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A Simple Method for Optimal Antenna Array Thinning using a Broadside MaxGain Beamformer

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***Abstract*—A simple and effective method for optimal antenna array thinning employing a broadside-scanned maximum gain beamformer is presented. Starting from a fully populated $\lambda/2$-spaced regular lattice, the array is thinned by progressively "turning off" the element(s) with the lowest weight(s) of the weight vector realizing maximum gain. The accuracy and effectiveness of the method is validated against a rigorous combinatorial search method that can be used to find the optimal irregular array configuration solution in small to moderate-sized arrays. Furthermore, to evaluate the robustness of the proposed approach, the effects of beam steering have been investigated for linear arrays consisting of 10–40 antenna elements as well. Good results can be obtained for close to broadside scanned arrays, which is of importance for the directly radiating arrays that are currently being considered as modern satellite systems.**

***Index Terms*—array design; sparse arrays; thinning; beamforming; signal processing; array configuration optimization**

I. Introduction

Arrays of active antennas have been increasingly employed in many wireless applications, including radar, radio astronomy and satellite communication systems. By deploying a group of radiating elements and adjusting the contribution for the elements through digital beamforming, it is possible to greatly enhance the global antenna performances including the resulting directivity, sidelobe level, polarization purity and bandwidth. These properties make the antenna array technologies very appealing for a new range of applications, but as new potentialities open to array technology, so do the challenges.

Thus far, the design of active beamforming arrays represents a complex and time-consuming problem; the solution space is very large and requires an appropriate choice of radiating element types, their number and positions in the array environment, as well as the selection of the optimal beamforming algorithm and its hardware implementation. This problem is even more challenging for arrays with randomly spaced elements and thinned arrays. Solving the former category of problems requires consideration of the whole continuous space for the element positions. These approaches typically rely on reference continuous tapering distributions [1] to position elements accordingly. In array thinning techniques, the problem is tackled starting from a regular fully populated array which continues by progressively removing elements based on some antenna performance criteria for obtaining a sparse irregular array and determining the optimal excitation coefficients. As a full combinatorial search for the optimal solution is intractable even for moderate sized arrays, several techniques have been proposed to alleviate this computational problem.

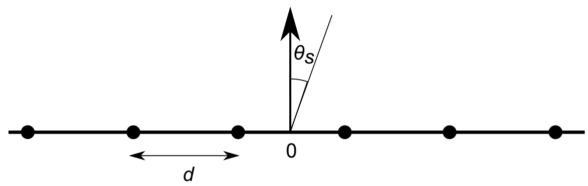

Fig. 1. Regular linear array with inter element distance d

Some of the earliest techniques addressed the problem through probabilistic array thinning, while a recent and more promising line of research is based on the attractive properties of the binary sequences, named Different Sets [2], [3]. The binary nature of the problem also encouraged the ample use of genetic algorithms [4], more recently combined with deterministic techniques to refine their solutions [5], [6]. While reportedly achieving successful results over previous generations, these methods are less robust for large array sizes and tend to provide suboptimal solutions. Another problem with this approach for large arrays is that the reference solution leading to the global optimum is difficult – if not impossible – to obtain; and therefore only relative improvements over previous results are commonly reported.

In this paper, we present a simple method to find the optimal solution for the array thinning problem that is based on the maximum gain beamformer (MaxGain) configured for a near-broadside-scanned direction and combined with the minimization of the number of non-zero terms of the weighting vector. Although the MaxGain beamformer has no control over the SLL and the overall pattern shape, it represents an important reference case that can be used to evaluate the accuracy and efficiency of numerical algorithms aimed at large array optimization problems. Furthermore, the performance upper bound (in terms of the gain) can be readily evaluated over a wide range of scan angles, so as to reduce the large space of thinning-related design parameters (such as the minimum aperture area of the array and the minimum number of elements). As shown in this paper, the proposed thinning approach has been numerically validated with respect to the rigorous combinatorial search method and demonstrated to lead to the same global optimum solution for the array sizes up to 40 elements (along one direction), within a limited scan range around the broadside direction. The following results apply in particular to satellite communication systems, where directly radiating arrays can consist of 100–1000 elements having relatively modest scan ranges.

II. Proposed Approach

The complex-valued array pattern of an N element regular linear array with inter-element distance d (*cf.* Fig. 1) can be defined as [7]

$$\mathbf{S}(\theta) = \sum_{n=1}^{N} w_n \mathbf{f}_n(\theta) e^{jknd\sin(\theta)} \tag{1}$$

with the weight coefficients $\{w_n\}_{n=1}^{N}$, the vector embedded element patterns $\{\mathbf{f}_n\}_{n=1}^{N}$, the propagation constant k, and the observation direction θ. Upon assuming that the co-polar component is dominant, it suffices to consider the scalar embedded element pattern $\{f_n\}_{n=1}^{N}$ and the scalar array pattern $S(\theta)$ instead.

In matrix notation – as is customary in the array signal processing community – we have that

$$S(\theta) = \mathbf{w}^H \mathbf{v}(\theta) \tag{2}$$

where $\mathbf{w}$ is the conjugated weighting vector, H the Hermitian operator, and $\mathbf{v}(\theta)$ is the array voltage response or steering vector, i.e.,

$$\mathbf{v}(\theta) = \begin{bmatrix} f_1(\theta)e^{jkd\sin(\theta)} \\ f_2(\theta)e^{jk2d\sin(\theta)} \\ \vdots \\ f_N(\theta)e^{jkNd\sin(\theta)} \end{bmatrix}. \tag{3}$$

The power that is radiated in a given direction is proportional to the square of the absolute value of the array pattern, and hence, to optimize the gain, it is required to maximize the power in a given direction θ_s with respect to the total radiated power, or, equivalently, to minimize the reciprocal form

$$\mathbf{w} = \min_{\mathbf{w}} \frac{\mathbf{w}^H \mathbf{B} \mathbf{w}}{\mathbf{w}^H \mathbf{A} \mathbf{w}} \tag{4}$$

where

$$\mathbf{A} = \mathbf{v}(\theta_s)\mathbf{v}^H(\theta_s) \tag{5a}$$

$$\mathbf{B} = \sum_{l=1}^{L} \mathbf{v}(\theta_l)\mathbf{v}^H(\theta_l) \tag{5b}$$

and where $\mathbf{A}$ accounts for the power radiated in the scanning direction θ_s, and $\mathbf{B}$ represents the total radiated power which is obtained by sampling the array pattern over the field of view at L points.

The above problem (4) is referred to as the maximum gain beamformer and can be defined as a linear programming problem, whose solution can be demonstrated to be [8]

$$\mathbf{w} = \text{principal eigenvector of } \mathbf{A}^{-1}\mathbf{B} \tag{6}$$

provided that $\mathbf{A}$ is invertible[1].

The problem of array thinning can be defined as the minimization of the ℓ_0-norm (the number of non-zero terms of its argument) of the weighting vector subject to some pattern constraints, i.e. [9],

$$\min_{\mathbf{w}} \|\mathbf{w}\|_{\ell_0} \text{ subject to a specified pattern mask.} \tag{7}$$

The present maximum gain beamformer does not employ pattern constraints, so that the problem should be reformulated as

$$\mathbf{w} = \min_{\mathbf{w}} \frac{\mathbf{w}^H \mathbf{B} \mathbf{w}}{\mathbf{w}^H \mathbf{A} \mathbf{w}} \text{ subject to } \|\mathbf{w}\|_{\ell_0} = (1 - TF) \times N \tag{8}$$

where TF is the thinning factor, defined as the number of removed elements relative to the number of array elements N of a fully filled array:

$$TF = \frac{N_{\text{filled array}} - N_{\text{thinned array}}}{N_{\text{filled array}}}. \tag{9}$$

Eq. (8) represents the rigorous formulation of the problem. The minimization of the objective function is a convex (linear) problem, however, the ℓ_0-norm constraint renders it nonlinear, so that only global optimization techniques – such as genetic algorithms – are able to solve the problem. As a remedy, alternative approaches have been proposed, some of which adopt a surrogate iterative weighed ℓ_1-norm to achieve sufficiently sparse solutions while preserving convexity [9]. Another approach is the herein proposed numerical technique as detailed in the next section.

III. Numerical Implementation and Results

The algorithm commences by applying the maximum gain beamformer for a given steering direction to a regular fully populated array of directive elements. The embedded element patterns are chosen to be of the type $\cos^n(\theta)$, where n is selected in accordance with the minimum required scan range of the array. Since the weights with the lowest magnitudes are expected to have the smallest effect on the solution, the element with the lowest weight is progressively removed during the thinning process in ascending order of the absolute value of the weighting vector.

An illustration of this iterative algorithm is provided in a stepwise manner in Fig. 2 for an $N = 20$ element broadside-scanned regular array with inter-element separation distance $\lambda/2$. The optimally thinned arrays are depicted as combinatorial reference solutions after each iteration. It is observed that these optimal solutions for all the thinning factors exactly match the ones obtained through selecting the elements based on the magnitudes of the weights.

The proposed approach has been tested for linear arrays of 10–40 antenna elements, however, to be able to obtain an optimal combinatorial reference solution for the largest arrays, only symmetric layouts have been considered. To validate the performance of the presented algorithm over the range

[1] If $\mathbf{A}$ is singular, one has to consider the principle eigenvector of the generalized eigenvalue equation $\mathbf{B}\mathbf{w} = G\mathbf{A}\mathbf{w}$, where $G = (\mathbf{w}^H\mathbf{B}\mathbf{w})/(\mathbf{w}^H\mathbf{A}\mathbf{w})$.

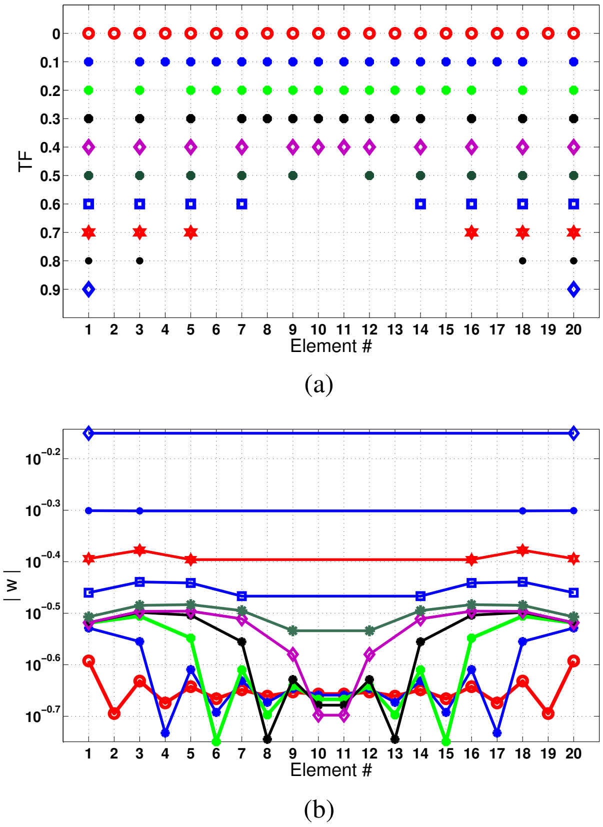

Fig. 2. (a) The optimal layouts of the linear irregular arrays with the thinning factor TF varying from 0 to 0.9 (view from top to bottom), as found by the combinatorial search procedure starting from the regular array of 20 half wavelength spaced antenna elements (see the red cirular markers for $TF = 0$) and maximizing the antenna gain in the broadside direction. (b) The magnitude of the weighting coefficients (view from bottom to top) of the regular array and the set of irregular arrays, as optimized by using the proposed thinning procedure that is based on progressively "removing" the element(s) with the lowest weights of the weight vector realizing the maximum gain. Note the agreement between the two optimal solutions.

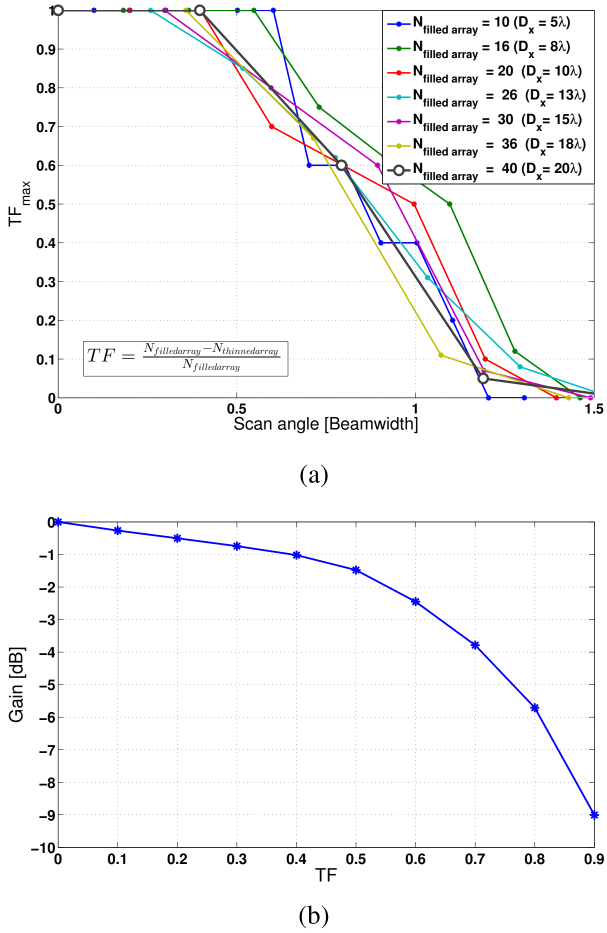

Fig. 3. (a) The maximum thinning factor for which the proposed algorithm returns the optimal array layout solution as a function of scanning angle (in beamwidths) for different array sizes. (b) The relative gain loss for increasing thinning factor as obtained with the proposed algorithm starting from the 20-element regular array.

of the considered array sizes and scan directions, we use the maximum thinning factor $TF_{\max}$ for which our algorithm returns the optimal solution as obtained with the reference method.

While this approach is promising for broadside scanned beams, it has proved to be inadequate for far off-broadside steered beams, since the optimal array configuration rapidly changes when the array is scanned beyond a critical scanning angle. This behavior is not modeled properly by the proposed approach since it progressively removes elements without restoring any afterwards. Fig. 3 presents the results for $TF_{\max}$ for irregular arrays with 10–40 elements as a function of the scan angle. As one can see, the proposed approach leads to the optimal thinning configurations for the broadside scan – for all tested array configurations. However, and as expected, for off-broadside direction, the range of the optimal solutions that our iterative method predicts rapidly decreases.

IV. Conclusions

It has been demonstrated that the proposed method is an accurate and effective way of synthesizing the optimal thinned array layout when the maximum gain performance is required to be near the broadside-scanned direction. While for most practical applications, additional constraints on the side-lobe level and pattern shape are commonly imposed, the present method is simple and easy to implement, and hence, is useful for studying design trade-offs of large-scale array antennas – with respect to their minimum size, number of elements and individual beamformer controls – as well as to serve as a reference case for testing other optimization techniques for achieving optimal array configurations. Further research is ongoing to extend this approach to deal with off-broadside scanning effects and to allow for asymmetric locations of the array elements. Also, investigation of the proposed strategy in combination with more practical beamformers accounting for SLL constraints and beam shaping is of interest.

Acknowledgment

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