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A conceptual model for energy, trade and economy

Master of Science Thesis in the
Complex Adaptive Systems Programme

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Department of Energy and Environment
Division of Physical Resource Theory
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Abstract

This thesis focuses on introducing and incorporating an explicit model for energy trade into a conceptual model for economy and energy. The inclusion of an energy market into the model where countries can buy and/or sell energy to a market price is something that was not done in the model that was used as a starting point for this study. In general, energy systems models use less explicit mechanisms to determine trade, like e.g. shadow prices from optimisation.

This kind of trading model is successfully developed and introduced into a more complex, dynamic model for economy and energy. The trading mechanism is analysed extensively in itself before introducing it into the more complex model. This to ensure that it exhibits the behaviours that one would expect to observe on a market.

The effects of introducing energy trade into such a more complex, dynamic model for economy and energy is then explored and it is shown that when at least one country is sitting on large enough fossil fuel assets, the possibility of energy trade will inhibit the development of renewable energy for all countries.

Moreover, the possibilities of avoiding finite time effects in model simulation by modifying the utility function that is subject to optimisation is studied. It is shown that it is possible to find a utility function that in many aspects avoid the finite time effects.

Keywords: energy, economy, trade, discrete, model, simulation

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1

Introduction

To understand the mechanisms of society is of utmost importance in order for us to make good decisions for the future. Therefore, it is valuable to create models that, as realistically as possible, captures the essence of the mechanisms that we want to study.

This thesis will focus on how different mechanisms linked to economy, energy and availability of resources interact with each other. Given the current situation in the world, with the energy supply heavily depending on fossil fuels, it is important to investigate how society can avoid damages on the economy as a result of climate change and dwindling supplies of fossil fuels. If we have a greater knowledge about the mechanisms coupled to these issues we have a better chance of creating a sustainable development for the future.

The ultimate goal of this thesis is to introduce a mechanism for modelling energy trade and incorporate it into a conceptual model for energy and economy that is a modified version of the model used by Brede and de Vries [4].

1.1 Background

Several models have allready been developed to study mechanisms linked to economy, energy and finite resources. In this thesis, the model from [4] has been used as a starting point although it has been modified and new mechanisms has been added.

Another closely related model is SUSCLIME [5] that describes the dynamics of, and the interactions between population, economy, energy and climate.

The model by Brede and de Vries [4] is similar to SUSCLIME in many aspects but both models are missing a way of modelling energy trade on a market and a way to set the energy price explicitly on such a market. Introducing and incorporating such a model for energy trade into a modified version of [4] is the main focus of this thesis. The main motivation for introducing an energy market is to account for the fact that, in case of a global energy shortage, countries sitting on abundant deposits of fossil fuels might not want to make these resources availible to other countries.

1.2 Objectives

In this thesis we want to introduce a trading mechanism that models energy trade between countries and study it isolated from other economic aspects. When extensively studied, we want to incorporate the trading mechanism into a more complex, dynamic economic model which will be a modified, discrete version of the model used by Brede and de Vries [4]. What we want to show is that the incorporation of a trading mechanism can have great effects on the qualitative behavior of the economies in the model.

The possibility of handling the effects of a finite optimisation period is also studied. A finite optimisation period in a dynamic economic model such as [4] gives rise to the problem that countries do not take wellfare after the optimisation period into consideration. Therefore, they stop all investments at the end of the optimisation period and turn all of their production into capital that is made availible for consumption. We want to study if it is possible to avoid this problem by modifying the utility function that is subject to the optimisation.

1.3 Thesis structure

The rest of this thesis is structured as follows. In chapter 2 we start by introducing a simple model for energy trade. This model is then thoroughly investigated throughout the rest of the chapter. Chapter 3 focuses on describing the dynamic economic model that is a discrete extension of the model used by Brede and de Vries [4]. In chapter 4 the concept of a utility function is described and strategies to avoid the effects of a finite optimisation period are examined. After that, in chapter 5, some model simulations are done and the effects of the energy trade in the model are discussed. Last, in chapter 6, the results are discussed, conclusions are presented and ideas for future work are presented.

2

Energy trade

In this chapter we will introduce a way of modelling a market where countries or regions can trade energy with each other. The model that will be introduced does not specifically demand that the traded commodity is energy. In this thesis we will however, only use the model in the context of energy trade as the goal of this thesis is to incorporate a model for energy trade in a conceptual model for economy and energy. The trade model will describe a market where a number of countries or regions have the possibility to export and/or import energy to a certain price that is the same for all players in the market. This price will be a direct product of both the total energy demand and the total energy supply on the market. First we will start in section 2.1 by deriving a price that is set by the total demand- and supply of energy. In section 2.2 we will analyse the price setting mechanism and present an example to get some intuition about how it works. Then, in sections 2.3 and 2.4, we will study how the behavior of countries and regions on the market give rise to certain dynamics in the price setting mechanism. Last, we will study a special case in section 2.5 where a monopoly situation arise as a product of the price setting mechanism and the behavior of the actors on the market.

2.1 Energy market price

In this section we will describe how the energy price is set on a market by the total supply and demand for energy. We will start from very simple assumptions and from these assumptions formulate an expression that gives us the price explicitly.

In this chapter we will only consider how the market price is set during a period of time under which the price, along with the supply and demand of energy, is constant. Let us consider a case when the trade occurs annually so that each country or region has both a demand for- and a supply of energy for the year as a whole and decides how much energy it will import and/or export during that year.

Moreover, let us assume that each country or region i has a net import B_i of energy over the year. Then, during this year, the net import $B_i = M_i - X_i$, where M_i is the amount of imported energy and X_i is the amount of exported energy for country i.

Furthermore we assume that there is no energy leakage in the trade so that:

$$\sum_{i=1}^{N} B_i = 0$$

Where N is the total number of countries.

This gives us:

$$\sum_{i=1}^{N} M_i - X_i = 0 \Rightarrow \sum_{i=1}^{N} M_i = \sum_{i=1}^{N} X_i$$

We can factor out the price (ρ) from M_i in the following way:

$$\sum_{i=1}^{N} \frac{\rho M_i}{\rho} = \sum_{i=1}^{N} X_i \Rightarrow \frac{1}{\rho} \sum_{i=1}^{N} \rho M_i = \sum_{i=1}^{N} X_i$$

And we get:

$$\rho = \frac{\sum_{i=1}^{N} \rho M_i}{\sum_{i=1}^{N} X_i}$$

If we assume that the import $M_i = \frac{(\sigma_M)_i Y_i}{\rho}$, where Y_i is the total goods production, or capital, of country i and $(\sigma_M)_i$ is the fraction of its total goods production the country uses to import energy, we get:

$$\rho = \frac{\sum_{i=1}^{N} (\sigma_M)_i Y_i}{\sum_{i=1}^{N} X_i}$$

Thus, the market price for energy can be determined for each year. If we furthermore assume that the exported energy of country i as $X_i = (\sigma_X)_i (E_{\text{produced}})_i$, where $(\sigma_X)_i$ is the fraction of energy to be exported and $(E_{\text{produced}})_i$ is the total amount of energy produced by country i, we get:

$$\rho = \frac{\sum_{i=1}^{N} (\sigma_M)_i Y_i}{\sum_{i=1}^{N} (\sigma_X)_i (E_{\text{produced}})_i} \tag{2.1}$$

2.2 Analysis of the price setting mechanism

To get some intuition about how the price setting mechanism in equation 2.1 works let us study a special case, where we only have two countries trading energy on the market. Then equation 2.1 is reduced to:

$$\rho = \frac{(\sigma_M)_1 Y_1 + (\sigma_M)_2 Y_2}{(\sigma_X)_1 (E_{\text{produced}})_1 + (\sigma_X)_2 (E_{\text{produced}})_2} \tag{2.2}$$

If we assume that both countries put all their energy production out on the market so that $(\sigma_X)_1 = (\sigma_X)_2 = 1$ equation 2.2 is reduced to:

$$\rho = \frac{(\sigma_M)_1 Y_1 + (\sigma_M)_2 Y_2}{(E_{\text{produced}})_1 + (E_{\text{produced}})_2} \tag{2.3}$$

From equation 2.3 we can see that the price will be given by a plane. The incline in the different directions will be given by Y_1 respective Y_2. The incline will also depend on the total energy flow into the market. Generally we can say two things about the price dynamics. First, the more money that goes into the market the higher the price will be and second, the more energy that flows into the market, the lower the price will be.

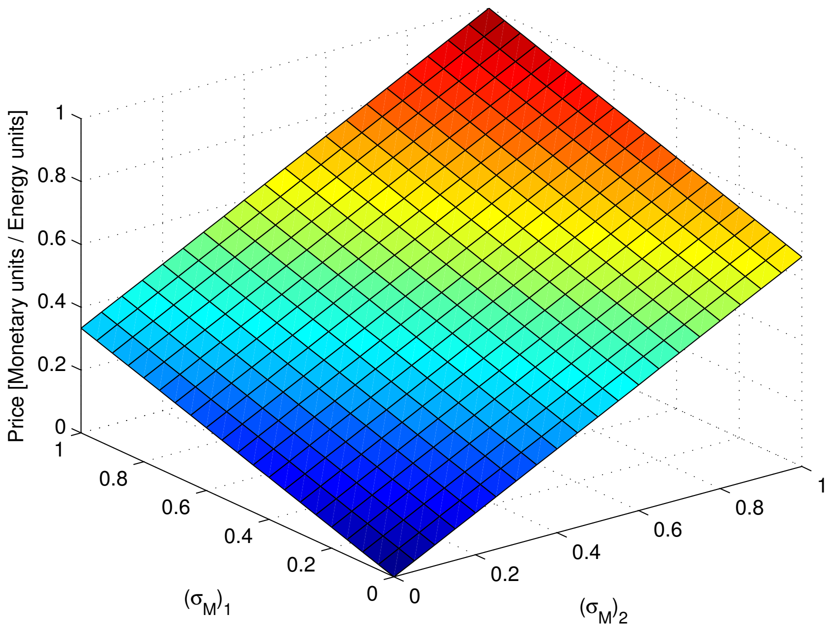


Figure 2.1: The price as a function of $(\sigma_M)_1$ and $(\sigma_M)_2$.

If we let for example $Y_1 = 500$, $Y_2 = 1000$, $(E_{\text{Produced}})_1 = 1000$ and $(E_{\text{Produced}})_2 = 500$ along with $(\sigma_X)_1 = (\sigma_X)_2 = 1$ we get the price as a function of $(\sigma_M)_1$ and $(\sigma_M)_2$. This function is, as we discussed earlier, a plane and it can be seen in figure 2.1.

If we go back to equation 2.2 we can see that if we lower $(\sigma_X)_1$ and/or $(\sigma_X)_2$ we get a lower energy flow into the market and correspondingly the incline of the plane in figure 2.1 will increase in both directions.

This reasoning can be extended to the general case when we have N countries and the price is given by equation 2.1. The price will then lie on a hyper plane in N dimensions if we consider only $(\sigma_M)_i$ as variables. The incline in direction i will be given by Y_i and the total flow of energy into the market.

2.3 Price dynamics

It is important to note that equation 2.1 does not determine the price in itself. It merely tells us that given all countries' decisions, there is a certain price that comes from the condition that there is a balance between inflow and outflow of energy on the market. The decisions are governed by a completely different mechanism.

To study how the price is set when we have many players on the market we can assume that each country or region is trying to optimise a utility function $U_i((\sigma_M)_i,(\sigma_X)_i)$ given its energy production $(E_{\text{produced}})_i$, energy demand $(E_{\text{demand}})_i$ and capital Y_i over a fixed period of time. Here, we will assume that each country or region is given a certain energy production and capital for a one year period which it can use to fill its energy demand over that same period. Let us study a very simple utility function, given by:

$$\begin{aligned}\text{Utility} = {} & (\text{Money left after trade})\cdot \\ & (\text{The amount of the energy demand filled by the energy supply})\end{aligned}$$

In mathematical terms, the utility is:

$$U_i = \Big[(1-(\sigma_M)_i)Y_i + \rho\cdot(\sigma_X)_i(E_{\text{produced}})_i\Big]\cdot\min\Big(1, \frac{(E_{\text{supply}})_i}{(E_{\text{demand}})_i}\Big) \tag{2.4}$$

With:

$$(E_{\text{supply}})_i = (1-(\sigma_X)_i)\cdot(E_{\text{produced}})_i + \frac{(\sigma_M)_i Y_i}{\rho}$$

So each country tries to maximise the amount of money it has left after the trade at the same time as it tries to fill its energy demand. Consequently, it is an optimisation problem for each country to maximise its utility by choosing favorable values for $(\sigma_M)_i$ and $(\sigma_X)_i$. Let us view it as a negotiation process where each country tries to maximise its utility given the decisions of all other countries in the previous negotiation step. The negotiation process is iterated in this way until an equilibrium is found, where no country can find a way to change its decisions to further increase its utility. The negotiation has then reached a non-cooperative Nash equilibrium. The real price is given by the end of this negotiation process. A negotiation process is said to converge if the maximal change in the price between one iteration to next has not exceeded 10^{-4} for 10 iterations. It is, however, not obvious that the negotiation process allways will converge. It could result in some kind of cycles or chaos. Therefore, we also look for cycles up to length $n = 1000$ iterations and assume that anything else is chaos.

Let us now simulate this negotiation process. First, we need to determine what the parameters of the utility function should be, i.e what each country's energy production $(E_{\text{produced}})_i$, energy demand $(E_{\text{demand}})_i$ and capital Y_i should be. Since there is real world data availible for these kinds of properties the most reasonable thing to do seems to be to use that data in our model. Using data from the international energy agency [1], we now study the negotiation process.

2.3.1 Regions

First, let us consider a simplified world where we have a few different regions that can trade with each other. The data for the different regions is given in table 2.1.

Table 2.1: Data for different regions taken from the international energy agency [1].

Region	GDP [billions of 2000 USD]	Energy production [Mtoe]	Energy demand [Mtoe]
OECD	32114	3807	5451
Middle East	1433	1561	610
Non-OECD Europe and Eurasia	2835	1645	1065
China	12434	2085	2390
Asia	9094	1310	1513
Latin America	3769	751	563
Africa	2565	1133	681

Consequently, we have seven different regions with different amounts of capital, different energy demands and different energy productions. Let us now study the negotiation process along with the price realisation during the negotiation.

First, let us consider a case where all regions are bound to put their produced energy out on the market so that all $(\sigma_X)_i = 1$. The realisation of such a constrained negotiation process can be seen in figure 2.2 with the corresponding energy price during the negotiation in figure 2.3.

If we now take away the constraint that regions are bound to put out all their energy on the market, we get a scenario where a region can choose to keep a

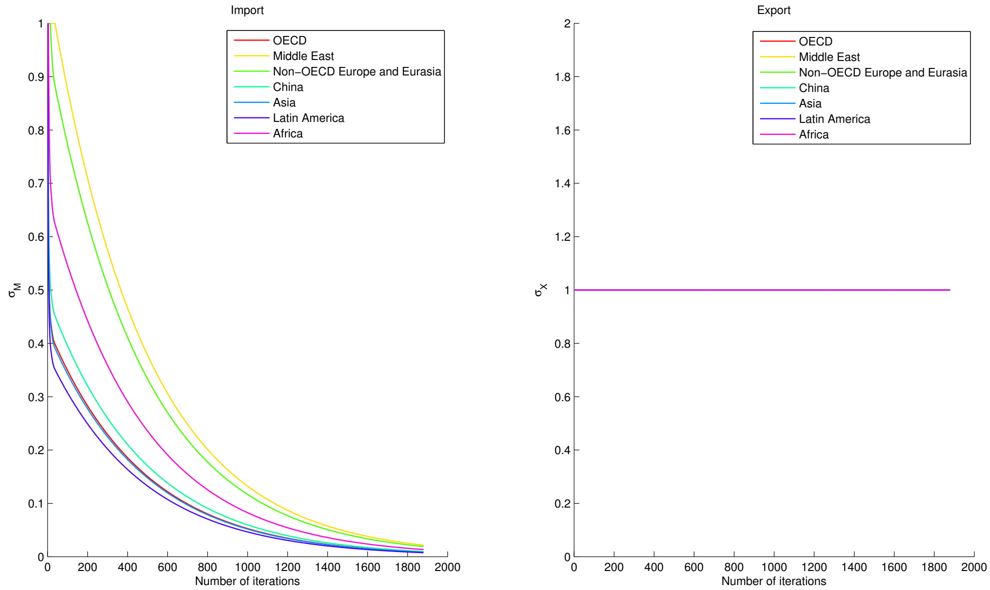


Figure 2.2: The realisation of a negotiation process for seven different regions where all countries are bound to put out all of their energy on the market.

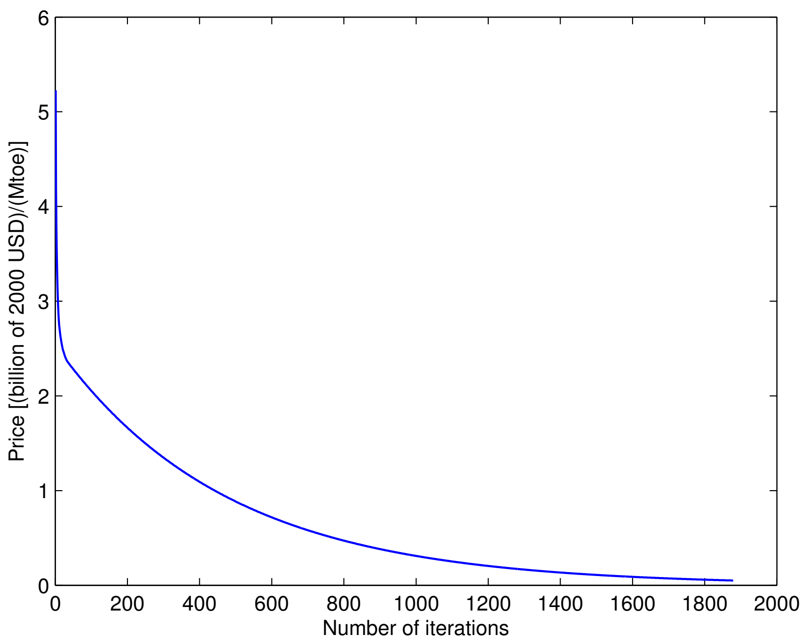


Figure 2.3: The price during the negotiation process in figure 2.2.

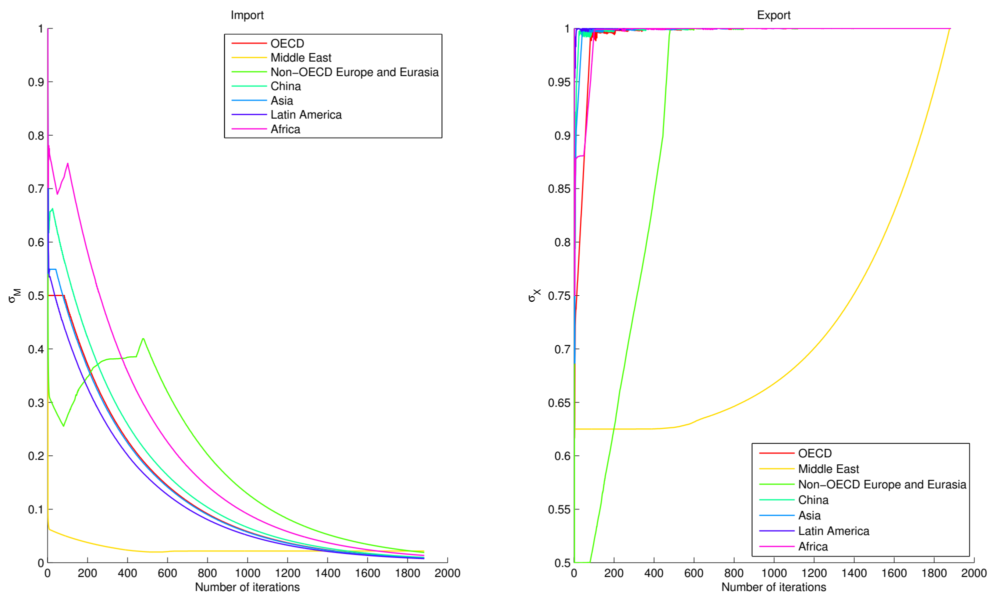


Figure 2.4: The realisation of a negotiation process for seven different regions where countries are free to refrain from putting all of their energy out on the market.

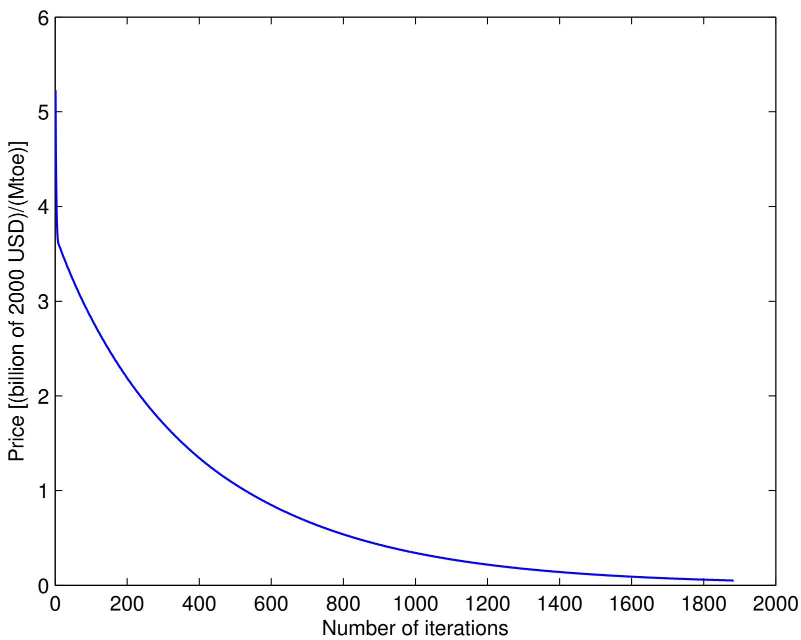


Figure 2.5: The price during the negotiation process in figure 2.4.

certain amount of its own energy production without letting it get priced on the market. This will result in a different negotiation process. The realisation of this unconstrained negotiation can be seen in figure 2.4 with the corresponding price in figure 2.5. By taking a look at figure 2.4 we can see that there are a number of peaks in the import decisions. Such a peak occurs if the energy price is low enough for a region to fill its energy demand, but high enough for the region to profit from exporting more energy. The region will then increase its export, which will lower the energy price. At the same time it must fill its energy demand, so the amount of money spent on import must be increased. Later in negotiation process, however, the price on energy will have decreased due to the other regions lowering their import investments and the region will then be able to fill its energy demand with its current import investment. After this, the region will follow the other regions on decreasing its import investment, lowering the energy price, so that it can get the same amount of energy for less money.

Let us now see what happens to the price at the end of the negotiation process if we change the total amount of energy that is produced. We can modify the total amount of energy produced by multiplying each region's energy production by a factor μ which we can call the availability of energy. If we have a low availability of energy, i.e a low value of μ, there will not be enough energy produced in the world to cover the total energy demand, which will lead to a higher energy price. If we, on the other hand, have a high availability of energy, there will be too much energy produced in the world, which will lead to a lower energy price.

The price as a function of the availability μ for both the constrained and the unconstrained case can be seen in figure 2.6. We can see that for low values of μ, the price is very high. If we increase μ, the price is decreasing succesively until we approach the case when $\mu = 1$. If we examine the data we can see that for the original case corresponding to $\mu = 1$, the total energy production is slightly higher than the total energy demand:

$$\text{Total energy production} = 3807{+}1561{+}1645{+}2085{+}1310{+}751{+}1133 = 12292 \text{ Mtoe}$$

$$\text{Total energy demand} = 5451+610+1065+2390+1513+563+681 = 12273 \text{ Mtoe}$$

$$\Rightarrow \text{Total energy production} - \text{Total energy demand} = 19 \text{ Mtoe}$$

There is accordingly an over production of 19 Mtoe in the world. Thus, the availability corresponding to an exactly filled energy demand is

$$\mu_{\text{Saturated}} = \frac{\text{Total energy demand}}{\text{Total energy production}} = \frac{12273}{12292} \approx 0.998$$

From figure 2.6 we can see that at approximately $\mu_{\text{Saturated}}$ we get a phase shift in the price where the price jumps to zero for $\mu > \mu_{\text{Saturated}}$. This jump is due

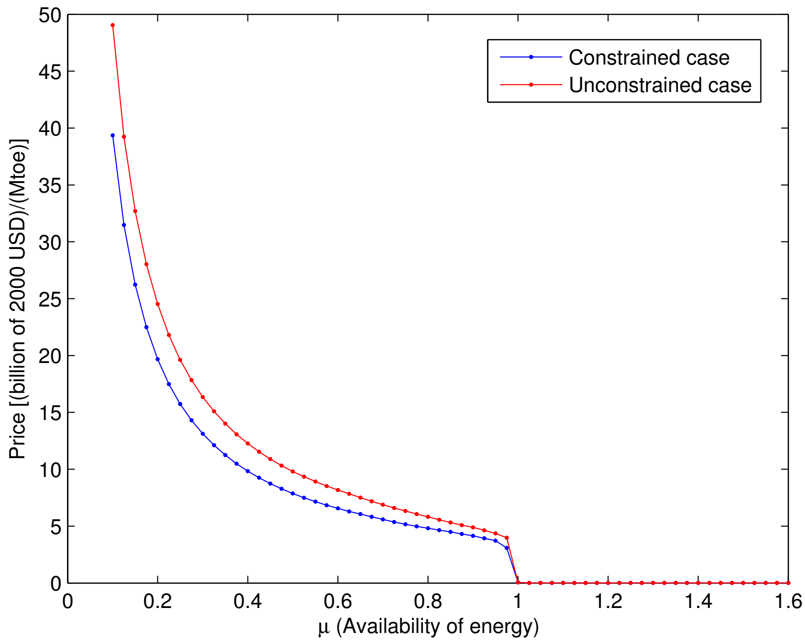


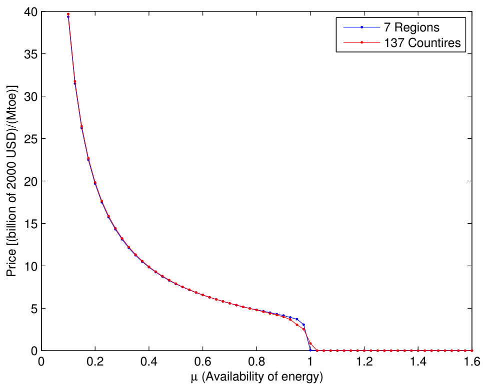
Figure 2.6: The price after the negotiation process as a function of the availability of energy μ for both the constrained and the unconstrained case.

to the definition of the utility function. The simple utility function that we have used here does not take into consideration the cost to produce the energy and thus the leftover energy is priced only by the total energy demand. If there is more energy availible than the total energy demand, it is possible for a region to lower the energy price by decreasing the amount of money it uses to buy energy. Thus it can get the same amount of energy for a slightly lower price. Other regions will then mimic this behavior and the price will fall to zero so that all regions get the energy they need for free. In a realistic model, where each region is able to control its energy production, this would not happen since if there was too much energy availible on the market it would not be profitable to produce energy since you could import it at a very low price. Such mechanisms will be taken into account in section 2.4.

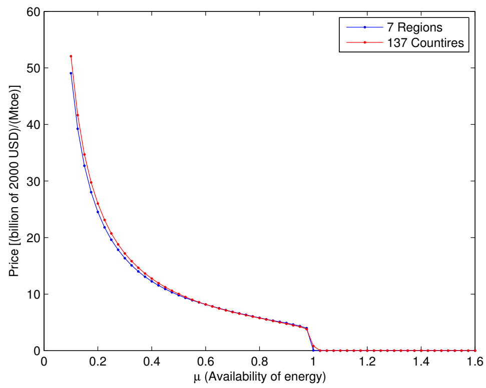
We can also note that in the case where we do not constrain $(\sigma_X)_i$ the curve is very similar to the constrained case. However, for values of $\mu < \mu_{\text{Saturated}}$ the price is higher than in the constrained case. This, due to the fact that regions with a surplus of energy can choose to keep some of their produced energy for themselves, thus lowering the inflow of energy on the market leading to an increased energy price. However, if such a region decreases its energy export too much, there may be other regions that are willing to export energy at the current price, thus increasing the energy price again. Accordingly, if we do not have a monopoly situation, the price will stabalize at a level given by the total demand- and supply of energy.

2.3.2 Countries

The assumption that there are seven regions acting on the market is of course a simplification. In the real world each country is a player in the market and is responsible for supplying itself with both money and energy. Instead of using data for the seven regions that we used earlier, we will therefore use data for 137 different countries. Again, we use data from [1]. This will, most likely, give us a more realistic idea of how the price is governed by the availability of energy. We again multiply each country's energy production with an availability factor μ and study how the price at the end of a negotiation process depends on this availability.



(a) All $(\sigma_X)_i = 1$



(b) All $(\sigma_X)_i$ are $0 \leq (\sigma_X)_i \leq 1$

Figure 2.7: The price after the negotiation process as a function of the availability of energy μ for both the case with seven regions and the case with 137 countries.

First, we constrain all $(\sigma_X)_i = 1$. Then the price as a function of μ can be seen in figure 2.7a for both the case with seven regions and the case with 137 countries. If we do not constrain $(\sigma_X)_i$, we instead get the curves seen in figure 2.7b. Again, we plot the case with the seven regions along with the case with 137 countries. If we examine the data, it turns out that in the original case, when $\mu = 1$, the total energy production is smaller than the total energy demand:

$$\text{Total energy production} = 12183 \text{ Mtoe}$$

$$\text{Total energy demand} = 12185 \text{ Mtoe}$$

$$\Rightarrow \text{Total energy production} - \text{Total energy demand} = -2 \text{ Mtoe}$$

There is accordingly an under production of 2 Mtoe in the world. Thus, the availability corresponding to an exactly filled energy demand is

$$\mu_{\text{Saturated}} = \frac{\text{Total energy demand}}{\text{Total energy production}} = \frac{12185}{12183} \approx 1.0001$$

Accordingly, the price should go to zero shortly after $\mu = 1$ in this case, which indeed seems to be the case in figure 2.7a and figure 2.7b. Apart from this, we can see that in both figure 2.7a and figure 2.7b the case with seven regions is very close to the case with 137 countries. This indicates that the number of players on the market does not have a great effect on the price at the end of the negotiation process. It is rather the availability of energy that is important.

It might seem strange that in the case with the seven regions we have an over production of energy in the world by 19 Mtoe while in the case with the 137 countries we have an under production of 2 Mtoe. We can also note that both the total energy demand and the total energy production differs between the two cases. This can be explained by the fact that the data only includes 137 countries, which is not all of the countries in the world. In the case with the seven regions, however, more countries are probably included in the data.

2.4 Price dynamics with a production cost

The scenario in the previous section is, as mentioned earlier, unrealistic in the sense that, in the real world, a country or region is not given an energy production for free, but must pay for the energy that it produces. Let us now instead see what happens if each player in the market can control its energy production.

Let us once again consider the case where we have seven regions as in section 2.3.1. The GDP, the energy production and the energy demand of each region can be seen in table 2.1. Now, let the energy production from table 2.1 instead stand for a maximum capacity for producing energy. If we again use the concept of availability μ, but instead of affecting the energy production as before, it instead limits the capacity for producing energy so that the maximal capacity for producing energy in region i is:

$$(E_{\text{capacity}})_i = \mu \cdot (E_{\text{capacity from data}})_i$$

Moreover, let us introduce a new decision variable for each region $(\sigma_P)_i$ that determines the energy production of region i by:

$$(E_{\text{produced}})_i = (\sigma_P)_i \cdot (E_{\text{capacity}})_i$$

We also have to change the utility function to include the total cost for each region to produce its energy so that:

$$\begin{aligned}\text{Utility} = &(\text{Money left after trade} - \text{Total cost for energy production})\cdot\\ &(\text{The amount of the energy demand filled by the energy supply})\end{aligned}$$

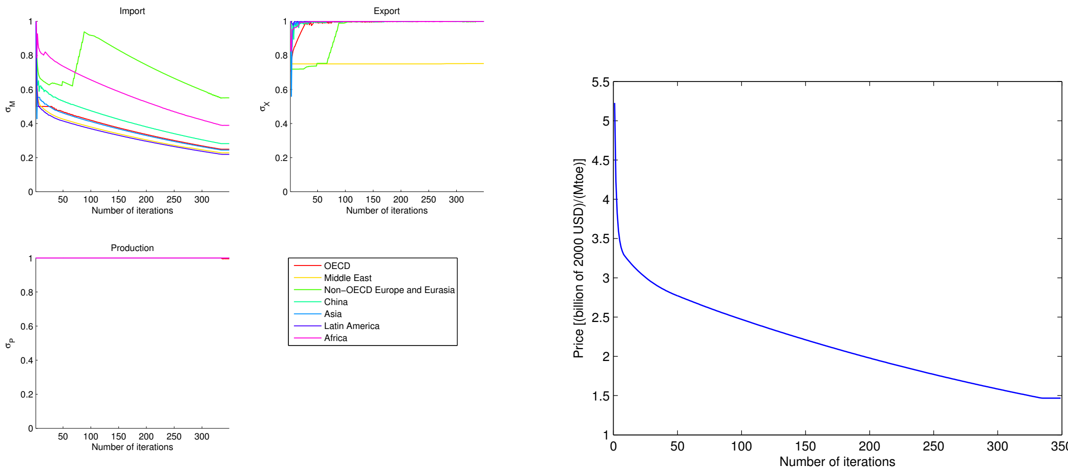
Which in mathematical terms is:

$$U_i = \Big[(1-(\sigma_M)_i)Y_i+\rho\cdot(\sigma_X)_i(E_{\text{produced}})_i-C\cdot(E_{\text{produced}})_i\Big]\cdot\min\Big(1,\frac{(E_{\text{supply}})_i}{(E_{\text{demand}})_i}\Big) \quad (2.5)$$

Where C is the cost to produce 1 Mtoe of energy and like in equation 2.4 we have:

$$(E_{\text{supply}})_i = (1-(\sigma_X)_i)\cdot(E_{\text{produced}})_i + \frac{(\sigma_M)_i Y_i}{\rho}$$

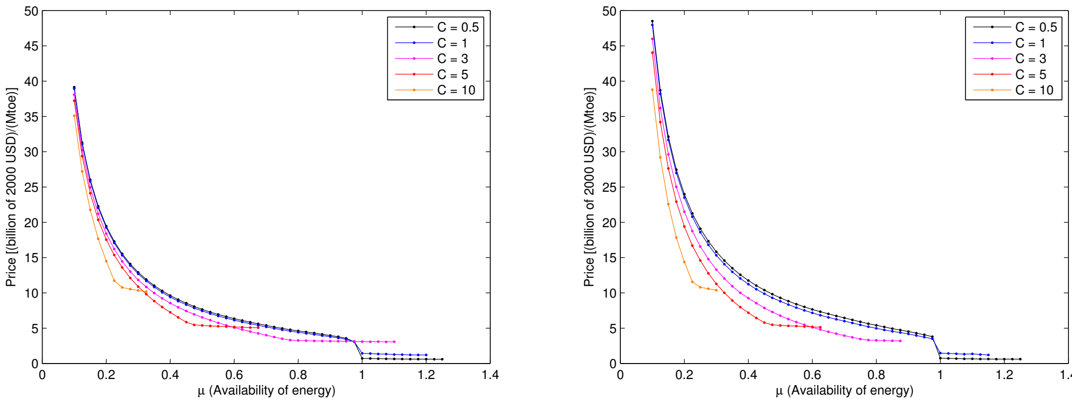
Let us see what the negotiation process looks like if we use the data from table 2.1. For the model to be realistic, the energy price should stabalize above the production cost, which in this case has been chosen to $C = 1$. In this case we also allow the regions to export as much or as little energy as they want.



(a) Decisons **(b)** Energy price

Figure 2.8: The realisation of the negotiation process when all regions are allowed to export as much or as little energy as they want. The production cost is taken to be $C = 1$.

The realisation of the negotiation process can be seen in figure 2.8. In figure 2.8b we can see that at the end of the negotiation process the price stabalises well above the production cost $C = 1$.



(a) All $(\sigma_X)_i = 1$.

(b) All $(\sigma_X)_i$ are $0 \leq (\sigma_X)_i \leq 1$

Figure 2.9: The energy price as a function of availability for the case when each region can control its own energy production. This function has been plotted for different production costs C.

With this new utility function, let us now see how the price at the end of the negotiation process varies as a function of the availability for different production costs. The result of such an experiment can be seen in figure 2.9. In figure 2.9a the export is constrained so that all $(\sigma_X)_i = 1$ and in figure 2.9b we have the case when each region can choose how much of its energy it wants to export so that all $(\sigma_X)_i$ can vary within $0 \leq (\sigma_X)_i \leq 1$.

First we can note that, unlike in section 2.3.1, the price does not at any time go to zero. In fact, with increasing availability, the price approaches the production cost. One can also observe that, when the price comes sufficiently close to the production cost, something happens and it is no longer possible to define an energy price. What happens is that the negotiation process does not converge to an equilibrium. Instead, the decisions are either cyclic as a function of steps in the negotiation process or chaotic. Since the negotiation process does not converge to an equilibrium it is not possible to define a meaningful price that corresponds to a certain set of decisions.

Let us first examine figure 2.9a. One could imagine that when the price comes to close to the production cost, it is no longer profitable to sell energy on the market, but since everyone is bound to put all of their produced energy on the market they are bound to trade with each other. If the energy price is slightly above the production cost it is profitable to produce energy so all regions that are able will want to produce more energy. This will, of course, lead to a lower energy price. If the new price is lower than the production cost it will no longer be profitable to produce energy and everyone will lower their energy production and start buying their energy instead. This will, in turn, lead to a higher energy price which, if it is higher than the production cost, will make us end up in the first case where everyone wants to produce more energy. Thus, if the price is sufficiently close to the production cost, it is not possible to reach an equilibrium price and we will have either cyclic or chaotic decisions.

Something similar happens in figure 2.9b. Here, the regions are not bound to trade with each other, but if the energy price is above the production cost it will be profitable to produce and export more energy, which will lower the energy price. If, on the other hand, the energy price is below the production cost, it will not be profitable to export energy and all regions will lower their export while adjusting their energy production to cover their own energy demand, consequently increasing the energy price. Thus, by the same arguments used for figure 2.9a, cycles or chaos will appear if the price at the end of the negotiation process comes sufficiently close to the production cost.

The energy price is set by the fraction between the total monetary and the total energy flow into the market. Since the money is fixed here, it is the total amount of energy availible for trade that sets the energy price. The total amount of energy is controlled by the availability, so that when the availability is increased, the total energy flow into the market is also increased. Consequently, the energy price will decrease until it comes sufficiently close to the production cost. That is when the cycles and chaos appear.

Another noteworthy thing about figure 2.9 is that for values of $\mu < \mu_{\text{saturated}}$ the energy price is lower the higher the production cost is. This might seem a bit counterintuitive since one might think that if the production cost is increased, the energy price should also increase since the energy producers would have greater expenses. This is indeed the case when $\mu > \mu_{\text{saturated}}$, which can be seen in figure 2.9. However, when $\mu < \mu_{\text{saturated}}$ the total capacity to produce energy is lower than the total energy demand. Thus, all regions will produce as much energy as they can. Regions with lower production capacity than their demand will do this to cover their energy demands and regions with a production capacity higher than their energy demand will do it to earn money, which will be possible as long as the energy price is higher than the production cost. Since all regions are producing at their full capacity, the total capital that is bound in the production of energy is $C \cdot \sum_{i=1}^{N} (E_{\text{capacity}})_i$ where C is again the cost for producing energy. Thus, the total capital left to buy energy on the market is $\sum_{i=1}^{N} (Y_i - C \cdot (E_{\text{capacity}})_i)$. Consequently, the larger the production cost for energy is the less money is left to buy energy on the market. This means that the energy price will decrease with an increasing production cost.

Since the energy price as a function of availability is decreasing faster for higher production costs, the energy price will converge to the production cost faster for higher production costs. By that it follows that the cycles and chaos is evoked for smaller values of μ the higher the production cost is. This is indeed what we can observe in figure 2.9.

2.5 Monopoly situations

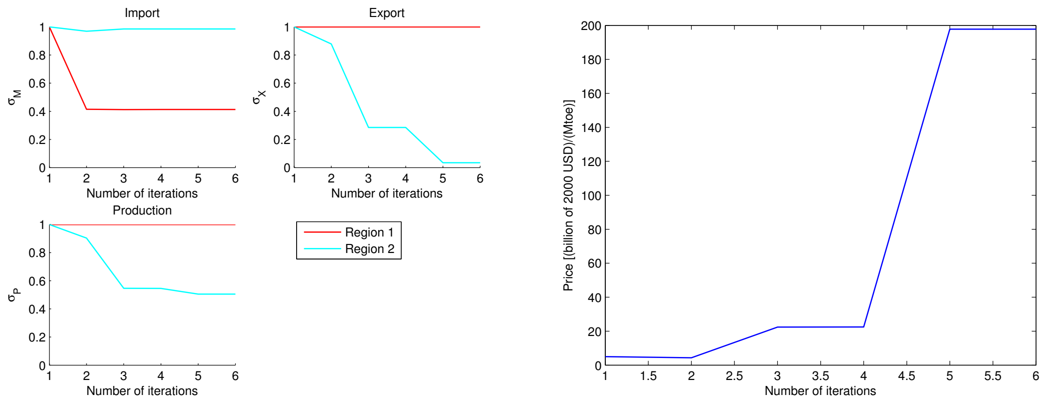
In the real world it is known that when one player in the market is sitting on resources that other players are in need of a monopoly situation arises. The player with the resources will then adjust how much resources it sells to earn as much money from the trade as possible. Consequently it will probably not produce or export enough resources to meet the total demand, even if it is able to do so. It is interesting to see if the iterated decision process along with the price setting mechanics and utility function used in section 2.4 captures this kind of behavior. If this is the case, we can conclude that our model corresponds even more to the real world in the sense that it captures this important aspect of a market.

Table 2.2: Made-up data for a monopoly situation.

Region	GDP [billions of 2000 USD]	Energy production capacity [Mtoe]	Energy demand [Mtoe]
Region 1	15000	0	3000
Region 2	15000	6000	3000

To study if the model captures this kind of behavior, let us make an example with energy as the resource and use the scenario described in table 2.2. The scenario is such that we have two regions, both with the same amount of money and the same energy demand. The difference is that region 1 has no ability to produce energy while region 2 has the ability to produce enough energy for both regions.

The resulting negotiation process corresponding to the data in table 2.2 can be seen in figure 2.10. The realisation during the negotiation process of the utility along with the individual parts of the utility function can be seen in figure 2.11. From these figures we can see that region 2 does not produce enough energy to fill the total energy demand of both regions. It instead chooses to keep enough energy for itself to fill its own energy demand while optimising the export to get as much money from the trade as possible. So we end up in a scenario where region 2 fills its energy demand while region 1 has to buy its energy for a very high price, consequently not being able to fill its own energy demand. What we can see here is clearly the effect of a monopoly situation, which is what we would expect in a similar real world scenario. From this we can conclude that the model does capture the aspects of a monopoly situation on the market.



(a) Decisions **(b)** Price

Figure 2.10: The realisation of the negotiation process.

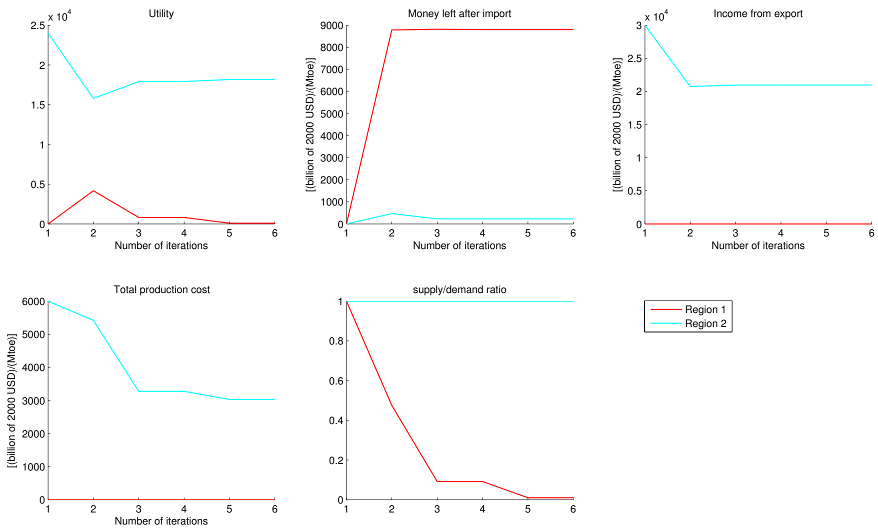


Figure 2.11: The utility during the negotiation process along with the individual properties making up the utility.

3

Model

In this chapter we formulate an economic model where there are a number of different economies that can produce their own goods and energy. They can also put some of their energy out on a world market where it will be priced by the total market demand and supply. It is also possible for countries to buy energy from the market in order for them to fill their internal energy demands. Each country can also regulate its own economy by a number of decisions regarding investments in a goods producing capital stock, a renewable energy producing capital stock and a fossil energy capital stock. A country can also decide how much of its goods production to use in order to buy energy on the energy market and how much of its internal energy production it wants to export. The amount of goods that is not invested in one of the capital stocks or used in energy trade is made availible for consumption by the countrys population in order to produce an internal welfare.

3.1 Model description

In the model we have a number of N countries. The economy of each country i is described by a number of coupled difference equations. They are all discrete with a time difference of one year. The economies are coupled by a global energy market on which each country can sell and/or buy energy. The model is in many aspects a discrete version of the model used by Brede and de Vries [4]. It does however differ from that model in some cases and where it does this will be stated. The model will now be described in detail.

The population growth of a country i depends in this model on both the current population (P) itself and the amount of consumption capital (C) availible per capita (C/P). A higher population in itself contributes to a higher population growth while the population growth decreases when C/P increases. Furthermore the population growth is bounded upwards so that the population growth at time $t+1$ can not exceed ten percent of the population at time t. The function determining the population is in mathematical terms:

$$(P^{[t+1]})_i = (P^{[t]})_i + \mathrm{Min}\Big((a(P^{[t]})_i\Big(\frac{(C^{[t]})_i}{(P^{[t]})_i}\Big)^b), 0.1 \cdot (P^{[t]})_i\Big) \tag{3.1}$$

Where $a = 0.00287$ and $b = -0.6$.

From the population a labour force L is given by:

$$(L^{[t]})_i = \frac{(P^{[t]})_i}{4} \tag{3.2}$$

The economy is comprised of a number of capital stocks and a goods production Y. The produced goods Y can either be reinvested in the capital stocks, be used to buy energy on the global energy market or be transformed to consumption capital C.

The goods producing capital stock K is governed by:

$$(K^{[t+1]})_i = (K^{[t]})_i + ((\sigma_Y^{[t]})_i(Y^{[t]})_i - \delta_K(K^{[t]})_i) \tag{3.3}$$

Where $(\sigma_Y^{[t]})_i$ is the savings rate at time t and $\delta_K = \frac{1}{10}$ is the deprecisation rate of goods producing equipment.

There are also two capital stocks for producing energy. One for renewable energy and one for fossil energy. In this context renewable energy is all energy forms that do not give greenhouse gases as a byproduct from the energy production and fossil energy refer to all energy forms that produce greenhouse gases.

The renewable energy capital stock K_R is governed by:

$$(K_R^{[t+1]})_i = (K_R^{[t]})_i + ((\sigma_R^{[t]})_i(Y^{[t]})_i - \delta_R(K_R^{[t]})_i) \tag{3.4}$$

Where $(\sigma_R^{[t]})_i$ is the investment rate at time t and $\delta_R = \frac{1}{10}$ is the deprecisation rate of renewable energy producing equipment.

The fossil energy capital stock K_F is governed by:

$$(K_F^{[t+1]})_i = (K_F^{[t]})_i + ((\sigma_F^{[t]})_i(Y^{[t]})_i - \delta_F(K_F^{[t]})_i) \tag{3.5}$$

Where $(\sigma_F^{[t]})_i$ is the investment rate at time t and $\delta_F = \frac{1}{10}$ is the deprecisation rate of fossil energy producing equipment.

The goods production Y is modelled by a Cobb-Douglas production function and depends on both the goods producing capital stock and the size of the labour force. It is also depending on a factor f_{Energy} that tells us how well the energy supply matches the energy demand. In mathematical terms we have:

$$(f_{\text{Energy}}^{[t+1]})_i = \begin{cases} 1 & \text{If } (E_{\text{Demand}}^{[t+1]})_i < (E_{\text{Supply}}^{[t]})_i \\ \frac{(E_{\text{Supply}}^{[t]})_i}{(E_{\text{Demand}}^{[t+1]})_i} & \text{Otherwise} \end{cases} \tag{3.6}$$

As you can see f_{Energy} is discretised so that the energy demand at time $t+1$ has to be filled by the energy supply from the year before, i.e at time t.

The goods production also depends on the income from energy export from the year before. The mathematical formulation for Y looks like:

$$(Y^{[t+1]})_i = e^{g(t+1)} Y_0 \Big((K^{[t+1]})_i\Big)^{\gamma} \Big((L^{[t+1]})_i\Big)^{1-\gamma} (f_{\text{Energy}}^{[t+1]})_i + \rho^{[t]} (E_X^{[t]})_i \tag{3.7}$$

Where $(E_X^{[t]})_i$ is the exported energy and $\rho^{[t]}$ is the energy price at time t. The parameters are $g = 0.007$, $Y_0 = 1$ and $\gamma = \frac{1}{2}$. Equation 3.7 differs from [4] in the way that the term $\rho^{[t]}(E_X^{[t]})_i$ has been added to model the incomes from energy trade.

From the goods production a certain amount of goods are made availible for consumption. This is called the consumption capital C and in mathematical terms it looks like:

$$(C^{[t+1]})_i = (\sigma_C^{[t+1]})_i (Y^{[t+1]})_i \tag{3.8}$$

Where $(\sigma_C^{[t+1]})_i$ is the fraction of the goods production that is made availible for consumption at time $t+1$.

The energy demand E_{Demand} is taken to be proportional to the goods production:

$$(E_{\text{Demand}}^{[t+1]})_i = \epsilon (Y^{[t+1]})_i \tag{3.9}$$

Where $\epsilon = 1$.

The energy supply E_{supply} depends on the internal energy production E_{Produced}, the energy export E_X and the energy import E_M like:

$$(E_{\text{Supply}}^{[t+1]})_i = (E_{\text{Produced}}^{[t+1]})_i - (E_X^{[t+1]})_i + (E_M^{[t+1]})_i \tag{3.10}$$

This differs from [4] in the way that the effects of energy export and energy import has been added to equation 3.10.

The energy import is an addition to [4] and depends on the total amount of goods used to buy energy and the current energy price. In mathematical terms:

$$(E_M^{[t+1]})_i = \frac{(\sigma_M^{[t+1]})_i (Y^{[t+1]})_i}{\rho^{[t+1]}} \tag{3.11}$$

Where $(\sigma_M^{[t+1]})_i$ is the fraction of the goods production used to buy energy and $\rho^{[t+1]}$ is the energy price at time $t+1$.

The exported energy is also an addition to [4] and is given by:

$$(E_X^{[t+1]})_i = (\sigma_X^{[t+1]})_i (E_{\text{Produced}}^{[t+1]})_i \tag{3.12}$$

Where $(\sigma_X^{[t+1]})_i$ is the fraction of the produced energy that is exported at time $t+1$.

The energy price at time $t+1$ is given by the sum of all countries' import investments divided by the sum of all countries' exported energy. In mathematical terms it looks like:

$$\rho^{[t+1]} = \frac{\sum_{i=1}^{N} (\sigma_M^{[t+1]})_i (Y^{[t+1]})_i}{\sum_{i=1}^{N} (\sigma_X^{[t+1]})_i (E_{\text{Produced}}^{[t+1]})_i} \tag{3.13}$$

The energy price is an effect of adding energy trade to the model by Brede and de Vries [4] and is not a part of that model.

All model equations regarding energy production, both fossil and renewable, along with the way of modelling energy productivity and fossil fuel assets have been modified from [4] and differ substantially from the corresponding parts in that model.

The produced energy depends on the fossil energy production E_f and the renewable energy production E_R as:

$$(E_{\text{Produced}}^{[t+1]})_i = (E_R^{[t+1]})_i + (E_F^{[t+1]})_i \tag{3.14}$$

The renewable energy production is given by:

$$(E_R^{[t+1]})_i = \phi_R (K_R^{[t]})_i$$

Where ϕ_R is a productivity factor of renewable energy that is assumed to be constant over time. This is, of course, a simplification. In practice the productivity of a technology is usually low when the technology is introduced and then increased with time to a maximum productivity level [2]. It is however, not within the scope of this thesis to try to model new technologies. ϕ_R can easily be related to a production cost, cost_R for renewable energy. Consider for example a case when we at time $t=0$ invest an amount A of goods in the renewable energy producing capital stock K_R and after that we stop investing. Then, from equation 3.4, we get that:

$$K_R^{[t+1]} = K_R^{[t]} - \delta_R K_R^{[t]} = (1-\delta_R) K_R^{[t]}$$

By using the fact about the inital investment we get:

$$K_R^{[0]} = A$$

$$K_R^{[1]} = (1-\delta_R) K_R^{[0]} = (1-\delta_R) A$$

$$K_R^{[2]} = (1-\delta_R)K_R^{[1]} = (1-\delta_R)^2 A$$
$$\vdots$$
$$K_R^{[t]} = (1-\delta_R)K_R^{[t-1]} = (1-\delta_R)^t A$$

Thus the total renewable energy producing capital generated from an investment of size A is given by:

$$A_{\text{Total}} = \sum_{t=0}^{\infty}(1-\delta_R)^t A = \frac{A}{1-(1-\delta_R)} = \frac{A}{\delta_R}$$

The total energy produced by A_{Total} is then $\phi_R \cdot A_{\text{Total}} = \phi_R \frac{A}{\delta_R}$. The production cost for renewable energy is then given by:

$$\text{cost}_R = \frac{\text{Money in}}{\text{Energy out}} = \frac{A}{\phi_R \frac{A}{\delta_R}} = \frac{\delta_R}{\phi_R}$$

Consequently the renewable energy production is given by:

$$(E_R^{[t+1]})_i = \frac{\delta_R}{\text{cost}_R}(K_R^{[t]})_i \tag{3.15}$$

The production cost, cost_R is of course constant over time as both ϕ_R and δ_R is time-independent. In all simulations in this thesis an arbitrarily chosen value of $\text{cost}_R = 0.5$ is used. It would in principle be possible to find a value of cost_R that is coupled to real world data, but finding such data is beyond the scope of this thesis.

The fossil energy production is proportional to the fossil energy producing capital stock but is limited by the remaining internal fossil fuel assets F. This is very natural since you can not extract resources that you do not possess. In mathematical terms the fossil energy production looks like:

$$(E_F^{[t+1]})_i = \text{Min}\Big(\phi_F(K_F^{[t]})_i, (F^{[t]})_i\Big)$$

Where ϕ_F is the productivity of the fossil energy production. The productivity ϕ_F can be associated with a production cost, cost_F for fossil energy in exactly the same way as ϕ_R was associated with cost_R. It then follows that:

$$\text{cost}_F = \frac{\delta_F}{\phi_F}$$

From this we get that:

$$(E_F^{[t+1]})_i = \text{Min}\Big(\frac{\delta_F}{\text{cost}_F}(K_F^{[t]})_i, (F^{[t]})_i\Big) \tag{3.16}$$

Again it is a simplification to assume that we have a production cost that is constant with time. A more realistic thing to do would be to have several fossil fuel assets with different production costs. Then a country would start by consuming the cheapest resource and then in turn continue with the more expensive ones. It is however, reasonable to assume that the production cost for one resource does not vary much with time [3]. In all simulations done in this thesis the arbitrarily chosen value $\text{cost}_F = 0.1$ is used. It would in principle be possible to find a value of cost_F that is coupled to real world data, but finding such data is beyond the scope of this thesis. The important thing is that we have $\text{cost}_F < \text{cost}_R$ so that it is cheaper to use fossil fuels than to use renewable energy sources.

As mentioned above we only consider one type of fossil fuel assets. This resource is modelled by a stockpile in the following way:

$$(F^{[t+1]})_i = \text{Max}\Big((F^{[t]})_i - \frac{\delta_F}{\text{cost}_F}(K_F^{[t]})_i, 0\Big) \tag{3.17}$$

The fossil fuel resource is decreased as the country extracts fossil fuel. When all of the resource has been extracted it is no longer possible for the country to produce fossil energy and it must either start producing renewable energy or import energy from other countries.

Each country's economy is consequently a product of the decisions it makes. The decision variables are:

$$0 \le (\sigma_Y^{[t]})_i \le 1$$
$$0 \le (\sigma_R^{[t]})_i \le 1$$
$$0 \le (\sigma_F^{[t]})_i \le 1$$
$$0 \le (\sigma_C^{[t]})_i \le 1$$
$$0 \le (\sigma_M^{[t]})_i \le 1$$
$$0 \le (\sigma_X^{[t]})_i \le 1$$

We also have the constraint that:

$$(\sigma_Y^{[t]})_i + (\sigma_R^{[t]})_i + (\sigma_F^{[t]})_i + (\sigma_C^{[t]})_i + (\sigma_M^{[t]})_i = 1$$

So that that all of the goods production of country i at time t is used in some way but not more and not less. Each country will try to optimise its decision variables over time in order to maximise its internal welfare. In order to make this optimisation procedure less computationally intense the decision variables are discretised in time on five year intervals so that, for example, $\sigma_Y^{[0]} = \ldots = \sigma_Y^{[4]} = D_Y^{[1]}$, $\sigma_Y^{[5]} = \ldots = \sigma_Y^{[9]} = D_Y^{[2]}$ and so on. Here, $D_Y^{[k]}$ is the decision variable for investment in the goods producing capital stock for period k. In other words, decisions about investments, trade and consumption can only be done every five years and remains constant in the meantime.

3.2 Decision making

Each country i is trying to optimise its decisions so as to maximise a utility function U_i that measures the welfare of country i over the time period considered. As the welfare of all countries are coupled by the global energy market, each country will take decisions based on the decisions of all other countries involved in the energy trade. Thus, all countries will need to go through a negotiation process in order to agree to some decisions. This negotiation process can be simulated by, at a certain iteration step in the negotiation process, letting every country optimise its utility U_i assuming that all other countries are using the same decisions as they did in the iteration before. This negotiation process goes on until no country is able to improve its utility by changing its decisions. When this happens the negotiation process has reached a noncooperative Nash equilibrium and is said to have converged.

This may seem all fine, but there are two major issues regarding this kind of iterated decision making process. The first is that it may not be easy to optimise U_i. In fact, as U_i is the product of a simulation and is depending on very many variables it is very likely that we will not be able to find a global optimum in a finite time. Thus, when each country is "optimising" its utility in each iteration it means in practice that each country is trying to improve its utility in each iteration by finding a local optimum starting from the decisions used in the previous iteration. Second, it is unlikely that the negotiation process will converge in the sense that no country will change its decisions. It is far more likely that the negotiation process will reach a state where only small changes in U_i can be accomplished and the utility will thus be subject to small oscillations around a constant level. Consequently, the negotiation process is said to have converged when no large change in U_i can be accomplished for any country i.

4

Utility and decision making

In order for the decision making to make any sense we must have a utility function U_i that is a realistic approximation of what a country is trying to optimise. The main assumption here is that each country tries to maximise the amount of consumtion capital per capita over time, which we here call wellfare. The first thing to do is then to find a utility function that generates that kind of behaviour. One such utility function is:

$$U_i = \frac{\sum_{t=0}^{T} (P^{[t]})_i \log\left(\frac{(C^{[t]})_i}{(P^{[t]})_i}\right)}{\sum_{t=0}^{T} (P^{[t]})_i} \tag{4.1}$$

This is a discretisation of the utility function used by Brede and de Vries [4] with zero discount rate. The utility function in equation 4.1 does however have a weakness. While it do maximise the quota $\frac{(C^{[t]})_i}{(P^{[t]})_i}$ over time it is not able to handle a limited time horizon in a simulation. What happens is that, when the time comes close to the end of the simulation, there is no future wellfare to consider anymore and all investments are disregarded and allmost all goods production is transformed into consumption capital.

One way to handle this is to run the simulation for a while longer and only take into consideration the first period of time. For example, if you want to simulate the economic system over 100 years you could run the simulation up to 150 years and then only consider the first 100 years which was done by Brede and de Vries [4]. This works well, but it is more computationally intense and it is a way to avoid the problem without trying to understand how to solve it.

4.1 Handling the finite time effect

If we want to avoid the effects of a finite time horizon in the simulation we need to expand the utility function to include, not only the wellfare over the simulation time, but also value future wellfare in some way. This can be done by including the elements that generate future wellfare into the utility function. What generates future wellfare is essentially the capital stocks. Thus we define:

$$(\kappa^{[t]})_i = (K^{[t]})_i + (K_R^{[t]})_i + \mathrm{Min}\Big((K_F^{[t]})_i, \frac{\mathrm{cost}_F}{\delta_F}(F^{[t]})_i\Big)$$

This is the total value of the all capital stocks at time t, with the constraint that the fossil energy producing capital stock can not be worth more than the total value of the remaining fossil fuel assets.

Now we want to incorporate $(\kappa^{[T]})_i$ into the utility function so that we value the total worth of all capital stocks at the end of the simulation where $t = T$. The most straight forward thing to do is to mimic equation 4.1 and add to the numerator:

$$(P^{[T]})_i \log\left(\frac{(\kappa^{[T]})_i}{(P^{[T]})_i}\right)$$

Then we take into consideration the worth of capital stocks per capita. If we add this term in the numeratur of equation 4.1 we get:

$$U_i = \frac{\sum_{t=0}^{T}(P^{[t]})_i \log\left(\frac{(C^{[t]})_i}{(P^{[t]})_i}\right) + (P^{[T]})_i \log\left(\frac{(\kappa^{[T]})_i}{(P^{[T]})_i}\right)}{\sum_{t=0}^{T}(P^{[t]})_i} \tag{4.2}$$

The problem with equation 4.2 is that in the first term of the numerator the consumption capital per capita is counted for each time step in the simulation while in the second term of the numerator we only take into consideration the value of the capital stocks at the end of the simulation. Consequently we will get a scaling problem where the first term is counted $T + 1$ times while the second term is counted one time. Therefore, we would like to add a multiplicator $\xi(T+1)$ to the second term in order to weight how much to value the capital stocks at the end of the simulation in comparison to the consumption capital per capita over the whole time period. If we add this multiplicator to equation 4.2 we get:

$$U_i = \frac{\sum_{t=0}^{T}(P^{[t]})_i \log\left(\frac{(C^{[t]})_i}{(P^{[t]})_i}\right) + \xi(T+1)(P^{[T]})_i \log\left(\frac{(\kappa^{[T]})_i}{(P^{[T]})_i}\right)}{\sum_{t=0}^{T}(P^{[t]})_i} \tag{4.3}$$

Let us now consider simulating an economic system with two countries where each country i is trying to optimise its decisions in order to maximise a utility function U_i over a 100 year time period. What we want to study is if there is any value of the parameter ξ for which the system give a similar response when using the utility in equation 4.3 and the optimisation period is 100 years as we would get from what we will call the baseline, using the utility in equation 4.1, optimising over a 150 year period and then throw away the last 50 years in order to avoid the finite time effect. A case with two countries is used here with the following inital conditions:

$$(K^{[0]})_1 = (K^{[0]})_2 = 1000$$
$$(P^{[0]})_1 = (P^{[0]})_2 = 1000$$

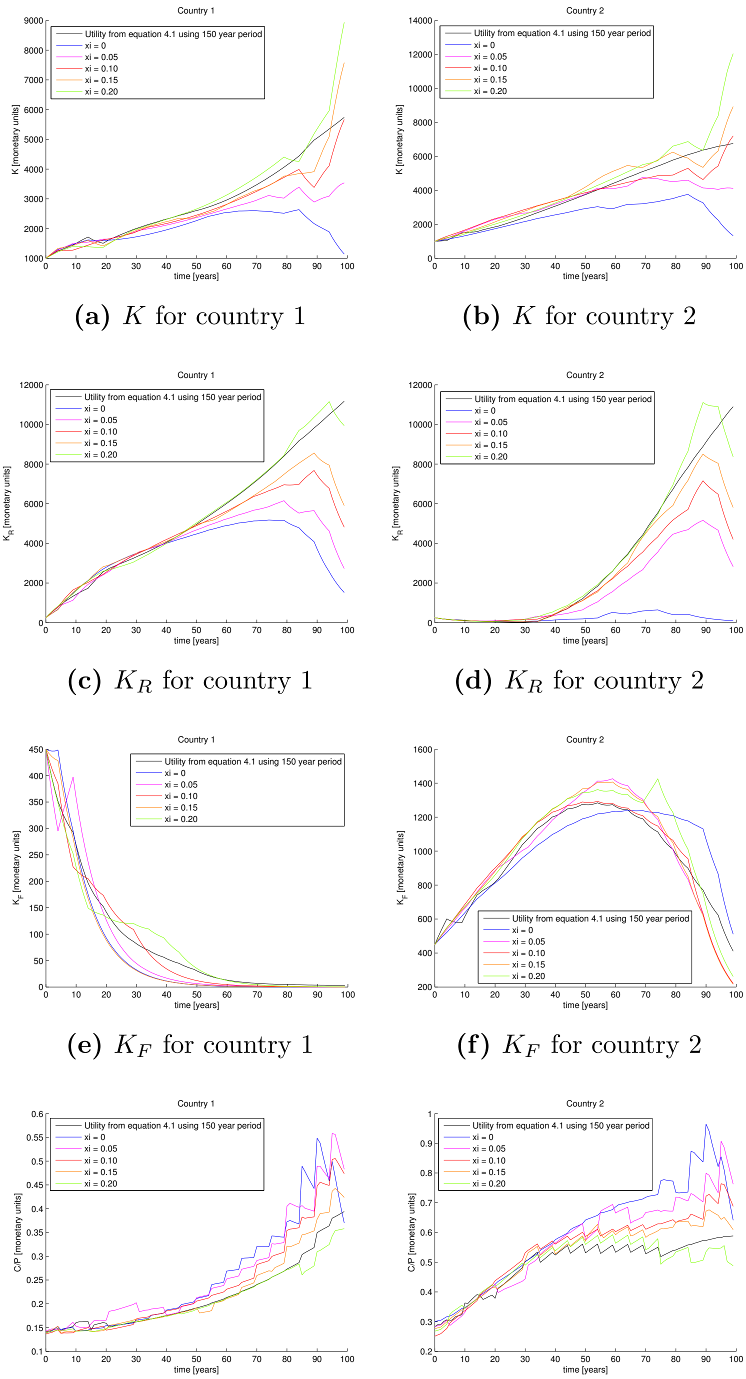
These initial conditions for K and P are the same as in [4]. In addition to this we have:

$$(F^{[0]})_1 = 10000\ ,\ (F^{[0]})_2 = 100000$$

The inital values of the fossil fuel assets are chosen such that country 1 will not have enough fossil energy assets to meet its internal energy demand with only fossil energy for a 100 year period while country 2 will have enough fossil energy assets to rely only on fossil fuels for about 100 years. Also, $\frac{1}{10}$ of the initial energy demand is covered by renewable energy sources while the rest of the inital energy demand is covered by fossil energy corresponding to the initial conditions for energy generating capital stocks used by Brede and de Vries [4]. Consequently, each country will at time $t = 0$ exactly fill its internal energy demand.

The result of such a study can be seen in figure 4.1. What we can see from this figure is that for $\xi = 0$, corresponding to optimising over 100 years and using the utility from equation 4.1, the effects of the finite time are very clear. At the end of the time period all capital stocks decrease dramatically due to the fact that all countries stop investing and start consuming instead. If we start to increase ξ we can see that this effect is avoided, even though a perfect match to the baseline seems to require further adjustments to equation 4.3. By just studying the qualitative behaviour of the plots in figure 4.1 we can see that $\xi = 0.10$ or $\xi = 0.15$ seems to be the best fits to the baseline. Increasing ξ beyond this seems to put to much weight on the capital stocks at the end of the time period so that the consumption during the whole time period is somewhat neglected.

Furthermore, we can see that the general problem with equation 4.3 is that the goods producing capital stock K dramatically increases at the end of the time period for values of ξ that avoids the finite time effect. Also, while the renewable energy producing capital stock K_R increases for the start and most of the time period we can still see a finite time effect, where the investments into this capital stock is neglected at the end of the time period, not taking into consideration the energy demand of the future. This would probably be the case for the fossil energy producing capital stock K_F, but due to the parameterisation of the fossil fuel assets, none of the countries have enough fossil fuel assets to provide fossil energy supply after 100 years of fossil energy use. Thus K_F will not be of any use at the end of the time period and will thus naturally decrease.



(a) K for country 1 **(b)** K for country 2

(c) K_R for country 1 **(d)** K_R for country 2

(e) K_F for country 1 **(f)** K_F for country 2

(g) C/P for country 1 **(h)** C/P for country 2

Figure 4.1: System responses for the two different countries for different values of ξ compared to the case when the utility of equation 4.1 is used with a simulation period of 150 years where the last 50 years are thrown away to avoid finite time effects

5

Model simulations

In order to study if the introduction of a trade mechanism into the model made by Brede and de Vries [4] is meaningful we can study the simulation respons to two different initial conditions. The first case consists of two countries with almost equal initial conditions. The only difference between the two countries is that country 1 only has enough fossil energy assets to meet its own energy demand for approximately ten years without using renewable energy sources while country 2 can rely solely on fossil energy for about 100 years. The following inital conditions are used for case 1:

$$(K^{[0]})_1 = (K^{[0]})_2 = 1000$$

$$(P^{[0]})_1 = (P^{[0]})_2 = 1000$$

$$(F^{[0]})_1 = 10000\ ,\ (F^{[0]})_2 = 100000$$

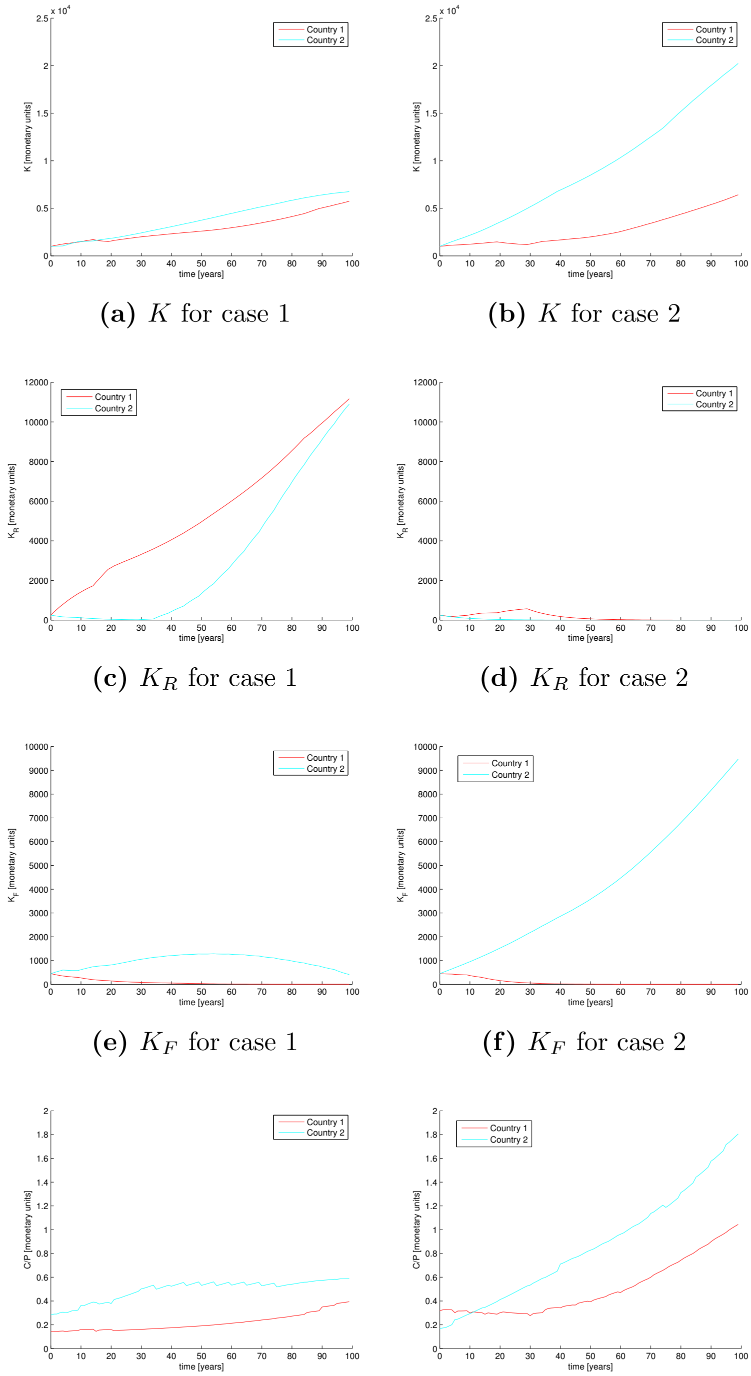
Moreover $\frac{1}{10}$ of the initial energy demand is covered by renewable energy sources while the rest of the inital energy demand is covered by fossil energy.

The second case is very similar to case 1 and differs only by the initial fossil fuel assets so that:

$$(F^{[0]})_1 = 10000\ ,\ (F^{[0]})_2 = 1000000$$

Consequently, the only difference is that country 2 now has very large fossil fuel assets, large enough to supply both economies with fossil energy for a substantially longer time than 100 years.

The decision making is based on the same iterative negotiation process as in previous chapters while using the utility function in equation 4.1, optimising over 150 years and then only showing the first 100 years in order to escape finite time effects.



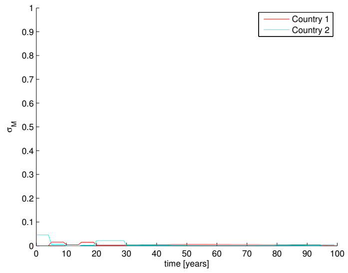
(a) K for case 1 **(b)** K for case 2

(c) K_R for case 1 **(d)** K_R for case 2

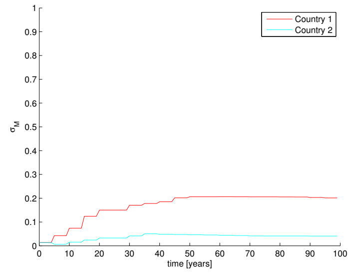
(e) K_F for case 1 **(f)** K_F for case 2

(g) C/P for case 1 **(h)** C/P for case 2

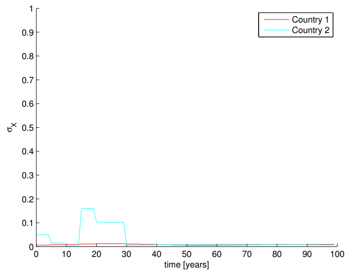
Figure 5.1: System resonses to the two different cases.



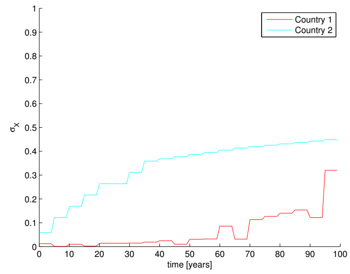
(a) σ_M for case 1



(b) σ_M for case 2



(c) σ_X for case 1



(d) σ_X for case 2

Figure 5.2: The decisions regarding energy trade in the two different cases.

The system reponses can be seen in figure 5.1 and the decisions regarding energy trade can be seen in figure 5.2. The first thing we can note from 5.2 is that in case 1 there is almost no energy trade between the two countries while in case 2 there is a lot of energy trade going on. This can be explained by the fact that in case 1 country 2, that is sitting on most of the fossil fuel assets, does not have enough fossil fuel assets to supply its own economy for more than 100 years. Consequently, it will not be advantageous for country 2 to sell fossil energy on the energy market. One could imagine that a monopoly situation like the one in section 2.5 could occur, but this is hindered by the fact that it will be advantageous for country 1 to start using renewable energy sources if the energy price gets higher than the production cost for renewable energy. Thus, country 2 will not be able to sell energy to a price higher than cost_R which means that it will be more advantageous for country 2 to use the fossil fuel assets to fill its indigenous energy demand and then, when the fossil fuel assets are dwindling, start a transition to renewable energy sources. Since country 1 will not be able to buy energy it is forced to start producing renewable energy right away. This behaviour is indeed what we can see in figure 5.1.

In case 2 on the other hande, there is a lot of energy trade going on, which can be seen in figure 5.2. This is due to the fact that country 2 do not have to worry about running out of fossil fuel during the optimisation period. Consequently, trading energy will be advantageous for both countries since country 2 can sell energy to a price higher than cost_F and country 1 can buy energy to a price lower than cost_R. In this case, the trade will disfavor the development of the renewable energy generating capital stock for country 1, which can be seen in figure 5.1d in relation to figure 5.1c, since it will be cheaper to buy energy from country 2 than to produce its own energy. Country 1 will consequently land in a situation where it is dependent on country 2 for energy supply. This will work since country 1 has the ability to produce its own energy if the energy price would increase beyond cost_R which means that country 2 can not exploit country 1's lack of fossil fuel assets. Thus, a mutually beneficial trade relation will arise between country 1 and country 2.

From this comparison we can conclude that there are some cases when the trading mechanism that has been added to the model by Brede and de Vries [4] can be neglected. In general, however, we can see that energy trade is an important part of the economic model which can have large qualitative impacts on the decision making of countries involved in the trade. It is also probable that if we would add a discount rate to equation 4.1 the energy trade would be of even more importance. This, because dicounting future wellfare would make it more attractive to make short term profits in relation to keep a high future wellfare.

6

Results, conclusions and future work

To summarize the work that has been done in this thesis we will start from the beginning. The ultimate goal of this thesis was to introduce a mechanism for modelling energy trade and incorporate it into the model by Brede and de Vries [4]. This was done by introducing a basic concept for energy trade in chapter 2 where it was also analysed separately from the more complex economic model.

A more complex, dynamic economic model was introduced that, in many aspects is a discretisation of the model by Brede and de Vries [4]. It has, however, been modified in some important ways, the most important being the introduction of energy trade.

Furthermore the concept of a utility function was discusses, i.e. the function to be used when optimising the decisions in the model from chapter 3. We also studied the possibility of avoiding finite time effects by modifying the utility function.

Last, we present some model simulations and study how the model responds to two different initial conditions.

6.1 Results and conclusions

In this thesis a trading mechanism is introduced and studied isolated from other economic aspects. This trading mechanism comprises a model setting the price on an energy market where different actors can buy or sell energy for money. Given a reasonable utility function, this trading mechanism is found to successfully model an energy market. More specifically the price on the energy market behaves in a way that one would expect prices to behave and we also show that this trading mechanism along with the simple utiltiy in section 2.4 can capture the phenomenon of monopoly situations on the market. Thus we can say that the trade model in itself is reasonable and useful.

Moreover, we have incorporated the trade model into a more complex, dynamic economic model that is a modified discrete version of [4]. We show that the trade mechanism that has been introduced has, in general, great effects on qualitative behavior of the actors in the economic model.

We also discuss the possibility of handling the effects of a finite optimisation time when countries are trying to optimise their behaviors in a dynamic economic model such as [4]. It was found that by choosing an appropriate utility function it was possible to partially avoid the effects of finite optimisation time. The problems with declining investments into production was possible to solve but the problem with declining energy investments still remained.

6.2 Future work

The most important thing do to in a continuation of the work presented in this thesis would be to reparameterize the model in chapter 3 in order for all units to correspond to units used to describe economic quantities in the real world. This would require an extensive search for data, but would make it much easier to relate the model to reality. In the same way, realistic inital conditions could be found from real data, which would make it possible to get more realistic simulations and help increase the knowledge of important mechanisms in the global economy.

Another important thing to further develop is understanding how to better avoid the finite time effects in chapter 4. In particular it is important to study the effects of declining energy investments at the end of the time period.

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