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# Comparing Two Approaches to Precompute Discharge Strategies for Plug-in Hybrid Electric Vehicles

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**Abstract:** This paper investigates two alternative approaches to precompute a discharge strategy for the main commuter route of a plug-in hybrid electric vehicle. The first approach is based on the idea of computing a state of charge reference trajectory by solving a convex program; while the second approach utilizes dynamic programming to determine an optimal cost-to-go function. During real-time operation the torque split is decided by an equivalent consumption minimization strategy where the main difference between the two approaches is how the equivalence factor is determined. With the first approach it is adapted to track the state of charge reference trajectory and in the second approach it is given by the partial derivative of the cost-to-go function with respect to state of charge. To evaluate the two approaches a simulation study is performed in the dynamic vehicle modelling software Autonomie using logged commuter driving data. The simulation results indicate no clear difference between the two approaches in terms of fuel economy and battery usage. Both approaches are, however, significantly better than a charge depleting charge sustaining discharge strategy.

Keywords: hybrid vehicles, energy management, convex optimization, dynamic programming

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## 1. INTRODUCTION

During trips that exceed the all electric range of a Plug-in Hybrid Electric Vehicle (PHEV), there is a degree of freedom concerning the battery discharge rate that can be exploited by the energy management system (EMS). The simplest discharge strategy is to operate in charge depletion mode until the battery is depleted and then proceed with charge sustaining operation. However, the overall energy cost can be reduced if the battery is discharged more gradually, to be depleted first towards the very end of the trip; this type of strategy is often called a *blended strategy* since use of electricity and gasoline is blended throughout the trip. A blended strategy can improve energy efficiency mainly since the average discharge current is reduced, thereby decreasing the resistive losses that are quadratic in current. Furthermore, a blended strategy can have additional advantages such as lower battery C-rate and Ah throughput.

The disadvantages with a blended strategy is the need for a priori information and the computations associated with finding an optimal discharge strategy. Trivial discharge strategies, i.e. discharging the battery linearly with remaining trip distance as done by Tulpule et al. [2010] and Larsson et al. [2012], works mainly if the topography is relatively flat and the driving conditions are uniform. If the topographic profile contains long downhill segments, a linear discharge rate might result in very poor performance, see Zhang and Vahidi [2011], since recovered

potential energy might not be depleted before the end of the trip.

Due to the computational demand it might not be practical to calculate an optimal discharge strategy online in a vehicle ECU. However, recent development trends within the automotive industry, mainly driven by safety and infotainment systems, is connecting the vehicle to the cellular network and the internet, see for example BMW ConnectedDrive and Volvo On Call. The idea of a connected vehicle opens up new possibilities, such as performing off-line computations on a server; e.g. for the main commuter route. The obtained solution can then be transmitted to the vehicle, thereby shifting the bulk of the computations outside of the real-time loop.

Limiting the scope to strategies where the torque/power-split is decided online by an Equivalent Consumption Minimization Strategy (ECMS), see Paganelli et al. [2002], the off-line computations can be grouped into two alternative approaches, differing mainly in how to achieve a feedback on the battery State of Charge (SoC) when updating the equivalence factor. The first approach is based on the notion of a SoC-reference trajectory, which can be obtained from the solution of a standard quadratic program with linear constraints, see Ambühl and Guzzella [2009], or from the solution of a Dynamic Programming (DP) problem as proposed by Yu et al. [2011]. The idea is then to track the SoC-reference in real time using an ECMS-strategy. The second approach does not consider any reference; instead it is based on DP and the fact that the ECMS-equivalence factor can be interpreted as the partial derivative of the

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cost-to-go function with respect to SoC, see for example Johannesson et al. [2009], Zhang and Vahidi [2011].

The main research question in this paper is to investigate benefits and drawbacks of the two different approaches to precompute a discharge strategy with SoC-feedback. To obtain the SoC-reference trajectory, used in the first approach, a convex optimization problem is solved using the methodology described by Murgovski et al. [2012a], where the powertrain is described by affine/quadratic expressions and the integer decisions, i.e. gear and engine state, are given by heuristics. Two different model complexities are considered, resulting in either a Quadratically Constrained Linear Program (QCLP) or a Semidefinite Program (SDP). The DP formulation, considered in the second approach, is also based on an affine/quadratic powertrain model; it can, however, also include the integer decisions in the problem formulation.

To compare the two approaches in a realistic scenario, a driving pattern consisting of logged commuter driving data is considered in a simulation study using the dynamic vehicle modelling tool *Autonomie*<sup>1</sup> developed by Argonne National Laboratory. The a priori information required for the precomputations is given as a Piecewise Linear (PWL) drive cycle, derived from historical driving data logged along the main commuter route used by the vehicle.

*Outline* The paper is divided into seven sections. After the introduction the simplified vehicle model is explained. The next section presents the two approaches to precompute a discharge strategy more in depth. In the following section it is described how a commuter route can be represented as a PWL drive cycle. Finally, the paper is ended with a simulation study, some discussion and conclusions.

## 2. SIMPLIFIED VEHICLE MODEL

This section presents the simplified vehicle model used during the precomputations. A post transmission parallel PHEV is considered, i.e. the electric motor (EM) is connected directly to the front axle and the engine (ICE) is coupled through a clutch and a stepped automatic transmission. The vehicle configuration is based on one of the predefined models in *Autonomie* and is shown in Figure 1, the only modification with respect to the original powertrain model is the number of cells in the battery pack. The simplified model, summarized in Table 1, is extracted from the dynamic *Autonomie* model, and powertrain components not described in this section are not considered during the precomputations.

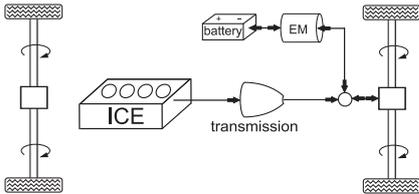


Fig. 1. The vehicle configuration.

The forces acting on the powertrain is calculated using an inverse simulation approach, meaning that the torque

<sup>1</sup> <http://www.autonomie.net/>

demanded at the wheels,  $T_d$ , to follow a given velocity and road slope trajectory is determined by

$$T_d = r_w(1/2\rho_a C_d A_f v^2 + mg(f_r \cos \theta + \sin \theta) + m_e a), \quad (1)$$

where  $r_w$  represents wheel radius,  $\rho_a$  density of air,  $C_d$  air drag coefficient,  $A_f$  frontal area,  $f_r$  rolling resistance,  $v$  velocity,  $a$  acceleration,  $\theta$  road slope,  $m$  vehicle mass and  $m_e$  equivalent vehicle mass, i.e. including moments of inertia of the rotating parts. The torque demand at the wheels must be fulfilled by the torque contributions from the EM  $T_{em}$ , the engine torque  $T_{ice}$  and the brake friction torque  $T_{fr}$ ,

$$T_d = \eta_f r_f (T_{em} + \eta_{gb,i} r_{gb,i} T_{ice}) + T_{fr}, \quad (2)$$

where  $r_f$  represents the final drive ratio. Note that the efficiency of the final gear  $\eta_f$  depends on the sign of the torque demand at the wheels; if the torque demand is positive  $\eta_f = \eta_{f0}$ , otherwise  $\eta_f = \eta_{f0}^{-1}$ . The gears,  $i = 1, \dots, 5$ , are represented by a drive ratio  $r_{gb,i}$  and a mechanical efficiency  $\eta_{gb,i}$ . Furthermore it is assumed that gear shifts as well as the engine state transitions are instantaneous and lossless. The electrical power of the EM  $P_{em}$  is assumed to be convex quadratic in torque,

$$P_{em} = d_0(\omega_{em}) T_{em}^2 + d_1(\omega_{em}) T_{em} + d_2(\omega_{em}), \quad (3)$$

and the fuel mass rate of the engine is approximated to be affine in torque. Consequently the instantaneous fuel cost of the engine is given by

$$\Pi = c_f (c_0(\omega_{ice}) T_{ice} + c_1(\omega_{ice})) \cdot e_{on}, \quad (4)$$

where  $e_{on}$  represents the engine state and  $c_f$  the price of fuel. The speed dependent coefficients are determined by linear least squares from steady state maps available in *Autonomie*; the resulting approximations are illustrated in Figure 2. Furthermore, the torque of the engine and EM and the battery power are constrained to be within the operating limits,

$$T_{ice} \in [0, T_{ice}^{max}(\omega_{ice})], \quad (5)$$

$$T_{em} \in [T_{em}^{min}(\omega_{em}), T_{em}^{max}(\omega_{em})] \quad (6)$$

$$P_{bat} \in [P_{bat}^{min}, P_{bat}^{max}]. \quad (7)$$

A Li-Ion battery is considered and it is modelled as an equivalent circuit consisting of  $n_c$  battery cells connected in series. Each cell is assumed to have a constant internal resistance  $R_{in}$  and an open circuit voltage  $V_{oc}$  that is affine in the state  $x$ , i.e. in SoC,

$$V_{oc} = a_0 x + a_1. \quad (8)$$

The requested net battery power is given by

$$P_{bat} = P_{em} + P_{aux} \cdot (2 - \eta_{pc}), \quad (9)$$

where  $\eta_{pc}$  represents the efficiency of the power converter and  $P_{aux}$  the auxiliary loads. Finally, the relationship between battery net power and cell current  $I$  is

$$P_{bat} = n_c (V_{oc} I - R_{in} I^2), \quad (10)$$

where the battery state, i.e. SoC, dynamics are given by

$$\dot{x} = -I/Q, \quad (11)$$

with  $Q$  denoting cell capacity.

## 3. PRECOMPUTING AN OPTIMAL STRATEGY

The problem formulation, for both approaches, is to minimize the total energy cost along a given route representation, while respecting the constraints of the powertrain model. Let  $u = [T_{ice}, T_{em}, e_{on}, r_{gb,i}]$  represent the control

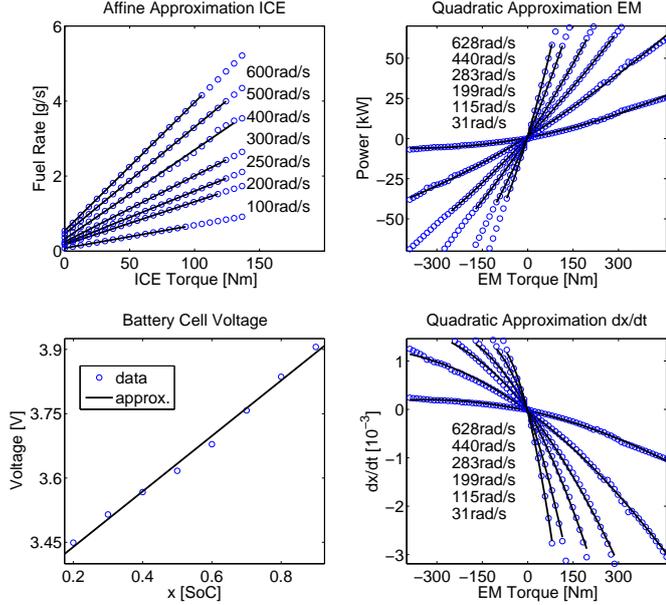


Fig. 2. The upper plots show approximations of the ICE and the EM. The lower left plot depicts the affine cell voltage and the lower right plot shows the approximation used by the simplified convex model.

Table 1. Vehicle data

Chassis Data	Symbol	Value
Mass, Area Front	$m, A_f$	1720 kg, 2.25m <sup>2</sup>
Wheel Rad./Inertia	$r_w, J_w$	0.3m, 4 kgm <sup>2</sup>
Air / Roll. res.	$C_d, f_r$	0.30, 0.008
Final gear Rat. / Eff.	$r_f, \eta_{f_0}$	4.4, 0.97
Gear box ratios	$r_{gb,i}$	2.6, 1.6, 1.0, 0.7, 0.5
Gear box eff.	$\eta_{gb,i}$	0.88 + 0.02 <i>i</i>
Aux. Load, Conv. eff	$P_{aux}, \eta_{pc}$	200W, 0.95
Battery Data		Li-Ion
Nom. Cell Voltage	$V_{nom}$	3.6V
Cell Res./Cap.	$R_{in}, Q$	2.1 mΩ, 37 Ah
Nr of series cells	$n_c$	60 (8kWh)
Peak Cell Curr.	$I_{min}/I_{max}$	-320/415 A
Initial/Final SoC	$x_0/x_f$	0.85/0.30
Max/Min SoC	$x_h/x_l$	0.90/0.25
ICE Data		4 Cyl. Spark Ignited
Max power	$P_{ice,max}$	65 kW @ 520 rad/s
Max torque	$T_{ice,max}$	137 Nm @ 393 rad/s
EM Data		Permanent Magnet
Max power	$P_{em,max}$	70 kW
Max torque	$T_{em,max}$	458 Nm @ [0, 130] rad/s

signal, i.e. choice of gear, engine state and torque split between EM and engine. Furthermore, assume that the route is represented by a velocity trajectory  $v(\cdot)$  and a road slope trajectory  $\theta(\cdot)$ .

The optimal control problem can then be formulated as

$$\begin{aligned}
 J^* = \min_{u(\cdot)} & \{c_e(x(t_0) - x(t_f)) + \int_{t_0}^{t_f} \Pi(u) dt\} \\
 \text{s.t. } & \dot{x} = f(x, u) \\
 & u \in U(x, v, \theta) \\
 & x \in [x_l, x_h] \\
 & x(t_0) = x_0, x(t_f) \geq x_f
 \end{aligned} \quad (12)$$

where  $f(x, u)$  is defined by Equations (8)-(11) and  $U(x, v, \theta)$  by Equations (1)-(11). The energy costs are rep-

resented by  $c_e = 0.12 \text{ €/kWh}$  for electricity and  $c_f = 1.75 \text{ €/liter}$  for gasoline.

### 3.1 Approach A: SoC-reference Trajectory

The methodology used to determine the SoC-reference trajectory is based on the convex optimization methodology introduced by Murgovski et al. [2012a]. The main idea is to convexify the vehicle model described in Section 2 and solve the optimal control problem, given by Equation (12), as a convex program; preferably on a server using some commercial optimization software. The SoC-trajectory given by the optimal solution can then be sent to the vehicle and be used as a SoC-reference in the online EMS.

To obtain a convex program the optimal control problem should be expressed on the following form

$$\begin{aligned}
 \min_y & f_0(y) \\
 \text{s.t. } & f_i(y) \leq 0, \quad i = 1, \dots, m \\
 & h_j(y) = 0, \quad j = 1, \dots, p \\
 & y \in \mathcal{Y}
 \end{aligned}$$

where the feasible domain  $\mathcal{Y} \subseteq \mathbb{R}^n$  is convex, the functions  $f_0(y)$  and  $f_i(y)$  are convex and  $h_j(y)$  are affine; for a detailed description of convex optimization theory see for example Boyd and Vandenberghe [2004].

To reformulate the optimal control problem into a tractable convex program, a number of simplifications must be done. First of all, integer decision variables cannot be treated in a convex problem formulation, meaning that the engine state and choice of gear must either be relaxed or predecided. Secondly, expressions that are not affine must be reformulated to obtain constraints on the form  $f_i(y) \leq 0$ .

*Integer Decision Variables* Since the engine and the EM do not share transmission, the choice of gear will mainly influence the operating point of the engine. Hence, the gear is chosen as the highest possible gear that will not cause engine stall when operating as a conventional vehicle, thereby maximizing the engine torque and thus also (in general) the efficiency. The engine state  $e_{on}$  is determined by the requested torque demand at the wheels; if the traction request exceeds the threshold  $T_{on}$  the engine is assumed to be on, otherwise it is assumed to be off

$$e_{on} = \begin{cases} 1, & T_d \geq T_{on} \\ 0, & T_d < T_{on}. \end{cases} \quad (13)$$

Due to this heuristic rule the convex program must be solved iteratively by varying the threshold until the optimal cost converges.

*Reformulations* The EM power, given by Equation (3), is convex quadratic but is given as an equality rather than an inequality. The equation can, however, be relaxed to greater or equal to without loss of optimality, since a solution where both sides are not equal would correspond to a situation where energy is wasted without generating any useful work. Furthermore, the affine relationship between battery voltage and SoC implies that a variable change is needed to obtain a convex problem formulation; rather than working with SoC as the energy state consider the battery energy

$$E = \frac{n_c Q (a_0 x + a_1)^2}{V_{nom}}, \quad (14)$$

with dynamics described by

$$\frac{dE}{dt} = -\frac{a_0}{R_{in}Q} \left( E - \sqrt{E \left( E - \frac{n_c R_{in} Q}{V_{nom}} P_{bat} \right)} \right). \quad (15)$$

The right hand side of Equation (15) is concave since the square root expression is on the form of a geometric mean, for a more in depth description of this modelling methodology see Murgovski et al. [2012b]. Moreover, Equation (15) can be relaxed to less or equal to using similar arguments as for Equation (3). The full convex problem formulation, denoted the detailed convex model, is stated in Table 2.

*Simplified Convex Model* The assumed affine voltage vs SoC relationship of the battery leads to the rather complicated nonlinear Equation (15), making the detailed convex model a computationally expensive SDP. Therefore, a simplified convex model is introduced to investigate how much the optimal SoC-trajectory is influenced by the model complexity used.

The simplified model is based on the real-time EMS implemented by Beck et al. [2007]. It is assumed that the open circuit voltage of the battery is constant and the entire electrified part of the powertrain, i.e. EM, battery, power converter and auxiliary load, is modelled by the concave expression

$$\dot{x} = b_o(\omega_{em})T_{em}^2 + b_1(\omega_{em})T_{em} + b_2(\omega_{em}), \quad (16)$$

which is illustrated in Figure 2. Note, however, that the fit shown is with respect to the simple vehicle model and not with measured data. Moreover, if Equation (16) is relaxed to less or equal to the problem is convex in  $x$ , i.e. SoC, and  $x$  can therefore be used as the energy state. Finally, the resulting QCLP is shown in Table 2.

*Discretization* To solve the problem numerically it has to be discretized. Equations (11) and (15) are time discretized using the Euler method. The route representation, however, is discretized uniformly in position rather than time; let the distance positions be indexed by  $k = 0, 1, \dots, N - 1$ , and denote the travel time between two positions  $t_k$ .

### 3.2 Approach B: DP cost-to-go function

Using Bellman's principle of optimality and DP, see Bellman and Dreyfus [1962], the idea is to determine the optimal state feedback law, along the route, as cost-to-go function. The methodology, described very briefly, is to discretize the optimal control problem and grid the energy state into  $M$  points described by  $x^d$ . The next step is to solve the recursive equation

$$J(z_k, x^d) = \min_{u_k \in U(v_k, \theta_k)} \left\{ t_k \Pi(u_k) + J(z_{k+1}, x_{k+1}^d) + \Gamma(x_{k+1}^d) \right\}, \quad (17)$$

backwards starting from the final position  $z_N$  with final cost  $J(z_N, x^d) = J_N$ , i.e. the price to recharge the battery. Furthermore, the constraints on  $x$  have been relaxed and are replaced with soft constraints represented by the smooth penalty function  $\Gamma$ . Note that the DP formulation, in contrast to the convex formulation, includes the integer decision variables in the control signal  $u$ . Finally, also in this approach the idea is to solve the problem on a server

Table 2. The convex program where the battery energy state is represented by  $s$ . The problem is solved numerically using CVX [2012].

Convex Problem Formulation	
	input variables: $T_d(\cdot), e_{on}(\cdot), r_{gb}(\cdot), \omega_{wh}(\cdot), t_k(\cdot)$
	main decision variables: $T_{ice}(\cdot), T_{em}(\cdot), s(\cdot)$
	minimize
a)	$c_f \sum_{k=0}^{N-1} t_k \{ c_0(\omega_{ice,k})T_{ice,k} + c_1(\omega_{ice,k}) \} e_{on,k} + c_e(s_1 - s_N)$
	subject to
b)	$(e_{on,k} \eta_{gb,k} r_{gb,k} T_{ice,k} + T_{em,k}) r_f \eta_{f,k} \geq T_d$
c)	$T_{ice,k} \in [0, T_{ice}^{max}(\omega_{ice,k})]$
d)	$T_{em,k} \in [T_{em}^{min}(\omega_{em,k}), T_{em}^{max}(\omega_{em,k})]$
e)	$s_k \in [s_l, s_h]$
f)	$s_1 = s_0$
g)	$s_N \geq s_f$
Detailed Convex Model - SDP ( $s = E$ )	
h)	$P_{em,k} \geq a_0(\omega_{em,k})T_{em,k}^2 + a_1(\omega_{em,k})T_{em,k} + a_2(\omega_{em,k})$
i)	$P_{bat,k} = P_{em,k} + P_{aux}(1 - \eta_{pc})$
j)	$P_{bat,k} \in [P_{bat}^{min}, P_{bat}^{max}]$
k)	$E_{k+1} \leq E_k - t_k \frac{a_0}{R_{in}Q} \left( E_k - \sqrt{(E_k - \frac{n_c R_{in} Q}{V_{oc}} P_{bat,k})} \right)$
Simplified Convex Model - QCLP ( $s = x$ )	
h <sub>s</sub> )	$x_{k+1} \leq x_k - t_k (b_o(\omega_{em,k})T_{em,k}^2 + b_1(\omega_{em,k})T_{em,k} + b_2(\omega_{em,k}))$

and then send the obtained cost-to-go function matrix,  $J \in \mathbb{R}^{N \times M}$ , to the vehicle and use it for state feedback, in  $x$  and position  $z$ , in the online EMS.

## 4. REPRESENTING THE COMMUTER ROUTE

To model the actual driving conditions along a commuter route, the representation should preferably be determined from logged driving data. The idea is to represent the driving trajectory, i.e. velocity and altitude, along the route as a sequence of PWL functions. Given  $Q$  logged driving trajectories, defined over a finitely gridded distance vector  $z \in \mathbb{R}^D$ , the objective is to find the best PWL route representation, i.e. a lower order representation. The discretized route consists of  $N - 1$  segments, each covering  $p$  distance positions in  $z$ , where a segment is defined by a slope  $\alpha_k$  and constant term  $\beta_k$ . Furthermore, assume  $D = pN$ ,  $p \in \mathbb{N}_+$ , and let  $u^q \in \mathbb{R}^D$  represent the logged trajectories, either velocity or altitude. The best PWL approximation,  $\hat{u} \in \mathbb{R}^D$ , is in this paper defined by the convex program

$$\begin{aligned} \min_{\alpha(\cdot), \beta(\cdot)} \quad & \sum_{q=1}^Q \|\hat{u} - u^q\|_2 + \epsilon \|\alpha\|_1 \quad (18) \\ \text{s.t.} \quad & \hat{u}_j = \alpha_k z_j + \beta_k, \quad k \in [0, N - 1], \quad j \in [k, p(k + 1)] \\ & \alpha_{k-1} z_{pk} + \beta_{k-1} = \alpha_k z_{pk} + \beta_k, \quad k \in [1, N - 1] \end{aligned}$$

where a small regularization term  $\epsilon \|\alpha\|_1$  is added to favour longer sections of constant speed. An example of a route representation is shown in Figure 3. Moreover, since each segment will be treated as one sample during the precomputations, it is assumed that the torque demand is constant along each segment. Hence, during segments with acceleration it is approximated that  $v_k \approx 1/2(v_{k+1} + v_k)$ .

## 5. SIMULATION STUDY

The simulation study is carried out in the Autonomie simulation environment for MATLAB/Simulink; a software based on a dynamic, forward-looking, modeling approach that features a driver model and transient responses for the key powertrain components.

### 5.1 Real-time Discharge Strategy in Autonomie

In Autonomie the so-called vehicle propulsion controller, deciding the engine state and the torque references for the engine and the EM, is modified from a rule based strategy to a conventional ECMS-strategy.

The ECMS-control signal is at a time sample  $i$  given by

$$u_i^* = \arg \min_{u_i \in U(v_i, \theta_i)} \left\{ h_s \Pi(u_i) + \lambda_i(z_i, x_i) \cdot (x_{i+1}(u_i) - x_i) + \delta \cdot \max[0, e_{on,i} - e_{on,i-1}] \right\}. \quad (19)$$

where  $h_s$  represents sample time and  $z$  the distance position along the route. The dynamics as well as the constraints are given by the simplified model in Section 2. Furthermore, the choice of gear is not included in the control signal,  $u = [e_{on}, T_{ice}^{ref}, T_{em}^{ref}]$ , since it is not decided by the vehicle propulsion controller in Autonomie. During the simulations the torque references are updated at 100Hz, i.e. the sample time of the simulation model, and the engine state at 0.5Hz. Furthermore, a small penalty  $\delta$  is added to decrease the number of engine starts.

The equivalence factor  $\lambda$  is determined differently depending on the approach used.

*Approach A: SoC reference Trajectory* The expression for the equivalence factor with the SoC-reference  $x_{ref}$  is

$$\lambda_i(z_i, x_i) = \lambda_0 + F_{PI}(x_{ref}(z_i) - x_i), \quad (20)$$

where the SoC feedback is given by a simple PI-controller  $F_{PI}$  and  $\lambda_0$  represents the initial guess for equivalence factor. A suitable initial guess can, for example, be obtained from the dual variables to the energy state in the convex program. The PI-controller is given on the standard form

$$F_{PI}(e(t)) = K_p e(t) + K_i \int_{t_0}^t e(\tau) d\tau, \quad (21)$$

where  $e$  represents the SoC tracking error.

*Approach B: DP cost-to-go function* In the DP approach the equivalence factor is determined as the partial derivative of the cost-to-go function  $J$  with respect to SoC, evaluated at the current SoC and distance position,

$$\lambda_i(z_i, x_i) = \left. \frac{\partial J}{\partial x} \right|_{x_i}^{z_i}. \quad (22)$$

*Nominal Strategy: CDCS* To illustrate the overall benefit of using a precomputed discharge strategy, the nominal Charge Depletion Charge Sustaining (CDCS) discharge strategy is also implemented. In the CDCS strategy the equivalence factor is given directly by the SoC

$$\lambda_i(\text{SoC}_i) = \begin{cases} 0, & \text{if } x_i > x_s \\ \lambda_0 + F_{PI}(x_f - x_i), & \text{if } x_i \leq x_s \end{cases} \quad (23)$$

where  $x_s = x_f + 0.025$  and the PI-controller is given by Equation (21).

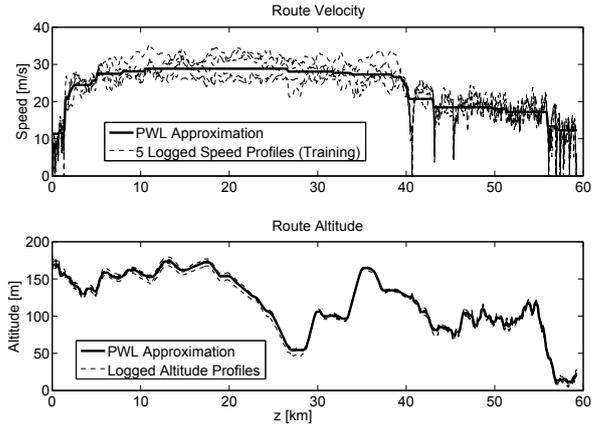


Fig. 3. The PWL representation of the commuter route, each segment is 100m long.

### 5.2 Simulation Setup

The driving pattern considered in this simulation study is taken from the Swedish Car Movement Database, see Karlsson [2013], containing driving data logged in the Gothenburg area in the western part of Sweden. A vehicle with a distinct commuting pattern was chosen to illustrate the benefits of having a precomputed discharge strategy for a commuter route exceeding the PHEV all electric range. Ten trips logged along the commuter route are considered, all going in the same direction. The trips were randomly divided into a training set and a validation set, five trips in each. Figure 3 shows the route representation obtained with the training set, using the methodology described in Section 4. The remaining trips, i.e. the validation set, are shown in the upper plot of Figure 4; these trips were used as speed reference trajectories during the simulations in Autonomie. To give comparable results the same parameters were used in Equation (20) and Equation (23), i.e.  $K_p = 250$ ,  $K_i = 0.5$  and  $\lambda_0 = -36.3$ . Furthermore, in the DP precomputation the SoC was discretized into 400 points.

### 5.3 Simulation Results

Starting with the SoC-reference trajectories shown in Figure 4, it is apparent that both the detailed and the simplified convex model result in almost overlapping references. The only noticeable difference is that the reference for the detailed model lies slightly higher, this since it accounts for the voltage drop with respect to SoC. Hence it favors a higher average SoC, i.e. voltage, thereby lowering the current and the resistive losses.

The simulation results for the five validation trips are summarized in Table 3 and the resulting discharge trajectories are shown in Figure 4. It is clear that both approaches, i.e. SoC-reference and DP cost-to-go, give fairly similar discharge behaviour and comparable fuel economy. However, comparing the precomputed strategies with the nominal CDCS, strategy it is evident that the precomputed strategies on average have about 6-7% lower fuel consumption along the commuter route. Furthermore, the precomputed strategies are also better from a battery point of view since the average c-rate and the total Ah-throughput are about 12-13% lower.

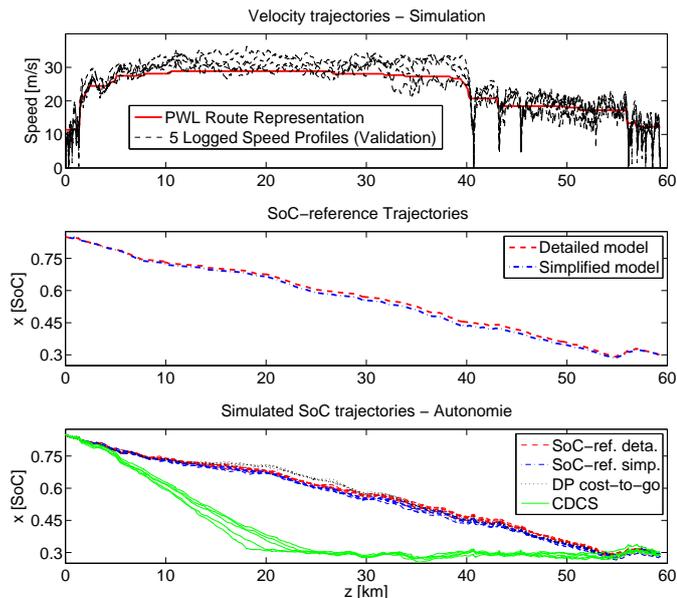


Fig. 4. Top, the speed profiles simulated in Autonomie. Middle, the SoC-references for the two model complexities. Bottom, the simulated SoC trajectories for the two approaches and the CDCS strategy.

Table 3. Simulation results for the two approaches and the CDCS strategy. Table shows: total battery Ah throughput, total fuel mass consumed, mean final SoC and mean c-rate.

Simulation Results - All 5 commuter Trips				
	$\int  I(t) dt$ [Ah]	$\int \dot{m}(t)dt$ [kg]	$SoC(t_f)$ (mean)	C-rate (mean)
SoC-ref Detailed	209	5.45	0.295	1.49
Soc-ref Simplified	210	5.43	0.291	1.49
DP cost-to-go	211	5.41	0.287	1.50
CDCS	241	5.85	0.296	1.71

## 6. DISCUSSION

The advantage with the convex optimization approach is that it, in contrast to DP, can consider several dynamic states, e.g. temperatures and battery state of health, without a significant increase in computational demand. Moreover, a SoC-reference trajectory is also beneficial from a data storage and transmission point of view, since it is one dimensional, a DP cost-to-go function will have a data storage requirement that is orders of magnitude higher.

However, a cost-to-go function has a very important benefit, it gives a state feedback based on Bellman's principle of optimality; meaning that disturbances in the past will not compromise the optimality of future control actions. For example, consider a scenario where the vehicle gets stuck in an unexpected traffic jam where the SoC drops due to auxiliary loads. If a SoC-reference is used the strategy would, once the trip continues, seek to reach the reference and thus use the engine to charge the battery, clearly not an optimal behaviour. Using a cost-to-go function would circumvent this behaviour since it contains the optimal state feedback law for the remainder of the trip, irrespective of any disturbances in the past. With a SoC-reference approach, a new reference trajectory must be determined

as soon as a major disturbance occurs anywhere along the route.

## 7. CONCLUSION

It is clear that it can be beneficial to use precomputed discharge strategies for frequently driven routes, such as the main commuter route. The results indicate that it is enough to have a vehicle model that catches the main characteristics of the powertrain; increased model complexity does not necessarily give better performance. Furthermore, the approach used to precompute the discharge strategy seems to play a minor role, at least in terms of fuel economy and battery usage. Which computational approach to use will most likely be decided by aspects such as robustness, computational demand and storage requirements, rather than fuel economy.

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