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MATHEMATICAL MODEL OF SEWAGE DISCHARGE INTO CONFINED, STRATIFIED BASINS - ESPECIALLY FJORDS

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ERRATA

Page 348, line 6, "dimensional" should be "non-dimensional"

line 9, "accoumt" should be "account"

Page 349, line 9, "Gaussion" should be "Gaussian"

last line" "pp 432-" should be "pp 423-"

Page 350, line 9 from the bottom, "10^{-9}" should be "10^{-8}"

line 14 from the bottom, "(5)" should be "(4)"

Page 352 line 2, "considrably" should be "considerably"

INTERNATIONAL ASSOCIATION FOR HYDRAULIC RESEARCH

DROGUE TRACKING - MEASURING PRINCIPLES AND DATA HANDLING

(Subject B.b.)

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SYNOPSIS

Drogues provide in many situations a suitable means of measuring currents and diffusive properties in coastal waters, estuaries and lakes. Guidelines for the choice of tracking method and proper design of the drogue are given together with a method to correct for the deviations of the drogue path from the true current path. The deviations are caused primarily by drag forces on the carrying and signal parts of the drogue. Drag coefficients have been established by laboratory measurements. For the processing and presentation of drogue data a computer program has been designed.

RESUME

Les drogues constituent en de nombreuses occasions un moyen auxiliaire adéquat pour mesurer les courants et la diffusion dans les eaux côtières, les estuaries et les lacs. Les lignes directrices de construction de drogues et de choix de méthodes de mesurage sont fournies avec une méthode permettant de corriger les déviations à partir du tracé de courant "correct". La déviation est principalement occasionnée par les forces de courant sur les parties portantes et la signalisation de drogues.
Les coefficients des forces des courants ont été déterminés par essais en laboratoires. Une programmation par calculateur électronique a été utilisée pour évaluation et présentation des relevés des drogues.

INTRODUCTION

A drogue is a device designed to measure the Lagrangian water movement at a predetermined, fixed depth. It consists of a submerged drag device, usually in the form of a cross connected by a cord to a surface float carrying a signal unit. Drogues provide in many situations a suitable means of measuring currents and diffusive properties in coastal waters, estuaries, and lakes [1]. This is especially true in areas with large spatial current variations, in wave-exposed upper layers and in simple engineering applications. Combinations with other measuring techniques may prove powerful, especially fixed-point measurements with self-contained current meters [2]. The latter provide information of the temporal current variations and the former of the spatial variations. In combination with tracer experiments with technical tracers like radioactive isotopes or fluorescent dyes, drogues may be very useful to locate the tracer cloud. Drogue tracking is an approximate current measuring method since drag forces acting on the carrying and signal parts give rise to deviations of the drogue path from the "true" current path.

The purpose of this paper is to provide information about drogue systems and a method for correction of the measured current vector [3].

DROGUE APPLICATIONS

The aim of the study determines the drogue release and tracking system to be used but also the resulting amount of data and its subsequent handling. Common drogue applications are:

- Single current measurements from an anchored boat. The drogue is allowed to float some fixed distance e.g. by attaching it to a cord.
- Flow field studies. -A number of free-floating drogues are needed, distributed over the area of interest. Trajectories are constructed from interpolations of the velocity vectors thus obtained.
- Water transport through a cross-section. - A number of drogues are repeatedly launched, well distributed over the cross-section. The method is often used to calibrate results from fixed current meters and is sometimes the only possible method in passages with ship traffic.
- Long range tracking with a limited number of drogues. - Parachutes are often used in open-sea measurements. In coastal and estuarine waters the non-tidal drift may be evaluated. The method puts heavy demands on the tracking capability.
- Measurement of horizontal turbulent diffusion. - A cluster of drogues is released within a small area and the growth and distortion of the group is measured repeatedly.

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TRACKING METHODS

Following the release of the drogues, the field work mainly consists of keeping control of the drogues and finding their successive positions. There are several methods and instruments available for measuring the positions of the drogues. There is an important distinction between visual and nonvisual methods. The former include theodolites, optical range finder, laser, bearing compass, sextant and aerial photography and the latter radar, radiolocation systems, and navigational aids such as the Decca Navigator System. Another important classification is if the instruments are land- or boat-based.

The choice of method depends primarily on the following factors:

Water area - size, wind, and wave exposure, distance to land. Tracking range from less than 1 km to tens of kilometers.

Visibility - requirement for optical tracking. Weather and light conditions affect the tracking range.

Accuracy - the demand is firmly related to the distance travelled between successive position findings.

Tracking capacity - restricts the number of drogues used simultaneously for a given tracking interval.

Costs - depend on the size of the staff, type of instrument for position finding, and number and size of boats. The relations between initial and operational costs are essential in connection with the scope of the study.

Availability - the most suitable instrument and other equipment may not be at hand. This is often the most restrictive factor besides the costs, especially for engineering work.

As a guidance for the choice of tracking method, significant parameters of performance, etc. for some common methods are given in Table 1.

Table 1. Drogue tracking methods, estimates of magnitude for significant parameters.

Instrument	Price, dollars	Maximum range, km	Accuracy	Capacity, trackings /hour	Minimum staff
Theodolite	2 x 1500	5 (<1) x	5 m/km	15-20	3
Optical range-finder, 0.4 m base	4000	1 (<1) x	1 m at 100 m 50 m at 1 km	15-40	2
Laser	25.000	5 (<1) x	5 m i.r.	15-40	2
Radar	7000	3 - 5	10 m/km	60	2
Bearing compass	150	1	25-100 m/km	10-20	2 (1)
Sextant	400	5	<5 m/km	10-20	2 (1)
Aerial photography	-		5 m i.r.	∞	4 (3)
Radiolocation	40.000	sea-line	5 m i.r.	10-30	2
Decca Navigation System	2000/year	∞	10-100 n independent of distance		2

x/ if not located and marked by an accompanying boat,
i.r. = independent of range

The table is based on experiences gained at various institutions in Sweden and Norway. The authors have personal experience from work with optical range finder, bearing compass, theodolite, aerial photography, and radar. The figures given should be considered as normal values from a practical point of view. In a specific case, the given performance data may very well be considerably improved. For example, for visual methods the measuring range can under certain good weather conditions be extended to the ultimate limit, the sea-line. The practical measuring range is, however, often limited to the order of 5 km for management reasons. Higher accuracy of position findings than 5 m is normally not significant as errors of this magnitude often arise from the method of tracking itself. Such errors could be the time recordings, position findings made on an accompanying boat instead of the drogue for visibility reasons, and several other factors.

The use of land-based instruments generally limits the investigational area to a few square kilometers along the coastline as a contrast to boat-based methods. In the latter case all parts of the drogue tracking, i.e. launching, position finding, and recovery of the drogues, are made from the same working place. The accuracy and tracking capacity is, however, often higher for land-based methods, and generally smaller boats can be used.

Among land-based tracking methods, the use of theodolites may be attractive for surveys of a few days´duration. Theodolites are standard geodetic instruments and easily available everywhere. The method is, however, laborious as it requires two instruments to be used simultaneously. Optical range finders and lasers provide both range and bearing information, and consequently only one land station is needed. The use of the range finder is restricted to distances within one kilometer, due to a rapid decrease in accuracy with range. The laser is a very useful tool for extensive surveys because of excellent accuracy and range. Instrumental costs, however, hampers a more common use of the method. An accompanying boat is usually needed to indicate the drogues when the range exceeds one kilometer for the above-mentioned methods. A land-based radar equipped with a robot camera taking photographs of the radar screen at equally spaced time intervals is powerful for very extensive field surveys. The instrument does not need contineous supervision, and moving pictures can be an expressive tool for visualization of the measurements.

The bearing compass can be used only if a dense net of reference points, landmarks or buoys, is available. The sextant requires only a few but easily discernable land-points and is very accurate. The observer, however, needs some experience in handling the instrument, especially when used in a small boat. Radiolocation systems such as the Decca Trisponder, using two land-based radio transmitters and a radio receiver in the surveying boat for range readings, give an excellent accuracy over an extensive investigational area. The system is very powerful and completely independent of sight if the drogues are searched out by radar. The instrument is, however, very expensive. For larger investigational areas and open sea measurements a combined use of radar for localization of the drogues and conventional navigational aids such as the Decca Navigation system for position findings is most suitable. Aerial photography is a very expensive method. The costs can usually be justified only when drogues are used for diffusion studies.

METHOD OF CORRECTION

The different parts of the drogue are subject to drag forces from the fluid velocity relative to the drogue and from mass forces from the relative acceleration. The mass forces are mainly due to waves. It can be shown, however, that the mass forces give rise to negligible drogue oscillations in in-shore waters with moderate waves. The effect of breaking waves would be a forward movement of the drogue due to impact forces on the signal parts, normally above the water surface. This force is difficult to estimate, but drogue measurements are, for other reasons, seldom performed in such heavy weather. If the mass forces are neglected the force balance of the drogue requires that (vector form):

$$F_w + F_s + F_c + F_d = 0 \qquad (1)$$

where F_w is the wind force on the signal parts, F_s is the surface current force on the float, F_c is the force on the cord, and F_d is the drag force on the cross from the deviating velocity.

The drag forces on different parts of the drogue are calculated by the well-known formula

$$F = \frac{1}{2} C_d \rho A \cdot |u - u_0| \cdot (u - u_0) \qquad (2)$$

where C_d is the drag coefficient, ρ the fluid density, for water and air respectively, A the projected area, u the fluid velocity vector and u_0 the drogue velocity vector. To facilitate the computations, we introduce a lumped drag force coefficient, $k = \frac{1}{2} C_d \cdot \rho \cdot A$.

The force on the cord is calculated by the expression

$$F_c = \int_0^{z_d} k_c \cdot |u-u_0| \cdot (u-u_0)dz \approx \sum_n k_{c_n} \cdot |u_n - u_0| \cdot (u_n - u_0) \qquad (3)$$

It is obvious that the entire current profile from the surface to the actual depth of measurement, z_d, needs to be known to reproduce correctly the forces on the cord.

Data requirements for surface float and cord correction are best provided for by making simultaneous drogue measurements at different depths including the surface, where a weighted float may be used. Equation (1) is then used to calculate the current at each depth, starting from the surface. It should be noted that the transport velocity of the wind waves is included in the surface current when measured this way.

DROGUE DESIGN

The proper design of a drogue should be based on the following criteria:

- Small deflections from the "true" current path. The drag force coefficient for the cross should be large as compared to that of the other parts of the drogue. The cross should thus have a large projected area, and the other parts should be made as small and streamlined as possible.
- Efficient signal device to allow for easy and reliable tracking.
- Properly balanced drogue system. The float should have little excess buoyancy and cut through the wave crests, and the cross should be heavy enough to avoid slack in the cord in wave throughs.
- The drogue should be easy to handle and transport and not be too expensive.

Obviously, the ultimate design of the drogue has to be a compromise between these requirements. The common design is that of a winged type of drag device, "cross", with 2-4 wings and projected areas of 1-3 m^2. In open coastal areas, parachutes with projected areas larger than 5 m^2 are commonly used.

The authors have conducted laboratory and field investigations on the winged type of drogue in order to find a proper design and determine the drag coefficients for different types of signal devices, surface floats and crosses. Drag measurements were carried out in a hydraulic flume and a wind tunnel and field studies of drogue behaviour under various field conditions in in-shore and coastal waters.

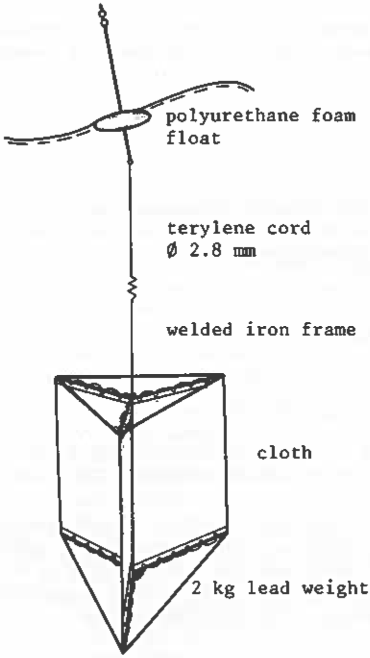

Fig.1. The Chalmers drogue.
Drogue de Chalmers.

As a result of the study, the three-winged type of drogue, Fig.1, (called the Chalmers drogue) was adopted. The cross is made of cloth, suspended between two welded iron frames and carried by a discus-like surface float. Field experience has revealed that the drogue shown in Fig.1 is very handy to use even from small boats. The three-winged "cross" has the advantage over the more common four-winged type to have less drag variation when turning different sides towards the current. Although the drogues have a tendency of rotating in the water due to turbulence and shear, the three-winged type normally faces the current with one wing and thus has a well-defined k-value.

Drag coefficients and lumped drag force coefficients for different parts of the Chalmers drogue are listed in Table 2. Coefficients for some other types of drogues are given in ref. [3].

The diameter of the cord should be as small as practically possible. (2.8 mm terylene line for the drogue in Fig. 1). The Reynolds number will thus become so small at common velocities that the drag coefficient will vary with the speed. For practical calculations, however, a constant value of $C_d = 1.6$ can be used.

The deflection of the drogue from the "true" current path, $\Delta u = u_d - u_o$, may be very significant in areas with weak currents, and in some cases the results may be misleading if not corrected. The main influence is usually from the wind which acts directly on the signal parts and indirectly on the surface float and carrying cord by the wind induced current.

Table 2. Drag coefficients, C_d, and lumped drag force coefficients, k, for the Chalmers drogue.

Drogue part	dimension,m	C_d	k,kg/m	notes
Signal stick	0.8 x 0.08	1.1	$4 \cdot 10^{-3}$	
Radar target	Ø 0.15	0.65	$7 \cdot 10^{-3}$	crossed aluminium discs.
Radar target	Ø 0.15		$5.3 \cdot 10^{-3}$	with centre hole Ø 0.03
Table-tennis ball	Ø 0.038		$0.2 \cdot 10^{-3}$	mounted on the stick
Stiff flag	0.2 x 0.1		$1 \cdot 10^{-3}$	
Cloth flag	0.2 x 0.1		$10 \cdot 10^{-3}$	
Discus float	0.3 x 0.1	0.42	5.1 } 5.9	volume 4.6 litres
Subsurface stick	0.15x0.08	1.35	0.8 } 5.9	
Chalmers cross	1 x 1	1.40	} 550	
-"-		1.09	} 550	recommended value

Wind and surface current components acting in the same direction cause the following correction:

$$u = \sqrt{\frac{k_w}{k_d} \cdot u_w^2 + \frac{k_s}{k_d} \cdot (u_s - u_0)^2} \qquad (4)$$

Examples of the magnitude of the corrections are given in Table 3.

Table 3. Examples of correction velocity, Δu, in cm/s, for the Chalmers drogue.
$k_d = 550$ kg/m, $k_s = 5.9$ kg/m and $k_w = 5 \cdot 10^{-3}$ kg/m

		u_w 2	6	10	15 m/s
$u_s - u_0$ cm/s	5	0.8	1.9	3.0	4.5
	10	1.2	2.1	3.2	4.6
	20	2.2	2.7	3.6	5.0
	50	5.2	5.5	6.0	6.9

For the Chalmers drogue the wind correction amounts to 0.4 % of the wind speed for the "best" optical signals increasing to 0.6 % for a flag and a little less for radar reflectors. The drag forces exerted by the wind and the wind driven surface current, respectively, are of the same magnitude when optical targets, except flags, are used. For the other targets the direct wind action dominates.

The forces acting on the carrying cord are usually smaller than the wind and surface current forces. The k-value of the cord is certainly rather high (2.6 m of the cord for the Chalmers drogue has the same value as that of the surface float and subsurface stick) but the square of the relative velocity, $(u-u_0)^2$, is usually much smaller than the corresponding value for the surface current. In estuaries and other density stratified waters, where multi-layered currents are a regular feature, the drag on the cord must, however, be carefully taken into account.

COMPUTER PROGRAM

The processing of the field data includes several tedious steps, especially the correction calculations. This procedure lends itself well to be done on a computer and a program written in FORTRAN IV has been designed for this purpose with all the calculations placed in subroutines. The main program comprises input and output procedures and directs the progress of the evaluation. This arrangement makes the program very clear and flexible, and it can easily be provided with other types of evaluations than available in the present version.

The evaluation is based on the following successive steps:

- Calculation of observed drogue positions from measured angles, distances, bearings, etc. Subroutines are available for several measuring systems as radar, bearing compass, and theodolite.
- Calculation of uncorrected velocity vectors from successive drogue positions and observational times.
- Calculation of corrected velocity vectors and drogue paths by adding corrected vectors. The program chooses as input the closest wind observation and the most representative current observation at each previously calculated depth.
- Calculation of the flow through a given cross section (cross sectional areas representative of each drogue must be given).

- Listing of all input parameters, observation times, drogue positions, and uncorrected and corrected velocity vectors. Corrected drogue paths are also listed but must, however, normally be interpreted with some caution since in a strict sense they are only valid in a homogenous flow field.
- Plotting of measured drogue paths and corrected velocity vectors (see Fig.2). To ensure a clear presentation one has to consider adequate choices of scales, possible division of each campaign into two or more drawings with respect to measuring depths etc. In fact, the presentation may also influence the performance of the measurements e.g. the pattern in which the drogues are launched.

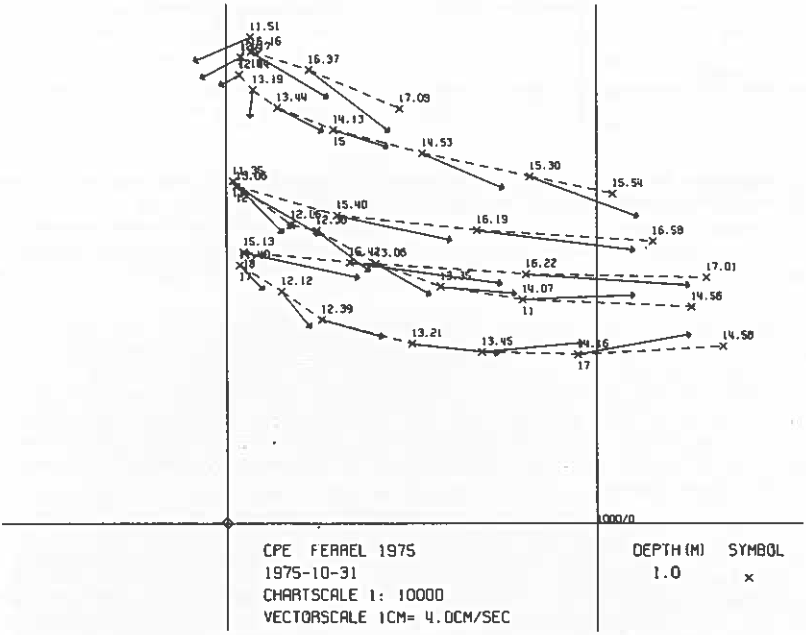

Fig. 2 Example of plotted drogue paths. The shoreline has been added separately.
Exemple des trajets de drogues dessinés.

Except for basic calculations of drogue positions, uncorrected velocity vectors, and listing of input parameters, one can freely choose among available evaluations. Preparations have been made for addition of further program blocks such as hourly mean values of the current during the tidal cycle.

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