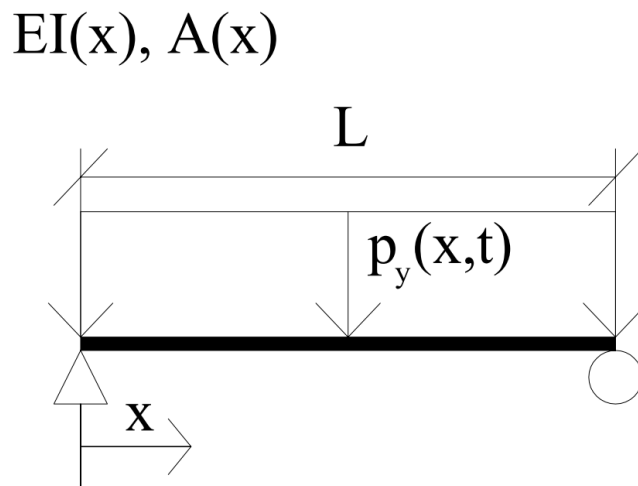


Figure 12 A simply supported Euler-Bernoulli beam.

The equation of motion can be written as:

$$\frac{\partial^2}{\partial x^2} \left( EI(x) \frac{\partial^2 v(x,t)}{\partial x^2} \right) + \rho A(x) \frac{\partial^2 v(x,t)}{\partial t^2} = p_y(x,t), 0 < x < L \quad (35)$$

Where	$EI(x)$	Flexural rigidity of the beam	$[\text{Nm}^2]$
	$A(x)$	Cross-sectional area of the beam	$[\text{m}^2]$
	$\rho$	Density of the beam	$[\text{kg/m}^3]$
	$p_y(x,t)$	Applied distributed load	$[\text{N/m}]$
	$v(x,t)$	Displacement of the beam	$[\text{m}]$

Consider an unloaded beam where the properties are constant for all  $x$ . The beam equation will be expressed as:

$$EI \left( \frac{\partial^4 v(x,t)}{\partial x^4} \right) + \rho A \frac{\partial^2 v(x,t)}{\partial t^2} = 0 \quad (36)$$

This is usually rewritten as:

$$\left( \frac{\partial^4 v(x,t)}{\partial x^4} \right) + \frac{\rho A}{EI} \frac{\partial^2 v(x,t)}{\partial t^2} = 0 \quad (37)$$

In order to simplify the notation, introduce the variable  $b$ , and let:

$$b^{-2} = \frac{\rho A}{EI} \quad (38)$$

which put into Equation (37) gives:

$$\left( \frac{\partial^4 v(x,t)}{\partial x^4} \right) + b^{-2} \frac{\partial^2 v(x,t)}{\partial t^2} = 0 \quad (39)$$

By assuming sinusoidal harmonic vibration of the beam, let the ansatz to the solution according to the assumed mode method be:

$$v(x,t) = V(x) \cos(\omega t - \alpha) \quad (40)$$

Put Equation (40) into Equation (39) to get:

$$\left( \frac{\partial^4 V(x)}{\partial x^4} \right) + \lambda^4 V(x) = 0 \quad (41)$$

where the eigenvalue is expressed as:

$$\lambda = \sqrt{\frac{\omega}{b}} \quad (42)$$

The general solution of the amplitude  $V(x)$  can be solved by the boundary conditions specified for the current case; here we consider the simply supported case.

$$V(x) = C_1 \sinh \lambda x + C_2 \cosh \lambda x + C_3 \sin \lambda x + C_4 \cos \lambda x \quad (43)$$

The solution to Equation (43) can be obtained using four boundary conditions. For a simply supported beam the boundary conditions are as follows:

$$V(0) = 0, \quad \left. \frac{d^2 V}{dx^2} \right|_{x=0} = 0 \quad (44, 45)$$

$$V(L) = 0, \quad \left. \frac{d^2 V}{dx^2} \right|_{x=L} = 0 \quad (46, 47)$$

Using the boundary conditions (44) and (45) in Equation (43) one can see that:

$$C_2 = C_4 = 0$$

The boundary conditions (46) and (47) inserted in Equation (43) will result in the following system of equations:

$$C_1 \sinh \lambda L + C_3 \sin \lambda L = 0 \quad (48)$$

$$\lambda^2 (C_1 \sinh \lambda L - C_3 \sin \lambda L) = 0 \quad (49)$$

By removing  $C_1$  and  $C_3$  and taking the determinant of the equation system expressed by Equation (48) and (49), the eigenvalues can be solved.

$$\begin{vmatrix} \sinh \lambda L & \sin \lambda L \\ \lambda^2 \sinh \lambda L & -\lambda^2 \sin \lambda L \end{vmatrix} = 0 \quad (50)$$

where the solution will become:

$$\sinh \lambda L \sin \lambda L = 0 \quad (51)$$

The eigenvalues can be determined from:

$$\sin \lambda L = 0 \quad (52)$$

Hence, the eigenvalues for simply supported beams are:

$$\lambda_r = \frac{r\pi}{L} \quad (53)$$

Where  $r = 1, 2, 3, \dots, n$

The natural frequencies are calculated by combining Equation (53) and (42) resulting in the following equation:

$$\omega_r = \left(\frac{r\pi}{L}\right)^2 \sqrt{\frac{EI}{\rho A}} \quad (54)$$

Where  $r = 1, 2, 3, \dots, n$

## 2.5 Analytic solution of a simply supported beam with a traversing point load

To display the dynamic behaviour of railway bridges the simply supported beam with a traversing point load will be shown. The derivation in this section is done in accordance to Frýba (1999).

Consider the damped simply supported Euler-Bernoulli beam subjected to a moving transverse point load in Figure 13 below.

$EI(x), A(x)$

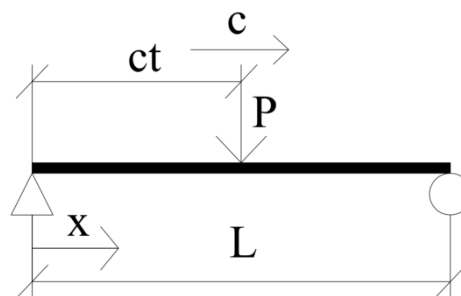


Figure 13 A simply supported beam subjected to a moving transverse load.



The governing differential equation for the beam is as follows, where the point force is expressed using the Dirac delta function.

$$\frac{\partial^2}{\partial x^2} \left( EI(x) \frac{\partial^2 v(x, t)}{\partial x^2} \right) + \rho A(x) \frac{\partial^2 v(x, t)}{\partial t^2} + 2\rho A(x) \omega_b \frac{\partial v(x, t)}{\partial t} = \delta(x - ct)P \quad 0 < x < L \quad (55)$$

Where	$EI(x)$	Flexural rigidity of the beam	[Nm <sup>2</sup> ]
	$\rho$	Density of the material	[kg/m <sup>3</sup> ]
	$A(x)$	Cross-sectional area of the beam	[m <sup>2</sup> ]
	$v(x, t)$	Displacement of the beam	[m]
	$\omega_b$	Circular frequency of the damping of the beam	[rad/s]
	$P$	Applied point load	[N]
	$\delta(x - ct)$	The Dirac delta function	[-]
	$c$	Constant speed of the applied load	[m/s]

The beam is initially at rest, and the boundary conditions are stated as:

$$v(0, t) = 0, \quad \left. \frac{d^2 v(x, t)}{dx^2} \right|_{x=0} = 0 \quad (56, 57)$$

$$v(L, t) = 0, \quad \left. \frac{d^2 v(x, t)}{dx^2} \right|_{x=L} = 0 \quad (58, 59)$$

The Dirac delta function is zero everywhere except in a certain point, where the magnitude is infinity. The total integral from minus infinity to infinity is equal to one. These properties make it possible to model a point force with a given location and magnitude using the Dirac delta function as follows.

$$\int_a^b \delta(x - ct)P dx = \begin{cases} 0 & \text{for } ct < a < b \\ P & \text{for } a < ct < b \\ 0 & \text{for } a < b < ct \end{cases} \quad (60)$$

To solve the differential Equation (55), the first step is to use a Fourier sine integral transformation that is defined by

$$V(n, t) = \int_0^L v(x, t) \sin\left(\frac{n\pi x}{L}\right) dx \quad (61)$$

Where  $n = 1, 2, 3, 4, \dots, n$

The back transform by definition becomes:

$$v(x, t) = \frac{2}{l} \sum_{n=1}^{\infty} V(n, t) \sin\left(\frac{n\pi x}{L}\right) \quad (62)$$

Where  $n = 1, 2, 3, 4, \dots, n$

Using the Fourier transform together with Equation (55) and the Fourier transform for the known Dirac delta function, the following equation is obtained:

$$\frac{n^4 \pi^4}{L^4} EIV(n, t) + \rho A \ddot{V}(n, t) + 2\rho A \omega_b \dot{V}(n, t) = P \sin\left(\frac{n\pi ct}{L}\right) \quad (63)$$

To simplify the problem, the circular frequency of a simply supported beam at the  $n$ -th mode of vibration is introduced as:

$$\omega_{(n)}^2 = \frac{n^4 \pi^4 EI}{L^4 \rho A} \quad (64)$$

and the circular frequency of the load is written as

$$\omega = \frac{\pi c}{L} \quad (65)$$

Inserting Equation (64) and (65) into Equation (63) one gets:

$$\ddot{V}(n, t) + 2\omega_b \dot{V}(n, t) + \omega_{(n)}^2 V(n, t) = \frac{P}{\rho A} \sin n\omega t \quad (66)$$

As can be seen, Equation (66) contains no spatial derivatives; therefore it is easier to deal with than the original equation of motion. By using Laplace-Carson integral transformation the time derivatives can be eliminated as well, resulting in a problem that is dependent of two new variables; the mode number  $n$  and an unknown variable in the complex plane,  $p$ . The Laplace-Carson transformation is in this case defined by:

$$V^*(n, p) = p \int_0^{\infty} V(n, t) e^{-pt} dt \quad (67)$$

The back transform has the following definition:

$$V(n, t) = \frac{1}{2\pi i} \int_{a_0 - i\infty}^{a_0 + i\infty} e^{tp} \frac{V^*(n, p)}{p} dp \quad (68)$$

Using the Laplace-Carson transform on Equation (66) one gets the following equation:

$$p^2 V^*(n, p) + 2\omega_b p V^*(n, p) + \omega_{(n)}^2 V^*(n, p) = \frac{Pn\omega}{\rho A} \frac{p}{p^2 + n^2\omega^2} \quad (69)$$

From Equation (69)  $V^*(n, p)$  can easily be solved. The solution to the original problem is then found by using the back transform of Laplace-Carson and Fourier. The final result is

$$\begin{aligned} v(x, t) = v_0 \sum_{n=1}^{\infty} & \left( \frac{1}{n^2(n^2(n^2 - \alpha^2)^2 + 4\alpha^2\beta^2)} (n^2(n^2 - \alpha^2) \sin n\omega t + \right. \\ & \left. - \frac{n\alpha(n^2(n^2 - \alpha^2) - 2\beta^2)}{(n^4 - \beta^2)^2} e^{-\omega_b t} \sin \omega'_{(n)} t + \right. \\ & \left. - 2n\alpha\beta (\cos n\omega t - e^{-\omega_b t} \cos \omega'_{(n)} t) \right) \sin \frac{n\pi x}{L} \end{aligned} \quad (70)$$

where the following simplifications have been introduced:

The initial deformation  $v_0$ :

$$v_0 = \frac{PL^3}{48EI} \quad (71)$$

$$\omega'^2_{(n)} = \omega_{(n)}^2 - \omega_b^2 \quad (72)$$

The two parameters  $\alpha$  and  $\beta$  are defined as:

$$\alpha = \frac{cl}{\pi} \left( \frac{\rho A}{EI} \right)^{0.5} = \frac{c}{c_{cr}} \quad (73)$$

$$\beta = \frac{\omega_b l^2}{\pi^2} \left( \frac{\rho A}{EI} \right)^{0.5} \quad (74)$$

$\omega_b$  is an expression of the damping frequency of the beam and is defined as follows:

$$\omega_b = \vartheta f_1 \quad (75)$$

where  $\vartheta$  is a function that characterizes the damping of the beam. This parameter is further discussed in Section 3.3, which focuses on damping of bridges.

## 2.6 Truncation of systems

In structural engineering the modes corresponding to the lowest eigenfrequencies are usually the most critical. Due to this, the governing equation for the system is often truncated to only contain the modes corresponding to the frequencies of interest. This is done by using what is called the mode-displacement approximation where the equation of motion is defined as:

$$\widehat{\Phi}^T \mathbb{K} \widehat{\Phi} \hat{\eta}(t) + \widehat{\Phi}^T \mathbb{C} \widehat{\Phi} \dot{\hat{\eta}}(t) + \widehat{\Phi}^T \mathbb{M} \widehat{\Phi} \ddot{\hat{\eta}} = \widehat{\Phi}^T \mathbb{F} \quad (76)$$

Where	$\widehat{\Phi}$	Eigenvectors of the original system
	$\hat{\eta}$	Displacement in new principle coordinates
	$\dot{\hat{\eta}}$	Velocity in new principle coordinates
	$\ddot{\hat{\eta}}$	Acceleration in new principle coordinates
	$\mathbb{K}$	Stiffness matrix of the original system
	$\mathbb{M}$	Mass matrix of the original system
	$\mathbb{C}$	Damping matrix of the original system
	$\mathbb{F}$	Load matrix of the original system

The system is solved in the same manner as in section 2.3 for the new principle coordinates  $\eta$  and can be translated to the time and spatial coordinates in the following manner:

$$\mathfrak{u}(t) \approx \widehat{\mathfrak{u}}(t) = \sum_{i=1}^{\widehat{N}} \phi_i \eta_i(t) = \widehat{\Phi} \hat{\eta}(t) \quad (77)$$

The acceleration of the system can be calculation by using the identification below:

$$\ddot{\mathfrak{u}}(t) \approx \ddot{\widehat{\mathfrak{u}}}(t) = \sum_{i=1}^{\widehat{N}} \phi_i \ddot{\eta}_i(t) = \widehat{\Phi} \ddot{\hat{\eta}}(t) \quad (78)$$

## 2.7 The Newmark- $\beta$ Method

The Newmark- $\beta$  method is an unconditionally stable method for direct time integration of transient problems. In an unconditionally stable method the solution is stable independent of the size of the time step used. The method is based on a generalised Taylor series approximation where the displacement vector  $\mathbb{u}_{n+1}$  can be written as:

$$\mathbb{u}_{n+1} = \mathbb{u}_n + \dot{\mathbb{u}}_n h + [(1 - 2\beta)\ddot{\mathbb{u}}_n + 2\beta\ddot{\mathbb{u}}_{n+1}] \frac{h^2}{2} \quad (79)$$

Where	$\mathbb{u}_n$	Displacement at step $n$
	$\mathbb{u}_{n+1}$	Displacement at step $n + 1$
	$\dot{\mathbb{u}}_n$	Velocity at step $n$
	$\ddot{\mathbb{u}}_n$	Acceleration at step $n$
	$\ddot{\mathbb{u}}_{n+1}$	Acceleration at step $n + 1$
	$h$	Time step
	$\beta$	Constant

The velocity can be expressed by:

$$\dot{\mathbb{u}}_{n+1} = \dot{\mathbb{u}}_n + [(1 - \gamma)\ddot{\mathbb{u}}_n + \gamma\ddot{\mathbb{u}}_{n+1}]h \quad (80)$$

Where	$\dot{\mathbb{u}}_{n+1}$	Velocity at step $n + 1$
	$\dot{\mathbb{u}}_n$	Velocity at step $n$
	$\ddot{\mathbb{u}}_n$	Acceleration at step $n$
	$\ddot{\mathbb{u}}_{n+1}$	Acceleration at step $n + 1$
	$h$	Time step
	$\gamma$	Constant

The equation of motion for the system, when applying the Newton- $\beta$  method can be seen below:

$$\mathbb{M}\ddot{\mathbb{u}}_{n+1} + \mathbb{C}\dot{\mathbb{u}}_{n+1} + \mathbb{K}\mathbb{u}_{n+1} = \mathbb{p}_{n+1} \quad (81)$$

By inserting the generalised equations for the displacement and the velocity into the governing equation of motion one can rearrange to get a linearly dependent equation for the new acceleration vector  $\ddot{\mathbb{u}}_{n+1}$ . One could also base the iterative calculations around the displacement vector  $\mathbb{u}_{n+1}$ . In this thesis the iteration is based on the displacement vector using  $\beta = \frac{1}{4}$  and  $\gamma = \frac{1}{2}$  which is called the Constant Average Acceleration Method (Craig Jr and Kurdila 2006).

### **3 Dynamic behaviour of railway bridges**

In this chapter the dynamic behaviour of railway bridges and the design requirements in Eurocode concerning bridges used in high-speed railway lines will be presented.

#### **3.1 Design requirements according to Eurocode**

In order to perform a dynamic analysis for a double track railway bridge, Eurocode states that the load should be applied on only one track at a time. The dynamic study has to include a study of the highest vertical acceleration of the bridge deck and all supporting structures. Calculations should take into account the frequencies up to the highest of either 30 Hz, 1.5 times the frequency of the first mode or the frequency of the third mode of vibration (Swedish Standards Institute 2010).

The allowed maximum vertical acceleration is  $5 \text{ m/s}^2$  for a railway bridge without ballast and  $3.5 \text{ m/s}^2$  for a bridge with ballast. The first natural frequency should preferably be higher than 1.2 Hz (Swedish Standards Institute 2010).

When designing a high speed railway bridge, Eurocode also states that the maximum vertical deflection of the bridge deck should be small enough to ensure that the vertical radius of the rails is acceptable. A recommended value of deflection is less than  $L/600$ , where  $L$  is the length of each span (Swedish Standards Institute 2010).

Further checks need to be performed during design, for example horizontal displacements of the bridge deck, rotation at supports etc. However, these are not within the scope of this thesis and are therefore excluded.

#### **3.2 Load model for design of high-speed railway lines according to Eurocode**

For the design of high-speed railway lines where the speeds exceeds 200 km/h a load model called High Speed Load Model (HSLM) has been developed and should be used according to Eurocode. The load model HSLM contains two different train models, HSLM-A and HSLM-B, which resemble the loads of high-speed trains (Swedish Standards Institute 2007).

The model HSLM-A should be used for spans longer than 7 meters and is characterised by a set of different train geometries designed to represent all real trains that are used in high-speed railway lines. The load of the train is represented by multiple point loads. A presentation of the HSLM-A trains can be seen in the figure below:

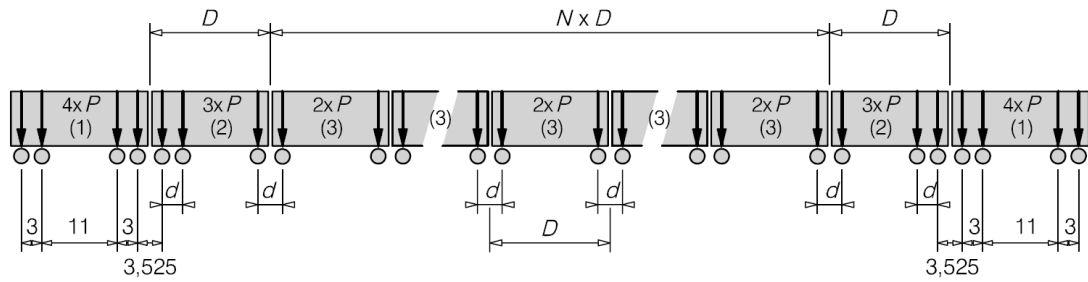


Figure 14 A presentation of the geometry of the modelled train that together with Table 1 form the ten trains in HSLM-A. From SS-EN 1991 (Swedish Standards Institute 2007)

The different trains can then be described by using the configurations in the table below as well as the figure above.

Table 1 The different geometries for the modelled trains in HSLM-A, taken from Eurocode (Swedish Standards Institute 2007)

Train	Number of carriages N	Length of carriage D [m]	Distance between bogie axles d [m]	Point load P [kN]
A1	18	18	2.0	170
A2	17	19	3.5	200
A3	16	20	2.0	180
A4	15	21	3.0	190
A5	14	22	2.0	170
A6	13	23	2.0	180
A7	13	24	2.0	190
A8	12	25	2.5	190
A9	11	26	2.0	210
A10	11	27	2.0	210

According to Eurocode, the design speed should be 1.2 times the highest allowed speed of the railway line. Another demand for the analyses is that the load model should be used for a series of train speeds from 40 m/s to the design speed. Noteworthy is that the load model is valid for a maximum speed of 350 km/h (Swedish Standards Institute 2007).

The resonance speed for a certain train can be calculated by the following equation:

$$v_{res} = n_0 \lambda \quad (82)$$

Where	$\lambda = \frac{D}{i}$	Excitation wavelength
	$D$	Length of carriage
	$v_{res}$	Resonance speed
	$n_0$	First natural frequency of the bridge
	$i$	1,2,3,4

A typical response for a beam-like bridge subjected to the HSLM-A load model is shown in Figure 15. The figure shows the resonance peaks from different trains.

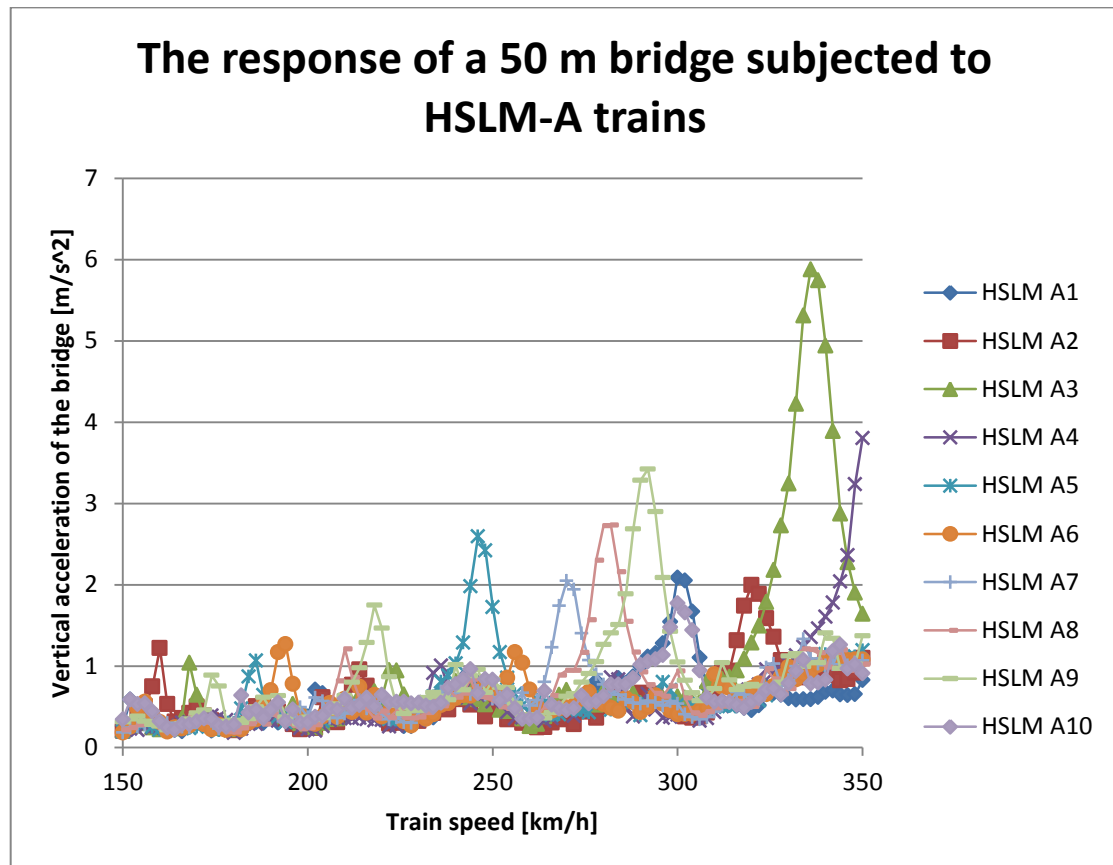


Figure 15 The response of a 50 m bridge subjected to HSLM-A trains with an increment of 2 km/h each point on the lines represents one considered speed.

In this thesis, the load model HSLM-A has been chosen for analysis and therefore the model of HSLM-B is not further explained here.



### 3.3 Damping of bridges

In order to calculate a realistic behaviour of a bridge, the damping factor of the bridge must be estimated. The damping can be calculated from the logarithmic decrement of damping for the beam,  $\vartheta$ , using the following empirical expression (Kaliyaperumal, Imam et al. 2011)

$$\vartheta = \frac{f_1}{10 \left(1 + \frac{10}{f_1}\right)} \quad (83)$$

The relation between the logarithmic decrement of the beam,  $\vartheta$ , and damping of the beam,  $\xi$ , can then be found as

$$\xi = \frac{\vartheta}{2\pi} \quad (84)$$

The vertical accelerations depend on the damping in a way that an increased damping gives lower vertical accelerations. However, the damping behaviour is complicated to predict. For example, two organisations did similar measurements on the St-Denis Bridge. The resulting damping factor from the two assessments, which both were proved to be correctly executed, became 0.4 % and 1.6 % respectively (ERRI D214 1999).

In Eurocode there is a lower bound approach which only takes the length of the bridge into account. By underestimating the damping, the calculated vertical accelerations will result in larger amplitudes than the real vibrations.

The following design values are specified in Eurocode for high speed railway bridges (Swedish Standards Institute 2007):

**Table 2** The design damping values to be assumed (Swedish Standards Institute 2007)

Bridge type	Damping $\xi$ [%]	
	L < 20 m	L > 20 m
Steel and composite	0.5 + 0.125 (20 - L)	0.5
Pre-stressed concrete	1.0 + 0.07 (20 - L)	1.0
Reinforced concrete	1.5 + 0.07 (20 - L)	1.5

### 3.4 Influence of weight

The weight is an important factor concerning the dynamic behaviour of a bridge. The mass has the effect that if increased, it will lower the global vertical acceleration of the bridge. However, the increased mass will also lower the resonance speed resulting in a higher risk that resonance might occur within the analysed interval. In order to account for both of these effects Eurocode requires two specific design cases considering mass (Swedish Standards Institute 2007).

- A lower bound estimation of the weight is used in order to account for the biggest possible acceleration of the bridge deck (Swedish Standards Institute 2007).
- A higher bound estimation of the weight is used in order to evaluate the lowest possible resonance speed of the system (Swedish Standards Institute 2007).

The figure below demonstrates the influence of the structural weight of a simply supported bridge subjected to the load HSLM-A1. In Figure 16 the decreased amplitude of the global vertical acceleration and the lowered resonance speed as a result of the increased mass can clearly be seen.

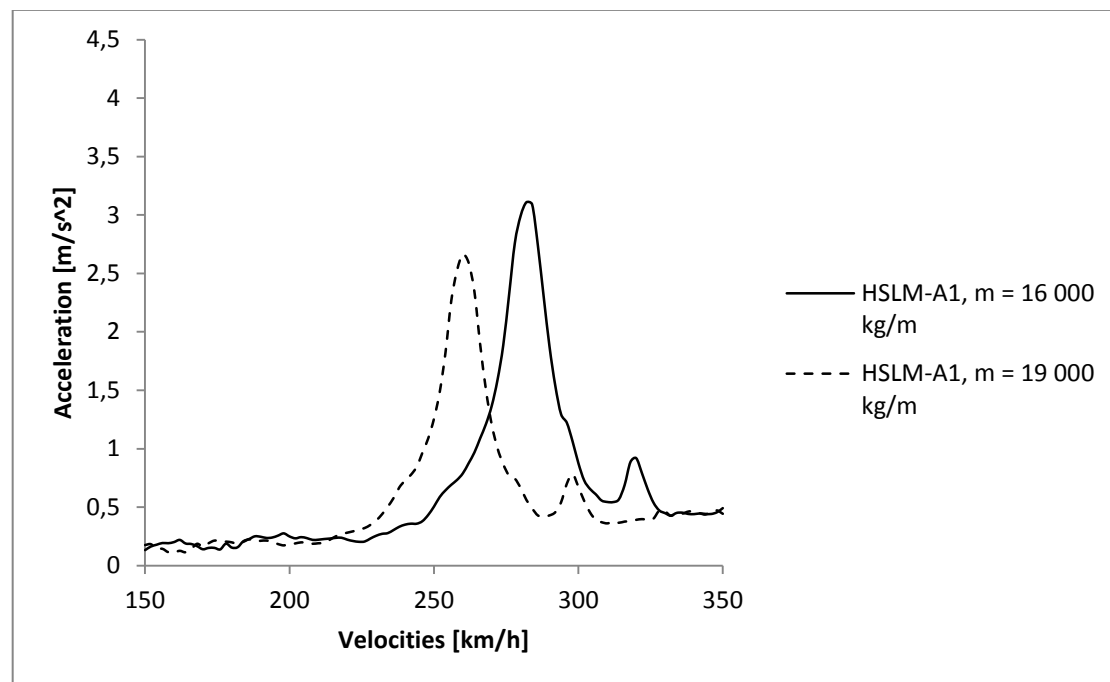


Figure 16 Illustration of the influence of the mass of the beam on the dynamic behaviour.

### 3.5 Influence of the flexural rigidity

The dynamic behaviour of a bridge depends on its flexural rigidity since an increase will raise the natural frequency and thereby also the resonance speed. Eurocode states that a lower bound value of the Young's modulus should be used since this is conservative (Swedish Standards Institute 2007). Noteworthy, is that the flexural rigidity of the bridge does not influence the amplitude of vertical acceleration, illustrated in Figure 17.

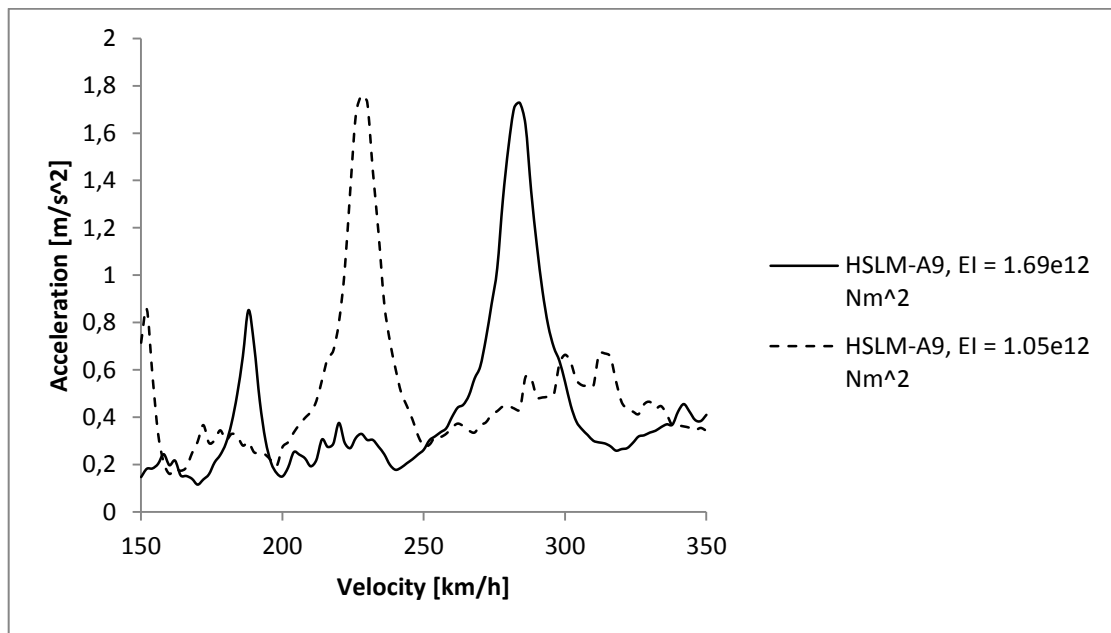


Figure 17 Illustration of the influence of the flexural rigidity of the beam on the dynamic behaviour.

## 4 Parametric study

In order to investigate the dynamic behaviour of simply supported bridges subjected to high speed train passages (HSLMA1-10) a parametric study was conducted. Three variables were investigated in order to evaluate the behaviour; the mass per meter, the flexural rigidity and the length of the bridge.

The parametric study was conducted using a number of MATLAB scripts. All scripts (except modaldamp.m which were developed by Prof Thomas Abrahamsson) were written specifically for this Master Thesis. The CALFEM 3.4 package was used during the development of the MATLAB scripts and is needed for usage.

The main script is called Parametricstudy.m. The physical properties of the beams were stated in the input file inputfile.m and can be chosen to represent any kind of Euler-Bernoulli beam. Further on, the main script generated the maximum vertical accelerations for the specified input parameters by using the following sub-routines:

- Loading2.m
- FEeig\_val\_solver.m
- modaldamp.m
- Newmark\_iteration.m
- HSLMAtrainset.m

The generated results were saved into .mat-files. The data was then imported to Excel using post\_processor.m. Contour plots were created by the scripts plotter.m and plottereigfreq.m which imports data from the excel-book. A more detailed description of each of the scripts is presented in the table found in Appendix A. The source code to all MATLAB scripts can be found Appendix B.

### 4.1 Finite element-model

The parametric study only considers simply supported beams. However, the MATLAB scripts were designed in a way that makes further studies with arbitrary boundary conditions possible. The bridge was modelled with Euler-Bernoulli beam elements using beam2d.m, found in the CALFEM 3.4 package. Beam2d.m generates the stiffness and mass matrices with three degrees of freedom for each node, see Figure 18.

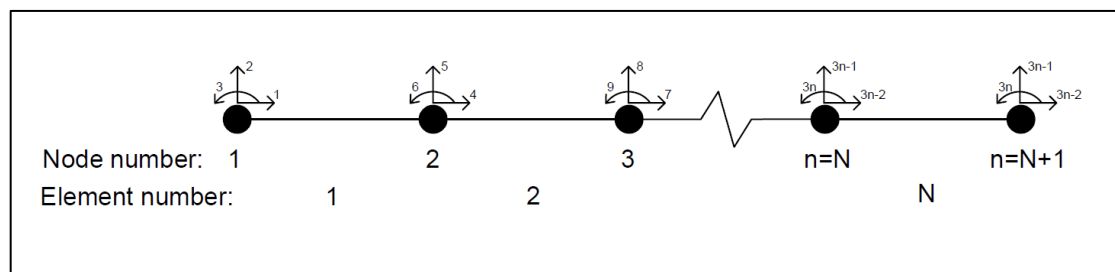


Figure 18 The node numbering of the original plane frame system from CALFEM.

In order to shorten the calculation time the horizontal degrees of freedom were reduced. This was possible since the calculation only considered vertical acceleration and the load at all time is orthogonal to the horizontal degrees of freedom.

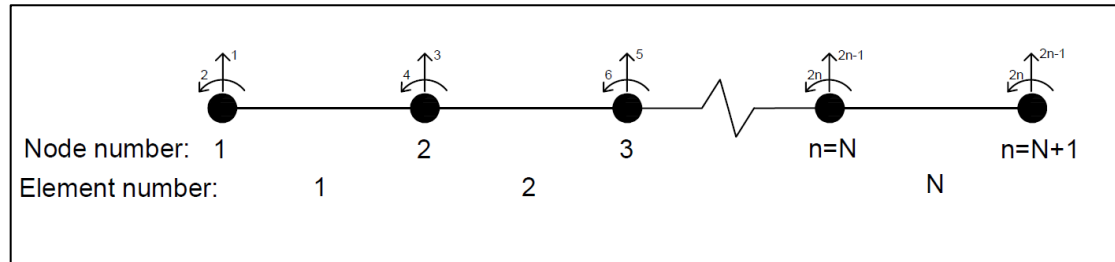


Figure 19 The node numbering of the reduced system.

The FE-model was truncated according to the requirements of Eurocode and calculations were performed using The Newmark- $\beta$  method. The truncation removes the high frequency vibrations. This cut-off is crucial for the convergence of the system. In a non-truncated system the time step will act as a cut-off giving higher accelerations for shorter time steps. Hence, a non-truncated system can be found difficult to converge. In practice, the truncated system will resemble reality well, since the accelerations at these high frequencies will not lead to instability of the bridge deck (ERRI D214 1999).

#### 4.1.1 The applied load

To minimise the resources needed for calculation, the results are computed for one point load representing one bogie axle of the train. The point load crosses the beam and the response of the decay of vibration in the beam is studied. In this parametric study, the total time considered is based on longest HSLM-A train. The time was chosen so that this train crosses the beam and travels an additional 50 meters along the line. Different train models may then be formed by phase shifting and superimposing the results, depending on the configuration of each train. This approach is possible since the system is linear.

A function called loading2.m was constructed in MATLAB. For each time step in the Newmark- $\beta$  iteration the point load moves across the length of the beam and distributes the load to the nodes at each element. The amplitude at each degree of freedom was computed using shape functions,  $\psi_i$ , from (Craig Jr and Kurdila 2006).

$$\psi_1(x) = 1 - 3\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3 \quad (85)$$

$$\psi_2(x) = x - 2L\left(\frac{x}{L}\right)^2 + L\left(\frac{x}{L}\right)^3 \quad (86)$$

$$\psi_3(x) = 3\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3 \quad (87)$$

$$\psi_4(x) = -L\left(\frac{x}{L}\right)^2 + L\left(\frac{x}{L}\right)^3 \quad (88)$$

The load is stored in a matrix containing the loading at all degrees of freedom at all time steps using the following definition:

$$F_i(t) = \int_0^L p(x,t)\psi_i(x)dx \quad (89)$$

The load effect from the train is approximated as a series of point loads travelling along the beam at a constant velocity (HSLM-A) as stated earlier. All ten HSLM-A trains are considered in the model. The MATLAB function HSLMAtrainset.m is used in order to implement the load model.

## 4.2 Convergence study

In order to confirm the validity and display the accuracy a convergence study was performed. The two most critical parameters are the number of elements and the size of the time step for iteration. Normally the mesh size is confirmed by doing a convergence study to see if the calculated eigenfrequencies have converged (Johansson, Andersson et al. 2010). The convergence study was performed in two parts. In the first part the influence of element size on the calculated natural frequencies was studied. The second part considers the size of the time step with respect to the displacements. The model was compared to an analytical solution collected from Frýba (1999).

### 4.2.1 Influence of mesh size on the eigenfrequencies

In the first part of the convergence study a series of different mesh sizes was used when calculating the eigenfrequencies. The geometry of the beam as well as the physical properties of the beam is shown in the Table 3. Noteworthy, is that the properties were chosen in order to perform the convergence study and was not intended to reflect a real beam.

Table 3 Input data for the convergence study of the mesh size

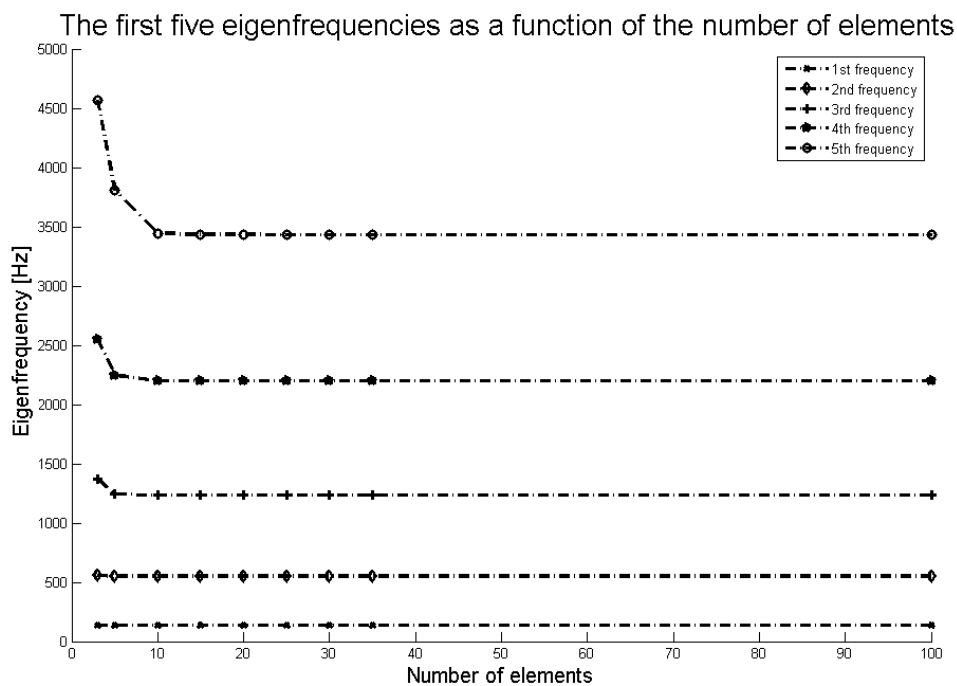
Input parameter		Input data
Area	$A$ [m <sup>2</sup> ]	0.0395
Second moment of area	$I$ [m <sup>4</sup> ]	1.7392
Young's modulus	$E$ [GPa]	210
Length	$L$ [m]	20
Material density	$\rho$ [kg/m <sup>3</sup> ]	7800
Damping ratio	$\xi$ [-]	0.005

Eight different mesh sizes were investigated in the study to evaluate the influence in calculation of the frequencies. The number of elements for each of these mesh sizes are shown in Table 4:

**Table 4** The influence of the mesh size on the fifth eigenfrequency of the system.

Mesh nr	Number of elements	Estimated error of the 5 <sup>th</sup> eigenfrequency
1	3	
2	5	16,51 %
3	10	9,55 %
4	15	0,31 %
5	20	0,05 %
6	25	0,002 %
7	30	0,001 %
8	35	<0,001 %
9	100	<0,001 %

The eigenfrequencies were computed, see Figure 20, for each mesh size and the first five were then plotted as a function of the mesh size. In engineering the lowest natural frequencies are the most important for the response of a structure which is the reason why the five first, i.e. lowest, natural frequencies are investigated in the study.



**Figure 20** The eigenfrequencies as a function of the mesh size

A mesh size finer than 15 elements does not have a significant influence on the results of the natural frequencies, see Figure 20.

#### 4.2.2 Influence of the time step

The time step for iteration with Newton- $\beta$  method needs to be small enough to get valid accuracy of the results. An analysis was performed with a number of different

time steps for a simply supported beam, subjected to moving point load with a constant speed. The input to the analysis can be seen in the following table:

**Table 5** Input parameters for the convergence study of the influence of the time step

Input parameter		Input data
Area	$A$ [m <sup>2</sup> ]	0.0395
Second moment of area	$I$ [m <sup>4</sup> ]	1.7392
Young's modulus	$E$ [GPa]	210
Length	$L$ [m]	20
Material density	$\rho$ [kg/m <sup>3</sup> ]	7800
Damping ratio	$\xi$ [-]	0.005

The analysis was made with respect to the maximum mid-span vertical acceleration. A series of time steps was considered, as can be seen in Table 6 and Table 7. From the convergence study concerning the influence of the number of elements a mesh size of 15 elements is chosen for further analysis. Since the parametric study is performed for train speeds from 150 to 350 km/h the calculations can be quite time consuming. Due to this fact, the convergence study of the time step is performed in two speed intervals.

**Table 6** The influence of the time step for velocities between 272 and 350 km/h

Velocity	Length of time step [s]	$a_{\max}$ [m/s <sup>2</sup> ]	Estimated error between time steps
<b>C=272-350 km/h</b>	0.1	0.1961	
	0.01	2.1658	1004%
	0.001	0.8302	61.67 %
	0.0001	0.8217	1.024%
	0.00001	0.8216	0.0122 %

**Table 7** The influence of the time step for velocities between 192 and 270 km/h

Velocity	Length of time step [s]	$a_{\max}$ [m/s <sup>2</sup> ]	Estimated error between time steps
<b>C=192-270 km/h</b>	0.1	0.1451	
	0.01	0.6527	349.8 %
	0.001	0.6380	2.25 %
	0.0001	0.6340	0.627 %
	0.00001	0.6339	0.0158 %

The time step chosen in the parametric study is  $h=0.0001$  for speeds between  $272 \leq c \leq 350$  km/h and  $h=0.001$  for  $c \leq 270$  km/h.



### 4.3 Speed increment in the analyses

As stated in Section 3.2, Eurocode states no requirements or recommendations concerning the increments of speed. Further on, The Swedish Transport Administration (Trafikverket) does not give any guidelines on this subject. In this study an increment of 2 km/h was used, see Figure 21, giving accuracy that the authors regard as sufficient.

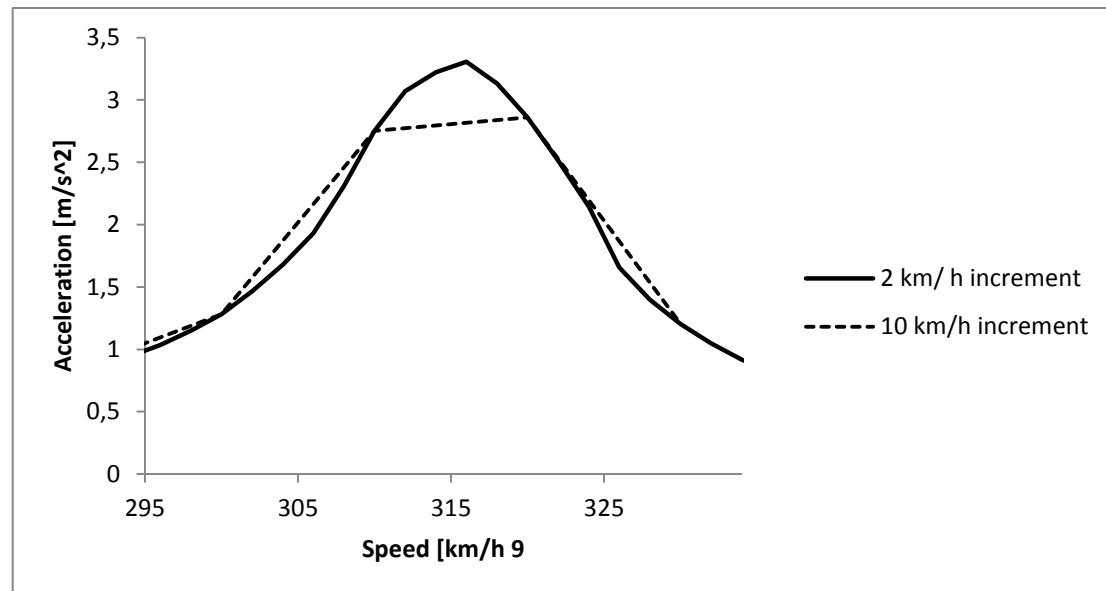


Figure 21 Illustration of the importance of the size of the speed increment in dynamic analyses

The figure above illustrates the importance of choosing a suitable time increment in analysis. If a too coarse speed increment is chosen, important acceleration peaks might be excluded from the analysis.

### 4.4 Studied parameters

A number of analyses have to be made in order to study the influence of different important parameters concerning the dynamic behaviour of bridges. As stated in the limitations of the thesis, only simply supported bridges are studied here.

Three different beam lengths (20, 35 and 50 meters) are analysed to examine how the length of a bridge will affect the dynamic behaviour.

Masses in the range of 1-40 metric tons/meter are analysed to see at which flexural stiffness,  $EI$ , the bridge fulfils the requirements as stated in section 3.1. All load configurations in the model HSLM-A are used for train speeds from 150 to 350 km/h and the length of each time step is chosen according to Section 4.2. The investigated parameters can be seen in Figure 22 below.

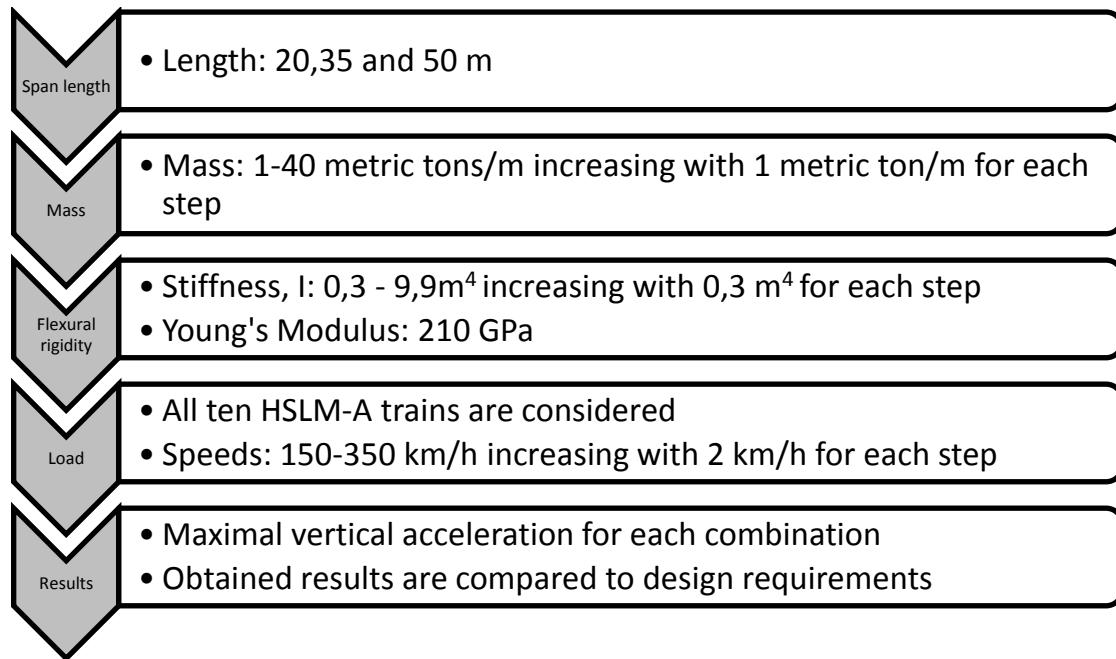


Figure 22 The parameters considered in the parametric study.

All acceleration data are stored together with the specified input data for each analysis. The results are then exported to excel where the design values for each length are defined and the data is plotted as contour plots in order to visualize trends and patterns.

## 4.5 Results and analysis

The results of each of the three considered lengths (20, 35 and 50 meter) are plotted in contour plots where the vertical acceleration is plotted for each combination of flexural rigidity and mass, see Figure 23, Figure 25 and Figure 27. In preliminary design, these contour plots can be useful to get a lower bound limit of the required flexural rigidity of a bridge.

When the eigenfrequency of a bridge is known, it is advantageous to use a plot where the acceleration is a function of the eigenfrequency and the mass. This is particularly useful in assessment of the possibility of using existing bridges in high-speed railway lines or during detailed development of a new bridge concept. Noteworthy, is that both types of contour plots are based on the same data, where the eigenfrequency is calculated using Equation 54. The vertical acceleration as a function of the eigenfrequency and mass for the considered lengths (20, 35 and 50 meter) is shown in Figure 24, Figure 26 and Figure 28.

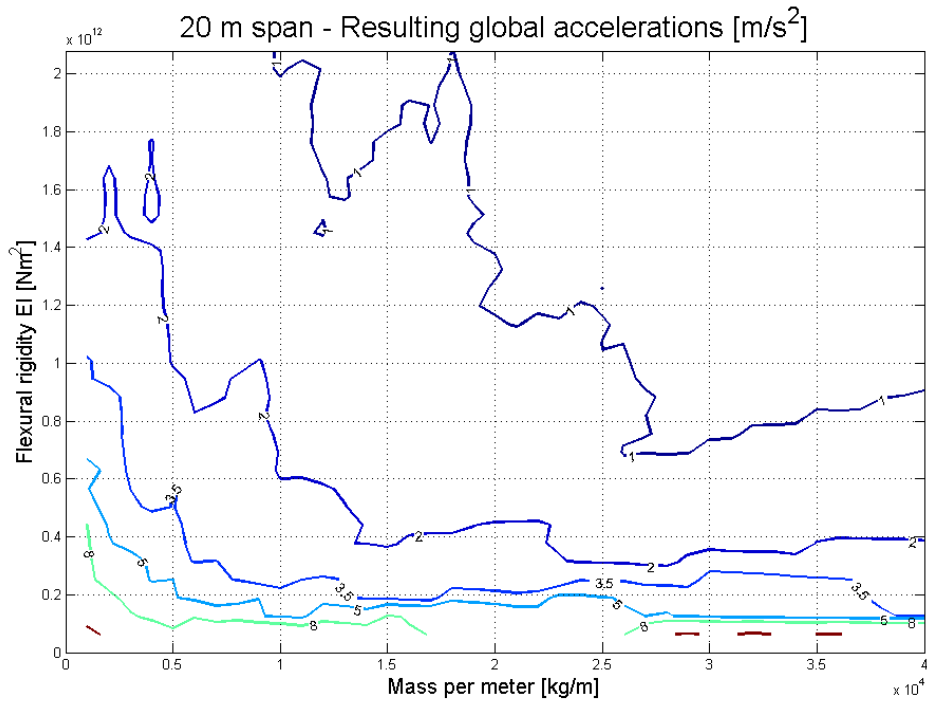


Figure 23 The resulting vertical accelerations as a function of the flexural rigidity and mass of a simply supported beam with span length 20 m.

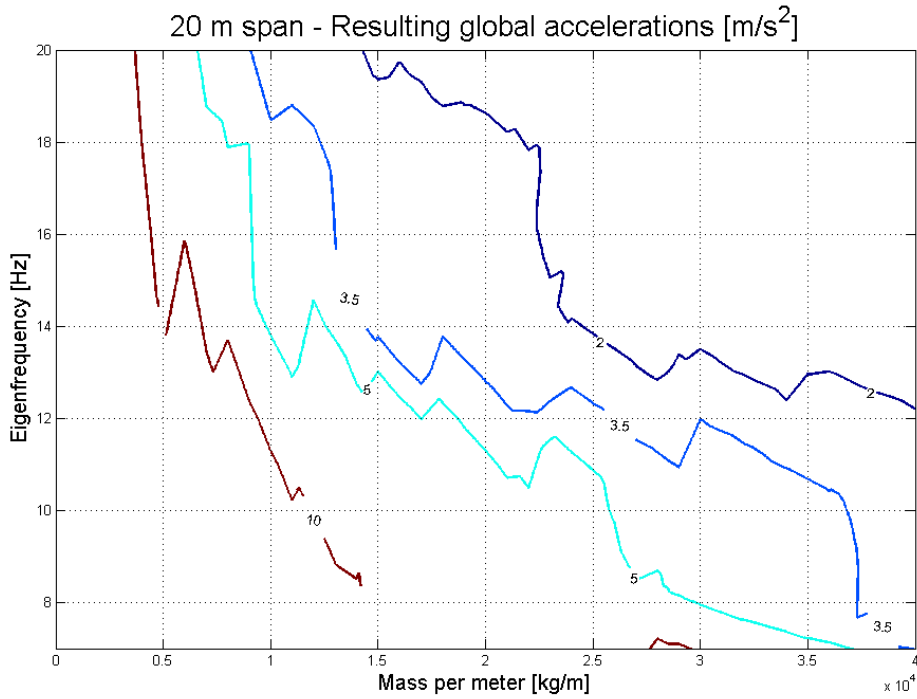


Figure 24 The resulting vertical acceleration as a function of the eigenfrequency and mass of a simply supported bridge with span of 20 m.

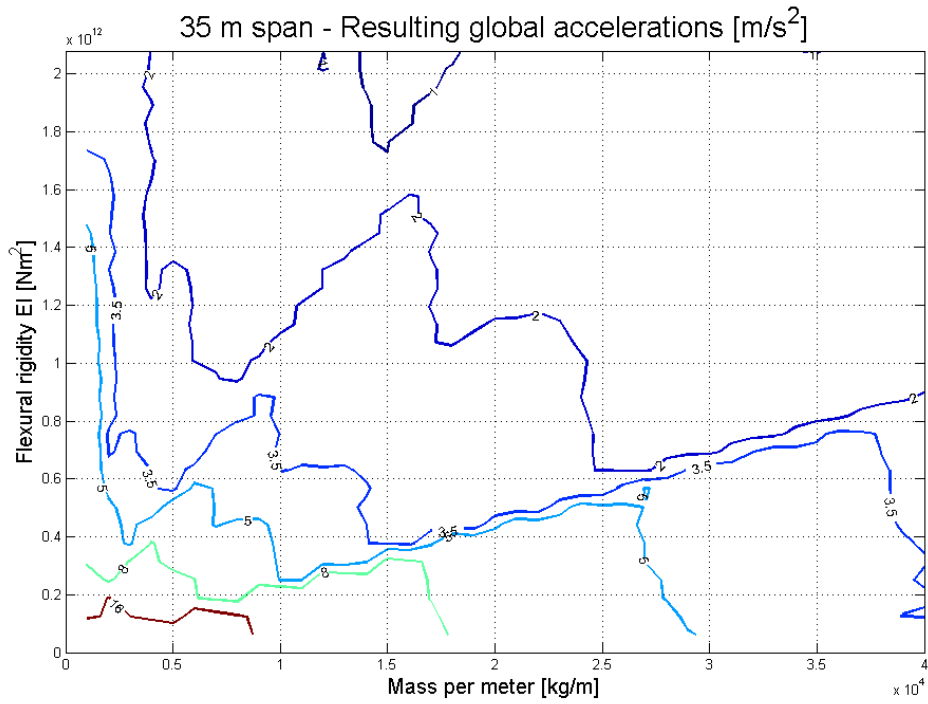


Figure 25 The resulting vertical accelerations as a function of the flexural rigidity and mass of a simply supported beam with span length 35 m.

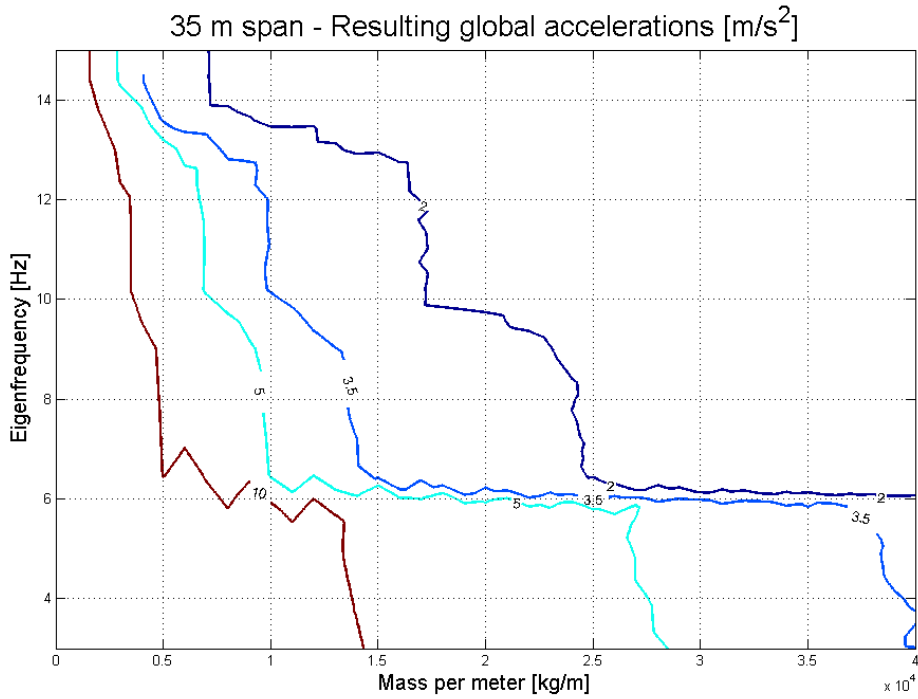


Figure 26 The resulting vertical acceleration as a function of the eigenfrequency and mass of a simply supported bridge with span of 35 m.

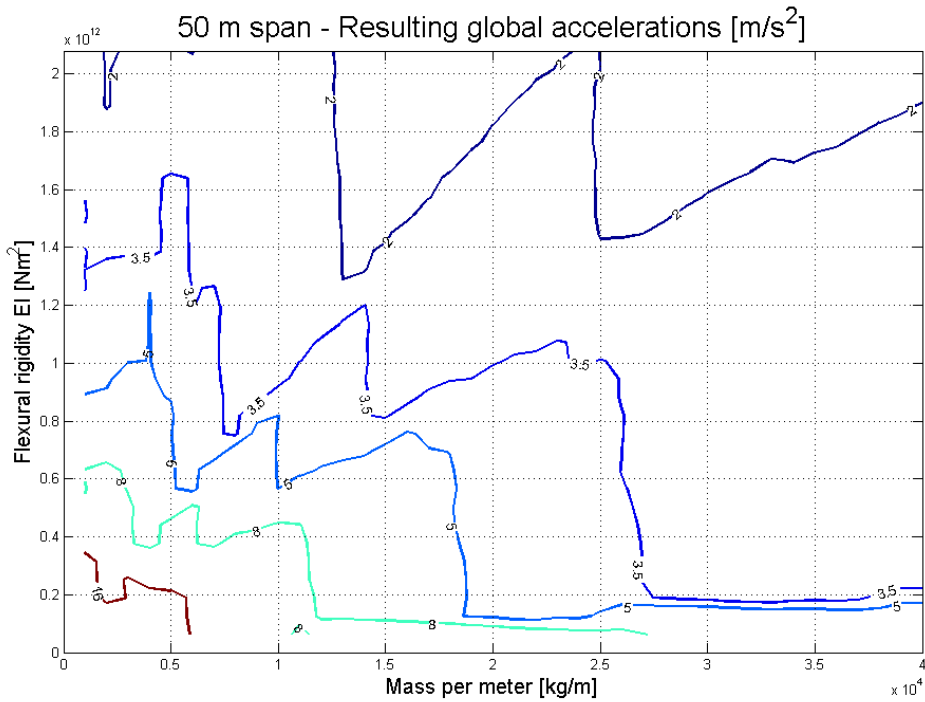


Figure 27 The resulting vertical accelerations as a function of the flexural rigidity and mass of a simply supported beam with span length 50 m.

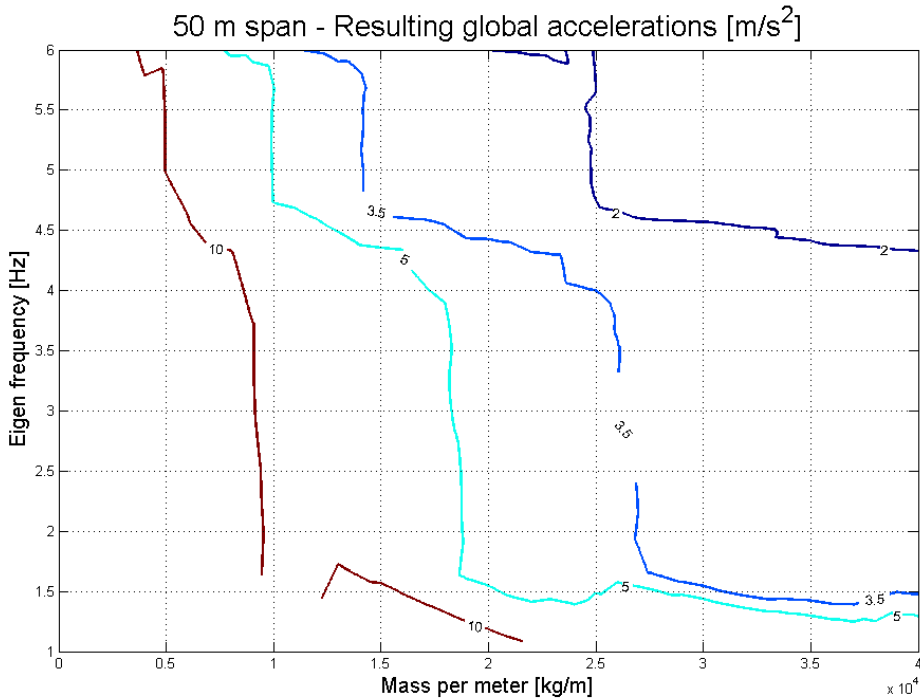
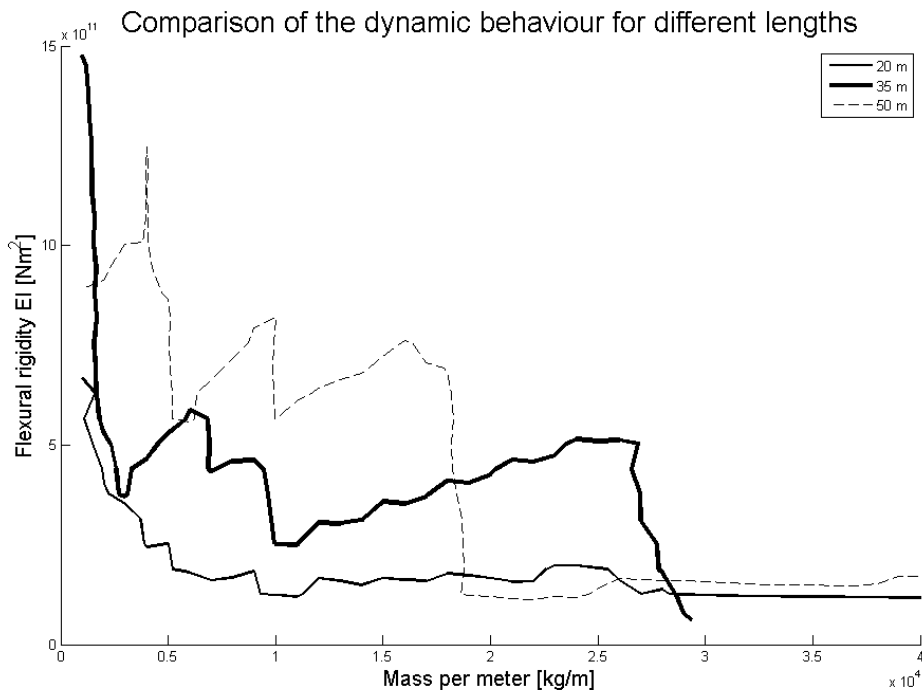


Figure 28 The resulting vertical acceleration as a function of the eigenfrequency and mass of a simply supported bridge with span of 50 m..

In order to compare the dynamic response for the different lengths the curves representing a vertical acceleration of  $5 \text{ m/s}^2$  was plotted together, see Figure 29.



**Figure 29** Comparison of the relations of flexural rigidity and mass for different lengths. For all three curves the resulting vertical acceleration is  $5 \text{ m/s}^2$ .

By looking at the three contour plots, concerning the flexural rigidity  $EI$ , it can be seen that the trend for all cases is that a heavier bridge needs less flexural rigidity. The decay is stepwise due to the load configuration.

The curve representing a vertical acceleration of  $5 \text{ m/s}^2$  in the contour plot of the 20 m span is much smoother than in the two other cases, which might be explained by the configuration of the bogies in the load model in relation to the length of the bridge. In the load model the length of each carriage varies between 18 and 27 meters which means that for the 20 meters case there is almost never more than one pair of bogies on the bridge at the same time. The effect is that the 20 meter beam resembles the idealised case of a beam subjected to a moving transverse point load. For the two longer cases investigated in this study superimposing and phase shifting of the response, coming from the presence of more than one bogie at the time, will influence the results and as a consequence the contour plots are less smooth.

The results also show that at some intervals in mass an increase in mass requires an increase in flexural rigidity. This is a very important observation, which especially can be seen in the case of 35 meter and 50 meter of length. These intervals need to be identified before design.

The explanation to the phenomenon described in the previous paragraph could be that new governing peaks of acceleration, coming from the load model, are introduced to the analysed speed interval when the mass is increased. In order to obtain a vertical acceleration beneath  $5 \text{ m/s}^2$  the flexural rigidity needs to be increased in order to exclude these governing peaks of acceleration.

Comparing the contour plots concerning the dynamic behaviour as a function of mass and flexural rigidity, shown in Figure 29, an important pattern can be observed. The required flexural rigidity is not linearly dependent on the length of the bridge, but rather on the relation between the length of the bridge and the length of the wagons in HSLM-A. For example, simply supported 35 meter bridges have considerably higher requirements of flexural rigidity within certain intervals in mass, compared with the other investigated cases. This is in contrast to the static behaviour where a longer bridge needs a higher flexural rigidity. In these cases the importance of considering the dynamic behaviour in an early phase is obvious.

In preliminary design of a structure, lower limits of the weight can be found in the contour plots. For example, in order to achieve a global acceleration of approximately  $5 \text{ m/s}^2$  a 50 meter bridge that weigh around 17 000 kg/m requires substantial higher flexural rigidity than a bridge that weights 19 000 kg/m. For bridges with eigenfrequencies between 1.5 and 4.5 Hz this can be seen as a lower mass limit.

## 5 Case study

The aims of this chapter are to design a bridge with a light cross-section and flexural rigidity high enough to ensure usage in high-speed railway lines. The concept considered will be a simply supported bridge with a length of 50 meter.

The design of the bridges in this thesis is done with guidance of the contour plots of the parametric study, see Section 4.5. Detailed analyses are done in the commercial software BRIGADE/Plus.

### 5.1 The Graphene Tube

The studied concepts were inspired by a graphene molecule, illustrated in Figure 30. The hexagonal shape is thought to keep the weight to a minimum but still conserve the strength.

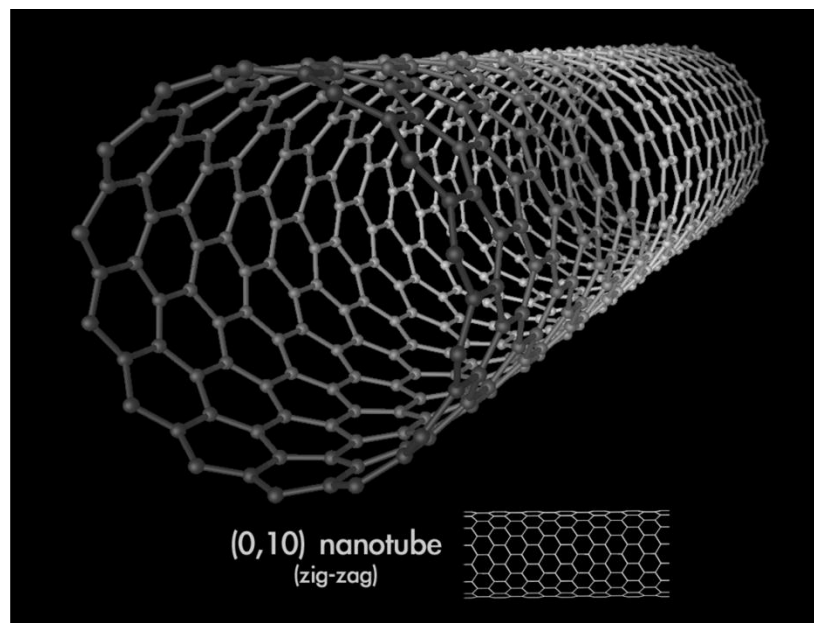


Figure 30 A principal picture of a graphene tube (Ströck 2006)

The circular cross-section gives good opportunities to manage torsion. For a double track railway the structural height will be at least 15 meter. The hexagonal pattern works very well in tension but is not an optimal shape in order to resist shearing of the structure.

Two different variants of the concept, inspired by a graphene tube, were investigated. The first bridge was designed as a graphene tube, and in the second bridge some additional structural elements were added to the original design to investigate the difference in response. The two investigated bridge designs are further presented in the following sections.

#### 5.1.1 The first draft

The aim of the design of the first concept, called the first draft, was to resemble the graphene tube to as great extent as possible. By doing so the suitability of a structure purely designed in accordance to the source of inspiration was investigated. In



addition, strengths and weaknesses of the structure could also be identified by the first draft.

### Structural design

The superstructure of the 50 meter long bridge is shown in Figure 31. Noteworthy, is the size of each hexagonal cell; they are 5 meters wide at its widest points.

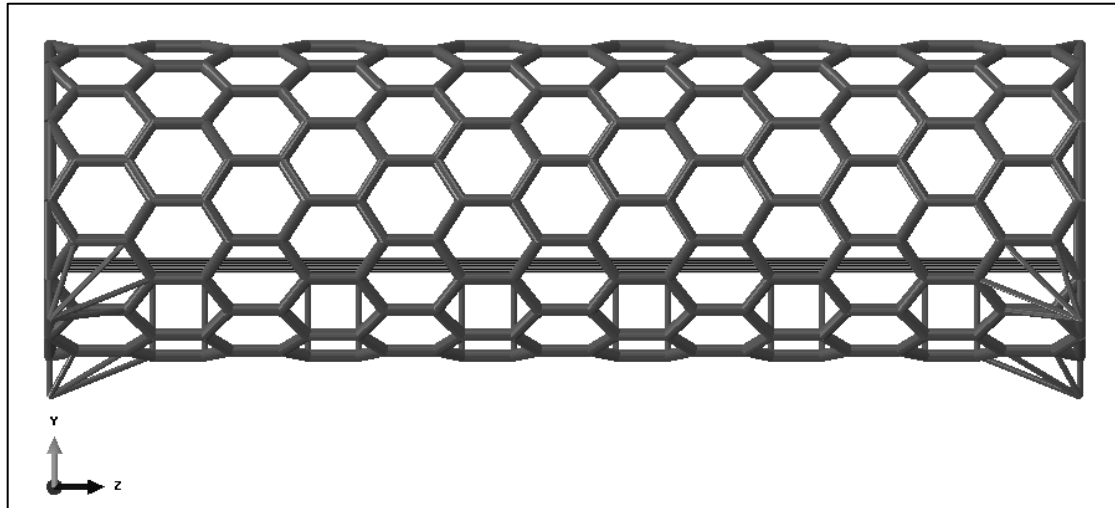
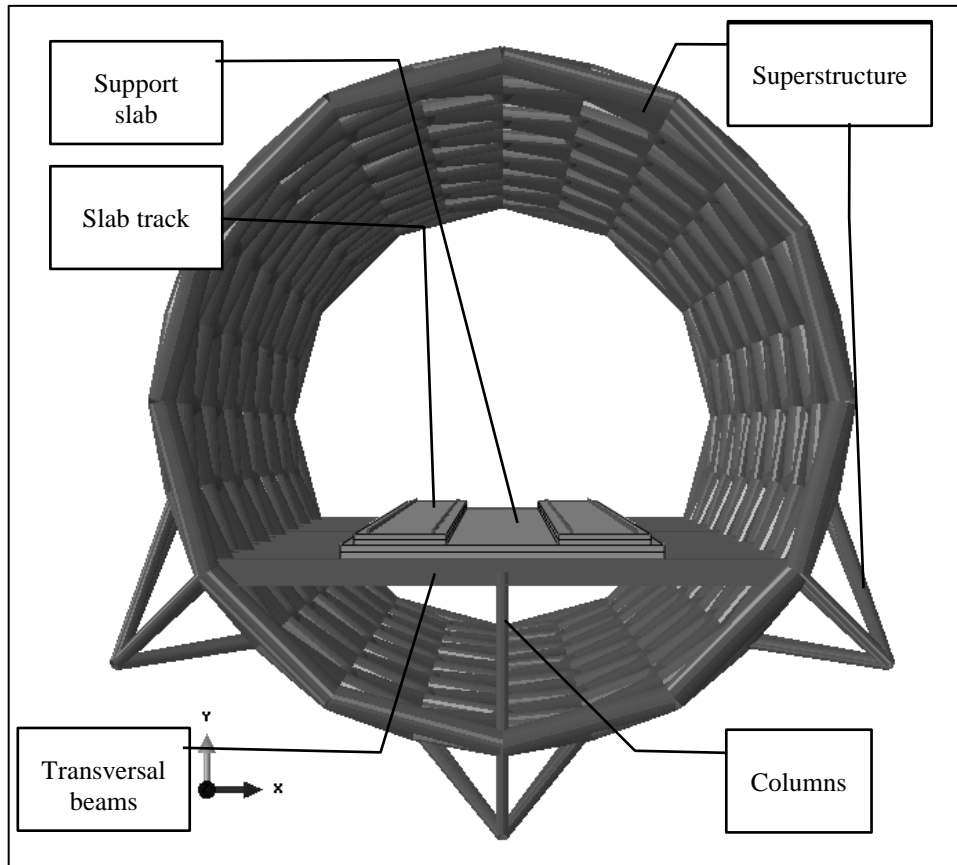


Figure 31 An elevation view of the first draft.

A sectional view of the first draft is shown below. The radius of the tube is 7.5 meter and it was designed for a minimum free height of 7 meters. The width of the tube was chosen in accordance with the requirements of the Swedish Traffic Administration for a double track railway bridge (11.5 meter).



**Figure 32** A sectional visualisation of the first draft.

The rails are placed on a slab track system. The slab track system consists of a concrete slab to which the rails are connected by grouting. Beneath the slab track system there is a support slab made out of concrete above transversal beams. The transversal beams are supported at three points; in the middle by columns and in each end by the superstructure. A presentation of the structural members and the size of them are shown in Table 8.

**Table 8 Properties of the structural members of the first draft.**

<b>Part</b>	<b>Structural member</b>	<b>Material</b>	<b>Dimension [m]</b>
<b>Superstructure</b>	Pipes	Steel	D=0.6 t=0.01
<b>Transversal beams</b>	HEA-Beam	Steel	HEA 600
<b>Columns</b>	Pipes	Steel	D=0.3 t=0.01
<b>Support slab</b>	Slab	Concrete	t=0.3
<b>Slab track</b>	Slab track	Concrete	t=0.24

### **Modelling**

In this thesis, the bridges were modelled using the commercial software BRIGADE/Plus. The superstructure was defined by a number of reference points representing the hexagonal connection points. All reference points can be found as an input script for BRIGADE/Plus in Appendix C. The FE-element types used for each structural part are shown in the table below.

**Table 9 Element types of structural members of the first draft.**

<b>Part</b>	<b>FE-element</b>
<b>Superstructure</b>	Beam
<b>Transversal beams</b>	Beam
<b>Columns</b>	Beam
<b>Support slab</b>	Shell
<b>Slab track</b>	Shell

All connection points were modelled as fixed, except between the support slab and the transversal beams. The vertical translation of the support slab and the beams were connected at all points directly above the transversal beams. Movement of the support slab relative to the transversal beams was prevented in the transversal and longitudinal direction at the end of the bridge. As a result the slab system did not contribute to the global flexural rigidity of bridge while the weight was included in the analysis. This was an assumption at the safe side, considering dynamic effects.

## Boundary Conditions

The bridge was prevented to move in the vertical direction in all six support points. Support 2 was also prevented to move in the longitudinal direction, while supports 1, 3, 4 and 6 were prevented to move in the transversal direction. Hence, the bridge has been given boundary conditions that represent a simply supported beam. The supports and their indexes are shown in Figure 33.

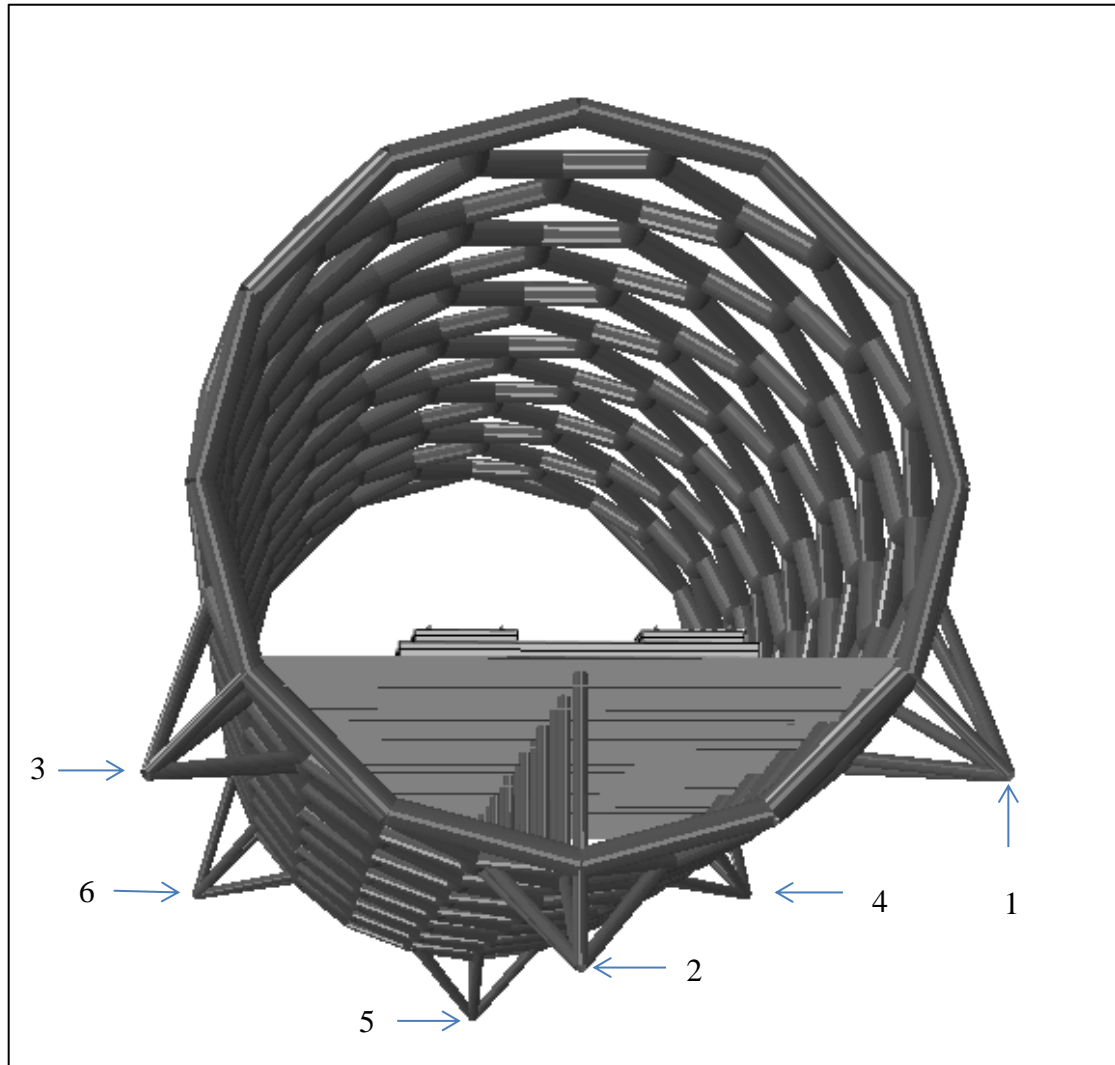


Figure 33 The numbering of the supports of the first draft.

## Convergence study

A convergence study was performed aiming to show the accuracy of the results. The eigenfrequencies were studied in order to show the bridge's physical behaviour in the same manner as in section 4.2.1. The mode closest to 30 Hz was chosen as an indicator of the convergence of the model. A MATLAB routine developed by Prof Thomas Abrahamsson was used in order to compare the mode shapes for different meshes to identify common modes. This method is called Modal Assurance Correlation (MAC). The results of the convergence study are shown in Table 10 below.

**Table 10** The influence of mesh size for modified graphene tube concept.

Mesh nr	Approximate mesh size [m <sup>2</sup> ]	Frequency of mode closest to 30 Hz [Hz]	Estimated error between meshes
1	1.25 x 1.25	29.754	
2	0.625 x 0.625	29.857	0.34%
3	0.3125x0.3125	29.842	0.05%

By looking at the results of the convergence study mesh number 2 was chosen for analysis.

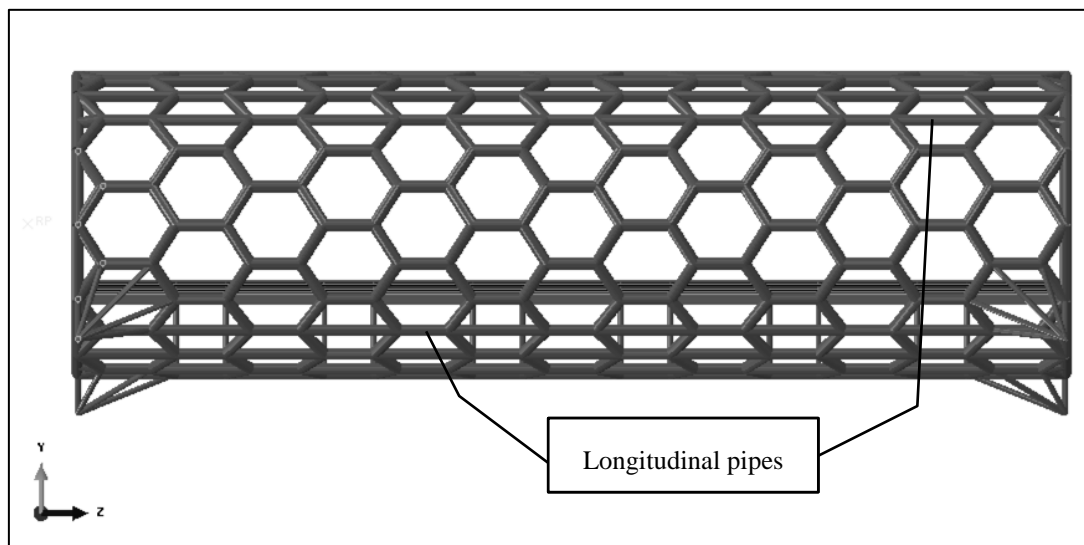
### 5.1.2 The final design

The second concept was a development of the first draft. It was developed in order to increase the flexural rigidity of the superstructure, as well as to obtain a more rigid bridge deck. During the analysis of the first draft, these properties were found to be weaknesses of the original structure.

The final design adapts the same requirements regarding the free height and width as the first draft, demonstrated in Section 5.1.1.

#### Structural design

The superstructure of the final design utilizes the same structural design as the first draft with the difference that strengthening parts were added in critical sections. In order to obtain a higher flexural rigidity, extra material was added in the top and bottom of the superstructure. This extra material consists of a total of 14 longitudinal pipes evenly distributed in the top and the bottom, shown in Figure 34.



**Figure 34** An elevation visualization of the final design.

For the purpose of minimising horizontal deformation, transversal beams were introduced in the top of the tube. In order to add extra weight to the bridge the supporting concrete slab was extended, see Figure 36. Moreover, the transversal beams supporting the bridge deck were stabilised by the introduction of two supporting columns connecting the beam and the tube. In addition to these columns,

the transversal support beams were also stabilized by longitudinal stiffeners, forming a beam-support system illustrated in Figure 35. In Figure 36 a sectional view of the final design is shown, where the described members can be seen.

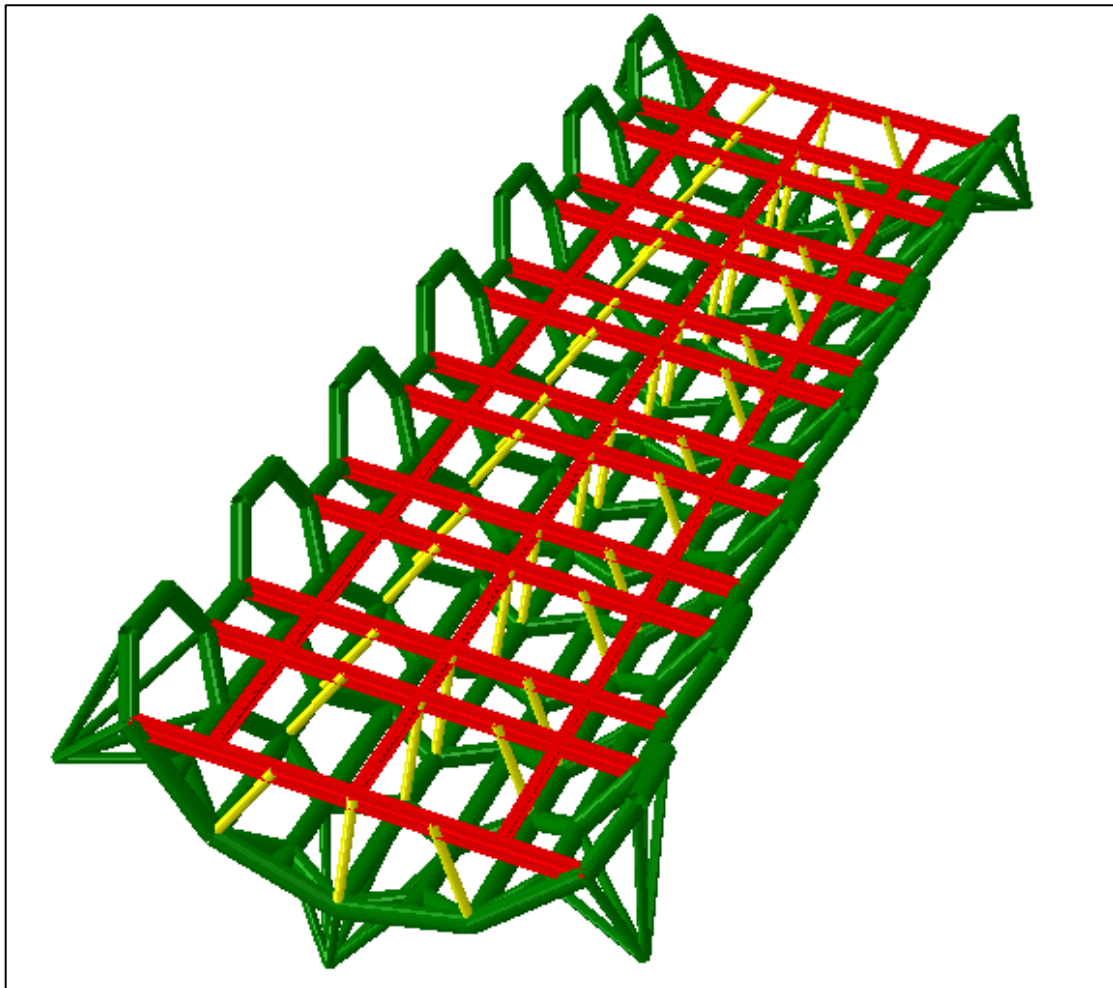


Figure 35 The modified beam-support system (in red).

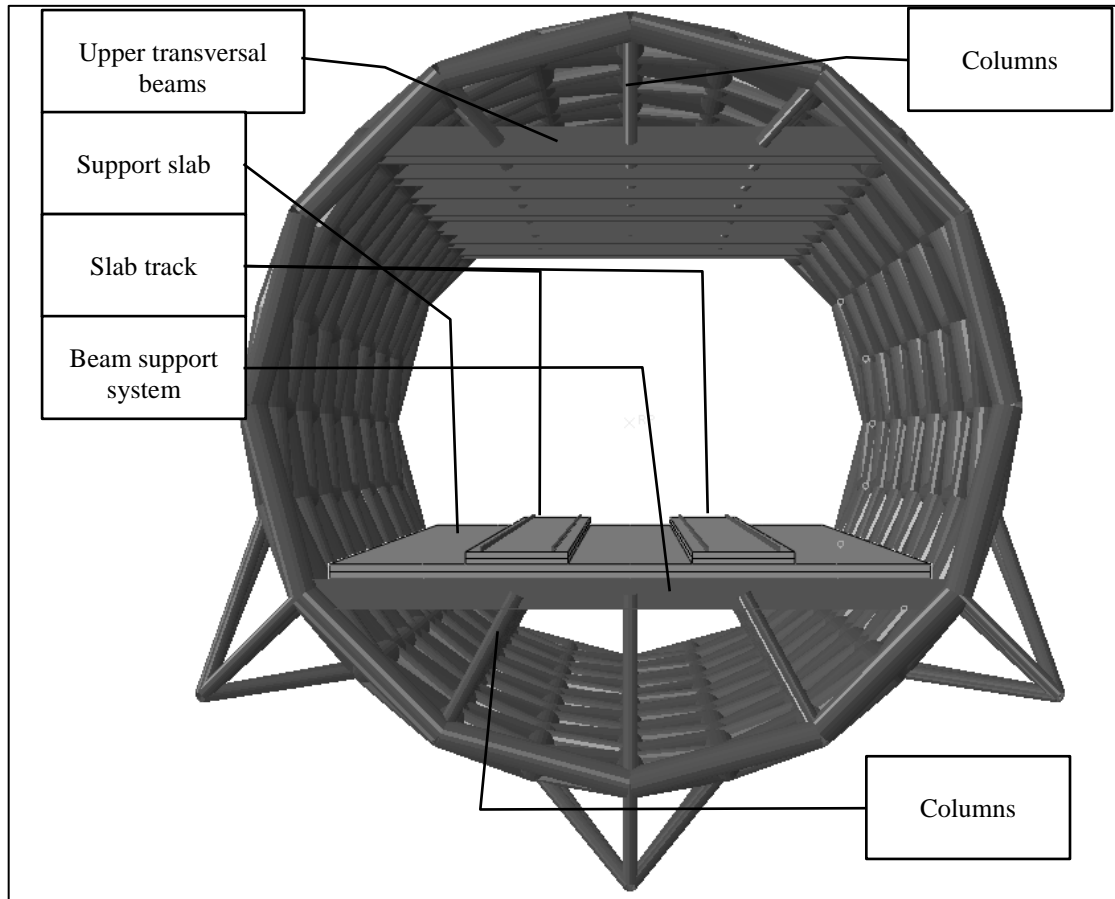


Figure 36 A sectional visualization of the final design.

The properties of the structural parts are presented in the table below.

**Table 11 Properties of the structural members of the final design.**

<b>Part</b>	<b>Structural member</b>	<b>Material</b>	<b>Dimension [m]</b>
<b>Superstructure</b>	Pipes	Steel	D=0.6 t=0.01
<b>Beam support system</b>	HEA-Beam	Steel	HEA 600
<b>Upper transversal beams</b>	HEA-beam	Steel	HEA 600
<b>Columns</b>	Pipes	Steel	D=0.3 t=0.01
<b>Support slab</b>	Slab	Concrete	t=0.3
<b>Slab track</b>	Slab track	Concrete	t=0.24

## Modelling

The final design was modelled in BRIGADE/Plus in a similar manner as the first draft. The superstructure was defined by a number of reference points representing the hexagonal connection points. All reference points can be found as a input script for BRIGADE/Plus in Appendix C. The FE-element types of each structural part are shown in Table 12.

**Table 12 The element types of structural members of the final design.**

<b>Part</b>	<b>FE-element</b>
<b>Superstructure</b>	Beam
<b>Beam support system</b>	Beam
<b>Upper transversal beams</b>	Beam
<b>Columns</b>	Beam
<b>Support slab</b>	Shell
<b>Slab track</b>	Shell



All connections were modelled as stiff similarly with the first draft, except between the support slab and the beam support structure. The vertical degrees of freedom of the support slab and the beam support system were connected at all points directly above the transversal beams. Movement of the support slab relatively to the beam support system was prevented in the transversal and longitudinal directions in the ends of the bridge. As a result the slab system did not contribute to the flexural rigidity of bridge while the weight was included in the analysis. This is an assumption at the conservative side, considering dynamic effects.

### Boundary conditions

The bridge was prevented to move in the vertical direction in all six support points. Support 2 was also prevented to move in the longitudinal direction, while supports 1, 3, 4 and 6 were prevented to move in the transversal direction. Hence, the bridge has been given boundary conditions that represent a simply supported beam, see Figure 37.

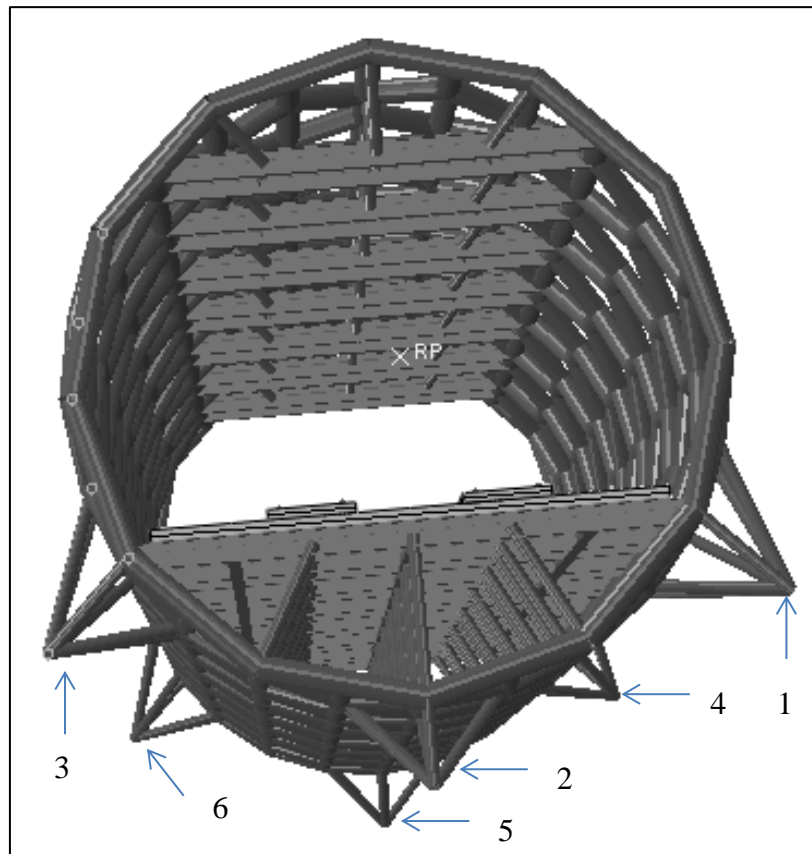


Figure 37 The numbering of the supports for the final design.

### Convergence study

A convergence study was performed in the same manner as for the first draft section 4.2.1. The results of the convergence study are shown in Table 13.

Table 13 The influence of mesh size for the modified graphene tube concept.

Mesh nr	Approximate mesh size	Frequency of mode closest to 30 Hz [Hz]	Estimated error between meshes
1	1.25 x 1.25	29.604	
2	0.625 x 0.625	29.845	0.8%
3	0.3125x0.3125	29.944	0.3%

By looking at the results of the convergence study mesh number 3 was chosen for analyses.

## 5.2 Performed analyses

Two analyses were performed in the commercial software BRIGADE/Plus. First a frequency analysis was made in order to truncate the system according to the recommendations in Eurocode. The first natural frequency in bending was also evaluated. Then a dynamic analysis was performed, in order to obtain the vertical accelerations of the bridge deck as well as global acceleration of the superstructure. The bridges were subjected to the HSLM-A load model for speeds from 150 to 350 km/h with an increment of 2 km/h. According to Eurocode the load was applied on one track, see Section 3.1.

The dynamic analysis was performed using modal analysis. For the lowest speed a time period of 16 seconds was used and for the highest speed a time period of 6 seconds was used. In both cases a time increment of 0.01 seconds was chosen. A direct damping of 0.5 % was used.

Gathering results of dynamic analyses in BRIGADE/Plus is time consuming. In order to reduce the total calculation time results were collected in the beam support system, representing the vertical accelerations of the bridge deck. Results were also collected from a hexagonal cell in the middle of the superstructure in order to represent global response of the superstructure, see Figure 38.

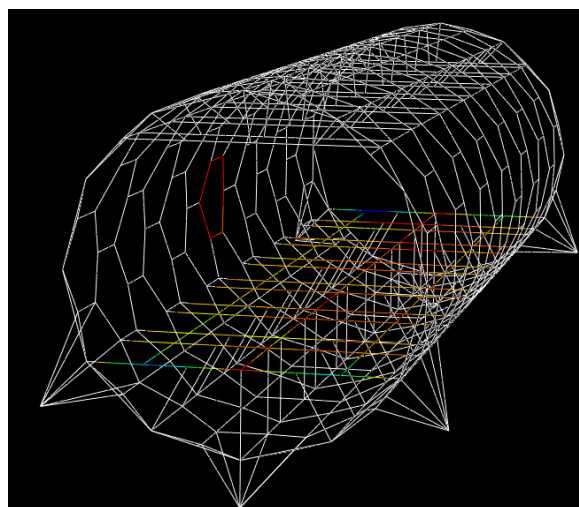


Figure 38 The results are extracted from the coloured parts of the structure above.

## 5.3 Results

The results from the analyses of the two concepts are presented and analysed in this section. The mass of each bridge as well as the first natural frequency are extracted and compared with the contour plots obtained in the parametric study, see Section 4.5.

Vertical accelerations were obtained on a global level to be compared with the results of the parametric study. The focus of the case study was to obtain global vertical accelerations beneath the limit of vertical accelerations in Eurocode of  $5 \text{ m/s}^2$ . In addition, vertical accelerations were extracted in the beam-support system as well to investigate local effects in the slab track system.

### 5.3.1 The first draft

In Table 14 the physical properties of the first draft is shown.

Table 14 The natural frequency and mass of the first draft.

The first draft	
First natural frequency $n_0$ [Hz]	2.03
Mass of the bridge [kg/m]	15 600

Since the weight and the natural frequency are obtained from the analysis the contour plot concerning vertical acceleration as a function of the weight and the eigenfrequency was used. The result of this comparison is shown in Figure 39.

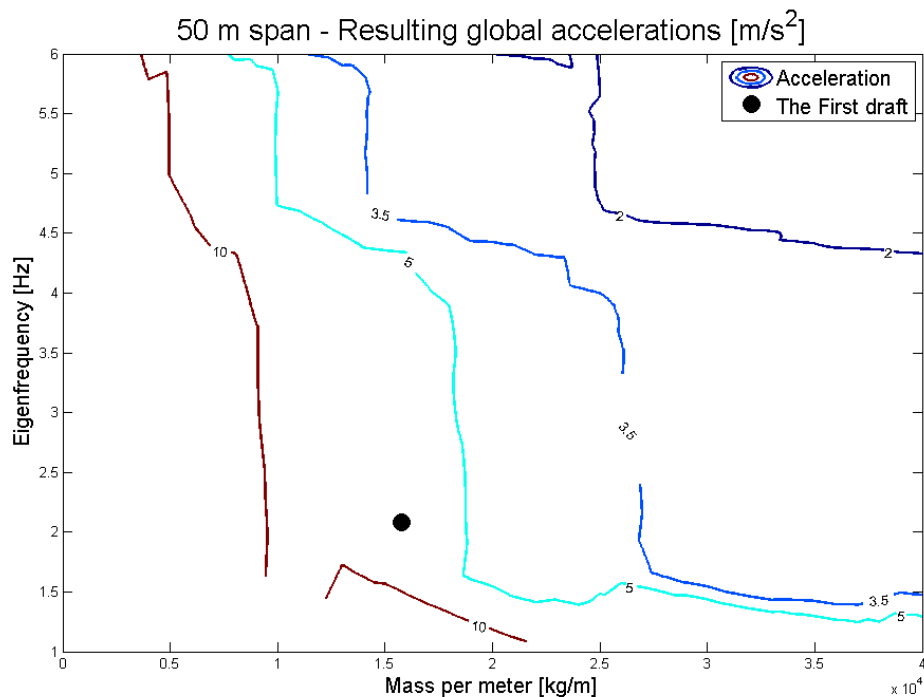


Figure 39 The properties of the first draft plotted in the parametric study

From the comparison above it is clear that the first draft has a natural frequency that is beneath the curve that represents a vertical acceleration of  $5 \text{ m/s}^2$ . This indicates that the concept will not be able to meet the dynamic requirements in Eurocode. In order to get a quick estimation of the amount of the vertical accelerations, an analysis was run for one train from the HSLM-A (HSLM-A10) at a speed close to resonance (197 km/h). The global acceleration for this concept was  $11.715 \text{ m/s}^2$ .

### 5.3.2 The final design

In Table 15 the physical properties of the final design is shown.

Table 15 The natural frequency and mass of final design.

The final design	
First natural frequency $n_0$ [Hz]	2.55
Mass of the bridge [kg/m]	18 800

This concept was also plotted in the contour plots obtained in the parametric study. The aim was to modify the final design in order to obtain a natural frequency that would result in a vertical acceleration of  $5 \text{ m/s}^2$  according to the contour plots. The final design was plotted in the contour plot of interest in Figure 40.

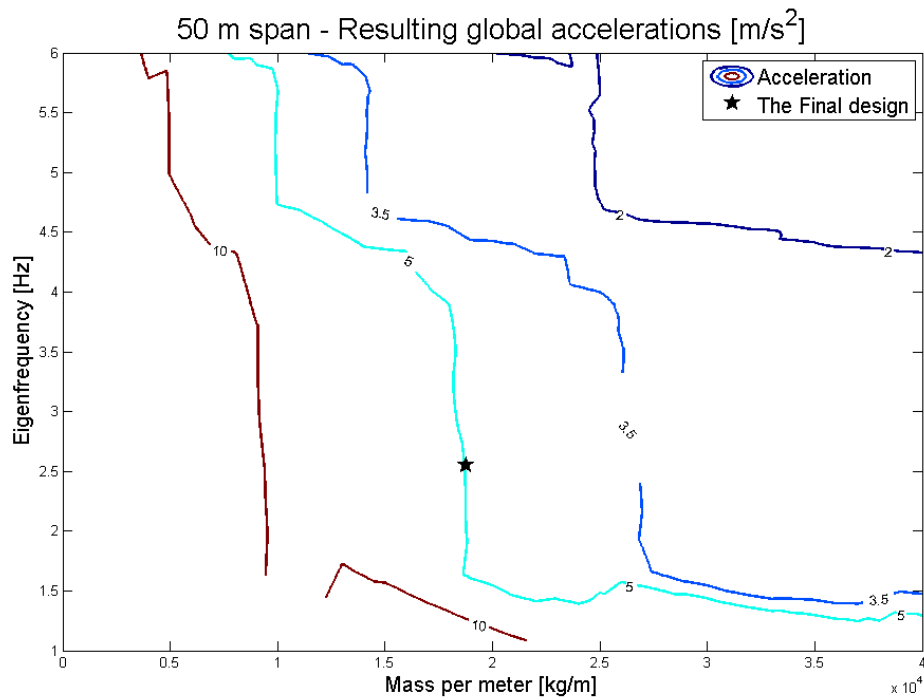


Figure 40 The properties of the final design plotted in the parametric study.

It can be seen in the figure that this structure is estimated to fulfil the requirement of maximum vertical acceleration of  $5 \text{ m/s}^2$ . A detailed dynamic analysis was performed using the entire HSLM-A for speeds from 150 km/h to 350 km/h. Both global and local vertical accelerations were extracted in this detailed analysis. The global

acceleration was obtained in the same manner as for the first draft and was  $7.90 \text{ m/s}^2$  for the final design.

Vertical accelerations were also extracted in the beam-support system below the support slab. These are extracted in order to investigate the local vertical acceleration in the slab-support system. The maximum vertical acceleration for each node is extracted for each train and all speeds considered in the analysis. These vertical accelerations are plotted in Figure 41.

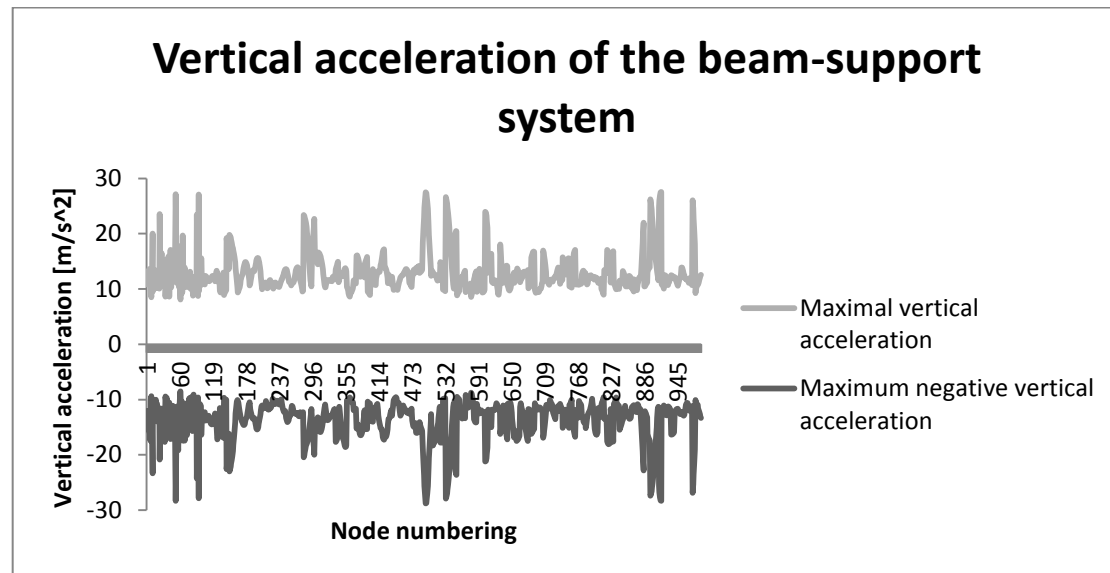


Figure 41 The vertical accelerations of the beam-support system for the final design.

It can clearly be seen that the maximal vertical accelerations are greater than the  $5 \text{ m/s}^2$  which is the upper limit according to Eurocode.

#### Centric loading of the final design

In order to investigate any influence of eccentric loading, an analysis was run with a centric loading. The global response was evaluated. For this case the global acceleration was  $5.5 \text{ m/s}^2$ .

### 5.4 Analysis of the Case study

The results show that none of the investigated bridges fulfilled the requirements, but some important tendencies can be found and these will be discussed in the following paragraphs.

The weight and natural frequency of the first draft, presented in Figure 39, indicate that the bridge will not fulfil the dynamic requirements according to Eurocode. This was confirmed by the simplified analysis where only one train at one resonance speed was considered. The global vertical acceleration was higher than the expected value from the contour plot. Noteworthy is that the global acceleration could be higher than the obtained value when all trains are considered at all speeds.

After development into the final design, by increasing the flexural rigidity and mass with the parametric study in mind, it can be seen in Figure 40 that the global vertical

acceleration might be beneath  $5 \text{ m/s}^2$ . By running the complete dynamic analysis it was shown that the vertical global acceleration was higher than the predicted value. However, by comparing the results a common trend is shown between the predicted vertical accelerations according to the contour plots and the obtained global vertical accelerations of the case study. An improvement according to the contour plots generates an improvement of the dynamic behaviour. The global vertical acceleration is seen as a lower bound value of the expected maximum vertical acceleration of the bridge deck

The global vertical acceleration from the case study was  $7.9 \text{ m/s}^2$  and the predicted acceleration was approximately  $5 \text{ m/s}^2$ . The difference is thought to result from rotation around the longitudinal axis of the bridge due to the eccentricity of the tracks. The twisting displacement is not taken into account in the parametric study and would result in higher vertical accelerations. The analysis run with a centric load indicates that any eccentricity of the load has a major effect of the vertical accelerations of the bridge.

## 6 Discussion

During the work conducted in this thesis a number of limitations were made. The assumed influence of the results due to these limitations is discussed in this chapter. Important issues as well as questions that arose during the work are also discussed.

- In this study only the dynamic behaviour of bridges has been studied, therefore bridges that are indicated to meet the requirements concerning dynamic behaviour must be checked concerning static response, buckling etc. Rail imperfections will also have an effect on the results, specific for each case.
- The parametric study in this report was calculated using a FE-model and a direct time-integration method. Another possible method might have been an analytical solution. The benefit of using an analytic solution would have been a shorter calculation time compared to the FE-method. However, the FE-approach makes it possible to extend the parametric study to treat different span configurations, boundary conditions and material properties. In future studies the FE-method could also be developed by adding effects of eccentric loading in order to represent a double track railway bridge. Concerning the load the result can easily be superimposed into any given train configuration, that is if the train can be represented as moving point loads.
- Eurocode only states that a series of speeds between 40 m/s and 1.2 times the design speed of the railway line should be considered. When working on this thesis it has become obvious that it is of great importance to choose small increments between speeds in order to capture all resonance speeds. The authors found that for the considered cases, 10 km/h in increment was not fine enough to catch the real behaviour of the bridges. The number of considered velocities had to be continuously updated due to this phenomenon. An increment of 2 km/h was considered accurate enough for this study.

Since the increments of the speeds had such a significant influence on the results, the authors would like to see a more detailed guideline in the design code. This would result in a better basis for standardised design of high-speed railway bridges.

- The assumption that the load is placed centrally on the beam in the parametric study is not representing a double track railway bridge. The results of the case study imply that the placement of the load plays a great role in the magnitude of the global vertical accelerations. The eccentric load results in twisting as well as bending modes giving contributions to the vertical accelerations. To be able to fully connect a parametric study to a bridge that does not behave as a beam with centric load, twisting modes must be incorporated in the parametric study.
- A lapsus has been found in the code when the maximal accelerations are collected from the parametric study, instead of taking the maximum absolute value of the acceleration, the maximum value is taken. This is a minor fault that does not lead to any significantly differences of the results since the damping is low.

- The local effects of the bridge have been treated very roughly in this thesis. These cannot be evaluated with the data from the parametric study. It has been found that the local effects differ greatly from case to case, concerning both the origin and the amplitude of the effect.
- In the case study two steel – concrete composite bridges were analysed. According to Eurocode a composite bridge has the same amount of damping as a bridge made completely out of steel. Table 2 shows that a concrete cross-section allows for higher damping to be used than a steel cross-section. This implies that a composite bridge could have a higher damping than a pure steel bridge, which would result in lower vertical accelerations than obtained in this thesis.
- The damping of the beam in the parametric study seems to have significant effect on the computed eigenfrequencies. This is on the contrary of the suspected response where only the magnitude of the response should be affected, see Section 3.3. The problem has been deeply investigated in collaboration with Prof. Thomas Abrahamsson without finding mistakes in the scripts.
- When analysing the results from the parametric study, it was seen that the maximal vertical accelerations were obtained some seconds after the train had left the beam. Since the considered time was the same for all trains the maximal values may not have been found for some of the trains considered. The reason for this was that the author did not realise that two super imposed decaying sinusoidal signals may give a response that increases with time.
- In its original form, the graphene tube is strong in pure axial tension while not optimal in bending. Extra strengthening seems to be needed in critical parts. This is indicated by the difference in vertical acceleration between the first draft and the strengthened final design.



## **7 Conclusions**

The dynamic response was shown to be hard to predict. This thesis shows the importance of a design based on the dynamic behaviour, where favourable intervals in the contour plots can be identified in an early stage of the design phase.

The extent of the case study is too limited to be able to establish any real conclusions. However, the results implies that the parametric study can be used to predict the global behaviour of real bridges and shows the importance of a parametric study that reflects the behaviour of the structure at hand.

The aim of the thesis was to investigate a design approach based on the dynamic behaviour of bridges. The result of the thesis has resulted in two suggested methodologies, one for the design phase and one for assessment of existing bridges. These are presented in the following sections.

### **7.1 Suggestion for usage of the parametric study in the design phase**

In order to use the results from the parametric study during the design of a high-speed railway bridge the following method is proposed:

- 1) The length of the bridge and an approximate weight per meter are chosen.
- 2) The required minimum flexural rigidity or eigenfrequency is chosen from the contour plots.
- 3) A detailed FE-analysis is performed on the bridge in order to obtain the vertical accelerations and investigate any local effects.

### **7.2 Suggestion to an estimation of the dynamic behaviour of existing bridges**

In order to use the results from the parametric study during assessment of existing bridges the following method is proposed:

- 1) A frequency analysis of the bridge is performed. It is important that the bridge structurally behaves as a beam.
- 2) By use of the contour plots, the first vertical eigenfrequency and the mass of the bridge, the global vertical acceleration is estimated. The estimation indicated if a more detailed and time consuming analysis is justifiable.
- 3) A detailed analysis of the bridge is performed in order to obtain the vertical accelerations as well as the local effects of the structure.

### **7.3 Further studies within this topic**

Based on this thesis, the authors suggest some issues for further studies.

- In the parametric study performed in this thesis, only beams with centric load were considered. In order to capture the behaviour of double-track bridges in a more realistic way, parametric studies should be performed with eccentric loading.
- In this thesis, only simply supported bridges have been investigated. The dynamic performance of continuous bridges is of great interest to the industry and should be further investigated.
- This thesis has focused on the global dynamic response. However, the results from the case study show that the local vertical accelerations are high. Further studies concerning local vibrations should be conducted concerning the grapheme tube concept.
- In this study only vertical accelerations have been considered. In future studies other parameters from Eurocode, such as rotation at supports, vertical displacement, among others, need to be examined. The static requirements should also be taken into consideration in future studies.

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# **Appendix A**

## **Description of the MATLAB scripts**

Name	Input parameters	Output parameters	Description
<b>Parametricstudy.m</b>		$a_{save}$	Generates the maximal acceleration for a given beam with a given load. The programme utilises the subroutines below and the Calfem 3.4 package.
<b>Inputfile.m</b>		$A_{in}$ $m_{in}$ $I_{in}$ $L$ $E$ $P0$ $c_{in}$ $nel$ $h1$ $h2$ $c_{limit}$ $greek_z (\xi)$	The specified input data.

<p><b>Loading2.m</b></p> <p><math>n_{newton}</math>  <math>c</math>  <math>L_{el}</math>  <math>P0</math>  <math>ndof</math>  <math>htimestep1</math></p>	<p><math>F_{load}</math></p> <p>Generates the load matrix for beams subjected to a transverse moving point load.</p>
<p><b>FE_eig_val_solver.m</b></p> <p><math>nel</math>  <math>ep</math>  <math>ex</math>  <math>ey</math>  <math>bc</math>  <math>Edof</math>  <math>ndof</math>  <math>Idof</math></p>	<p><math>\mathbb{K}</math>  <math>\mathbb{M}</math>  <math>eigfreq</math>  <math>eigfreq_{Hertz}</math>  <math>vec</math></p> <p>Generates the eigenfrequencies of beams as well as the eigenvectors of the same beams. Stiffness and mass matrices are also generated</p>
<p><b>Modaldamp.m</b></p> <p><math>\mathbb{K}_{red}</math>  <math>\mathbb{M}_{red}</math>  <math>greek_z (\xi)</math></p>	<p><math>\mathbb{C}</math></p> <p>Written by Prof. Thomas Abrahamsson, this script generates the damping matrix for beams using modal damping.</p>

<p><b>Newmark_iteration.m</b></p> <p><i>Cd</i> <i>Mid</i> <i>nel</i> <i>N</i> <i>n_newton</i> <i>t_total</i></p>	<p>Generates the vertical acceleration of beams subjected to moving transverse point loads using the Newmark-<math>\beta</math> method.</p> <p><i>unew</i> <i>anew</i></p>
<p><b>HSLMAtrainset.m</b></p> <p><i>unew</i> <i>anew</i> <i>train</i> <i>n_newton</i> <i>t_total</i> <i>P0</i> <i>v_dof</i></p>	<p>Generates the resulting vertical acceleration of beams when subjected to the load model HSLM-A. The results from the Newmark_iteration.m are phase shifted and superimposed to generate the acceleration for each train in the load model.</p> <p><i>u_train</i> <i>a_train</i></p>



<p><b>Post_processor.m</b></p> <p>Stores the obtained results into Excel.</p>	<p><b>Plotter.m</b></p> <p>Loads the stored results from Excel and computes contour plots of the vertical acceleration as a function of mass and flexural rigidity.</p>	<p><b>Plottereigfreq.m</b></p> <p>Loads the stored results from Excel and computes contour plots of the vertical acceleration as a function of mass and eigenfrequency.</p>
---	---	---



## **Appendix B**

### **MATLAB source code**

This appendix contains the MATLAB source code of the following scripts:

- Parametricstudy.m
- Inputfile.m
- Loading2.m
- FEeig\_val\_solver.m
- Modaldamp.m
- Newmark\_iteration.m
- HSLMAtrainset.m
- Post\_processor.m
- Plotter.m
- Plottereigfreq.m

# Parametricstudy.m

```
%% Parametricstudy.m
% Parametric study for bridges subjected to HSLM-A
% Program that calculates the vertical acceleration for beams
% subjected to HSLM_A load model from Eurocode. The c
% calculations are computed using direct time integration
% using the Newmark-beta method.
%
%
% Written by: Carl Westerlund and Anders Pagoldh Johansson
% as a part of their Master Thesis
% "The global dynamic behaviour of
% steel and composite bridges
% for high-speed railway lines"
%
% IN ORDER TO RUN THIS PROGRAMME THE CALFEM PACKAGE IS NEEDED.
%
% The output files are saved in .mat files in the following
% manner:
% LXXmYYEIZZZZ.mat where XX represents the length and YY
% represents the considered mass and ZZZZ represents the
% limits of EI used. ZZZZ must be specified manually see
% row 214.
%
%
% Copyright 2013-02-21

clear all
close all
clc

% Stating the geometry and properties of the beam.
[A_in,m_in,I_in,L,E,P0,c_in,nel,h1,h2,c_limit,greek_z]=...
    inputfile;

% Start a timer
tic

%% Calculation of the FE-Model

% Define the coordinates of the beam elements
% in x- and y-direction

for i= 1:nel
    ex(i,:)=[(i-1)*L/nel i*L/nel];
end
ey=        zeros(nel,2);

% Calculate the E dof which shows how the degrees of freedom
% are connected to each other.
```

```

for i= 1:nel
    Edof(i,:) = [i      3*i-2 3*i-1 3*i 3*i+1 3*i+2 3*i+3];
end

ndof=          max(max(Edof)); % number of degrees of freedom.
Ndof=1:ndof;
nnod=nel+1;           % number of nodes.
idof=2*nnod;         % "interesting degrees of freedom".
Idof=Ndof;
L_el=L/nel;          % Length of one element.

% Specify the horizontal degrees of freedom.
for q=1:nnod;
    horisont(q,1) = 3*q-2;
end

% Remove the horizontal degrees of freedom.
Idof(horisont)=[];

% Define the bc's.
bc= [1 idof-1 ]'; % For a SS-beam

% The parametric study is performed in these loops:
for massa=1:length(m_in);
for velocity=1:length(c_in);

    c=c_in(velocity);

        if c<=c_limit;
            himestep1=h1;
        else
            himestep1=h2;
        end

% The total analysis time and the number of timesteps is
% calculated.
t_total=(2*L+410)/c;
n_newton=ceil(t_total./himestep1);

% The Load vector is specified
[F_load]=loading2(n_newton,c,L_el,P0,ndof,himestep1);

for mass=1:length(I_in);

    % Define A and m for the ith loop.
    A=A_in(massa);
    m=m_in(massa);
    I=I_in(mass);

```

```

% Set a_dim as a start
a_dim=20;

% Calculate the element length
L_el=L/nel;
count=1;

% Define vector ep, used in Calfem..
ep=      [E A I m];

%% Solving the eigenfrequencies using FEeig_val_solver.m
[K,M,eigfreq,eigfreq_Hertz,vec]=...
    FEeig_val_solver(nel,ep,ex,ey,bc,Edof,ndof,Idof);

% Truncate the system according to the requirements in
% Eurocode. "Calculations should consider the frequencies up
% to the highest of either 30 Hz, 1,5 times the frequency of
% the first mode or the frequency of the third mode of
% vibration."

if eigfreq_Hertz(3)>30 && eigfreq_Hertz(3)>...
    1.5*eigfreq_Hertz(1)M;
    N=3;
elseif eigfreq_Hertz(3)<30;
    for i=1:length(eigfreq_Hertz);
        if eigfreq_Hertz(i)>30;
            if eigfreq_Hertz(i-1)>1.5*eigfreq_Hertz(1);
                N=i-1;
                break
            elseif eigfreq_Hertz(i)>1.5*eigfreq_Hertz;
                N=i;
                break
            end
        end
    end
end
end

% Trunkation of the system
vec_trunk=vec(:,1:N);
Kd=vec_trunk'*K*vec_trunk;
Md=vec_trunk'*M*vec_trunk;
Kred=K;
Kred(bc,:)=[];
Kred(:,bc)=[];
Mred=M;
Mred(bc,:)=[];
Mred(:,bc)=[];

%% Introducing damping in the beam.
% Calculate the C Matrix with modal damp.m

```

```

[C]= modalDamp(Kred,Mred,[greek_z]);
vec_trunk_c=vec_trunk;
vec_trunk_c(bc,:)=[];
Cd=vec_trunk_c'*C*vec_trunk_c;
disp('THIS PROGRAM IS RUN WITH DAMPING')

%% Solving the FE system using Newmark-beta method

% using Newmark_iteration.m
[unew,anew]=...
    Newmark_iteration(L,c,Kd,Cd,Md,nel,...
    N,n_newton,t_total,vec_trunk,Idof,F_load);

v_dof=1:2:Idof;
tnew=linspace(0,t_total,n_newton);

% Stop the timer.
toc

% Specify which HSLM-A trains that is tested by changing 1:10
for train=1:10
    [u_train, a_train]=...
        HSLMAtrainset(unew,anew,train,c,...
        n_newton,t_total,P0,v_dof);

    % The designing value for each train. (This was used)
    a_max(train)=max(max(a_train(v_dof,:)));
    % This should have been used, please uncomment:
    % a_max(train)=max(max(abs(a_train(v_dof,:))));

end

% The designvalue of the vertical acceleration.
a_dim=max(a_max);
fprintf('Max acc of the bridge deck is %3.2f m/s^2',a_dim)

a_save(velocity,mass)=a_dim;
Isave(velocity,mass)=I;
end

end

% The variables is saved
save(['L',num2str(L),'m',num2str(m),'EI0399.mat'],...
    'a_save','I_in','nel','c_in','m');

end
% The end

```

## Inputfile.m

```
function [A_in,m_in,I_in,...
    L,E,P0,c_in,nel,h1,h2,c_limit,greek_z]=inputfile
%% Input file to the Parametricstudy.m
%
% Input to the parametricstudy.m
%
% All Parameters will be specified in this input file.
%
% Written by: Carl Westerlund and Anders Pagoldh Johansson
% as a part of their Master Thesis
% "The global dynamic behaviour of
% steel and composite bridges
% for high-speed railway lines"
%
% Copyright 2013-02-21

% Input Parameters.
A_in=[1000:1000:40000]/7800; % [m^2] - The area of the
% cross-section.
% It is allowed to choose as many
% values as the user wants.

rho=7800; % [kg/m^3] - Density of the
% material used in the beam.

m1=0; % [kg/m] - Extra weight, that is
% put upon the structure which is
% calculated as non-bearing.

m_in=rho*A_in+m1; % [kg/m] - Total load per meter.

I_in=[0.3:0.3:9.9]; % [m^4] - The flexural rigidity
% of the beam.

% The user is allowed to choose
% as many values as the user
% wants.

L=50; % [m] - Length of the beam, only
% one length per run is allowed.
% In the thesis 20,35 and 50 was
```



```

% considered.

E=210e9; % [GPa] - The Youngs modulus of
% the beam.

P0=-250e3; % [N] - The load from the point
% load. When the HSLMAttrainset.m
% script is used, P0 can be set
% to anything since the result
% will be scaled.

c_in=[150:2:350]/3.6; % [m/s] - The speeds considered
% in the analysis. It is possible
% to chose as many as the user
% wants.

nel=15; % The number of elements chosen.

% The timestep is defined according to the convergence study
% in the report h1 is the time step for speeds below 270 km/h,
% h2 is sthe timestep for speeds above 270km/h.

h1=0.001; % [s] - The chosen timestep for
h2=0.0001; % the Newmark-beta integration.

c_limit=270/3.6 % [m/s] - The speed where the
% timestep is changed from the
% short one to the longer.

% The damping for steel and composite bridges as it is
% specified in Eurocode.

if L <= 20
    greek_z=(0.5+0.125*(20-L))*10^-2;
    disp('Beam length shorter than 20 m')
else
    greek_z=0.5*10^-2;
    disp('Beam length longer than 20 m')

```

end

## Loading2.m

```
function [F]=loading2(n,c,L_el,P0,ndof,htimestep)

%
% loading.m
%
% A function that gives the load matrix.
%
% Written by: Carl Westerlund and Anders Pagoldh Johansson
% as a part of their Master Thesis
% "The global dynamic behaviour of
% steel and composite bridges
% for high-speed railway lines"
%
% Copyright 2013-02-21

% Predefine the time and the load matrix.
tload=0;
F=spalloc(ndof,n,4*n);

% Waitbar for the patient student to watch
Hwb=waitbar(0,'');
set(Hwb,'Position',[10 15 280 45],'Name',...
    'Calculating Load vector - Please wait','Resize','on');

for i=1:n

    % Coordinate of the point load, for a given time.
    x=c*tload;

    % The element, where the load is situated at a given
    % coordinate.
    el=ceil((x/L_el));

    % If el==0 the point load is situated on the first element
    if el==0
        el=1;
    end

    % Define how long into a element the load has come.
    while x >= L_el
        x=x-L_el;
    end

    % For each degree of freedom.
    for j=1:4

        % The shape function for the degrees of freedom that
```

```

% are loaded.
N1= 1-3*(x/L_el)^2+2*(x/L_el)^3;
N2= x-2*L_el*(x/L_el)^2+L_el*(x/L_el)^3;
N3= 3*(x/L_el)^2-2*(x/L_el)^3;
N4= -L_el*(x/L_el)^2+L_el*(x/L_el)^3;

N=[N1 N2 N3 N4];

% The magnitude of load at each active degree of
% freedom.
P(j)=P0*N(j);
end

% Put the values of P into right places in F.
adof= [3*el-1 3*el 3*el+2 3*el+3];

F(adof(1),i)=P(1);
F(adof(2),i)=P(2);
F(adof(3),i)=P(3);
F(adof(4),i)=P(4);

% Update the time.
tload= tload + himestep;

% Update waitbar.
waitbar(i/(n));
end

close(Hwb)

% Reduce F according to the boundary conditions.
F=F(1:ndof,:);

```

## FE\_eig\_solver.m

```
function [K,M,eigfreq,eigfreq_Hertz,vec]=...
    FEEig_val_solver(nel,ep,ex,ey,bc,Edof,ndof,Idof)
% FE_eig_val_solver.m
% A function that computes K and M and eigenfrequencies for an
% arbitrary system.
%
% Written by: Carl Westerlund and Anders Pagoldh Johansson
% as a part of their Master Thesis
% "The global dynamic behaviour of
% steel and composite bridges
% for high-speed railway lines"
%
% Copyright 2013-02-21

% Predefining stiffness and mass matrices.
K=      zeros(ndof);
M=      zeros(ndof);

% Calculate K and M for the undamped case.
for i=1:nel;
    [Ke, Me] = beam2d(ex(i,:),ey(i,:),ep);
    [K]      = assem(Edof(i,:),K,Ke);
    [M]      = assem(Edof(i,:),M,Me);
end

% Reduce K and M into only rotational and vertical degrees of
% freedom
K=K(Idof,Idof);
M=M(Idof,Idof);
disp('Calculation of K and M done')

% Calculate the eigenvalues.
[lambda, vec] = eigen(K,M,bc);

% Calculate the eigenfrequencies.
for i=1:length(lambda);
    eigfreq(i,1)=sqrt(lambda(i));
end

% Calculate the eigenfrequencies in Hertz.
eigfreq_Hertz=eigfreq/(2*pi);

% Recalculate K and M as sparse matrices, to save calculation
% time.
K=      spalloc(ndof,ndof,6*ndof);
```

```
M=                spalloc(ndof,ndof,6*ndof);

for i=1:nel;
    [Ke, Me] = beam2d(ex(i,:),ey(i,:),ep);
    [K]      = assem(Edof(i,:),K,Ke);
    [M]      = assem(Edof(i,:),M,Me);
end

K=K(Idof,Idof);
M=M(Idof,Idof);
```

## Modaldamp.m (written by Prof Thomas Abrahamsson)

```

function [C,Ds]=modaldamp(K,M,z,zind,z0)
%MODALDAMP Establish the viscous damping matrix from given
% modal dampings
%Inputs: K,M - Stiffness and mass matrices
% z - Relative modal dampings, either as scalar
% (meaning same damping for all modes) or as
% vector
% zind - (optional) index vector for damping values of
% z, z(zind(I)) specifies I:th mode's damping
% value
% z0 - Damping of modes not specified in zind
% (default: 0.01);
%Output: C - Viscous damping matrix
%Call: C=modaldamp(K,M,z[,zind,z0])

%Written: 2012-09-04, Thomas Abrahamsson
%Modified: 2012-09-27, Added default damping /TA

%Reference:
%Craig & Kurdila, Fundamentals of Structural Dynamics, p305

%% Initiate and test
N=size(K,1);
if nargin<4,zind=[];end
if ~isempty(zind)
    if length(z)~=length(zind)
        error('z and zind need to be of same length');
    end
end
zdef=0.01;

%% Solve eigenvalue problem and sort
[V,D]=eig(full(K),full(M));
[Ds,inds]=sort(diag(D));Ds(Ds<0)=0;
V=V(:,inds);

%% Prepare damping value vector
if length(z)==1
    z=z*ones(N,1);
else
    zs=z;
    try z=z0*ones(N,1); catch z=zdef*ones(N,1);end
    z(zind)=zs;
end

%% Compute damping matrix
Md=diag(V'*M*V);iMd=diag(1./Md);
Cd=diag(2*z.*sqrt(Ds).*Md);
C=M*V*iMd*Cd*iMd*V.'*M;

```

## Newmark\_iteration.m

```
function
[unew, anew]=Newmark_iteration(L,c,K,C,M,nel,N,n,t_total,vec_trunk,Ido
f,F_load)
%
% Newmark_iteration.m
%
% A function that iteratively computes the displacements and
% the acceleration for the given system.
%
%
% Written by: Carl Westerlund and Anders Pagoldh Johansson
% as a part of their Master Thesis
% "The global dynamic behaviour of
% steel and composite bridges
% for high-speed railway lines"
%
%
% Copyright 2013-02-21

% Using special case Constant Average Acceleration Method.

gamma      =      1/2;      % According to Craig et al. p 514
beta       =      1/4;      % According to Craig et al. p 514

% The timestep is defined, aswell as the length of one element
% and the number of timesteps per element.
L_el       =      L/nel;      % [m]
h          =      t_total/n;    % [s]
n_ts       =      L_el/(c*h);  % [-]

% Calculate u n-1:

% Define the IC's (system is at initial rest).
u_dotdot_0= zeros(N,1);
u0=        zeros(N,1);
u_dot_0=   zeros(N,1);
u_         = u0-h*u_dot_0+(h^2/2)*u_dotdot_0;

% State un.
unc        =      zeros(N,1);
u          =      [u_unc];
u_dot      =      u_dot_0;
u_dot_dot  =      u_dotdot_0;

% Put a counter to one.
```



```

count      =      1;

%%
% Get the load truncate it
F_load=F_load(Idof,:);
F=vec_trunk'*F_load;

% Waitbar cosmetics....To see how the iteration has come.
Hwb=waitbar(0,'');
set(Hwb,'Position',[10 15 280 45],'Name',...
    'Calculating Newmark-beta method','Resize','on');

while count <= n
    % Displacement for timestep n-1 and n
    u_      =      u(:,count);
    un     =      u(:,count+1);

    % Velocity and acceleration for timestep n
    u_dotn=      u_dot(:,count);
    u_dot_dotn=  u_dot_dot(:,count);

    % Load the Load into P.
    if length(F(1,:))<=n
        P=F(:,count);
    end

    % Solve the displacement for step n+1:
    RHS =      -K*un-(C+h*K)*u_dotn-...
              (h*(1-gamma)*C+h^2/2*(1-2*beta)*K)*u_dot_dotn;

    a_plus1 =      (M+gamma*C+beta*h^2*K)\(RHS+P);
    u_plus1 =...
    un+u_dotn*h+((1-2*beta)*u_dot_dotn+2*beta*a_plus1)*h^2/2;

    v_plus1 =...
    u_dotn+((1-gamma)*u_dot_dotn+gamma*a_plus1)*h;

    % Save these into vectors ( the next time step)
    u(:,count+2)=      u_plus1;
    u_dot(:,count+1) =      v_plus1;
    u_dot_dot(:,count+1)=      a_plus1;

```

```
% add one time step to the count
count = count+1;

    waitbar(count/(n));
end

% Reduce u from timestep n-1 and n
u(:,1:2)=[];

% Reduce u_dot_dot from timestep n-1 and n
u_dot_dot(:,1)=[];

% Put the values into unew and anew.
unew=vec_trunk*u; %unew=u_result
anew=vec_trunk*u_dot_dot;
close(Hwb)
```

## HSLMAtrainset.m

```
function [u_result, a_result]=HSLMAtrainset(uneq, anew, train,
c, n_newton, t_total, P0, v_dofacc);
%
% HSLMAtrainset.m
%
% A function that by the help of the super position method
% will solve the response of a bridge when subject to multiple
% load that are simulating a train. (HSLM-A1-10)
%
% Input data is the response from one point load crossing the
% bridge and the geometric indata for the train.
%
%
% Written by: Carl Westerlund and Anders Pagoldh Johansson
% as a part of their Master Thesis
% "The global dynamic behaviour of
% steel and composite bridges
% for high-speed railway lines"
%
%
% Copyright 2013-02-21

% Number of coaches
N= [ 18 17 16 15 14 13 13 12 11 11];

% Length of one coach
D= [ 18 19 20 21 22 23 24 25 26 27];

% distance between boggies
d= [2.0 3.5 2.0 3.0 2.0 2.0 2.0 2.5 2.0 2.0];

% The load (scaled from P0)
scale=[170 200 180 190 170 180 190 190 210 210]*10^3/P0;

h=t_total/n_newton; % The timestep in seconds

% Calculate the distance between the loads for the two first
% and the two last carriages.

two_first = ...
[3 14 17 20.525 20.525+d(train) 18.7625+D(train)-d(train)/2];

two_last_char = [18.7625+D(train)-d(train)/2 ...
20.525+d(train) 20.525 17 14 3 0];

A = two_first(end);

for j=1:N(train);
    carriage(2*j-1) = A+d(train);
    carriage(2*j) = A+D(train);
    A = carriage(2*j);
end
```

```

% The total train length
total_trainL=2*20.525+2*D(train)+N(train)*D(train);

% The last carriage
two_last=total_trainL-two_last_char;

% The total train
traindist=[two_first carriage two_last];

% The time between the first axle and the n:th.
traintime= traindist/c;

pos=      ceil(traintime/h);

a_train=   anew;
u_train=   unew;

% Superimpose the results.
for i=1:length(pos)
    a_train(:,pos(i):end)=...
        a_train(:,pos(i):end)+anew(:,1:(end-(pos(i)-1)));

    u_train(:,pos(i):end)=...
        u_train(:,pos(i):end)+unew(:,1:(end-(pos(i)-1)));

    % Here one can see the response for different trains if
    % the rows belo is uncommented.

    % figure(11)
    % plot(1:length(anew),scale(train)*a_train(v_dofacc,:))
end

% Output
a_result=scale(train)*a_train;
u_result=scale(train)*u_train;

```

## Post\_processor.m

```
%% A Post processor that saves the results from the Parametric
% study, stored as .mat files, into a .xls-file.
%
% In order to work correctly the file names be named as
% follows:
%
% LXXmYYEIZZZZ.mat where XX represents the length and YY
% represents the considered mass and ZZZZ represents the
% limits of EI used.
%
%
% Written by: Carl Westerlund and Anders Pagoldh Johansson
% as a part of their Master Thesis
% "The global dynamic behaviour of
% steel and composite bridges
% for high-speed railway lines"
%
%
% Copyright 2013-04-21
%
%

clear all
close all
clc

E=210*10^9;
count=1;

for i=1000:1000:40000; % Define which masses that should be
    % used.
    for j=[20 35 50]; % Define which lengths that

        % Watch carefully that the file name is named
        % according to this
        % definition:

        load(['L',num2str(j),'m',num2str(i),'EI0399','.mat'])

        EI=I_in*E;
        Eurocode=5*ones(1,length(I_in));
        Designvalue=max(a_save);

        %% Store the obtained results. One tab for each run.
        xlswrite(['Parametricstudy',num2str(date),'.xls'],...
            a_save,['L',num2str(j),'m',num2str(i)],'D4')

        xlswrite(['Parametricstudy',num2str(date),'.xls'],...
            'I',['L',num2str(j),'m',num2str(i)],'C2')
```

```

xlswrite(['Parametricstudy', num2str(date), '.xls'], ...
        c_in'*3.6, ['L', num2str(j), 'm', num2str(i)], 'C4')

xlswrite(['Parametricstudy', num2str(date), '.xls'], ...
        I_in, ['L', num2str(j), 'm', num2str(i)], 'D2')

xlswrite(['Parametricstudy', num2str(date), '.xls'], EI, ...
        ['L', num2str(j), 'm', num2str(i)], 'D3')

xlswrite(['Parametricstudy', num2str(date), '.xls'], 'L', ...
        ['L', num2str(j), 'm', num2str(i)], 'A1')

xlswrite(['Parametricstudy', num2str(date), '.xls'], ...
        num2str(j), ['L', num2str(j), 'm', num2str(i)], 'B1')

xlswrite(['Parametricstudy', num2str(date), '.xls'], 'm', ...
        ['L', num2str(j), 'm', num2str(i)], 'C1')

xlswrite(['Parametricstudy', num2str(date), '.xls'], 'm', ...
        ['L', num2str(j), 'm', num2str(i)], 'A2')

xlswrite(['Parametricstudy', num2str(date), '.xls'], m, ...
        ['L', num2str(j), 'm', num2str(i)], 'B2')

xlswrite(['Parametricstudy', num2str(date), '.xls'], ...
        Designvalue, ['L', num2str(j), 'm', num2str(i)], 'D106')

xlswrite(['Parametricstudy', num2str(date), '.xls'], ...
        Eurocode, ['L', num2str(j), 'm', num2str(i)], 'D107')

% Store the obtained results, Design values only.
xlswrite(['Parametricstudy', num2str(date), '.xls'], ...
        Designvalue, ...
        ['Acc function of m', num2str(j), 'meter'], ...
        ['E', num2str(count+4)])

xlswrite(['Parametricstudy', num2str(date), '.xls'], ...
        m, ['Acc function of m', num2str(j), 'meter'], ...
        ['D', num2str(count+4)])

xlswrite(['Parametricstudy', num2str(date), '.xls'], ...
        EI, ['Acc function of m', num2str(j), 'meter'], 'E4')

xlswrite(['Parametricstudy', num2str(date), '.xls'], ...
        I_in, ['Acc function of m', num2str(j), 'meter'], 'E3')

xlswrite(['Parametricstudy', num2str(date), '.xls'], 'm', ...
        ['Acc function of m', num2str(j), 'meter'], 'C5')

xlswrite(['Parametricstudy', num2str(date), '.xls'], 'L', ...
        ['Acc function of m', num2str(j), 'meter'], 'A1')

xlswrite(['Parametricstudy', num2str(date), '.xls'], ...

```

```
        num2str(j), ...
        ['Acc function of m', num2str(j), 'meter'], 'B1')

    xlswrite(['Parametricstudy', num2str(date), '.xls'], 'm', ...
            ['Acc function of m', num2str(j), 'meter'], 'C1')

    end
    count=count+1;
end
```

## Plotter.m

```
%% Plotter that creates contour plots of EI and m from data
% found in .xls files.
%
% The resulting accelerations should be stored in cells in the
% following way:
%
%           EI1    EI2    EI3    EI4
% mass1    acc    acc    acc    acc
% mass2    acc    acc    acc    acc
% mass3    acc    acc    acc    acc
% mass4    acc    acc    acc    acc
%
% Written by: Carl Westerlund and Anders Pagoldh Johansson
% as a part of their Master Thesis
% "The global dynamic behaviour of
% steel and composite bridges
% for high-speed railway lines"
%
%
% Copyright 2013-04-21
%
%

clear all
close all
clc

options.Format = 'tiff';
v=[5 5]

FigHandle = figure('Position', [100, 100, 1049, 895]);
figure(1)

num = xlsread('Plotterdata.xlsx', '50m');
[C1,h]=contour(num(2:end,1),num(1,2:end),num(2:end,2:end)',v);
set(h,'ShowText','on','Linewidth',2)
title('50 m span - Resulting global accelerations [m/s^2]',...
      'FontSize', 20);
ylabel('Flexural rigidity EI [Nm^2]','FontSize', 15)
xlabel('Mass per meter [kg/m]','FontSize', 15)
grid on
AXIS([0 40000 0 9.9*210e9])
hgexport(1,'contour50.tif',options)
set(gca,'XTickLabel',num2str(get(gca,'XTick').'))
set(gca,'fontsize',12)

FigHandle = figure('Position', [100, 100, 1049, 895]);
figure(2)

hum = xlsread('Plotterdata.xlsx', '35m');
[C2,h]=contour(hum(2:end,1),hum(1,2:end),hum(2:end,2:end)',v);
set(h,'ShowText','on','Linewidth',2)
```



```

title('35 m span - Resulting global accelerations [m/s^2]',...
      'FontSize', 20);
ylabel('Flexural rigidity EI [Nm^2]', 'FontSize', 15)
xlabel('Mass per meter [kg/m]', 'FontSize', 15)
grid on
AXIS([0 40000 0 9.9*210e9])
hgexport(2, 'contour35.tif', options)
set(gca, 'XTickLabel', num2str(get(gca, 'XTick').'))
set(gca, 'fontsize', 12)

```

```

FigHandle = figure('Position', [100, 100, 1049, 895]);
figure(3)
dum = xlsread('Plotterdata.xlsx', '20m');
[C3, h]=contour(dum(2:end,1), dum(1,2:end), dum(2:end,2:end)', v);
set(h, 'ShowText', 'on', 'LineWidth', 2)
title('20 m span - Resulting global accelerations [m/s^2]',...
      'FontSize', 20);
ylabel('Flexural rigidity EI [Nm^2]', 'FontSize', 15)
xlabel('Mass per meter [kg/m]', 'FontSize', 15)
grid on
AXIS([0 40000 0 9.9*210e9])
hgexport(3, 'contour20.tif', options)
set(gca, 'XTickLabel', num2str(get(gca, 'XTick').'))
set(gca, 'fontsize', 12)

```

```

figure
FigHandle = figure('Position', [100, 100, 1049, 895]);
hold on
plot(C3(1,2:end), C3(2,2:end), 'k', 'LineWidth', 2)
plot(C2(1,2:73), C2(2,2:73), 'k-', 'LineWidth', 3)
plot(C1(1,2:end), C1(2,2:end), 'k--', 'LineWidth', 1)
legend('20 m', '35 m', '50 m')
title(...
      'The dynamic behaviour for different lengths',...
      'FontSize', 20);
ylabel('Flexural rigidity EI [Nm^2]', 'FontSize', 15)
xlabel('Mass per meter [kg/m]', 'FontSize', 15)

```

## Plottereigfreq.m

```
%% Plotter that creates contour plots of eigenfrequencies and
% m from data found in .xls files.
%
%
% The resulting accelerations should be stored in cells in the
% following way:
%
%           EI1    EI2    EI3    EI4
% mass1    acc    acc    acc    acc
% mass2    acc    acc    acc    acc
% mass3    acc    acc    acc    acc
% mass4    acc    acc    acc    acc
%
% The eigenfrequency for each combination of EI and mass
% should be stored in cells in the following way, with one
% sheet per length.
%
%           EI1    EI2    EI3    EI4
% mass1    eigfreq eigfreq eigfreq eigfreq
% mass2    eigfreq eigfreq eigfreq eigfreq
% mass3    eigfreq eigfreq eigfreq eigfreq
% mass4    eigfreq eigfreq eigfreq eigfreq
%
% Written by: Carl Westerlund and Anders Pagoldh Johansson
% as a part of their Master Thesis
% "The global dynamic behaviour of
% steel and composite bridges
% for high-speed railway lines"
%
%
% Copyright 2013-04-21
%
%

clear all
close all
clc

% Choose format of the picture.
options.Format = 'tiff';

% Read all accelerations and eigenfrequencies.
num20= xlsread('Plotterdatawitheigenfrequencies.xlsx','20m');
numfreq20= xlsread('Plotterdatawitheigenfrequencies.xlsx',...
    '20mEigenfrequencies');

num35= xlsread('Plotterdatawitheigenfrequencies.xlsx','35m');
numfreq35= xlsread('Plotterdatawitheigenfrequencies.xlsx',...
    '35mEigenfrequencies');

num50= xlsread('Plotterdatawitheigenfrequencies.xlsx','50m');
numfreq50= xlsread('Plotterdatawitheigenfrequencies.xlsx',...
```

```

    '50mEigenfrequencies');

% Removes texts and the values of mass and EI from the table.
numfreq20= numfreq20(3:end,2:end);
numfreq35= numfreq35(3:end,2:end);
numfreq50= numfreq50(3:end,2:end);

%% For 20m
count=1;

% Sort the values so that we get a table with
% [mass1 eigenfrequency1 accel].
for i=2:length(num20(1,:))
    for j=2:length(num20(:,1))
        values20(count,:)=...
            [num20(j,1) numfreq20(j,i) num20(j,i)];
        count=count+1;
    end
end

% Define x-, y- and z-collumn.
x20=values20(:,1);
y20=values20(:,2);
z20=values20(:,3);

% Create a mesh of values interpolated between the calculated.
a20=size(values20);
b20=a20(:,1);
xlin20=linspace(min(x20),max(x20),b20);
ylin20=linspace(min(y20),max(y20),b20);
[X20,Y20]=meshgrid(xlin20,ylin20);
Z20=griddata(x20,y20,z20,X20,Y20);

% Plot the contourplot
figure(1)
FigHandle = figure('Position', [100, 100, 1049, 895]);
u20=[2 3.5 5 10];
[C,h]=contour(X20,Y20,Z20,u20);
set(h,'ShowText','on','Linewidth',2)
title('20 m span - Resulting global accelerations [m/s^2]',...
    'FontSize', 20);
ylabel('Eigenfrequency [Hz]','FontSize', 15)
xlabel('Mass per meter [kg/m]','FontSize', 15)
axis([0 40000 7 20])
grid on
hgexport(2,'contoureig20.tif',options)

%% For 35 m

```

```

count=1;
% Sort the values so that we get a table with
% [mass1 eigenfrequency1 accl].

for i=2:length(num35(1,:))
    for j=2:length(num35(:,1))
        values35(count,:)=...
            [num35(j,1) numfreq35(j,i) num35(j,i)];
        count=count+1;
    end
end

% Define x-, y- and z-collumn.
x35=values35(:,1);
y35=values35(:,2);
z35=values35(:,3);

% Create a mesh of values interpolated between the calculated.
a35=size(values35);
b35=a35(:,1);
xlin35=linspace(min(x35),max(x35),b35);
ylin35=linspace(min(y35),max(y35),b35);
[X35,Y35]=meshgrid(xlin35,ylin35);
Z35=griddata(x35,y35,z35,X35,Y35);

% Plot the contourplot
figure(2)
FigHandle = figure('Position', [100, 100, 1049, 895]);
u35=[2 3.5 5 10];
[C,h]=contour(X35,Y35,Z35,u35);
set(h,'ShowText','on','Linewidth',2)
title('35 m span - Resulting global accelerations [m/s^2]',...
    'FontSize', 20);
ylabel('Eigenfrequency [Hz]','FontSize', 15)
xlabel('Mass per meter [kg/m]','FontSize', 15)
axis([0 40000 3 15])
grid on
hgexport(3,'contoureig35.tif',options)

%% For 50 m
count=1;
% Sort the values so that we get a table with
% [mass1 eigenfrequency1 accl].

for i=2:length(num50(1,:))
    for j=2:length(num50(:,1))
        values(count,:)=..
            [num50(j,1) numfreq50(j,i) num50(j,i)];
        count=count+1;
    end
end

```

```
end
```

```
% Define x-, y- and z-collumn.
```

```
x=values(:,1);
```

```
y=values(:,2);
```

```
z=values(:,3);
```

```
% Create a mesh of values interpolated between the calculated.
```

```
a=size(values);
```

```
b=a(:,1);
```

```
xlin=linspace(min(x),max(x),b);
```

```
ylin=linspace(min(y),max(y),b);
```

```
[X,Y]=meshgrid(xlin,ylin);
```

```
Z=griddata(x,y,z,X,Y);
```

```
% Plot the contourplot
```

```
figure(3)
```

```
FigHandle = figure('Position', [100, 100, 1049, 895]);
```

```
u=[2 3.5 5 10];
```

```
[C,h]=contour(X,Y,Z,u);
```

```
set(h, 'ShowText', 'on', 'Linewidth',2)
```

```
title('50 m span - Resulting global accelerations [m/s^2]',...
```

```
'FontSize', 20);
```

```
ylabel('Eigenfrequency [Hz]','FontSize', 15)
```

```
xlabel('Mass per meter [kg/m]','FontSize', 15)
```

```
axis([0 40000 1 6])
```

```
hgexport(3, 'contoureig50.tif', options)
```



# Appendix C

# Python script developed for Abaqus / BRIGADE/Plus.  
# Returns the datum points for the nodes of the 50 m graphene tube treated in the master thesis  
# Copyright Carl Westerlund and Anders Pagoldh Johansson 2013.

```
# -*- coding: mbcs -*-
from job import *
from step import *
from part import *
from material import *
from section import *
from assembly import *
from interaction import *
from load import *
from mesh import *
from optimization import *
from sketch import *
from visualization import *
from connectorBehavior import *

from abaqus import *

#Datumpunkter for en kolnanotub med radie 7.50000 m

#Datumpunkter

#Ny sektion
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(7.50000, 0.00000, 2.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(6.49519, 3.75000, 2.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(3.75000, 6.49519, 2.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(0.00000, 7.50000, 2.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-3.75000, 6.49519, 2.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-6.49519, 3.75000, 2.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-7.50000, 0.00000, 2.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-6.49519, -3.75000, 2.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-3.75000, -6.49519, 2.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-0.00000, -7.50000, 2.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(3.75000, -6.49519, 2.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(6.49519, -3.75000, 2.50000))

#Ny sektion
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(7.24444, 1.94114, 3.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(5.30330, 5.30330, 3.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(1.94114, 7.24444, 3.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-1.94114, 7.24444, 3.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-5.30330, 5.30330, 3.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-7.24444, 1.94114, 3.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-7.24444, -1.94114, 3.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-5.30330, -5.30330, 3.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-1.94114, -7.24444, 3.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(1.94114, -7.24444, 3.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(5.30330, -5.30330, 3.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(7.24444, -1.94114, 3.75000))

#Ny sektion
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(7.24444, 1.94114, 6.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(5.30330, 5.30330, 6.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(1.94114, 7.24444, 6.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-1.94114, 7.24444, 6.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-5.30330, 5.30330, 6.25000))
```













```
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-1.94114, 7.24444, 48.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-5.30330, 5.30330, 48.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-7.24444, 1.94114, 48.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-7.24444, -1.94114, 48.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-5.30330, -5.30330, 48.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-1.94114, -7.24444, 48.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(1.94114, -7.24444, 48.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(5.30330, -5.30330, 48.75000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(7.24444, -1.94114, 48.75000))
```

#Ny sektion

```
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(7.24444, 1.94114, 51.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(5.30330, 5.30330, 51.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(1.94114, 7.24444, 51.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-1.94114, 7.24444, 51.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-5.30330, 5.30330, 51.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-7.24444, 1.94114, 51.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-7.24444, -1.94114, 51.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-5.30330, -5.30330, 51.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-1.94114, -7.24444, 51.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(1.94114, -7.24444, 51.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(5.30330, -5.30330, 51.25000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(7.24444, -1.94114, 51.25000))
```

#Ny sektion

```
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(7.50000, 0.00000, 52.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(6.49519, 3.75000, 52.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(3.75000, 6.49519, 52.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(0.00000, 7.50000, 52.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-3.75000, 6.49519, 52.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-6.49519, 3.75000, 52.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-7.50000, 0.00000, 52.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-6.49519, -3.75000, 52.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-3.75000, -6.49519, 52.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(-0.00000, -7.50000, 52.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(3.75000, -6.49519, 52.50000))
mdb.models['Model-1'].parts['Part-1'].DatumPointByCoordinate(coords=(6.49519, -3.75000, 52.50000))
```