CHALMERS

Chiral Extensions of the MSSM

Master's Thesis in Theoretical Physics

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Abstract

This thesis is based on the paper [1]. We present the construction and analysis of supersymmetric models. We begin by giving a short description of the Minimal Supersymmetric Standard Model (MSSM) and pointing out its two main problems. Motivated to resolve them we construct a class of MSSM extensions characterized by a fully chiral field content (no μ-terms) and no baryon or lepton number violating terms in the superpotential due to an extra $U'(1)$ gauge symmetry. The minimal models consist of the usual matter sector with family dependent $U'(1)$ charges, six Higgs weak doublets, and four charged singlets required to give masses to the Higgsinos, cancel anomalies and allow for commensurate charges. These models are characterized by a discrete set of solutions for the charges. The models with right handed neutrino superfields are also presented.

As a different issue we briefly discuss the SUSY breaking mechanism in gauge mediation scenario, where we show how an extra gaugino $\tilde{Z}'$ can be used to mediated SUSY breaking from the Hidden Sector.

Analysing the models, we discuss their general features, e.g. classical vacuum, CP-violation, Electro-Weak symmetry breaking and running coupling constants. In the end we investigate the decays of Z and Z' -bosons and study in detail experimental constraints on Flavour Changing processes.

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1

Introduction

Supersymmetry (SUSY) is a deep theoretical idea whose relevance to Electro-Weak (EW) symmetry breaking is now being tested at the LHC. In this context, most of the attention has been focused on the minimal supersymmetric extension of the Standard Model (MSSM) consisting of a supermultiplet for each known matter field and gauge boson and a pair of Higgs doublets (for a review see [2]). Minimality is certainly appealing for many reasons. To begin with, if SUSY is realized in nature, all of the particles of the MSSM are guaranteed to exist. Furthermore, minimality allows to control, at least to some extent, the vast parameter space coming when SUSY is spontaneously broken and to set up benchmark points to be used as guidelines in the search for new physics.

In the case of the MSSM however, minimality comes at a price. The accidental symmetries that follow from gauge invariance and chirality in the Standard Model and prevent e.g. proton decay no longer arise automatically in the MSSM and need to be enforced by imposing R-parity or other discrete symmetries. Furthermore, one of the main reasons to believe that SUSY is relevant to physics at the TeV scale is the fact that it solves the "hierarchy problem" arising in the presence of light fundamental scalars such as the Higgs. In the MSSM however, this solution is only partial, since one is required to introduce the μ-term in the superpotential, which, while technically natural begs the question of why it should be at the same scale as EW breaking. Lastly, in view of the 2011 LHC run, hinting at a 125 GeV Higgs and failing to observe colored superparticles, the pure MSSM itself is starting to look quite fine-tuned (see e.g. [3] for a recent review).

The μ problem is due to the fact that the MSSM matter content is not "fully chiral" since the two Higgs doublets H_u and H_d together form a real representation of the gauge group. Chirality has certainly served us well in the Standard Model, preventing bare fermion masses and unwanted couplings but it is not fully exploited in the MSSM. Most of the popular extensions of the MSSM, such as the NMSSM, are also non chiral and one wonders what the minimal fully chiral model might look like. This question needs to be made more specific in order to be properly analyzed, so in this work we address the following issue: what is the minimal extension of the gauge group and the Higgs sector of the MSSM for which all SUSY masses (μ-terms) and baryon number (B) and lepton number (L) violating terms are forbidden? We shall see that it is possible to satisfy these requirements by extending the gauge group by a single $U(1)'$ and the Higgs sector by a total of six Higgs doublets (instead of the two of the MSSM) and four $U(1)'$ charged singlets. (Six in the presence of right handed neutrinos.)

We shall work in a strictly "bottom-up" approach and give up manifest grand-unification. Our approach necessarily leads to some amount of tree level Flavor Changing Neutral Currents (FCNC) which are notoriously strongly constrained by many experimental observations. We will discuss this point in section 4, but one interesting point worth mentioning already is that only the weak singlets are allowed to have family dependent charges, thus evading the strongest constraints on FCNC.

The MSSM with extended gauge group by an extra $U'(1)$ is often denoted by UMSSM [4, 5, 6] (see [7] for a review). The literature on the subject is vast. Some previous versions of chiral UMSSM (different from ours) are given in e.g. [8, 9, 10, 11, 12, 13, 14, 15]. In almost all these models it is required to include additional colored superfields (exotics) for anomaly cancelation.

1.1 Outline

In chapter 2 we recall the general form of supersymmetric $SU(N)$ gauge invariant lagrangian. This is a sufficient basis to construct any supersymmetric model with arbitrary gauge group and a matter content. SUSY must be broken in Nature, thus we pay some attention to SUSY breaking in the gauge mediated scenario and show how to generate masses of sparticles through radiative corrections. Finally, we present the structure of Minimal Supersymmetric Standard Model and briefly point out its problems.

In chapter 3 we construct a new class of the MSSM extensions characterized by a fully chiral field content (no μ-terms) and no baryon or lepton number violating term in the superpotential due to an extra $U'(1)$ gauge symmetry. The minimal models consist of the usual matter sector with family dependent $U'(1)$ charges, six Higgs weak doublets, and four charged singlets (six in a presence of right handed neutrinos) required to give masses to the Higgsinos, cancel anomalies and allow for commensurate charges. These models are characterized by a discrete set of solutions for the charges.

In chapter 4 we perform analysis of constructed models. We start by deriving the classical potential of the theory. We show its stability and presence of CP-violating phases. The Electro-Weak breaking is analysed in details; we show the presence of flavour violating processes due to family non-universal charges. Finally we discuss running of the coupling constants.

In chapter 5 we derive decay rates of Z-boson into fermions and Z'-boson into scalar particles (we assume that Z' mass is big enough to allow for the decay into some superparticles). In the last section we analyse phenomenological constraints on the models.

1.2 Notation

Relativity and Tensors

The metric tensor:

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{1.1}$$

Contravariant four-vectors (e.g. positions and momenta) and covariant four-vectors (e.g. derivatives):

$$x^\mu = (t, \vec{x}), \tag{1.2}$$

$$p^\mu = (E, \vec{p}), \tag{1.3}$$

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = (\frac{\partial}{\partial t}, \vec{\nabla}) \tag{1.4}$$

Relation between energy and momentum for the massive particle:

$$p^2 = p^\mu p_\mu = E^2 - |\vec{p}|^2 = m^2. \tag{1.5}$$

Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{1.6}$$

generators of SU(2) are defined as: $\tau^j \equiv \frac{1}{2}\sigma^j$ and $\tau^\pm \equiv \frac{1}{2}(\sigma^1 \pm i\sigma^2)$.

Two-component spinors

The spinor indices are raised and lowered by antisymmetric epsilon symbol:

$$\varepsilon^{12} = -\varepsilon^{21} = \varepsilon_{21} = -\varepsilon^{12} = 1. \tag{1.7}$$

Spinor indices are contracted by a convention:

$${}^{\alpha}{}_{\alpha} \quad \textit{and} \quad {}_{\dot{\alpha}}{}^{\dot{\alpha}}. \tag{1.8}$$

Lorentz vectors can be constructed from the spinors by introducing the sigma matrices $\sigma^\mu_{\alpha\dot{\beta}}$ and $\bar{\sigma}^{\mu\dot{\alpha}\beta}$:

$$\sigma^0 = \bar{\sigma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \sigma^1 = -\bar{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{1.9}$$

$$\sigma^2 = \bar{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = -\bar{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1.10}$$

The Standard Model fields

Any Dirac fermion can be written as:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}. \tag{1.11}$$

Instead of ψ we can use two independent left-handed fermions $f \equiv \psi_L$ and $\bar{f} \equiv i\sigma^2\psi_R^*$. It is convenient to combine left-handed quarks and leptons in to SU(2) doublets, as follows:

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad L = \begin{pmatrix} \nu \\ e \end{pmatrix} \tag{1.12}$$

where u, d are up and down quarks, and ν, e are neutrino and electron respectively. Symbol $\Box$ ($\bar{\Box}$) - means that the field transforms under fundamental (anti-fundamental) representation of the gauge group. Superpartner of the fermion f we denote as $\tilde{f}$.

2

SUSY models and SUSY breaking

THIS chapter is devoted to the construction of supersymmetric models. In the first section we recall the general form of supersymmetric $SU(N)$ gauge invariant lagrangian and discuss how to construct any supersymmetric model with arbitrary gauge group and a matter content. In the second section we discuss SUSY breaking in the gauge mediated scenario and show how to generate masses of sparticles through radiative corrections. In the third section we present the structure of the Minimal Supersymmetric Standard Model, we point out its two basic problems: the so called μ-problem and an ad hoc requirement on imposing a discrete symmetry (R-parity).

2.1 General structure of SUSY models

To begin with, let us note that in supersymmetric models every fermionic field ψ^i ($i = 1..n$, n-number of such fields) with spin $s = 1/2$ has its own scalar supersymmetric partner ϕ^i with spin $s = 0$. As usually, we require the lagrangian to be gauge invariant for example under the group $SU(N)$, with generators T^a (index $a = 1..(N^2 - 1)$). In a presence of SUSY all gauge bosons A^a_μ with spin $s = 1$ get their own superpartners - gauginos λ^a with spin $s = 1/2$. Except for spin the superpartners have exactly the same properties including masses. Let us recall the most general supersymmetric gauge invariant form of the lagrangian ([2], Chapter 3)

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} - \sqrt{2}g(\phi^*_i T^a \psi^i)\lambda^a - \sqrt{2}g(\psi^*_i T^a \phi^i)\lambda^a. \tag{2.1}$$

The gauge part of the lagrangian is given by the expression:

$$\mathcal{L}_{gauge} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + i\lambda^{\dagger a} \bar{\sigma}^\mu \nabla_\mu \lambda^a, \tag{2.2}$$

where $F_{\mu\nu} = F^a_{\mu\nu} T^a$ is a usual field strength $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$; gaugino covariant derivative is $\nabla_\mu = \partial_\mu + g f^{abc} A_\mu$ and g is a coupling constant to the gauge field.

The matter part of the lagrangian is given by:

$$\mathcal{L}_{matter} = i\psi_i^\dagger \bar{\sigma}^\mu D_\mu \psi^i - D^\mu \phi_i^* D_\mu \phi^i - \frac{1}{2} M_{ij} \psi^i \psi^j - \frac{1}{2} M^{ij*} \psi_i^\dagger \psi_j^\dagger - \\ - \frac{1}{2} y_{ijk} \phi^i \psi^j \psi^k - \frac{1}{2} y^{ijk*} \phi_i^\dagger \psi_j^\dagger \psi_k^\dagger - V(\phi^i, \phi_j^\dagger), \quad (2.3)$$

where $D_\mu = \partial_\mu - igA_\mu^a T^a$, M_{ij} - mass of the fermions ψ and y^{ijk} Yukawa couplings. The allowed fermionic mass terms and Yukawa couplings are fully controlled by the auxiliary function W, called superpotential. The superpotential is defined as a holomorphic gauge invratiant function of the scalar fields in the model:

$$W = \frac{1}{2} M_{ij} \phi^i \phi^j + \frac{1}{6} y_{ijk}\, \phi^i \phi^j \phi^k, \quad (2.4)$$

where the first term generates fermionic and bosonic masses and the second generates Yukawa interactions. The connection between (2.3) and (2.4) are well explained in [2] (Chapter 3).

Finally the scalar potential has the form:

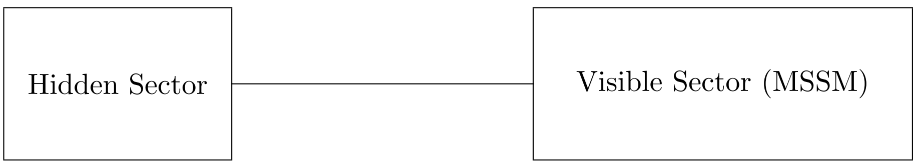
$$V = V_F + V_D = W^k W_k^* + \frac{1}{2} g^2 \sum_a (\phi_i^\dagger T^a \phi^i)^2, \quad (2.5)$$

where the V_F - term comes from the superpotential ($W^k \equiv \partial W / \partial \Phi_k$). The V_D-term is closely related to the gauge structure of the model.

To build any SUSY model we need first to specify the matter content, second to set a gauge group and third to fix the form of the superpotential (2.4). For the MSSM we will clarify this steps in section 2.3.

2.2 SUSY breaking

It is obvious from the experiment that known fermions in Nature do not have superpartners with the same masses, thus SUSY if it exists must be somehow broken. There are several known scenarios of SUSY breaking. Usually it is assumed that this breaking occurs for very high energy scale in the so called hidden sector (sector with new matter and gauge fields). This sector is assumed to be connected to our visible MSSM sector very weakly through the messengers (mediators). The most popular scenarios are gravity mediation and gauge mediation. But we will discuss gauge mediation only.



Suppose for example we have an extra gauge symmetry $U(1)'$ (it results in a Z'-gauge boson and $\tilde{Z}'$-gaugino). The SUSY breaking mediated by the Z' boson was discussed in [16]. If SUSY were unbroken $m_{Z'} = m_{\tilde{Z}'}$. After SUSY breaking in the hidden sector the gaugino $\tilde{Z}'$ gets different mass from the Z' boson ($m_{Z'} \neq m_{\tilde{Z}'}$). The $\tilde{Z}'$ even being extremely heavy can now participate in the loop effects for ordinary MSSM particles and give rise to radiatively induced mass terms for sparticles. Let us then show this calculations in details for the fermions and gauginos separately. We will need the diagrammatic notation which is shown on the figure 2.1. In this section we will work in two-component notation, the structure of diagrams, form of propagators and rules are briefly given in A.1.

Figure 2.1: a - fermion f, b - sfermion $\tilde{f}$, c - gauge boson λ, d - gaugino $\tilde{\lambda}$

2.2.1 One-loop Sfermion Masses

The propagator of sfermion is given by the ordinary expression for a scalar particle:

$$\frac{i}{k^2 - m_{\tilde{f}}^2}, \tag{2.6}$$

where $m_{\tilde{f}} = m_f$ is a bare mass of the sfermion, which is the same as for corresponding fermion. Let us consider now the effective sfermionic propagator shown on the figure 2.2 and let us calculate effects due to $\tilde{Z}'$-fermion presence in the loop.

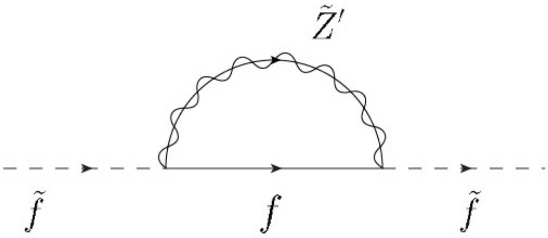


Figure 2.2: Effective propagator of the sfermion $\tilde{f}$. The $\tilde{Z}'$-fermion and fermion f are present in the loop.

First, let us note that if the propagator of massive $\tilde{Z}'$ gaugino is present on any Feynman diagram, its contribution is partially cancelled by the propagator of massless superpartner Z' boson (we assume for simplicity case with unbroken $U(1)'$):

$$= \frac{ik \cdot \sigma_{\alpha\dot{\beta}}}{k^2 - m^2} = \frac{ik \cdot \sigma_{\alpha\dot{\beta}}}{k^2}(1 + \frac{m^2}{k^2} + \frac{m^4}{k^4} + \dots) =$$

$$= \frac{ik\cdot\sigma_{\alpha\dot\beta}}{k^2} + \frac{im^2 k\cdot\sigma_{\alpha\dot\beta}}{k^4}(1+\frac{m^2}{k^2}+\frac{m^4}{k^4}+\dots) = \underbrace{\frac{ik\cdot\sigma_{\alpha\dot\beta}}{k^2}}_{cancelled\ by\ SUSY} + \frac{im^2 k\cdot\sigma_{\alpha\dot\beta}}{k^2(k^2-m^2)}. \tag{2.7}$$

The diagram at the figure 2.2 can be interpreted as "one Particle Irreducible" (1IP) insertion to the tree level propagator (more details are given in appendix A.2):

$$i\Pi^2(0) = \int (-ig)^2 \frac{d^4k}{(2\pi)^4}\frac{-ik\cdot\bar\sigma^{\dot\beta\alpha}}{k^2-\underbrace{m_f^2}_{\approx 0}}\frac{im^2_{\tilde Z'}k\cdot\sigma_{\alpha\dot\beta}}{k^2(k^2-m^2_{\tilde Z'})} = [\bar\sigma^{\mu\ \dot\beta\alpha}\sigma^{\nu}_{\alpha\dot\beta} = tr(\bar\sigma^\mu\sigma^\nu) = 2\eta^{\mu\nu}] =$$

$$= -2g^2m^2_{\tilde Z'}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k^2-m^2_{\tilde Z'})} = [k^0 = ik^0_E,\ \vec k = \vec k_E;\ d^4k = ik^3_E dk_E d\Omega_3,\ \Omega_3 = 2\pi^2] =$$

$$= -\frac{ig^2m^2_{\tilde Z'}}{4\pi^2}\int_0^\Lambda \frac{k_E dk_E}{k_E^2+m^2_{\tilde Z'}} = -\frac{ig^2m^2_{\tilde Z'}}{4\pi^2}\ln(\frac{\Lambda}{m_{\tilde Z'}}), \tag{2.8}$$

where Λ is a cut-off, energy scaly of SUSY breaking. Finally using the formula (A.7) the result of (2.8) lead to the effective mass of the sfermion:

$$m^2_{\tilde f} = \frac{g^2m^2_{\tilde Z'}}{4\pi^2}\ln(\frac{\Lambda}{m_{\tilde Z'}}). \tag{2.9}$$

2.2.2 Two-loop Gaugino Masses

Gauginos corresponding to the Standard Model gauge bosons (gluons, W-bosons, Z-boson and photon) are called gluinos, winos, zino and photino. The gauginos can obtain masses through Majorana terms $m_{\tilde\lambda}\tilde\lambda\tilde\lambda$. In $\tilde Z'$ gauge mediation approach it is possible to generate Majorana masses through two-loop radiative correction (coupling gauginos with massive gaugino $\tilde Z'$). The structure of two such possible diagrams is given in figure 2.3. Again following the approach outlined in appendix A.2, we can calculate 1IP insertion according to the presented diagrams.

First diagram

$$i\Pi_1(0) = \int\frac{d^4k}{(2\pi)^4}\int\frac{d^4l}{(2\pi)^4}(-ig_\lambda)^2(-ig_{\tilde Z'})^2\frac{-ik\cdot\sigma_{\beta\dot\alpha}}{k^2-m_f^2}\cdot\frac{i}{(k-l)^2-m^2_{\tilde f}}\cdot\frac{im_{\tilde Z'}\delta^{\dot\alpha}_{\dot\beta}}{l^2-m^2_{\tilde Z}}\cdot$$

$$\cdot\frac{-i(k-l)\cdot\bar\sigma^{\dot\beta\beta}}{(k-l)^2-m_f^2}\cdot\frac{i}{k^2-m^2_{\tilde f}} = \frac{2im_{\tilde Z'}g^2_\lambda g^2_{\tilde Z'}}{(2\pi)^8}J_1, \tag{2.10}$$

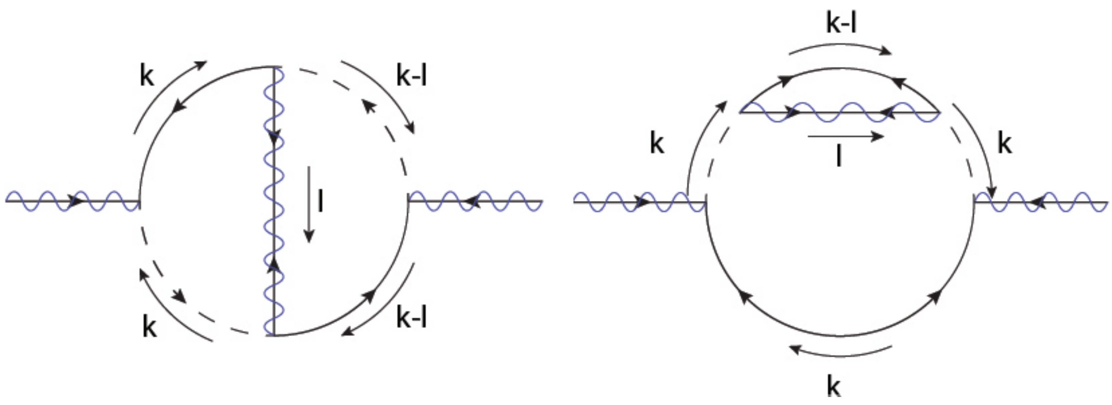


Figure 2.3: Two-loop diagrams with virtual $\tilde{Z}'$ inside the gaugino propagator λ^a.

where

$$J_1 = \int \frac{d^4 l}{l^2 - m^2_{\tilde{Z}'}} \int d^4 k \, \frac{k \cdot (k-l)}{[k^2 - m_f^2][k^2 - m^2_{\tilde{f}}][(k-l)^2 - m_f^2][(k-l)^2 - m^2_{\tilde{f}}]}. \tag{2.11}$$

Second diagram

$$i\Pi_2(0) = \int \frac{d^4 k}{(2\pi)^4} \int \frac{d^4 l}{(2\pi)^4} (-ig_\lambda)^2 (-ig_{\tilde{Z}'})^2 \frac{i}{k^2 - m^2_{\tilde{f}}} \cdot \frac{-im_f}{(k-l)^2 - m_f^2} \cdot \frac{im_{\tilde{Z}'}}{l^2 - m^2_{\tilde{Z}'}} \cdot$$
$$\cdot \frac{i}{k^2 - m^2_{\tilde{f}}} \cdot \frac{-im_f}{k^2 - m_f^2} = \frac{im_{\tilde{Z}'} m_f^2 g_\lambda^2 g^2_{\tilde{Z}'}}{(2\pi)^8} J_2, \tag{2.12}$$

where

$$J_2 = \int \frac{d^4 k}{[k^2 - m_f^2][k^2 - m^2_{\tilde{f}}]^2} \int \frac{d^4 l}{[(k-l)^2 - m_f^2][l^2 - m^2_{\tilde{Z}'}]}. \tag{2.13}$$

The difference between masses of the particles and their superpartners arises only trough loop corrections. Again, at tree level we consider them to be the same: $m_f = m_{\tilde{f}}$ and in the case of unbroken electroweak symmetry $m_f = m_{\tilde{f}} = 0$. It forces the second diagram to be 0; J_1 could be written as:

$$J_1 = \int \frac{d^4 l}{l^2 - m^2_{\tilde{Z}'}} \int d^4 k \, \frac{k \cdot (k-l)}{k^4 (k-l)^4}. \tag{2.14}$$

Introducing Feynman parameter, we can modify the denominator as follows:

$$\frac{1}{k^4 (k-l)^4} = 6 \int_0^1 xy \, dx \, dy \, \delta(x+y-1) \frac{1}{(xk^2 + y(k-l)^2)^4} =$$
$$= [x + y = 1, t \equiv k - yl] = 6 \int_0^1 xy \, dx \, dy \, \delta(x+y-1) \frac{1}{(t^2 + xy \, l^2)^4}. \tag{2.15}$$

Using (2.15) J_1 gets the form:

$$J_1 = 6\int_0^1 xy\,dx\,dy\,\delta(x+y-1)\underbrace{\int\frac{d^4l}{l^2-m_{\tilde{Z}'}^2}\int d^4t\,\frac{t^2+(y-x)\,t\cdot l - xy\,l^2}{(t^2+xy\,l^2)^4}}_{J_1'}. \tag{2.16}$$

And after applying Wick rotation:

$$J_1' = -\int\frac{d^4l_E}{l_E^2+m_{\tilde{Z}'}^2}\int d^4t_E\,\frac{t_E^2+(y-x)\,t_E\cdot l_E - xy\,l_E^2}{(t_E^2+xy\,l_E^2)^4}. \tag{2.17}$$

Consider 4 dimensional Euclidean space: $d^4t_E = t_E^3 dt_E d\Omega_3$, $t_E\cdot k_E = t_E k_E\cos\theta$. Let us denote $a \equiv \sqrt{xy}l_E$, then:

$$\begin{aligned}
J_1' = &-\int\frac{d^4l_E}{l_E^2+m_{\tilde{Z}'}^2}[\Omega_3\underbrace{\int_0^\Lambda dt_E\,\frac{t_E^5}{(t_E^2+a^2)^4}}_{\approx 1/6a^2}+ \\
&+(y-x)\,l_E\underbrace{\int_0^\Lambda dt_E\,\frac{t_E^4}{(t_E^2+a^2)^4}}_{(3\arctan\Lambda/a)/48a^3}\int\cos\theta d\Omega_3 - xy\,l_E^2\Omega_3\underbrace{\int_0^\Lambda dt_E\,\frac{t_E^3}{(t_E^2+a^2)^4}}_{1/12a^4}] = \\
&= -[\frac{\Omega_3^2}{6xy}\underbrace{\int_0^\Lambda\frac{l_E\,dl_E}{l_E^2+m_{\tilde{Z}'}^2}}_{\ln\frac{\Lambda}{m_{\tilde{Z}'}}}+\frac{3(y-x)\Omega_3}{48xy\sqrt{xy}}\int\cos\theta d\Omega_3\underbrace{\int_0^\infty\frac{l_E\arctan\frac{\Lambda}{\sqrt{xy}l_E}dl_E}{l_E^2+m_{\tilde{Z}'}^2}}_{\frac{\pi}{2}\ln\frac{\Lambda}{\sqrt{xy}m_{\tilde{Z}'}}}- \\
&-\frac{\Omega_3^2}{12xy}\underbrace{\int_0^\Lambda\frac{l_E\,dl_E}{l_E^2+m_{\tilde{Z}'}^2}}_{\ln\frac{\Lambda}{m_{\tilde{Z}'}}}] = -\frac{\Omega_3}{12xy}[\Omega_3\ln\frac{\Lambda}{m_{\tilde{Z}'}}+\frac{3\pi(y-x)}{8\sqrt{xy}}\ln\frac{\Lambda}{\sqrt{xy}m_{\tilde{Z}'}}\int\cos\theta d\Omega_3].
\end{aligned} \tag{2.18}$$

Applying results of (2.18) to (2.16) we get:

$$J_1 = -\frac{\Omega_3^2}{2}\ln\frac{\Lambda}{m_{\tilde{Z}'}} - \frac{9\pi}{4}\int\cos\theta d\Omega_3\underbrace{\int_0^1 dx\frac{(1-2x)}{\sqrt{x(1-x)}}\ln\frac{\Lambda}{\sqrt{x(1-x)}m_{\tilde{Z}'}}}_{=0}. \tag{2.19}$$

Then imposing $\Omega_3 = 2\pi^2$ we get the final result:

$$J_1 = -2\pi^4\ln\frac{\Lambda}{m_{\tilde{Z}'}}. \tag{2.20}$$

And the final expression for the gluino masses using (A.8) can be written as:

$$m_{\tilde{\lambda}} \approx -\Pi(0) = \frac{m_{\tilde{Z}'}g_\lambda^2 g_{\tilde{Z}'}^2}{64\pi^4}\ln\frac{\Lambda}{m_{\tilde{Z}'}}. \tag{2.21}$$

2.3 The MSSM

The structure of the MSSM is given in the table 2.1. We have ordinary Standard Model Q,u,d,L,e fields (the fermions and their SUSY partners denoted by the same symbol). The bars above u,d and e fields mean that they transform as left-handed fermions (this fields are related to ordinary "right" handed fields by the charge conjugation, see section 1.2).

	SU(3)	SU(2)	U(1)
Q	$\square$	$\square$	$\frac{1}{6}$
$\bar{u}$	$\bar{\square}$	-	$-\frac{2}{3}$
$\bar{d}$	$\bar{\square}$	-	$\frac{1}{3}$
L	-	$\square$	$-\frac{1}{2}$
$\bar{e}$	-	-	1
H_u	-	$\square$	$\frac{1}{2}$
H_d	-	$\square$	$-\frac{1}{2}$

Table 2.1: MSSM table of content

We should note that in the MSSM there are two Higgs weak doublets H_u and H_d compared to the Standard Model. The presence of two doublets is required because of the holomorphy of the superpotential and anomaly cancellation requirement.

The superpotential in the MSSM is specified as follows:

$$W_{MSSM} = Y_u^{ij}\bar{u}_i Q_j H_u - Y_d^{ij}\bar{d}_i Q_j H_d - Y_e^{ij}\bar{e}_i L_j H_d + \mu H_u H_d, \tag{2.22}$$

where the first three Yukawa terms are required to generate masses of quarks and leptons after Electro-Weak symmetry breaking. The last term is called μ-term and it is responsible for the Higgsino masses (Higgsino - is a fermionic fields, superpartners of the Higgs bosons).

If SUSY exist in Nature it must be broken, because we haven't found any superpartners. The mechanism of SUSY breaking could be very complicated, but for phenomenological purposes it is enough to introduce this breaking explicitly giving masses (soft masses) to the superpatners. A bit more generally SUSY breaking terms also include Yukawa terms; the soft part of the lagrangian for the MSSM (part with explicit SUSY breaking) is thus

$$\mathcal{L}_{soft} = -(\frac{1}{2}M_a\lambda^a\lambda^a + \underbrace{\frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i}_{Superpotential\ W\ with\ different\ coefficients}) + cc - (m^2)^i_j\phi^{j*}\phi_i, \tag{2.23}$$

where M_a - soft masses of gauginos and $(m^2)^i_j$ - soft masses of the scalars.

The MSSM is analysed in full details in [2], Chapter 6. To conclude our very brief discussion about this model let us just point out its two main problems:

1. The last term in the superpotential (2.22) and SUSY breaking terms (2.23) lead to the following supersymmetric masses of Higgses and Higgsinos:

$$\mathcal{L}_{Higgsino\,mass} = -\mu H_u H_d + c.c., \tag{2.24}$$

$$\mathcal{L}_{Higgs\,mass} = -\underbrace{|\mu|^2(|H_u|^2 + |H_d|^2)}_{supersymmetric\ masses} - \underbrace{m_{H_u}^2|H_u|^2 - m_{H_d}^2|H_d|^2}_{SUSY\ breaking\ masses}. \tag{2.25}$$

   As we can see from (2.25) the minimum of the potential is given by $H_u = H_d = 0$, thus we cannot understand Electro-Weak symmetry breaking without including negative supersymmetry breaking squared-masses for the Higgs bosons. Then we can immediately see from (2.25), that μ cannot be to big in order to keep Electro-Weak breaking and not to introduce fine tuned cancellation between $|\mu|^2$ and SUSY breaking masses. In contrary equation (2.24) put the lower limit on μ (Higgsinos haven't been discovered yet). This ad hoc requirement on parameter μ in the model called the μ-problem.

2. The most general superpotential, beside terms already given in (2.22), should also include $\bar{u}\bar{d}\bar{d}$, which violate baryon number (B) and terms $LL\bar{e}$, $LQ\bar{d}$ and LH_u, which violate lepton number (L). Presence of all these terms leads to very fast unobserved proton decay. The solution in the MSSM is to add a new discrete symmetry, called R-parity, which forbids all dangerous terms. This ad hoc feature of the model is the second problem.

3

MSSM Extension

Here we construct a new class of the MSSM extensions characterized by a fully chiral field content (no μ-terms) and no baryon or lepton number violating term in the superpotential due to an extra $U'(1)$ gauge symmetry. The minimal models consist of the usual matter sector with family dependent $U'(1)$ charges, six Higgs weak doublets, and four charged singlets required to give masses to the Higgsinos, cancel anomalies and allow for commensurate charges. These models are characterized by a discrete set of solutions for the charges. Right handed neutrino superfields can also be added.

3.1 Construction

Our attempt of extending the MSSM is motivated by the two problems outlined in 2.3. The weak Higgs doublets H_u and H_d are allowed to form a "μ-term" because they have an opposit $U(1)$ hypercharge. To forbid this term we are forced to extend the gauge group by an extra $U(1)'$ (the simplest case) and to give them $U(1)'$ charges q_{H_u} and q_{H_d} respectively, where $q_{H_u} \neq -q_{H_d}$. The second requirement is to obtain R-parity as an accidental symmetry arising from the gauge symmetry. Thus, we want also to give $U(1)'$ charges to the matter sector. The "matter sector" consists of the usual three Q^i,u^i,d^i,L^i,e^i families, (i = 1,2,3 family number). The scalar components are denoted by the same symbol as fermionic components. Additional (weak singlet) neutrino superfields v^i could also be added in order to construct Dirac masses. We will present our analysis for two cases with and without right-handed neutrinos.

The matter sector acquires a mass by renormalizable couplings to an extended "Higgs sector", comprising of a number of Higgs pairs (H_u^a, H_d^a), $a = 1 \ldots m$ carrying the usual MSSM quantum numbers and a $U(1)'$ charge. Finally, we need a number of $U(1)'$ charged MSSM singlets S^r, $r = 1 \ldots n$, to give mass to the Higgsinos via a coupling of type $H_u H_d S$ and to cancel the anomalies. We refer to these fields as "the singlet sector".

	$SU(3)$	$SU(2)$	$U(1)$	$U'(1)$
Q^i	$\square$	$\square$	$\frac{1}{6}$	q_Q^i
$\bar{u}^i$	$\bar{\square}$	-	$-\frac{2}{3}$	q_u^i
$\bar{d}^i$	$\bar{\square}$	-	$\frac{1}{3}$	q_d^i
L^i	-	$\square$	$-\frac{1}{2}$	q_L^i
$(\bar{\nu}^i$	-	-	-	$q_v^i)$
$\bar{e}^i$	-	-	1	q_e^i
H_u^a	-	$\square$	$\frac{1}{2}$	$q_{H_u}^a$
H_d^a	-	$\square$	$-\frac{1}{2}$	$q_{H_d}^a$
S^r	-	-	-	q_S^r

Table 3.1: The content of EMSSM

The structure of our extended MSSM is shown in the table 3.1,

We will impose that the extra $U(1)'$ symmetry automatically forbids any dimension-full coupling in the superpotential W, namely any linear combination of the following terms

$$S^r,\ S^rS^s,\ H_u^aH_d^b,\ H_u^aL^i \notin W. \tag{3.1}$$

This condition translates to the set of linear constraints on the charges:

$$q_S^r \neq 0,\ q_S^r + q_S^s \neq 0,\ q_{H_u}^a + q_{H_d}^b \neq 0,\ q_{H_u}^a + q_L^i \neq 0. \tag{3.2}$$

Furthermore, we impose that the same gauge symmetry forbids dimension three B or L violating terms in the superpotential, i.e. any linear combination of terms like

$$u^id^jd^k,\ L^iL^je^k,\ L^iQ^jd^k,\ L^iH_u^aS^r \notin W, \tag{3.3}$$

what gives us following constraints:

$$q_u^i + q_d^j + q_d^k \neq 0,\ q_L^i + q_L^j + q_e^k \neq 0,\ q_L^i + q_Q^j + q_d^k \neq 0,\ q_L^i + q_{H_u}^a + q_S^r \neq 0. \tag{3.4}$$

Condition (3.3), which essentially forbids the same (dimension four) terms as those usually called R-parity violating (RPV), might need to be relaxed in light of the recent LHC searches failing to see large amounts of missing energy. RPV is a very active research area at this moment and our strategy could easily be modified along these lines.

Thus, the most general form of the superpotential in our construction is

$$W = y_{ija}^uQ^iu^jH_u^a + y_{ija}^dQ^id^jH_d^a + y_{ija}^eL^ie^jH_d^a + \kappa_{abr}H_u^aH_d^bS^r + \lambda_{rst}S^rS^sS^t. \tag{3.5}$$

The first three terms are required to give masses to the matter fields. We will assume that all such terms are actually present (no “textures”), although even this condition could be relaxed if needed. As mentioned before, the fourth term is required to give a mass to the Higgsinos while the last term is not required but is allowed in principle.

The requirement (3.5) that all the Yukawa couplings are $U(1)'$ invariant translates into the set of linear equations on the charges:

$$q_Q^i + q_u^j + q_{H_u}^a = 0,\ q_Q^i + q_d^j + q_{H_d}^a = 0,\ q_L^i + q_e^j + q_{H_d}^a = 0,\ q_{H_u}^a + q_{H_d}^b + q_S^r = 0. \quad (3.6)$$

Lastly, we require the $U(1)'$ charges to be commensurate (i.e. mutually rational). Irrational charges require "non-compact $U(1)$ groups" which cannot be embedded into a compact GUT gauge group. In our model, we do not require manifest grand unification (the coupling constants do not meet without additional matter at a higher scale) and one could hope that "string GUT" allows for more options than "gauge GUT". Even in this more general case however, there are strong arguments against incommensurate charges [17].

The presence of a new gauge group introduces a whole new set of anomaly cancelation conditions namely: $SU(3)_C^2U(1)'$, $SU(2)_W^2U(1)'$, $U(1)_Y^2U(1)'$, $\text{Grav}^2U(1)'$, $U(1)_YU(1)'^2$ and $U(1)'^3$. In the obvious notation for the charges, they read

$$\sum_{i=1}^{3}\left(2q_Q^i + q_u^i + q_d^i\right) = 0, \quad (3.7)$$

$$\sum_{i=1}^{3}(3q_Q^i + q_L^i) + \sum_{a=1}^{m}(q_{Hu}^a + q_{Hd}^a) = 0, \quad (3.8)$$

$$\sum_{i=1}^{3}(q_Q^i + 8q_u^i + 2d^i + 3q_L^i + 6q_e^i) + \sum_{a=1}^{m}(3q_{Hu}^a + 3q_{Hd}^a) = 0, \quad (3.9)$$

$$\sum_{i=1}^{3}(6q_Q^i + 3q_u^i + 3q_d^i + 2q_L^i + (q_\nu^i) + q_e^i) + \sum_{a=1}^{m}(2q_{Hu}^a + 2q_{Hd}^a) + \sum_{r=1}^{n} q_S^r = 0, \quad (3.10)$$

$$\sum_{i=1}^{3}(q_Q^{i2} - 2q_u^{i2} + q_d^{i2} - q_L^{i2} + q_e^{i2}) + \sum_{a=1}^{m}(q_{Hu}^{a2} - q_{Hd}^{a2}) = 0, \quad (3.11)$$

$$\sum_{i=1}^{3}(6q_Q^{i3} + 3q_u^{i3} + 3q_d^{i3} + 2q_L^{i3} + (q_\nu^{i3}) + q_e^{i3}) + \sum_{a=1}^{m}(2q_{Hu}^{a3} + 2q_{Hd}^{a3}) + \sum_{r=1}^{n} q_S^{r3} = 0. \quad (3.12)$$

To begin with, we can make the following general statement: It is impossible to fulfill the chirality conditions (3.2) with only one or two Higgs pairs, i.e. we must take $m = 3$ in the minimal case. (In non-SUSY models the case with three different Higgs fields assigned to each family is discussed in [18].)

This fact is true regardless of the number n of singlets. It is easy to see that just one single pair (H_u, H_d) will not work by adding all the Yukawa conditions for the quarks

$$\forall i,\, j: \quad q_Q^i + q_u^j + q_{Hu} = q_Q^i + q_d^j + q_{Hd} = 0. \quad (3.13)$$

Comparing with (3.7) we see that $q_{Hu} + q_{Hd} = 0$ violating (3.2). The same conclusions can be reached for two Higgs pairs ($m = 2$). One has to consider all possible independent positions of the Higgs fields in the Yukawa terms and use only the anomaly conditions that do not involve the singlets. (The linear conditions (3.7), (3.8) and (3.9) are enough.)

With $m = 3$ one can satisfy (3.2), (3.4) and (3.6), together with (3.7), (3.8), (3.9) and (3.11) in many ways corresponding to different distributions of the Higgs fields into the Yukawa couplings. We believe not much is gained by considering all these combinations and we will instead chose the combination that seems most promising phenomenologically, namely the one that gives to the matter weak doublets Q and L family independent $U(1)'$ charges. (The reason why this is preferable is explained in e.g. [7] and we will discuss this issue in section 4.) Thus, from now on, the Yukawa matter couplings in (3.5) are taken to be $y^u_{ij} Q^j u^i H^i_u$, $y^d_{ij} Q^j d^i H^i_d$ and $y^e_{ij} L^j e^i H^i_d$ where we use the same index for the Higgs fields since they are associated to the family.

We can now start adding singlets. The systematic approach is to add one singlet at the time trying to preserve the above conditions, while also requiring at least one $H_u H_d S$ term for each Higgs.

Again, we can make a general statement: It is impossible to fulfill the chirality condition (3.2) with less than three singlets. Consider the case of one singlet S. Its vacuum expectation value (vev) must give a mass to all Higgsinos and thus we require a coupling of the type $S(H^1_u H^1_d + H^2_u H^2_d + H^3_u H^3_d)$ or similar terms involving permutations of the H_d fields. Imposing $U(1)'$ invariance of all coupling and eqs. (3.7), (3.8), (3.9) and (3.11) forces some of the Higgs fields to form a non-chiral representation. The same conclusion can be reached for two singlets after some combinatorial analysis. We will thus restrict the fourth term in the superpotential (3.5) to be $\kappa_i H^i_u H^i_d S^i$. Once again one could consider permuting some of the H_d's but nothing substantial seems to be gained by this generalization. Just as we did for the Higgs, we use the same index to label these singlets and we will consider them "as part of the family".

We must now impose the last two conditions (3.10) and (3.12). Without additional singlets other than S^i above, there are solutions but we were able to prove that they are always not commensurate, the simplest set belonging to the extension of $\mathbb{Q}$ by $\sqrt{3}$. While an interesting curiosity, we do not believe they are promising models. Luckily, the addition of one more spectator singlet, denoted by S^0 in the following, is enough to close the system and give rise to commensurate charges obeying all our requirements.

Let us briefly describe how the solutions are found: To begin with, all effects from the scalars are confined in equations (3.10), (3.12) and the Yukawa constraints coming from the terms $H^i_u H^i_d S^i$. First we solve equations (3.7), (3.8), (3.9) together with the Yukawa constraints which do not involve any singlets. This allows to solve the quadratic equation (3.11) in terms of a rational function of the charges. We than solve the remaining linear constraints and are left with cubic homogeneous equation (3.12). The second observation is that it is possible to set $q^i_L = 0$. This charge is family independent for the Higgs couplings we have chosen and can be set to zero using non-anomalous hypercharge. The final step is that of searching for integer solutions of equation (3.12) using Mathematica© and checking that they obey the inequalities (3.1) and (3.3). There is in fact an infinite but a discrete set of solutions and we will present explicitly some of the simplest charge assignments $\hat{q}$ for each particular construction.

3.2 Models without r.h. neutrinos

One particular solution for the case without right-handed neutrinos is presented in the table 3.2.

Family	Q	u	d	L	e	H_u	H_d	S	S^0
$i=1$	0	3/8	−1/8	0	−1/8	−3/8	1/8	1/4	
$i=2$	0	−1/12	−19/24	0	−19/24	1/12	19/24	−7/8	1
$i=3$	0	17/24	−1/12	0	−1/12	−17/24	1/12	5/8	

Table 3.2: *One possible solution for the values of $\hat{q}$ for the model discussed in the text.*

Regardless of the particular model, given a solution obtained by the approach outlined above, another set of solutions with $q_L^i \neq 0$ is given by

$$q_\phi = Y_\phi + r\hat{q}_\phi, \tag{3.14}$$

where ϕ is any field in the model, Y_ϕ its hypercharge and r a rational constant. In table 3.2 the charges are normalized in a such way that the largest charge is equal to 1. There is also an obvious symmetry of the equations under family number permutation. Setting $r = 0$ in (3.14) reintroduces non-chiral models. Since, as we will discuss in section 4, family dependent charges give rise to additional Flavor Violating processes, we could in principle set $r \ll 1$ making flavor violating processes unobservable while keeping the model chiral, but this sounds rather ad hoc and unnatural.

There is in fact another interesting type of solutions, which should be mentioned, when $q_S^1 = q_S^0$ (q_S^2 or q_S^3). We present example of such charge assignment in the table 3.3.

Family	Q	u	d	L	e	H_u	H_d	S	S^0
$i=1$	0	3/7	−5/7	0	−5/7	−3/7	5/7	−2/7	
$i=2$	0	5/21	16/21	0	16/21	−5/21	−16/21	1	−2/7
$i=3$	0	−20/21	5/21	0	5/21	20/21	−5/21	−5/7	

Table 3.3: *One possible solution for the values of $\hat{q}$ for the model discussed in the text.*

3.3 Models with r.h. neutrinos

Here we present a class of models with right-handed neutrinos v^i. Let us first discuss the modification of the equations we need to introduce to the above discussion. First, the list of forbidden terms in (3.1) should be enlarged by

$$\bar{\nu}^i,\ \bar{\nu}^i\bar{\nu}^j,\ \bar{\nu}^i S^r. \tag{3.15}$$

Second, we require right handed neutrinos to obtain Dirac masses through Yukawa couplings to the Higgses, thus we should add an extra term $y^{\nu}_{ija}L^i\nu^j H^a_u$ to the superpotential (3.5).

The minimal realization requires three singlets X^j in addition to the three S^i fields, generalizing the S^0 field in the previous section. This fact can easily be seen from the equation (3.10) after substituting the solution coming from the all Yukawa constraints and three anomaly equations (3.7)-(3.9):

$$\sum_{j=1}^{p} q^j_X = 0, \tag{3.16}$$

where p is a number of extra X fields. For $p = 1$ and $p = 2$ it results in $q_X = 0$ and $q^1_X = -q^2_X$, immediately breaking the chirality condition. One particular example is given in the table 3.4. (This particular solution also allows for some L violating terms involving only neutrinos, like $S^1X^1v^2$. This terms are forbidden for other charge realizations.)

Family	Q	u	d	L	v	e	H_u	H_d	S
I	0	1/3	2/3	0	1/3	2/3	−1/3	−2/3	1
II	0	−3/4	−1/12	0	−3/4	−1/12	3/4	1/12	−5/6
III	0	7/12	−3/4	0	7/12	−3/4	−7/12	3/4	−1/6

	X^1	X^2	X^3
q^j_X	−1/4	2/3	−5/12

Table 3.4: *The values of $\hat{q}$ charges for the model with right-handed neutrinos and three singlets X.*

Another interesting fact is an automatic presence of anomaly free $U_{B-L}(1)$ symmetry (which could in principle also be gauged). The charges under $U_{B-L}(1)$ are given in the table 3.5.

matter	Q^i	$\bar{u}^i$	$\bar{d}^i$	L^i	$\bar{\nu}^i$	$\bar{e}^i$
$U_{B-L}(1)$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	-1	1	1

Table 3.5: $U_{B-L}(1)$ charges of the matter sector. Other fields left uncharged.

Extra anomaly conditions are presented by equations

$$U_{B-L}U_{B-L}U'(1) \Rightarrow \sum_{i=1}^{3}(2q_Q^i + q_u^i + q_d^i + 6q_L^i + 3q_v^i + 3q_e^i) = 0, \tag{3.17}$$

$$U_{B-L}U'(1)U'(1) \Rightarrow \sum_{i=1}^{3}(2q_L^{i2} - q_u^{i2} - q_d^{i2} - 2q_L^{i2} + q_v^{i2} + q_e^{i2}) = 0. \tag{3.18}$$

They are satisfied by the solution coming just from four Yukawa constraints (which do not involve singlets S) and three anomaly equations (3.7)-(3.9). Thus, the most general solution for the $U'(1)$ charges in this case is given by

$$q_\phi = Y_\phi + t(B-L)_\phi + r\hat{q}_\phi, \tag{3.19}$$

where t and r are rational parameters, which can be chosen arbitrarily (except some particular values breaking the chirality conditions and violating R-parity).

For such type of models there is also a possibility to generate Majorana masses of the right-handed neutrinos through the coupling to X fields. In this case, if the X fields acquire a big vev, we could incorporate the seesaw mechanism. It can be shown that this requires a minimum of four extra singlets X, one example is given in the table 3.6. The Majorana mass for the right-handed neutrinos is generated from the following terms in the superpotential: $X^1v^2v^3$, $X^2v^1v^3$ and $X^3v^1v^2$.

Family	Q	u	d	L	v	e	H_u	H_d	S
I	0	$19/32$	$13/32$	0	$19/32$	$13/32$	$-19/32$	$-13/32$	1
II	0	$-25/32$	$17/32$	0	$-25/32$	$17/32$	$25/32$	$-17/32$	$-1/4$
III	0	$-1/32$	$-23/32$	0	$-1/32$	$-23/32$	$1/32$	$23/32$	$-3/4$

	X^1	X^2	X^3	X^4
q_X^j	$13/16$	$-9/16$	$3/16$	$-7/16$

Table 3.6: *The values of $\hat{q}$ charges for the model with right-handed neutrinos and four singles X.*

4

Analysis

In this chapter we perform a more detailed analysis of the constructed model. We start by deriving the classical potential of the theory. We show its stability and presence of extra CP-violating phases. The Electro-Weak breaking is presented in details, we derive the mass spectrum of gauge bosons, Z-Z' mixing angle and show the presence of flavour violating processes due to family non-universal charges. In the end we discuss the running of the coupling constants in our model.

4.1 Scalar Potential

We now discuss some of the basic generic features of the models presented in the previous section. The actual numerical value of the charges is unimportant for the generic dynamical features and so our comments apply to any solution, nevertheless to be more demonstrative let us work with charges given in table 3.2. One common feature of all models is that they allow a term cubic in the S fields and thus the form of the superpotential (3.5) can now be written as

$$W = y^u_{ij} Q^i u^j H^j_u + y^d_{ij} Q^i d^j H^j_d + y^e_{ij} L^i e^j H^j_d + \kappa_i H^i_u H^i_d S^i + \lambda S^1 S^2 S^3. \tag{4.1}$$

The same function, interpreted as a function of the scalars only and with different coefficients, can be used to describe the A-terms of the model. The superpotential (4.1) is the most general potential allowed by the solution provided in table 3.2. As we mentioned, there are other possible charge assignments and, for some of them, additional cubic terms like $S^0 S^2 S^3$ are also allowed.

Thus, not only the μ-terms but also the $B\mu$-terms are absent and instead replaced by the vevs of the singlets. The only masses that are always allowed are the non-holomorphic, SUSY breaking diagonal scalar masses as they preserve all gauge symmetries by construction. Being part of the soft lagrangian they can be naturally at the TeV scale if generated via dynamical SUSY breaking. In this case however the extra $U(1)'$

is not enough to prevent another source of LFV such as $m^2_{Lij} L^{i\dagger} L^j$, but we will assume that the soft terms are generated in a gauge mediation framework, where such problems do not arise[1].

The classical potential given in (2.5) of the model should be extended by the soft-terms and in general can be written (soft-term, F-term and D-term):

$$V = V_S + V_F + V_D = V_S + W^{*i} W_i + \frac{1}{2} \sum_g \sum_a g^2 (\phi^{*i} T^a \phi^i)^2, \tag{4.2}$$

where W is the superpotential and $W_i = \partial W / \partial \phi_i$. The Higgs fields can be explicitly written as:

$$H_u^i = \begin{pmatrix} H_u^{+i} \\ H_u^{0i} \end{pmatrix}, H_d^i = \begin{pmatrix} H_d^{0i} \\ H_d^{-i} \end{pmatrix}, \tag{4.3}$$

where H_u^{+i} and H_d^{-i} are charged fields, and H_u^{0i} and H_d^{0i} are neutral fields (the charges of the Higgses follow from the Gell-Mann relation $Q = I_3 + Y$). Also we drop terms involving "QudLe" scalars and analyse the classical potential without them. Then the superpotential gets the form

$$W = \kappa_i S_i H_u^i H_d^i + \lambda S_1 S_2 S_3, \tag{4.4}$$

if we put back $SU(2)$ indices:

$$H_u^i H_d^i = H_{u\alpha}^i H_{d\beta}^i \epsilon^{\alpha\beta} = H_u^{+i} H_d^{-i} - H_u^{0i} H_d^{0i}. \tag{4.5}$$

We assume the summation under repeated indices, where $i = 1..3$ family index and $j = 0..3$ index of scalar fields. From the definition (4.2) using (4.4) F-term of the scalar potential can be written as:

$$\begin{aligned} V_F = \left|\frac{\partial W}{\partial H_u^{+i}}\right|^2 + \left|\frac{\partial W}{\partial H_d^{-i}}\right|^2 + \left|\frac{\partial W}{\partial H_u^{0i}}\right|^2 + \left|\frac{\partial W}{\partial H_d^{0i}}\right|^2 + \left|\frac{\partial W}{\partial S^i}\right|^2 = \\ = |\kappa_i|^2 |S_i|^2 \left(|H_d^{-i}|^2 + |H_u^{+i}|^2 + |H_d^{0i}|^2 + |H_u^{0i}|^2\right) + \\ + |\kappa_1 H_u^1 H_d^1 + \lambda S_2 S_3|^2 + |\kappa_2 H_u^2 H_d^2 + \lambda S_1 S_3|^2 + |\kappa_3 H_u^3 H_d^3 + \lambda S_1 S_2|^2. \end{aligned} \tag{4.6}$$

Let us denote g_2, g_1 and g_1' to be coupling constants of the $SU(2)$, $U(1)$ and $U'(1)$ gauge fields respectively. Using the definition (4.2) the D-term can be written as:

$$\begin{aligned} V_D = \frac{1}{8} g_2^2 \left(H_u^{i\dagger} \sigma^a H_u + H_d^{i\dagger} \sigma^a H_d^i\right)^2 + \frac{1}{8} g_1^2 \left(H_u^{i\dagger} H_u^i - H_d^{i\dagger} H_d^i\right)^2 + \\ + \frac{1}{2} g_1'^2 \left(H_u^{i\dagger}\, q^i_{H_u}\, H_u^i + H_d^{i\dagger}\, q^i_{H_d}\, H_d^i + S^{j*} q_S^j\, S^j\right)^2, \end{aligned} \tag{4.7}$$

[1]This is not strictly true in our context, since, as we will discuss in section 5.2, the Z' couplings do violate flavor symmetry and thus will generate subleading off diagonal terms.

where σ^a are the Pauli matrices. Applying algebraic transformations to the (4.7) we can get the final expression:

$$V_D = \frac{1}{8}g_2^2\left(4\left|H_u^{+i}H_u^{0i*} + H_d^{0i}H_d^{-i*}\right|^2 + \left(|H_u^{+i}|^2 - |H_u^{0i}|^2 + |H_d^{0i}|^2 - |H_d^{-i}|^2\right)^2\right) + \frac{1}{8}g_1^2\left(|H_u^{+i}|^2 + |H_u^{0i}|^2 - |H_d^{0i}|^2 - |H_d^{-i}|^2\right)^2 + + \frac{1}{2}g_1'^2\left(q^i_{H_u}(|H_u^{+i}|^2 + |H_u^{0i}|^2) + q^i_{H_d}(|H_d^{0i}|^2 + |H_d^{-i}|^2) + q^j_S|S^j|^2\right)^2. \quad (4.8)$$

We will take a bottom-up approach and simply parameterize the SUSY breaking effects by the most generic soft terms allowed by the gauge symmetries. The scalars in the matter sector can always be arranged to be non tachyonic by suitably large soft masses. We thus set them to zero and analyze the scalar potential in the Higgs sector alone.

$$V_S = (m_i^{Hu})^2\left|H_u^i\right|^2 + (m_i^{Hd})^2\left|H_d^i\right|^2 + (m_i^S)^2\left|S^i\right|^2 + (m_0^S)^2\left|S^0\right|^2 + b_iH_u^iH_d^iS^i + aS^1S^2S^3. \quad (4.9)$$

Let us now ignore all charged fields H_u^{+i}, H_d^{-i}, because they cannot acquire vacuum expectation value, then our potential gets the form:

$$V = |\kappa_i|^2|S_i|^2\left(|H_u^{0i}|^2 + |H_d^{0i}|^2\right) + + |\kappa_1H_u^{01}H_d^{01} - \lambda S_2S_3|^2 + |\kappa_2H_u^{02}H_d^{02} - \lambda S_1S_3|^2 + |\kappa_3H_u^{03}H_d^{03} - \lambda S_1S_2|^2 + + \frac{1}{8}(g_1^2 + g_2^2)\left(|H_u^{0i}|^2 - |H_d^{0i}|^2\right)^2 + \frac{1}{2}g_1'^2\left(q^i_{H_u}|H_u^{0i}|^2 + q^i_{H_d}|H_d^{0i}|^2 + q^k_S|S^k|^2\right)^2 + + \left(m^2_{H_ui}|H_u^{0i}|^2 + m^2_{H_di}|H_d^{0i}|^2 + m^2_{Sk}|S_k|^2\right) + \left(-b_iS_iH_u^{0i}H_d^{0i} + aS_1S_2S_3 + c.c.\right). \quad (4.10)$$

Some of the "soft"-masses $m^2_{H_ui}$, $m^2_{H_di}$ and m^2_{Si} can be negative. It is important to note that there is no danger of a classical instability at large fields. In the MSSM, the only F-terms in the Higgs sector are those coming from the μ-term leading to a quadratic term in the potential. This gives rise to classical instabilities along the D-flat directions (such directions in parameter space where $V_D = 0$) if the soft terms are sufficiently negative. There is no such danger in this model since the superpotential is cubic and generically lifts all the flat directions giving a quartic contribution to V_F. Soft terms cannot reverse that as they are at most cubic in the scalars.

Now we need to analyse the scalar potential (4.10).

Minimum of the potential

One should find the classical minimum of our theory to get the vacuum expectation values of the neutral fields responsible for the Electro-Weak symmetry breaking. The main problem is a lack of experimental data on the parameters (especially terms coming from the soft part of the lagrangian).

It is possible to assume to first approximation that only the third generation of Higgses and four scalars can acquire big vev:

$$\langle H_u^{01}\rangle \approx \langle H_u^{02}\rangle \approx \langle H_d^{01}\rangle \approx \langle H_d^{02}\rangle \approx 0, \quad (4.11)$$

and

$$\langle H_u^{03}\rangle \equiv v_u,\ \langle H_d^{03}\rangle \equiv v_d,\ \langle S_j\rangle \equiv v_s^j. \tag{4.12}$$

Then the scalar potential gets the following form:

$$\begin{aligned} V = {} & \kappa^2|v_s^3|^2(|v_u|^2+|v_d|^2)+\lambda^2|v_s^2|^2|v_s^3|^2+\lambda^2|v_s^1|^2|v_s^3|^2+|\kappa\, v_u v_d-\lambda v_s^1 v_s^2|^2+ \\ & +\frac{1}{8}(g_1^2+g_2^2)(|v_u|^2-|v_d|^2)^2+\frac{1}{2}g_1'^2(q_{H_u^3}|v_u|^2+q_{H_d^3}|v_d|^2+q_{S^1}|v_s^1|^2+q_{S^2}|v_s^2|^2+q_{S^3}|v_s^3|^2)^2+ \\ & +m_u^2|v_u|^2+m_d^2|v_d|^2+m_{s1}^2|v_s^1|^2+m_{s2}^2|v_s^2|^2+m_{s3}^2|v_s^3|^2+m_{s0}^2|v_s^0|^2- \\ & -bv_s^3v_uv_d-av_s^1v_s^2v_s^3-b^*v_s^{3*}v_u^*v_d^*-av_s^{1*}v_s^{2*}v_s^{3*}. \end{aligned} \tag{4.13}$$

This potential could be minimized numerically, however one must be careful, because some of the ignored fields can become tachyons (fields with the negative squared mass indicating, that the given vacuum is unstable - not true vacuum of the system)

CP-violation

Analysing the scalar potential (4.10) we can take a look at the part that might contain CP-violating phases:

$$\begin{aligned} \mathcal{L} \supset {} & -\kappa_1\lambda^* H_u^{01}H_d^{01}S_2^*S_3^*-\kappa_2\lambda^*H_u^{02}H_d^{02}S_1^*S_3^*-\kappa_3\lambda^*H_u^{03}H_d^{03}S_1^*S_2^*- \\ & -b_1S_1H_u^{01}H_d^{01}-b_2S_2H_u^{02}H_d^{02}-b_3S_3H_u^{03}H_d^{03}+aS_1S_2S_3+cc. \end{aligned} \tag{4.14}$$

Since λ always appears multiplied by κ_i we can chose it to be real and positive without loss of generality. The phases of a and κ_i can then be rotated away by a redefinition of the fields leaving the three phases of b_i. There is thus a CP violating contribution from the soft terms in the Higgs sector. To remind, there is no such contribution in the MSSM.

4.2 Electroweak Breaking

The covariant derivative is defined as follows:

$$D_\mu = \partial_\mu - ig_2A_\mu^a\tau^a - ig_1YB_\mu - ig_1'qB_\mu'. \tag{4.15}$$

Suppose we give a vacuum expectation value to the neutral Higgs fields and singlets S, where $i = 1..3$ and $j = 0..3$:

$$\langle H_u^i\rangle = \begin{pmatrix} 0 \\ v_u^i \end{pmatrix}, \langle H_d^i\rangle = \begin{pmatrix} v_d^i \\ 0 \end{pmatrix}, \langle S^j\rangle = v_s^j, \tag{4.16}$$

where v_u, v_d and v_s are complex constants (determined from the minimum of the scalar potential). Electroweak symmetry breaking arises from the following term in the lagrangian:

$$(D_\mu\phi^i)^\dagger D^\mu\phi_i \to (D_\mu H_u^i)^\dagger D^\mu H_u^i + (D_\mu H_d^i)^\dagger D^\mu H_d^i + (D_\mu S_i)^\dagger D^\mu S_i =$$
$$= \langle H_u^i\rangle^\dagger (g_2 A_\mu^a \tau^a + \frac{1}{2} g_1 B_\mu + g_1' q_u^i B_\mu')^2 \langle H_u^i\rangle + \langle H_d^i\rangle^\dagger (g_2 A_\mu^a \tau^a - \frac{1}{2} g_1 B_\mu + g_1' q_d^i B_\mu')^2 \langle H_d^i\rangle +$$
$$+ \langle S^j\rangle^\dagger (g_1' q_S^j B_\mu')^2 \langle S^j\rangle, \quad (4.17)$$

where using the definition of Pauli matrices (1.6)

$$g_2 A_\mu^a \tau^a \pm \frac{1}{2} g_1 B_\mu + g_1' q^\Phi B_\mu' = \frac{1}{2} \begin{pmatrix} g_2 A_\mu^3 \pm g_1 B_\mu + 2 g_1' q^\Phi B_\mu' & g_2(A_\mu^1 - i A_\mu^2) \\ g_2(A_\mu^1 + i A_\mu^2) & -g_2 A_\mu^3 \pm g_1 B_\mu + 2 g_1' q^\Phi B_\mu' \end{pmatrix} \quad (4.18)$$

Relation (4.18) together with (4.17) provides the part of the lagrangian responsible for the masses of the vector bosons:

$$\mathcal{L} \supset \frac{1}{4} g_2^2 v_1^2 (A_\mu^1 - i A_\mu^2)(A_\mu^1 + i A_\mu^2) + \begin{pmatrix} B_\mu & A_\mu^3 & B_\mu' \end{pmatrix} M^2 \begin{pmatrix} B^\mu \\ A^{3\mu} \\ B'^\mu \end{pmatrix} \quad (4.19)$$

where M^2 is a mass matrix for the neutral gauge bosons:

$$M^2 = \frac{1}{2} \begin{pmatrix} g_1^2 v_1^2 & -g_1 g_2 v_1^2 & -2 g_1' g_1 v_2^2 \\ -g_1 g_2 v_1^2 & g_2^2 v_1^2 & 2 g_1' g_2 v_2^2 \\ -2 g_1' g_1 v_2^2 & 2 g_1' g_2 v_2^2 & 4 g_1'^2 v_3^2 \end{pmatrix}. \quad (4.20)$$

As already was mentioned g_2, g_1 and g_1' are the coupling constants to the $SU(2)$, $U(1)$ and $U'(1)$ gauge fields respectively. It turns out that only three different combinations of "vevs" v_1, v_2 and v_3 enters the mass matrix:

$$v_1^2 = \sum_i \left(|v_u^i|^2 + |v_d^i|^2 \right), \quad v_2^2 = \sum_i \left(q_{Hu}^i |v_u^i|^2 - q_{Hd}^i |v_d^i|^2 \right),$$
$$v_3^2 = \sum_i \left((q_{Hu}^i)^2 |v_u^i|^2 + (q_{Hd}^i)^2 |v_d^i|^2 + (q_S^i)^2 |v_S^i|^2 \right) + (q_S^0)^2 |v_S^0|^2. \quad (4.21)$$

Notice that it is reasonable to assume $v_2 < v_1 \ll v_3$ because of possible cancellations in v_2 and the presence of singlet vevs in v_3. It is easy to see that the charged vector bosons have the same masses as in the Standard Model where v_1 takes the place of the single Higgs vev: $m_W^2 = g_2^2 v_1^2 / 2$.

Diagonalization

To obtain the mass eigenstates of the gauge bosons we need to diagonalize the mass matrix (4.20). First, let us note that the 2×2 block matrix in the left upper corner of (4.20) is exactly the Standard Model mass matrix and can be diagonalized by the rotation matrix

$$\begin{pmatrix} \cos\theta_W & -\sin\theta_W & 0 \\ \sin\theta_W & \cos\theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4.22}$$

Then the part of (4.19) responsible for the masses of neutral gauge fields can be rewritten as

$$\mathcal{L} \supset \begin{pmatrix} A_\mu & Z_\mu^{\mathrm{SM}} & B'_\mu \end{pmatrix} M'^2 \begin{pmatrix} A^\mu \\ Z^{\mathrm{SM}\mu} \\ B'^\mu \end{pmatrix}, \tag{4.23}$$

where

$$M'^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2}(g_1^2+g_2^2)v_1^2 & \sqrt{g_1^2+g_2^2}g'_1 v_2^2 \\ 0 & \sqrt{g_1^2+g_2^2}g'_1 v_2^2 & 2g_1'^2 v_3^2 \end{pmatrix}. \tag{4.24}$$

The photon is still massless and given by the same linear combination as in the Standard Model: $A_\mu = \cos\theta_W B_\mu + \sin\theta_W A_\mu^3$, where $\sin\theta_W = g_1/\sqrt{g_1^2+g_2^2}$ as usual and the Standard Model Z-boson $Z_\mu^{\mathrm{SM}} \equiv -\sin\theta_W B_\mu + \cos\theta_W A_\mu^3$, which in this case also mixes with B'_μ.

To complete the diagonalization we also need to use a second rotation matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\eta & -\sin\eta \\ 0 & \sin\eta & \cos\eta \end{pmatrix}, \tag{4.25}$$

where η is the Z_μ^{SM} - B'_μ mixing angle and its value is given by the expression:

$$\tan 2\eta = \frac{4\sqrt{g_1^2+g_2^2}\,g'_1 v_2^2}{(g_1^2+g_2^2)v_1^2 - 4g_1'^2 v_3^2}. \tag{4.26}$$

Finally, the remaining two mass eigenstates Z_μ and Z'_μ are given in terms of B'_μ and the original Standard Model Z-boson as:

$$Z_\mu = \cos\eta Z_\mu^{\mathrm{SM}} + \sin\eta B'_\mu, \quad \text{and} \quad Z'_\mu = -\sin\eta Z_\mu^{\mathrm{SM}} + \cos\eta B'_\mu. \tag{4.27}$$

The bosons masses are respectively

$$m_Z^2 = \frac{1}{4}\left((g_1^2+g_2^2)v_1^2 + 4g_1'^2 v_3^2 - \sqrt{((g_1^2+g_2^2)v_1^2 + 4g_1'^2 v_3^2)^2 + 16(g_1^2+g_2^2)g_1'^2(v_2^4 - v_1^2 v_3^2)}\right), \tag{4.28}$$

$$m_{Z'}^2 = \frac{1}{4}\left((g_1^2+g_2^2)v_1^2 + 4g_1'^2 v_3^2 + \sqrt{((g_1^2+g_2^2)v_1^2 + 4g_1'^2 v_3^2)^2 + 16(g_1^2+g_2^2)g_1'^2(v_2^4 - v_1^2 v_3^2)}\right).$$

To first non trivial order in $1/v_3^2$:

$$m_Z^2 \approx \frac{1}{2}(g_1^2+g_2^2)v_1^2 - \frac{1}{2}(g_1^2+g_2^2)\frac{v_2^4}{v_3^2}, \qquad m_{Z'}^2 \approx 2g_1'^2 v_3^2 + \frac{1}{2}(g_1^2+g_2^2)\frac{v_2^4}{v_3^2} \tag{4.29}$$

and η-mixing angle

$$\eta \approx -\frac{\sqrt{g_1^2+g_2^2}\, v_2^2}{2g_1' v_3^2}, \;\; |\eta| \ll 1. \tag{4.30}$$

Fermion mass eigenstates

In terms of the new fields our covariant derivative can be written as:

$$\begin{aligned} D_\mu = \partial_\mu - \frac{ig_2}{\sqrt{2}}(W_\mu^+\tau^+ + W_\mu^-\tau^-) - ig_{em}QA_\mu - \\ - i\frac{g_2}{\cos\theta_W}\left(\left(\cos^2\theta_W\, Q - Y\right)\cos\eta + q^\Phi \underbrace{\frac{g_1'\cos\theta_W}{g_2}\sin\eta}_{\approx -v_2^2/(2v_3^2)}\right) Z_\mu - \\ - ig_1'\left(q\cos\eta - \frac{g_2}{g_1'\cos\theta_W}\left(\cos^2\theta_W\, Q - Y\right)\sin\eta\right) Z_\mu', \end{aligned} \tag{4.31}$$

where q^Φ is a $U'(1)$ charge of the chiral superfield Φ and

$$\tau^\pm \equiv \frac{1}{2}(\sigma^1 \pm i\sigma^2), \;\; Q \equiv \tau^3 + Y, \;\; g_{em} = g_2\sin\theta_W.$$

Using the definition of the covariant derivative (4.31) the kinetic terms of the "visible" fermions in the model can be written as:

$$\begin{aligned} \sum_\psi i\psi_i^\dagger\bar{\sigma}^\mu D_\mu\psi^i \to \sum_\psi i\psi_i^\dagger\bar{\sigma}^\mu\partial_\mu\psi^i + g_2 J_{W^+}^\mu W_\mu^+ + g_2 J_{W^-}^\mu W_\mu^- + \\ + g_2 J_Z^\mu Z_\mu + g_1' J_{Z'}^\mu Z_\mu' + g_{em} J_{em}^\mu A_\mu, \end{aligned} \tag{4.32}$$

where, as usually, we assume the summation by family index i of the fermionic field ψ. The currents have the following form:

$$\begin{aligned} J_{W^+}^\mu = \frac{1}{\sqrt{2}}\sum_\psi \psi_i^\dagger\bar{\sigma}^\mu\tau^+\psi^i, \; J_{W^-}^\mu = \frac{1}{\sqrt{2}}\sum_\psi \psi_i^\dagger\bar{\sigma}^\mu\tau^-\psi^i, \; J_{em}^\mu = \sum_\psi \psi_i^\dagger\bar{\sigma}^\mu Q\psi^i, \\ J_Z^\mu = \frac{1}{\cos\theta_W}\sum_\psi \psi_i^\dagger\bar{\sigma}^\mu\left(\left(\cos^2\theta_W\, Q - Y\right)\cos\eta + q_i^\psi\frac{g_1'\cos\theta_W}{g_2}\sin\eta\right)\psi^i, \\ J_{Z'}^\mu = \sum_\psi \psi_i^\dagger\bar{\sigma}^\mu\left(q_i^\psi\cos\eta - \frac{g_2}{g_1'\cos\theta_W}\left(\cos^2\theta_W\, Q - Y\right)\sin\eta\right)\psi^i, \end{aligned} \tag{4.33}$$

note that some of the charges q^ψ are family dependent. The fields ψ are gauge eigenstates. To get the final expressions for the currents we need to go from the gauge to mass eigenstates. Here we will briefly outline this procedure.

Terms in the Lagrangian responsible for the Dirac masses of the fermions from "matter sector" are:

$$\mathcal{L}_{f.m.} = -\frac{1}{2} y^{ijk}\phi_i\psi_j\psi_k - \frac{1}{2} y^*_{ijk}\phi^{\dagger i}\psi^{\dagger j}\psi^{\dagger k}, \tag{4.34}$$

giving the vacuum expectation values for the Higgses

$$\mathcal{L}_{f.m.} = -\frac{1}{2}\left(\underbrace{Y_u^{ij}v_u^j}_{\lambda_u^{ij}} u^i\bar{u}^j + \underbrace{Y_d^{ij}v_d^j}_{\lambda_d^{ij}} d^i\bar{d}^j + \underbrace{Y_e^{ij}v_d^j}_{\lambda_e^{ij}} e^i\bar{e}^j + h.c.\right), \tag{4.35}$$

where λ are off-diagonal mass matrices. Any matrix can be diagonalized by the following transformation: $\mu = V_L^\dagger \lambda V_R \to \lambda = V_L \mu V_R^\dagger$, where μ is a diagonal and λ is a non-diagonal mass matrices, V_L and V_R are unitary matrices, which perform the diagonalization. Rotating the fields as follows:

$$u_m^i = (V_L^u)^{ij}u^j, \quad \bar{u}_m^i = (V_R^u)^{\dagger ij}\bar{u}^j, \tag{4.36}$$
$$d_m^i = (V_L^d)^{ij}d^j, \quad \bar{d}_m^i = (V_R^d)^{\dagger ij}\bar{d}^i, \tag{4.37}$$
$$e_m^i = (V_L^e)^{ij}e^j, \quad \bar{e}_m^i = (V_R^e)^{\dagger ij}\bar{e}^j, \tag{4.38}$$

we get the mass eigenstates, denoted by the index "m".

After rotating the fields to their gauge eigenstates, the neutral currents coupled to Z and Z' bosons can be then written as (where we dropped index m):

$$J_Z^\mu = \frac{1}{\cos\theta_W}\sum_\psi \psi_i^\dagger\bar{\sigma}^\mu\left(\left(\cos^2\theta_W\, Q - Y\right)\cos\eta\,\delta_{ij} + B_{ij}^\psi \frac{g_1'\cos\theta_W}{g_2}\sin\eta\right)\psi_j, \tag{4.39}$$
$$J_{Z'}^\mu = \sum_\psi \psi_i^\dagger\bar{\sigma}^\mu\left(B_{ij}^\psi\cos\eta - \frac{g_2}{g_1'\cos\theta_W}\left(\cos^2\theta_W\, Q - Y\right)\sin\eta\,\delta_{ij}\right)\psi_j. \tag{4.40}$$

where the indices i, j run from 1 to 3 and stand for family number. The coefficients B_{ij} can be defined as:

$$B_{ij}^\psi = \sum_{k=1}^{3} V_{ik}^\psi q_k^\psi V_{kj}^{\psi\dagger}, \tag{4.41}$$

where $V^\psi = V_L^{\psi\dagger}$ for $\psi = u,d,e$ and $U^\psi = V_R^\psi$ for $\psi = \bar{u},\bar{d},\bar{e}$. If all families have the same charges, the matrices B are diagonal, because $VV^\dagger = I$. In our model for some fields this is not true, what gives rise to the flavour changing processes. Let us note that the values of $V_L^{\psi\dagger}$ are of the same order as the Cabibbo-Kobayashi-Maskawa matrix elements, because of its form $V_{CKM} = V_L^u V_L^{d\dagger}$. The values of V_R^ψ are completely unknown.

4.3 Running Coupling Constants

The β-function can be defined as:

$$\beta(g_i) \equiv \frac{dg_i}{dt}, \, t = \ln \frac{\mu}{\mu_0}, \tag{4.42}$$

where g_i is the coupling constant of the gauge group "i", μ is the energy scale of the process and μ_0 is the energy scale of Electroweak-Breaking (EW). $\mu_0 = 246\, GeV$ for EW breaking scale and the Planck scale $\mu_{Pl} = 1.22 \cdot 10^{19}\, GeV$ (this number translates into $t_{Pl} \approx 38.4 \cdot$).

To one loop the β-function has the following form:

$$\beta(g_i) = \frac{1}{16\pi^2} b_i g_i^3, \tag{4.43}$$

where b_i are determined by the matter content. Expression (4.43) allows one to determine the running of the coupling constants g_i. However, it is more convenient to work with $\alpha_i \equiv \frac{g_i^2}{4\pi}$. In terms of α_i, we can rewrite the differential equation for the running coupling constants as follows:

$$\frac{d}{dt} \alpha_i^{-1} = -\frac{b_i}{2\pi}. \tag{4.44}$$

The solution to this equation is:

$$\alpha_i^{-1}(t) = \alpha_i^{-1}(EW) - \frac{b_i}{2\pi} t. \tag{4.45}$$

The values of $\alpha_i^{-1}(EW)$ are given by the experimental data:

$$\alpha_1^{-1}(EW) = 58.98 \pm 0.08, \tag{4.46}$$

$$\alpha_2^{-1}(EW) = 29.60 \pm 0.04, \tag{4.47}$$

$$\alpha_3^{-1}(EW) = 8.47 \pm 0.22. \tag{4.48}$$

The coefficients b_i for the $SU(N)$ gauge group are given by the analytic expression (the case of $U(1)$ is special and shell be discussed separately):

$$b_i = -\frac{11}{3} C(Adj) + \underbrace{\frac{2}{3} \sum_i C(\Psi_i)}_{fermions} + \underbrace{\frac{1}{6} \sum_i C(\phi_i)}_{real\ scalars}, \tag{4.49}$$

where $C(r)$ is the index of the representation. For $SU(N)$ group we use the following convention: $C(\square) = \frac{1}{2}$, where symbol $\square$ denotes the fundamental representation and $C(Adj) = N, \forall N \neq 1$.

The $U(1)$ group is special. The details are given in [19]. For the non-supersymmetric models the coefficient b_1 is given by the expression:

$$b_1 = \frac{2}{3} \sum_f q_f^2 + \frac{1}{3} q_s^2, \tag{4.50}$$

where q_f are the $U(1)$ charges of chiral fermions and q_s are $U(1)$ charges of scalars. For the supersymmetric models (4.50) translates simply into

$$b_1 = \sum_{sup} q_{sup}^2, \tag{4.51}$$

where q_{sup} are the charges of supermultiplets in the model. The values of these charges are taken to be proportional to the hypercharge $q = c \cdot Y$ with proportionality coefficient $c = \sqrt{\frac{3}{5}}$.

Applying formulas (4.49) and (4.50), we can get the coefficients b_i for different models, where we denote number of generations by n_g and number of Higgs doublets by n_h.

For the Standard Model:

$$b_1 = \frac{4}{3}n_g + \frac{1}{10}n_h, \tag{4.52}$$

$$b_2 = -\frac{22}{3} + \frac{4}{3}n_g + \frac{1}{6}n_h, \tag{4.53}$$

$$b_3 = -11 + \frac{4}{3}n_g. \tag{4.54}$$

For the MSSM:

$$b_1 = 2n_g + \frac{3}{10}n_h, \tag{4.55}$$

$$b_2 = -6 + 2n_g + \frac{1}{2}n_h, \tag{4.56}$$

$$b_3 = -9 + 2n_g. \tag{4.57}$$

Thus for $n_g = 3$ in the Standard Model ($n_h = 1$), MSSM ($n_h = 2$) and for our class of models ($n_h = 6$):

$$\{b_1, b_2, b_3\} = \begin{cases} \{\frac{41}{10}, -\frac{19}{6}, -7\}, & SM \\ \{\frac{33}{5}, 1, -3\}, & MSSM \\ \{\frac{39}{5}, 3, -3\}, & extended\ MSSM \end{cases} \tag{4.58}$$

The result of plotting $\alpha^{-1} = \alpha^{-1}(t)$ for the MSSM and our extension are given in the figure 4.1. As we can see at first glance the running coupling constants do not meet in our extension compared to the MSSM, due to an enlarged Higgs sector.

Let us now discuss the running coupling constant g_1' of the $U(1)'$ gauge group. From the expression (4.45) easily follows the definition of Landau pole - energy scale when the coupling constant become infinitely big. The critical condition for that is given by $b\,\alpha(EW) = (2\pi)/t$. Landau pole is not a problem if this occurs above the Planck energy scale (because our Quantum field theory description breaks down due to strong effects of gravity), thus we demand the following condition on b: $b\,\alpha(EW) \leq (2\pi)/t_{Pl}$ or using

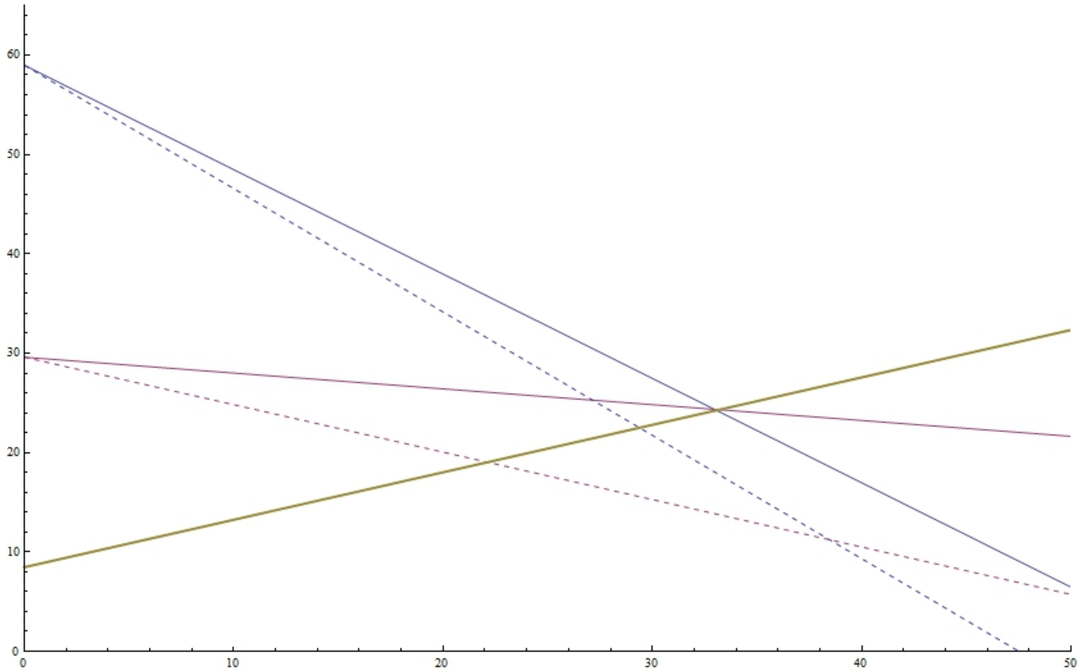


Figure 4.1: The blue, purple and brown lines correspond to $U(1)$, $SU(2)$ and $SU(3)$ couplings respectively. Solid lines correspond to MSSM, dashed - to extended MSSM.

the definition of α: $\sqrt{b}\, g(EW) \lesssim 1.13$. This requirement is fulfilled for all the models given in the previous section.

We can note that the scale of Grand Unification in the MSSM is $\mu_{Gr} = 5.3 \cdot 10^{16}\, GeV$ ($t_{Gr} \approx 33$). It is obvious also from the plot 4.1 that the running coupling constants of $SU(2)$ and $U(1)$ gauge groups hit the Landau pole also after the Plank scale ($t_{Pl} \approx 38.4 \cdot$).

5

Phenomenology

We derive decay rates of the Z-boson into fermions and Z'-boson into scalar particles (we assume the mass of Z'-boson is big enough to decay into some superpartners and some Higgs fields). In the last section we analyse the phenomenological constraints on the models coming from the experimental bounds on flavour violating processes allowed by our construction.

5.1 Decay rates

In our models we have a new gauge group, which results in additional vector boson Z' giving correction to the already well measured parameters of the Standard Model Z-boson. Luckily this correction is very small. For example the mass of the Z-boson (4.29) differ from the Standard Model Z-boson mass by a very small number due to large value of the parameter v_3. Another parameter of the Z-boson which can be affected by a presence of $U(1)'$ is its decay width. This correction arises due to new possible decays, like $Z \to \mu^- e^+$ (much more suppressed in the SM). While it is impossible for the Z-boson to decay into sfermions due to its small mass, the Z' could be heavy enough to do so. That is why it is of interest to derive analytic expressions for the decay rates of neutral gauge bosons into fermions and scalars. Here and in the rest of this section we prefer to work in four component Dirac notation (following the convention of [20]).

The differential decay rate of a particle into two particles is given by

$$d\Gamma = \frac{1}{32 m_Z \pi^2} \frac{d^3k}{E_k} \frac{d^3p}{E_p} |M|^2 \delta(E_l - E_k - E_p) \delta^{(3)}(\vec{l} - \vec{k} - \vec{p}) \tag{5.1}$$

Very important relations are

$$E_j = \sqrt{m_j^2 + k^2}, \quad k = |\vec{k}| \tag{5.2}$$

$$\delta(m_Z - E_i - E_j) = \frac{E_i E_j}{k(E_i + E_j)}[\delta(k - k_0) + \delta(k + k_0)], \tag{5.3}$$

$$k_0 = \frac{\sqrt{m_Z^4 + m_i^4 + m_j^4 - 2m_Z^2 m_i^2 - 2m_Z^2 m_j^2 - 2m_i^2 m_j^2}}{2m_Z}. \tag{5.4}$$

We always assume a decaying particle to in the rest frame ($\vec{l} = 0$). Integrating over momentum $\vec{p}$ and applying formulas (5.2)-(5.4) in the (5.1) we get

$$\Gamma = \frac{1}{8m_Z\pi}|M|^2 \int_0^\infty \frac{k\,dk}{E_i + E_j}\delta(k - k_0) = \frac{k}{8m_Z^2\pi}|M|^2\bigg|_{k=k_0}. \tag{5.5}$$

5.1.1 Z boson decay

The matrix element of the Z-boson decay into fermion and antifermion is given by

$$i\tilde{M}[Z(l) \to f_i(k)\bar{f}_j(p)] = -ig_2 C_{ij}\bar{u}(k)_i^{s1}\gamma^\mu P_L v(p)_j^{s2}\epsilon(l)_\mu, \tag{5.6}$$

where f ($\bar{f}$) is a fermion (antifermion); i and j are family indices; $s1$ and $s2$ are spin indices of the particles; $\epsilon(l)_\mu$ - polarization of the Z-boson; l, p and k are momenta of the particles, $P_L = (1 - \gamma^5)/2$ ($P_L^2 = P_L$) and finally the coefficient coming from the expression (4.39)

$$C_{ij} = \frac{1}{\cos\theta_W}\left(\left(\cos^2\theta_W\, Q - Y\right)\cos\eta\,\delta_{ij} + B_{ij}^\psi \frac{g_1'\cos\theta_W}{g_2}\sin\eta\right). \tag{5.7}$$

The probability amplitude of given process is (no summation among repeated indices here and later on in this section)

$$|\tilde{M}|^2 = g_2^2 C_{ij}C_{ij}^*\bar{u}(k)_i^{s1}\gamma^\mu P_L v(p)_j^{s2}\epsilon(l)_\mu^{(\lambda)}\bar{v}(p)_j^{s2}\gamma^\nu P_L u(k)_i^{s1}\epsilon(l)_\nu^{(\lambda)*}, \tag{5.8}$$

summing over all possible spin configurations and averaging over 3 possible spin states (polarizations) of the Z boson, we get

$$|M|^2 = \frac{g_2^2}{3}\sum_{s1}\sum_{s2}\sum_{\lambda} C_{ij}C_{ij}^*\bar{u}(k)_i^{s1}\gamma^\mu P_L v(p)_j^{s2}\epsilon(l)_\mu^{(\lambda)}\bar{v}(p)_j^{s2}\gamma^\nu P_L u(k)_i^{s1}\epsilon(l)_\nu^{(\lambda)*}. \tag{5.9}$$

For the Z boson we can write

$$\sum_\lambda \epsilon(l)_\mu^{(\lambda)}\epsilon(l)_\nu^{(\lambda)*} = -\eta_{\mu\nu} + \frac{l_\mu l_\nu}{m_Z^2}. \tag{5.10}$$

Let us also recall the usual relations for the fermions:

$$\sum_{s2} v(p)_j^{s2} \bar{v}(p)_j^{s2} = p_\sigma \gamma^\sigma + m_j, \tag{5.11}$$

$$\sum_{s1} u(k)_i^{s1} \bar{u}(k)_i^{s1} = k_\sigma \gamma^\sigma - m_i. \tag{5.12}$$

Using (5.11) and (5.12) in (5.10), and using the definition of the trace we get:

$$|M|^2 = \frac{g_2^2}{3} |C_{ij}|^2 Tr[(k_\sigma \gamma^\sigma - m_i)\gamma^\mu P_L (p_\rho \gamma^\rho + m_j)\gamma^\nu P_L (-\eta_{\mu\nu} + \frac{l_\mu l_\nu}{m_Z^2})]. \tag{5.13}$$

After simple algebraic transformations the previous expression gets the form

$$|M|^2 = \frac{2}{3} g_2^2 |C_{ij}|^2 [(k \cdot p) + \frac{2}{m_Z^2}(k \cdot l)(p \cdot l)]. \tag{5.14}$$

We always assume Z boson is at rest ($\vec{l} = 0$). Setting $\vec{p} = -\vec{k}$ because of delta-function in (5.1) we get the final form of the squared probability amplitude:

$$|M|^2 = \frac{2}{3} g_2^2 |C_{ij}|^2 [3E_i E_j + \vec{k}^2]. \tag{5.15}$$

The masses of almost all known fermions are very small compared to the mass of the Z-boson, thus to a very good accuracy we can set $m_i = m_j = 0$. Combining (5.5) and (5.15) we get the final expression for the decay rate

$$\Gamma \approx \frac{m_Z {g_2}^2}{24\pi} |C_{ij}|^2. \tag{5.16}$$

5.1.2 Z' decay

Z' can decay the same way as an ordinary Z-boson into fermions, this case is described then exactly by the same expression (5.16). Nevertheless the huge difference in mass opens the possibility to decay also into heavy scalar particle. Let us then investigate such processes. The matrix element is given by the expression

$$i\tilde{M}[Z(l) \to \phi_i(k)\ \phi_j^*(p)] = -ig_1' K_{ij} \epsilon(l)_\mu^{(\lambda)} (k^\mu - p^\mu), \tag{5.17}$$

where the coefficient K_{ij} comes from (4.40):

$$K_{ij} = B_{ij}^\psi \cos\eta - \frac{g_2}{g_1' \cos\theta_W} \left(\cos^2\theta_W\, Q - Y\right) \sin\eta\, \delta_{ij}. \tag{5.18}$$

Averaging over 3 possible spin states (polarizations) of the Z' boson we get

$$|M|^2 = \frac{1}{3} {g_1'}^2 K_{ij} K_{ij}^* (k^\mu - p^\mu)(k^\nu - p^\nu) \sum_\lambda \epsilon(l)_\mu^{(\lambda)} \epsilon(l)_\nu^{(\lambda)*}. \tag{5.19}$$

Applying (5.10) to the previous expression we get

$$|M|^2 = \frac{1}{3}{g'_1}^2|K_{ij}|^2[-m_i^2 - m_j^2 + 2(k\cdot p) + \frac{1}{m_{Z'}^2}(l\cdot(k-p))^2]. \tag{5.20}$$

Setting $\vec{l} = 0$ and $\vec{p} = -\vec{k}$ as in the previous discussion, we get:

$$|M|^2 = \frac{4}{3}{g'_1}^2|K_{ij}|^2\vec{k}^2. \tag{5.21}$$

Combining (5.21) and (5.5) we get finally the decay rate:

$$\Gamma = \frac{{g'_1}^2|K_{ij}|^2}{6m_Z^2\pi}k_0^3, \tag{5.22}$$

where k_0 is given by (5.4) with m_Z replaced by $m_{Z'}$.

5.2 Experimental Constraints

Apart from the obvious lower mass bounds on the Z' recently set by the LHC [21, 22], and from the mass and width of the SM Z boson set by LEP [23], the strongest experimental constraints arise from the potential flavor changing effects. Such processes can arise at tree level even if the charge matrix is diagonal after rotating the matter fermions into their mass eigenstates. Their effects on the low energy physics have been analyzed in all generality in [24] and we will use their formalism to test our model, apart for the following notational changes from their paper to our work: $\theta \to \eta$, for the Z' mixing, $g_1 \to g_2/\cos\theta_W$ and $g_2 \to g'_1$ for the couplings to the Z and Z' and $\epsilon^{(2)} \to q$ for all the $U(1)'$ charges.

We thus define, following [24]

$$\rho_1 = \frac{m_W^2}{m_Z^2\cos^2\theta_W}, \quad \rho_2 = \frac{m_W^2}{m_{Z'}^2\cos^2\theta_W} \tag{5.23}$$

and

$$w = \frac{g'_1\cos\theta_W}{g_2}\sin\eta\cos\eta(\rho_1-\rho_2), \quad y = \left(\frac{g'_1\cos\theta_W}{g_2}\right)^2(\rho_1\sin^2\eta + \rho_2\cos^2\eta). \tag{5.24}$$

Many of the experimental bounds can be expressed in terms of these last two dimensionless quantities.

Moreover, denoting by V_L^ψ and V_R^ψ ($\psi = u, d, e$) the unitary matrices that diagonalize the Yukawa couplings after EW breaking and by q_L^ψ and q_R^ψ the 3×3 diagonal matrices of the charges (see e.g. table 3.2) we write, referring to (3.14)

$$B_L^\psi \equiv V_L^\psi\, q_L^\psi\, V_L^{\psi\dagger} = Y_L^\psi \quad \text{and} \quad B_R^\psi \equiv V_R^\psi\, q_R^\psi\, V_R^{\psi\dagger} = Y_R^\psi + r\, V_R^\psi\, \hat{q}_R^\psi\, V_R^{\psi\dagger}. \tag{5.25}$$

$m_{Z'} =$	2 TeV	3 TeV	4 TeV
$g'_1\sqrt{\mathrm{Re}((B^d_R)^2_{12})} <$	$6. \times 10^{-5}$	$9. \times 10^{-5}$	$1. \times 10^{-4}$
$g'_1\sqrt{\mathrm{Re}((B^u_R)^2_{12})} <$	$8. \times 10^{-4}$	$1. \times 10^{-3}$	$2. \times 10^{-3}$
$g'_1\sqrt{\mathrm{Re}((B^d_R)^2_{13})} <$	$4. \times 10^{-4}$	$6. \times 10^{-4}$	$9. \times 10^{-4}$
$g'_1\sqrt{\mathrm{Re}((B^d_R)^2_{23})} <$	$2. \times 10^{-3}$	$3. \times 10^{-3}$	$4. \times 10^{-3}$
$g'_1\sqrt{\mathrm{Im}((B^d_R)^2_{12})} <$	$5. \times 10^{-6}$	$7. \times 10^{-6}$	$1. \times 10^{-5}$
$g'^2_1\lvert(B^e_R)_{12}\rvert <$	$6. \times 10^{-5}$	$1. \times 10^{-4}$	$3. \times 10^{-4}$

Table 5.1: *Bounds on various combinations of g'_1 and B^ψ_R from meson mass splitting, CP violation and muon conversion. The bound in the last line is mildly dependent on v_2 and we have chosen $v_2 = 100.$ GeV.*

The flavor changing effects are induced by the off diagonal terms in the matrices (5.25). By construction, the matrices B^ψ_L are diagonal for all fermions since q^ψ_L are proportional to the identity matrix and V^ψ_L is unitary. In the "best case" scenario, where the diagonalization of the Yukawa couplings is achieved almost entirely by V^ψ_L and $V^\psi_R \approx \mathbf{1}$, the matrices B^ψ_R are also close to be diagonal. Lacking any experimental input on the matrices V^ψ_R (recall that only $V^u_L\, V^{d\dagger}_L$ are observable in the SM) we will set bounds on them from current experiments.

We consider three benchmark values for the mass of the Z': $2, 3$ and 4 TeV, out of the latest ATLAS and CMS searches but still within reach of the future runs. In this regime the mass of the Z' is essentially independent on v_2 and this observation allows us to fix v_3 in terms of the coupling g'_1: $v_3 = m_{Z'}/\sqrt{2}g'_1$.

We begin with the mass splitting between neutral mesons. For the four neutral mesons $P = K, D, B, B_s$ the mass splitting between mass eigenstates in our models reads

$$\Delta m_P = \frac{4\sqrt{2}}{3} G_F m_P F_P^2 y \mathrm{Re}(B_P^2), \tag{5.26}$$

where G_F is the Fermi constant, m_P and F_P the average mass and decay constant of the meson and $B_P = (B^d_R)_{12}$, $(B^u_R)_{12}$, $(B^d_R)_{13}$, $(B^d_R)_{23}$ for $P = K, D, B, B_s$ respectively. For these range of Z' masses, not only $m_{Z'}$ but also y is essentially independent on v_2 and it is in fact proportional to g'^2_1 allowing us to set a bound on the product $g'_1\sqrt{\mathrm{Re}(B^2_P)}$ shown in the first four lines of table 5.1 using the data from [23]. Bounds on the imaginary part of B^2_P can be set from indirect CP violation from $P\,\bar{P}$ mixing. So far this has been firmly established only for Kaons, and a computation similar to the above yields the values in the fifth line of table 5.1. It would also be interesting to analyze in detail the contribution of our model to direct CP violation in D and B mesons that has attracted much attention lately.

Bounds on the off-diagonal elements of B_R^e come from various LFV processes such as $\mu^- \to e^- \gamma$ [25] or muon conversion. It turns out that the strongest bound can be obtained by considering $\mu^- \to e^-$ conversion in Ti nuclei and we will restrict ourselves to this process. The Sindrum-II collaboration established a bound on the branching ratio [26] of 4.3×10^{-12}. In our model this translates to

$$\frac{G_F^2 \alpha^3 m_\mu^5}{2\pi^2 \Gamma_{\rm capt}} \frac{Z_{\rm eff}^4}{Z} F_P^2 |(B_R^e)_{12}|^2 \Big| w((Z-N)/2 - 2Z \sin^2\theta_W)$$
$$+ y((2Z+N)(1/6 + (B_R^u)_{11}) + (Z+2N)(1/6 + (B_R^d)_{11}))\Big|^2 < 4.3 \times 10^{-12} \qquad (5.27)$$

where, for Ti: $Z = 22$, $N = 26$, $Z_{\rm eff} = 17.38$, $F_P = 0.54$ and $\Gamma_{\rm capt} = 1.73 \times 10^{-18}$ GeV. (See e.g. [27, 28, 29] for details.) In this case, the presence of w, which mildly depends on v_2, and of some diagonal B_R^u, B_R^d terms in (5.27) does not allow a general determination of a bound on $g_1'^2 |(B_R^e)_{12}|$ but we can get an estimate by setting $(B_R^u)_{11} = 2/3$, $(B_R^d)_{11} = -1/3$ (eq. (5.25) to leading order in r) and choosing $v_2 = 100$ GeV. For these values the bound is shown in the last line of table 5.1.

In the above, we assumed that the main sources of flavor violation were ultimately the standard Yukawa couplings. In a SUSY theory additional flavor violation may arise from the structure of the soft terms. We assumed throughout that they are subleading, as in the case of gauge mediated SUSY breaking.

6

Conclusion

In this thesis we presented a new class of supersymmetric models. This class of models was constructed on the basis of the MSSM by enlarging the Higgs sector, adding the "singlet" sector and adding a new $U(1)'$ gauge symmetry (all fields in the model are charged under $U(1)'$, some of the charges are family depend). We demanded that such extension is minimal (in a sense there are no new coloured fields), which allow to solve the μ-problem and justify the presence of R-parity (in our case R-parity arises as an accidental symmetry from the gauge symmetry).

Lacking the experimental data on the supersymmetric part of the model, we mainly focused our attention on the Electro-Weak breaking and the flavour violating processes (arising in the model due to the family non-universal charges). Another important issue is the analysis of the classical vacuum in the model. We presented analytic expression for the classical potential, but the problem of searching its vacuum is highly complicated (due to the presence of large number of unknown constants) and was postponed.

In future it might be interesting to search for minimum of the classical potential, setting some benchmark values for the constants. Following the bottom-up approach we did not require a manifest grand unification. The very interesting problem could be to find the grand unified version of the fully chiral models if it exists.

A

Appendix

A.1 Two Component Spinor Techniques and Feynman Diagrams

In two-component spinor notation there are four different types of propagators for the fermions, the details are given in [30]. They are given on the figure A.1. The external arrows correspond to momentum p of the propagating particle with mass m. The arrows on the propagator indicate the spinor structure. Their direction is chosen by convention; “spinor” arrows are going from dotted $\dot{\alpha}$ indices to undotted β. To remind the definition of dotted and undotted indices let us recall the form of the Lorentz group: $SU(2)_L \times SU(2)_R$. Left-handed fermions transform under $SU(2)_L$, while right handed fermions transform under $SU(2)_R$. To distinguish between two $SU(2)$ groups we denote spinor indices of left-handed fermions by α and spinor indices of right-handed fermions by $\dot{\alpha}$.

Figure A.1: Four types of fermionic propagators.

Scalars do not have a spinor structure, nevertheless it is convenient to introduce direction of analyticity flow. By convention the arrows are going from the field ϕ to complex conjugate field ϕ^*.

For computation of Feynman diagrams in two-component notation, it is also necessary to have expressions for traces of alternating products of σ and $\bar{\sigma}$. For our purposes it is enough to write down the following expressions:

$$Tr[\sigma^\mu \bar{\sigma}^\nu] = 2g^{\mu\nu}, \tag{A.1}$$
$$Tr[\sigma^\mu \bar{\sigma}^\nu \sigma^\rho \bar{\sigma}^\kappa] = 2\left(g^{\mu\nu}g^{\rho\kappa} - g^{\mu\rho}g^{\nu\kappa} + g^{\mu\kappa}g^{\nu\rho} + i\varepsilon^{\mu\nu\rho\kappa}\right), \tag{A.2}$$
$$Tr[\bar{\sigma}^\mu \sigma^\nu \bar{\sigma}^\rho \sigma^\kappa] = 2\left(g^{\mu\nu}g^{\rho\kappa} - g^{\mu\rho}g^{\nu\kappa} + g^{\mu\kappa}g^{\nu\rho} - i\varepsilon^{\mu\nu\rho\kappa}\right). \tag{A.3}$$

A.2 Radiative Corrections

To compute soft SUSY breaking masses from a mediation mechanism we need to study the radiative corrections of the theory. On the fig. A.2 we define an effective propagator which consist of an ordinary tree level propagator and a series of corrections, where blobs indicate all possible quantum effects which can occur inside the propagator. Performing particular calculations we will need to specify the structure of the blob, but for the current discussion we can abstract from its particular form. In QFT such blobs we call 1–particle–irreducible (1PI) insertion into the propagator and denote by $i\Pi^2(p)$, where p is momentum of propagating particle.

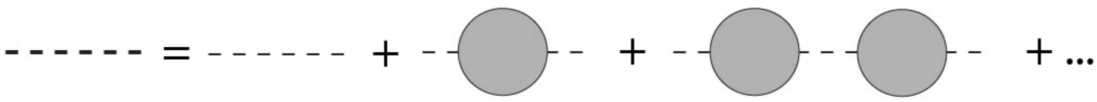


Figure A.2: 1PI insertion to the propagator of scalar particle.

We denote by $\tilde{D}_F$ - fourier transformed Feynaman propagator. According to fig. A.2, the effective propagator can be written as

$$\begin{aligned}\tilde{D}_{F\,eff} &= \frac{i}{p^2 - m^2 + \Pi^2(p)} = \frac{i}{p^2-m^2} + \frac{i}{p^2-m^2} i\Pi^2(p) \frac{i}{p^2-m^2} + \\ &\quad + \frac{i}{p^2-m^2} i\Pi^2(p) \frac{i}{p^2-m^2} i\Pi^2(p) \frac{i}{p^2-m^2} + \cdots = \\ &= \frac{i}{p^2-m^2}\left(1 - \frac{\Pi^2(p)}{p^2-m^2} + \left(\frac{\Pi^2(p)}{p^2-m^2}\right)^2 + \dots\right) = \frac{i}{p^2-m^2} \frac{1}{1+\frac{\Pi^2(p)}{p^2-m^2}},\end{aligned} \tag{A.4}$$

Suppose we can make an expansion around small momentum p of propagating particle:

$$\Pi^2(p) \approx \Pi^2(0) + p^2 \Pi'^2(0) + \dots. \tag{A.5}$$

Then the effective lagrangian gets the form:

$$\mathcal{L} \approx \frac{1}{2}(1 + \Pi'^2(0))\partial\phi\partial\phi - \frac{1}{2}(m^2 - \Pi^2(0))\phi^2. \tag{A.6}$$

Thus, $i\Pi'^2(0)$ can be interpreted as field renormalization and $i\Pi^2(0)$ as mass renormalization. For our application to the generating of soft SUSY breaking terms $\Pi^2(0)$ is finite. Than the effective mass of the scalar can be defined as follows:

$$m_{eff}^2 \approx m^2 - \Pi^2(0). \tag{A.7}$$

For the fermions we denote the blob by $i\Pi(p)$, where as usual p is a momentum of the propagating fermion; then effective fermionic mass gets the form:

$$m_{eff} \approx m - \Pi(0). \tag{A.8}$$

A.3 Gauge Anomalies

In this section we very briefly discuss triangle ABJ - anomalies. The pioneering work on this subject was made by Adler, Bell and Jackiw [31], [32].

According to the Noether's theorem there is always a current J^μ corresponding to a symmetry, which is conserved ($\partial_\mu J^\mu = 0$). At the quantum level it means:

$$\langle 0|T\{\partial_\mu J^\mu O_1 O_2 \dots\}|0\rangle = \partial_\mu \langle 0|T\{J^\mu O_1 O_2 \dots\}|0\rangle + (contact\ terms) = 0, \tag{A.9}$$

where T is time-ordering. If the current corresponding to the symmetry is not conserved, then the symmetry is called anomalous. This anomaly leads to an inconsistency (e.g. gives negative probabilities).

Suppose we have a theory with gauge group $SU(N)$ (with (N^2-1) generators T^a) and a set of left-handed fermions ψ_i coupled to the gauge group. At the classical level the associated conserved current to the symmetry is:

$$J^{\mu a} = \bar{\psi}\gamma^\mu T^a \psi. \tag{A.10}$$

It turns out that in $D = 4$ at the quantum level the "dangerous" terms possibly violating (A.9) have the following structure:

$$\partial_\mu \langle 0|T\{J^{\mu a} J^{\nu b} J^{\rho c}\}|0\rangle = ? \tag{A.11}$$

In diagrammatic representation the expression (A.11) is given by two diagrams of the type shown at the figure A.3, where gauge currents coupled to the three vertices and all fermions of the theory coupled to gauge group going in to the loop.

After performing general calculation of this diagrams we get the expression (A.11) to be zero only if the following condition is fulfilled:

$$\sum_\psi Tr(T^a\{T^b, T^c\}) = 0. \tag{A.12}$$

For semisimple gauge groups like in the Standard Model ($SU(3) \otimes SU(2) \otimes U(1)$) expression (A.12) is still valid, but we are also forced to check triangle diagrams with gauge currents associated to $SU(3)SU(3)SU(2)$, $SU(3)SU(2)U(1)$ symmetries etc. We

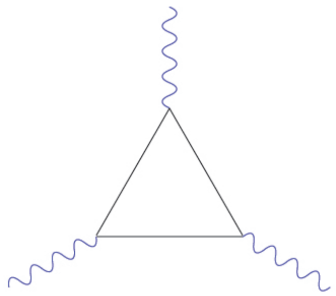


Figure A.3: Triangle diagrams

can notice that the currents associated to non-Abelian groups should be present two times to give non-trivial conditions. For example if τ^a- generators of SU(2) and T^a - generators of SU(3):

$$SU(2)SU(3)SU(3) \to \underbrace{Tr(\tau^a)}_{=0} Tr(\{T^b, T^c\}) = 0, \tag{A.13}$$

because all generators are traceless.

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