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AN ANALYSIS OF PARAMETERS IN A KINEMATIC WAVE MODEL OF OVERLAND FLOW IN URBAN AREAS

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ABSTRACT

A numerical model based on the kinematic wave approximation is discussed and dimensionless hydrographs computed by the model and the shallow water equations are compared. An example of simulation of runoff from a surface using the numerical model is given.
It was found that:

a) the kinematic wave approximation performs well in simulating urban surface runoff
b) the numerical solution is sensitive to space step which must be choosen small.

INTRODUCTION

A general description of runoff is given by the shallow water equations, which are valid for surface flow, gutter flow and the flow in the sewer system. The shallow water equations are two partial differential equations derived from the laws of conservation of mass and momentum. The equations have no general analytical solution and must be solved by numerical methods. These methods are quite complex and need a lot of computer time. This fact, in combination with the limited accuracy in input data such as rainfall intensity, slope and surface area makes it desirable to find simplified computation methods.

One simplified approach is the kinematic wave approximation which has been used successfully for both surface runoff and sewer routing. The purpose of this paper is to:

- discuss the kinematic wave approach

- present a simplified numerical method of solution based on this approach
- discuss the parameters involved.

THE KINEMATIC WAVE APPROXIMATION

Basic equations

The shallow water equations can be derived from the laws of conservation of mass and conservation of momentum (or energy), which are given in many standard texts, for instance Eagleson

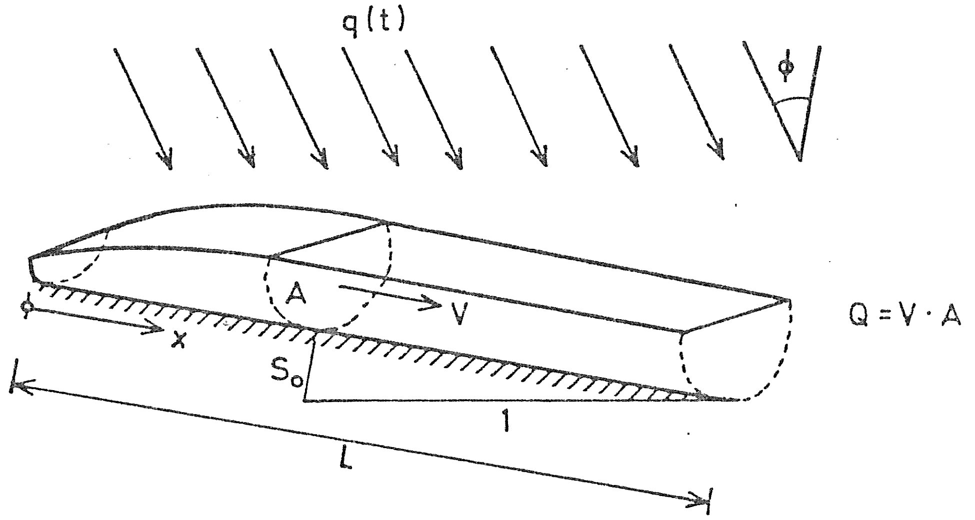

Figure 1. Surface runoff - arbitrary, prismatic cross section.

(1970). With notations according to Fig. 1 the equations can be written:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \qquad \text{(continuity)} \qquad (1)$$

$$\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{\partial Q}{\partial t} - gA\left(S_o - S_f - \frac{\partial Y}{\partial x}\right) - q \cdot U \cdot \cos \emptyset = 0,$$

$$\text{(equation of motion)} \qquad (2)$$

where Q = flow rate

A = cross sectional flow area

q = lateral inflow (rain or flow from upstream surface)

$\emptyset$ = angle between directions of lateral inflow and main flow

U = velocity of lateral inflow
Y = water depth
S_o = slope in flow direction
S_f = friction slope
x = coordinate in flow direction
t = time

The friction slope can be expressed in the general form $S_f = C_1 \cdot Q^{C_2} \cdot Y^{C_3}$ and may involve friction losses due to bottom friction, impact of rain falling on the water table and wind shear stress.

The equations can be rendered dimensionless by the substitutions

$A^* = A/A_o$
$Q^* = Q/Q_o = Q/q_o \cdot L$
$Y^* = Y/Y_o$ (water depth)
$q^* = q/q_o$
$V^* = V/V_o$ (average flow velocity)
$U^* = U/U_o$
$x^* = x/L$ (L = length of plain)
$t^* = t \cdot V_o/L$

where index o refers to a choosen suitable flow situation, for instance normal flow at a lateral inflow of q_o and * denotes a dimensionless variable. The equations (1) and (2) become:

$$\frac{\partial Q^*}{\partial x^*} + \frac{\partial A^*}{\partial t^*} = q^* \quad (3)$$

$$\frac{\partial}{\partial x^*}\left(\frac{Q^{*2}}{A^*}\right) + \frac{\partial Q^*}{\partial t^*} - K_o \cdot A^* \cdot \left(1 - \frac{S_f}{S_o}\right) + \frac{1}{F_o^2} A^* \cdot \frac{\partial Y^*}{\partial x^*}$$

$$- M_o \cdot U^* \cdot q^* = 0 \quad (4)$$

where $F_o = \frac{V_o}{(Y_o \cdot g)} 1/2$ (the Froude number)

$K_o = \frac{L \cdot S_o}{Y_o \cdot F_o^2}$ (the kinematic flow number)

$M_o = U_o \cdot \cos \emptyset / V_o$ (reflects the contribution to momentum of lateral inflow)

The equation of motion is governed by three dimensionless parameters K_o, M_o and $1/F_o^2$. If the kinematic flow number K_o is great compared with the other parameters, all terms but one can be neglected, and the equation of motion is reduced to $(1-S_f/S_o) = 0$. Thus the friction slope equals the bottom slope in the flow direction, and for each flow section there is a unique relation between flow rate and flow area. This is the definition of the kinematic wave approximation.

For ordinary urban surfaces and storm situations, there will be a great difference in magnitude between the parameters K_o, M_o and $1/F_o^2$. For instance, runoff from a paved urban surface with $S_o > 0.01$ and $L > 3$ m has $K_o > 20$ and $1/F_o^2 < 2$ (Manning's formula is used). A surface with $S_o > 0.03$ and $L > 20$ m has $K_o > 550$ and $1/F_o^2 < 1$.

The lateral inflow parameter M_o will be significant compared with K_o only if strong wind causes the angle $\emptyset$ to be great, resulting in values of M_o that could exceed 20.

In gutters having ordinary slopes and shapes, the corresponding values of K_o, $1/F_o^2$ and M_o are slightly lower but K_o still dominates.

Discussion of dimensionless rising hydrographs

Woolhiser (1967) analysed the rising hydrograph from a surface in the case of uniform rain intensity. The dimensionless shallow water equations were solved by the method of characteristics ('exact' solution (Abbot, 1975)) assuming that $\emptyset = 90^o$ and using the Chezy relation for the friction slope. Dimensionless rising hydrographs with different values of K_o and F_o were then compared with the solution obtained by the kinematic approximation (Fig. 2 and 3).

Fig. 2 shows how the 'exact' solution rapidly approaches the kinematic approximation for increasing values of K_o. Woolhiser found that the error introduced using the kinematic approximation was approximately 10% when $K_o = 10$ and stated that the approximation is very good for $K_o > 20$. If the Chezy relation is used, F_o and K_o can be expressed:

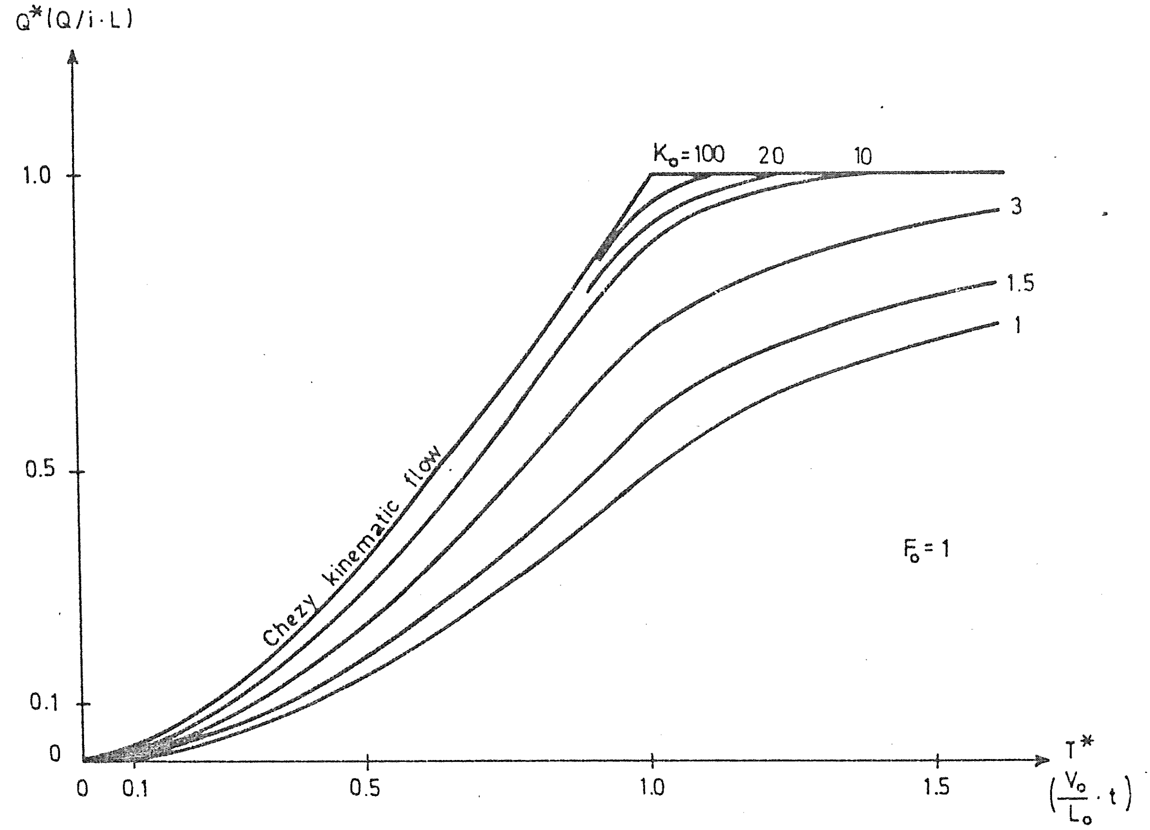

Figure 2. Dimensionless rising hydrographs - variation in K_o if F_o = 1. (After Woolhiser).

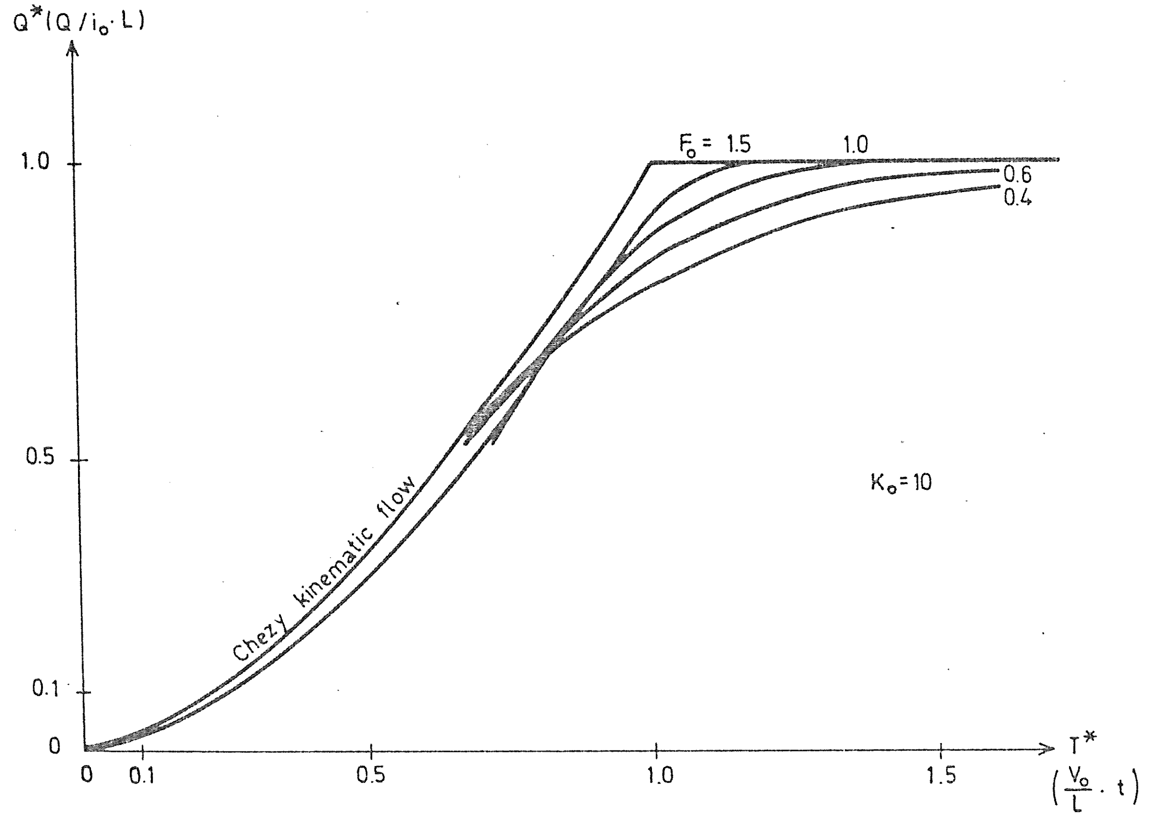

Figure 3. Dimensionless rising hydrographs - variation in F_o if K_o = 10. (After Woolhiser).

$$F_o = C \cdot \sqrt{\frac{S_o}{g}} \tag{5}$$

$$K_o = \frac{(L \cdot S_o)^{1/3}}{(i_o \cdot C^2)^{2/3}} \quad , \tag{6}$$

where C = the Chezy roughness factor and
i_o = the rain intensity.

It is evident from (5) and (6) that the error introduced using the kinematic approximation will decrease when:

length L	increases
slope S_o	increases
roughness $1/C$	increases
rain intensity i_o	decreases

If an urban surface is steep so called roll waves may occur which completely change the flow characteristics (Yen et al., 1977). This will happen when the Froude number exceeds 1.5 to 2.0 (slopes greater than approximately 0.05). Under these conditions, the shallow water equations are no longer valid, and the numerical solution of the equations may become instable. Consequently, Woolhiser has used Froude numbers less than 1.5 (Fig. 3).

Though it is possible to use a method of solution where all terms in the shallow water equations are considered, there are several reasons (some mentioned in the introduction) for using a simplified approach for surface runoff. The most important advantage of using the kinematic approximation is its ability ot simulate the mean velocity of a real wave. In other words the celerity of a wave moving downstream on a surface, gutter or sewer is very close to the kinematic wave velocity. This ability is very important in simulating runoff from urban areas during storms of short duration.

The major difficulty of the kinematic approach, despite its simplicity, is to find a suitable numerical method of solution (discussed in the next section).

SIMPLIFIED NUMERICAL SOLUTION

The kinematic wave equations are usually given by (for example Eagleson, 1970):

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \tag{7}$$

$$Q = a\ (A)^b, \tag{8}$$

where Q = flow rate
A = flow area
q = lateral inflow (rain included)
a,b = parameters

a and b can be regarded as constants empirically chosen or calculated using a friction relation such as Manning's or Chezy's formulas. Effects of roughness due to impact of rain on the water table, wind shear stress and also change in flow regime can easily be included in the numerical solution of the kinematic approximation. That is if a and b are functions of rain intensity, wind and Reynolds number. In the first part of this paper the numerical model described below is compared with the complete shallow water equations using dimensionless analysis and uniform inflow. Here the additional friction factors can be neglected and any friction law used. The Chezy relation has been chosen here, only to make comparisons with Woolhiser's dimensionless hydrographs possible. In the last section, a friction relation that considers the effects of both rain drops and flow regime is used in the numerical model.

The attenuation of a wave moving over a surface is in the shallow water equations reflected by the dynamic terms. In the kinematic wave approximation, these are neglected, and Eqs. (7) and (8) represent a wave movement with no attenuation. The $\partial Q/\partial x$ and $\partial A/\partial t$ can be approximated in terms of finite differences (box scheme):

$$\frac{\partial Q}{\partial x} = \left(\frac{Q_{j+1}^{m+1} + Q_{j+1}^{m}}{2} - \frac{Q_{j}^{m+1} + Q_{j}^{m}}{2}\right)/\Delta x \tag{9}$$

$$\frac{\partial A}{\partial t} = \left(\frac{A_{j+1}^{m+1} + A_{j}^{m+1}}{2} - \frac{A_{j+1}^{m} + A_{j}^{m}}{2}\right)/\Delta t, \tag{10}$$

where j and j+1 refer to the inlet and outlet sections of the finite volume, m and m+1 to time t and t+Δt respectively. The lateral inflow $\Delta x \cdot q = \Delta x \cdot q^{m+1}/2 + \Delta x \cdot q^{m}/2$ can be included in Q_j^{m+1} and Q_j^m respectively.

If the continuity equation (7) is approximated by means of (9) and (10) a difference scheme is obtained which may give uncontrolable parasitic oscillations in the solution. This is probably the reason why the difference scheme has not been used in any known runoff model (Brakensiek, 1967, Sjöberg, 1976).

If (10) is replaced by:

$$\partial A/\partial t = (A_n^{m+1} - A_n^m)/\Delta t \qquad (11)$$

where A_n^m is the stored water volume at time step m per unit length in flow direction, the difference solution of Eqs. (7) and (8) is:

$$\frac{Q_{j+1}^{m+1} + Q_{j+1}^{m} - Q_j^{m+1} - Q_j^m}{2\Delta x} + \frac{A_n^{m+1} - A_n^m}{\Delta t} = 0 \qquad (12)$$

$$A_n = a^{-1/b} \cdot (Q_{j+1})^{1/b} \qquad (13)$$

This simplified difference equation will result in an increased truncation error compared with Eqs. (9) and (10) and will give numerically stable solutions.

It should be noted that if $\Delta x = L$ the numerical model is equivalent to a non-linear reservoir model.

It can be shown by Taylor series expansion that the difference equation (12) is a better approximation of the equation:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} + \frac{\Delta x}{2} \cdot \frac{\partial^2 A}{\partial x \partial t} = q \qquad (14)$$

than the continuity equation (7). The third (diffusive) term will cause the numerical attenuation discussed below.

If Eq. (8) is inserted in (14) the dimensionless expression will be:

$$\frac{\partial Q^*}{\partial x^*} + \frac{\partial (Q^*)^{1/b}}{\partial t^*} + \frac{1}{2} \cdot \frac{\Delta x}{L} \cdot \frac{\partial^2 (Q^*)^{1/b}}{\partial x^* \partial t^*} = q^* \qquad (15)$$

When $\Delta x/L \to 0$ the third term will disappear and we get the kinematic wave solution shown in Fig. 2 and 3. In Fig. 4 dimensionless rising hydrographs computed by equations (12) and (13) are compared with the corresponding kinematic hydrograph.

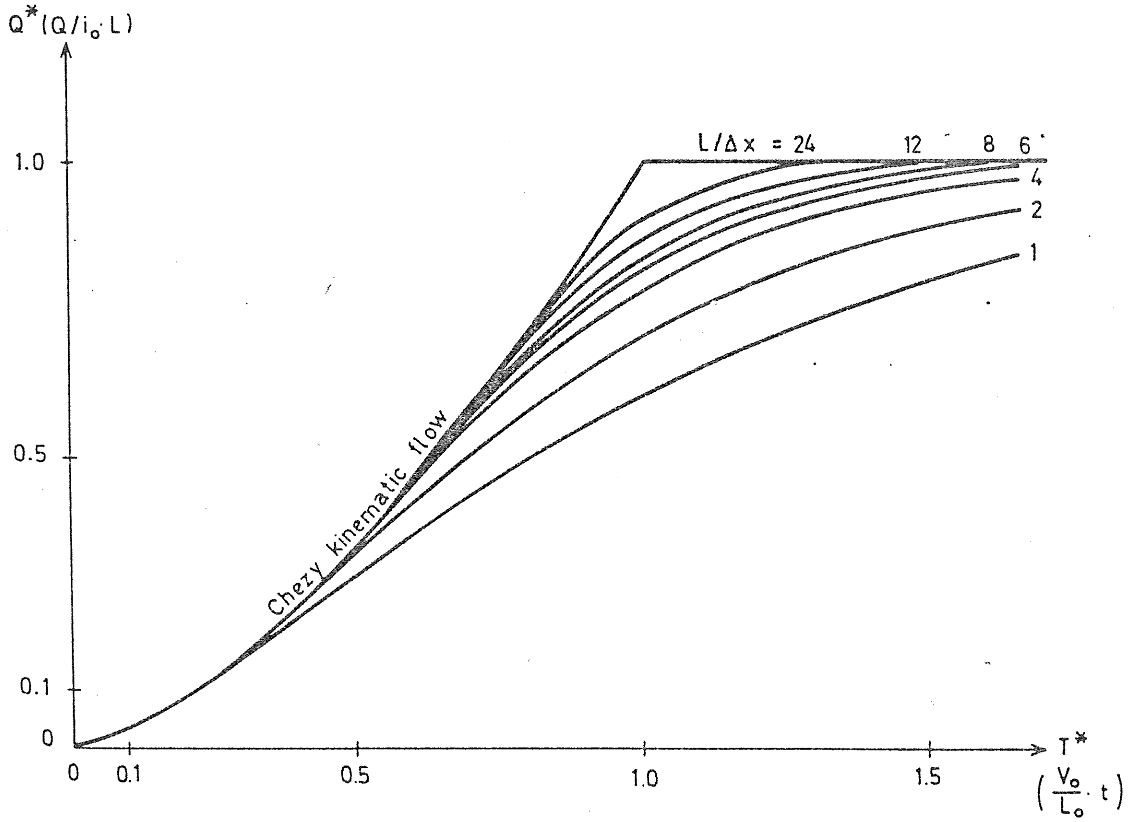

Fig. 4 Dimensionless rising hydrographs computed by equations (12) and (13).

All input parameters were varied and it was found that only $\Delta x/L$ had any significant influence on the shape of the hydrographs. This could, of course, have been concluded directly from Eq. (15). We can see in Fig. 2 and 4 that the parameters $\Delta x/L$ and K_o have almost the same influence on the hydrographs. In principle, it would be possible to choose a suitable $\Delta x/L$ for each value of K_o and obtain a wave shape very similar to that computed by the complete shallow water equations. Referring to the discussion in the previous section about the magnitude of K_o, $\Delta x/L$ should be decreased when length, slope and roughness increases and rain intensity decreases. Fig. 2 and 4 indicates that the use of the proposed numerical model on ordinary urban surfaces requires $\Delta x/L < 1/24$.

The numerical model has also been used to simulate dimensionless hydrographs using Manning's formula and the laminar friction law. These hydrographs were compared with experimental hydrographs presented by Morgali (1970). For both the Manning-turbulent and the laminar case it was necessary to use $\Delta x/L \leq 1/8$ to achieve similarity between simulated and experimental hydrographs. The difference between turbulent and laminar hydrographs was quite pronounced.

So far we have only discussed uniform rain intensity. In the case of variable rain intensity, it seems reasonable to assume that the gradient of the rain intensity should affect the choise of $\Delta x/L$. It should also be noted that the assumption of sheet flow in the analyse may give an overestimation of K_o compared to "real" values. It is evident that the choise of $\Delta x/L$ must be further investigated in simulation of runoff from real surfaces and storms.

RUNOFF SIMULATIONS

The numerical solution of the kinematic wave equations presented in previous sections has been used in simulating runoff from a small composit urban catchment described in Fig. 5. A heavy storm of short duration (Fig. 6) was simulated using Manning's and Darcy-Weisbach's relations for friction losses. The friction factor f in Darcy-Weisbach's formula was related to rainfall intensity and flow regime according to Li et al (1975):

$$f^{lam} = \frac{K_1 + K_2 \cdot i}{R_e} \quad \text{when } R_e < R_e^l \qquad (16)$$

$$f^{turb} = \frac{K_3}{R_e^{1/4}} \quad \text{when } R_e < R_e^t , \qquad (17)$$

where i = rain intensity

K_1, K_2, K_3 = constants

R_e = Reynolds number

R_e^l, R_e^t = lower and upper limits of the transition zone.

The model was run using different values of R_e^l and R_e^t. It was

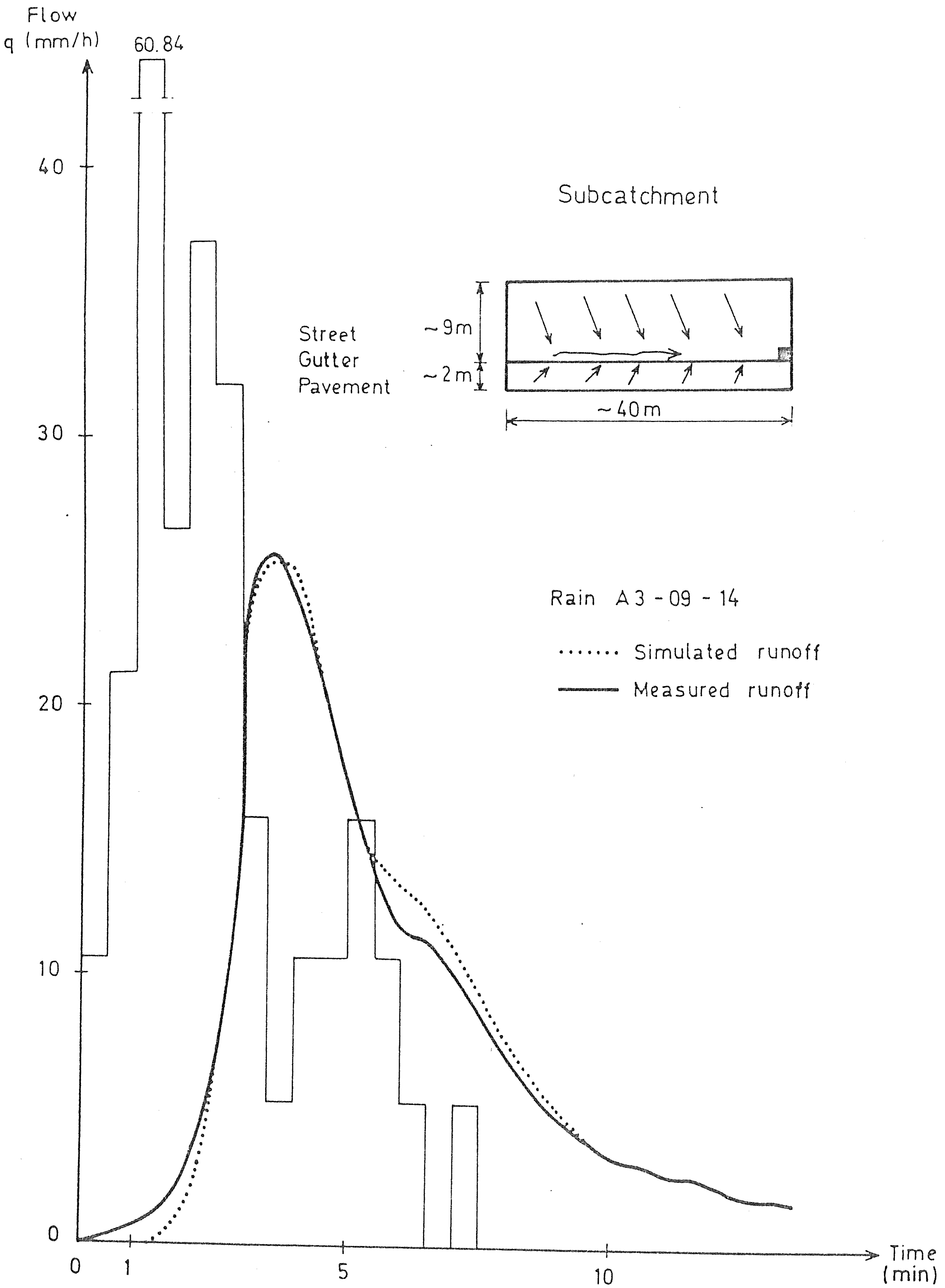

Figure 5. Simulated runoff from a street,pavement and gutter (average slope 0.045).

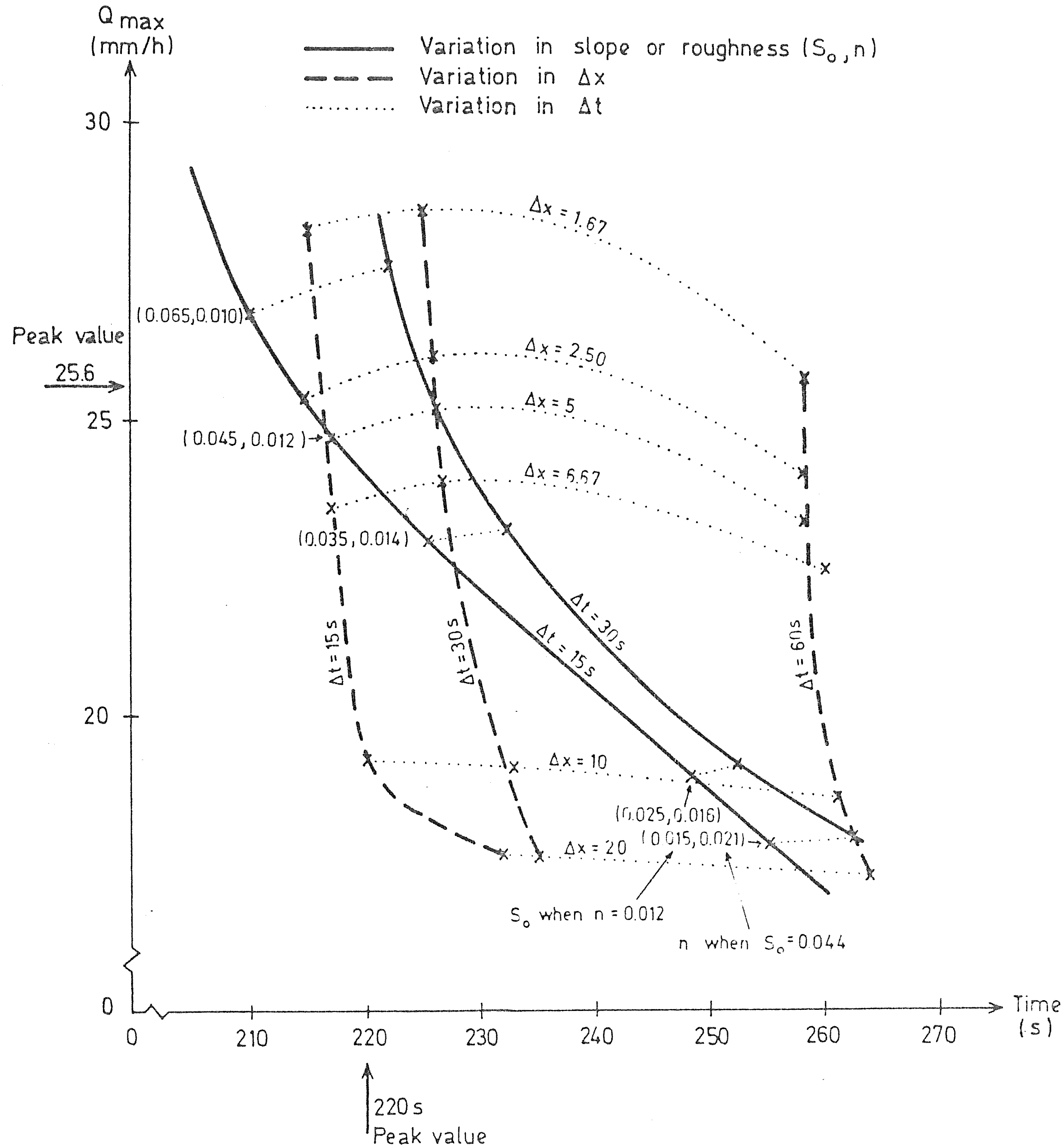

Figure 6. Effects on the peak value due to variations in Δx, Δt, S_o and n.

found that no part of the hydrograph was properly simulated with the laminar flow friction factor (16). Very good and almost identical results were achieved using Darcy-Weisbach's relation with the turbulent friction factor (17), Chezy's- or Manning's formula. The hydrograph in Fig. 5 is simulated using equation (17) and $\Delta x/L = 1/8$.

No general conslusions could, of course, be made from only one storm and one subcatchment, but there is an indication that the transition from laminar to turbulent flow will take place for substantially lower values of Reynold's number than reported in literature, (Woolhiser 1975). This might be an explanation to the successful use by several investigators of Manning's formula in surface routing (Arnell and Lyngfelt 1975, Brandstetter 1976). If the flow becomes turbulent for low values of the Reynold's number, the friction relation may be improved by also relating the turbulent friction factor to rainfall intensity.

To show in a real case the effect of variations in input parameters, hydrographs from the storm in Fig. 5 were simulated with different values of roughness (n), slope (S_o), space step (Δx) and time step (Δt) using Manning's formula. In Fig. 6, the position of each simulated peak flow is indicated by crosses. The variation of one parameter is indicated by lines. Slope and roughness affect the numerical solution in exactly the same way. Variations of these parameters are thus in Fig. 6 represented on the same line. Figures (within brackets) along this line gives (left hand values) variations in slope when $n = 0.012$ or (right hand values) variations in roughness when $S_o = 0.044$. It is evident that:

increasing Δx will attenuate the hydrograph
increasing Δt will delay the hydrograph
decreasing S_o or $1/n$ will both delay and attenuate the hydrograph.

It should be noted that a variation of Δt has almost no direct influence on the attenuation of the flow peak. However when great time steps are used, the rain intensity <u>input</u> will not be properly described resulting in an indirect attenuation (in our case when $\Delta t = 60$ s).

The dimensionless analysis in the previous section showed that the numerical model requires $\Delta x/L \sim 1/8$ to reproduce a proper attenuation. Runoff simulations from the storm in Fig. 5 indicates a similar ratio. For practical purposes it is interesting to examine possibilities of increasing the ratio $\Delta x/L$ and still obtain good results. According to Fig. 6, this can be done if the roughness is decreased, but then we will get a hydrograph that is not properly delayed.

CONCLUSIONS

Simulations of runoff from most urban surfaces are well performed by the kinematic wave approximation of the shallow water equations. The proposed simplified numerical solution will introduce an artificial attenuation, governed by the ratio $\Delta x/L$, that "simulates" the dynamic terms in the full equations. Then if characteristics of the surface and the rain are taken into account in choosing $\Delta x/L$, the numerical solution should perform even better than the kinematic equations. Comparisons with measured hydrographs indicate an overall approximate ratio $\Delta x/L \approx 1/8$.

The friction factor must basically be selected according to the flow regime. But several investigators report excellent results using only a turbulent friction relation, which indicates transition from laminar to turbulent flow at a very early stage of runoff. If this is generally valid for urban surfaces remains to be proved. No turbulent friction relation was found superior to the other.

The friction factor affects both the delay and attenuation of the simulated hydrographs, while the ratio $\Delta x/L$ only affects the attenuation. The results of increasing $\Delta x/L$ and decreasing the friction factor will be too "fast" a hydrograph. If and how small step lengths can be replaced by small friction factors should also be further investigated.

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