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# Empirical Formulas for Designing Gap-Waveguide Hybrid Ring Coupler

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Abstract—A new type of hybrid ring coupler (or rat-race coupler) - the gap-waveguide ring hybrid has been developed in this work. The advantages of the new gap-waveguide ring hybrid are low manufacturing cost since the good electrical contact between metal plates is not required, and lower transmission loss compared to microstrip line since there is no dielectric material needed. Therefore, the gap-waveguide technology opens up a possibility of waveguide integration circuit from millimeter wave up to terahertz. We present an empirical formula for designing the gap-waveguide ring hybrid based on simulated data and has been verified by measurements of a prototype of the ring hybrid realized at 16 GHz. The prototype has an operating frequency band of 2 GHz around 16 GHz, with a reflection coefficient below -10 dB and a phase difference between two output ports of  $182^{\circ} \pm 2^{\circ}$ .

*Index Terms*—Hybrid coupler, rat-race coupler, gap-waveguide.

## I. INTRODUCTION

ANY feeding networks for antennas involve 3dB inphase  $(0^\circ)$  or out-of-phase  $(180^\circ)$  couplers. There are many different types of geometries for such couplers. The hybrid ring coupler (rat-race coupler) provides both in-phase and out-of-phase equal power division at its output ports with good isolation between them [1]-[3].

The hybrid ring coupler can be realized in waveguide or microwave strip-line. However, for high frequency applications, such as in millimeter wave, sub-millimeter wave and up to terahertz, there are some drawbacks to use these conventional technologies. By using rectangular waveguide, the tolerance requirement and good electrical contact for the metal waveguide walls impose difficulties for fabrication. By using planar microstrip circuits on printed circuit board (PCB), the loss in dielectric substrate is often unacceptable.

In this paper, we propose a new solution to hybrid ring coupler, potentially for high frequency applications - the gapwaveguide ring hybrid. Gap waveguide is a new microwave circuit technology introduced recently. For the basic theory of the gap waveguide, please refer to [4]-[7]. By using the gap-waveguide technology, there is no need to have electrical contact between the top metal plate and the low metal texture plate, and there is no dielectric substrate. Therefore, the new gap-waveguide hybrid has potential to have low manufacture cost and low transmission loss in high frequency applications.

The focus of this paper is to present empirical formulas obtained from simulated data for designing the gap-waveguide ring hybrid. The formulas provide good initial values of the dimensions when designing a gap-waveguide hybrid ring coupler, which therefore leads to an efficient design and optimization whenever a numerical EM solver is employed in the design.

A prototype of the gap-waveguide ring hybrid [8] designed by using the empirical formulas has been fabricated and presented in the paper for the sake of verification.



Fig. 1. Dimension definition of the gap-waveguide ring hybrid.

# II. EMPIRICAL FORMULAS

All dimensions for the gap-waveguide ring hybrid are defined in Fig. 1.

A. Dimensions of the pins According to [7], pins can be designed by

$$d_{pin} = 0.15\lambda, \qquad h_{pin} = 0.25\lambda$$
  
$$p_{pin} = 0.175\lambda, \qquad h \le 0.015\lambda.$$
 (1)

Note that h is the gap height between the top metal plate and the ridge, and  $\lambda$  the free space wavelength at the operating frequency.

B. Characteristic impedance of a straight ridge gapwaveguide The characteristic impedance  $Z_0$  of a straight ridge gap-waveguide can be expressed according to [7] as:

$$Z_0 = \frac{\eta}{2} \cdot \left[\frac{W_e}{2h} + 0.441\right]^{-1}$$
(2)

where

$$\frac{W_e}{2h} = \frac{W}{2h} - \begin{cases} 0, \quad W/(2h) > 0.35\\ 0.35 - (W/(2h))^2, \quad W/(2h) < 0.35 \end{cases}$$
(3)

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and  $\eta \approx 377$  Ohm is the free space impedance.

C. Characteristic impedance of a circularly curved ridge gap-waveguide The characteristic impedance of a circularly curved ridge gap waveguide, needed for designing a ring hybrid, has not been studied before. In this work, we use a conformal mapping transform to obtain the formula to calculate the characteristic impedance of a circularly curved ridge gap waveguide.



Fig. 2. Conformal mapping.

The characteristic impedance of a circularly curved microstrip line was analyzed in [9] by using a conformal mapping transform from z-plane to w-plane, defined as

$$w = lnz = lnR + j\phi$$

$$z = Re^{j\phi}$$
(4)

The geometry presentation of this transform is shown in Fig. 2. Then, a circularly curved microstrip line in z-plane can be converted into a rectangular microstrip line in w-plane by the above conformal mapping, where the equivalent width of the microstrip line in W-plane is  $W = \ln(r_1/r_2)$ . It was shown in [10] that capacitances, inductances, resistances, etc. evaluated in the w-plane have the same values as they are evaluated directly in the z-plane from the potential distribution.

However, the conformal mapping transform defined in (4) will not be valid when  $r_2 \rightarrow \infty$  because the equivalent width  $W = ln(r_1/r_2)$  of the straight line in *w*-plane is close to zero when  $r_2 \rightarrow \infty$ , while the equivalent width of the curved microstrip line should be close to  $W_r = r_1 - r_2$  because now the curved line becomes the straight line. Therefore, we introduce a modified conformal-mapping function here defined as

$$w = \sqrt{zz^*} + Clnz = R + ClnR + jC\phi \tag{5}$$

where C is a constant to be determined empirically. By the above definition, the width of the microstrip line in w-plane becomes  $W_r + CW'_r$ , where

$$W_r = r_1 - r_2$$
  

$$W'_r = ln\left(\frac{r_1}{r_2}\right).$$
(6)

We can see that when  $r_2 \to \infty$ ,  $W_r + CW'_r \to W_r$ , which agrees with the fact that the curved line becomes the straight line when  $r_2 \to \infty$ .

Therefore, the characteristic impedance  $Z_0^{ring}$  of a circularly curved ridge gap-waveguide is

$$Z_0^{ring} = \frac{\eta}{2} \cdot \left[\frac{W_{re}}{2h} + 0.441\right]^{-1},\tag{7}$$

where

$$\frac{W_{re}}{2h} = \frac{W_r + CW'_r}{2h} - \begin{cases} 0, & \frac{W_r}{2h} > 0.35\\ & \\ 0.35 - \left(\frac{W_r + CW'_r}{2h}\right)^2, & \frac{W_r}{2h} < 0.35 \end{cases}$$
(8)

We re-write  $r_1$  and  $r_2$  as

$$r_1 = r_0 + W_r/2, r_2 = r_0 - W_r/2.$$
(9)

The circumference of the ridge ring should be  $3\lambda_g/2$ , where  $\lambda_g$  is the guide wavelength at the operating frequency in gap waveguide.  $\lambda_g$  is very close to the free space wavelength; see [11]. We have  $r_0 = 3\lambda_g/(4\pi)$  in this work. Then, C = -1.485 is determined by changing the value of C to fit the calculated curve of  $Z_0^{ring}$  from (7) to the simulated curve by using CST MS [12], as shown in Fig. 3.



Fig. 3. Simulated and calculated characteristic impedance  $Z_0$  of a circular curved ridge gap-waveguide when  $r_0 = 3\lambda_g/(4\pi)$ .

#### **III. SIMULATED AND MEASURED RESULTS**

A prototype of a gap-waveguide ring hybrid at 16 GHz was designed according to the formulas discussed above. The radius of the ridge ring is  $r_0 = 3\lambda_g/4\pi$ , where  $\lambda_g = 19.9$  mm is the guide wavelength at 16 GHz. The distances between



Fig. 4. Prototype of the 16 GHz gape waveguide ring hybrid. The texture plate shows the ring hybrid surrounded by metal pins.

 TABLE I

 DIMENSIONS OF THE GAP-WAVEGUIDE HYBRID RING COUPLER

$r_1 \text{ (mm)}$	$r_2 \text{ (mm)}$	h (mm)	W (mm)
5.75	3.8	0.3	2.1
$d_{pin}$ (mm)	$h_{pin}~(\mathrm{mm})$	$p_{pin} \ (\mathrm{mm})$	$W_r  ({ m mm})$
3.0	5.0	3.5	1.95

ports 1 and 2, ports 2 and 3, and ports 3 and 4 are the same as  $\lambda_q/4$ , whereas the distance between ports 1 and 4 is  $3\lambda_q/4$ .

According to [13], the characteristic impedance for the ridge gap-waveguide ring  $Z_0^{ring}$  should be equal to  $\sqrt{2}Z_0$ , where  $Z_0$  is the characteristic impedance of the straight ridge gap-waveguide given in (2). Using (7) and the above mentioned length requirement, we can calculate the radius  $r_1$  and  $r_2$  of the ring as shown in Table I.

A prototype of the design was manufactured. The ring and pins were made by milling a copper plate, as shown in Fig. 4.

Measurements and simulations (by CST MS) are shown in Figs. 5 - 6. It can be observed that the gap-waveguide ring hybrid has a good performance over the band of 15.5 - 17.5 GHz: the reflection coefficient ( $S_{11}$ ) is below -10 dB, the isolation ( $S_{31}$ ) is larger than 20 dB for port 3, the power balance between two output ports 2 and 4 ( $S_{21}, S_{41}$ ) is good, the ohmic loss is about -0.3 dB (after taking away the losses of 0.2 dB in the SMA connectors [7]) which is a bit larger than that in [7] due to the smaller air-gap height h, and the phase differences of  $182^{\circ}\pm2^{\circ}$  between ports 2 and 4 for out-of-phase when port 1 is excited and of  $-2^{\circ}\pm4^{\circ}$  for in-phase when port 3 is excited. The discrepancy between the simulated and the measured reflection coefficient is mainly due to the tolerance of the manufactured SMA-to-gap-waveguide transition.

## **IV. CONCLUSION**

In the paper, empirical formulas for designing gapwaveguide ring hybrid were presented. A prototype of the ring hybrid was manufactured. The design with the formulas has been verified by simulated and measured data.

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Fig. 5. Simulated and measured transmission coefficient between ports.



Fig. 6. Simulated and measured transmission coefficient between port 1 and ports 2&4.

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