Abstract—We consider a frequency division multiplexed system where each user adopts a continuous phase modulation (CPM) and multiuser detection is employed at the receiver side. The spectral efficiency (SE) is used as a performance measure to compare, from an information-theoretic point of view, different modulation formats. More precisely, we consider the new CPM formats recently included in the DVB-RCS2 standard. We describe a framework for computing the information rate (IR) and the SE of such systems, and use it for optimizing the channel spacing between adjacent users and the CPM phase response. Our analysis reveals that modulation formats adopted in the DVB-RCS2 standard are suboptimal in terms of SE, while if we allow multiuser detection, excellent performance can be achieved by using simple binary formats and highly frequency packed signals. Furthermore, we consider practical schemes, where CPMs are serially concatenated with an outer code, and a low-complexity multiuser receiver is employed, showing that the theoretical limits predicted by the IR analysis can be approached.

I. INTRODUCTION

Spectral efficiency (SE) of frequency-division-multiplexed (FDM) systems can be increased by reducing the spacing between adjacent channels, thus allowing overlap in frequency and hence admitting a certain amount of interference [1]. This aspect has been investigated from an information-theoretic point of view for linear [2] as well as continuous phase modulations (CPMs) [3], showing that a significant improvement can be obtained through packing even when at the receiver side a single-user detector is employed. When a multiuser receiver is adopted, the benefits in terms of SE can be even larger and the signals can be packed denser and denser [1]–[3].

In this paper, we focus on FDM systems where each user employs a CPM (FDM-CPM), which is appealing for satellite systems for its robustness to nonlinear distortions, stemming from the constant envelope, for its power and spectral efficiency, and for its recursive nature which allows to employ it in serially concatenated schemes [5], [6]. CPMs are often employed in satellite communications and they have been recently included in the 2nd-generation Digital Video Broadcasting - Return Channel Satellite (DBV-RCS2) standard [4], with the purpose of enabling the use of cheaper amplifier components in the Return Channel Satellite (DVB-RCS2) standard [4], with the aim of improving SEs. Thus, our results show that the modulation formats adopted in the DVB-RCS2 standard are suboptimal in terms of SE. Moreover, we demonstrate that if we allow multiuser detection, excellent performance can be achieved by using simple binary formats and highly frequency packed signals. Finally, we consider practical serially concatenated CPM (SCCPM) schemes employing the reduced-complexity multiuser detector described in [7] as inner soft-input soft-output (SISO) detector. This new algorithm outperforms all other suboptimal multiuser detection algorithms in both performance and complexity. We analyze the convergence behavior of joint multiuser detection/decoding by means of an extrinsic information transfer (EXIT) chart analysis and show that the theoretical limits predicted by the information-theoretic analysis can be approached by SCCPM schemes.

II. SYSTEM MODEL

We assume that the channel is shared by $2U+1$ independent users. Each user employs a code serially concatenated with a CPM modulator through a bit interleaver. Without loss of generality, we consider synchronous users, all employing the same modulation format, and AWGN channels. Extensions to the cases of asynchronous users, of a channel with unknown (possibly time-varying) phase, and of a known time-invariant frequency-selective channel are possible but beyond the scope of this paper. The information data $\mathbf{v}^{(u)}$ of user $u$, encoded by encoder $C_u$ into codeword $\mathbf{x}^{(u)}$, which is interleaved and mapped onto a sequence of $N$ M-ary symbols. We denote by $\alpha_0^{(u)} \in \{\pm 1, \pm 3, \ldots, \pm(M-1)\}$ the symbol transmitted by user $u$ at discrete-time $n$. Moreover, $\mathbf{\alpha}^{(u)} = (\alpha_0^{(u)}, \ldots, \alpha_{N-1}^{(u)})$ is the vector of the $N$ symbols transmitted by user $u$ and we also define $\mathbf{\alpha} = (\mathbf{\alpha}_0, \ldots, \mathbf{\alpha}_{N-1})^{\dagger}$. The complex envelope of the received signal can be written as

$$r(t) = \sum_{u=-U}^{U} s^{(u)}(t, \mathbf{\alpha}^{(u)}) \exp\{j 2\pi f^{(u)} t\} + w(t), \quad (1)$$

1Throughout the paper, $(\cdot)^\dagger$ denotes transpose and $(\cdot)^H$ transpose conjugate.
where $w(t)$ is a zero-mean circularly symmetric white Gaussian noise process with power spectral density (PSD) $2N_0$, $f^{(u)}$ is the difference between the carrier frequency of user $u$ and the frequency assumed as reference for the computation of the complex envelope, and $s^{(u)}(t, \alpha^{(u)})$ is the CPM information-bearing signal of user $u$, given by

$$s^{(u)}(t, \alpha^{(u)}) = \sqrt{2E_s^{(u)}} \exp \left\{ j2\pi h \sum_{n=0}^{N-1} \alpha^{(u)}_n q(t-nT) \right\},$$

where $E_s^{(u)}$ is the energy per information symbol of user $u$, $T$ the symbol interval, common to all users, $h = r/p$ the modulation index ($r$ and $p$ are relatively prime integers), and $q(t)$ the phase-smoothing response. The derivative of $q(t)$ is the so-called frequency pulse $g(t)$, of length $L$ symbol intervals. In this paper, we consider frequency pulses which are given by the linear combination of the RC and REC pulses [4], namely

$$g(t) = \rho g_{RC}(t) + (1-\rho)g_{REC}(t)$$

where the real parameter $\rho \in [0,1]$ is the pulse factor, and $g_{RC}(t)$ and $g_{REC}(t)$ are the classical RC and REC frequency pulses, respectively of duration $L$ symbols. When $\rho = 0$ (resp. $\rho = 1$) we obtain the REC (resp. RC) frequency pulse. In the generic time interval $[nT, nT+T)$, the CPM signal of user $u$ is completely defined by symbol $\alpha^{(u)}_n$ and state $\phi^{(u)}_n = \omega^{(u)}_n q_n(\alpha^{(u)}_n)$, where $\omega^{(u)}_n = (\alpha^{(u)}_{n-1}, \alpha^{(u)}_{n-2}, \ldots, \alpha^{(u)}_{n-L+1})$ is the correlative state and $\phi^{(u)}_n$ is the phase state which can be recursively defined as $\phi^{(u)}_n = [\phi^{(u)}_{n-1} + \pi h \omega^{(u)}_{n-L+1}]2\pi$, where $[\cdot]2\pi$ denotes the “modulo $2\pi$” operator, and takes on $p$ values.

III. SPECTRAL EFFICIENCY AND EXIT CHARTS

In order to evaluate the ultimate performance limits of FDM-CPM systems we evaluate the IR and the SE. This analysis is an extension of the approach described in [3] to a multiuser detector. The described framework allows us to find the optimal spacing between adjacent channels.

We assume that all users transmit at the same power ($E_s^{(u)} = E_s$, $u = -U, \ldots, U$), employ the same modulation format and are equally spaced in frequency. Under these conditions, the frequency spacing is a measure of the signal bandwidth and the SE can thus be computed. In order to avoid boundary effects, we assume $U \rightarrow \infty$. However, for complexity reasons we consider a multiuser detector that assumes the presence of only $2U+1$ users and treats the other remaining users as additional noise. In other words, the channel model assumed by the receiver is

$$r(t) = \sum_{u=-U}^{U} s^{(u)}(t, \alpha^{(u)}) \exp\{j2\pi f^{(u)}(t)+n(t)\}, \quad (2)$$

where $n(t)$ is a zero-mean circularly symmetric white Gaussian process with PSD $2(N_0+N_f)$, $N_f$ being a design parameter which will be optimized through computer simulations.

Notice that the goal here is to evaluate the ultimate performance limits achievable by a receiver designed for the auxiliary channel (2) when the actual channel is that in (1) with $U \rightarrow \infty$. This problem is an instance of mismatched decoding [8], and can be solved by means of the simulation-based method described in [9]. The method in [9] requires the existence of an algorithm for exact maximum a posteriori (MAP) symbol detection over the auxiliary channel (2), i.e., the optimal multiuser MAP symbol detector for $2U+1$ users for the AWGN channel [7] in this case.

Denoting by $r$ a set of sufficient statistics for the detection of $\alpha$, we first compute the IR for user $u$ as

$$I(\alpha^{(u)}; r) = \lim_{N \rightarrow \infty} \frac{1}{N} E \left\{ \log \frac{p(r|\alpha^{(u)})}{p(r)} \right\} \left[ \begin{array}{c} b \text{ ch. use} \end{array} \right]. \quad (3)$$

The probability density functions $p(r|\alpha^{(u)})$ and $p(r)$ can be computed by a forward recursion of the optimal multiuser MAP symbol detector [9]. In (3), the expectation is with respect to the input and output sequences generated according to the model in (1). Assuming a system with an infinite number of users, we can define the system bandwidth as the separation between adjacent channels $F = |f^{(i)} - f^{(i-1)}|$ and use it in the definition of the achievable SE

$$n^{(u)} = \frac{1}{FT} I(\alpha^{(u)}; r) \quad [\text{bps/Hz}].$$

Notice that the approach described in this section, due to the use of a (optimal) mismatched multiuser detector, leads to achievable lower bounds on IR and SE.

The optimal multiuser detector required to compute the IR and the SE of FDM-CPM schemes is prohibitively complex for practical purposes, since its computational complexity grows exponentially with the number of users. Therefore, for the design of practical schemes, we consider the suboptimal multiuser detector for MAP symbol detection proposed in [7].

We analyze the convergence behavior of this multiuser SCCPM scheme by means of an EXIT-chart analysis [10]. The aim is to evaluate the convergence behavior of different joint detection/decoding schemes, and to assess the tradeoff between high SE and actual performance.

The joint detection/decoding scheme consists of $2U+1$ a posteriori probability decoders $C_{-U}^{-1}, \ldots, C_U^{-1}$, matched to encoders $C_{-U}, \ldots, C_U$ of the $2U+1$ users, and a multiuser detector module $C_{MU}^{-1}$ consisting of $2U+1$ SISO blocks (one for each user) to separate signals from different users. The $2U+1$ SISO blocks within the multiuser detector exchange soft information among each other as well as with the $2U+1$ decoders in an iterative fashion. We distinguish between local iterations, within the multiuser detector, and global iterations, between the multiuser detector and the $2U+1$ SISO decoders. A global iteration consists of a single activation of decoders $C_{-U}, \ldots, C_U$ and a single local iteration of $C_{MU}^{-1}$. Accordingly, the iterative decoding process can then be tracked using a multi-dimensional EXIT chart. Alternatively, the EXIT functions of the constituent decoders and of the multiuser detector can be properly combined and projected into a two-dimensional chart.

Let $I_{e_{x(j)}}^{C_j}$ denote the extrinsic mutual information (MI) generated by decoder $C_j^{-1}$ of user $j$ on codeword $x^{(j)}$ at the output of $C_j$. Also, let $I_{e_{x(j)}}^{C_{MU}}$ be the extrinsic MI generated by the multiuser detector on codeword $x^{(j)}$ (more precisely on its interleaved version). Correspondingly, denote by $I_{a_{x^{(j)}}}^{C_j}$ and $I_{a_{x^{(j)}}}^{C_{MU}}$ the a priori MI at the input of decoder $C_j^{-1}$ and of
\[ \mathcal{C}_{MU}^{-1} \] respectively. \( I_{\mathcal{C}}^{C_{i}}(U_{x}(j)) \) is a function of \( I_{\mathcal{C}}^{C_{i}}(U_{x}(j)) \), while \( I_{\mathcal{C}}^{C_{MU}}(U_{a}(x)) \) is a function of \( I_{\mathcal{C}}^{C_{MU}}(U_{a}(x)) \) for \(-U' \leq j \leq U'\) and of the channel signal-to-noise ratio (SNR) \( \gamma = E_s/N_0 \):

\[
I_{\mathcal{C}}^{C_{i}}(U_{x}(j)) = T_{x}(I_{\mathcal{C}}^{C_{i}}(U_{x}(j))), \\
I_{\mathcal{C}}^{C_{MU}}(U_{a}(x)) = T_{x}(I_{\mathcal{C}}^{C_{MU}}(U_{a}(x), \gamma)).
\]

Notice that \( I_{\mathcal{C}}^{C_{i}}(U_{x}(j)) = I_{\mathcal{C}}^{C_{MU}}(U_{a}(x)) \) and \( I_{\mathcal{C}}^{C_{MU}}(U_{a}(x)) = I_{\mathcal{C}}^{C_{i}}(U_{x}(j)) \). Combine now the EXIT functions of encoders \( i \in \{-U', \ldots, U'\}/j \) and of the multiuser detector in a single EXIT function, which we denote by \( I_{\mathcal{C}}^{C_{i}}(U_{x}(j), \gamma) \), where \( I_{\mathcal{C}}^{C_{i}}(U_{x}(j)) = I_{\mathcal{C}}^{C_{i}}(U_{x}(j)) \). Now, the iterative process can be tracked by displaying in a single plot the EXIT function of user \( j \), \( I_{\mathcal{C}}^{C_{i}}(U_{x}(j)) = T_{x}(I_{\mathcal{C}}^{C_{i}}(U_{x}(j))) \), and the EXIT function \( I_{\mathcal{C}}^{C_{i}}(U_{x}(j)) = T_{x}(I_{\mathcal{C}}^{C_{i}}(U_{x}(j), \gamma)) \). \( I_{\mathcal{C}}^{C_{i}}(U_{x}(j)) \) can be computed for all values \( 0 \leq I_{\mathcal{C}}^{C_{i}}(U_{x}(j)) \leq 1 \) by activating all \( 2U' \) decoders \( C_{i}^{-1}(U_{x}(j)), C_{i}^{-1}(U_{x}(j) - 1), \ldots, C_{i}^{-1}(U_{x}(j) - 1), C_{i}^{-1}(U_{x}(j)) \) and \( C_{i}^{-1}(U_{x}(j)) \) until \( I_{\mathcal{C}}^{C_{i}}(U_{x}(j)) \) has converged to a fixed value.

**IV. Numerical Results**

Due to lack of space, we consider only binary and quaternary CPM formats. This choice is justified by the need to illustrate the relevant concepts and by the results which show that we can design transmission schemes with very high SE using simple CPMs. We consider binary CPMs with \( h = 1/3 \), and quaternary formats with \( h = 1/4 \). All considered schemes are characterized by a frequency pulse of length two symbol intervals (\( L = 2 \)). The shape of the frequency pulse is specified by means of the pulse factor \( \rho \).

The following results are obtained by simulating a system with a large number of users and by using a receiver working on the \( 2U' + 1 \) central ones. Moreover, we consider the performance of the channel with index 0, when the detector takes into account \( U' \) adjacent channels on each side.

Figs. 1 and 2 show the SE as a function of the spacing \( F \) for the considered binary and quaternary CPM formats, respectively. In particular, we compute the achievable SE for a given value of \( E_s/N_0 \), when a multiuser detector for different values of \( U' \) is employed. The curves with \( U' = 0 \) refer to the use of a single-user detector. For the binary CPM schemes, we also evaluate the optimal spacing when a detector with \( U' = 1 \) is employed; however, the curves are not plotted in Fig. 1 for the sake of clarity. These figures show that for each considered CPM scheme, the multiuser detection strategy allows to achieve a much higher SE than that achievable by the single-user detector. Also, the frequency spacing which maximizes the SE is lower in the multiuser detection case. Moreover, we can also observe that schemes with pulse factor \( \rho < 0.5 \) achieve the highest SE values, showing that the formats in the DVB-RCS2 standard [4], for which \( \rho \) ranges from 0.625 to 0.98, are suboptimal from the SE point of view.

In the following, we consider the optimal modulation formats, corresponding to \( \rho = 0 \) for \( M = 2 \) (B-\( \rho_0 \)) and \( \rho = 0.25 \) for \( M = 4 \) (Q-\( \rho_0 \)), in the case of multiuser (\( U' > 0 \)) and single-user detection (\( U' = 0 \)), with channel spacing resulting from the previous optimization—we noticed that the optimal spacing only slightly depends on the considered value of SNR.

In the same figure, we compare our optimized schemes with the formats in the DVB-RCS2 standard. In particular, among all formats in the standard we considered the quaternary scheme with \( L = 2 \), \( h = 1/5 \), \( \rho = 0.625 \), \( FT = 0.974 \) (DVB-RCS2 mod1) providing the highest SE for high SNR values and the scheme with \( L = 2 \), \( h = 2/7 \), \( \rho = 0.75 \), \( FT = 1.21 \) (DVB-RCS2 mod2) providing the highest SE for low to medium SNR values. We can see that, as expected, the multiuser schemes strongly outperform the DVB-RCS2 formats. Moreover, the proposed quaternary scheme based on single-user detection allows achieving SE values higher than those obtained with the DVB-RCS2 formats.

We now consider practical SCCPM schemes by concatenating the CPM modulator with an outer code through a random interleaver. Instead of the optimal multiuser detector,
at the receiver we employ the suboptimal multiuser detector described in [7]. Here, we employ a 16-state convolutional code for moderate rates $R$ and eBCH codes for very high rates ($R > 0.75$) [6]. Notice that the code rate is completely determined by the SE and the employed channel spacing.

In Table I we report the iterative convergence thresholds for binary and quaternary SCPPM schemes for several values of the SE and $U' = 1, 3$. In all cases, we use the optimized channel spacing. The corresponding code rate is also reported in the table. For each CPM format, the highest value of SE which has been considered is slightly lower than the maximum achievable SE since we consider suboptimal reduced-complexity detection schemes. In Table I, we also report the achievable IR threshold in $E_b/N_0$ for the mismatched multiuser MAP detector obtained through the information-theoretic analysis. For the suboptimal detector, an optimization of the noise variance $N_f$ has been carried out by simulation.

The EXIT-chart analysis reveals that for each scheme the gap with respect to the theoretical limits is bigger for increasing values of the SE. Despite the use of a suboptimal detector, for $U' = 3$ all schemes perform between $1.15 - 1.8$ dB away from the achievable IR. Among the considered schemes, the best tradeoff between SE and actual performance is given by the binary format with $U' = 3$ and code rate $R = 0.8$. This is an interesting result, which shows that by optimizing the channel spacing and by using multiuser processing, we can achieve high SEs with simple binary CPMs and no improvement can be obtained, when the complexity is constrained, by resorting to higher-order modulations.

In Fig. 4, we plot the bit error rate (BER) curves of the binary optimized schemes with SE= 2.33 bps/Hz and 2.66 bps/Hz. Here, $U' = 3$, $K = 64000$ information bits, and the curves for the middle user are reported. A maximum of 20 iterations between the detector and the decoders is allowed. In the figure, the iterative convergence thresholds are also reported, showing that the BER results are in agreement with the EXIT-chart analysis. It can be also observed that the proposed scheme performs quite close to the theoretical limit.

V. CONCLUSIONS

We investigated the ultimate performance limits of FDM-CPM systems in terms of IR and SE. In particular, we considered CPM formats consisting of a linear combination of RC and REC pulses, adopted in the DVB-RCS2 standard. We optimized the channel spacing of FDM-CPM systems using binary and quaternary CPMs to maximize the SE. We showed that, for low spectral efficiencies, the performance of binary and quaternary CPMs is similar. However, with constrained complexity, binary multiuser schemes allow achieving higher spectral efficiencies. Our analysis showed that the modulation formats adopted in the DVB-RCS2 standard are suboptimal in terms of SE. We also considered practical serially concatenated CPM schemes using a suboptimal multiuser detector. We analyzed the convergence thresholds of serially concatenated multiuser CPM through an EXIT-chart analysis and showed that the theoretical limits can be approached.

REFERENCES


