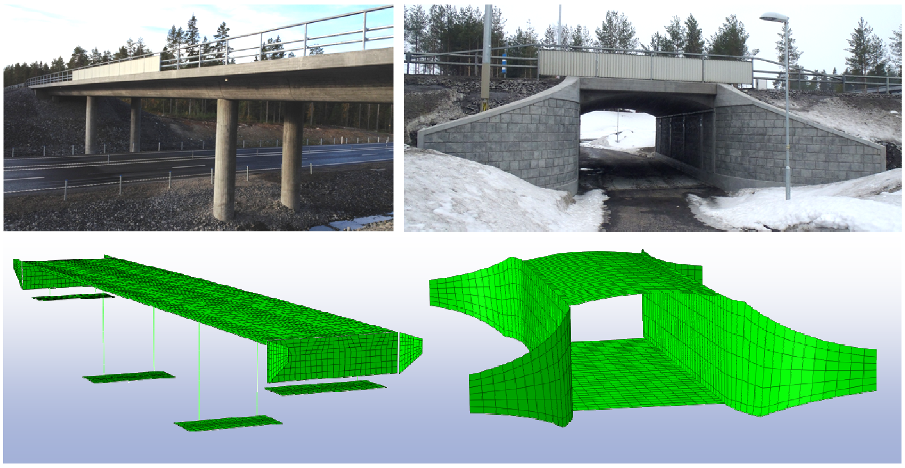


# Structural analysis and design of concrete bridges

## Current modelling procedures and impact on design

*Master of Science Thesis in the Master's Programme Structural Engineering and Building Performance Design*


MATTIAS GRAHN

Department of Civil and Environmental Engineering
*Division of Structural Engineering*
*Concrete Structures*
CHALMERS UNIVERSITY OF TECHNOLOGY



Göteborg, Sweden 2012
Master's Thesis 2012:162

MASTER'S THESIS 2012:162

# Structural analysis and design of concrete bridges

Current modelling procedures and impact on design

*Master of Science Thesis in the Master's Programme Structural Engineering and Building Performance Design*

MATTIAS GRAHN

Department of Civil and Environmental Engineering
*Division of Structural Engineering*
*Concrete Structures*
CHALMERS UNIVERSITY OF TECHNOLOGY

Göteborg, Sweden 2012

Structural analysis and design of concrete bridges
Current modelling procedures and impact on design

*Master of Science Thesis in the Master's Programme Structural Engineering and Building Performance Design*
MATTIAS GRAHN



Examensarbete / Institutionen för bygg- och miljöteknik,
Chalmers tekniska högskola 2012:162

Department of Civil and Environmental Engineering
Division of Structural Engineering
Concrete Structures
Chalmers University of Technology
SE-412 96 Göteborg
Sweden
Telephone: + 46 (0)31-772 1000

Cover:
Visualization of structural analysis models of bridges studied in studied in the thesis.

Chalmers reproservice / Department of Civil and Environmental Engineering
Göteborg, Sweden 2012

Structural analysis and design of concrete bridges
Current modelling procedures and impact on design

*Master of Science Thesis in the Master's Programme Structural Engineering and Building Performance Design*

MATTIAS GRAHN
Department of Civil and Environmental Engineering
Division of Structural Engineering
Concrete Structures
Chalmers University of Technology

## ABSTRACT

Current practice for design of reinforced concrete bridges is based on a linear elastic structural analysis in which a suitable distribution of sectional forces is sought. In Sweden such an analysis is today required to account for the structural response in its entirety, implying a demand for three-dimensional (3D) models capable of describing the force distribution in longitudinal and transverse directions.

To account for structural behaviour in a model there is an abundance of different modelling techniques which can be utilised. With modern user-friendly 3D finite element analysis software the amount of available modelling techniques becomes even more apparent. This thesis investigates different methods for 3D structural analysis of concrete bridges at a design stage. The aim is to illuminate differences and the impact different modelling procedures and choices made in the modelling process has on the resulting design of a bridge.

Case studies have been performed where different modelling techniques was investigated. Different models have been established and the results from structural analyses compared.

Resulting sectional forces and reinforcement design show only small differences between different modelling techniques. The results show that the choice of structural model for analysis at a design stage has little impact on the results. As long as a structural model doesn't introduce errors in the evaluation of the response, other parameters are more important than the accuracy in the model. Such parameters are for example user friendliness, verifiability and interpretation of results.

The results also indicate that there is a lack of established procedures and guidelines for modelling and verification. Such guidelines would simplify the work for engineers when establishing structural analysis models and set the foundation for collaboration and common working procedures within the industry.

Key words: Structural analysis, design of bridges, concrete structures, finite element analysis, modelling procedures, 3D analysis, Brigade/Plus, Abaqus CAE

Systemanalys och dimensionering av betongbroar
Modelleringsmetoder och dess inverkan på dimensionering

*Examensarbete inom Structural Engineering and Building Performance Design*

MATTIAS GRAHN
Instutitionen för bygg- och miljöteknik
Avdelningen för konstruktionsteknik
Betongbyggnad
Chalmers tekniska högskola

SAMMANFATTNING

Praxis för utformning av armerade betongbroar baseras idag på en linjärelastisk systemmodell där en lämplig fördelning av snittkrafter eftersöks. Trafikverket ställer sedan införandet av bronormen TK-bro krav på att en sådan modell ska kunna beskriva konstruktionens verkningssätt i sin helhet. I praktiken innebär detta att tredimensionella (3D) modeller efterfrågas, där kraftfördelningen erhålls i relevanta riktningar.

Det finns flera metoder för att ta hänsyn till en konstruktions verkningssätt vid modellering. Detta märks inte minst vid 3D-modellering där modern finit element-programvara ger användaren stora möjligheter att upprätta och styra en modells verkningssätt. Det här examensarbetet undersöker olika metoder för 3D strukturanalys av betongbroar vid dimensionering. Målet är att belysa skillnader och undersöka vilken inverkan på utformning olika val gjorda vid modellering har på den slutliga utformningen av konstruktionen.

Fallstudier, där skillnader mellan olika modelleringstekniker studeras, har utförts. Resultat från upprättade systemmodeller har studerats och jämförts.

Endast små skillnader mellan resulterande snittkrafter och armeringsutformning har kunnat påvisas. Resultaten tyder på att valet av strukturmodell har liten inverkan på resultatet. Så länge direkta fel i modellen undviks visar resultaten på att andra parametrar än modellens noggrannhet är viktigare vid modellens utformning, exempelvis användarvänlighet och möjligheten att verifiera och tolka resultat.

Vidare har det identifierats en avsaknad av etablerade riktlinjer för modellering och verifiering av strukturanalysmodeller. Sådana riktlinjer skulle förenkla arbetet för konstruktörer och granskare samt sätta grunden för tydligare, branschgemensamma arbetsätt vid konstruktionsarbete.

Nyckelord: Systemanalys, strukturanalys, dimensionering av broar, betongbroar, finit element analys, FEM, modelleringsmetoder, 3D analys

# Contents

# Preface

This thesis has been written as the concluding part of the MSc. programme in "Structural Engineering and Building Performance Design" at Chalmers University of Technology. The thesis has been written at the Bridge- and Civil group of Skanska Teknik in Göteborg in collaboration with the division of Structural Engineering at Chalmers.

When designing bridges it is today required that a structural analysis describes the actions of the structure in its entirety. In practice this means that a 3D-model has to be established. However, the practice of using 3D-models for design has only recently been widespread and therefore the practice lacks clear guidelines as to how a model should be established. Therefore, several procedures exist and often differ between different companies, level of education and designer.

By investigating different modelling procedures and comparing the results it is the aim of this thesis to illustrate some differences which arise when a model is established.

It has from the beginning been my wish as author that this work may come to some use for practicing engineers. The aim of the thesis was not to thoroughly investigate all aspects of structural analysis, but rather to illuminate the difficulties of structural analysis, and the impact a structural model has on the design. In the thesis the structural analysis process of two common types of bridges are studied in order to determine differences. The purpose was not determine "right" and "wrong", but rather to illustrate the impact of modelling and perhaps point out the direction of "more right" or "less wrong". In short, the results showed that it is more important to avoid errors when modelling than it is to model with complete accuracy.

Examiner and supervisor from the University for the thesis has been associate professor PhD Mario Plos. PhD Helén Broo has served as supervisor from Skanska Teknik, whose assistance with everything from modelling and design of bridges to report writing and presentation of results has proved invaluable. Everyone else at Skanska Teknik should also be thanked for providing a stimulating and inspiring working environment during which this thesis was written.

The knowledge I have gained during the writing of this thesis has given me confidence to begin my career as a practising structural engineer. It is my hope that at least parts of the work in this thesis may serve as an inspiration to anyone who reads it.

Göteborg 2012

Mattias Grahn

# 1 Introduction

Design of reinforced concrete bridges is normally done on the basis of a structural analysis. The purpose of the analysis is to find a distribution of sectional forces which fulfils equilibrium and is suitable for design. In the past structural analyses were often done with simplified models, for example two-dimensional (2D) equivalent beam or frame models. Such a model is not able to describe the distribution of forces in transversal directions. Therefore a design according to a 2D equivalent model will not be according to the true linear elastic distribution, even though the design might fulfil requirements in ultimate limit state (ULS) after sufficient plastic redistribution.

With the recent introduction of Eurocode and the Swedish Transport Administrations, Trafikverkets, new technical requirements for bridges TK Bro, Trafikverket (2009a), the demands on structural analysis has been updated. A model for structural analysis has to be able to describe the response of the structure in its entirety. In practice this implies that 2D equivalent models are not sufficient and a 3D analysis describing the forces in multiple directions is needed.


*Figure 1.1. Example of documents governing structural analysis and design of concrete bridges in Sweden. From the left: SS-EN 1991-2 Eurocode 1 Part 2: Traffic loads on bridges, SS-EN 1991-2 (2007), SS-EN 1992-1 Eurocode 2 Part 1-1: Design of concrete structures – General rules and rules for buildings, SS-EN 1992-1 (2008), SS-EN 1992-2 Eurocode 2 Part 2: Design of concrete structures – Concrete bridges – Design and detailing rules, SS-EN 1992-2 (2005) and TK Bro, technical requirements for bridges in Sweden, Trafikverket (2009a).*

Even though 3D-models have been used for design of bridges to a varying degree for quite some time, it is only recently that it actually has been set as a requirement. In order to cope with these demands new methods are used for structural analysis. Generally these methods are more sophisticated and advanced e.g. 3D finite element (FE) models, where the designer has large freedom in constructing the model. The choices available in the modelling procedure are large; hence the same structure can be modelled in several different ways.

Today there are no clear guidelines for designers when establishing 3D-models, which presents problems since the impact of choices made during modelling stage has not been properly investigated. This thesis aimed towards investigating and illuminating some effects these choices in establishing a structural model might have on the design of a structure. The study was focused on concrete road bridges, though several of the concepts and modelling procedures are general and should be applicable to other types of concrete structures.

## 1.1 Aim

The aim of the study was to investigate and compare current methods in analysis of bridge structures. This has been done in order to investigate and illuminate actual differences between modelling procedures and how choices made in a modelling stage impact the resulting design of reinforcement in a reinforced concrete road bridge.

## 1.2 Methodology

In order to investigate the resulting differences between modelling methods, two case studies was performed; one including an integral slab frame bridge and one including a 3-span double beam bridge. The bridges are designed according to the different models and current praxis. Sectional forces and the resulting designs are compared in order to illustrate the impact different choices in a modelling stage have on resulting design.

The established models are chosen on basis of a short literature study where handbooks and articles concerning finite element modelling and design of concrete structures and bridges are studied. The survey focuses on current modelling procedures and covers some possible choices in a modelling stage.

The software used for FE-analyses in this thesis is Brigade/Plus 3.4-5 and 4.1-2. Brigade uses the well-known Abaqus solver, and anyone familiar with the Abaqus environment should be able to use Brigade/Plus. In addition to the Abaqus CAE graphical users interface which allows for complex geometry modelling, the software uses well-developed methods for handling traffic loads and load combinations. This makes it suitable for state-of-the-art bridge modelling, which is why the software is widely used for analyses of bridges in Sweden. For more information please see to the Brigade/Plus, Scanscot Technology AB (2010), and Abaqus user’s manual, Dassault Systèmes (2008).

## 1.3 Limitations

The bridges modelled in the thesis are subjected to a simplified combination of loads in the ultimate limit state. These loads include;

- Self-weight and surfacing
- Shrinkage
- Earth pressure, including earth pressure increase due to horizontal loading
- Surcharge
- Uniform temperature change and temperature gradient on superstructure

- Traffic loads, load models 1 and 2 according to Eurocode including lateral and horizontal loading due to acceleration and braking
- Wind load

Load combinations included in the analyses include load combination for ULS as used in design of bridges in Sweden. Since the current practice in design of concrete bridges involves linear elastic analysis, non-linear analysis will not be included in the thesis.

The thesis has for time-constraint reasons been limited to analysis of the superstructure only. Calculation of required reinforcement amounts has been performed for the main reinforcement in longitudinal and transverse directions. The substructure has been included in the studied models, though results from substructure construction elements have not been analysed.

## 1.4 Outline of the thesis

Chapter 2 begins by shortly describing aim and theoretical background for structural analysis of load carrying structures. In the chapter the concepts of linear elastic modelling and finite element method are also presented.

Chapter 3 concerns design of bridges on the basis of a linear elastic structural analysis model and, more specifically, reinforcement design according to element sectional forces. For those not familiar with traffic loading on bridges a short introduction to load models according to Eurocode is also presented.

Chapter 4 introduces modelling and problem areas identified in literature when establishing structural analysis models with finite element method (FEM). Presented issues concerns modelling of corner and discontinuity regions, modelling of soil-structure interaction, modelling of haunches and load application procedures. In the chapter issues concerning choice of elements are also briefly presented.

Chapter 5 describes investigation and results of different modelling procedures for an integral slab frame bridge. The chapter presents modelling from a practical point of view, presents differences between structural analysis models and the impact different models has on resulting sectional forces and design. Short reflections on the results are also presented.

Chapter 6 describes investigation of a double beam bridge. Two models are investigated for modelling of the bridge deck and main beams; a combination of beam and shell elements and modelling with a box-like spatial assembly of shell elements.

Chapter 7 summarizes and presents discussions on results and recommendations for further work.

# 2 Structural analysis of concrete bridges

The purpose of a structural analysis is to evaluate the response of a structure. This can be of interest for many reasons, where the most common are design of new structures or assessment of existing ones. In design one aims towards providing a distribution of sectional forces, see Figure 2.1, while assessment aims towards accurately describing the response during loading and/or failure.

In the current European design code for concrete structures, Eurocode 2, SS-EN 1992-2 (2005), four examples of methods for structural analysis are presented. These are:

- Linear elastic analysis
- Linear elastic analysis with limited redistribution
- Plastic analysis
- Non-linear analysis

Out of these methods it is only the non-linear which is capable of accurate prediction of the response during loading and describe the complex force redistribution taking place when cracking of concrete and yielding of reinforcement occurs, Engström (2011a). This means that it is only the non-linear analysis which accurately predicts the behaviour of the structure in service state, and the mode of ultimate failure. However, the non-linear analysis requires substantial effort in establishment and post-processing of the model, as well as a large computational effort. It also requires the knowledge of the complete layout of the structure beforehand, making it a method suitable for accurate assessment of existing structures but not suitable for design purposes since this knowledge is not available at a design stage. Another major drawback for non-linear modelling in a design stage of reinforced concrete bridges is that non-linear analysis does not allow for load superposition. For bridge design applications with many different loads and load combinations it is essential from a practical point of view that load superposition is possible.

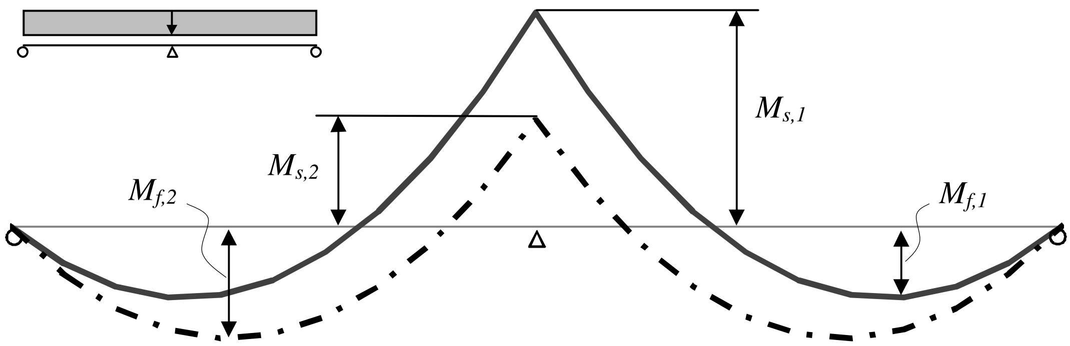



*Figure 2.1. Possible moment distributions for design in ultimate limit state of a uniformly loaded continuous beam.*

When performing a structural analysis at a design stage the aim is to provide a realistic and suitable distribution of sectional forces which fulfils equilibrium and can be used for design of the cross-sections in ULS. Since most bridge structures are statically indeterminate structures, there are many distributions (in fact an infinite number) that fulfils equilibrium, though of course not all are suitable for design with regard to plastic redistribution and serviceability.

## 2.1 Linear elastic modelling

A linear elastic model assumes that the behaviour of the structure is linearly dependant of the applied load. This is a simplification of the behaviour of reinforced concrete. The linear elastic model will only describe the “true” distribution of forces under certain conditions, such as uncracked sections. Those conditions are generally never achieved, since concrete structures often crack even for relatively low service loads. Concrete will crack and force redistribution will take place in the structure (providing that the structure is statically capable of redistribution). Consequently the designer can choose a distribution in ultimate limit state. The structure will then, providing that it has sufficient plastic rotational capacity, adapt to the provided capacities in each sections. Hence, the distribution in ULS will approach the distribution provided by a linear elastic analysis if reinforcement is designed accordingly.

## 2.2 The finite element method and element formulations

The finite element method (FEM) is a general tool for solving differential equations suitable for structural engineering applications. FEM is capable of handling large structural mechanics problems by discretisising the problem into a finite number of elements, which in turn is governed by equations. Since the element equations govern the model and the results, it is important that designers have a fair understanding of the underlying assumptions of these elements.

The elements used in FEM can roughly be categorized in three different categories; continuum elements, structural elements and special purpose elements. Continuum elements describe the structure as a continuum, and give the stress-state in the structure. This includes 3D solid elements and 2D plane stress/strain elements. Since these elements work with the stresses of the structure they describe the real behaviour of the structure and lack some of the limitations of the structural elements. However, the output from an analysis based on such elements is often massive and difficult to apply on reinforced concrete design since the design of such structures are more easily done based on sectional forces. In order to design concrete structures according to current praxis on basis of 3D-volume elements, integration of cross-sections has to be performed in order to get the sectional reactions.

Structural elements are based on the equations of for example beam and plate theory. This makes structural elements suitable for design since they provide sectional forces directly for each cross-section. Structural elements also allows for a simpler and more intuitive modelling process, see Figure 2.2.

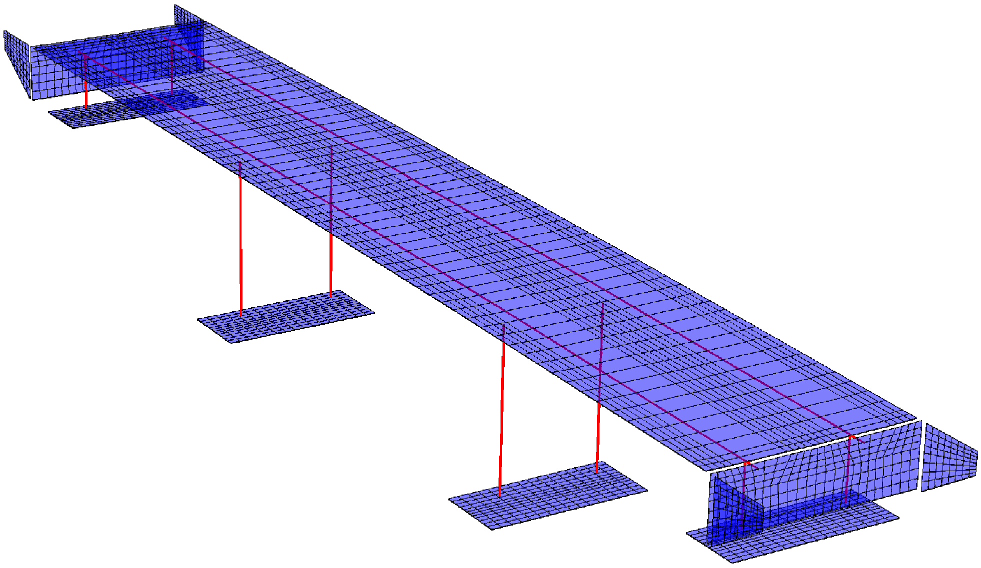


*Figure 2.2. Example of finite element structural analysis model of a double beam bridge. Beam elements shown as lines (red) represent main beams and columns, shell elements shown as surfaces (blue) represent bridge deck, end shields, wing walls and foundation slabs.*

The basic assumption of beam- and shell elements is that they rely on linear strain distribution over the cross-section (e.g. plane sections remain plane during loading, Bernoulli-theorem). This assumption has large impact on certain areas of a structure, such as frame corners, deep beams or holes, where the strain distribution diverge from the assumed linear distribution. In order to cope with this a designer might use different techniques, discussed further in chapter 4.

Special purpose elements are used to describe certain conditions in the model, such as interaction between structural parts or foundations.

# 3 Design of reinforced concrete bridges on the basis of linear elasticity

The response of reinforced concrete is highly non-linear, Plos (1996), where effects such as cracking and yielding of reinforcement have large impact on the stiffness of the structure. This causes redistribution of sectional forces when stiffer sections become more stressed, which is why a linear elastic model is unable to describe the behaviour of a reinforced concrete structure. However, thanks to the structures ability to redistribute forces linear elastic models can be used for design. If the structure has sufficient capacity for plastic rotation, which governs the ability to redistribute forces, the structure will be able to utilize its capacity in all critical sections before a mechanism is formed and failure occurs.

By this reasoning any distribution of sectional forces fulfilling equilibrium is valid for design in ultimate limit state, as long as the structure is able to fully develop the provided capacity in critical sections by yielding of weaker sections. If sufficient ductility is provided in yielded sections, redistribution will continue until all critical sections yield and a mechanism is formed causing structural failure.

Design according to linear elastic analysis is generally considered as a good design approach. It is normally assumed that a design according to linear elastic analysis will require only small amounts of plastic rotation in order for the sections to utilize their maximum capacities, Engström (2011a).

## 3.1 Critical section

The critical cross sections for design depend on the expected modes of failure. For a cast connection, such as a frame corner or monolithic column, the critical crack for bending failure will form along the column or wall surface, Sustainable Bridges (2007); hence this is where reinforcement can be expected to yield first. When designing a structure according to a linear elastic moment distribution, it is therefore sufficient to design for the moment at the column or wall face.

The critical shear crack for such cast connections can be assumed to have an inclination less than 45 degrees. The critical section for shear will therefore be situated at a distance equal to the internal lever-arm from the face of the column or wall, Sustainable Bridges (2007), or approximated to the thickness of the member, Rombach (2004). The shear force inside of that section will be carried to the column/wall and not be critical for shear failure.

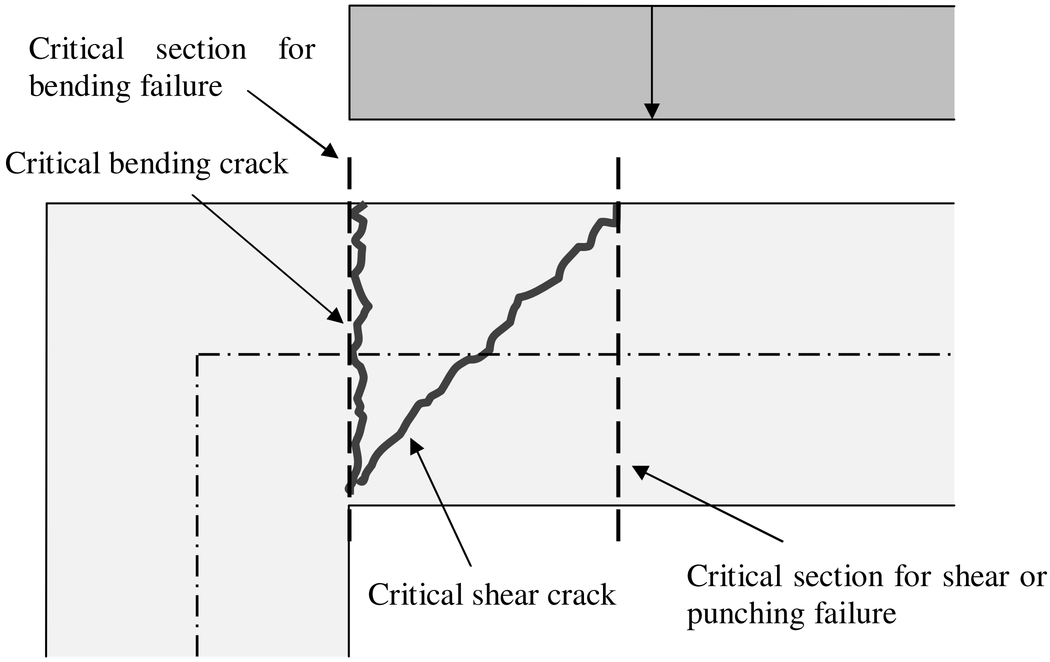



*Figure 3.1. Illustration of critical sections at a support region of a frame corner loaded on the top surface.*

## 3.2 Redistribution of moments

Due to the non-linear behaviour and the capacity of a structure to redistribute forces and moments, a redistribution of moments compared to the linear elastic solution is possible. Redistribution will take place when concrete cracks and when reinforcement yields, Engström (2011a). Continuous redistribution will also take place in the concrete due to its non-linear behaviour after cracking (stage II). When a concrete slab cracks, torsional moments will be taken as bending moments in the sections. Redistribution will also take place between sections, e.g. from support to field sections, and within sections, e.g. from primary to secondary reinforcement directions in a slab.

In Eurocode the allowable redistribution of sectional moments is governed by the need for plastic rotation and ductility of the cross sections, SS-EN 1992-1 (2008). When a design has been made of a cross-section the ductility can be checked with relative ease by using the ratio between the compressive zone of concrete, $x_u$, and the effective height of the cross-section, $d$, see equation 3.1.

$$\begin{aligned} &\frac{x_u}{d} \leq \frac{0.80\delta - 0.35}{\beta_c} \quad \text{for concrete grades C12/15 to C50/60} \\ &\frac{x_u}{d} \leq \frac{0.80\delta - 0.45}{\beta_c} \quad \text{for concrete grades C55/67 and higher} \end{aligned} \tag{3.1}$$

In addition the ductility of the reinforcement steel should allow for sufficient redistribution, SS-EN 1992-2 (2005), according to equation 3.2.

$$\begin{aligned} &\delta \geq 0.7 \quad \text{for reinforcement steel of ductility class B and C} \\ &\delta \geq 0.8 \quad \text{for reinforcement steel of ductility class A} \end{aligned} \tag{3.2}$$

## 3.3 Design of beams

Design of beam elements can be made on the basis of sectional forces acquired from a linear elastic structural analysis. For this a non-linear cross-sectional analysis can be used, Sustainable Bridges (2007).

The forces applied to a cross-section are to be resisted by the reactions in the concrete compressive zone and the reactions in tensile reinforcement. Compressed concrete will together with the tensile reinforcement form a force couple to balance the moment acting on the section.

For preliminary simplified design the required reinforcement area can be estimated using simplified expressions, Al-Emrani, et al. (2008). By assuming an internal lever arm and yielding of tensile reinforcement the resisting moment can be calculated and required amount of reinforcement estimated.

Estimation of internal lever arm of a ductile section subjected to pure bending, see equation 3.3.

$$z \approx 0.9 \cdot d \tag{3.3}$$

Estimation of moment capacity with yielding of reinforcement can then expressed according to equation 3.4.

$$M_{Rd} \approx f_{yd} \cdot A_s \cdot 0.9d \tag{3.4}$$

Which gives an estimation of required reinforcement area according to equation 3.5.

$$A_s \approx \frac{M_{Ed}}{f_{yd} \cdot 0.9d} \tag{3.5}$$

## 3.4 Design of membranes, plates and shells

An optimal layout of reinforcement would be in the directions of the principal tensile stresses, Fib Bulletin 45 (2008). However, these directions will shift when a structure is loaded differently and it may also result in an unpractical reinforcement arrangement. Therefore it is more suitable to place the reinforcement orthogonally, along the structures primary longitudinal and transversal axes.

A 3D-plate element subjected to bending, see Figure 3.2, will have 5 stress resultants consisting of bending and twisting moments $m_x$, $m_y$ and $m_{xy}$ as well as out-of-plane shear forces $v_x$ and $v_y$.

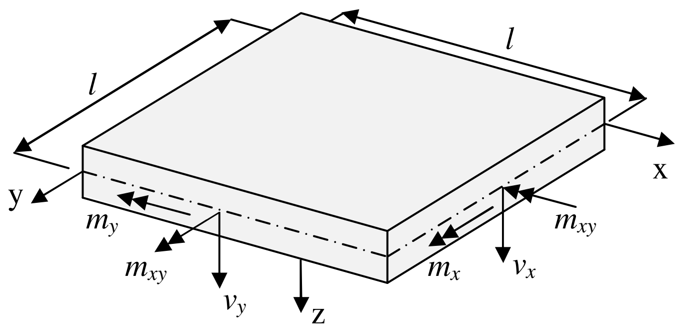



*Figure 3.2. Stress resultants on a 3D-plate element subjected to bending. Adopted from Blaauwendraad (2010).*

In order to achieve a reasonable reinforcement arrangement with longitudinal reinforcement placed in x- and y-directions the torsional moment has to be resisted by the provided reinforcement in those directions. The torsional moment is included into the ordinary moments in x- and y-directions rather easily with equation 3.6, Sustainable Bridges (2007).

$$
\begin{aligned}
m_{xt,pos} &= m_x + \mu_1 \cdot |m_{xy}| \\
m_{xt,neg} &= m_x - \mu_2 \cdot |m_{xy}| \\
m_{yt,pos} &= m_y + \frac{1}{\mu_1} \cdot |m_{xy}| \\
m_{yt,neg} &= m_y - \frac{1}{\mu_2} \cdot |m_{xy}|
\end{aligned}
\tag{3.6}
$$

where $m_x$, and $m_y$ are the plane bending moments, $m_{xy}$ is the twisting moment. $\mu_1$ and $\mu_2$ are factors chosen, for simplicity often set to 1.

In a similar way the in-plane shear force is included with the ordinary membrane forces for membrane elements according to equation 3.7, Blaauwendraad (2010).

$$
\begin{aligned}
n_{xt} &= n_x + |n_{xy}| \\
n_{yt} &= n_y + |n_{xy}| \\
n_c &= -2|n_{xy}|
\end{aligned}
\tag{3.7}
$$

where $n_x$, and $n_y$ are the membrane forces and $n_{xy}$ is the in-plane shear force. $n_c$ is the compressive force in the compressed concrete strut, see Figure 3.3.

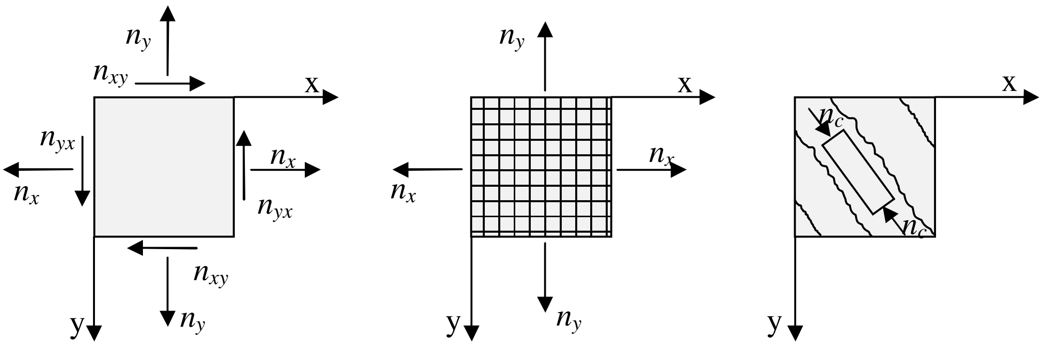



*Figure 3.3. Stress resultants in a 3D-membrane element, resulting reinforcement forces and compressive strut in concrete. Adopted from Blaauwendraad (2010).*

A general shell element is subjected to a combination of membrane and bending actions. The design of a combined membrane/bending-element can be done by combining the plate-bending and membrane action models into a sandwich model, see Figure 3.4, Fib Bulletin 45 (2008), SS-EN 1992-2 (2005). In the sandwich model, the cross section is divided into three layers where the outer layers are subjected to membrane action and the middle layer resists the out-of-plane shear force and provides a lever arm for the outer membranes to resist the bending moment.

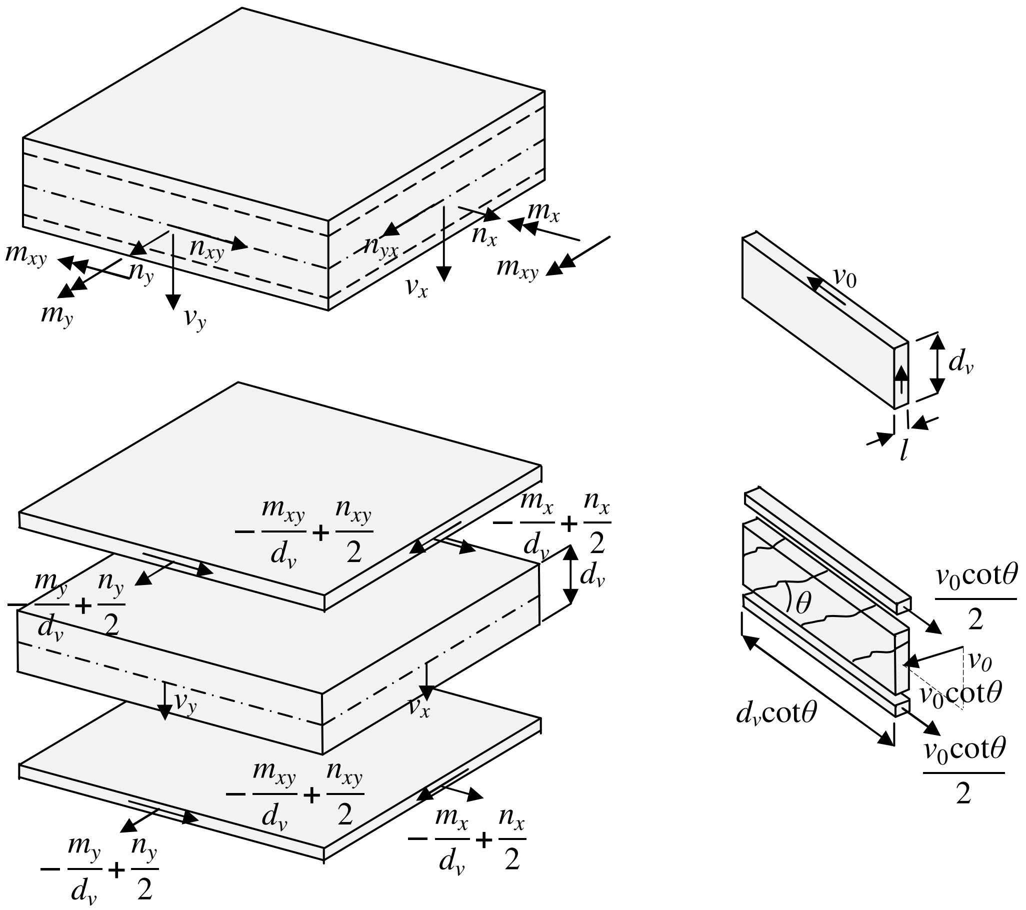


*Figure 3.4. Stress resultants in a shell element (top left), layers and force distribution in the sandwich model (bottom left). To the right, contribution to outer layer membrane forces due to inclined shear cracking in the mid-layer. Adopted from Fib Bulletin 45 (2008).*

By combining the models above for plate- and membrane elements using the middle layers thickness, $d_v$, as internal level arm the expressions in equations 3.8 are obtained.

$$\begin{aligned}
a_{sx} f_y &\geq \frac{m_x}{d_v} + \frac{n_x}{2} + \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} \right| \\
a_{sy} f_y &\geq \frac{m_y}{d_v} + \frac{n_y}{2} + \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} \right| \\
a'_{sx} f_y &\geq -\frac{m_x}{d_v} + \frac{n_x}{2} + \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} \right| \\
a'_{sy} f_y &\geq -\frac{m_y}{d_v} + \frac{n_y}{2} + \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} \right|
\end{aligned} \tag{3.8}$$

where $a_{s\text{-}}$ is the required reinforcement area and $f_y$ is the yield limit of the reinforcement steel. If the middle core layer contains transverse shear cracks the expressions are modified to equations 3.9.

$$\begin{aligned}
a_{sx} f_y &\geq \frac{m_x}{d_v} + \frac{n_x}{2} + \frac{v_x^2}{2v_0} + \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0} \right| \\
a_{sy} f_y &\geq \frac{m_y}{d_v} + \frac{n_y}{2} + \frac{v_y^2}{2v_0} + \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0} \right| \\
a'_{sx} f_y &\geq -\frac{m_x}{d_v} + \frac{n_x}{2} + \frac{v_x^2}{2v_0} + \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0} \right| \\
a'_{sy} f_y &\geq -\frac{m_y}{d_v} + \frac{n_y}{2} + \frac{v_y^2}{2v_0} + \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0} \right|
\end{aligned} \tag{3.9}$$

where $v_0$ is the shear resultant. In the above expression the common choice of crack inclination $\theta=45°$ has been chosen.

This model is somewhat simplified since it is assumed that the core layer does not contribute to the transfer of membrane forces. The internal level arm is assumed to be the same for both reinforcement directions. This can be handled by modifying the model, Blaauwendraad (2010), which results in a more complicated iterative model. However, for the scope of this thesis the simplified model presented above is sufficient.

## 3.5 Traffic loads on bridges

In order to simulate the loading situation due to traffic Eurocode uses a combination of concentrated and distributed loads, SS-EN 1991-2 (2007). The carriageway of the structure is divided into discrete traffic lanes where the loads are applied, see Figure 3.5.

Loads are moved along and between the traffic lanes in order to capture the response for vehicles being situated at different positions. The results are then combined into an envelope which gives the maximum reaction on each element.

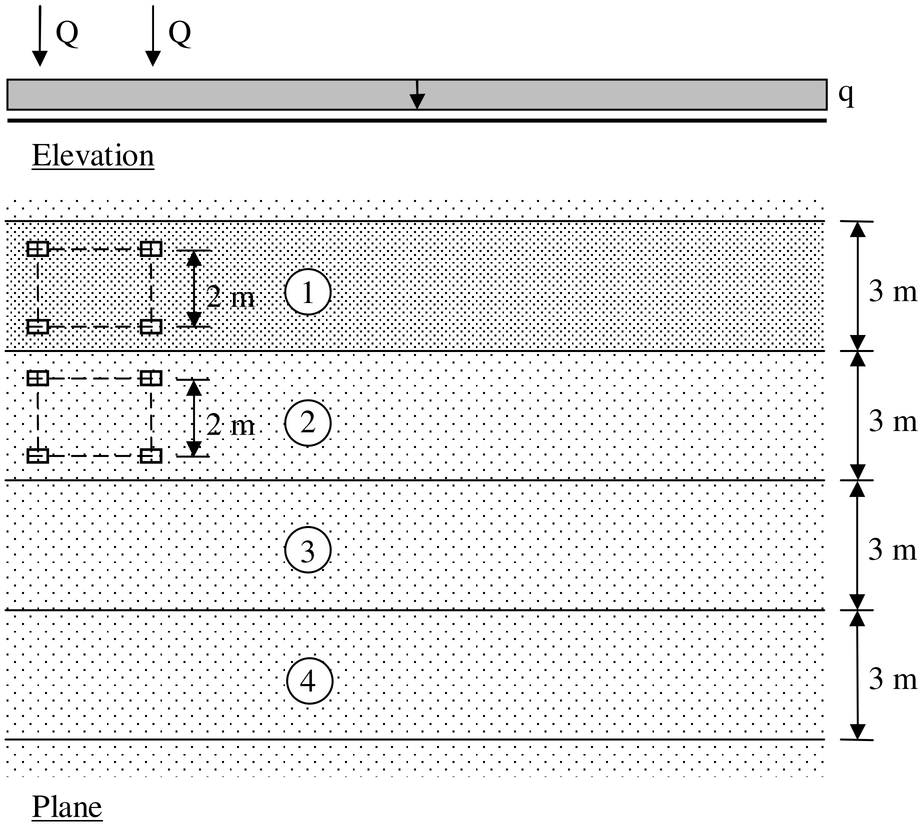



*Figure 3.5: Illustration of lane division of a bridge carriageway. All lanes are loaded with a distributed load, while some lanes are loaded with point loads simulating vehicle positions according to certain rules. Figure adopted from, SS-EN 1991-2 (2007).*

Concentrated loads are applied in groups simulating wheel pairs (axle loads) and bogie loads, see Figure 3.5. Each axle load consists of two concentrated loads, and two axle loads make up one bogie load. Distributed loads are applied onto the whole traffic lane. Values for axle and distributed loads according to Eurocode and Swedish annex are presented in Table 3.1.

*Table 3.1. Values for axle and distributed loads for traffic load model 1 according to Eurocode and Swedish annex.*

| **Traffic lane** | **Distributed load q [kN/m$^2$]** | **Axle loads Q [kN]** |
|---|---|---|
| 1 | 6.3 | 270 |
| 2 | 2.5 | 180 |
| 3 | 2.5 | 0 |
| Remaining lanes | 2.5 | 0 |
| Remaining surface | 2.5 | 0 |

Horizontal loads on bridges (loads in the bridge plane) comes from vehicles braking, accelerating or turning on the bridge. Braking loads are calculated as a part of the total vertical loads acting on a traffic lane. Acceleration loads are defined as the braking loads acting in opposite direction, which practically means that the load can have both positive and negative sign.

# 4 Practical modelling

Performing 3D structural analysis of concrete bridges with FEM allows the user to control several aspects of the modelling. These aspects range from choice of geometric and element representation to choosing to what extent and how closely the model needs to resemble the structural system.

## 4.1 Element mesh

The finite element mesh needs to be sufficiently dense in order to capture the proper response of the structure. A general rule of thumb says that the element size for shell elements should be equal to or smaller than the thickness of the elements.

Near critical sections there has to be a sufficient amount of elements between singular peak values (such as pinned connections) and the critical section. Mesh dependency studies have shown that for different meshes the difference in results is small only one element away from the peak value and two elements away it is negligible, Davidson, (2003) and Sustainable Bridges (2007).

## 4.2 Structural element types

As stated before, structural elements are suitable for structural analysis in a design stage of bridges and structures since the output (sectional forces) allows for simple and intuitive design of the structure. Structural elements are also rather effective at describing the actions of the structure, which is important for analysis of bridge structures where design codes requires analysis of several load cases, positions and load combinations.

## 4.3 Discontinuity regions and frame corners

A structural system can be divided into B- and D-regions. This is done in order to distinguish between areas in a structure where the state of strain diverge from the plane strain assumption (Bernoulli hypothesis) under which beam and plate theory are valid. Hence, D-regions (or discontinuity regions) are areas in a structure where the strains no longer remain linear over the cross-section, see Figure 4.1.

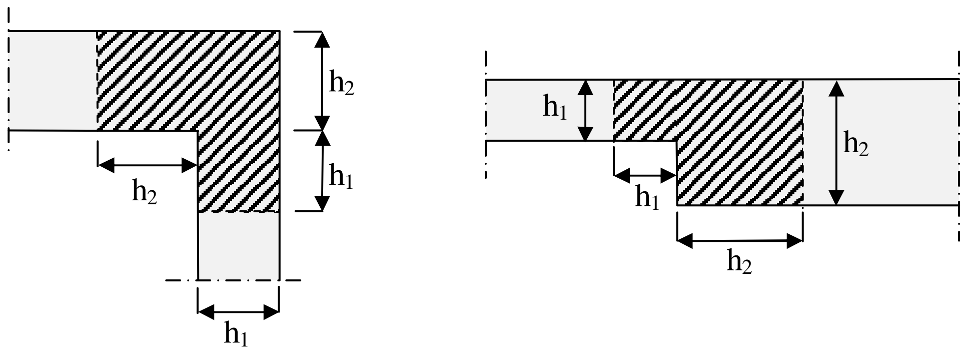



*Figure 4.1. Examples of discontinuity regions, D-regions, where the strain distribution will differ from the linear strain distribution predicted by beam theory. The extent of the discontinuity is often assumed to be equal to the width of the element. Adopted from Engström (2011b)*

These regions may not only affect the response locally, but the modelling of them is also important to correctly assess the response globally. In for example a frame corner, see Figure 4.2, the strain distribution no longer remains linear and, when modelled in detail, the elements within the corner cannot move independently of each other. Often the centre line is modelled with beam or shell elements and, since the corner more or less behaves like a diaphragm, its stiffness becomes underestimated. Therefore the elements within the corner region should be coupled to simulate this effect.

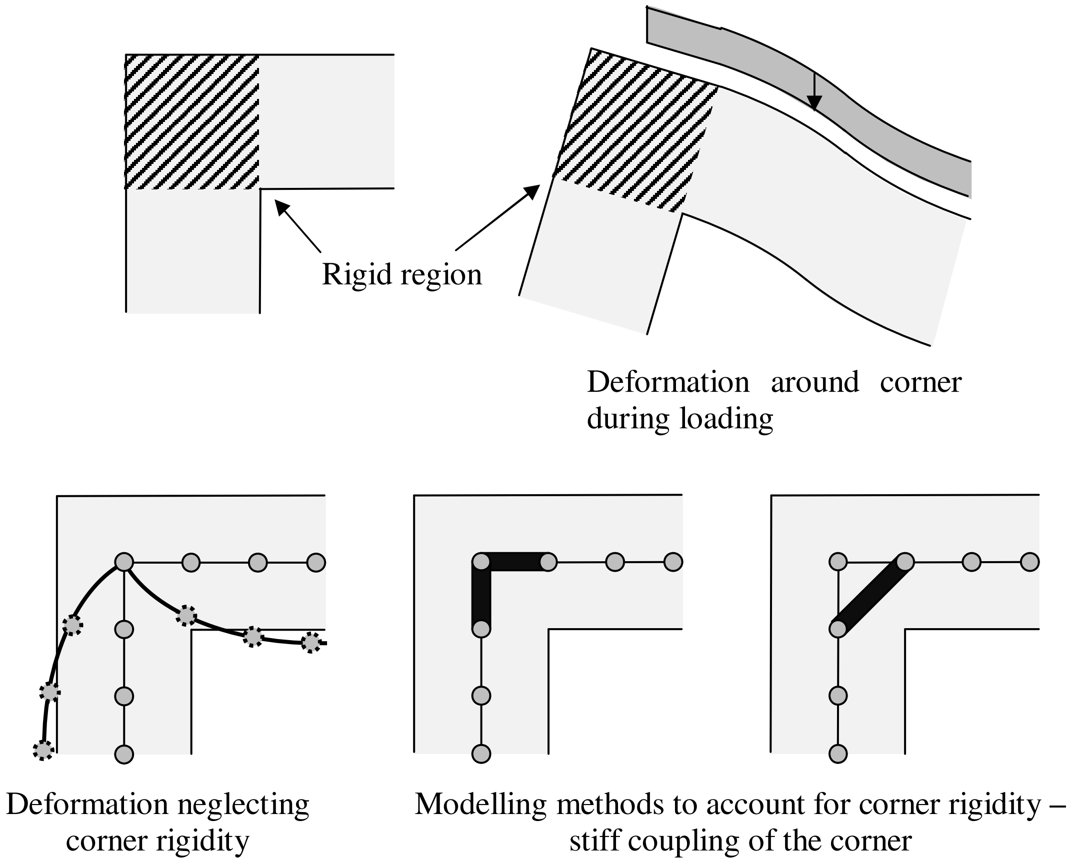



*Figure 4.2. Response of a frame corner during loading and the response of a model neglecting frame corner rigidity. Alternative models for accounting frame corner rigidity are also shown, adopted from Rombach (2004)*

The stiffness increase can be modelled using stiff connections, either assigning the corner elements with rigid properties (very large/infinite stiffness) or introducing an additional, inclined, stiff truss element into the corner region, Rombach (2004).

When assigning stiff material certain care has to be taken, since if the stiffness is to great in relation to the ordinary stiffness, numerical problems when performing the analysis might occur, Rombach (2004). Also when assigning stiff properties for shell elements it is important to model the stiffness increase in the correct direction. In for example a slab frame bridge modelled with shell elements; the stiffness increase is only relevant in the normal direction of the frame. Increasing the stiffness in the transverse direction will cause the stiffness of the wall to increase, causing errors in load cases where bending of the walls occur. However, these effects might be difficult to spot; since the structures principal direction of action is in the longitudinal direction the increased transverse stiffness will only have an effect on certain load cases where transverse bending of the walls will occur. Therefore the effect might be difficult to detect if the response is not evaluated for these individual load cases. Equation 4.2 shows the constitutive relationship for orthotropic material in plane stress elements, such as shells subjected to in-plane loading.

$$
\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \frac{1}{t} \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{bmatrix} \tag{4.1}
$$

## 4.4 Soil-structure interaction

In many cases the supports will deform to some extent when loaded, for example in shallow foundations. Here the soil will deform under the foundation slab when loaded, Rombach (2004), which will influence the structural response. Consequently it should be included in the structural model, Trafikverket (2009a). The soil material and its influence on the structural system can be included in different ways; some examples are presented in Figure 4.3.

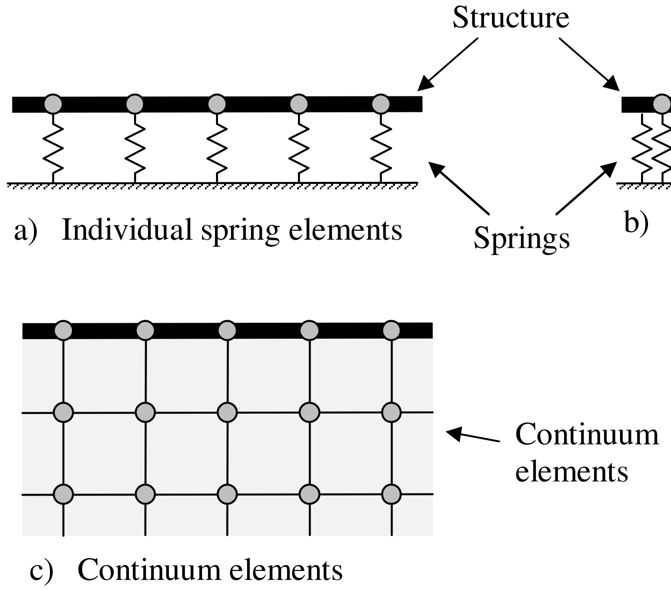



*Figure 4.3. Various models for elastic support. The simplest and most convenient methods to include the foundation materials deformation are by springs (models a – b). A more advanced model would be to represent the surrounding soil material by a continuum. Adopted from Rombach (2004).*

The deformation of the soil material can be included by representing it as springs, either with discrete springs situated at element nodes, see Figure 4.3a, or with an element formulation including continuous bedding, see Figure 4.3b. The stiffness of the springs is characterised by a bedding modulus, representing the normal stiffness of the soil. When using individual spring elements at element nodes, see Figure 4.3a, the spring stiffness will also be dependent on the spacing between the nodes, i.e. it will be element mesh dependant. For the continuous bedding model the spring stiffness is included in the element stiffness matrix and therefore not mesh dependent. However, with such a model it is generally only possible to define springs normal to the elements. It should be noted that the spring methods will give the same results, provided that the element mesh is sufficiently dense.

A simple way to assess the bedding modulus is to perform a simplified settlement calculation with an arbitrary load. Methods for this are for example presented in TR Bro, Trafikverket (2009b). The bedding modulus can then be estimated as a linear relation between settlement and applied load, equation 4.2.

$$k = F / d \tag{4.2}$$

Methods based on spring foundations (or Winkler foundations) will not account for shear interaction between springs, leaving surrounding soil stress-less. This can often be neglected, though it might be important to account for in some situations. Examples are adjacent plates where one foundation does not settle independently of the other or shallow uniformly loaded foundations where an ordinary spring model will not result in any member forces. There are methods for including the shear stiffness of the soil, for example with a so called Pasternak foundation, Blaauwendraad (2010) and Caselunghe & Eriksson (2012), though it is not covered further in this thesis.

Since linear material parameters are generally used, a spring model is unable to describe uplift of the foundation correctly since such a case will result in tension in the springs, not present in the real case where the slab will instead lift from the soil. By using non-linear springs this could be handled though, according to the reasoning in chapter 2, such a model is incompatible with design of bridges.

Interaction with surrounding soil material is relevant also for backfilled vertical members, such as frame walls or end shields on a bridge. The backfill material will provide resistance against horizontal deflection of the structural member, affecting the structural response and should therefore be included in a structural analysis. Backfill can be modelled in a similar manner as foundation interaction, where springs describe the soil in a very simplified manner. Another way to include the backfill is by adding an external load simulating the increased earth pressure due to deflection, Figure 4.4. Such a model is presented in Trafikverkets recommendations document for construction of bridges and other similar civil structures, Trafikverket (2009b).

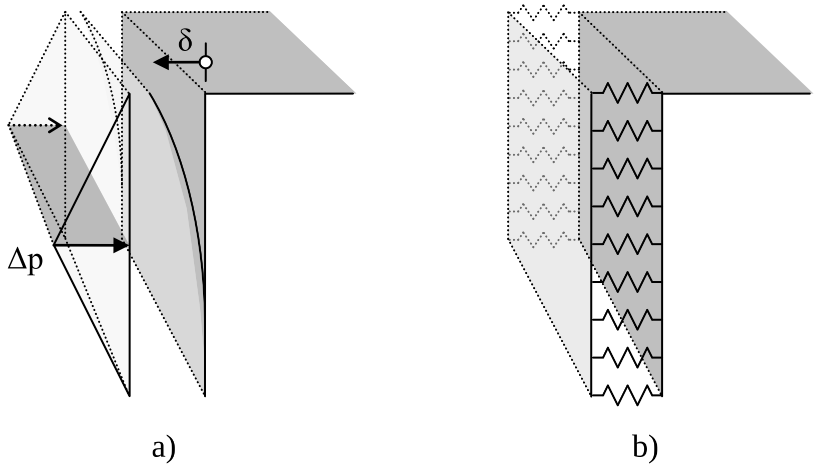



*Figure 4.4. Earth pressure increase models; (a) model according to TR Bro, Trafikverket (2009b) where an additional load, Δp, is added against the vertical member dependant on the members deflection at the upper edge, δ. (b) Model similar to spring foundation models described in 4.4.*

In the model presented by TR Bro the increased earth pressure is given a maximum value at the mid height of the vertical member and the value *0* at top and bottom edge of the member. The load *Δp* is defined as expressed in equation 4.3.

$$\Delta p = c \cdot \gamma \cdot z \cdot \beta \tag{4.3}$$

where:

$$c = \begin{cases} 300 & \text{if earth pressure is favourable} \\ 600 & \text{if earth pressure is unfavourable} \end{cases}$$

$\gamma$ density of soil material

z depth from surface

$\beta = \dfrac{\delta}{h}$ ratio between top deflection, $\delta$, and height, $h$, of the member

## 4.5 Load application

When loaded against a corner region, a structural model represented by member centrelines will not account for loading outside of the centrelines, for example frame corners. This load can be accounted for in different ways presented in Figure 4.5.

- Extending elements over the corner
- Adding point load and moment to account for load and eccentricity of load, Rombach (2004)

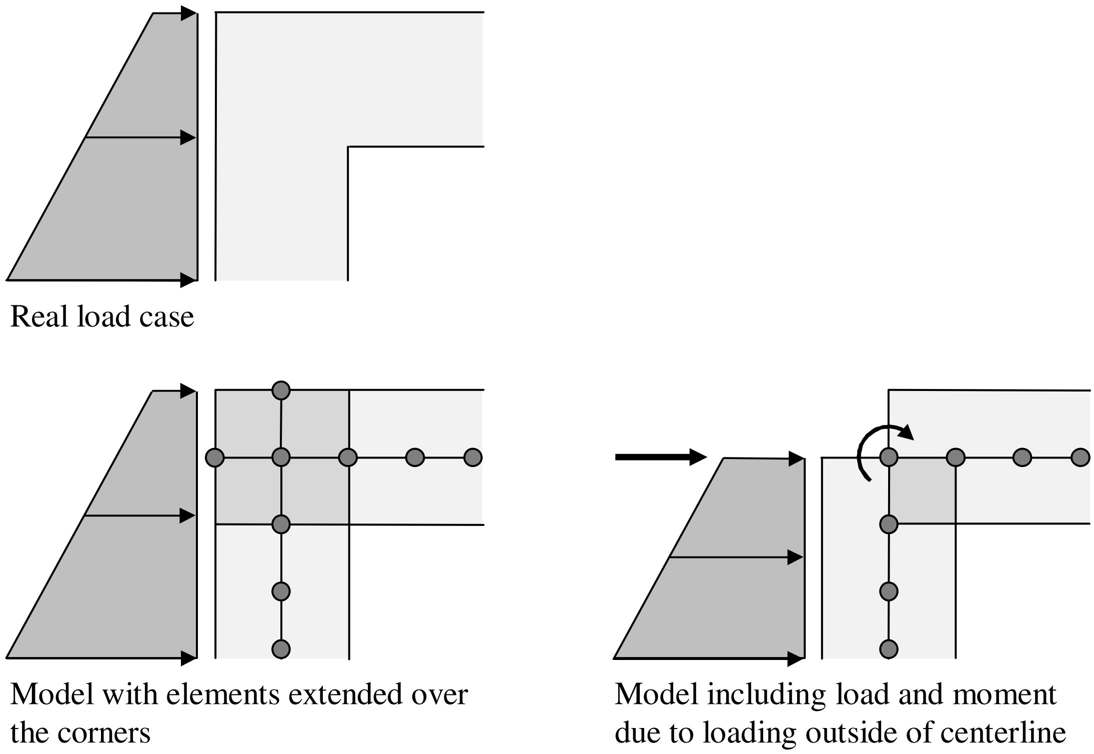



*Figure 4.5. Various models for accounting loading outside of centrelines.*

In principle both should account for the load sufficiently well, though both has their difficulties. When extending the elements over the corner one has to be careful when defining the element properties so that they do not influence the structural response. This becomes increasingly difficult for 3D-models where stiffnesses in multiple directions have to be handled. Adding point loads and moments makes the load definition more complex, with increased risks for errors.

# 5 Comparison of modelling techniques for an integral slab frame bridge

An integral slab frame bridge, see Figure 5.1, was studied in order to determine the influence of the structural analysis models on the design of the bridge deck. The studied aspects of modelling include; influence of corner region stiffness, influence of soil-structure interaction against frame walls, influence of haunch modelling and influence of load application. How these aspects of modelling are compared is described in detail in section 5.3. The bridge was modelled and analysed using the FE software Brigade/Plus, see section 1.2.

*Figure 5.1. Example of an integral slab frame bridge with curved wing walls and circularly haunched bridge deck. The geometry of the studied bridge is loosely based on this one, where a swedish national highway crosses a walk- and bicycle path.*

## 5.1 Layout and geometry of studied bridge

The bridge consists of a highway – walk- and bicycle pathway crossing. It is situated outside of the town Umeå, on the east coast of northern Sweden.

The studied bridge has a closed foundation slab and is founded directly on the ground. The top slab forming the bridge deck is circularly haunched in the longitudinal direction and cambered by 2.5 % in the transversal direction, hence a section in the middle of the bridge is about 0.18 m thicker than a section near the edge beam. It has a free opening of 5 m, is 13.6 – 14.1 m wide and the walls have an average height of 4 m. Thicknesses of the structural parts is presented in Table 5.1.

*Table 5.1. Table over thicknesses for structural members. Varying thickness of the bridge deck is due to the circular haunch and 2.5 % camber of bridge deck surface.*

| **Structural part** | | **Thickness** |
|---|---|---|
| Foundation slab | | 450 mm |
| Frame walls * | | 400 mm |
| Wing walls | | 450 mm |
| Bridge deck | at walls | 740 mm – 920 mm |
| | in midspan | 240 mm – 420 mm |

** Frame walls 450 mm thick with 50 mm cutout.*

## 5.2 Modeling

3D shell elements with thicknesses defined according to Table 5.1 were used to model the geometry of the bridge, see Figure 5.2. The varying thickness of the bridge deck was defined according to an analytical expression using the "Analytical field" tool in Brigade/Plus. The element size was chosen to 0.25 by 0.25 m in the deck and 0.4 by 0.4 in the remaining parts of the bridge. This is in accordance with the general rule of thumb that the element size should not be chosen larger than the thickness.

Since the study was based on linear-elastic material properties the model was based on uncracked gross concrete sections. The material properties was therefore set to C35/45 concrete with modulus of elasticity 34 GPa, poissons ratio 0.2 and coefficient of thermal expansion $10^{-5}$.

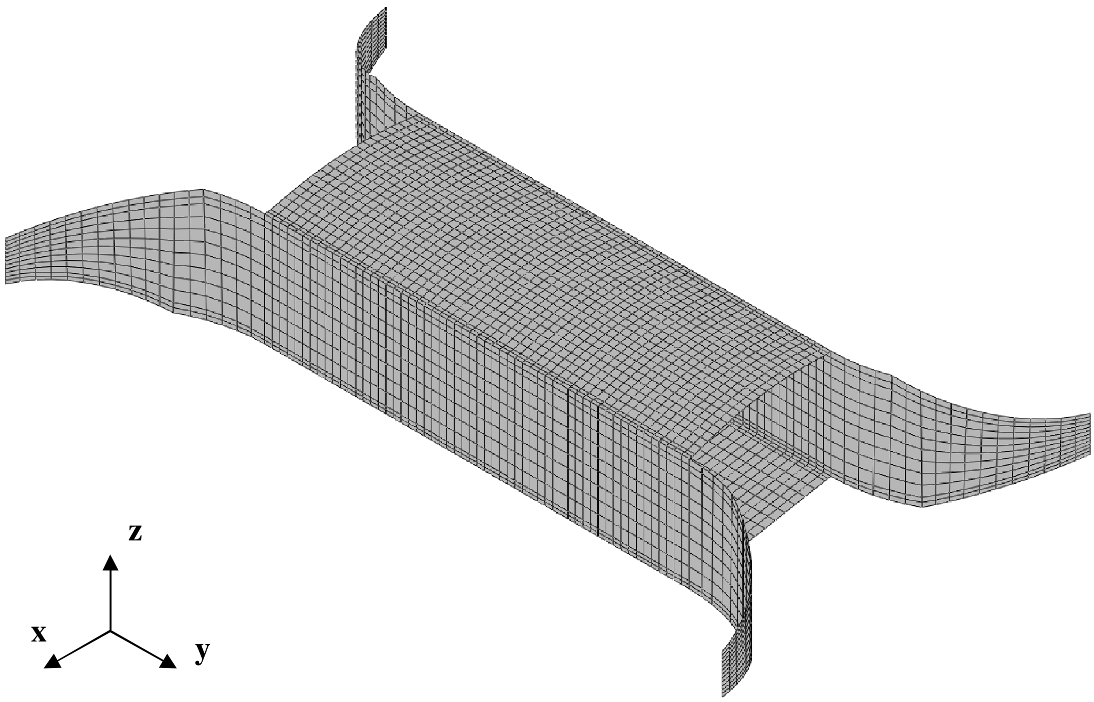



*Figure 5.2. Visualization of the integral slab bridge model studied. The bridge is modelled with 3D-shell elements and is here shown in an isometric perspective.*

### 5.2.1 Boundary condition and foundation

The foundation was modelled with spring-elements given a stiffness of 3.0 MPa/m in all directions. The stiffness has been estimated by a simple settlement calculation where a load of 20 kPa has been introduced onto the soil. The bedding modulus has then been estimated by the simple relationship $E=\sigma/\delta$.

### 5.2.2 Load modelling and load combinations

The loads covered in the analysis are based on load models from Eurocode, SS-EN 1991-2 (2007) and Swedish requirements on bridge structures, Trafikverket (2009a). Covered loads are presented below:

- **Self-weight.** The self-weight was applied uniformly as a material parameter on the concrete with an effect of 25 kN/m$^3$. This was done by setting material density to 2500 kg/m$^3$ and acceleration of gravity to 10 m/s$^2$.
- **Surfacing.** The surfacing of the road was given a thickness of 70 mm and a weight of 24 kN/m$^3$, for the tunnel surfacing corresponding values is set to 60 mm and 23 kN/m$^3$. The applied load was 0.07×24=1.68 kN/m$^2$ on the bridge deck and 0.06×23=1.38 kN/m$^2$ on the foundation slab.
- **Earth pressure.** Earth pressure was applied on frame and wing walls with earth density of 22 kN/m$^3$ and frictional angle $\varphi_k$=45°. This gives a characteristic value for the earth pressure coefficient at rest to 0.29 and a design value of 0.39.
- **Traffic loads.** The traffic loads used include traffic load model 1 and 2. Traffic load lines are defined according to Figure 5.3. Adjusted axle loads lane 1-3 load model 1 are according to Eurocode and Swedish National Annex 270 kN, 180 kN and 0 kN respectively. The adjusted distributed loads were 6.3 kN/m$^2$ for lane 1 and 2.5 kN/m$^2$ for remaining bridge area. Adjusted single axle load for load model 2 were 360 kN.

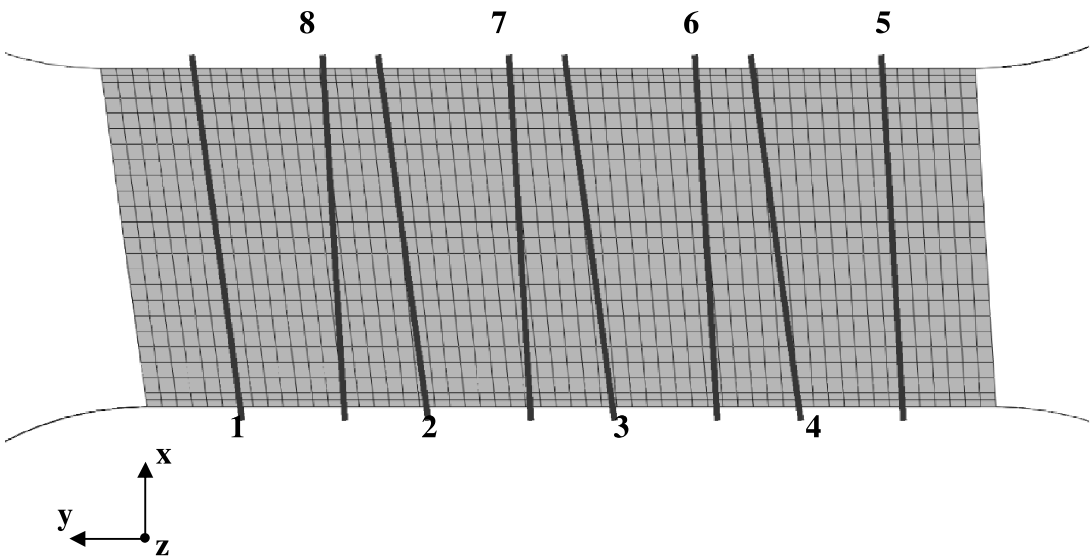



*Figure 5.3. Definitions of traffic load lines used for live load analysis. Line 1 and 5 is spaced 1.5 m from the edges, lines 2 - 4 are spaced on 3 m intervals from line 1. Lines 6 - 8 are spaced in the same manner away from line 5.*

- **Braking, acceleration and lateral traffic loads.** Horizontal (braking and acceleration in the decks primary direction) and lateral (loading in the decks secondary direction) loads were applied as line loads on the lanes according to Figure 5.3 at the level of the surfacing. The lanes loaded were tied to the bridge deck, thus the moment due to eccentricity of the load against the centreline was accounted for. The braking force axle component was 324 kN and the distributed component 11 kN. The corresponding components for lateral load was 81 kN and 3 kN respectively. All horizontal and lateral traffic loads were applied as distributed line loads over the lane length 5.9 m.
- **Surcharge.** A surcharge due to loading on the carriageway behind the walls was applied as a pressure on the frame walls according to TK Bro, Trafikverket (2009a). Earth pressure increase on the opposite side due to single sided loading was included.
- **Temperature.** Temperature loads included temperature difference causing expansion and contraction as well as a temperature gradient applied to the bridge deck. Expansion and contraction was modelled as a temperature increase/decrease with a 15°C difference between sub- and superstructure according to Eurocode. On frame walls in between the foundation and the bridge deck a linear variation was assumed in order to limit restraint effects and obtain a more “natural” temperature distribution, see Figure 5.4.

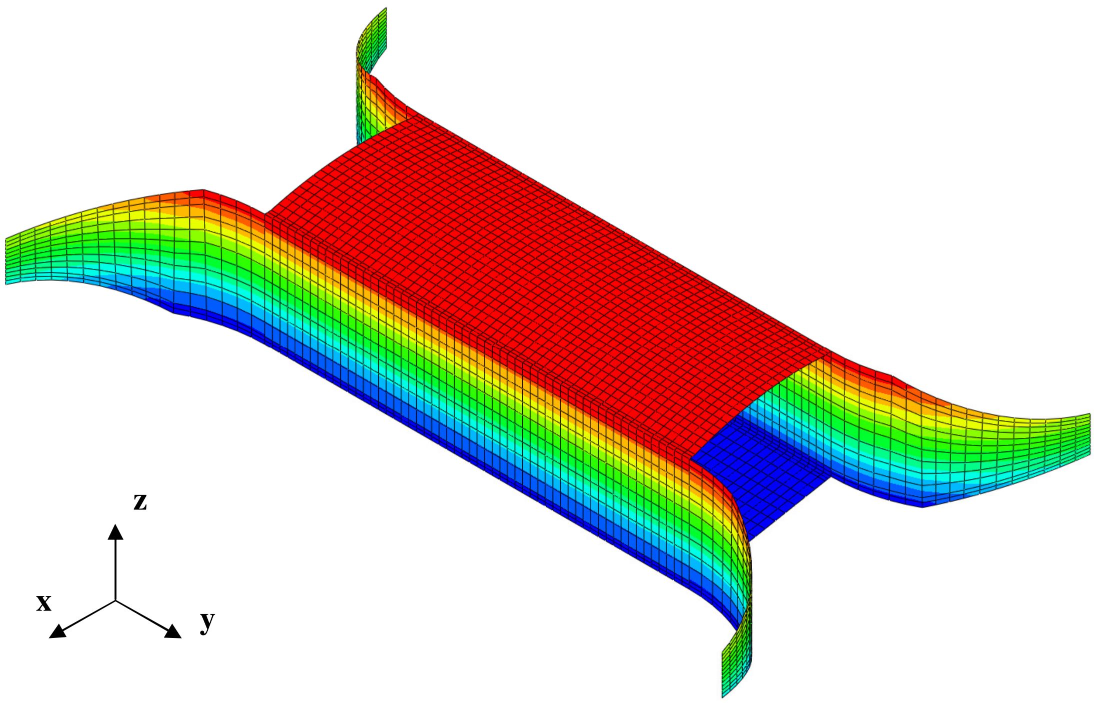



*Figure 5.4. Temperature distribution over the bridge for load case expansion. The top slab is heated by 20°C while the foundation slab is heated by 15°C less, e.g. 5°C. A linear temperature variation is defined over the walls.*

The maximum temperature increase (expansion) for the deck was calculated to 20°C, and maximum decrease (contraction) was -40°C. Temperature gradient with heated surface was $\Delta T=10.5°C$ and with cooled surface $\Delta T=8°C$.

- **Wind load.** Wind load was applied horizontally on the sides of the bridge deck as a shell edge load. The eccentricity of the load was accounted for by application of an additional moment calculated for a mean distance to the deck centreline.

- **Earth pressure increase.** An increase in earth pressure was modelled for relevant horizontal loads (braking, one-sided surcharge and temperature expansion/contraction). This was applied according to the model presented in TR-bro, Trafikverket (2009b), as dependant on the deflection of the frame wall at surface level.
- **Load combination.** The bridge was studied for ULS load combination according to Eurocode and Trafikverket, variable loads governing. In this load case either traffic loads or other loads can be the leading one.

## 5.3 Methodology of comparison

Resulting sectional forces and required reinforcement amounts were compared in ultimate limit state load combination for the different models. Reinforcement moments, see equation 5.1, and accompanying membrane forces, see equation 5.2, were used in order be able to do the comparison with relative ease. Torsional moments and in-plane shear forces were added to the in-plane moments and normal forces respectively according to section 3.4.

Reinforcement moments are defined as:

$$\begin{aligned} m_{xt,pos} &= m_x + \left|m_{xy}\right| \\ m_{xt,neg} &= m_x - \left|m_{xy}\right| \\ m_{yt,pos} &= m_y + \left|m_{xy}\right| \\ m_{yt,neg} &= m_y - \left|m_{xy}\right| \end{aligned} \qquad (5.1)$$

And the accompanying normal forces as:

$$\begin{aligned} n_{xt} &= n_x + \left|n_{xy}\right| \\ n_{yt} &= n_y + \left|n_{xy}\right| \end{aligned} \qquad (5.2)$$

The required reinforcement amounts were calculated according to the sandwich model presented in section 3.4. In order to check if a tensile force contribution to the reinforcement due to inclined shear cracking was needed, inclined shear cracking was checked in a simplified manner according to equation 5.3.

$$\begin{aligned} v_0 &= \sqrt{v_x^2 + v_y^2} \\ v_{Rd,c} &= (v_{\min} + k_1\sigma_{cp})b_w d \\ v_{\min} &= 0.035k^{3/2} \cdot f_{ck}^{\ 1/2} \\ \sigma_{cp} &= N_{Ed} / A_c < 0.2 f_{cd} \\ v_0 &< v_{Rd,c} \end{aligned} \qquad (5.3)$$

$v_0$ is the shear resultant and $v_{Rd,c}$ is the design shear resistance for structural parts without shear reinforcement according to Eurocode 2, SS-EN 1992-2 (2005). The results of the check showed that $v_0 < v_{Rd,c}$ in all sections of the bridge deck, hence no tensile force contribution to the reinforcement needed to be accounted for. Design equations according to the sandwich model will be according to equation 5.4.

$$\begin{aligned}
a_{sx} f_y &\geq \frac{m_x}{d_v} + \frac{n_x}{2} + \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} \right| \\
a_{sy} f_y &\geq \frac{m_y}{d_v} + \frac{n_y}{2} + \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} \right| \\
a'_{sx} f_y &\geq -\frac{m_x}{d_v} + \frac{n_x}{2} + \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} \right| \\
a'_{sy} f_y &\geq -\frac{m_y}{d_v} + \frac{n_y}{2} + \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} \right|
\end{aligned} \tag{5.4}$$

The internal level arm, $d_v$, was chosen so that the midplane of the outer sandwich layers coincide with the reinforcement plane. The concrete cover was set to 40 mm and the reinforcement diameter was chosen to 12 mm when determining layer thickness. This method assumes dominating bending action, i.e. that the membrane forces are small. Furthermore it is assumed that bending failure is governed by the yielding of reinforcement.

Critical sections has been identified, presented in Figure 5.5. Primary longitudinal reinforcement follows the x-direction and the secondary longitudinal reinforcement the y-direction. The chosen sections provide a reasonable illustration of the overall response of the bridge deck slab and comprise of the mid-sections, 1-1 and 3-3 and sections along the edges, sections 2-2 and 4-4.

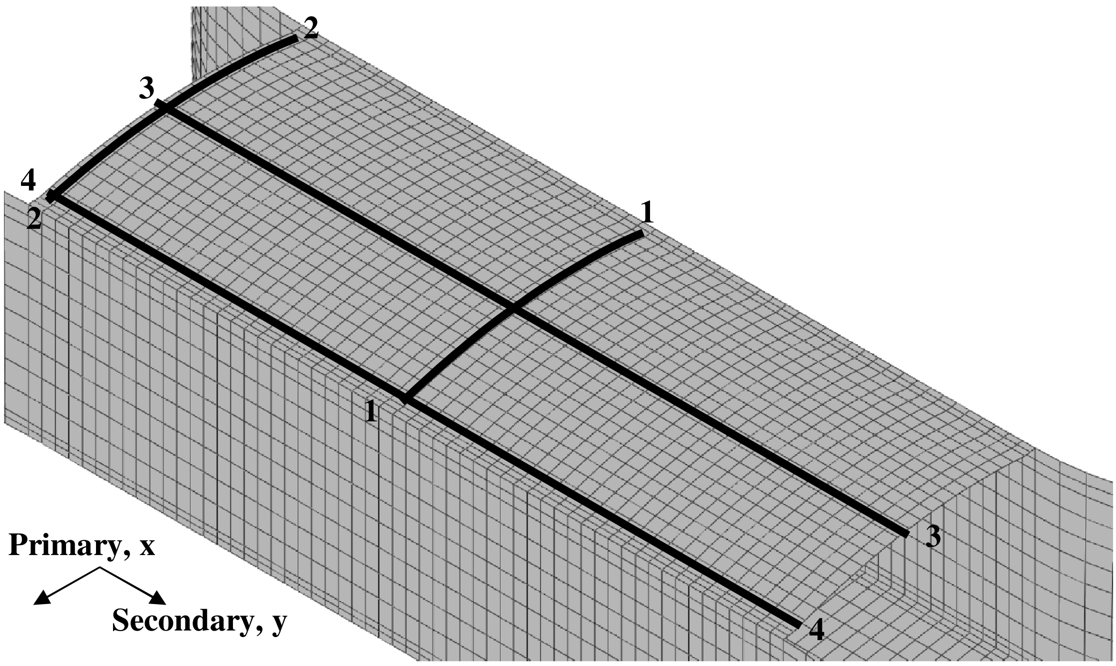



*Figure 5.5. Sections analysed in the bridge deck. In addition, the response is studied between critical sections for design.*

## 5.4 Influence of corner stiffness modelling

To study the influence of the structural model's corner stiffness three different models with the corners modelled according to different principles was established. The different models were; coupling of corner elements through increased element stiffness properties in the primary direction, coupling corner nodes in the critical section through inclined stiff link and disregarding the increased corner stiffness entirely, see Figure 5.6.

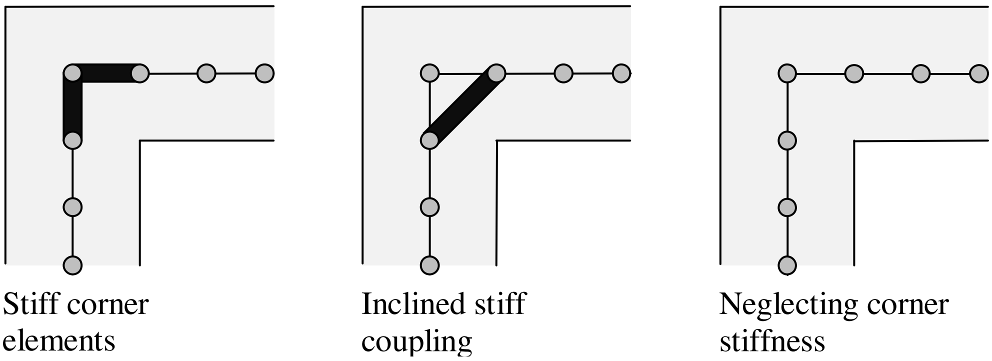



*Figure 5.6. Investigated techniques for modelling of corners in a frame. Standard model (left), where corner rotational stiffness was accounted for by increasing the element stiffness in the primary direction. The stiffness was accounted for by diagonal coupling of degrees of freedom in the nodes at critical sections (middle) and disregarding increased corner stiffness (right).*

### 5.4.1 Modelling technique

The first model was established by assigning ortotropic material properties for the corner region. In the primary direction, Figure 5.5, the stiffness was "infinitely" higher (factor $10^{10}$) compared to the ordinary material stiffness in the structure. The material was defined as "lamina", which in Brigade/Plus allows definition of orthotropic plane stress elastic materials, see Equation 5.5. Local direction 1 was defined to coincide with the primary direction x, direction 2 will then coincide with secondary direction y.

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \frac{1}{t} \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{bmatrix} \tag{5.5}$$

$E_1$ was given the increased stiffness, while $E_2$ retained the stiffness of the rest of the structure ($E=34$ GPa for C35/45 concrete). Poissons ratio $\nu_{12}$ is set to $0.2$ and the shear modulus $G_{12}$ was calculated according to Equation 5.6 with $E=34$ GPa and $\nu=0.2$. $G_{12}$ then became $14.2$ GPa.

$$G = \frac{E}{2(1+\upsilon)} \tag{5.6}$$

The diagonal coupling model was established by coupling the nodes in the critical sections with stiff links. This was done with the Tie-tool, where a master- and slave region is defined. The nodes in the slave region are coupled to the closest node in the master region, effectively creating the desired coupling condition. For more information about the Tie tool, see the Brigade and Abaqus manuals, Dassault Systèmes (2008).

### 5.4.2 Primary reinforcement moment in the bridge deck

Differences between the reinforcement moments were in line with what was expected; negligible to no differences was found for the total reactions, while some differences could be seen between the distribution of moments. In the maximum envelope of reinforcement moment, resisted by bottom reinforcement, the moment in the field section 3-3 was slightly lower for the model accounting for the corner stiffness, Figure 5.7 to Figure 5.9. However, for the minimum envelope of reinforcement moment, resisted by top reinforcement, the moment was slightly higher in the support sections for these models, Figure 5.8.

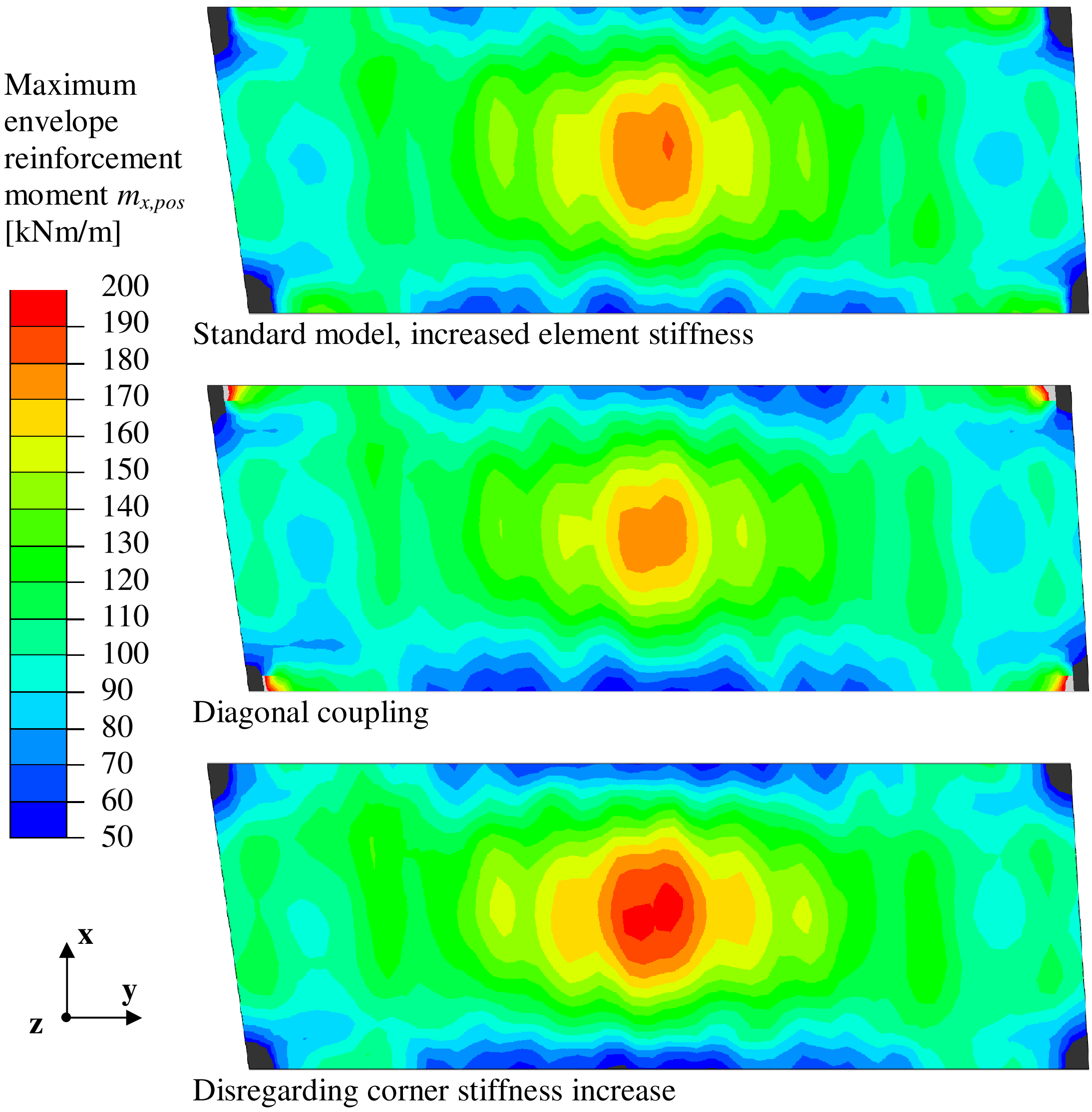



*Figure 5.7. Maximum envelope reinforcement moment in the bridge deck (resisted by bottom reinforcement). Corner stiffness modelled with increased material stiffness (top), corner nodes in the critical sections coupled diagonally (middle) and corner stiffness increase neglected (bottom). Local peak values due to torsion near the slabs corners are not shown.*

In section 1-1, midspan section see Figure 5.5, for the minimum envelope, Figure 5.8, the reinforcement moment was higher for the models accounting for an increased corner rotational stiffness. This diverged from the expected results since stiffer corners should attract more moment, reducing the field section moment. However, it can be explained since the favourable effect of permanent loads such as self-weight is more substantial for the model disregarding the corner stiffness. When combined with permanent loads, the effect of loads acting unfavourable in the minimum envelope will be reduced, i.e. favourable effects in the minimum envelope of permanent loads are larger for the models with disregarding corner stiffness. When individual load cases were studied the moments in mid-sections were higher (positive or negative bending) for all load cases when disregarding the increased corner stiffness.

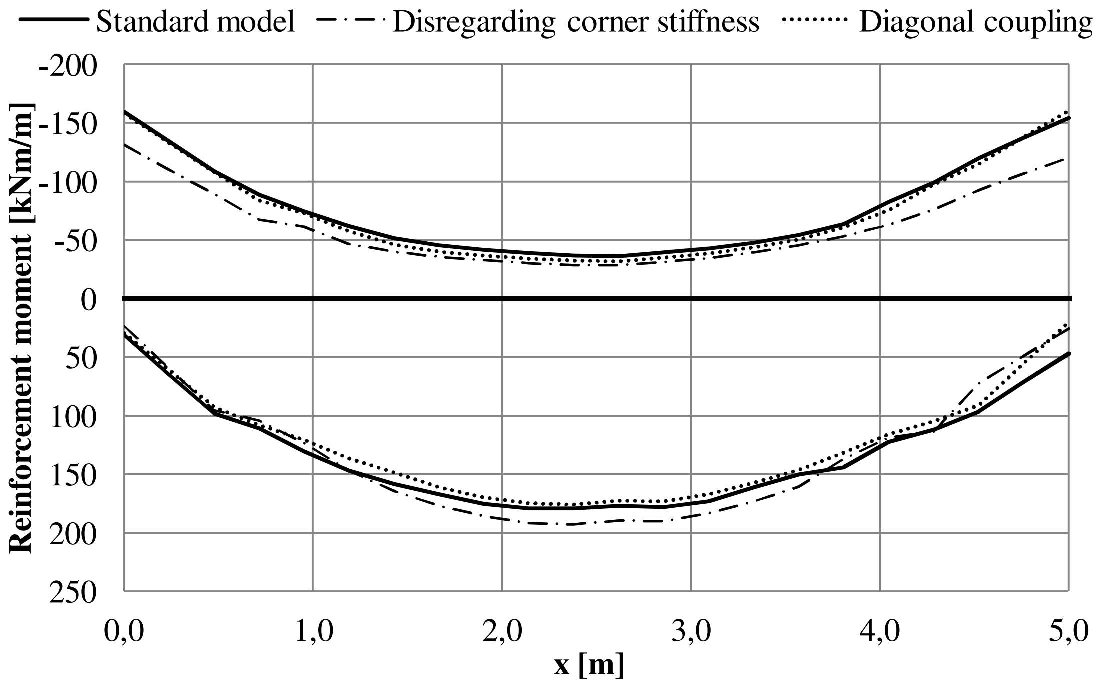



*Figure 5.8. Primary reinforcement moment envelopes in section 1-1, see Figure 5.5, in the middle of the bridge deck. Negative moment is resisted by top reinforcement and positive moment is resisted by bottom reinforcement.*

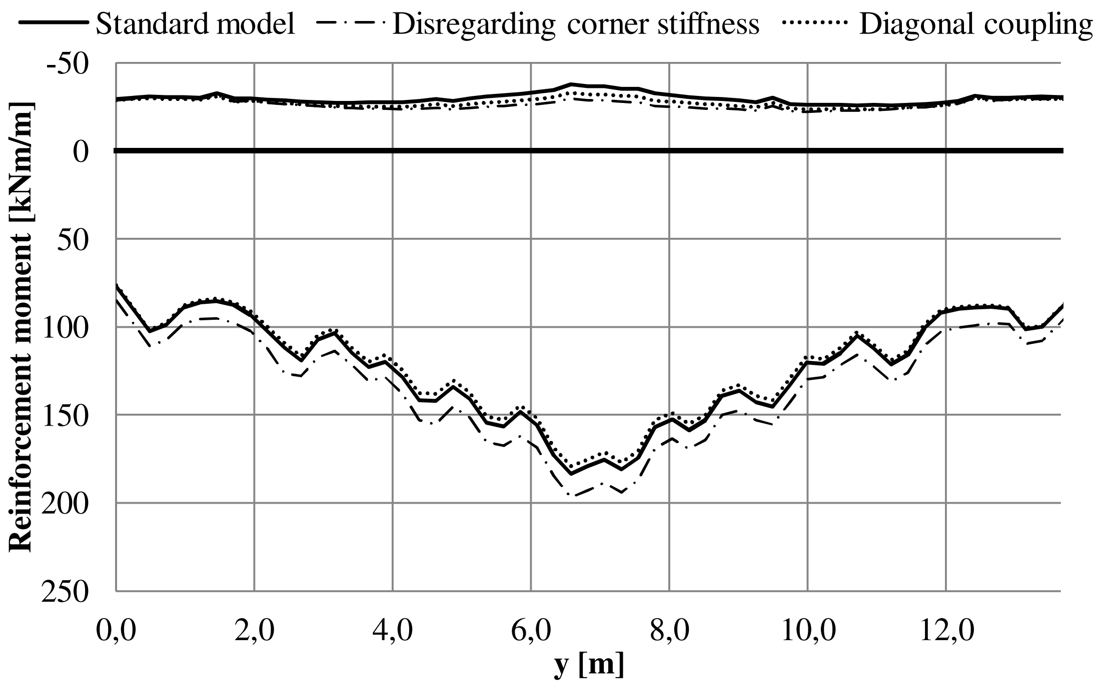



*Figure 5.9. Primary reinforcement moment in section 3-3, see Figure 5.5, along the midspan of the bridge deck.*

The most marked differences between reinforcement moments was noticed locally near the corners of the bridge deck, see Figure 5.10. This is where the torsional moment in the slab has greatest impact and increases the reinforcement moment in all models substantially. However, this increase is larger for the stiff corner models since those obtain a markedly higher torsional moment than the models without stiff corners, as illustrated for the simple self-weight load case in Figure 5.11.

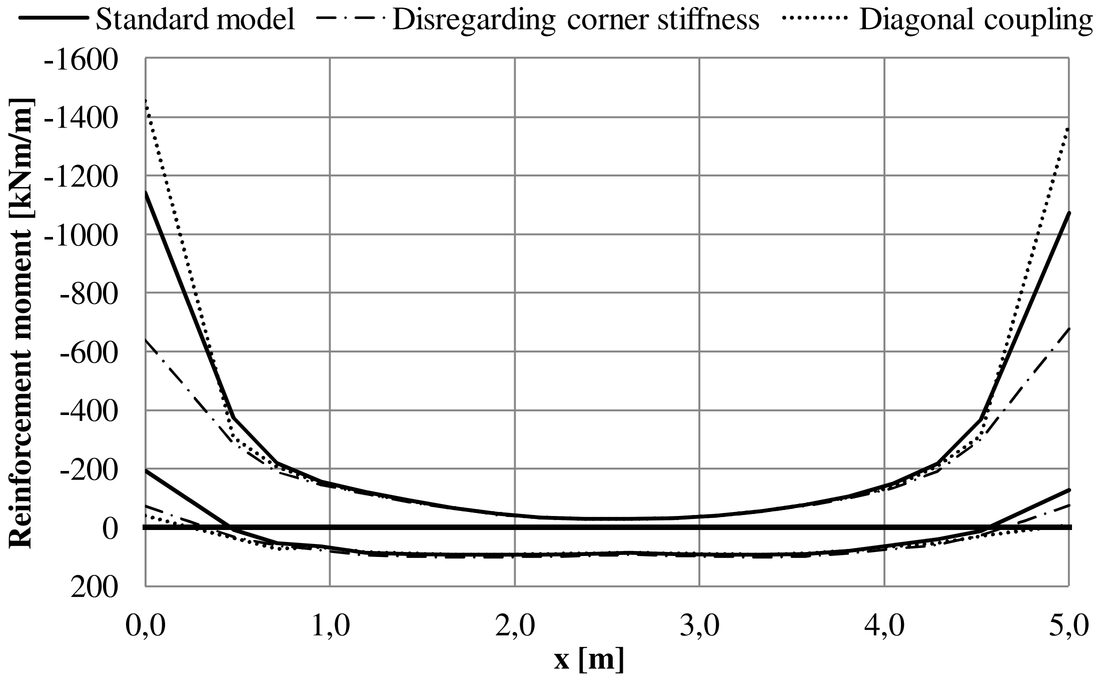



*Figure 5.10. Primary reinforcement moment in section 2-2, see Figure 5.5, near the edge beam of the bridge deck.*

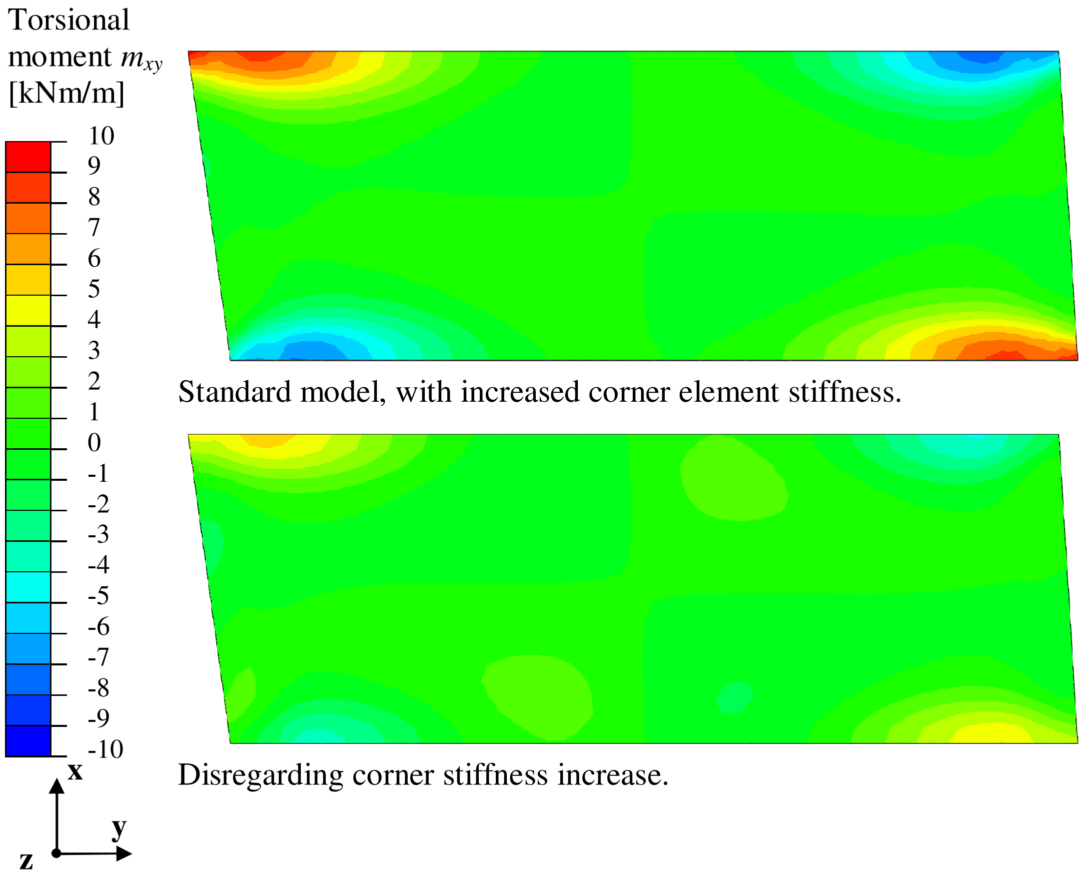



*Figure 5.11. Difference in torsional moment in the bridge deck for load case self-weight. The model neglecting support stiffness (bottom) shows substantially lower torsional moment near the deck corners than the models accounting for support stiffness (top).*

### 5.4.3 Secondary reinforcement moment in the bridge deck

In the secondary reinforcement moment direction there was no discernible differences between any of the models, see Figure 5.12, except for near the bridge decks corners where torsion moment had a large influence, see Figure 5.11.

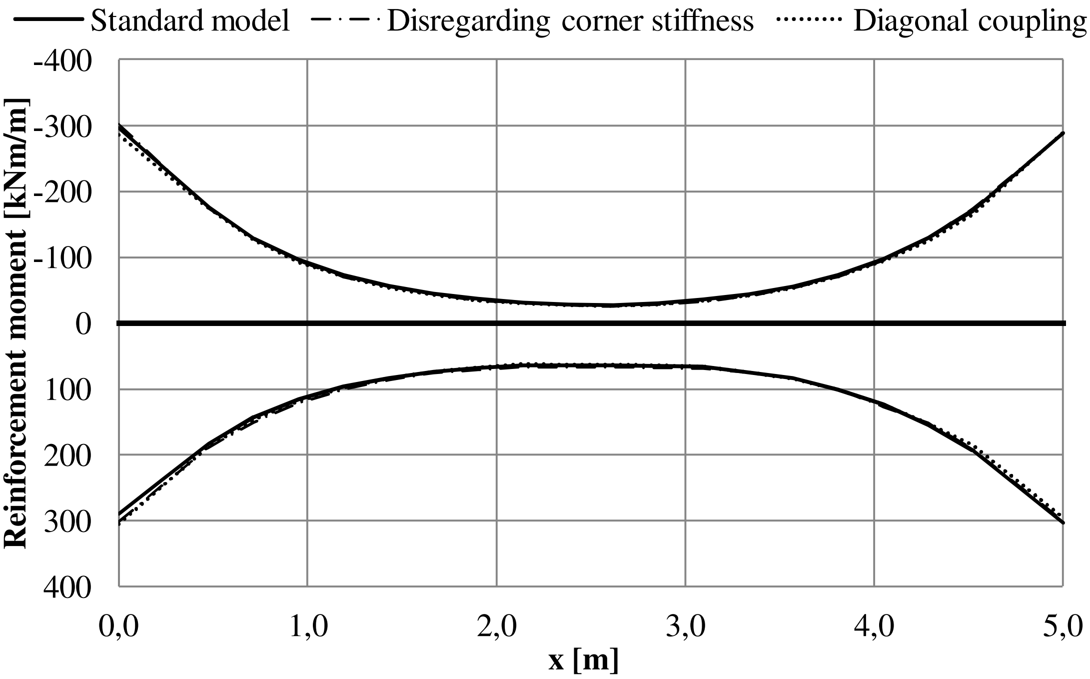



*Figure 5.12. Secondary reinforcement in section 1-1, a section between the frame walls in the mid of the bridge deck.*

An interesting modelling effect presented in all models is the variation of maximum envelop secondary moment in the midspan section 3-3, see Figure 5.13. This is an effect of traffic loading in discrete loading lines, see Figure 5.3, in a design scenario this should be smoothed out using the peak values in order to account for the loading applied anywhere on the deck.

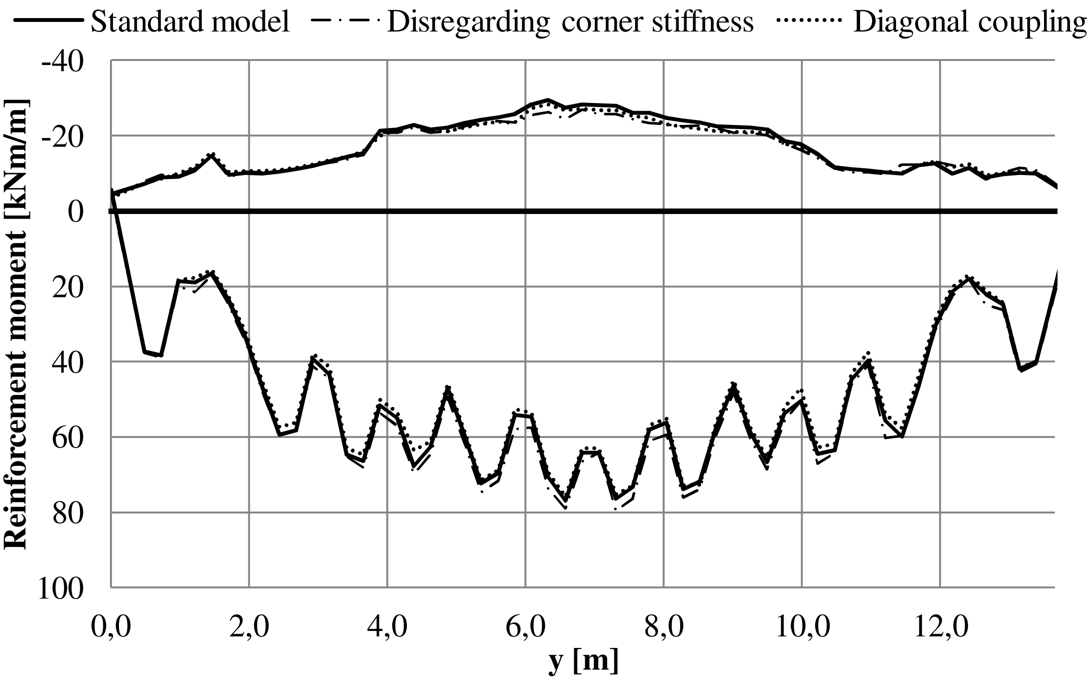



*Figure 5.13. Secondary reinforcement moment in section 3-3. Notice the variation of moment in the maximum envelope (lower curves).*

### 5.4.4 Required reinforcement amount in the bridge deck

Required reinforcement for the bridge deck was calculated according to section 5.3. By calculating and comparing reinforcement amounts the deck cross section and associated sectional forces was accounted for, further illustrating the impact of modelling procedure.

The difference in moment distribution presented in section 5.4.2 resulted in a demand for more primary bottom reinforcement in the model disregarding corner stiffness, see Figure 5.14, and more top reinforcement in the models accounting for the corner stiffness, see Figure 5.15.

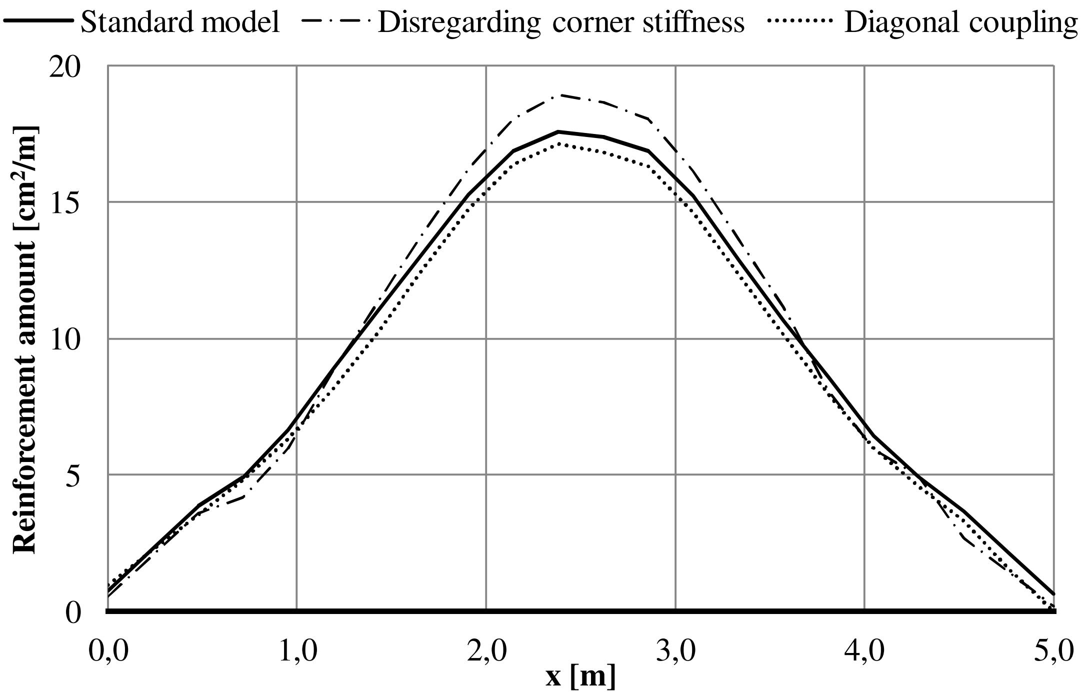



*Figure 5.14. Required primary bottom reinforcement in section 1-1. Model neglecting corner stiffness showed higher requirement in the midspan, as was expected from section 5.4.2.*

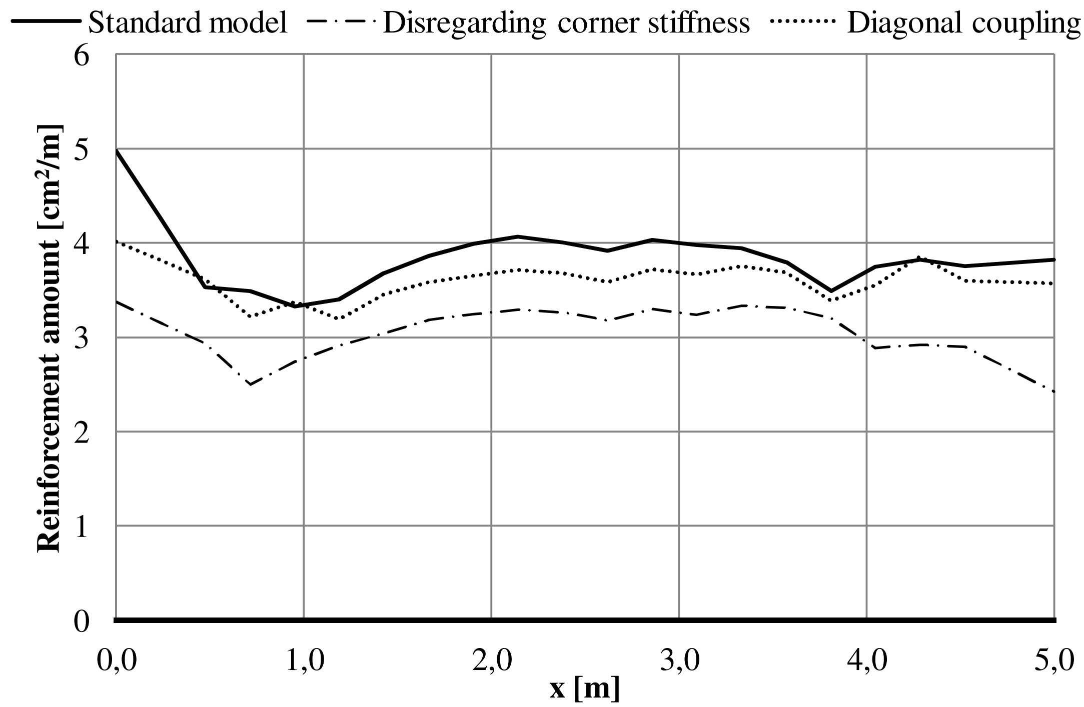



*Figure 5.15. Required primary top reinforcement in section 1-1. Model neglecting corner stiffness showed higher requirement in the midspan, as was expected from section 5.4.2.*

By studying the sum of top and bottom primary reinforcement requirement it could be seen that the differences between the models indeed only resulted in a different reinforcement distribution in the deck, see Figure 5.16.

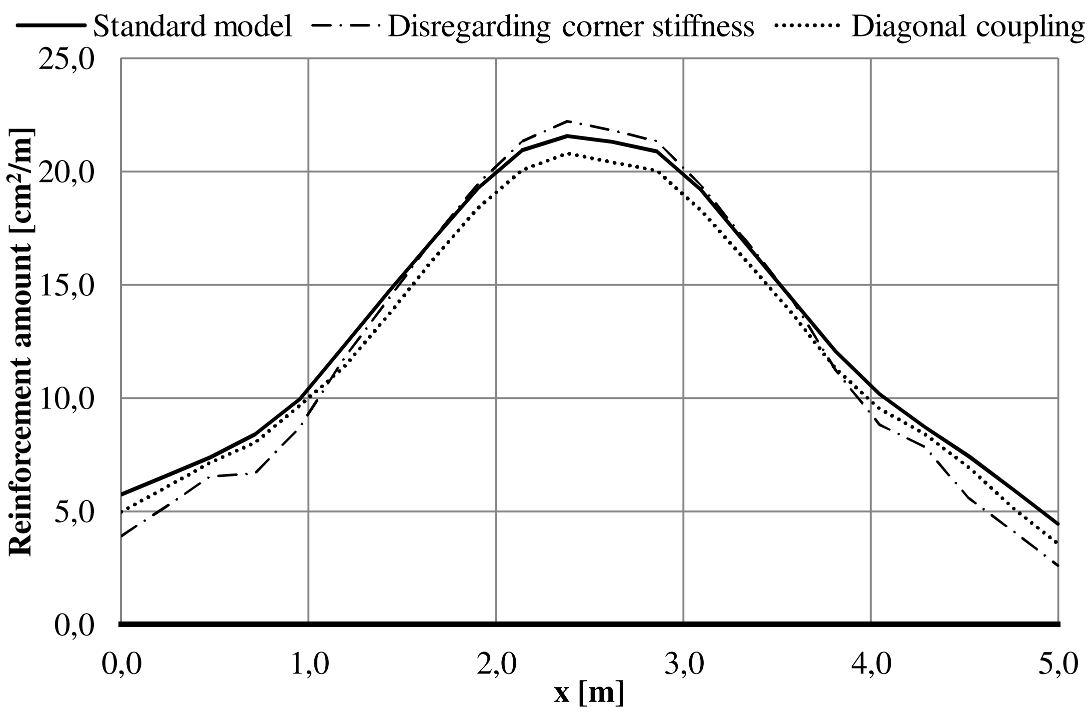



*Figure 5.16. Total required primary reinforcement in section 1-1. Only very small differences can be seen between the different models.*

## 5.4.5 Modelling with isotropic stiffness increase in the corner

As mentioned earlier, the stiffness increase in the corner region was assigned in the primary direction, whereas the secondary direction retained the stiffness of ordinary concrete. The effect of assigning stiffness to the secondary direction was studied, where an additional model was assigned with a uniform isotropic stiffness increase for the corner shell elements in both directions.

In most load cases the response of the structure leads primarily to bending of the bridge deck in its primary direction (e.g. for self-weight). In these load cases little to no effect was seen when comparing models with and without stiffness increase in the secondary direction.

However, for load cases which results in transversal bending of the structure, and mainly of the frame walls, the response differed significantly between the different models, see Figure 5.17. The load case with temperature expansion resulted in two very different moment distributions over the bridge deck, where both the distribution and the moment magnitude differed considerably, see Figure 5.18.

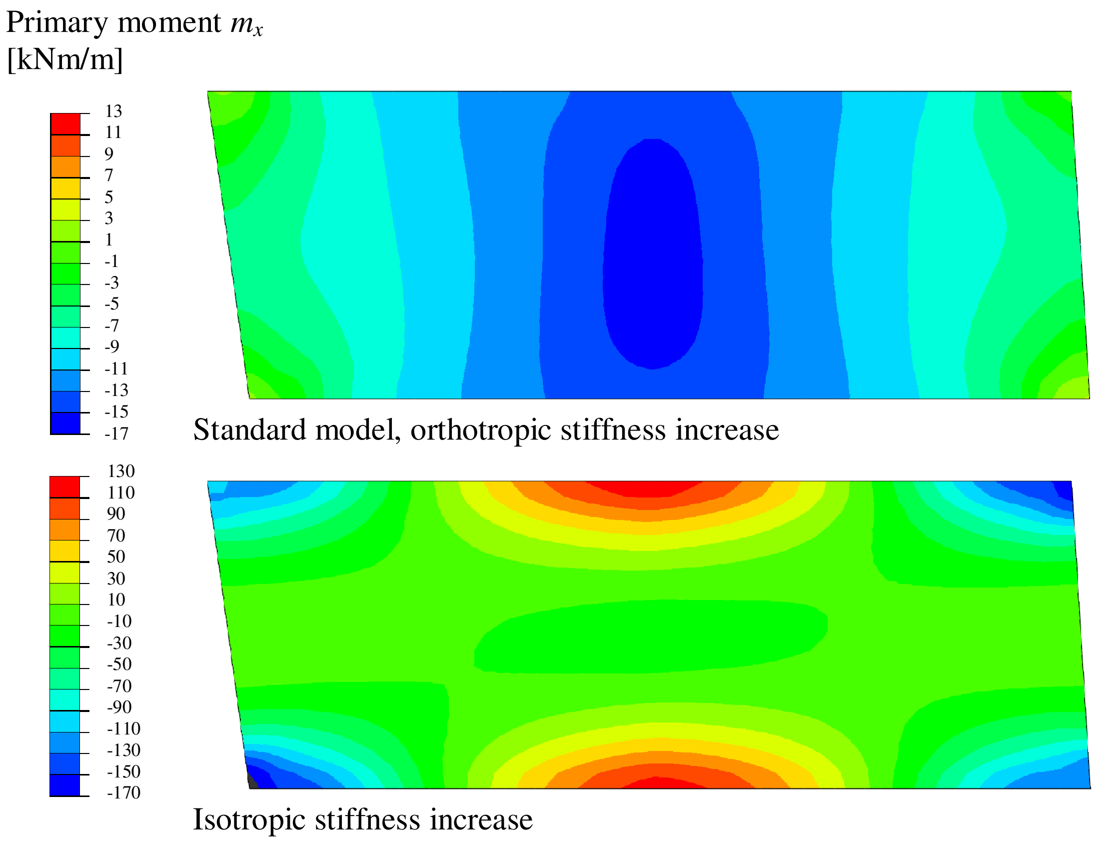



*Figure 5.17. Primary moment in the bridge deck for the load case with temperature expansion for models with corner stiffness increase in the primary direction only (top) and with isotropic stiffness increase in both primary and secondary directions (bottom). Note the difference in both distribution and magnitude of the reactions.*

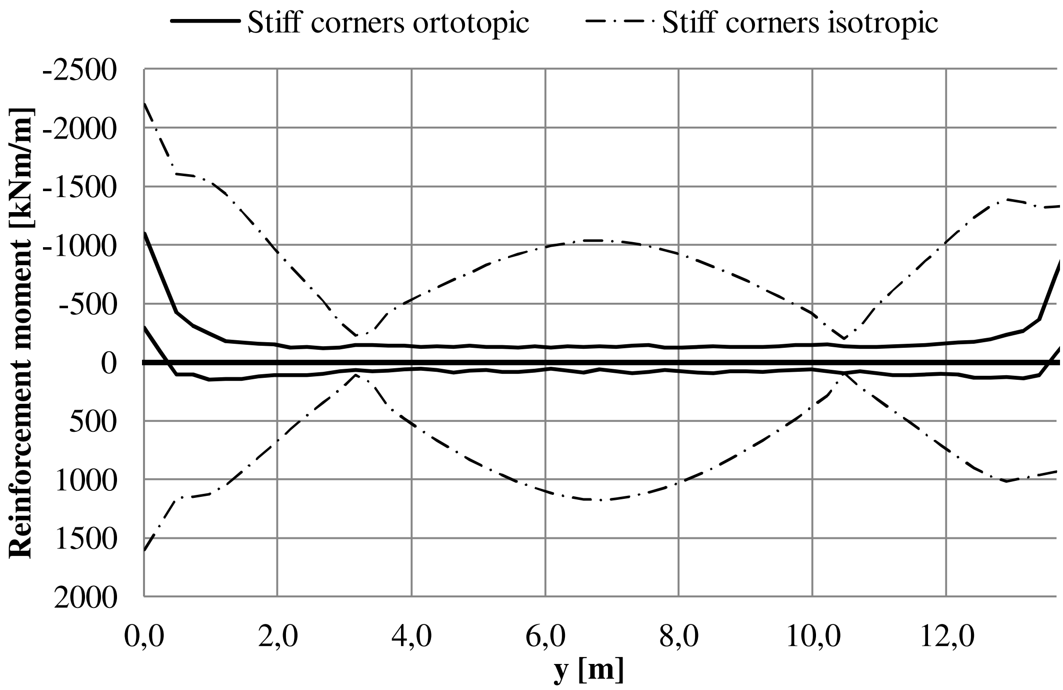



*Figure 5.18. Primary reinforcement moment in section 4-4 near the support. Large differences in both magnitude and shape (distribution) of the curves.*

### 5.4.6 Discussions on results of frame corner stiffness modelling

By summarizing the total reactions and required reinforcement amounts it can be seen that there was no difference between the different corner stiffness modelling procedures, as long as the stiffness is increased in the primary direction only. Therefore, it can rather safely be assumed that the total load carrying capacity should not be influenced by this. However, as expected, there was a difference between the distribution of sectional forces and reinforcement requirements, both between maximum and minimum envelopes (top/bottom reinforcement) and the distribution between field and support sections. Even so, this difference was rather small and should only influence the service behaviour to a limited extent. This can be further illustrated by comparing the maximum field moments between models disregarding or accounting for increased support stiffness. The difference between maximum field moments was in the order of magnitude 7 %, which is well within the limits for redistribution with regard to reinforcement ductility class, see section 3.2.

It is important to note that it is rather easy to make mistakes when adjusting the stiffness of the structure, especially in the transverse direction. This has rather significant consequences in the design and it can also result in difficulties for designers to evaluate load effects and review results for verification.

## 5.5 Influence of frame wall earth pressure increase modelling

The walls of an integral frame bridge are supported by a backfill. Therefore, the backfill provides some resistance for deflection of the frame walls. The earth pressure will increase in the soil when the frame walls deflect horizontally. This effect may be included in the structural analysis model in different ways, in this thesis two different methods were investigated.

### 5.5.1 Modelling

The first model, see Figure 5.19a, was according to Trafikverkets recommendations document TR Bro, Trafikverket (2009b), where an external load is applied dependant on the deflection of the frame wall. In the second model, see Figure 5.19b, the frame walls were modelled as supported against a spring foundation (a so-called winkler-foundation). Two versions of the spring-supported wall model was set up, spring model 1 with the same spring stiffness as the foundation, k = 3 MPa/m. Spring model 2 with a higher stiffness, simulating a more compacted material used as backfill k = 10 MPa/m. Finally a model disregarding all earth pressure increase due to deflection of the frame walls was used for reference.

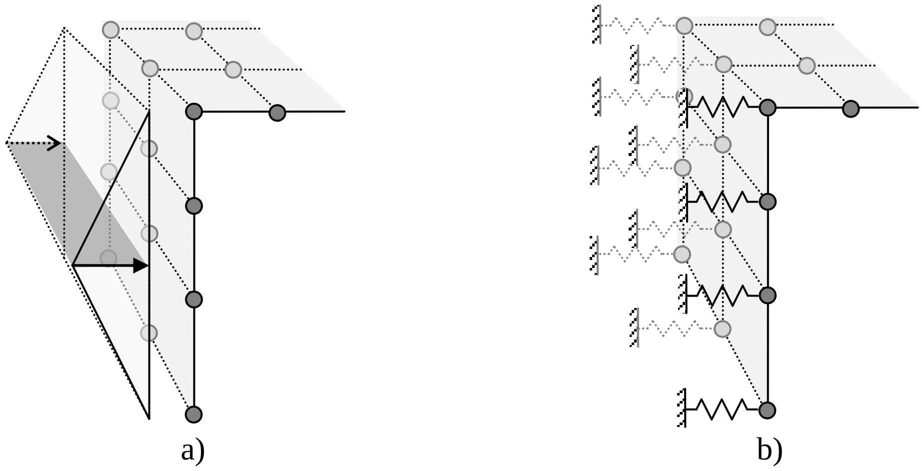



*Figure 5.19. The standard model with earth pressure increase according to TR-bro, Trafikverket (2009b), (a) and the alternative model frame walls supported against springs (b). See also Figure 4.4.*

### 5.5.2 Sectional forces in the bridge deck

By studying the primary reinforcement moment in the bridge deck it was clear that the model suggested by Trafikverket had the largest impact, compared to the model where earth-pressure increase was disregarded. This way of modelling provided a slightly favourable effect in the maximum envelope, see Figure 5.20 and Figure 5.21, with a fairly constant decrease to the moment to be resisted by bottom reinforcement. In the minimum envelope the increased earth pressure increases the moment, therefore acting unfavourable. This increase was mainly present in the mid-sections of the bridge deck.

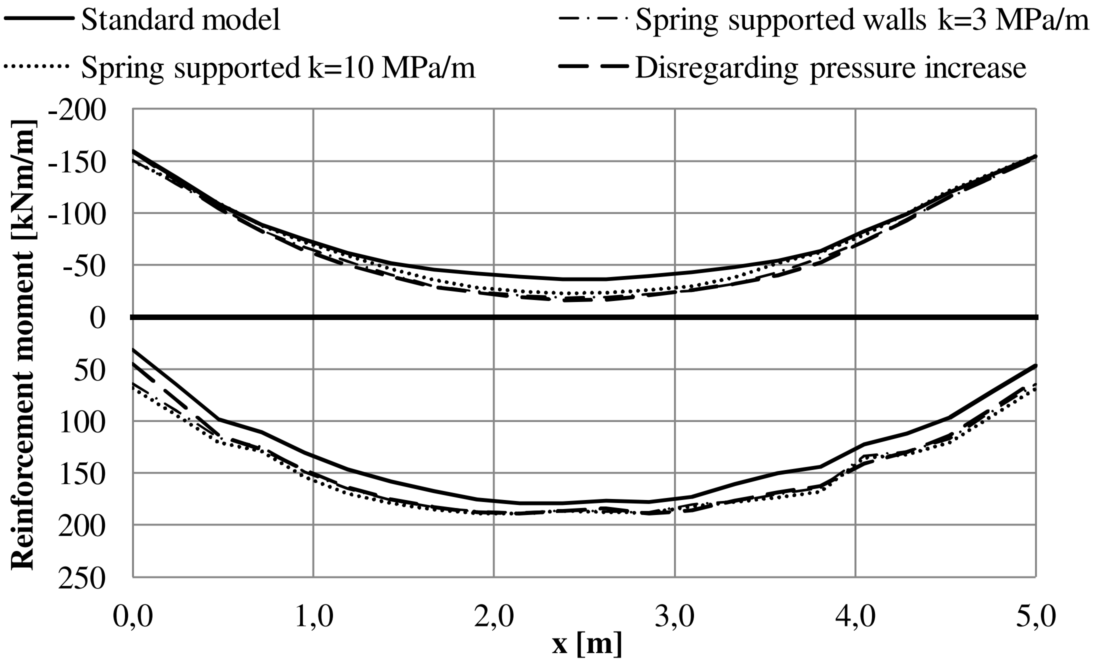



*Figure 5.20. Primary reinforcement moment in the bridge deck section 1-1.*

The spring supported models showed only a slight difference in the bridge deck compared to the model disregarding all earth pressure increase.

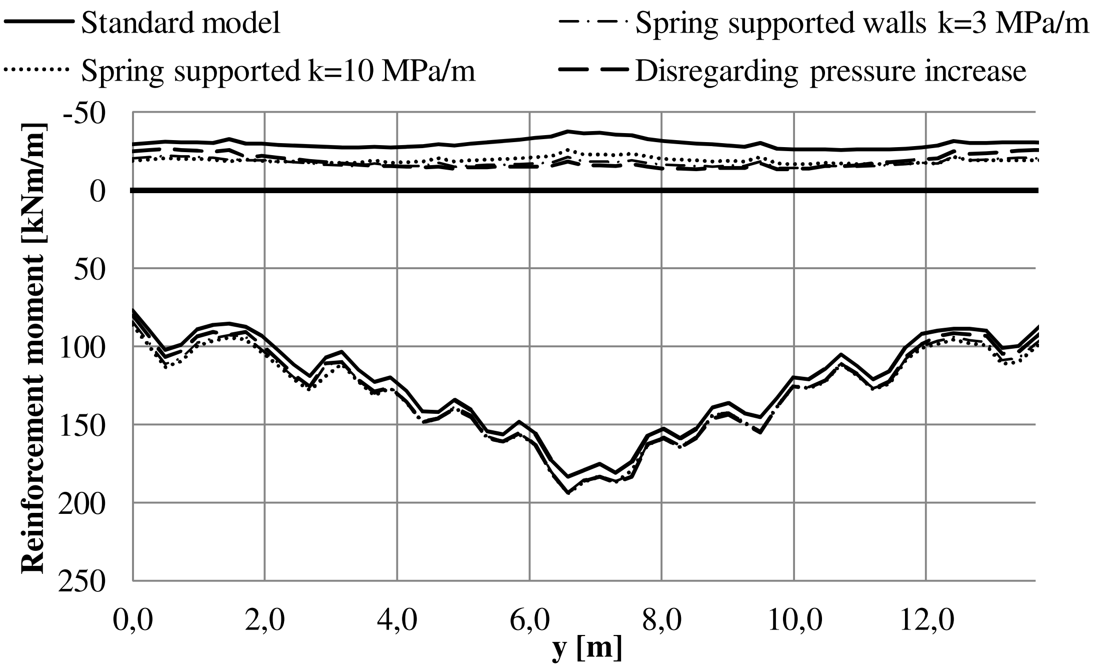



*Figure 5.21. Primary reinforcement moment in the bridge deck section 3-3.*

As in the case with modelling of corner stiffness, there was no discernible difference between the reinforcement moments in the secondary direction of the bridge deck.

### 5.5.3 Required reinforcement in the bridge deck

As with the sectional forces in the deck, required amount of reinforcement showed little difference between the models. A slight increase in the requirement for bottom reinforcement in the span was noted for the spring supported wall models and the model neglecting earth pressure increase, see Figure 5.22.

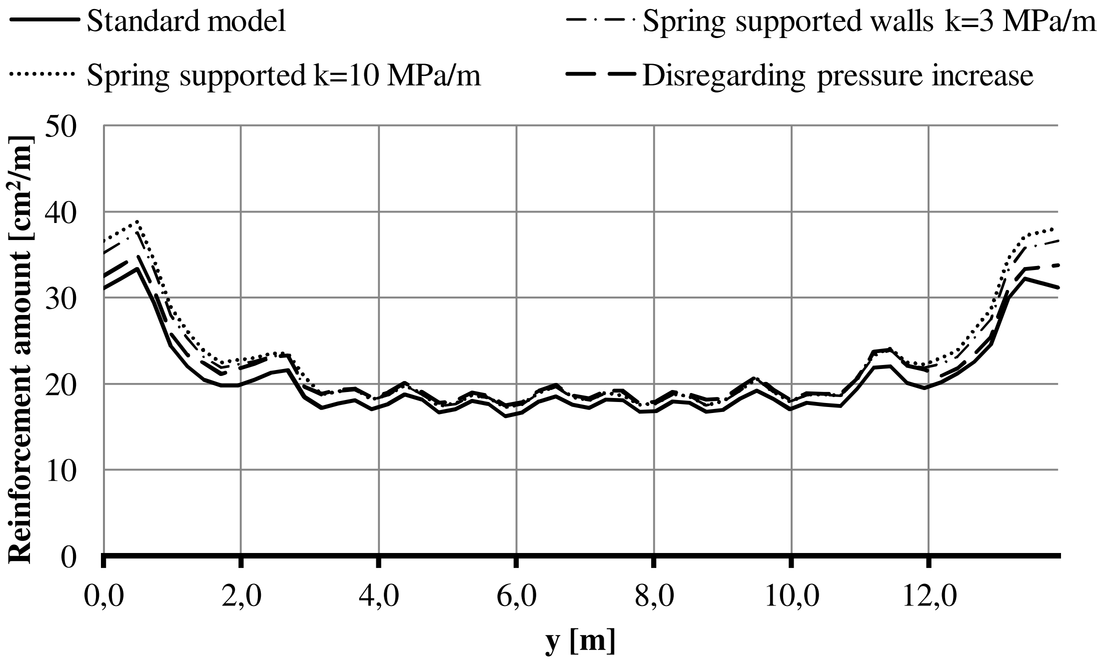



*Figure 5.22. Required primary bottom reinforcement in section 3-3. Only very small differences between the differences could be seen.*

The standard model accounting for earth pressure increase by additional loading according to TR-Bro, Trafikverket (2009b), showed a slight increase in required top reinforcement compared to the other models, see Figure 5.23.

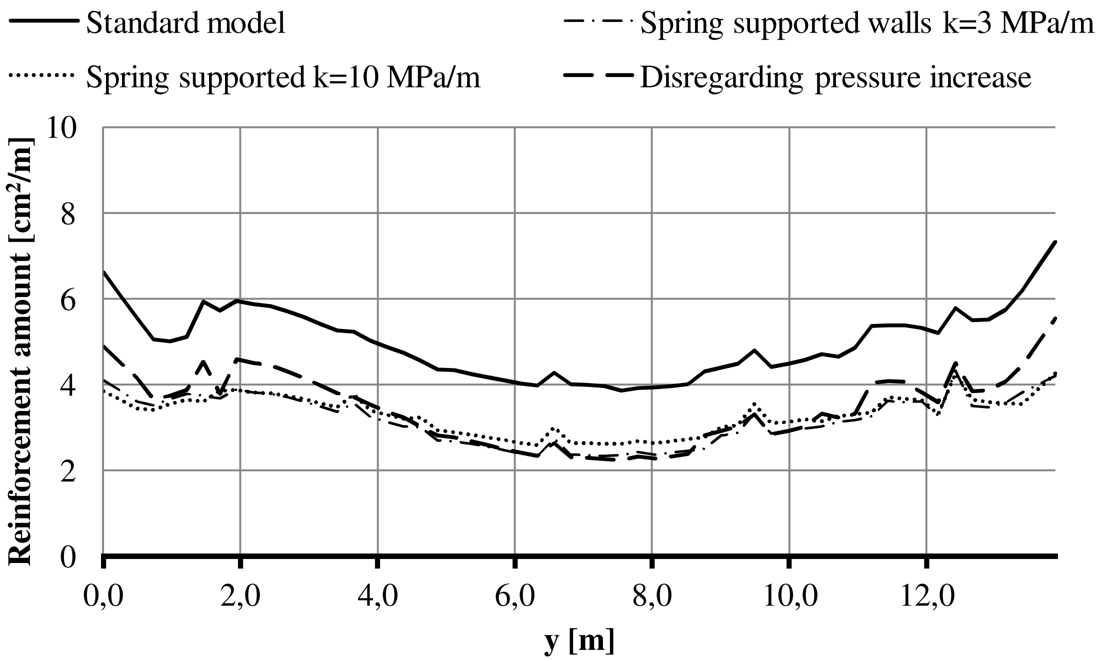



*Figure 5.23. Required primary top reinforcement in section 3-3. Some differences could be seen, though since the requirement was so small it would have little impact on the design.*

In total these differences seemed to balance each other, resulting in an equal total amount of required reinforcement, see Figure 5.24.

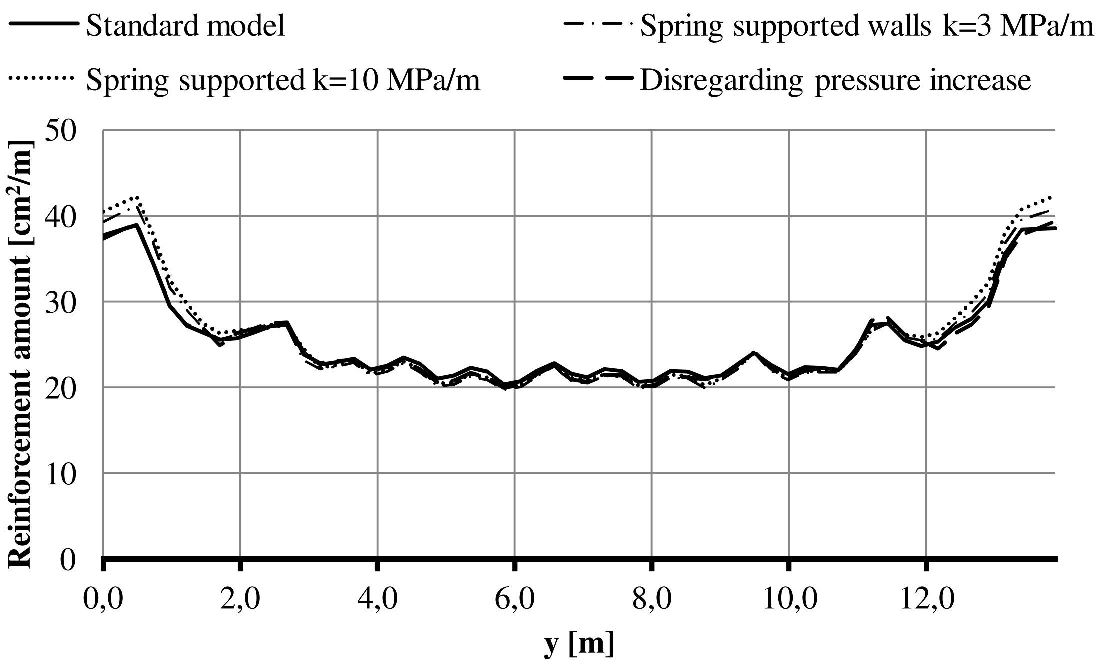
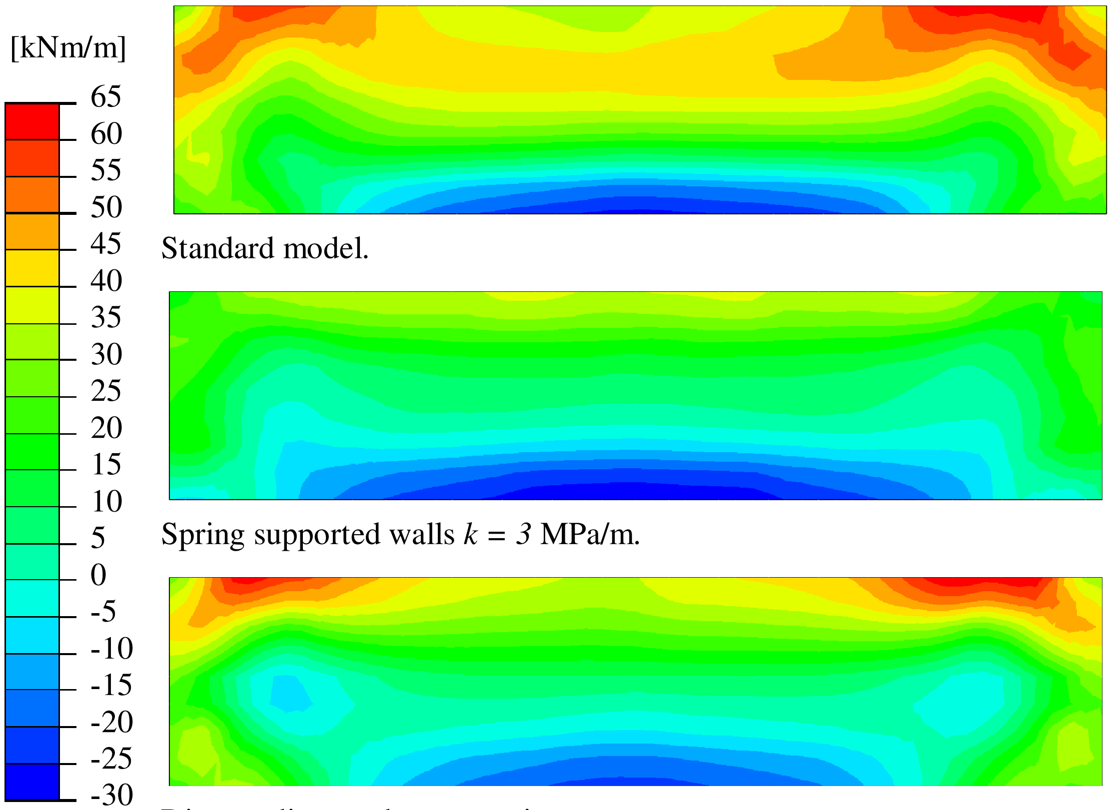



*Figure 5.24. Total required primary reinforcement in section 3-3. Almost no differences between the models could be seen.*

### 5.5.4 Reinforcement moment distribution in the frame walls

It can be expected that the influence of modelling choice should be bigger on the frame walls. This was confirmed by studying contour plots of the reinforcement moment over the frame walls, see Figure 5.25. The design moment was slightly reduced for models with spring supported walls. Local peak values near the corner of the upper edge were also reduced, while they for the standard model using the model from TR-bro, Trafikverket (2009b), were increased. This reduction in the spring supported wall models might be because the springs help distribute forces more evenly along the frame walls.




Standard model.

Spring supported walls *k* = *3* MPa/m.

Disregarding earth pressure increase.

*Figure 5.25. Main reinforcement moment distribution over the frame walls in maximum envelope. From the top: Standard model with earth pressure increase according to TR-bro (top), Spring supported wall model with spring stiffness k = 3 MPa/m (middle) and model neglecting increase of earth pressure against the frame walls (bottom)*

### 5.5.5 Discussions on the results of frame wall earth pressure increase modelling

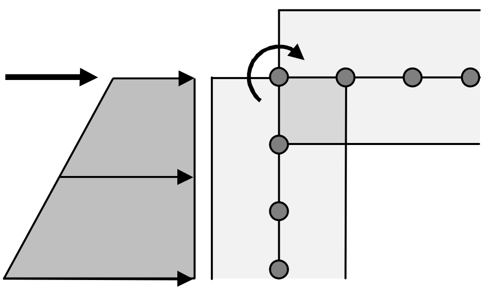
It was found that the frame wall earth pressure increase modelling had a very small influence on the analysis results in the bridge deck. For the frame walls, however, the influence seems to be rather significant, at least locally.

It should also be noted that the model presented in TR-bro, Trafikverket (2009b), requires more effort to implement in modern 3D structural analysis models compared to the models with spring supports. This is because the earth pressure increase can only be decided after the deflection of the frame walls is known, thus the analysis has to be performed in steps. Furthermore the earth pressure is only included for some load cases which give a significant horizontal deformation. In this thesis it has been included for temperature expansion and contraction, one-sided surcharge and braking and acceleration loading. In reality, the frame walls will deflect for all load cases, also vertical loads will cause deformation of the frame walls due to rotation of the corner. Including the earth pressure increase for such loads should increase the rotational stiffness of the corner, allowing more moment to be resisted by the corners than in the field.

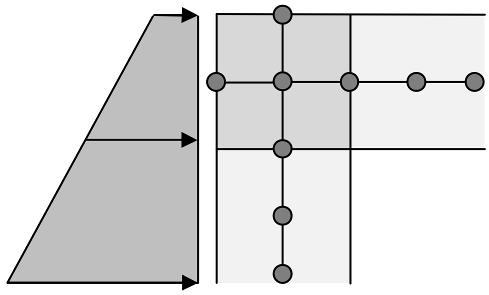
Modelling with spring-supported walls is simpler to implement in 3D structural analysis models; though it can prove difficult to assess parameters for the spring stiffness, O'Brien & Keogh (1999). Also, springs will normally resist deflection in both ways by either tension or compression of the springs, while in reality more resistance should be provided in compression requiring the use of non-linear springs. This might be handled if the springs could be controlled as active or inactive in different load cases. However that would require that the designer beforehand can estimate for which load cases the springs should be activated, which is not possible in most commercial FE-softwares (e.g. Brigade/Plus). Most loads acting on the structure should provide outwards deflection of the frame walls (i.e. compression in the springs). However, loads acting directly on the frame walls, such as earth pressure, should be adjusted to account for the resistance of the springs.

## 5.6 Influence of load application over corner regions

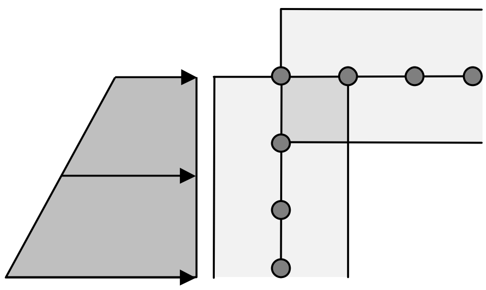
A model where the elements extend over the corner regions was created in order to study the influence load application outside of the corner centreline has on the analysis results, see Figure 5.26. The extended elements were given weak stiffness properties in the direction along the frame corner and no density in order not to influence the stiffness properties or the self-weight of the structure. Loads extending over the edges were then applied on the extended shell elements.



Model including load and moment due to loading outside of centerline



Model with elements extended over the corners



Model disregarding loading outside of centerline

*Figure 5.26. Models investigating loading over corner region. Standard model (top left) including earth pressure loading outside of the corner centerline by adding extra load and moment on the corner node. Model including loading over corner centreline by extending elements in the corner region over the corner (top right). Reference model disregarding all loading outside of corner centerline (bottom left)*

### 5.6.1 Modelling

For the standard model only earth pressure was included over the frame corner. Since line loads and moments cannot be assigned on common shell edges in Abaqus or Brigade a "weak" (no stiffness) beam element was created and tied to the nodes at the corner, see Figure 5.27. The location of the beam element was adjusted so that the applied moment should become correct.

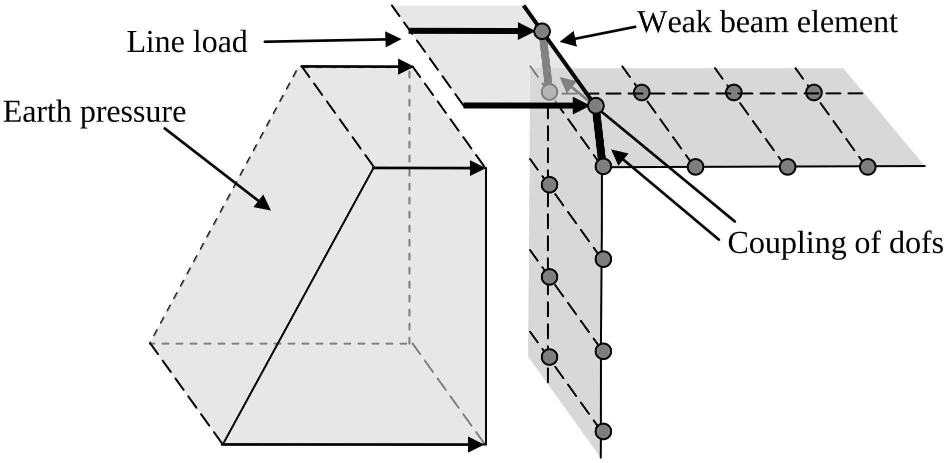



*Figure 5.27. Illustration of how earth pressure loading over the corner centerline is applied in the standard model. The eccentricity of the loaded weak beam element is adjusted so that the moment on the corner is correct. For illustrative purposes the model is not to scale and the elements in the deck are shown as straight.*

For the model with extended elements over the corner region, see Figure 5.28, the extended elements were given high stiffness in the primary direction to ensure correct coupling of the corner and no stiffness in the secondary, transverse direction. This was done in order to ensure that the stiffness of the frame wall was not altered due to introduction of elements above the corner centreline. The additional elements should also be massless in order not to introduce any additional load, not present in reality.

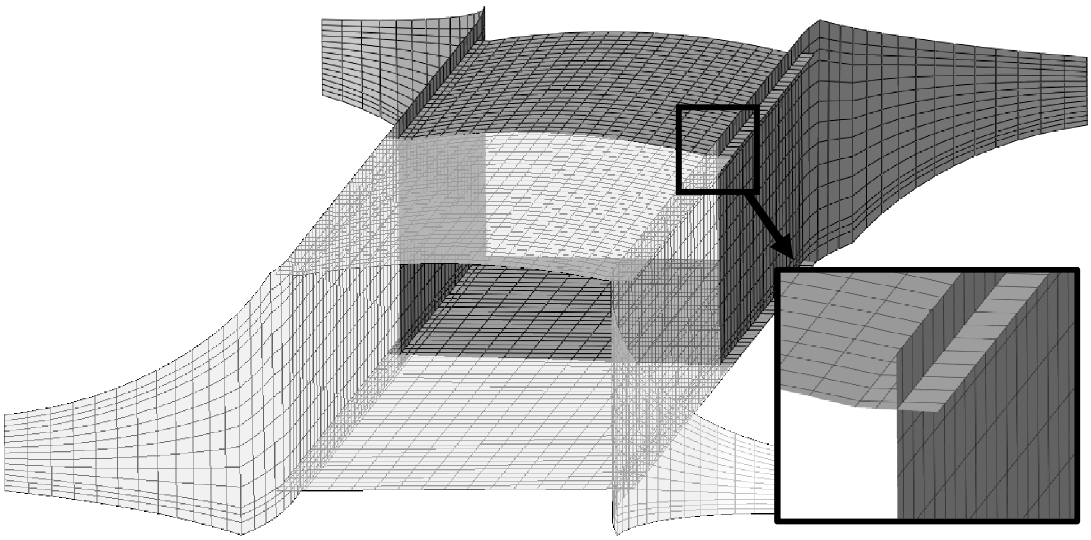


*Figure 5.28. The figure shows how the shells are extended over the corner in order to account for loading outside of the centerline.*

The model disregarding loading outside of the corner centerline was similar to the standard model though without the extra load applied outside the corner node.

### 5.6.2 Sectional forces in the bridge deck

By studying contour plots and reinforcement moment diagrams, see Figure 5.29 and Figure 5.30, it was clear that there are no discernible differences between the results from the studied load application modelling techniques. This shows that the influence of loading outside of the corner centerline is too small to be significant for design for this type of structures.

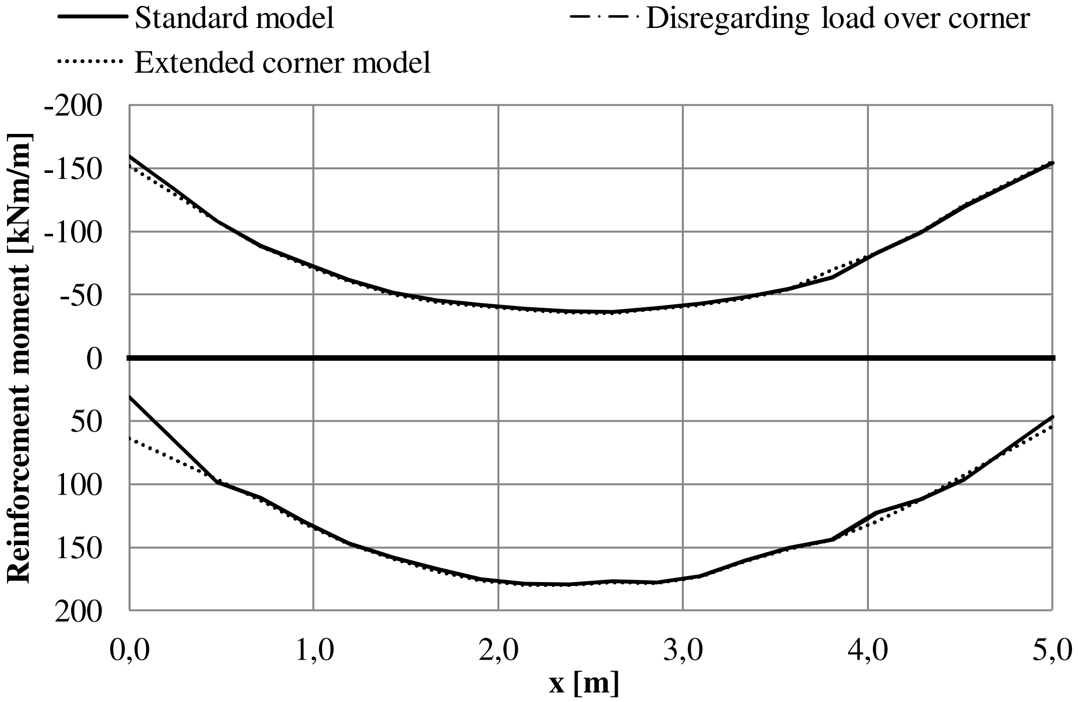



*Figure 5.29. Primary reinforcement moment in section 1-1.*

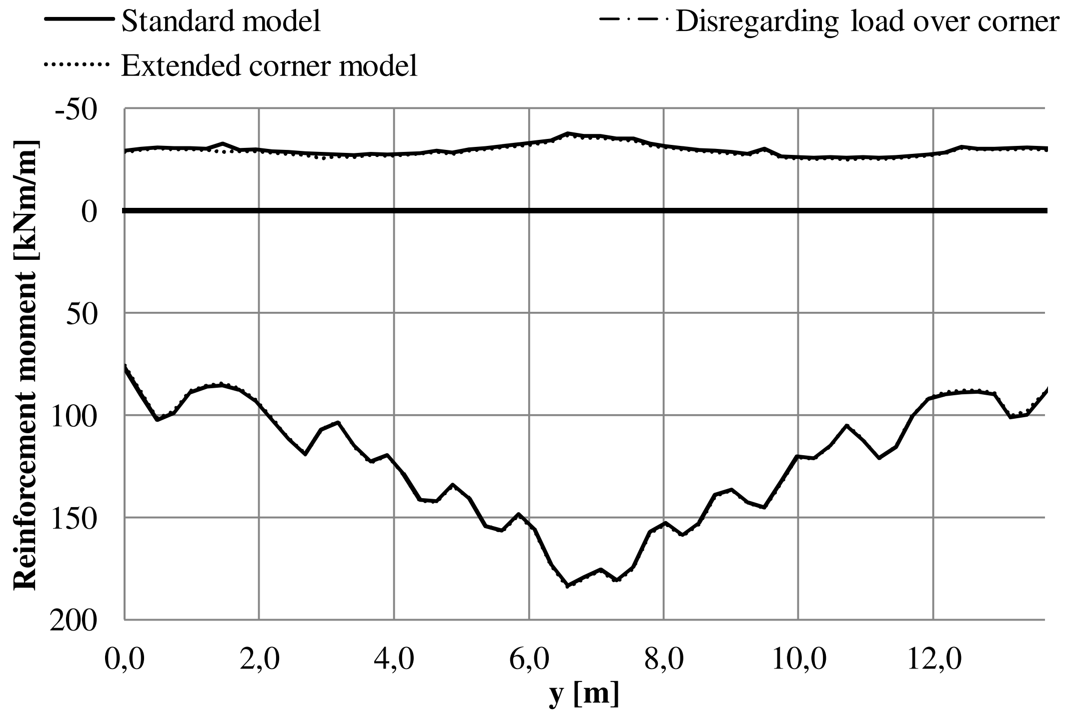



*Figure 5.30. Primary reinforcement moment in section 3-3.*

As can be seen in the figures above there was no significant difference between the moments in primary direction. The same can be said when studying graphs for other sections and in the transverse direction.

### 5.6.3 Required reinforcement in the bridge deck

As expected from section 5.6.2, no differences in required total amounts of reinforcement could be seen between the models, see Figure 5.31 and Figure 5.32.

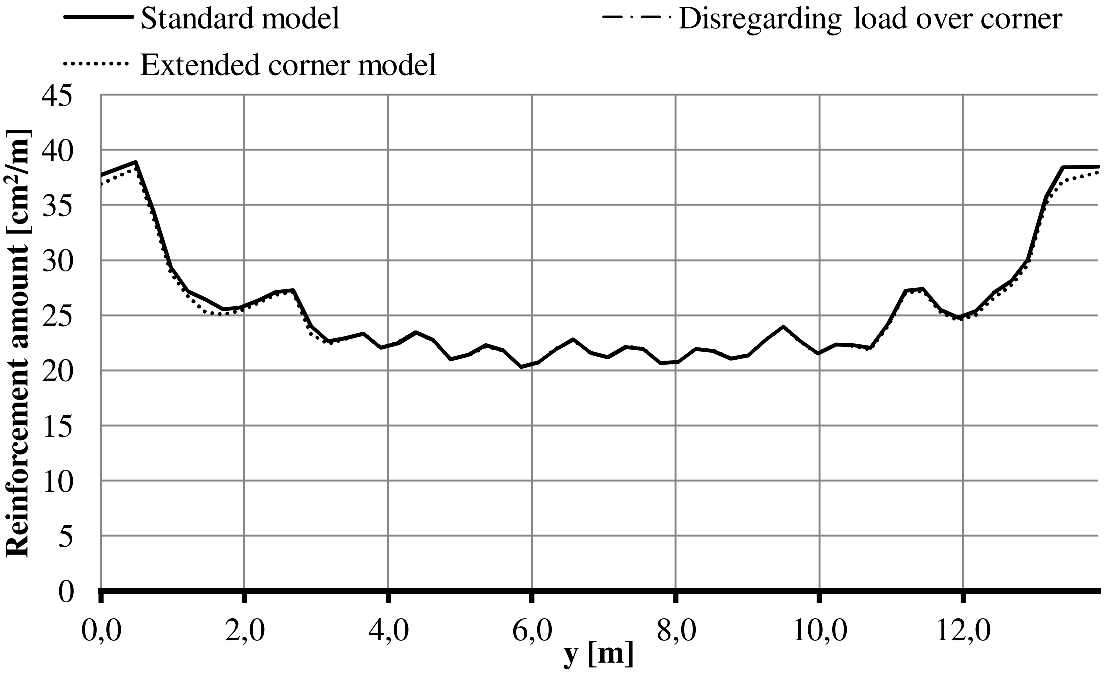



*Figure 5.31. Total required primary reinforcement in section 3-3. No differences between the models could be seen.*

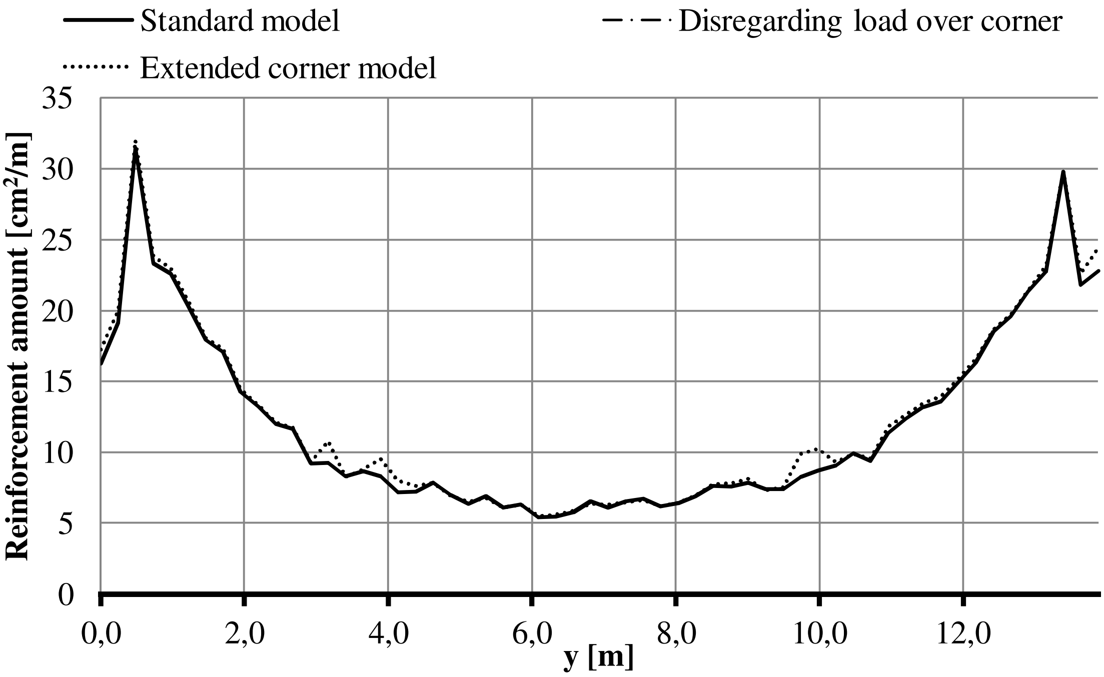



*Figure 5.32 Total required primary reinforcement in section 4-4. No differences between the models could be seen.*

### 5.6.4 Discussions on the results of load application over corner region

The loads applied outside of the corner region do not seem to have any influence at all. These loads are simply too small to influence the structural behaviour. However, as with the modelling of frame corner stiffness in section 5.4 it can be seen that the stiffness assigned can have an impact on the behaviour, which prompts for difficulties in assigning the correct stiffness. If elements that are extended over the corners are given the wrong stiffness, the structural behaviour is changed and the model will give incorrect results similar to those presented in section 5.4.5.

## 5.7 Influence of inclined haunch modelling

The bridge deck was circularly haunched, which gives an opportunity to study the influence of how the geometry is modelled, more specifically the curvature of the elements. The different models for investigating this are presented in Figure 5.33 and include modelling the deck's centreline's arch-profile, disregarding the curvature modelling it as straight, and modelling the deck as straight with eccentrically defined element thickness.

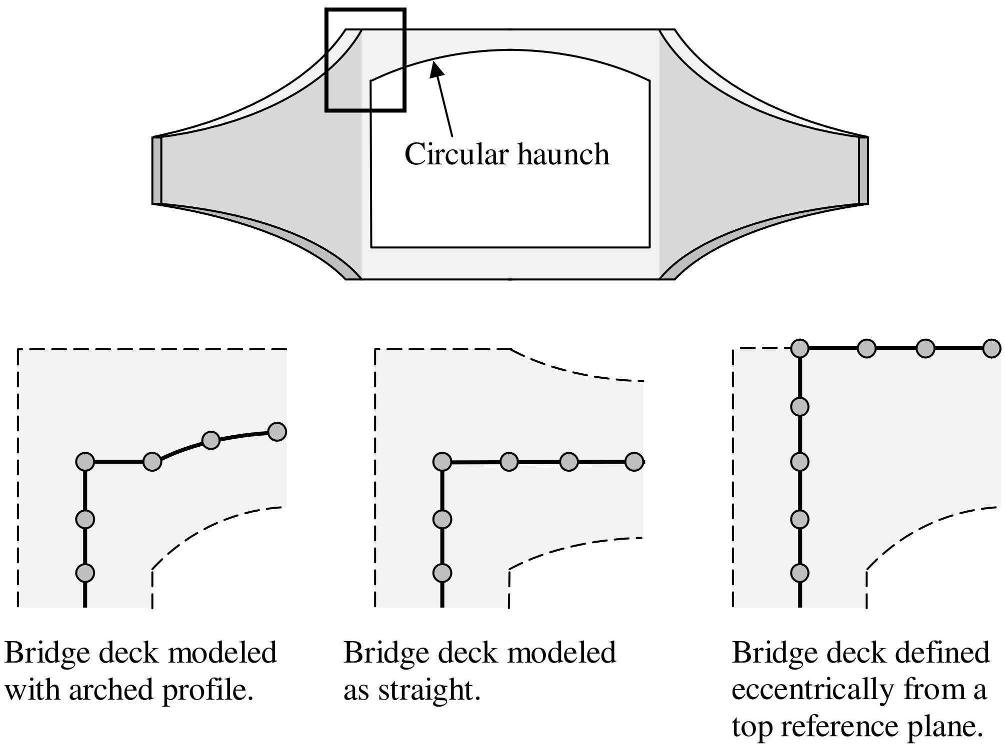



*Figure 5.33. Sketch of bridge profile illustrating the haunching of the deck (top). Models investigating influence of haunch modelling (bottom). The deck modelled with elements following the centreline of the haunch (left), modelled with straight deck geometry (middle) and modelling with thickness defined eccentrically from reference plane on the top (right).*

### 5.7.1 Modelling

The difference between the models lies with how they account for the curvature of the bridge deck. The model used as standard model for comparison had the bridge deck elements placed in the centreline, which gave a radius of about 12 m for the centreline curvature. In the second model used for comparison the centreline was modelled as straight, while retaining the elements thicknesses, thus not changing the properties of individual elements. The third model placed the reference plane for the deck on the top surface, while defining the elements thickness downwards from the reference plane, thus retaining the geometry.

### 5.7.2 Sectional forces in the bridge deck

Some differences between the models could be noted. To start with it was seen that the model including the arch-profile of the deck showed slightly smaller moments, see Figure 5.34, due to the slight arch action in the deck. The model which modelled the deck with an eccentric thickness definition resulted in some local variations near the edges of the deck, see Figure 5.35.

These variations were probably due to introduction of a fictitious lever arm, which affected the general structural behaviour. For external loads, such as traffic loads, the structural behaviour was generally similar, with the exception of slightly less stressed mid sections for the curved model with arch action. For load cases which originated from the internal movements of the structure, such as temperature, the difference was larger. Here, we can see that the geometry can have a substantial impact on the loading due to restraining effects.

In ULS the difference was less since traffic loading is dominating for design. Therefore the impact of the modelling is smaller, but still visible. Especially in the minimum envelope where the impact of temperature is more distinct for design, see Figure 5.36.

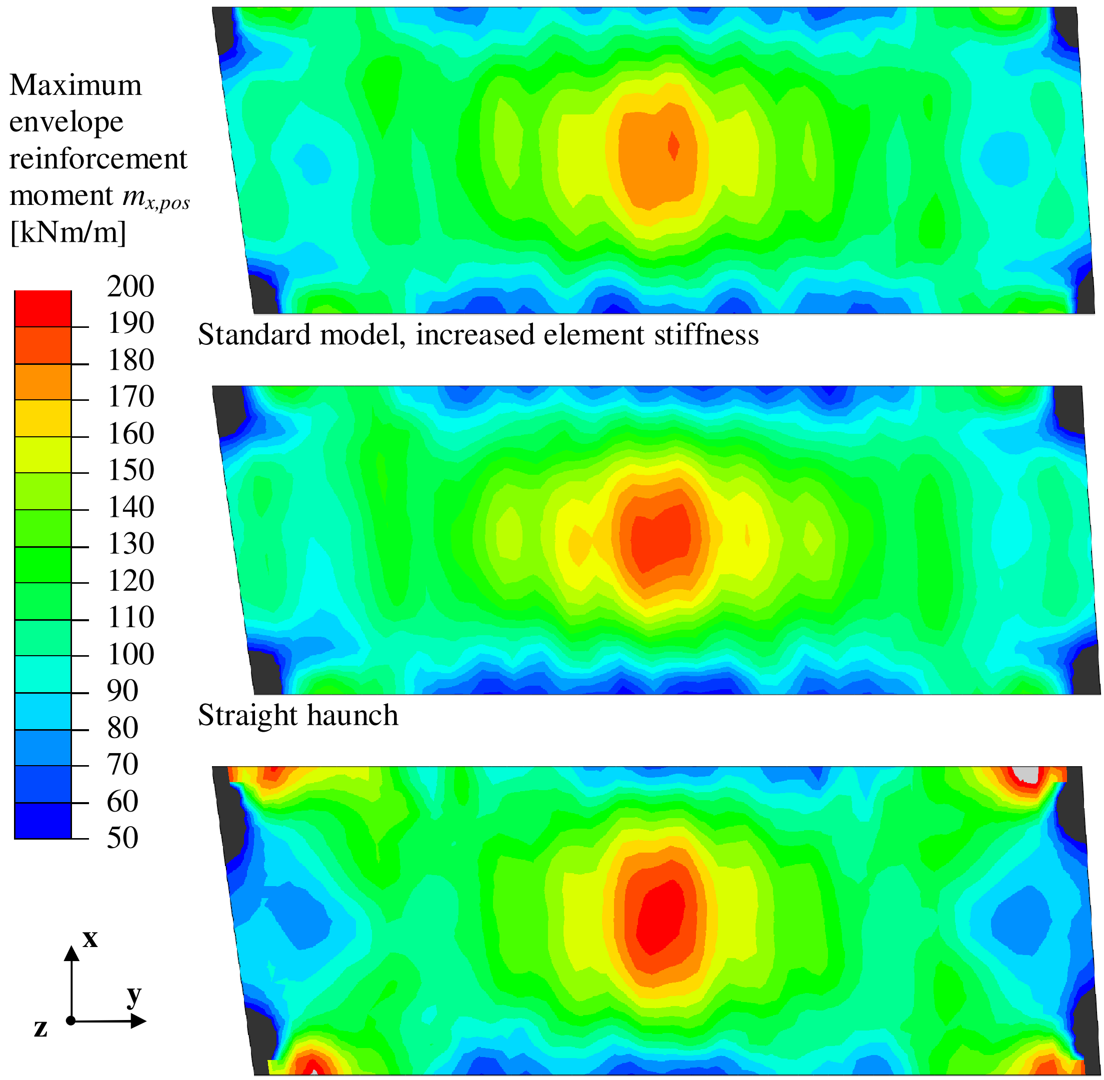



*Figure 5.34. Primary reinforcement moment in the bridge deck. The difference in mid field section was only marginal, but near the edges the difference was much more substantial.*

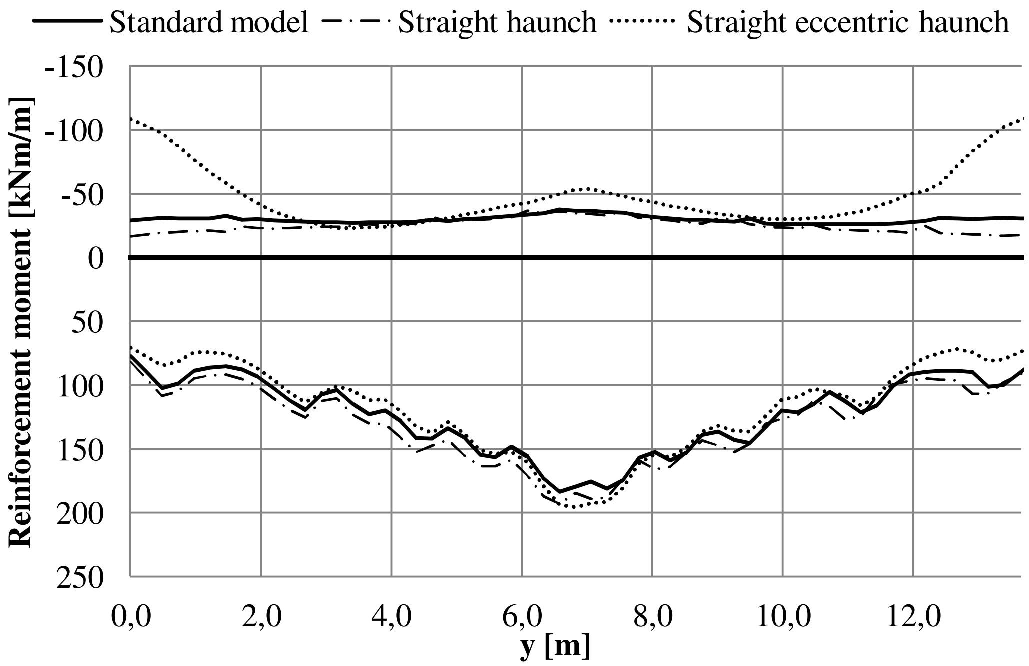



*Figure 5.35. Main reinforcement moment in section 3-3 along the midspan of the bridge deck. Notice how the moment differs in the minimum envelope for the model with eccentric haunch.*

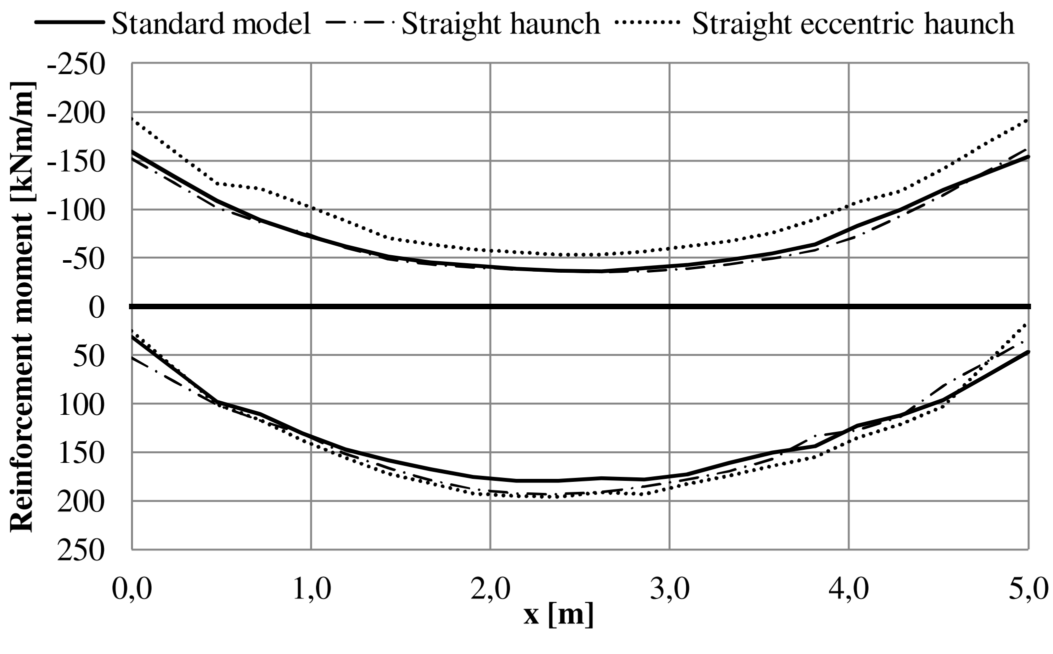



*Figure 5.36. Main reinforcement moment in section 1-1 over the span of the bridge deck. Notice how the moment differs in the minimum envelope for the model with eccentric haunch.*

### 5.7.3 Required reinforcement in the bridge deck

Differences between total reinforcement amounts could also be noticed between the models. The model with eccentric thickness show a requirement for more reinforcement in the midspan sections, see Figure 5.37. This increase was larger than the increase of reinforcement moment shown in section 5.7.2 indicating a larger influence of accompanying tensile membrane forces.

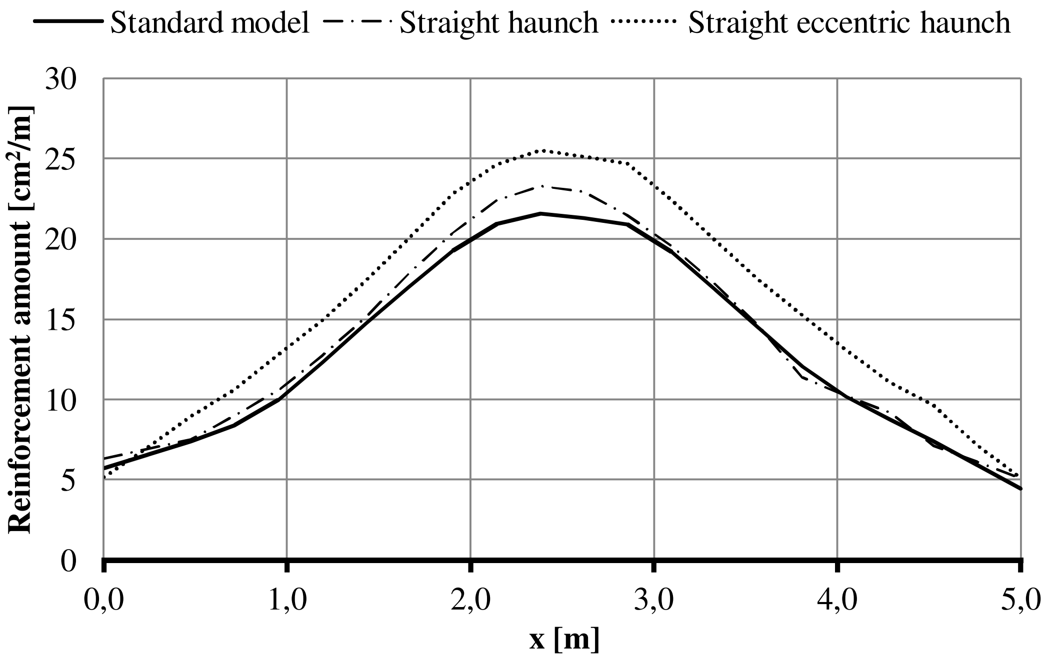



*Figure 5.37. Total amount of required reinforcement in section 1-1. A clearly visible increase for the model with eccentrically defined thickness.*

However, the increase was most marked in a section near the edge beams, see Figure 5.38, most probably due to the inadvertent restraining effects discussed in section 5.7.2.

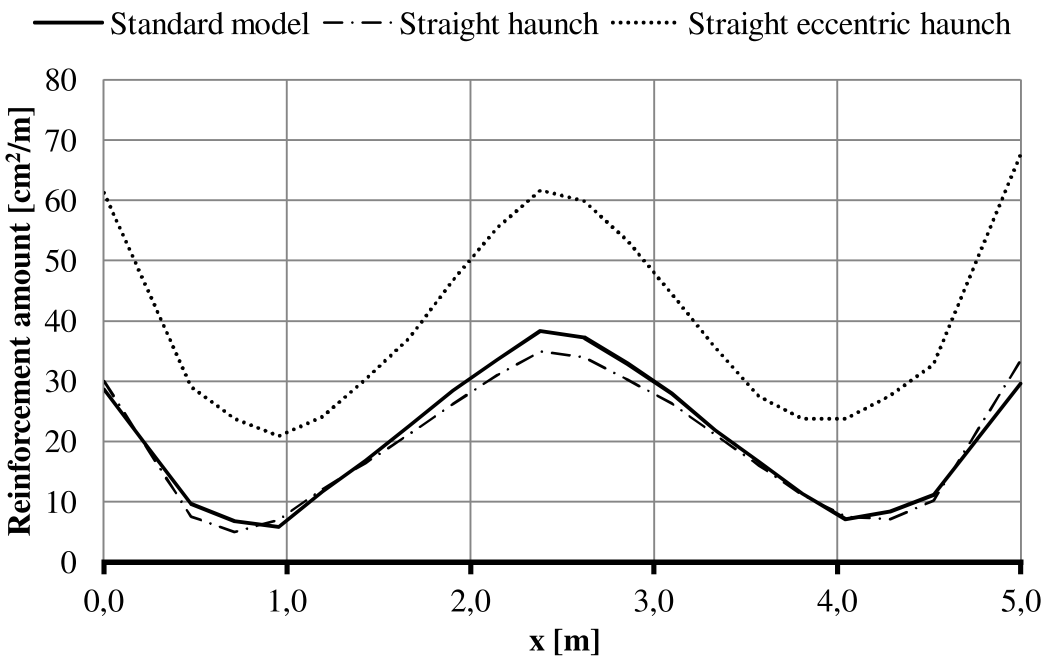



*Figure 5.38. Total amount of required reinforcement in section 2-2. The model with eccentric thickness shows a large increase (factor 2) in required reinforcement.*

### 5.7.4 Discussion on the results of inclined haunch modelling

It can be seen that there is a difference between accounting for the curvature and disregarding it, even though for the case studied, where only a slight curvature was present, the difference was limited. Though it is outside the range of this thesis, it might have been interesting to study the effects on shear and shear response, since that relates to the structural purpose of the haunch. When modelling the bridge deck with eccentricity it could be seen that the response was altered, consequently it should be avoided.

The technique of using elements which are not placed in the centreline of the structure is often adopted when different levels of detailing are used. Then, a simplified model of the structure surrounding an area of interest can be modelled with simplified geometry while the area of interest is modelled more in detail, Broo, et al. (2008). However, in such a case, one is not interested in evaluating the response in the areas modelled with simplified geometry, the purpose is merely to more accurately describe boundary conditions and to capture loading surrounding the detailed area of interest. In such cases it does not matter if the response in the simplified areas provides results which are not accurate. However, a recommendation based on the results above is that eccentricity of reference elements should not be used when it is of interest to evaluate the response of the region modelled with eccentricity.

# 6 Comparison of modelling techniques for double beam bridge

A three-span double-beam bridge, Figure 6.1, was the object for the second study in this thesis. Here, alternatives to modelling are studied, with focus on choice of elements and how the structural model is assembled. Two models were studied; the first model with beam elements for the main beams of the bridge and the second model with a spatial assembly of shell elements for the main beams. The substructure was modelled in the same way for both models, with shell elements as slabs and walls and the columns modelled with beam elements.

*Figure 6.1. Example of a three-span double-beam bridge. The geometry of the studied bridge is loosely based on this one.*

## 6.1 Layout and geometry of studied bridge

The three-span bridge had span lengths 17.0 – 23.5 – 17.0 m, see Figure 6.2, and a width of 10 m. The deck was cambered by 2.5 % on each side of the centerline. The abutments were integrated with the bridge deck and beams, they were cast together allowing the transfer of moment. The mid-support columns, support 2 and 3, were monolithically joined with the beams while the end support 1 and 4 were pinned to the beams.

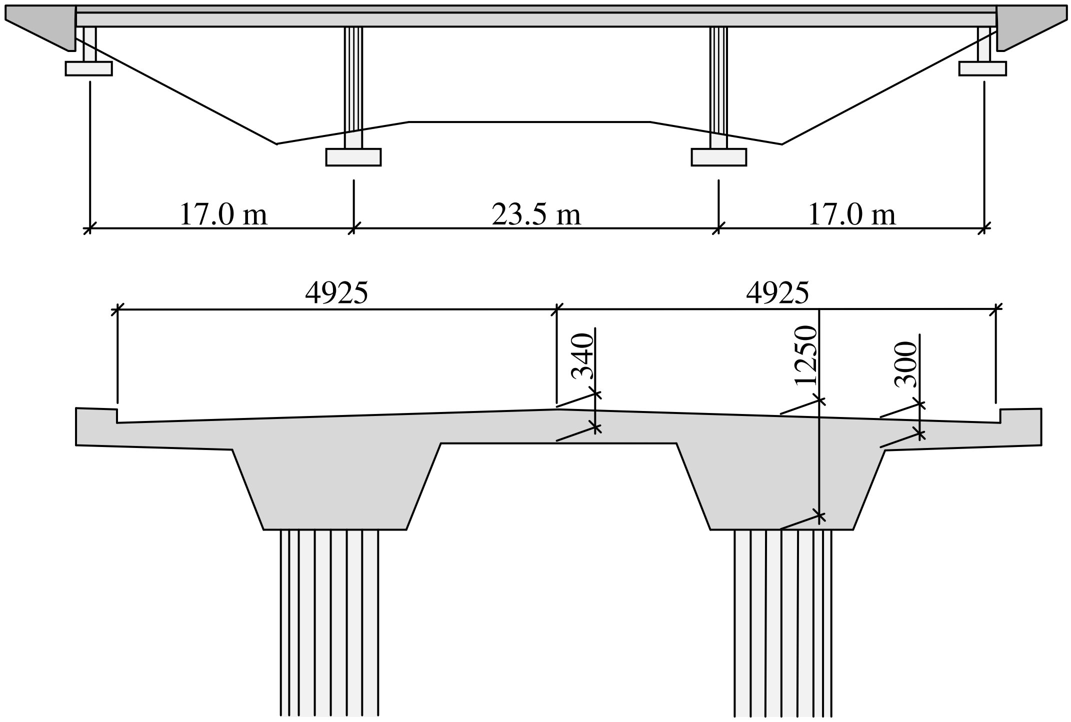



*Figure 6.2. Sketch of the studied three-span beam bridge. (Top) view of the bridge showing span-lengths, (bottom) section of the bridge deck and main beams.*

## 6.2 Modelling

Common for the compared models were that the substructure was modelled in the same way. Slabs and walls were modelled with shell elements, while columns were modelled using beam elements. The bridge deck was in both models modelled with shell elements, while the beams were modelled as beam elements or a spatial assembly of shell elements for the different models, see sections 6.2.2 and 6.2.3 respectively.

In a similar manner as the modelled slab frame bridge, see section 5.2, linear elastic uncracked gross concrete sections were used to model the material behaviour. The material properties were therefore set as C35/45 concrete with a modulus of elasticity of 34 GPa, a poissons ratio of 0.2 and a coefficient of thermal expansion of $10^{-5}$.

As with the previously modelled slab frame bridge, see section 5.2.1, the foundation was modelled with spring-elements with stiffnesses based on a simplified settlement analysis. The stiffness was set to 35 MPa for supports 2 and 3 (middle supports), and 25 MPa for supports 1 and 4 (end supports).

### 6.2.1 Loads

The bridge was studied for the same loads as the frame bridge in Chapter 5. Below is a short summary of included loads:

- **Self weight** was included as a material density 2500 kg/m$^3$ (reinforced concrete) and acceleration of gravity 10 m/s$^2$. For the shell model the self weight of the beams were applied as a surface load on the bridge deck above the beams. The edge beams were included as an edge load and an edge moment acting on the edges of the bridge deck.

- **Surfacing** was applied as a uniform surface load on the bridge deck. The surfacing consisted of a 70 mm thick concrete cover with density 2400 kg/m$^3$ (plain concrete). Applied surface load became $0.07 \times 24 = 1.68$ kN/m$^2$.
- **Earth pressure** was applied on wing walls and endshields of the abutments with soil density of 20 kN/m$^3$ and characteristic soil friction angle $\varphi_k$=40°. This gives a characteristic value for the earth pressure coefficient at rest to 0.36 and a design value of 0.46.
- **Surcharge** due to loading on the carriageway behind the abutments was applied as pressure on the endshields according to TK Bro, Trafikverket (2009a). The load was averaged and applied as a uniform surface load on the entire surface.
- **Temperature.** Expansion or contraction due to temperature increase or decrease and temperature gradient were included for the bridge deck and main beams. Temperature expansion corresponded to an increase of 20°C while contraction corresponded to a decrease of -40°C. Gradients for heated and cooled surfaces are $\Delta$T=10.5°C and $\Delta$T=8°C, respectively.
- **Wind load** was applied on bridge deck edges as a horizontal edge load and a moment due to eccentricity.
- **Traffic loads** used includes load model 1 (distributed and axle loads) and 2 (single axle load).
- **Load combination.** The bridge was studied for ULS load combination according to Eurocode and Trafikverket.

### 6.2.2 Beam model

In this model, beam elements were used to represent the main beams. Connections between the columns and superstructure were made with "stiff" beam elements between the beam centrelines and column tops, see Figure 6.3. The beam elements were then placed below the shell elements representing the bridge deck. They were connected by rigid links, allowing full interaction. A single row of weak shell elements were used to span the bridge deck area above the beam, see Figure 6.4, allowing transfer of load to the rest of the structure.

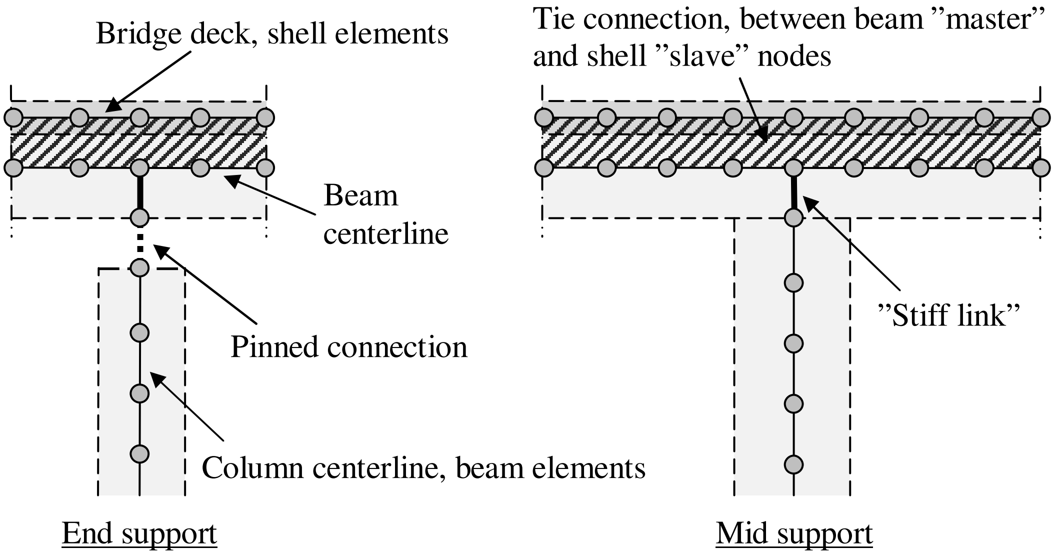



*Figure 6.3. Description of static system for column connections for shell-beam model. Connections for model with only shell elements is made analogously by setting a stiff beam connection between shell and column tops.*

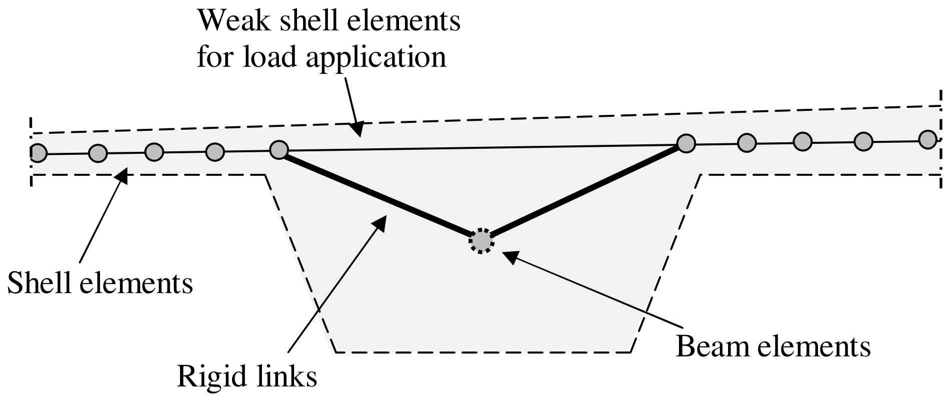



*Figure 6.4. Deck cross section for shell-beam model. The weak shell elements above the beam distribute loads to the beam elements.*

Since the beams were cast together with the abutments, the beams were connected with rigid links to the surface of the end shields of abutments. This captured the structural behaviour in that region and avoided connection of the beams to a single point of the end shields.

### 6.2.3 Shell model

The shell model used a box-like assembly to represent the main beams, where the bottom layer became tensioned and the top layer compressed during bending in the midspan, see Figure 6.5. In principle, membrane actions will dominate allowing for a simple design of the cross sections.

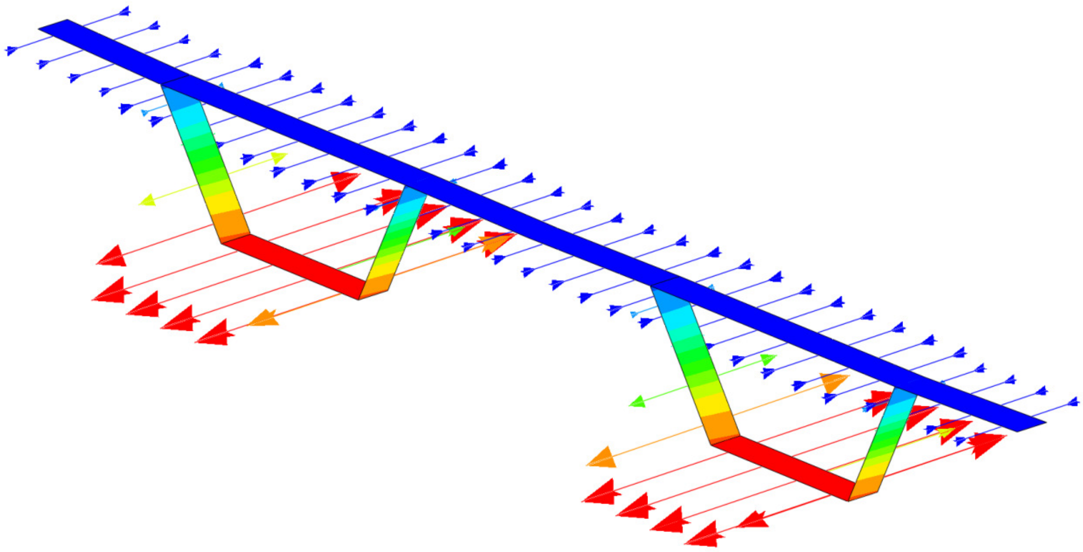


*Figure 6.5. Illustration showing the response of a cutout part of the bridge section during bending. Lower parts of the main beams are tensioned and the top parts are compressed.*

In order for the box assembly to behave as a single beam, the elements in the assembly were coupled to a dummy node, see Figure 6.6. This ensured that plane sections remained plane and that the sections deformed as a unit.

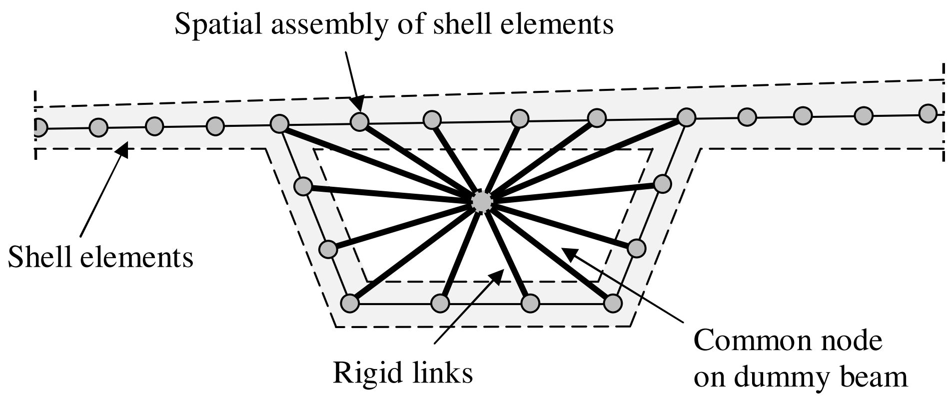



*Figure 6.6. Deck cross section for spatial assembly shell model. Nodes in plane sections are coupled in order to ensure beam action of the cross section.*

## 6.3 Methodology of comparison

Comparison of the models was made in a simplified manner focusing on the required reinforcement in the main beams. This was calculated for the beam model according to the simplified method presented in section 3.3, again presented as equation 6.1.

$$A_s \approx \frac{M_{Ed}}{f_{yd} \cdot 0.9d} \tag{6.1}$$

For the shell model, the top and bottom reinforcement was designed as the result of the membrane forces in the top and bottom elements in the beam, see equation 6.2.

$$n_{xt} = n_x + |n_{xy}|$$
$$A_s \approx \frac{n_{xt}}{f_{yd}} \tag{6.2}$$

## 6.4 Resulting primary reinforcement in bridge main beams

In order to verify the models the total support reactions are compared in Table 6.1. Since the differences are less than 1 %, it is practically negligible.

*Table 6.1. Comparison of support reactions for support 1 (end support) and support 2 (mid support). Reactions evaluated at the bottom of the foundation slabs. In general the differences are small, the largest difference is noticed in the moment around an axis perpendicular to the supports.*

| **Support reaction** | | **Beam** | **Shell** | **Diff. [%]** |
|---|---|---|---|---|
| **Self weight** | | | | |
| Vertical support 1 | [kN] | -2387 | -2381 | 0% |
| Vertical support 2 | [kN] | -4752 | -4752 | 0% |
| Parallell support 2 | [kN] | 70 | 71 | 1% |
| Moment perpendicular support 2 | [kNm] | 118 | 120 | 2% |
| | | | | |
| **Trafic characteristic** | | | | |
| Vertical support 1 | [kN] | -1239 | -1240 | 0% |
| Vertical support 2 | [kN] | -1715 | -1715 | 0% |
| Parallell support 2 | [kN] | -109 | -110 | 1% |
| Moment parallell support 2 | [kNm] | -1928 | -1902 | -1% |
| Perpendicular support 2 | [kN] | -170 | -158 | -7% |
| Moment perpendicular support 2 | [kNm] | -290 | -293 | 1% |
| | | | | |
| **ULS** | | | | |
| Vertical support 1 | [kN] | -5731 | -5937 | 4% |
| Vertical support 2 | [kN] | -9401 | -9399 | 0% |
| Parallell support 2 | [kN] | -105 | -106 | 0% |
| Moment parallell support 2 | [kNm] | -3044 | -3010 | -1% |
| Perpendicular support 2 | [kN] | -268 | -250 | -7% |
| Moment perpendicular support 2 | [kNm] | -370 | -372 | 1% |

Similarities between the models are also visible when studying the sectional moment in the bridge deck as well as the deflections, shown in Figure 6.7 for the self weight load case.

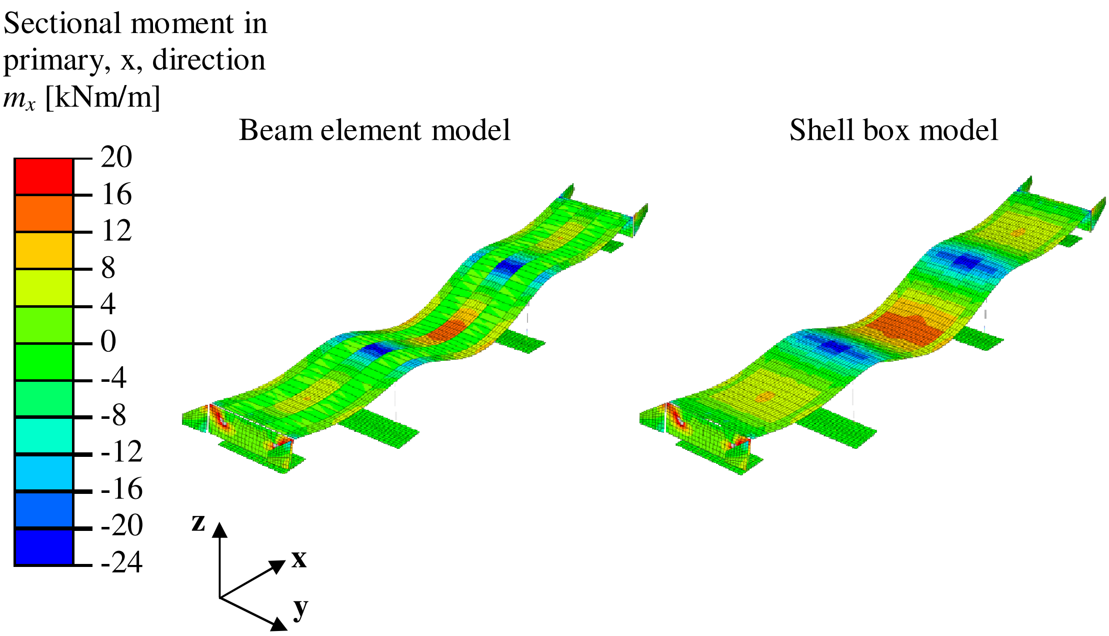



*Figure 6.7. Simple comparison of primary sectional moment and deformations in self weight load case. The models show similar behaviour in both parameters. The area above the beams in the beam element model shows no stress since the load in that area is carried by the beam elements below the deck. Deflections scaled in both models with a factor 500.*

Also when studying the main reinforcement requirement it becomes evident that the difference between the modelling methods is small, see Figure 6.8 and Figure 6.9. One small difference which can be observed is that the shell model shows a requirement for more bottom reinforcement, and less top reinforcement indicating a different distribution between span and support sections.

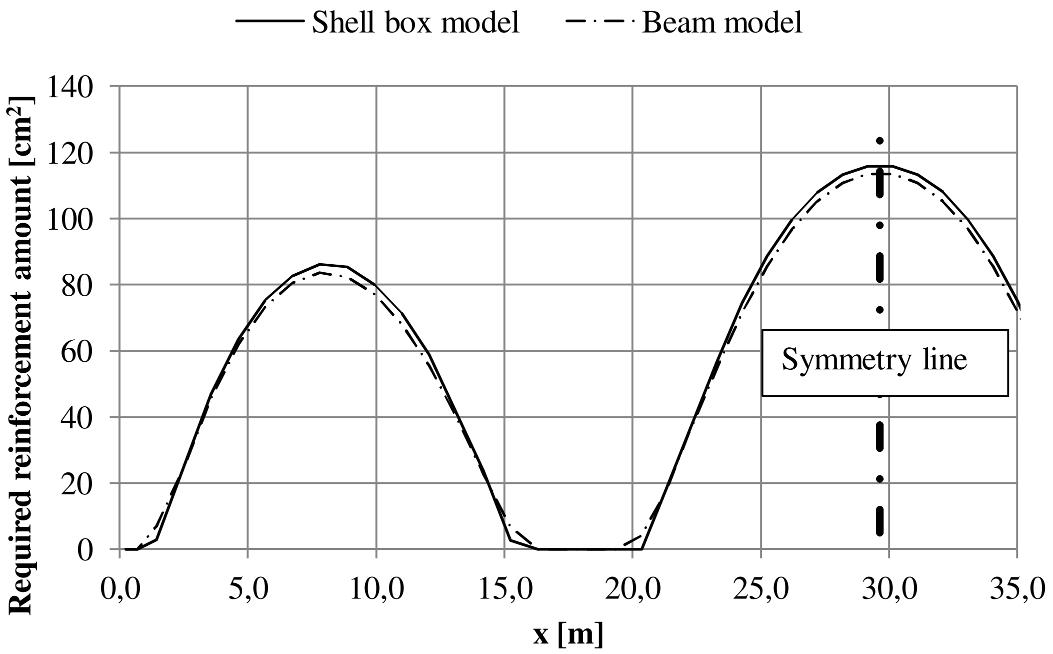



*Figure 6.8. Required main bottom reinforcement in one of the beams. Slightly higher requirement for the shell model than the beam element model.*

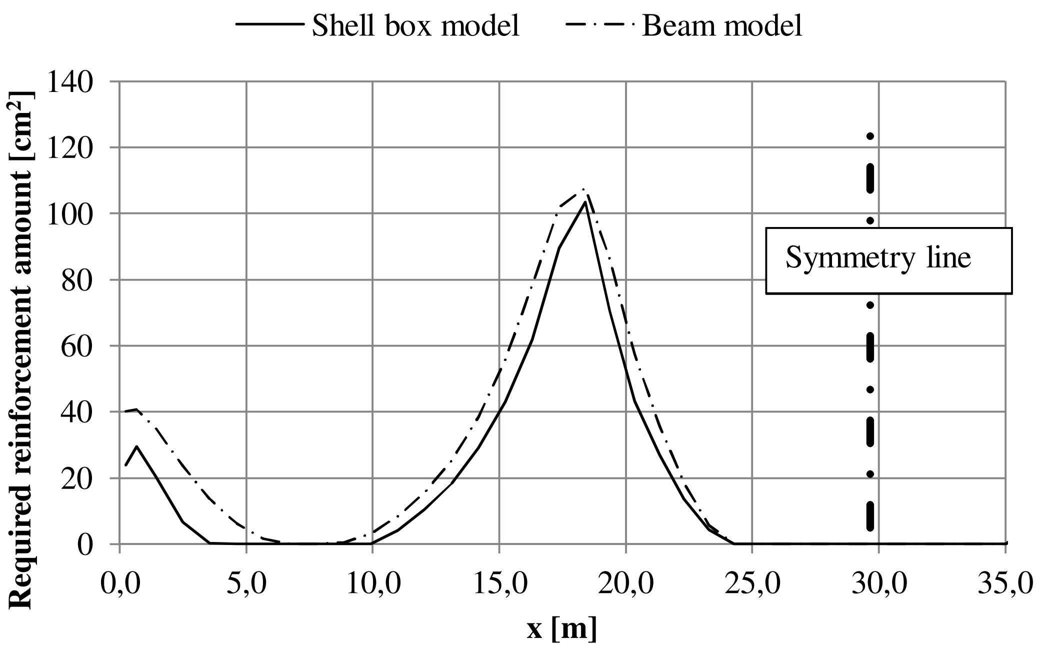



*Figure 6.9. Required main top reinforcement in one of the beams. The beam element model shows a slightly higher reinforcement requirement than the shell element model.*

Some differences could also be seen in a closer study of the response in the bridge deck, see Figure 6.10. By studying the maximum envelope normal membrane force in the bridge deck primary direction it was possible to observe slight differences between the reactions of the models. It is possible that these differences are the result of different sectional force distributions within the sections.

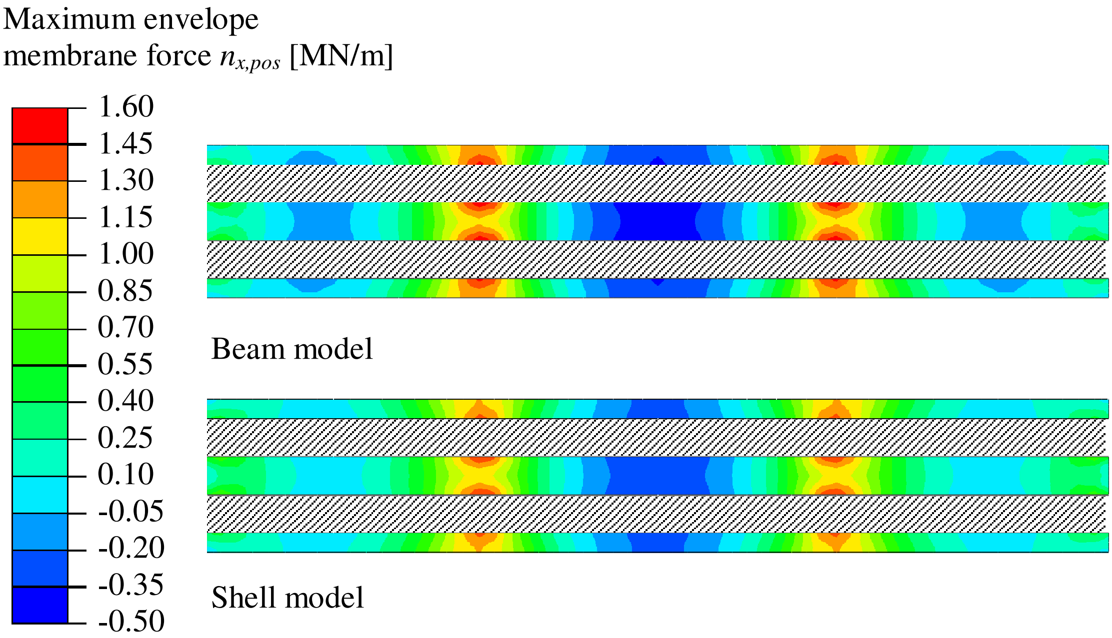



*Figure 6.10. Maximum envelope SF1 (tensile membrane force in main longitudinal direction). The deck of the beam model shows more tension than the shell model, implying a need for more reinforcement in the deck. Areas of the deck above the beams are not shown since the results in those areas represent the beams.*

## 6.5 Discussion on modelling with shell elements

The main objective of the study performed in Chapter 6 was to illustrate an alternative modelling method, and it was shown that it is possible to model beam structures with shell elements. However, in order to draw more accurate conclusions a more thorough analysis of the results is needed. This could be done for example by analysing and comparing different sections in both main and transverse directions.

In general only very small differences was noticed between the models, and since the total support reactions of both models are practically equal it is safe to assume that both models are loaded in the same way. Most distinct observed differences between the models are differences in sectional force distribution, both between and within cross-sections. Since it has been shown that both models describes the same loading situation both models should be valid for design, providing that the checks for the cross-sections capability to develop their full capacity are made.

One main advantage of the shell model is that it probably simplifies the design of the intermediate areas between main beams and bridge deck. Since the transition between these areas for the shell model is smoother, and doesn't involve any coupling of different element types, the shell model provides more accurate information about the response and requirement of reinforcement in these areas. This indicates that these areas can be designed in a more intuitive way which better reflects the needs in the structure.

# 7 Conclusion

In this thesis different modelling procedures for structural analysis in design of concrete bridges are studied. The results show that in most cases there are only small differences between the models and procedures for structural analysis. However, it was identified that when modelling according to some principles inadvertent restraint might be introduced to the model if certain care was not taken. This might alter the response drastically, while still be difficult to detect when studying envelopes in ULS.

Introduction of inadvertent restraint was mainly an issue when modelling with stiff shell elements in frame corners. It was identified that lack of verification and difficulties in interpreting results could easily lead to mistakes and undesired results.

It was also noticed that some load models give differences in results. Though it was not studied in detail, it could be seen that different methods of modelling frame wall and soil interaction gave different moment distributions over the frame walls. The load model for earth pressure increase presented in Trafikverkets recommendations document, Trafikverket (2009b), was more difficult and less intuitive to use in 3D analyses.

In general it could be said that verification and interpretation of 3D models can be difficult. It is also difficult to assess the behaviour of a 3D model under certain types of loading beforehand, for example temperature load effects. Therefore it is always important to carefully and critically study the results in order to assess their credibility. Since modelling in 3D is in principle a requirement for structural analysis today, there is a need for guidelines and easy to use verification methods.

The response of concrete structures is in reality non-linear, while the design of such a structure is made on the basis of a linear structural analysis. This is possible due to the structures ability to adapt, provided capacities by redistribution of sectional forces to stiffer regions in the structure. As the purpose of a structural analysis for design is not to accurately describe the response of the structure, other parameters for the structural analysis model should be prioritised, such as usability. Important features regarding usability include; model construction, verification and interpretation.

Since the impact of choices in model construction is relatively small, as long as errors are avoided, a structural engineer could be relatively free in constructing structural analysis models. As the difference between different models is small it is more important to focus on that a model doesn't introduce errors than it is entirely accurate. In short, one could say that it is more important to avoid errors than it is to model accurately. Therefore it is important that an analysis model is easily verified with simpler models, since errors otherwise easily arise in 3D modelling.

It is important to point out that these suggestions are only valid for design of structures, when evaluating the response of existing structures different approaches are necessary.

## 7.1 Suggestions for further work within the subject

- Development of guidelines for modelling in 3D, together with easy to use methods for model verification.
- Adapt current design codes to 3D-modelling, for example load models in TK-bro.
- Further studies on the impact of structural analysis on the design, for example by verifying design based on linear elastic structural analysis with non-linear FE-models.
- Study the influence of design based on linear elastic structural analysis on verification of response in SLS.

# 8 References


Al-Emrani, M., Engström, B., Johansson, M. & Johansson, P. (2008): *Bärande konstruktioner Del 1* (Load bearing structures part 1. In Swedish). Department of Civil and Environmental Engineering, Chalmers University of Technology, Göteborg.

Blaauwendraad, J. (2010) *Plates and FEM - Surprises and Pitfalls.* Springer, Dordrecht.

Broo, H., Lundgren, K. & Plos, M. (2008): *A guide to non-linear finite element modelling of shear and torsion in concrete bridges.* Department of Civil and Environmental Engineering, Chalmers University of Technology, Göteborg.

Caselunghe, A. & Eriksson, J. (2012): *Structural Element Approaches for Soil-Structure Interaction.* MSc. Thesis. Department of Civil and Environmental Engineering, Chalmers University of Technology, Göteborg.

Dassault Systèmes (2008) *Abaqus User's Manual.* Dassault Systèmes, United States of America.

Davidson, M. (2003): *Strukturanalys av brokonstruktioner med finita elementmetoden - Fördelning av krafter och moment* (Structural analysis of bridge structures with the finite element method - Distribution of forces and moments. In Swedish). Brosamverkan Väst, Göteborg.

Engström, B. (2011a): *Design and analysis of continuous beams and columns,* Department of Civil and Environmental Engineering, Chalmers University of Technology, Göteborg.

Engström, B., (2011b). *Design and analysis of deep beams, plates and other discontinuity regions.* Department of Civil and Environmental Engineering, Chalmers University of Technology, Göteborg.

Fib Bulletin 45 (2008): *Practitioner's Guide to Finite Element Modelling of Reinforced Concrete Structures.* CEB-FIB, Lausanne.

O'Brien, J. E. & Keogh, L. D. (1999): *Bridge deck analysis.* E & FN Spon, Cornwall.

Plos, M. (1996): *Finite element analyses of reinforced concrete structures.* Department of Civil and Environmental Engineering, Chalmers University of Technology, Göteborg.

Rombach, G. A. (2004): *Finite element design of concrete structures.* Thomas Telford, Cornwall.

Scanscot Technology AB (2010): *Brigade/Plus User's Manual.* Scanscot Technology AB, Sweden.

SS-EN 1991-2 (2007). *Eurokod 1: Laster på bärverk - Del 2: Trafiklast på broar* (Eurocode 1: Actions on structures - Part 2: Traffic loads on bridges. In Swedish). SIS Förlag AB, Stockholm.

SS-EN 1992-1-1, 2008. *Eurokod 2: Dimensionering av betongkonstruktioner - Del 1-1: Allmänna regler och regler för byggnader* (Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings. In Swedish). SIS Förlag AB, Stockholm.

SS-EN 1992-2 (2005): *Eurokod 2: Dimensionering av betongkonstruktioner - Del 2: Broar* (Eurocode 2: Design of concrete structures - Part 2: Concrete bridges - Design and detailing rules. In Swedish). SIS Förlag AB, Stockholm.

Sustainable Bridges (2007): *Non-Linear Analysis and Remaining Fatigue Life of Reinforced Concrete Bridges.* Sustainable Bridges - Assessment for Future Traffic Demands and Longer Lives.

Trafikverket (2009a): *TK Bro* (Technical Requirements for Bridges. In Swedish). Trafikverket, Borlänge.

Trafikverket (2009b): *TR Bro* (Technical Recommendations for Bridges. In Swedish). Trafikverket, Borlänge.