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# Using Akaike Information Criterion for Selecting the Field Distribution in a Reverberation Chamber

Xiaoming Chen

**Abstract**—Previous studies on modeling the random field (amplitude) in a reverberation chamber (RC) were conducted either by fitting a given distribution to measured data or by comparing different distributions using goodness-of-fit (GOF) tests. However, the GOF tests are inappropriate for comparing different distribution candidates in that they are meant to check if a given distribution provides an adequate fit for a set of data or not and they cannot provide correct relative fitness between different candidate distributions in general. A fair comparison of different distributions in modeling the RC field is missing in the literature. In this paper, Akaike's information criterion (AIC), which allows fair comparisons of different distributions, is introduced. With Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions as the candidate set, the AIC approach is applied to measured data in an RC. Results show that the Weibull distribution provides the best fit to the field in an undermoded RC and that the Rayleigh distribution provides the best approximation of the field in an overmoded RC. In addition, it is found that both the Rician and Weibull distributions provide improved approximations of the field in an RC loaded with lossy objects. This study provides correct complementary results to the previous RC studies.

**Index Terms**—Akaike's information criterion (AIC), field distribution, goodness-of-fit (GOF), reverberation chamber (RC).

## I. INTRODUCTION

THE reverberation chamber (RC) has been used for electromagnetic compatibility (EMC) tests as well as over-the-air (OTA) measurements of wireless devices [1]–[16]. Due to the complicated test conditions (e.g., inhomogeneous test objects, irregular mode stirrers, changing boundary conditions, etc.), various RC measurements are ubiquitously studied from a statistical point of view. Since the overmoded RC represents a rich scattering environment, it is natural to assume that the complex (electromagnetic) field inside is Gaussian distributed. In other words, the amplitude of the field in an overmoded RC is Rayleigh distributed [1]–[4], [10]. On the other hand, there are studies showing that the field in an undermoded RC follows the Weibull or Bessel K distribution [5]–[8]. However, the study in [9] claims that the Weibull distribution fits the field distribution of the overmoded RC but not the undermoded one.

Interestingly, the superiority of the Weibull fit has been shown based on measurements in indoor/outdoor multipath environments [17], [18]. In the course of finding a distribution from a set of candidates to fit the measured data, the relevant prior studies (e.g., [9], [17], and [18]) used various goodness-of-fit (GOF) tests [19].

GOF tests are basically special versions of hypothesis tests. The general procedure for testing a null hypothesis  $H_0$  is to partition the sample space into a rejection region and an acceptance region based on a test statistic. Usually, a significance level is chosen to ensure a small probability that the true  $H_0$  is rejected. A good GOF test should minimize the probability that a false  $H_0$  is accepted for a given significance level. Powerful as they are for examining a specific distribution, GOF tests are not suitable for examining a set of distribution candidates. In other words, GOF tests are meant to check if a given distribution provides an adequate fit for a set of data or not and they do not provide a relative measure of how good the fit actually is. For this reason, it is usually suggested that several types of GOF tests should be applied to examine the given distribution [19]. Nevertheless, there are studies applying the GOF test for several distributions in the candidate set, where the percentage with which each fitted distribution passes the test is used as the performance metric. Such studies assume that the GOF test in use is equally powerful for all distributions, which does not hold in general [20], [21].

While [1]–[7] did not compare different distributions, [9] used a GOF test for comparing Rayleigh and Weibull distributions. To overcome the drawbacks of GOF tests in distribution selection, this paper uses Akaike's information criterion (AIC) [20] to test the field distribution in an RC. The AIC approach falls into the category of model selection [21], which is suitable for choosing the best fitted distribution among the candidate set for the random variable under test. The AIC approach has been used in [22] and [23] for model selection for wireless fading channels. In this paper, based on the measurements in an RC and with Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions as the candidate set, it is found that the Weibull distribution offers the best fit for the undermoded RC and that the Rayleigh distribution provides the best fit for the overmoded RC.

It is shown in [5] and [7] by fitting the Weibull distribution to RC measurements that the field in an undermoded RC is Weibull distributed. On the contrary, using GOF tests [9] shows that the Weibull distribution well approximates the field in an overmoded RC but not that in an undermoded RC. It is believed that the reason of the superiority of the Weibull distribution over the Rayleigh one in [9] is partially due to the facts that the Weibull distribution has two scalar parameters while the

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The author is with the Department of Signals and Systems, Chalmers University of Technology, SE-412 96 Gothenburg, Sweden (e-mail: xiaoming.chen@chalmers.se).

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Rayleigh distribution has only one scalar parameter and that GOF tests cannot take the overfitting (i.e., the parameter vector with a large  $p$  tends to offer more flexibility in fitting specific data and the nice fitting tends to break down for another independent data) problem of the Weibull distribution into account. On the other hand, different distributions with possibly different scalar parameter numbers can be fairly compared using the AIC approach [20]. In addition, based on measurements with different RC loadings, it is found in this paper that the Weibull distribution provides better fit to the measured data with increasing loading. Another explanation for the disagreement between this work and that in [9] may be that a loaded RC has relatively more unstirred components (and thus resembles an RC with less effective mode stirrers or an undermoded RC) [24]. This work provides correct complementary results to the previous RC studies [1]–[9].

## II. AIC APPROACH

The selection of a suitable distribution out of a given candidate set  $J$  involves the calculation of the discrepancy between the true cumulative distribution function (CDF or simply distribution)  $F$  and each candidate distribution  $G_{j|\theta}$ ,  $j = 1, \dots, |J|$ , where  $|J|$  denotes the cardinality of  $J$  and  $\theta$  represents the  $p \times 1$  parameter vector (with  $p$  being a positive integer). The detailed derivation of the AIC approach can be found in [20]–[23]. For the sake of conciseness, the AIC is given here directly

$$\text{AIC}_j = -2 \sum_{n=1}^N \ln g_{j|\hat{\theta}}(x_n) + 2p \quad (1)$$

where  $\ln$  denotes the natural logarithm,  $g_{j|\hat{\theta}}$  is the corresponding probability density functions (PDF) of  $G_{j|\hat{\theta}}$ , and  $x_n$  denotes the  $n$ th sample of the measured  $N$  sample. The corresponding maximum likelihood (ML) parameter estimator is

$$\hat{\theta} = \arg \max_{\theta} \frac{1}{N} \sum_{n=1}^N \ln g_{j|\theta}(x_n). \quad (2)$$

The AIC values are difficult to interpret directly in that the AIC values of different (reasonable) candidate distributions are usually of the same order of magnitude. Similarly, comparisons of the empirical CDF of the measured data and those of the distribution candidates do not provide interpretable distinctions (the corresponding results are therefore omitted). Therefore, one has to resort to the AIC weights [20] for better distinctions. The AIC weights are defined as

$$w_j = \frac{\exp(\phi_j/2)}{\sum_{l=1}^J \exp(\phi_l/2)} \quad (3)$$

where  $\phi_j = \text{AIC}_j - \min_l \{\text{AIC}_l\}$ . It represents relative feasibilities of different candidates, ranging from 0 (the worst fit) to 1 (the best fit). In other words, a larger AIC weight means a better fit.

Note that (3) is only accurate for a reasonably large  $N/p$  and that for  $N/p < 40$  a correction term should be added to the

AIC [25]

$$\text{AICC}_j = \text{AIC}_j + \frac{2p(p+1)}{N-p-1}. \quad (4)$$

The weights of the AIC with the correction term (AICC) can be obtained by replacing  $\text{AIC}_j$  with  $\text{AICC}_j$  in (3).

There are different information criteria in the literature, e.g., the Bayesian information criterion (BIC). A comparison of AIC/AICC and BIC is given in [26], showing that AIC/AICC has theoretical advantages over BIC in that AIC/AICC is derived from principles of information theory [29]. It was also shown by simulations that suggest AICC tends to have practical performance advantages over BIC [26]. Further comparison of AIC and BIC shows that AIC is asymptotically optimal in selecting the model with the least mean square error, under the assumption that the true model is not in the candidate set (as is virtually always the case in practice) and that BIC is not asymptotically optimal under the assumption [27]. Therefore, this paper will use the AIC approach.

## III. CANDIDATE DISTRIBUTION SET

The Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions are believed to be the most relevant models. Their PDFs and corresponding free parameter ML estimators are given in the following sections.

### A. Rayleigh Distribution

If the field in an RC is complex Gaussian distributed with a zero mean, then the amplitude of the field is Rayleigh distributed. The Rayleigh distribution is probably the most common statistical model for an overmoded RC [1], [2], [10]. Its PDF can be expressed as (for the sake of notational convenience, the subscript  $_{j|\theta}$  is dropped hereafter)

$$g(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad (5)$$

where the free parameter is  $\theta = \sigma$ . Thus, the Rayleigh distribution has only one scalar parameter, i.e.,  $p = 1$ . The ML estimator of  $\sigma$  is [28]

$$\hat{\sigma} = \sqrt{\frac{1}{2N} \sum_{n=1}^N x_n^2}. \quad (6)$$

### B. Rician Distribution

If the field in the RC is complex Gaussian distributed with a nonzero mean (representing an unstirred component [3], [4]), then the amplitude of the field is Rician distributed. The PDF of the Rician distribution is

$$g(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2 + v^2}{2\sigma^2}\right) I_0\left(\frac{xv}{\sigma^2}\right) \quad (7)$$

where  $I_0$  is the modified Bessel function of the first kind with order zero and the free parameter vector is  $\theta = [v \ \sigma]^T$  (the superscript  $T$  denotes transpose). Thus, the Rician distribution has two scalar parameters, i.e.,  $p = 2$ . The well-known Rician

K-factor  $K$  is related to these parameters as  $K = v^2/\sigma^2$ . As will be shown in the next section, a vector network analyzer (VNA) is used to sample the complex field  $S_{21}$  ( $x = |S_{21}|$ ). Denote the total power of the field as  $\Omega = 2\sigma^2 + v^2$ , whose ML estimator is [28]

$$\hat{\Omega} = \frac{1}{N} \sum_{n=1}^N x_n^2. \quad (8)$$

The ML estimator of  $v$  is

$$\hat{v} = \left| \frac{1}{N} \sum_{n=1}^N S_{21n} \right|. \quad (9)$$

It is then natural to use  $(\hat{\Omega} - \hat{v}^2)/2$  as the estimator of  $\sigma^2$ , but this is not an ML estimator. Therefore, we have to resort to the numerical ML estimation [28], which utilizes the *fminsearch* function in MATLAB.

### C. Nakagami Distribution

The Nakagami distribution (that includes Rayleigh and Rician distributions as special cases) is a popular statistical model. The PDF of the Nakagami distribution is

$$g(x) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m x^{2m-1} \exp\left(-\frac{mx^2}{\Omega}\right) \quad (10)$$

where  $\Gamma$  is the gamma function and the free parameter vector is  $\theta = [m \ \Omega]^T$ . Hence, the Nakagami distribution has two scalar parameters, i.e.,  $p = 2$ . For  $m = 1$ , the Nakagami distribution reduces to the Rayleigh distribution. The Nakagami distribution can well approximate the Rician distribution by letting  $K = m - 1 + \sqrt{m^2 - m}$ . The ML estimator of  $\Omega$  is given by (8). The ML estimator of  $m$  is [28]

$$\hat{m} = \left( 2 \ln \hat{\Omega} - \frac{2}{N} \sum_{n=1}^N \ln x_n^2 \right)^{-1}. \quad (11)$$

### D. Bessel K Distribution

It is shown in [8] that the Bessel K distribution can be used to model the field in an imperfect RC. The PDF of the Bessel K distribution is

$$g(x) = \frac{t^{M+1}}{2^{M-1}\Gamma(M)} x^M K_{M-1}(tx) \quad (12)$$

where the free parameter vector is  $\theta = [M \ t]^T$ , and  $K_{M-1}$  denotes the modified Bessel function of the second kind with the order of  $M - 1$ . The closed-form ML parameter estimator does not exist for the Bessel K distribution. Hence, we resort to the numerical ML estimation  $\hat{\theta}$  [28] (based on the *fminsearch* function in MATLAB).

### E. Weibull Distribution

The Weibull distribution was originally proposed to model failure rate (see [9] and reference therein). It also finds applications in modeling random fields in the RC [5]–[7], [9]. The

TABLE I  
ML ESTIMATORS FOR THE CANDIDATE DISTRIBUTIONS

Candidate distributions	ML parameter estimator
Rayleigh	(6) [28]
Rician	numerical estimator [28]
Nakagami	(8) and (11) [28]
Bessel K	numerical estimator [28]
Weibull	MATLAB function <i>wblfit</i>

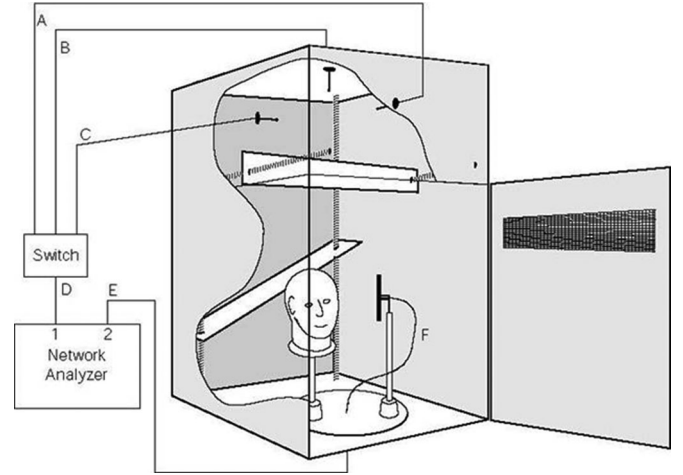


Fig. 1. Drawing of Bluetest reverberation chamber with two mechanical plate stirrers, a platform, and three wall antennas.

PDF of the Weibull distribution is

$$g(x) = ba^{-b} x^{b-1} \exp(-(x/a)^b) \quad (13)$$

where the free parameter vector is  $\theta = [a \ b]^T$ . For  $b = 2$ , the Weibull distribution reduces to the Rayleigh distribution. A closed-form ML estimator for the Weibull distribution does not exist in the literature. Thus, one has to resort to the numerical ML estimation [28]. In this case,  $\hat{\theta}$  can be obtained by calling the available function *wblfit(x)* in MATLAB, where  $\mathbf{x}$  is a vector of the measured field amplitudes.

All the aforementioned ML estimators are summarized in Table I.

## IV. MEASUREMENTS AND RESULTS

Although the main focus of this paper is to apply the AIC approach for a fair distribution selection, it is helpful to present the GOF tests results for the sake of comparison of GOF and AIC for distribution selection. Thus, in this section, the results of GOF tests are presented briefly prior to the AIC results.

### A. Measurements

Measurements were performed from 500 to 2000 MHz in an RC with a size of  $1.80 \times 1.75 \times 1.25 \text{ m}^3$  (a drawing of which is shown in Fig. 1). Its fundamental mode resonance frequency is  $f_0 = 119 \text{ MHz}$ , giving a lowest usable frequency (LUF) of about  $6f_0 = 717 \text{ MHz}$  (see [24] and reference therein). Note that this LUF corresponds to a well-stirred and unloaded RC. Provided that the stirrers are less effective and/or the RC is loaded, the actual LUF should be larger than 717 MHz. It has two plate mode



Fig. 2. Photograph of the *load2* configuration in the RC.

stirrers (that are equivalent to the translated stirrer in [30]), a turn-table platform (on which a wideband discone antenna is mounted), and three antennas mounted on three orthogonal walls (referred to as wall antennas hereafter). The wall antennas are actually wideband half-bow-tie (or triangular sheet) antennas. The measurement setup (or stirring sequence) of the RC is chosen such that the turn-table platform was step-wisely moved to 20 platform positions evenly distributed over one complete platform rotation; at each platform position the two plates were simultaneously and step-wisely moved to 50 positions (equally spanned on the total distances that they can move). At each stirrer position and for each wall antenna a full frequency sweep was performed by the VNA with a frequency step of 1 MHz, during which the  $S_{21}$  is sampled as a function of frequency and stirrer position. Thus, there are  $20 \times 50 = 1000$  stirrer positions per frequency point. The same measurement procedure were repeated for three loading conditions: *load0* (unloaded RC), *load1* (head phantom that is equivalent to a human head in terms of microwave absorption), and *load2* [the head phantom plus three Polyvinyl Chloride (PVC) cylinders filled with microwave absorbers cut in small pieces]. Fig. 2 shows a photograph of the *load2* configuration (where the three gray colored PVC tubes are mounted orthogonally in the corner). Hereafter measured data from these different loading configurations are simply referred to as *load0*, *load1*, or *load2* data, whose corresponding Q-factors are shown in Fig. 3.

In the postprocessing, only the  $S_{21}$  samples corresponding to one of the wall antennas are used (the statistics of the samples corresponding to the other two wall antennas are quite similar). As mentioned in Section III, the random field amplitude is denoted as  $x = |S_{21}|$  and the measured ( $N = 1000$ ) amplitude samples are stacked into one column vector denoted as  $\mathbf{x}$ . Both transmit and receive antennas have a moderately low average reflection coefficient. Fig. 4 shows the  $|\langle S_{11} \rangle|$  of the transmit antenna for all loading conditions, where  $\langle \cdot \rangle$  denotes average over all the stirrer positions. The receive antenna has slightly lower  $|\langle S_{11} \rangle|$  and are omitted for conciseness. The average re-

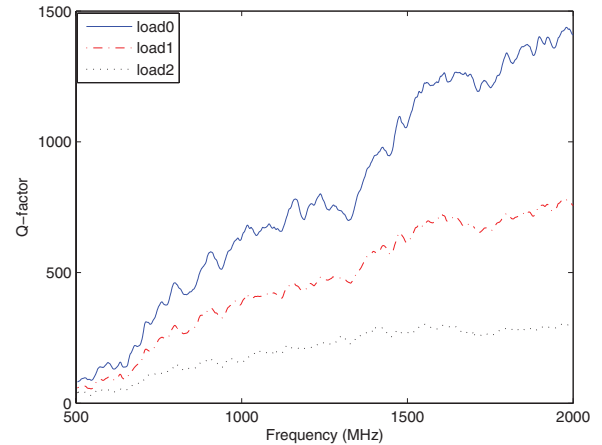


Fig. 3. The Q-factor for measurement data at *load0*, *load1*, and *load2* cases. A frequency stirring (see [24] and reference therein) of 20 MHz (corresponding to 20 frequency points) was applied to smoothen the curves.

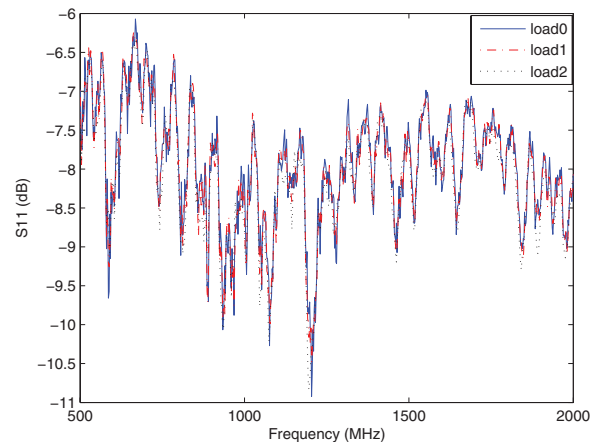


Fig. 4. Reflection coefficient (averaged over all the stirrer positions) of the transmit antenna at *load0*, *load1*, and *load2* cases (all three curves almost overlap).

lection coefficient of the antenna is almost not affected by the loading (which affects the field distribution, as can be seen in Section IV-C).

### B. GOF Tests

The Kolmogorov–Smirnov (KS) GOF test and the Anderson–Darling (AD) GOF test have been used in [9] for comparison of the fitness of Rayleigh and Weibull distributions to the RC measurements. The AD GOF test makes use of the specific distribution in calculating the critical values [19]. Therefore, it allows more sensitive tests (the drawback is that the critical values must be calculated for each distribution), compared with the KS GOF test. Therefore, the AD GOF test is employed in this paper; and we confine the tests to Rayleigh and Weibull distributions for conciseness and direct comparison with previous RC work.

It is shown in [24] that the AD GOF test rejects not only fields in the low-frequency range, but also in the high-frequency range. In this paper, we use the rejection significance level (as in [24]) to present the test results. Fig. 5 shows the rejection significance

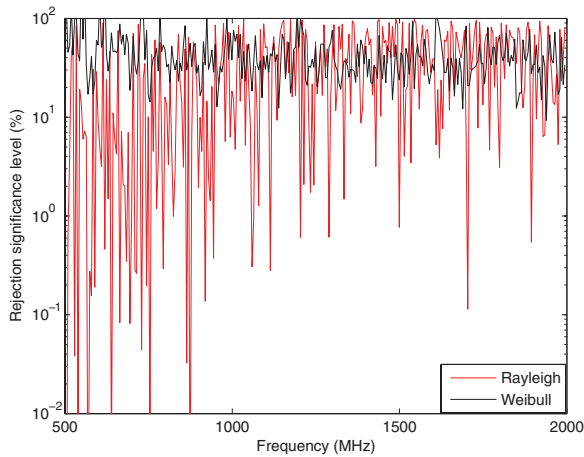


Fig. 5. Rejection significance levels of Rayleigh and Weibull distributions obtained by applying the AD GOF test to the *load0* data.

levels (where 100% means the best fit and 0% means the worst fit) of Rayleigh and Weibull distributions by applying the AD GOF test to the measurement data  $\mathbf{x}$  of the *load0* case. The result of the Rayleigh test agrees with that in [24]. In addition, it shows that its rejection rate is larger at the low frequencies, which is reasonable in that it is well known that the field in an overmoded (higher frequencies) RC is Rayleigh distributed and that the field of an undermoded (lower frequencies) RC is not. The results are also in agreement with that of [5], which shows that the Weibull distribution fits the data of the undermoded RC better than that of the Rayleigh distribution. Nevertheless, it should be noted that the comparison is only valid when the GOF test is equally powerful in both distributions, which does not hold in general. In addition, the GOF test also suffers from the overfitting problem (cf., Section I), which prevents fair comparison of different distributions with different parameter numbers (i.e., the Weibull distribution has two scalar parameters and the Rayleigh distribution has only one scalar parameter). Therefore, one has to resort to the AIC test for fair comparisons of different candidate distributions.

### C. AIC Tests

Since the AIC test is more suitable for the selection of different candidate distributions (as explained in Sections I and II), it will be studied in more detail in this subsection by applying the AIC test to various measurement data.

Fig. 6 shows the comparison of the AIC weights for the candidate distribution based on the *load0* data. It can be seen that in the higher frequencies the Rayleigh distribution provides the best fit (the largest AIC weight) and that in the lower frequencies both Bessel K and Weibull distributions provide better fit. Note that, for an AIC test, the best candidate may not necessarily have an AIC weight of 1 and that the best fit simply corresponds to the largest AIC weight since the AIC test provides relative fitness. This implies that, for the unloaded RC, the field in an undermoded RC (at lower frequencies) is more likely to be Weibull or Bessel K distributed and that the field in an overmoded RC (at higher frequencies) is more probable to be Rayleigh distributed.

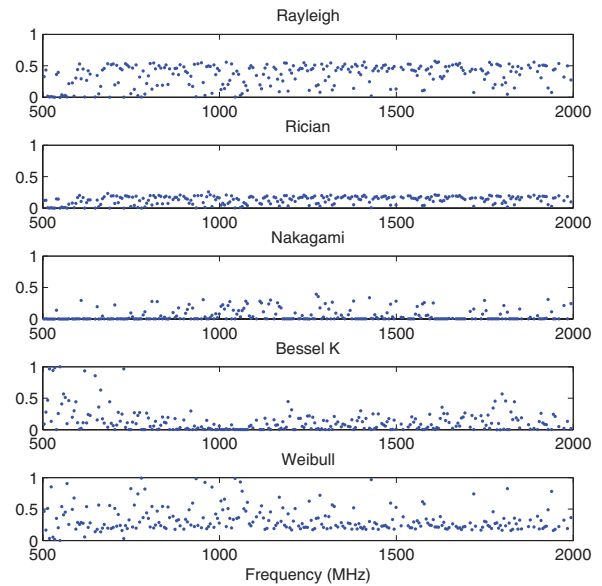


Fig. 6. Comparison of AIC weights for Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions based on the *load0* data.

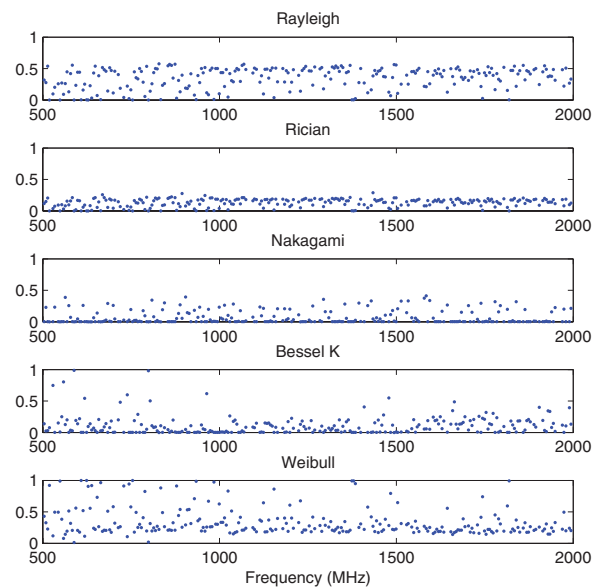


Fig. 7. Comparison of AIC weights for Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions based on the *load1* data.

It is shown that by loading the RC it is possible to create Rician distributed field. (According to [15], the unstirred multipath component (UMC) has the same effect as the line-of-sight (LOS) component [15]. By locating the loads in the corners of the RC, they reduce only the scattered power not the LOS or UMC power. Hence, the K-factor can be increased by loading.) In order to study, the feasibility of the Rician distribution in the RC, the AIC test is applied to the *load1* and *load2* data. The corresponding AIC weights are shown in Figs. 7 and 8, respectively. It is seen that even with increasing loading, the Rician distribution is not feasible for the RC field. The reasons why the Rician distribution is inferior to the Rayleigh distribution are that both transmit and receive antennas are nondirective and

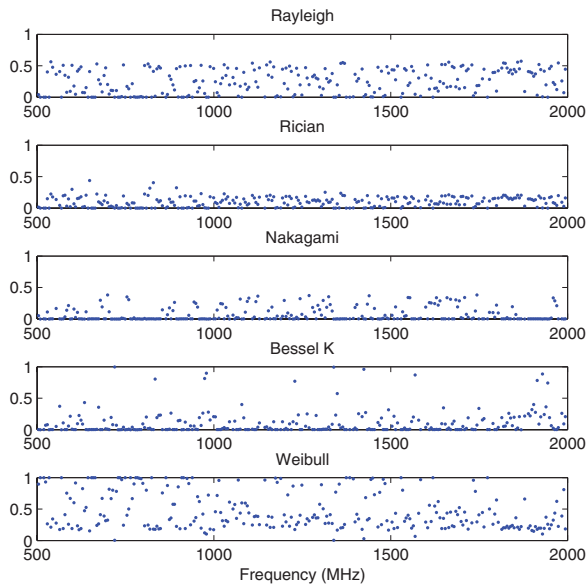


Fig. 8. Comparison of AIC weights for Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions based on the *load2* data.

that the platform stirring effectively reduces the potential unstirred components even with increasing loading [31]. On the other hand, the Weibull distribution shows better fit to measurements in the loaded RC.

In order to be able to observe the Rician distributed field, one has to restrict the platform position to one and with one wall antenna, because the LOS components are different with different platform position and wall antennas [31]. Instead of doing another set of measurements with one platform position, a subset of the measured data corresponding to one platform position is selected. By doing so, the number of samples reduces to  $N = 50$  (i.e., 50 plate stirrer positions). The corresponding reduced data are referred hereafter as a platform subset of the data (and different platform subsets have very similar statistics).

Note that since  $N/p < 40$  for this case, the AICC (4) has to be used. Figs. 9 and 10 show the corresponding AICC weights of a platform subset of *load1* and *load2* data, respectively. It can be seen that the AICC weights of the Rician distribution increases with increasing loading and that it almost becomes comparable to that of the Rayleigh distribution for a platform subset of the *load2* data. Note that Bessel K's AICC weight reduces by restricting the platform position number to one even in the lower frequencies. This is probably due to the fact that the ML estimator for the Bessel K distribution is more sensitive to sample number than the other ones and that by limiting  $N = 50$  its parameter estimation degrades, which in turn degrades its AICC performance [32].

From Figs. 6–10, it can be seen that the feasibility frequency of the Weibull distribution (i.e., the frequency above which the Weibull distribution reasonably fits the measured data) increases with increasing loading. This implies that the RC's actual LUF is affected by the loading (and the effectiveness of the mode stirrers). This observation agrees with the result in [24]. The

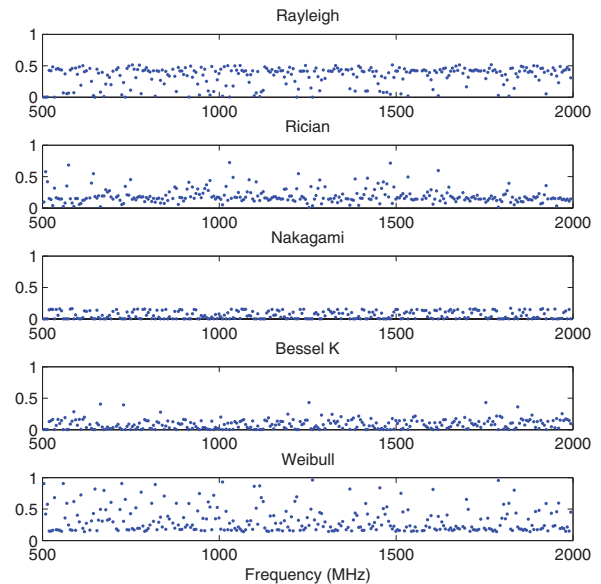


Fig. 9. Comparison of AICC weights for Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions based on a platform subset of the *load1* data.

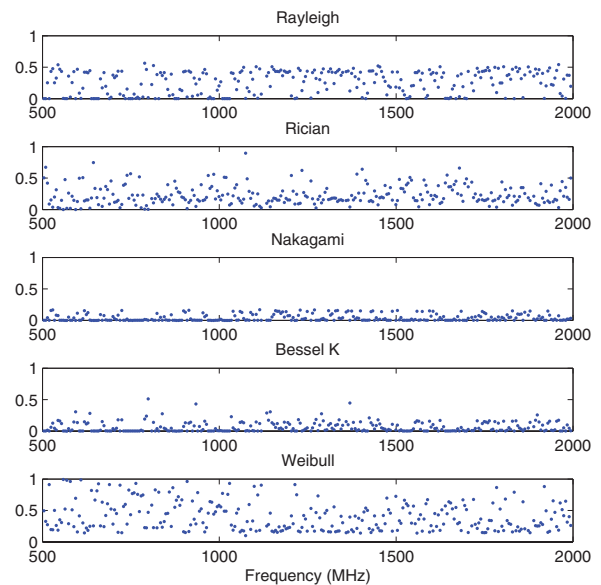


Fig. 10. Comparison of AICC weights for Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions based on a platform subset of the *load2* data.

Nakagami distribution, however, gives the worst overall performance in almost all cases.

## V. CONCLUSION

In this paper, the AIC approach is introduced to select the best approximating distribution for the field in an RC. Unlike the GOF tests, the AIC approach provides fair comparisons between different distribution candidates (with possibly different scalar parameter numbers). With the Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions as the candidate set, the AIC approach is applied to the measured data in an RC. It is found that the field in an undermoded RC is most fitted by the Weibull distribution and that the Rayleigh distribution

approximated the field in an overmoded RC the best. By restricting the platform position to one, it is shown that the Rician distribution provides better approximation with increasing loading. It is also found that the Weibull distribution provides better fits to the measured data with larger loading. The intuitive explanation for this is that with increasing loading the RC becomes less stirred (or equivalently undermoded).

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**Xiaoming Chen** received the B.Sc. degree from Northwestern Polytechnical University, Xi'an, China, in 2006, and the M.Sc. and Ph.D. degrees from the Chalmers University of Technology, Gothenburg, Sweden, in 2007 and 2012, respectively.

He is currently a Research Associate in the Department of Signals and Systems, Chalmers University of Technology. His current research interests include reverberation chamber measurements, multiantenna channel characterization, and statistical electromagnetics.

Dr. Chen is a reviewer of the IEEE TRANSACTION ON ANTENNAS AND PROPAGATION, IEEE TRANSACTION ON ELECTROMAGNETIC COMPATIBILITY, IEEE TRANSACTION ON WIRELESS COMMUNICATIONS, and IEEE TRANSACTION ON VEHICULAR TECHNOLOGY.