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Abstract—**In the previous measurement uncertainty work by Kildal, et al. (2012), the random line-of-sight (LOS) component in a reverberation chamber (RC) was assumed to be Gaussian distributed without quantitative justification. In the present paper, we apply goodness-of-fit (GOF) test to the measured LOS component samples to check its Gaussianity. It is shown that the Gaussianity hypothesis of the distribution of the random LOS component is accepted at most of the frequencies, which justifies the Gaussian LOS component assumption.**

Index Terms—goodness-of-fit (GOF); line-of-sight (LOS); reveration chamber (RC)

I. Introduction

Reverberation chambers (RCs) has been used for electromagnetic compatibility (EMC) tests as well as over-the-air (OTA) measurements of wireless devices [1]-[12]. Due to the complicated test conditions (e.g. inhomogeneous test objects, irregular mode stirrers, changing boundary conditions, etc), RC measurements are usually studied from a statistical point of view. It is well known that the magnitude of the electromagnetic field inside a well-stirred RC is Rayleigh distributed [1]-[3] and that in the presence of an unstirred component it is Rician distributed [4]-[7]. In other words, the complex field is Gaussian distributed. Based on this and the assumption that the random line-of-sight (LOS) component (which is equivalent to the unstirred component [6]) is also Gaussian distributed, a RC measurement uncertainty formula was derived in [8]. Although the uncertainty formula was verified by extensive measurements, the Gaussianity assumption of the random LOS component has not been verified directly. Thus, the purpose of this work is to verify this assumption using the goodness-of-fit (GOF) test.

The GOF test is basically a special version of the hypothesis test [13]. The general procedure for testing a null hypothesis H_0 is to partition the sample space into a rejection region and an acceptance region based on a test statistic. Usually a significance level is chosen to ensure a small probability that the true H_0 is rejected. A good GOF test should minimize the probability that a false H_0 is accepted for a given significance level. It is powerful for examining a specific distribution and therefore it has been used to verify the distribution of the stirred field in the RC [14], [15]. However, to the best knowledge of the authors, a GOF test of the unstirred component does not exist in the literature so far. Hence, in this work, we use it to check if the random LOS component is Gaussian distributed as well.

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II. GOF Test

The Kolmogorov-Smirnov (KS) GOF test and the Anderson-Darling (AD) GOF test are probably two of the most common GOF tests [13]. The AD GOF test makes use of the specific distribution in calculating the critical values. Therefore, it is more sensitive (powerful) than the KS GOF test. The drawback of the AD GOF test is that the critical values must be calculated for each distribution and therefore it is only available for a few specific distributions. Fortunately, Gaussian distribution is among these distributions. Therefore, the AD GOF test is chosen in this work.

In this work, the null hypothesis H_0 is that "the random LOS component in a RC is Gaussian distributed." The AD statistic is [13]

$$AD = -\frac{\sum_{n=1}^{N}(2n-1)\left[\ln F\left(x_n\right)+\ln\left(1-F\left(x_{N+1-n}\right)\right)\right]}{N} - N \quad (1)$$

where N denotes the sample number, x_n is the nth realization of the random LOS component in ascending order, ln is the natural logarithm, and F represents the Gaussian cumulative distribution function (CDF), i.e.

$$F\left(x\right) = \frac{1}{2}\left[1 + erf\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)\right] \quad (2)$$

where μ and σ represents the mean and standard deviation (STD) of the random LOS component, respectively, and the error function *erf* can be expressed as

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt. \quad (3)$$

Note that in a GOF test μ and σ needs to be estimated using the maximum likelihood estimator [16], i.e.,

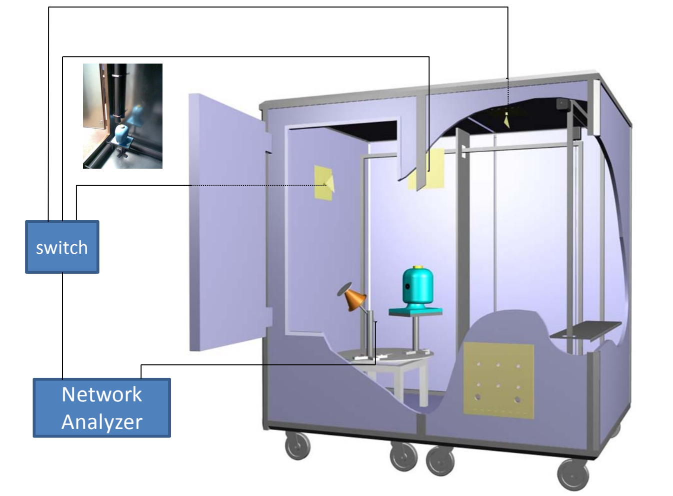


Fig. 1. Drawing of Bluetest HP RC with two mechanical plate stirrers, one platform, and three wall antennas.

$$\hat{\mu} = \frac{1}{N}\sum_{n=1}^{N} x_n,$$
$$\hat{\sigma} = \sqrt{\frac{1}{2N}\sum_{n=1}^{N}|x_n|^2}. \quad (4)$$

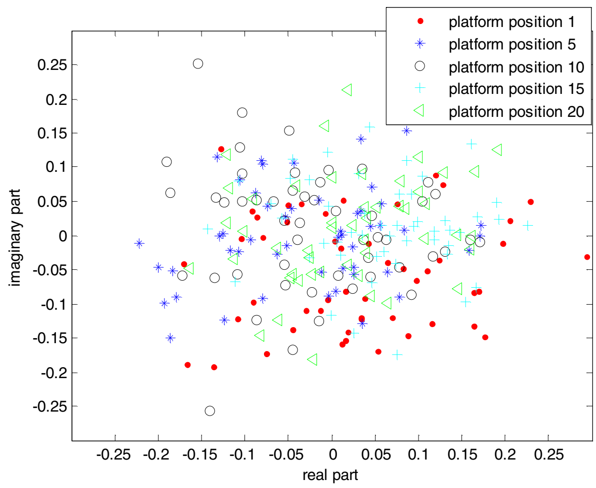
Normally a correction term is multiplied to the AD statistic to take into account of the finite sample number, i.e.

$$ADC = AD(1+\frac{0.75}{N}+\frac{2.25}{N^2}). \quad (5)$$

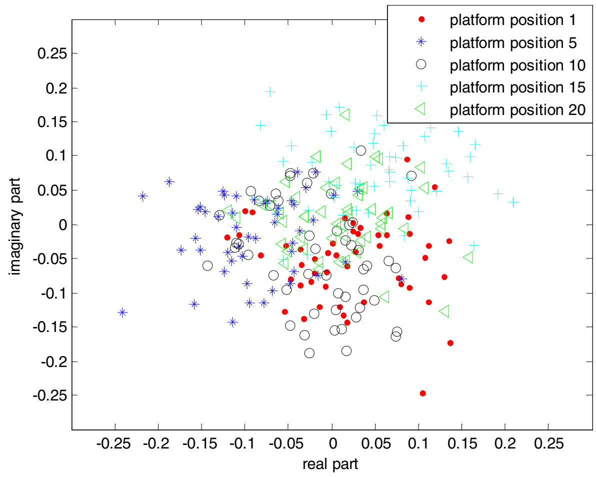
Note that this correction term is different for a different distribution. The rejection significance level P is simply the cumulative CDF of the AD statistics. The calculated rejection significance level P is then compared with the level of significance α. If $P < \alpha$, H_0 is rejected; otherwise it is accepted. It is shown that a good choice of α is 0.05 [13].

III. MEASUREMENT AND RESULT

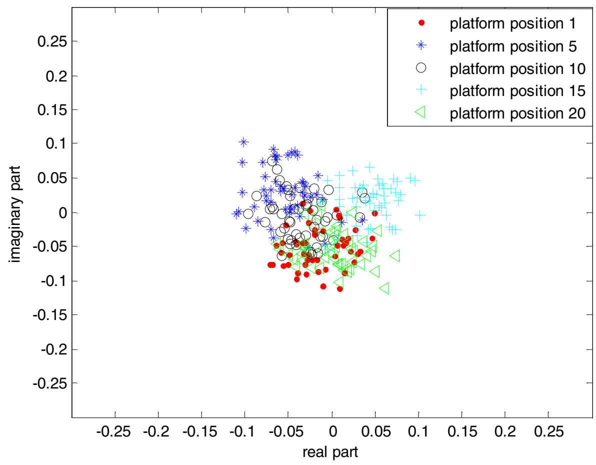
Measurements were performed from 500 to 2000 MHz in Bluetest HP RC with a size of 1.80 × 1.75 × 1.25 m^3 (a drawing of which is shown in Fig. 1). It has two plate mode-stirrers, a turn-table platform (on which a wideband discone antenna is mounted), and three antennas mounted on three orthogonal walls (referred to as wall antennas hereafter). The wall antennas are actually wideband half-bow-tie antennas. The measurement setup (or stirring sequence) of the RC is chosen such that: The turn-table platform was step-wisely moved to 20 platform positions equally spaced by 18°; at each platform position the two plates were simultaneously and step-wisely moved to 50 positions (equally spanned on the total distances that they can travel). At each stirrer position and for each wall antenna a full frequency sweep was performed by the VNA with a frequency step of 1 MHz, during which the channel are sampled as a function of frequency and stirrer position.



(a)

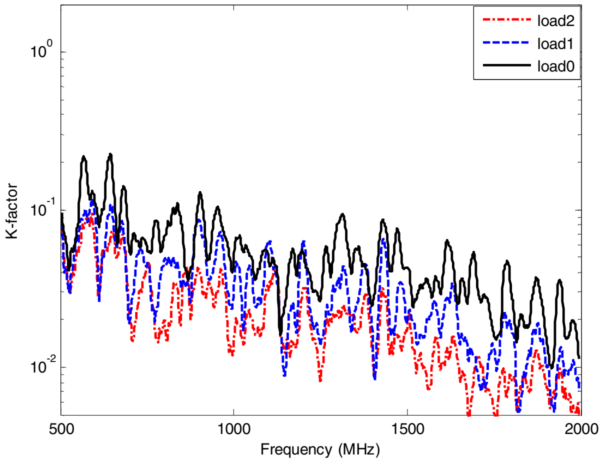


(b)

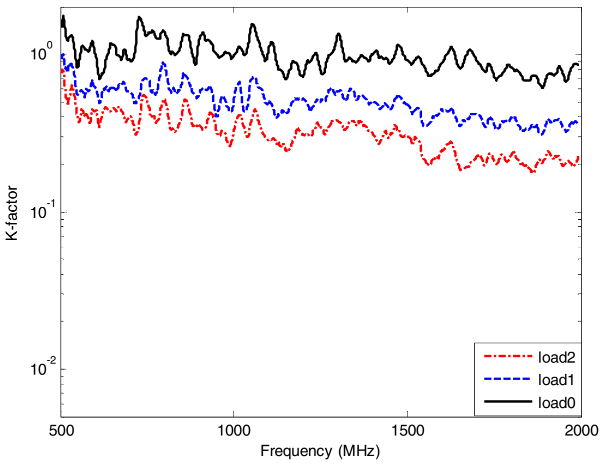


(c)

Fig. 2. Scatter plots of the measured channel transfer function at 1 GHz for 5 different platform positions under *load0* (a), *load1* (b) and *load2* (c).

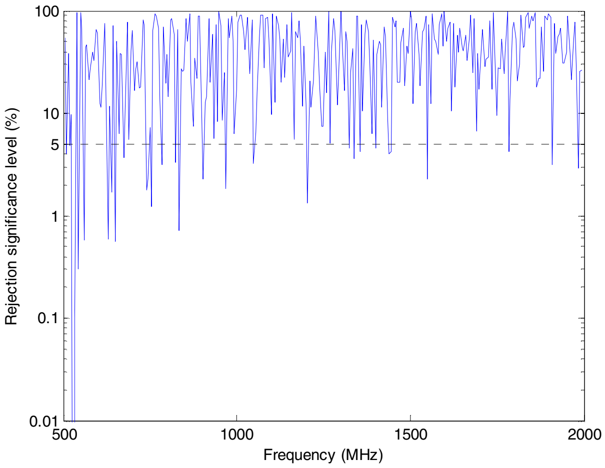


(a)

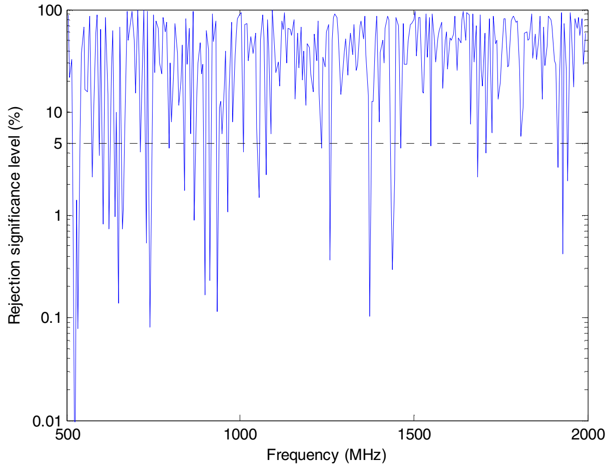


(b)

Fig. 3. Measured K-factors: (a) channel mean calculated over all plate and platform positions; (b) channel mean calculated over all plate positions and the calculated K-factors are averaged over all the platform positions.



(a)



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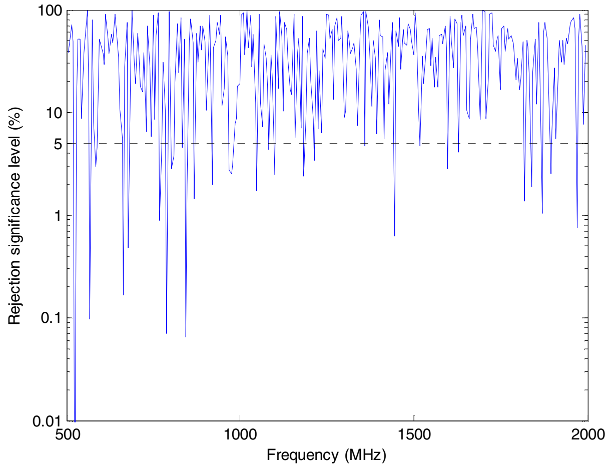

(c)

Fig. 4. The rejection significance level obtained by applying the AD GOF test to the LOS samples for *load0* (a), *load1* (b) and *load2* (c).

The same measurement procedure was repeated for three loading conditions: *load0* (unloaded RC), *load1* (head phantom that is equivalent to a human head in terms of microwave absorption), and *load2* (the head phantom plus three Polyvinyl Chloride (PVC) cylinders filled with electromagnetic absorbers cut in small pieces). The inserted photo in the upper left corner of Fig. 1 shows the *load2* configuration. Hereafter measured data from these different loading configurations are simply referred to as *load0*, *load1*, or *load2* data.

The LOS component can be calculated from the measured channel transfer functions by taking a sample mean of them over the 50 plate positions at each platform position and for each wall antenna. Fig. 2 shows the scatter plot of the measured channel transfer function at 1 GHz under different loading configurations. Different color (and shape) represents different platform positions. For the same group of scatters with the same color (and shape), each individual scatter denotes a distinct plate position. It can be seen that, on one hand, by loading the chamber, the average power transfer function reduces (i.e., the scatter plot shrink toward the origin), on the other hand, the K-factor [6] increases (i.e., the relative separation between different scatter groups increases). Fig. 3 shows the calculated K-factor as a function of frequency for all loading configurations. From the previous scatter plots, it can be seen that the calculated K-factor value depends on whether distinguishing platform positions or not. When calculating the K-factor over all plate and platform positions, the resulting K-factor value is rather small, because the LOS component then is stirred by the platform stirring. The resulting K-factor is shown in Fig. 3a. If we calculate the K-factor at different platform positions by averaging over the plate positions only, the resulting K-factor will be much larger, as shown in Fig. 3b. A detailed discussion of K-factor calculations can be found in [7] (based on a different measurement setup). Nevertheless, it can be shown that by loading the RC, the K-factor increases.

The obtained LOS samples are stacked into a column vector, denoted as $\mathbf{x}$. As a result, we have 60 (20 × 3) LOS samples at each frequency. Note that since the LOS samples need be sorted in ascending order for the GOF test (cf. Sec. II), the complex vector $\mathbf{x}$ has to be converted into real vector with double length, i.e. $[\mathrm{Re}(\mathbf{x})\ \mathrm{Im}(\mathbf{x})]^T$ before further processing, where Re and Im represent the real and imaginary parts of the argument, respectively. It is easy to show that the statistics of these two vectors are the same [17]. Therefore there is no loss of information in this vector conversion. We apply the AD GOF test to the measured LOS sample vector. Fig. 4 shows the rejection significance level P of the measured LOS samples under different loading configurations. It is shown that the null hypothesis (i.e. the random LOS component in a RC is Gaussian distributed.) is accepted at most of the frequencies, especially in the higher frequencies where the RC is better stirred, almost regardless of the loading configurations. Therefore, it is sensible to assume the random LOS component in the RC to be Gaussian distributed (to be exact, circular symmetric Gaussian distributed [17]). Note that [15] shows that the AD GOF test rejects not only stirred fields in the low frequency range, but also in the high frequency range. The observation here is in agreement with that in [15] in this sense, even though it is the random LOS component (not the stirred field) that is tested here.

IV. Conclusion

In this paper, we applied the powerful AD GOF test to measured LOS samples in a RC. It is shown that the Gaussianity of the random LOS component is accepted at most of the frequencies. The result from this work verified the critical assumption in the RC measurement uncertainty model in [8].

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