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EFFECTS OF DISTORTION IN PHYSICAL  
MODELS OF COOLING WATER DISCHARGE

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## INTERNATIONAL ASSOCIATION FOR HYDRAULIC RESEARCH

EFFECTS OF DISTORTION IN PHYSICAL MODELS OF  
COOLING WATER DISCHARGE

(SUBJECT C.c.)

by

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## SYNOPSIS

The practical modelling of surface discharges of cooling water is discussed. The effects of having different vertical and horizontal geometric scale ratios are estimated by applying inspectional analysis on a mathematical model of surface buoyant jets. By knowing what these effects are it is possible to transform the results of an investigation in a distorted model into prototype conditions.

## RESUME

Le modelage physique des rejets d'eau de refroidissement est discuté. Les effets d'une échelle différente pour l'axe vertical par rapport à l'axe horizontal, d'une analyse sont déduits de l'application d'inspection sur un modèle mathématique de rejets de surface. Quand ces effets sont connus il est possible de transférer les résultats de mesures sur un modèle distordu à des conditions prototypes.

## 1. INTRODUCTION

In order to predict the warm-water spread from surface discharge of cooling water from steam power plants, physical models have been used frequently during the last decades. These models have, in most cases, been distorted i.e. they have been constructed with different vertical and horizontal scale factors. Because of the distortion, the warm-water spread cannot be correctly reproduced. Therefore distorted model investigations have often been criticized. Although scaling rules have been previously discussed (e.g. Silberman and Stefan 1970), very little has so far been done to estimate in what way and how much the distortion affects the results of an investigation. The work by Haggström (1978), seems to be the first one that tries to give an overall picture of the problems encountered.

During the last decade, several mathematical models have been presented by different authors. Most of these models are based on numerical solution techniques, for instance, the models developed by Stolzenbach and Harleman (1971), Prych (1972), and Shirazi and Davis (1974). The latter model, called the PSD-model, is to a great extent based on the Prych model and is considered by Dunn et.al (1975) to be one of the best models available. However, the accuracy of the numerical models is considered poor.

A few models with analytical solutions are, however, available (Stolzenbach and Harleman 1971), (Pande and Rajaratnam 1975), (Engelund and Pedersen 1973), and (Engelund 1976). The models developed by Stolzenbach and Harleman as well as that developed by Engelund seem to be the most reliable ones (Haggström).

Silberman, E., and Stefan, H., (1970): "Physical (Hydraulic) Modeling of Heat Dispersion in Large Lakes. A Review of the State of the Art". Project Report No 115 St Anthony Falls Hydraulic Laboratory, University of Minnesota (1970).

Haggström, S.H. (1978): "Surface Discharge of Cooling Water. Effects of Distortion in Model Investigations". Report Series A:3, Department of Hydraulics, Chalmers University of Technology, Göteborg, Sweden.

Stolzenbach, K.D. and Harleman, D.R.F. (1971): "An Analytical and Experimental Investigation of Surface Discharges of Heated Water", Report 135, Ralph M. Parsons Lab. for Water Res. and Hyd., M I T. Feb 1971.

Prych, E.A.: "A Warm Water Effluent Analyzed as a Buoyant Surface Jet", Swedish Meteorological and Hydrological Institute, Series Hydrology, Nr 21, Stockholm, Sweden. 1972.

Shirazi, M., and Davis, L. (1974): "Workbook of Thermal Plume Prediction, Volume 2, Surface Discharge", Pacific Northwest Environmental Research Lab., Report EPA-R2-72-005b, Corvallis, Ore, May 1974.

Dunn, W.E., Policastro, A.J. and Paddock, R.A. (1975): "Surface Thermal Plumes. Evaluation of Mathematical Models for the Near and Complete Field". ANR/WR-75-3, Argonne National Laboratory, Argonne, Illinois. May 1975.

Pande, B.B.L. and Rajaratnam, N. (1975a): "A Similarity Analysis of Heated Surface Discharges into Quiescent Ambients". Report No HY-1975-TPR1. Department of Civil Engineering. The University of Alberta Edmonton, Canada, May 1975.

Engelund, F.A. and Pedersen, F.B. (1973): "Surface Jet at Small Richardson Numbers" ASCE, Journal of the Hydraulics Division. Vol.99, No. HY3, March 1973.

Engelund, F.A. (1976): "Hydraulics of Surface Buoyant Jet", ASCE, Journal of the Hydraulics Division, Vol. 102, No. HY9, Sept. 1976.

Stolzenbach and Harleman, however, do not take buoyancy effects into consideration. All the analytical models disregard ambient diffusion and ambient currents and are thus of limited value for practical prediction of warm-water spread.

Mathematical models are, thus, considered to give poor estimates of, for example, excess temperature areas. However, this does not imply that they cannot give an approximately correct picture of the relative importance of different parameters. We thus believe that existing mathematical models can be used for an approximate evaluation of distortion effects. Because of this, the prediction of warm-water spread in physical model investigations can be improved.

The effect of distortion in physical models has been evaluated by Haggström (1978) by means of different mathematical models, laboratory experiments and case studies. In this paper, we will only demonstrate the technique to be used when founding the evaluation on Engelund's analytical solution. As this model, like most other models, is mainly based on jet entrainment theory, the discussion is limited to the near-field where jet diffusion is important.

## 2. ENGELUND'S ANALYTICAL SOLUTION TO THE SURFACE BUOYANT JET PROBLEM

In this theory, Engelund applies the entrainment principle to a buoyant surface jet with buoyancy affecting vertical entrainment as well as longitudinal and lateral spread. Horizontal entrainment is disregarded and the ambient water is considered quiescent, homogeneous, and infinitely deep. Engelund obtains the following set of equations

$$\frac{u}{u_0} = \left(\frac{x}{L}\right)^{-7/9} \dots\dots\dots (1)$$

$$\frac{T}{T_0} = \left(\frac{x}{L}\right)^{-7/9} \dots\dots\dots (2)$$

$$\frac{B}{L} = c_1 Ri_0^{1/2} \left(\frac{x}{L}\right)^{13/9} \dots\dots\dots (3)$$

$$\frac{H}{D} = \left(\frac{x}{L}\right)^{1/9} \dots\dots\dots 4)$$

and as an additional condition

$$\frac{9}{7} \frac{e \cdot L}{Ri_0 D} = 1 \dots\dots\dots (5)$$

- where
- u = velocity at centre-line of jet
  - x = coordinate along centre-line of jet
  - L = characteristic horizontal length of jet
  - T = excess temperature at centre-line of jet
  - B = width of jet
  - Ri =  $c_2 \frac{TH}{u^2}$  bulk Richardson number
  - H = thickness of jet

- D = characteristic vertical length of jet
- $e_o$  = entrainment factor corresponding to  $Ri=1.0$  (constant)

Index o indicates that the parameters are from a reference section. The characteristic length parameters D and L are unknown functions of the flow, velocity and excess temperature (density deficit) of the reference section. Therefore Eqs (1)-(5) cannot be used directly for an inspectional analysis.

We introduce another control section, index c, in the flow which gives us the following simple set of equations

$$\frac{u}{u_c} = \left(\frac{x}{x_c}\right)^{-7/9} \dots\dots\dots (6)$$

$$\frac{T}{T_c} = \left(\frac{x}{x_c}\right)^{-7/9} \dots\dots\dots (7)$$

$$\frac{B}{B_c} = \left(\frac{x}{x_c}\right)^{13/9} \dots\dots\dots (8)$$

$$\frac{H}{H_c} = \left(\frac{x}{x_c}\right)^{1/9} \dots\dots\dots (9)$$

If we can express  $x_c$ ,  $H_c$  and  $B_c$  in the parameters  $Q_c$ ,  $u_c$ , and  $T_c$ , where Q is the volume flux, then it is possible to apply the inspectional analysis to Eqs (6)-(9). In order to do this, we must apply Eqs (1)-(4) to our control section, using the definition of Richardson number, Equation (5) and the following expression of the volume flux.

$$Q = c_3 B H u \dots\dots\dots (10)$$

By using arithmetic we obtain the following expression for  $x_c$ ,  $H_c$  and  $B_c$

$$x_c = c_4 T_c^{1/7} Q_c^{4/7} u_c^{-6/7} \dots\dots\dots (11)$$

$$H_c = c_5 T_c^{-3/7} Q_c^{2/7} u_c^{4/7} \dots\dots\dots (12)$$

$$B_c = c_6 T_c^{3/7} Q_c^{5/7} u_c^{-11/7} \dots\dots\dots (13)$$

Equations (6)-(9) will then take the form

$$\frac{u}{u_c} = c_7 T_c^{1/9} Q_c^{4/9} u_c^{-2/3} x^{-7/9} \dots\dots\dots (14)$$

$$\frac{T}{T_c} = c_8 T_c^{1/9} Q_c^{4/9} u_c^{-2/3} x^{-7/9} \dots\dots\dots (15)$$

$$B = c_9 T_c^{2/9} Q_c^{-1/9} u_c^{-1/3} x^{13/9} \dots\dots\dots (16)$$

$$H = c_{10} T_c^{-4/9} Q_c^{2/9} u_c^{2/3} x^{1/9} \dots\dots\dots (17)$$

In this form, the equations can be used for an inspectional analysis.

3. INSPECTIONAL ANALYSIS

Before the inspectional analysis is introduced, the following assumptions are made

- o Physical modelling of the flow in the control section is made according to the Froude scaling law i.e.  $u_r = \lambda_v^{1/2}$ ,  $Q_r = \lambda_v^{3/2} \lambda_h^{1/2}$ ,  $T_r = 1.0$ .  
with index r indicating model to prototype ratio, and where  $\lambda_v$  and  $\lambda_h$  are vertical and horizontal model to prototype geometric scale ratios, respectively.
- o The depth and the width of the jet flow of the control section are supposed to follow the theory of Engelund. Then, it follows from Eqs (12)-(13) that  $(B_c)_r \neq \lambda_h$  and  $(H_c)_r \neq \lambda_v$ . It also follows from Equation (11) that  $(x_c)_r \neq \lambda_h$  e.g. the origo of the x-coordinate in the model and in the prototype, respectively, have different positions. The theory thus has to be applied so far from the control section that the influence of the different positions can be disregarded. As the only purpose of this paper is to evaluate effects of distortion, knowledge of the absolute position of the control section is not necessary.

With these assumptions it is possible to use Eqs (14)-(17) to calculate the effects of having different vertical and horizontal scale ratios.

$$\left(\frac{u}{u_c}\right)_r = \lambda_v^{1/3} \lambda_h^{-1/3} \dots\dots\dots (18)$$

$$\left(\frac{T}{T_c}\right)_r = \lambda_v^{1/3} \lambda_h^{-1/3} \dots\dots\dots (19)$$

$$B_r = \lambda_h^{4/3} \lambda_v^{-1/3} \dots\dots\dots (20)$$

$$H_r = \lambda_h^{1/3} \lambda_v^{2/3} \dots\dots\dots (21)$$

The equations above show that the effects of distortion are as follows:

The centre-line velocity is incorrect by a factor of  $\lambda_v^{1/3} \lambda_h^{-1/3}$  (too great)

The centre-line temperature is incorrect by a factor of  $\lambda_v^{1/3} \lambda_h^{-1/3}$  (too great)

The flow width is incorrect by a factor of  $\lambda_v^{-1/3} \lambda_h^{1/3}$  (too small)

The thickness of flow is incorrect by a factor of  $\lambda_v^{-1/3} \lambda_h^{1/3}$  (too small)

This evaluation is valid only as long as the theory is valid, which, according to Engelund, is, in the case of a Richardson number, greater than 0.1. An upper limit is not given. This corresponds to a Froude number,  $F_\Delta = (u^2 / g \frac{\Delta \rho}{\rho} H)^{1/2}$ , of the order of 3 or less. Since the Richardson

number increases with the distance from the outlet, a warm-water flow with a higher outlet densimetric Froude number can follow Engelund's theory except close to the outlet.

#### 4. COMPARISON WITH LABORATORY DATA

There are very few laboratory investigations of surface buoyant jet where the width to height ratio ( $A$ ) of the outlet is the only parameter varied, which is a presumption for the data to be used for evaluating effects of distortion. In this case, the conditions for Engelund's theory must at the same time be fulfilled. One data set of Stolzenbach and Harleman (1971) fulfil these requirements if we consider the last statement in Section 3.

The centre-line excess temperature has been measured for two surface jets and is plotted in Figure 1 against the dimensionless distance  $x/B_d$  from discharge ( $B_d$  = width of rectangular outlet). Straight lines are fitted to data with the method of least squares.

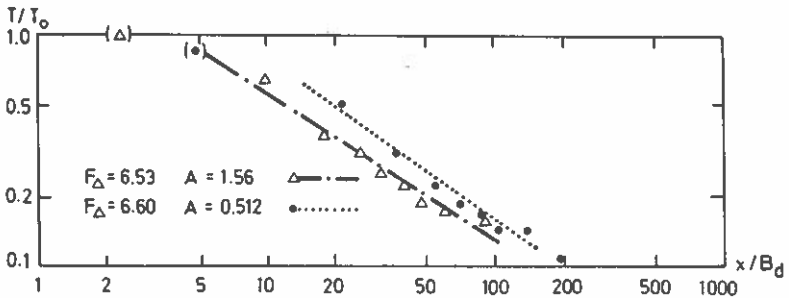


Figure 1. Centre-line excess temperature versus distance from outlet for surface discharges with  $F_{\Delta} = 6.53$  and  $A = 1.56$  and  $F_{\Delta} = 6.60$  and  $A = 0.51$  respectively. ( $B_d$  = width of rectangular outlet). Data from Stolzenbach and Harleman.

Figure 1. Excès de température le long de l'axe central en fonction de la distance du point de rejet respectivement avec  $F_{\Delta} = 6.53$  et  $A = 1.56$  et avec  $F_{\Delta} = 6.60$  et  $A = 0.51$  ( $B_d$  = largeur du canal de rejet rectangulaire). Résultats de mesures d'après Stolzenbach et Harleman.

The experiment with outlet densimetric Froude number  $F_{\Delta} = 6.60$  and aspect ratio  $A = 0.512$  is regarded as a reproduction of the experiment with  $F_{\Delta} = 6.53$  and  $A = 1.56$  at a distortion of  $1.56/0.512 = 3.1$ . This leads to the result that the centre-line excess temperature is too great by a factor of 1.2-1.4 in this reproduction. This number should be compared with the corresponding number calculated from Equation (19) giving  $3.1^{1/3} = 1.46$ . Thus the theoretical and experimental results correspond well.

## 5. CONCLUDING REMARKS

In most cases, the nature of the warm-water spread is very complex, much more complex than can be described by theoretical models. Evaluation of the effects of distortion based on one mathematical model is not sufficient to get reliable results. Other models and methods should be used and if possible in combination with field measurements on existing discharges. Figure 2 shows a transformation of model results into prototype conditions based on calculations using the Prych model and measurements in the distorted model ( $\lambda_h = 1/400$ ,  $\lambda_v = 1/50$ ) and in the prototype of a cooling water discharge ( $25 \text{ m}^3/\text{s}$ ) (Rahm and Haggström 1976).

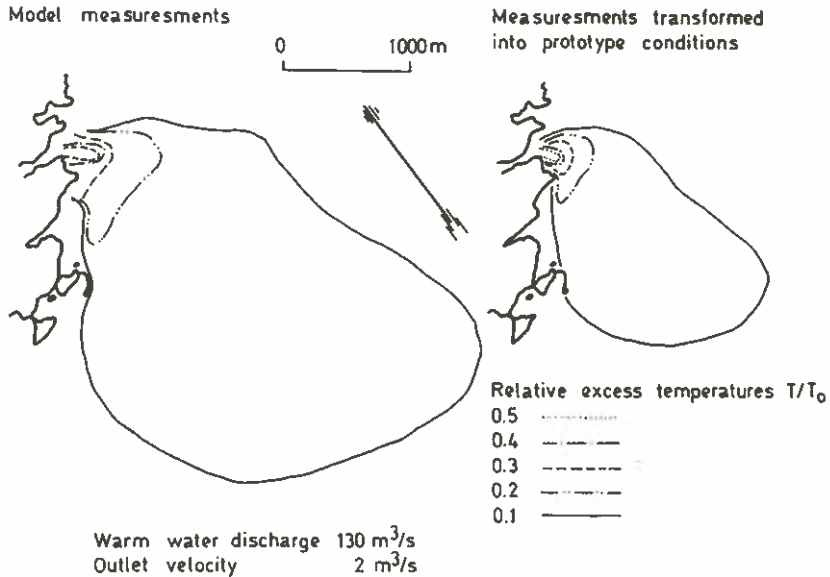


Figure 2. Excess temperature  $T/T_0$  as measured in model and after transformation into prototype conditions for a discharge of  $130 \text{ m}^3/\text{s}$ .

Figure 2. Excès de température  $T/T_0$  mesurés sur un modèle physique et après adaptation aux conditions prototypes pour un rejet de  $130 \text{ m}^3/\text{s}$ .

Rahm, L., and Haggström, S. (1976): "Oskarshamn's Nuclear Power Plant. Model Studies of Cooling Water Spread at Future Expansion". Part I—Main Report. Part II—Appendices. Bulletin 86, Department of Hydraulics, Chalmers University of Technology, Gothenburg, 1976 (in Swedish).





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