

Spatial Diversity Performance of Multiport Antennas in the Presence of a Butler Network

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Abstract—The main concern of this paper is to study the presence of a cascaded Butler network in diversity antenna systems. The advantages of a cascaded Butler network in 2D multipath environments have been already shown. These advantages are limited to either correlated or nonuniform multipath environments with a line-of-sight component. In the frame of this paper, for the first time, we show a further advantage of a cascaded Butler network in 3D multipath environments, including the effects of coupling. This advantage is associated with an improved convergence in received signals' covariance matrix in a multipath environment wherein the number of independent incoming waves is limited, i.e., non-rich multipath environment. For this purpose, we use a simulation tool in which a multipath scenario with certain number of incoming waves is emulated and some selected multiport antennas are exposed to it. Afterwards, the simulated random received signals are used to yield different performance metrics like spatial correlation. This advantage achieves considerable practical interest since a typical actual multipath scenario is most likely a non-rich multipath environment.

Index Terms—Multiport antennas, Butler network, beam-forming, multiport matching efficiency, effective diversity gain, richness threshold.

I. INTRODUCTION

DISCRETE Fourier transformation is a contemporary concern for its advantages in multiple-input and multiple-output (MIMO) wireless communication systems. In practice, a discrete Fourier transformation over spatially received signals can be realized in the RF domain by an ideal Butler network [1]. Firstly, in [1] the authors studied the effects of implementing a Butler network in a MIMO communication system in both correlated and uncorrelated multipath environments. From a communication engineer's point of view, they showed the benefits of a cascaded Butler network in correlated multipath environments. Later, A. Grau *et al.* investigated the matter from an antenna standpoint and highlighted the advantages inherent in using a Butler network [2]–[4].

However, some additional developments of the aforementioned works may become useful. For instance, the multipath model used to simulate their results is in 2D, e.g. a two-ring model located at the transmitter and receiver with the scatterers distributed uniformly in the azimuth plane [2], [3]. Moreover, the developments appear to be limited to cases of negligible coupling among radiating elements. In contrast, we simulate

some proposed antennas in 3D multipath environments including coupling among nearby radiating elements. In addition to these, we analyze the behavior of the proposed multiport antennas in the presence and absence of a Butler network in two distinct multipath environments: (a) a rich multipath environment wherein the received signals at different ports of the antennas have converged joint probability density function and (b) a multipath environment with finite number of incident waves which is, henceforth, referred to as a *non-rich* multipath environment [5].

In general, the advantages of a cascaded Butler network can be viewed from two perspectives. From the first standpoint, an advantage is linked to an improved mean effective gain (MEG). This advantage is likely only realizable in certain non-uniform multipath environments with limited angular spread, regardless of the number of incident waves [4]. On the contrary, in this paper, we reveal a further advantage of a Butler network in diversity antenna systems, which to the best of our knowledge has not been appreciated before. In brief, it is clarified that in general the spatial correlation between the received random signals depends also on the number of incident waves. We reveal that a Butler network facilitates the convergence of the spatial correlation and thus can potentially enhance the overall diversity performance in a non-rich multipath environment.

Throughout this paper we restrict ourselves to uniform multipath environments in order to focus on the effects of a cascaded Butler network solely upon correlation convergence and not MEG enhancement. Even so, the extracted conclusions are general and hold for non-uniform multipath environments too. This study ultimately leads to a fundamental conclusion saying that, as long as the diversity gain is concerned, the shape of the embedded patterns plays more critical role than the element separation in non-rich multipath environments. We stress that our study is limited to random received signals which have converged Rayleigh distributions.

The structure of the paper is as follows. Section II develops a general compact formula to calculate the total embedded efficiencies in the presence of an arbitrary microwave network. The total embedded efficiencies are used to normalize the associated embedded patterns, which play an important role in later sections. A general description of the microwave component under study as well as the simulation approach is presented in Section III. Section IV addresses the effects of Butler network on diversity gain performance in a rich multipath environment. This is followed in Section V applying a similar study in a non-rich multipath circumstance.

To set the notation, letters with an over-bar sign denote the column vectors, whereas the bold letters represent matrices.

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The matrix \mathbf{I} denotes the identity matrix. The dagger sign † stands for Hermitian transpose while the superscript \top for transpose. The incident and reflected wave vectors are normalized such that their square is the associated mean power [6, pp. 182].

II. MULTI-PORT MATCHING EFFICIENCY FORMULATION

In order to normalize the far field patterns used in our simulations we need to know the total embedded element efficiencies. In lossless structures, the total embedded element efficiencies equate the multiport matching efficiencies [7]. Thus, the main goal in this section is to derive a compact formula for multiport matching efficiency in an n -port antenna system in the presence of an arbitrary passive microwave network. Indeed, the details presented here are the base for generalization of multiport matching efficiency in the presence of two or more number of cascaded networks [8]. We start with briefly reviewing the antenna modeling upon which the required formulations are developed.

A. Antenna Modeling: Background

Assume an n -port antenna system characterized by its scattering matrix, \mathbf{S}_{ant} . The antenna's scattering matrix can be divided into four submatrices, as shown in (1) [9], [10]. Submatrix \mathbf{S}_a contains the input reflection coefficients as well as the coupling among different element ports. Regarding the other submatrices, which are all functions of angular direction, \mathbf{S}_b and \mathbf{S}_c link the element ports with the two orthogonal radiation ports, whereas submatrix \mathbf{S}_d describes the structural antenna scattering with the element ports being matched-terminated.

$$\mathbf{S}_{\text{ant}} = \begin{bmatrix} \mathbf{S}_a \ (n \times n) & \mathbf{S}_b \ (n \times 2) \\ \mathbf{S}_c \ (2 \times n) & \mathbf{S}_d \ (2 \times 2) \end{bmatrix}_{(n+2) \times (n+2)} \quad (1)$$

To construct a general formula for the multiport matching efficiency, we are solely concerned about the input ports' scattering matrix, \mathbf{S}_a . In a similar way, the middle microwave network in Fig. 1 is characterized by its S-matrix, \mathbf{S}_m . The sub-blocks of this matrix are identified as

$$\mathbf{S}_m = \begin{bmatrix} \mathbf{S}_{11} \ (n \times n) & \mathbf{S}_{12} \ (n \times n) \\ \mathbf{S}_{21} \ (n \times n) & \mathbf{S}_{22} \ (n \times n) \end{bmatrix}_{(2n \times 2n)} \quad (2)$$

B. Derivation of Multiport Matching Efficiencies

Multiport matching efficiency is the ratio between the accepted power by a multiport antenna, P_{acc} , and the maximum available power from the source exciting it, P_{avs} [7]. The primary target is to derive both of them in terms of the source voltages. The necessary parameters in the following derivations are shown in Fig. 1. First, we need to derive an expression for the accepted power. To this end, let us start by expressing the incident waves at the input of the microwave network, \bar{a}_m , in terms of the sources' normalized voltage vector, \bar{a}_s . Using the voltage division rule in the fundamental circuit theorem, we can write

$$\bar{a}_m = (\mathbf{I} + \mathbf{\Gamma}_1)^{-1} \mathbf{Q}_m^{-1} \bar{a}_s = \mathbf{T}_s \bar{a}_s \quad (3)$$

wherein \mathbf{Q}_m is defined as

$$\mathbf{Q}_m = [(\mathbf{I} + \mathbf{\Gamma}_s)(\mathbf{I} - \mathbf{\Gamma}_s)^{-1}(\mathbf{I} - \mathbf{\Gamma}_1)(\mathbf{I} + \mathbf{\Gamma}_1)^{-1} + \mathbf{I}] \quad (4)$$

with

$$\mathbf{\Gamma}_s = (\mathbf{Z}_s + \mathbf{Z}_o)^{-1}(\mathbf{Z}_s - \mathbf{Z}_o) \quad (5)$$

in which \mathbf{Z}_o is an $n \times n$ diagonal matrix of the characteristic impedance of the system. The matrix \mathbf{T}_s can best be called the *source transmission matrix*. Moreover, the reflection coefficient matrices at both ends of the microwave network are obtained by

$$\begin{aligned} \mathbf{\Gamma}_1 &= \mathbf{S}_{11} + \mathbf{S}_{12}\mathbf{S}_a(\mathbf{I} - \mathbf{S}_{22}\mathbf{S}_a)^{-1}\mathbf{S}_{21} \\ \mathbf{\Gamma}_2 &= \mathbf{S}_{22} + \mathbf{S}_{21}\mathbf{\Gamma}_s(\mathbf{I} - \mathbf{S}_{11}\mathbf{\Gamma}_s)^{-1}\mathbf{S}_{12} \end{aligned} \quad (6)$$

Now, by virtue of (3), the input power to the microwave network, denoted by P_m , is achieved by

$$\begin{aligned} P_m &= \bar{a}_m^\dagger \bar{a}_m - \bar{b}_m^\dagger \bar{b}_m = \bar{a}_m^\dagger (\mathbf{I} - \mathbf{\Gamma}_1^\dagger \mathbf{\Gamma}_1) \bar{a}_m \\ &= \bar{a}_s^\dagger \mathbf{T}_s^\dagger (\mathbf{I} - \mathbf{\Gamma}_1^\dagger \mathbf{\Gamma}_1) \mathbf{T}_s \bar{a}_s \end{aligned} \quad (7)$$

On the other hand, the incident waves at the antenna ports are shown to be

$$\bar{a}_a = (\mathbf{I} - \mathbf{S}_{22}\mathbf{S}_a)^{-1}\mathbf{S}_{21} \bar{a}_m \quad (8)$$

Therefore, the accepted power can be written as

$$\begin{aligned} P_{\text{acc}} &= \bar{a}_a^\dagger (\mathbf{I} - \mathbf{S}_a^\dagger \mathbf{S}_a) \bar{a}_a \\ &= \bar{a}_s^\dagger \mathbf{T}_t^\dagger (\mathbf{I} - \mathbf{S}_a^\dagger \mathbf{S}_a) \mathbf{T}_t \bar{a}_s \end{aligned} \quad (9)$$

wherein, by use of (8) and (3), the total transmission coefficient of the cascaded system, \mathbf{T}_t , is defined as

$$\mathbf{T}_t = (\mathbf{I} - \mathbf{S}_{22}\mathbf{S}_a)^{-1}\mathbf{S}_{21} (\mathbf{I} + \mathbf{\Gamma}_1)^{-1} \mathbf{Q}_m^{-1} \quad (10)$$

To achieve the multiport matching efficiency, we also need to obtain the maximum available power from the source. For this purpose, we recall that the accepted power in (7) equals the maximum available power as long as $\mathbf{\Gamma}_1 = \mathbf{\Gamma}_s^\dagger$. That is

$$\begin{aligned} P_{\text{avs}} &= P_m \Big|_{\mathbf{\Gamma}_1 = \mathbf{\Gamma}_s^\dagger} \\ &= \bar{a}_s^\dagger \mathbf{T}_m^\dagger (\mathbf{I} - \mathbf{\Gamma}_s \mathbf{\Gamma}_s^\dagger) \mathbf{T}_m \bar{a}_s \end{aligned} \quad (11)$$

where

$$\mathbf{T}_m = [(\mathbf{I} + \mathbf{\Gamma}_s)(\mathbf{I} - \mathbf{\Gamma}_s)^{-1}(\mathbf{I} - \mathbf{\Gamma}_s^\dagger) + (\mathbf{I} + \mathbf{\Gamma}_s^\dagger)]^{-1} \quad (12)$$

Having obtained all the required parameters, now we can write an expression for the multiport matching efficiency. For a certain excitation scheme, \bar{a}_s , the multiport matching efficiency becomes

$$e_{\text{mp}} \Big|_{\bar{a}_s} = \frac{\bar{a}_s^\dagger \mathbf{T}_t^\dagger (\mathbf{I} - \mathbf{S}_a^\dagger \mathbf{S}_a) \mathbf{T}_t \bar{a}_s}{\bar{a}_s^\dagger \mathbf{T}_m^\dagger (\mathbf{I} - \mathbf{\Gamma}_s \mathbf{\Gamma}_s^\dagger) \mathbf{T}_m \bar{a}_s} \quad (13)$$

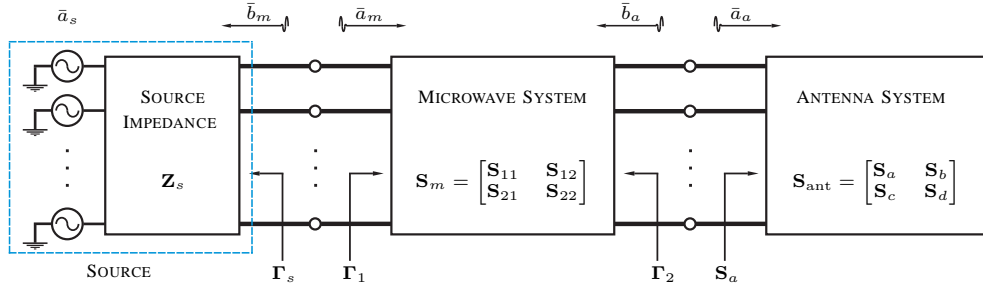


Fig. 1. Microwave circuit model of a multiport antenna system connected to a set of sources through an arbitrary microwave network.

with \mathbf{T}_t and \mathbf{T}_m defined in (10) and (12), respectively. The definition given above is quite general and can be applied to any arbitrary excitation scheme, \bar{a}_s . Since the diversity characterization of multiport antennas in receive mode is our main concern, the excitation vector of a single non-zero entry is of particular interest.

As an example, assume we wish to design a matching network for maximizing the multiport matching efficiency, e_{mp} . For this purpose, the ultimate goal is to simultaneously achieve $\Gamma_1 = \Gamma_s^\dagger$ and $\Gamma_2 = \mathbf{S}_a^\dagger$. Note that, under these constraints, the S_{11} and S_{22} in (6) have to be determined such that the associated expressions hold.

III. SIMULATION DESCRIPTION

The goal in this section is to describe and motivate different presumptions in the upcoming simulations upon which we carry out our study. In the first part, the configurations of two multiport antennas, which are chosen arbitrarily, are depicted. In the second subsection, the characteristics of an ideal Butler network are highlighted. Later, we briefly explain the way we obtain different desired parameters from the derived formulas in Section II. A review of the approach used to calculate the received random signals and the associated metrics finalizes this section.

A. Multiport Antennas under Study

In this study, we arbitrarily choose two different four-port antennas. Firstly, a case of four parallel equidistant monopoles (4MP) above a perfect electric conductor (PEC) plane is investigated. The schematic of this structure is depicted in Fig. 2 in which an arbitrary element separation, d , is chosen. The resonance frequency of these identical elements in isolation state is slightly more than $f_0 \approx 1$ GHz ($\lambda_0 \approx 0.3$ m).

The second structure under study is four equidistant horizontal parallel dipoles (4HD) at $h = 0.15\lambda_0$ height above a PEC plane. The resonance frequency of these similar dipoles while isolated is around that of the preceding counterpart structure. The 4HD radiating elements present higher directivity (e.g., around 7–9 dBi dependent on the element separation and that whether the elements are internal or lateral) in comparison with 4MP.

All simulations are performed using a method of moment full-wave simulator. Note that, although in these figures the PEC plane is shown to be finite, it extends to infinity in our simulations.

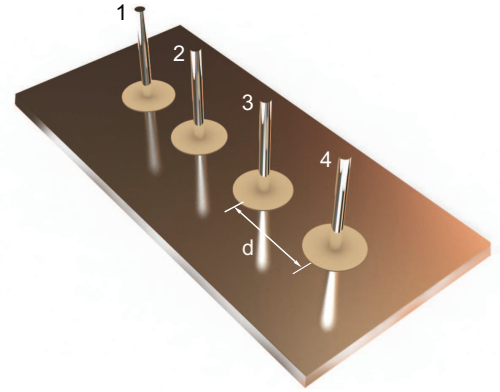


Fig. 2. Four equidistant $\lambda_0/4$ monopoles (4MP) above a PEC plane.

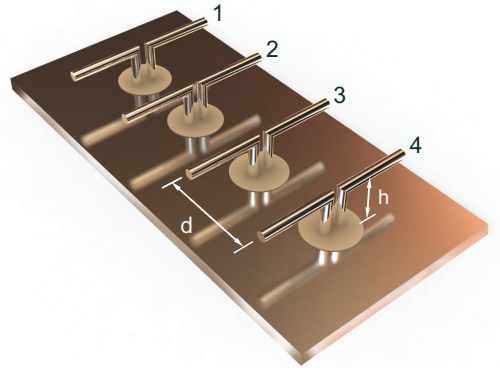


Fig. 3. Four equidistant $\lambda_0/2$ dipoles (4HD) above a PEC plane.

B. Ideal Butler Network

To focus solely on the effects of beam-forming upon the diversity gain performance, we presume an ideal Butler network throughout this paper. The Butler network is a lossless, reciprocal and transparent network (i.e., it is matched at both ends) within the desired bandwidth. Being matched at both ends means that the submatrices \mathbf{S}_{11} and \mathbf{S}_{22} in \mathbf{S}_m (Equation (2)) are both zero matrices. The S-matrix of this network is [11]

$$\mathbf{S}_m = \begin{bmatrix} \mathbf{0} & \mathbf{T} \\ \mathbf{T}^\top & \mathbf{0} \end{bmatrix}_{8 \times 8} \quad (14)$$

wherein

$$\mathbf{T} = \frac{1}{2} \begin{bmatrix} \exp(j\frac{\pi}{4}) & -1 & \exp(-j\frac{\pi}{4}) & j \\ 1 & \exp(j\frac{\pi}{4}) & j & \exp(j\frac{3\pi}{4}) \\ \exp(j\frac{3\pi}{4}) & j & \exp(j\frac{\pi}{4}) & 1 \\ j & \exp(-j\frac{\pi}{4}) & -1 & \exp(j\frac{\pi}{4}) \end{bmatrix}$$

For simplicity, we set the following terminology: In contrast to the ports at the antenna system, which are called the element ports, the ports at the input of the cascaded Butler network (at its left hand side in Fig. 1) are called the *beam ports*. Similar to the element ports, the beam ports are also numbered from left to right.

C. Total Embedded Efficiencies

Furthermore, to focus on the effects of beam-forming, we also need to abandon the influence of non-matched source impedances, i.e., we require $\mathbf{Z}_s = \mathbf{Z}_o$. In order to calculate the total embedded efficiency, one has to use a single non-zero entry excitation vector in (13) rendering the corresponding multiport matching efficiency. As a reminder, since the structure is assumed lossless, the multiport matching efficiencies equal the total embedded element efficiencies. To obtain these efficiencies at the element ports, it is sufficient to replace \mathbf{S}_{12} and \mathbf{S}_{21} in (2) by \mathbf{I} , and the associated \mathbf{S}_{11} and \mathbf{S}_{22} by $\mathbf{0}$, and consequently substitute the resultant into (13). After some simplifications, the desired parameters at the element ports become the diagonal entries in

$$\mathbf{e}_{\text{tot}} = \text{diag}(\mathbf{I} - \mathbf{S}_a^\dagger \mathbf{S}_a) \quad (\text{element ports}) \quad (15)$$

In a similar way, the total embedded efficiencies at the beam ports are achieved. Inserting the corresponding terms given in (14) into (13) and after some algebra, the required efficiencies at the beam ports become the diagonal entries in

$$\mathbf{e}_{\text{tot}} = \text{diag}(\mathbf{I} - \mathbf{\Gamma}_1^\dagger \mathbf{\Gamma}_1) \quad (\text{beam ports}) \quad (16)$$

where

$$\mathbf{\Gamma}_1 = \mathbf{T} \mathbf{S}_a \mathbf{T}^\top$$

Using the above expression, the total embedded efficiencies of 4MP antenna versus frequency achieved from (15) and (16) are plotted in Fig. 4. These results belong to the case of $d = 0.2\lambda_o$. Note that at this element separation, the neighboring elements present a fair amount of coupling, which is the source of their deviations from those in an isolated state. Although the efficiencies at the element and beam ports differ significantly, the arithmetic mean efficiencies in both cases are the same.

D. Spatial Correlation

Recall that we have restricted ourselves to lossless single-mode radiation structures. This restriction allows us to most conveniently calculate the spatial correlation in a uniform multipath environment at the element ports through the input network parameters [12]. To obtain the corresponding correlations at the beam ports, we use the expressions in [13, Equations 20 & 31]. One can show that the equations in (15) and (16) also yield similar results. Note that these correlations are credible in *rich* isotropic Rayleigh multipath environments

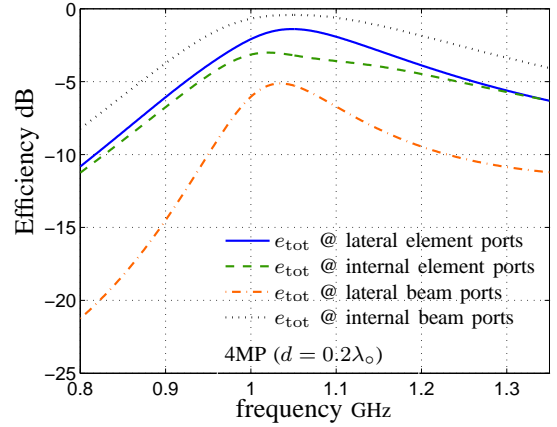


Fig. 4. Total embedded efficiencies versus frequency at element ports and beam ports for four collinear monopoles above a PEC plane.

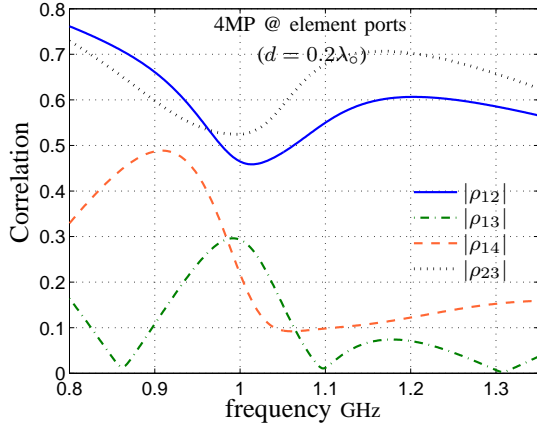
wherein the term *rich* shall soon be clarified. In this paper, we study the square root of received powers' correlation and refer to it simply as *correlation*. It is known that the latter metric approximately equals the absolute value of complex correlation between the received voltage signals in a *rich* Rayleigh multipath environment [14], [15, Chapter 2].

As an example, Fig. 5 illustrates the correlations versus frequency for the 4MP antenna with $d = 0.2\lambda_o$. In this figure, the correlations at the element ports are denoted by ρ , whereas those at the beam ports are shown by ϱ . In addition to discrepancies in the total embedded efficiencies as seen before, the differences between the corresponding correlations at the element and beam ports are noticeable.

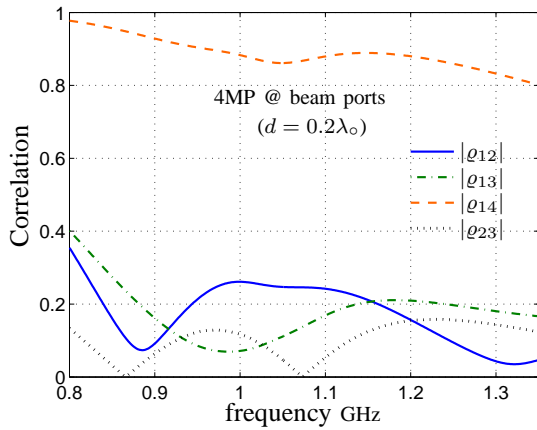
E. Effective Diversity Gain

In a multiport diversity antenna system used in a multipath environment, different performance metrics are derived from random received signals at its ports. In particular, the effective diversity gain is calculated through the cumulative distribution function (CDF) of the received random power signals [16]. Therefore, throughout this section, we are concerned about the simulation of received random power vectors. For this purpose, we use a classic approach in which a random multipath is emulated and then the embedded patterns of antennas are exposed to it. Consequently, the random received power signals are calculated, which are accordingly used to yield different metrics such as the effective diversity gain being henceforth referred to as *diversity gain*. The method has been used in [17] and later generalized and further detailed in [15, Chapter 4], [5]. Therefore, we avoid detailing it again and refer the interested readers to these references. Nevertheless, there are a few points that need to be cleared.

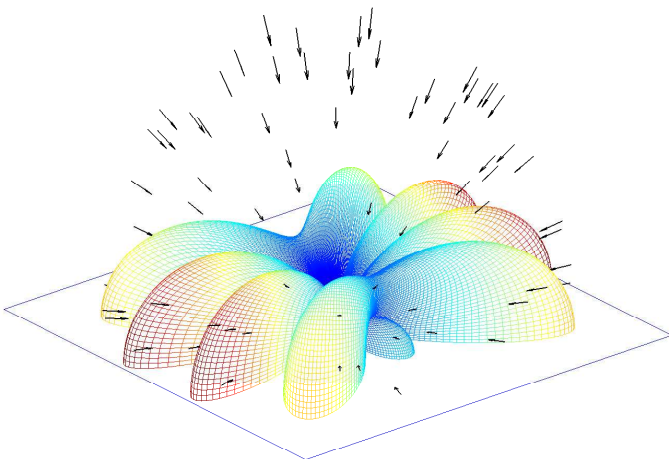
Concerning the angle of arrival (AoA) distribution, since a moving multiport antenna can have any arbitrary orientation in space, the effective distribution of AoA can *on average* be presumably uniform. The same kind of reasoning is applied equally to the effective polarization of the incoming waves. Thus, a uniform multipath environment of balanced polarization (i.e., isotropic multipath) stands as a generic



(a) Correlations at the element ports.



(b) Correlations at the beam ports.

 Fig. 5. Correlations versus frequency for four monopoles above a PEC plane with $d = 0.2\lambda_0$.

 Fig. 6. A sample realization in which embedded far field functions of four monopoles (4MP) above a PEC ($d = 0.5\lambda_0$) with a cascaded Butler network are exposed to 80 random incoming waves with uniform AoA within $\theta \in [0, \pi/2]$ and $\psi \in [0, 2\pi]$.

representation of the actual multipath scenarios being retained for all simulations conducted in this paper. More importantly,

the choice of uniform AoA will also nullify the impact of MEG variations caused by a cascaded Butler network. This serves our purpose best in the frame of the current study.

Furthermore, the number of random incoming waves in each realization indicates the *richness* of the multipath environment. This metric plays a central role in the remainder of this paper. In Fig. 6, a sample of such a scenario is depicted wherein 80 uncorrelated uniform random waves are incident upon the embedded patterns of 4MP antenna with a cascaded Butler network. In addition to this, the total number of realizations is associated with the accuracy of the results. Here, this number exceeds a million rendering a relative error of less than 0.05dB accordingly to [18, Table 1].

As a final point, note that in parallel with the proposed radiation elements, we also need to expose an arbitrary reference antenna to the same set of incident waves at each scenario, and use its received average power for normalization purpose. In this study, we used an ideal isotropic antenna as the reference.

IV. DIVERSITY PERFORMANCE IN RICH MULTIPATH ENVIRONMENTS

A *rich* multipath environment is an environment in which random received signals present a converged joint probability density function. In a *rich* isotropic Rayleigh environment, the diversity gain is a complex function of the correlation and the total embedded element efficiencies at different ports [18]. Figs. 4 and 5 illustrate the total embedded efficiencies and correlations versus frequency for 4MP antenna with a specific element separation. Apparently, at $f = 1050$ MHz the 4MP structure presents an optimum mean radiation efficiency. Hence, we choose $f = 1050$ MHz and study both radiation efficiencies and correlations versus element separation, d . The results are shown in Figs. 7 and 8, respectively. Once more, we observe significant differences between the associated metrics at the two sets of ports. Despite all these considerable discrepancies, since the rows and columns of \mathbf{T} in (16) are orthogonal, the eigenvalues of the resultant covariance matrix is identical to those of (15). This requires the associated diversity gains be similar at the element and beam ports. In fact, our simulations verify this requirement. Thus, regardless of the amount of coupling among the radiation elements, in a rich isotropic environment, the ultimate diversity performances at the element and beam ports are the same. This is also known from measurements in [19]. Nevertheless, in non-rich multipath environment, the results are in general different which is the subject of the next section.

V. ADVANTAGES OF A BUTLER NETWORK IN NON-RICH MULTIPATH ENVIRONMENTS

Up to now we have concluded that in a rich multipath environment the presence of a Butler network does not prove beneficial. However, the performance of the aforementioned network should also be examined in a non-rich scattering environment, where the number of random incident waves is finite.

To this end, and for a fair comparison between performances at the element and beam ports, we make some presumptions.

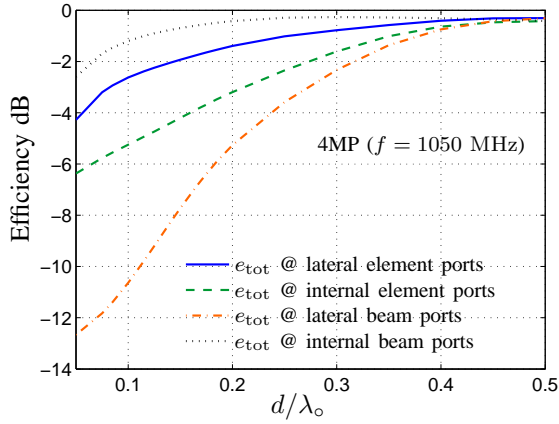
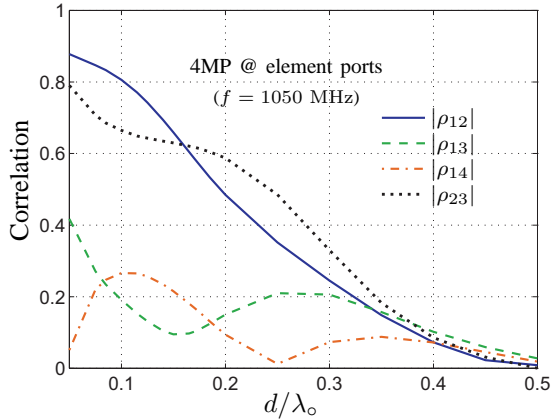
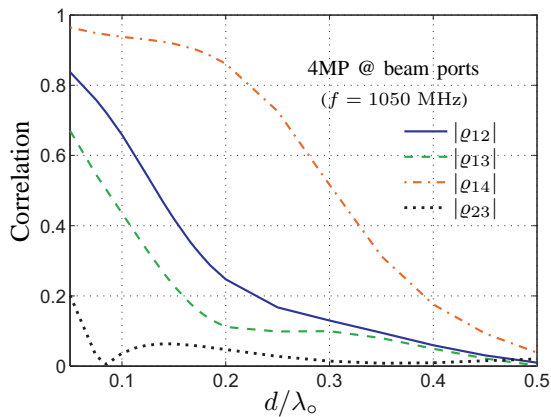


Fig. 7. Total embedded efficiencies versus element separations at element and beam ports for the 4MP antenna ($f = 1050$ MHz).



(a) Correlations at the element ports.



(b) Correlations at the beam ports.

Fig. 8. Spatial correlations for four monopoles above a PEC plane.

Recall that in a non-rich multipath environment, the joint distributions of random received signals do not converge. Convergence in joint distribution functions is linked to convergence in distributions of signals at different ports as well as convergence in their covariance matrix. To focus on the antenna characterization and alleviate its complexity, we assume that the distribution of the received signals at different ports is complex Gaussian. This assumption holds regardless of the number of incident waves in each realization. Thus, convergence in the joint distribution function will solely depend on convergence in the received signals' covariance matrix. The latter parameter depends on the shapes of the embedded patterns as well as the array configuration which are antenna engineers' concern. Bear in mind that the aforementioned constraint minimizes the least number of independent incoming waves yielding a converged joint distribution for the received signals. If we relax this restriction, a larger number of incident waves is necessary to cause convergence. This number is thus dependent not only on the antenna properties, but also on the distribution of the incoming waves which is beyond the intended scope.

To perform the simulation, we first choose a certain separation between the elements and a frequency of operation e.g., $d = 0.5\lambda_0$ and $f = 1050$ MHz. The 50Ω embedded patterns and the input parameters are achieved by the method of moment for both structures under study. It is known that proportional to the dimension of the Butler network, the directivity of the embedded patterns increases at the beam ports compared with those of the element ports [1]. Here, the variable is the number of incident waves in each realization. As an interesting start, let us study the CDFs of the simulated received signals by the 4MP antenna ($d = 0.2\lambda_0$) for two different numbers of incident waves, e.g., $K = 5$ and $K = 30$. Fig. 9 illustrates the results. Since the number of incident waves does not affect the normalized average received power, under the assumption of Rayleigh distributed received signals, the CDFs of the signals at different ports are independent of this number. Furthermore, due to the symmetry of the structure, the two lateral and internal ports have similar CDF curves. Nevertheless, the CDF curve of the maximum ratio combining (MRC) diversity signal depends on the number of incident waves. While with $K = 30$ number of incident waves, the CDFs of the diversity signals at both element and beam ports are almost the same, for $K = 5$ the CDF of the diversity signal associated with the beam ports shows better performance compared to that of the element ports. Fig. 10, shows the MRC diversity gain at the 1% CDF level for this particular case of $d = 0.2\lambda_0$ versus the number of incident waves in each realization. Similar results associated with 4HD antennas ($d = 0.5\lambda_0$) are shown in Fig. 11.

It is important to stress that the corresponding total embedded efficiencies simulated in this study are independent of the number of independent incident waves. But, the covariance matrix of the received signals varies with respect to this variable. Fig. 12 shows convergence in correlations at element and beam ports for 4HD antennas at $d = 0.5\lambda_0$ and $f = 1050$ MHz. An identical observation can be made for correlations in 4MP antennas which is neglected here for the sake of conciseness.

From a diversity performance point of view, in these sim-

TABLE I
RICHNESS THRESHOLD VERSUS ELEMENT SEPARATION AT -0.5 dB
LEVEL FOR 4HD ANTENNAS ($f = 1050$ MHz).

Element separation	Richness Threshold (waves)	
	Element ports	Beam ports
$0.05\lambda_0$	15	13
$0.10\lambda_0$	19	6
$0.15\lambda_0$	26	7
$0.20\lambda_0$	25	10
$0.25\lambda_0$	27	13
$0.30\lambda_0$	32	13
$0.40\lambda_0$	31	7

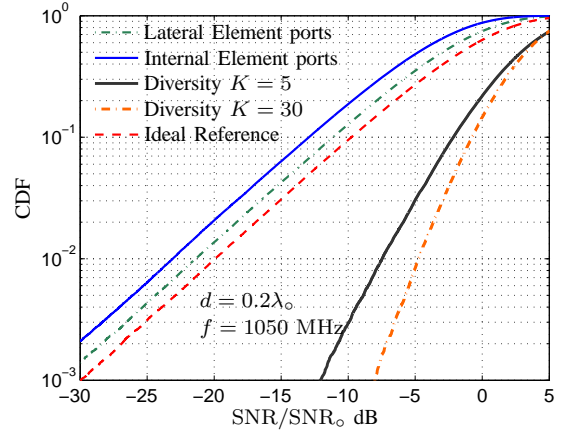
ulations the optimum diversity gain cannot be realized unless the covariance matrix of the received signals converges. This occurs when the number of incident waves exceeds a threshold. As the number of the incident wave increases beyond this threshold, no extra diversity gain can be achieved. In our preliminary work, we referred to this novel performance metric as the *richness threshold* [5]. If the distribution of the incoming waves is complex Gaussian, richness threshold becomes a unique measure rendering some information about the overall performance of a multipoint diversity antenna system. As a criterion, the less the richness threshold, the better the diversity system performance.

In order to quantify the enhancement caused by the Butler network, we study the richness threshold. For this purpose let us choose the threshold as the number of independent incident waves by which the diversity gain exceeds -0.5 dB level from its asymptotic (or converged) value. Fig. 13 illustrates this metric versus element separation for 4MP antenna at $f = 1050$ MHz. There is no doubt that the Butler network reduces the richness threshold considerably. Based on this figure, the best performance belongs to the cases of $d = 0.4\lambda_0 - 0.5\lambda_0$ where only four independent incident waves are sufficient for the antenna to realize 90% of its optimum performance. For the sake of comparison, similar results for 4HD antenna are summarized in Table I. Regardless of the element separation and in the absence of a Butler network, apparently the 4HD antenna requires more number of independent waves to represent an effective diversity antenna.

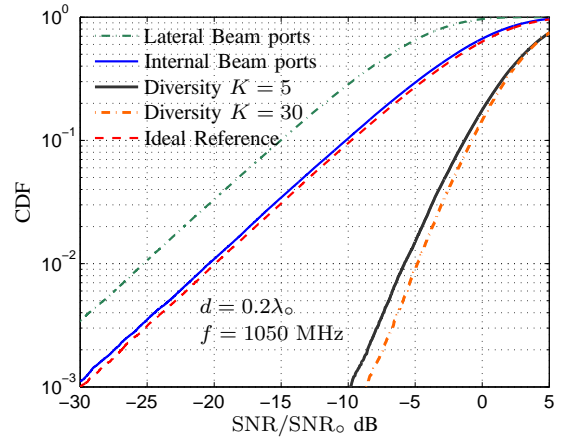
In general, the significance of richness threshold may be understood more when one recalls that in a typical multipath environment, the frequently reported number of incoming waves at an outdoor receiving antenna is five or six [20], [21], [22]. A number of 20 incoming waves in an indoor scattering environment seems also presumably acceptable [23]. But, for numerous available multipoint antennas, this number of incident waves is insufficient to realize the best diversity performance. Therefore, in practice, most of the multipoint antennas are working below their richness threshold and thus cannot realize their optimum performance.

VI. CONCLUSION

This paper provides a general - yet compact - expression for calculation of multipoint matching efficiencies in the presence of a microwave network. In a lossless structure, the multipoint matching efficiencies equal the corresponding total embedded



(a) CDF curves at the element ports for 4MP antenna.



(b) CDF curves at the beam ports for 4MP antenna.

Fig. 9. CDFs of the received signals and diversity signals for four monopoles above a PEC plane. K is the number of incident waves and SNR_0 is the average received power by the reference antenna.

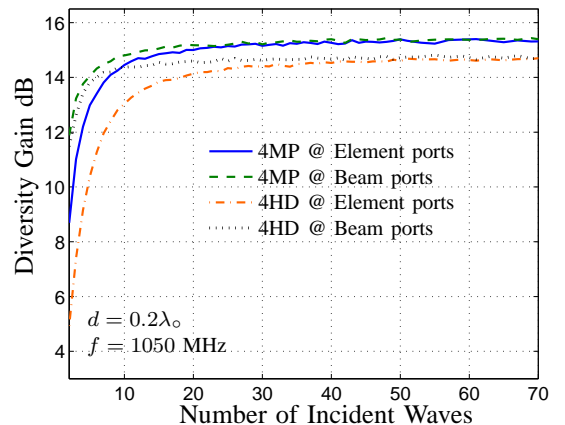


Fig. 10. MRC Effective diversity gain at the 1% CDF level versus the number of incident waves for the two antennas under test with $d = 0.2\lambda_0$.

efficiencies. These efficiencies are used for normalization of the embedded pattern in the presence of an arbitrary cascaded network. The main purpose of this paper is to investigate the impacts of a cascaded Butler network in a diversity antenna

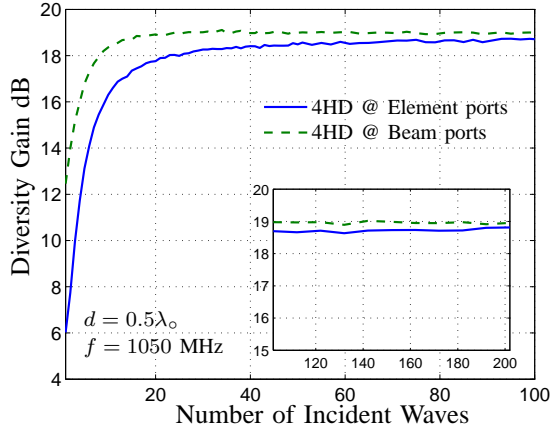


Fig. 11. MRC Effective diversity gain at the 1% CDF level versus the number of incident waves for the 4HD antenna under test with $d = 0.5\lambda_0$. The labels and the legends of the internal figure are the same as those in the main figure.

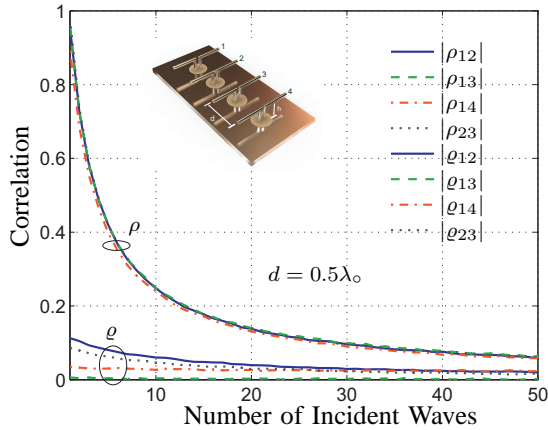


Fig. 12. Convergence in correlation for 4HD antenna with $d = 0.5\lambda_0$. ρ signifies correlations at the element ports, whereas $\rhō$ those of the beam ports.

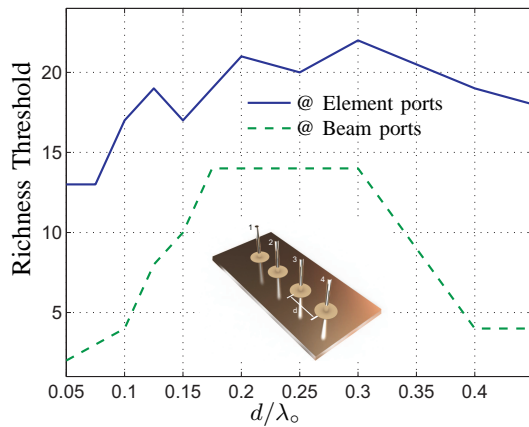


Fig. 13. Richness threshold for 4MP antenna at -0.5 dB level versus element separation at the element port and beam ports ($f = 1050$ MHz).

system. Limiting ourselves to lossless structures in a uniform multipath environment, for four monopoles above a PEC plane,

we present a study of correlation and total embedded element efficiencies both in the presence and absence of a cascaded Butler network. The results show considerable differences between the aforementioned metrics. Nevertheless, in a rich isotropic multipath, regardless of the frequency of operation and the amount of coupling between different elements, we have an identical diversity performance at both element and beam ports. In contrast, in a non-rich multipath environment, which represents actual scenarios better, the two proposed multiport antennas show a reduced performance. The deteriorated performance in the non-rich multipath is linked with the convergence in covariance matrix of the received signals at different ports. We show by numerical simulation that under this circumstance the cascaded Butler network to the antenna system proves beneficial. To quantify the advantage of the cascaded Butler network, we study the richness threshold at the element and beam ports, which is a novel performance metric. In general, the lower the richness threshold, the better the antenna's overall diversity performance. In this regard, the cascaded Butler network decreases the richness threshold and is thus quite advantageous in most practical environments wherein the typical number of incident waves is less than twenty. We emphasize that the results presented in this paper is under presumption of received signals of Rayleigh distribution, regardless of the number of incident waves. A further study is required to reveal the impact of a cascaded Butler network upon convergence in distribution of the random received signals.

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