AdS backgrounds from black hole horizons

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Abstract

We utilize the classification of IIB horizons with 5-form flux to present a unified description for the geometry of AdS_n, $n = 3, 5, 7$ solutions. In particular, we show that all such backgrounds can be constructed from 8-dimensional 2-strong Calabi-Yau geometries with torsion which admit some additional isometries. We explore the geometry of AdS_3 and AdS_5 solutions but we do not find AdS_7 solutions.

1 Introduction

The gravitational duals of gauge/gravity correspondences [1] and flux compactifications [2] are (warped) products of AdS and Minkowski spaces with some other "internal" manifold. Because of this, they have been the focus of intensive investigations in the literature. Most of the work has been done on supersymmetric solutions. More recently the attention has been shifted to a systematic construction of such solutions and several approaches have been proposed to find such solutions. One is to make an ansatz for the fields and Killing spinors that respect the symmetries of the solution, for a review see [3] and references within. However, such choices are special and it is not apparent that all solutions can be described in this way. In [4] an alternative approach has been used which again involves the use of an ansatz for the fields but now combined with the spinor bilinears technique for solving the KSEs [5]. This avoids the problem of making a special choice for the Killing spinors. However because of the complexity of solving the KSEs in this way, the fluxes of some backgrounds are not always chosen to be the most general ones allowed by the symmetries, see e.g. [6]. So again the most general solutions may not have been described. Moreover, all investigations so far have been done on a case by case basis and no overall picture has emerged for the geometry of all AdS_n backgrounds independently of n.

It is advantageous to remove all the assumptions made for constructing gravitational duals of gauge/gravity correspondences and flux compactification backgrounds, and at the same time find a way to describe all solutions in a uniform way. In this paper, we shall focus on the second question. For this, we shall use a straightforward coordinate transformation, which is described in appendix A, to bring the metric of all (warped) products of AdS and Minkowski spaces with an "internal" manifold X to a form which is included in the standard near horizon geometry of extreme black holes [7]. Then we shall utilize the classification results for supersymmetric near horizon geometries in 10- and 11-dimensional supergravity theories [8, 9, 10], based on the spinorial technique for solving Killing spinor equations [11], to give a unified description for the geometry of all these warped products.

The classification of near horizon geometries is centered around the description of near horizon spatial sections $\mathcal{S}$. The "near horizon section" for the wrapped product $AdS_n \times_w X$ is $\mathcal{S} = H_{n-2} \times_w X$, where H is a hyperbolic space, while for $\mathbb{R}^{n-1,1} \times_w X$ is $\mathcal{S} = \mathbb{R}^{n-2} \times_w X$. As a consequence the geometry of all $N \times_w X$, $N = \mathbb{R}^{n-2}, H_{n-2}$, and in particular that of X, can be determined as a special case of the geometry of spatial horizon sections. Thus the classification of gravitational duals and vacua of compactifications with fluxes can be viewed locally as a special case of that of near horizon geometries for black holes.

The examples which we shall present in detail are the AdS solutions in IIB supergravity. In particular, we shall utilize the classification of the near horizon geometries of IIB supergravity [9] with only 5-form flux, see also [12], to present a unified description of $M \times_w X$, $M = AdS_n, \mathbb{R}^{n-1,1}$ geometries in this theory. We shall mostly focus on $M = AdS_n$. This is because the $\mathbb{R}^{n-1,1} \times_w X$ spacetimes can be viewed as a special case of $AdS_n \times_w X$ in the limit where the radius of AdS goes to infinity. In particular, we shall show that the spacetimes with $AdS_3 \times_w X^7$ and $AdS_5 \times_w X^5$ can be constructed from 8-dimensional 2-strong Calabi-Yau manifolds with torsion, i.e. 2-SCYT manifolds. This is the geometry of spatial sections of IIB horizons [9]. In addition, we shall demonstrate that there are no $AdS_7 \times_w X$ solutions in this class.

Furthermore, our construction allows for $AdS_n \times_w X^{10-n}$ backgrounds for which the $SO(n-1,2)$ isometry group of the metric is broken by the 5-form flux[1]. To restore the full $SO(n-1,2)$ symmetry for the whole background, one has to impose additional restrictions on the geometry which we investigate. It turns out that for the AdS_3 and AdS_5 backgrounds which have full $SO(n-1,2)$ symmetry, the geometry of X coincides with that which has already been found in the literature. In particular, X^5 is Sasaki-Einstein for AdS_5 backgrounds [13], and X^7 is a fibration over a Kähler manifold for the AdS_3 backgrounds [4] for which the Ricci scalar and Ricci tensor satisfy a certain differential equation. Thus we establish a relation between 5-dimensional Sasaki-Einstein geometry and the geometry on 6-dimensional Kähler manifolds of [4] with the 2-SCYT geometry on 8-manifolds. We point out though that in the latter case, the relationship between the above Kähler geometry and the 2-SCYT geometry which arises from the horizon analysis [9] is somewhat involved. The two differential systems which characterize the geometries are rather distinct.

The other examples which we shall explore are $AdS \times_w X$ solutions in heterotic supergravity. A direct inspection of the classification results for supersymmetric solutions in [17] and that of near horizon geometries [8] reveals that there are two classes with one class consisting of non-trivial $SL(2,\mathbb{R})$ fibrations over a 7-dimensional base manifold. Therefore the geometry of AdS_3 backgrounds can be read off from that of heterotic horizons. The non-trivial fibration breaks the $SO(2,2)$ isometry group of AdS_3 to a subgroup which always contains one of the $SL(2,\mathbb{R})$ subgroups. We describe the geometry of the AdS_3 solutions which exhibit the full $SO(2,2)$ symmetry. We find that in this case the fibration is trivial and the solutions are direct products $AdS_3 \times X^7$. Depending on the geometry of X^7, the AdS_3 backgrounds which arise as a special case of heterotic horizons preserve 2, 4, 6 and 8 supersymmetries. This is in agreement with the results of [18].

Before we proceed, we would like to point out that the classification of near horizon geometries is carried out under two assumptions. One is the identification of the timelike or null Killing vector field of the near horizon geometry with the timelike or null Killing spinor bilinear of a supersymmetric background. The other is that the horizon section is compact which is instrumental to solving the field equations. Neither of these two assumptions necessarily hold in the investigation of $AdS \times_w X$ solutions. The first assumption on the identification of the two Killing vector fields is technical and it can be removed, see [14] for the near horizon geometries in 4-dimensional supergravity. The compactness of the horizon section does not apply in the $AdS \times_w X$ case if X is not compact. However, the compactness of $\mathcal{S}$ is not always used in the analysis of near horizon solutions. For example although it has been instrumental in the understanding of heterotic horizons in [8], it has not been used in the analysis of IIB horizons in [9] admitting a pure Killing spinor. As a result our examples of $AdS \times_w X$ spaces based on IIB supergravity horizons are not restricted by the compactness assumption.

This paper has been organized as follows. In section two, we state the geometry of IIB horizons adapted for the investigation of AdS backgrounds. In sections 3, 4 and

[1] The symmetry preserved by the background does not contain an $SO(n-1,1)$ subgroup and so a gauge/gravity duality interpretation of such backgrounds is not apparent.

5, we examine the geometry of solutions which are direct products of AdS_3, AdS_5 and AdS_7 spaces. In sections 6, 7 and 8 we examine the geometry of solutions which are warped products of AdS_3, AdS_5 and AdS_7 spaces, respectively. In particular in section 6, we establish the relation between 2-SCYT geometry and the Kähler geometry of [4]. In appendix A, we present the coordinate transformation which relates AdS solutions with near horizon geometries, and in appendix B, we describe the AdS_3 backgrounds of heterotic supergravity.

2 IIB CFT gravitational duals

2.1 IIB near horizon geometry

The near horizon black hole geometries of IIB supergravity with only 5-form flux have been classified in [9]. The spatial horizon section $\mathcal{S}$ is an 8-dimensional manifold equipped with a 2-strong Calabi-Yau structure with torsion. This means that $\mathcal{S}$ is a Hermitian manifold, with Hermitian form ω, such that

$$\hat{\rho} = 0 \ , \qquad d(\omega \wedge H) \equiv \partial\bar{\partial}\omega^2 = 0 \ , \tag{2.1}$$

where $\hat{\rho}$ is the Ricci form of the connection with skew-symmetric torsion $H = -i(\partial - \bar{\partial})\omega$.

The spacetime geometry can be summarized as follows. The spacetime metric and 5-form flux can be written as

$$\begin{aligned} ds^2 &= 2du(dr + rh) + ds^2(\mathcal{S}) \\ F &= e^+ \wedge e^- \wedge Y - \star_8 Y \ , \end{aligned} \tag{2.2}$$

where

$$Y = \frac{1}{4}\left(d\omega - \theta_\omega \wedge \omega\right) \ , \tag{2.3}$$

$$h = \theta_\omega \tag{2.4}$$

and θ_ω is the Lee form of $\mathcal{S}$. From now on instead of stating F, we shall give Y for simplicity.

Therefore to construct solutions, one has to find Hermitian manifolds $\mathcal{S}$ which satisfy the conditions (2.1). Note that the Ricci form in (2.1) can be expressed as

$$\hat{\rho} = -i\partial\bar{\partial} \log\det g - d i_I \theta_\omega \ , \tag{2.5}$$

where g is the Hermitian metric of $\mathcal{S}$.

2.1.1 Geometry of gauge/gravity duals

Consider first the direct product $AdS_n \times X^{10-n}$ backgrounds. The straightforward transformation which brings the metric of the above background into a near horizon form is explained in appendix A. One also finds that the spatial horizon section $\mathcal{S}$ is the direct product of a hyperbolic space with X^{10-n}, $\mathcal{S} = H_{n-2} \times X^{10-n}$, and the metric can be written as

$$ds^2(\mathcal{S}) = (Z^1)^2 + \sum_k (Z^k)^2 + ds^2(X^{10-n}) \ . \tag{2.6}$$

To continue, we must identify the rest of the geometric structure on $\mathcal{S} = H_{n-2} \times X^{10-n}$ and in particular the Hermitian form ω. There is not a natural way to do this. However, suppose in addition that AdS_n is odd dimensional. In such a case X^{10-n} is also odd-dimensional and so it admits a no-where vanishing vector field. After possibly multiplying this vector field with a function of X^{10-n}, the metric on X^{10-n} can be written as

$$ds^2(X^{10-n}) = w^2 + ds^2_{(9-n)} \ , \tag{2.7}$$

where w is the 1-form dual to the vector field and $ds^2_{(9-n)}$ is orthogonal to w. In this case a Hermitian form can be defined on $\mathcal{S}$ as

$$\omega = Z^1 \wedge w - \sum_k Z^{2k} \wedge Z^{2k+1} + \omega_{(9-n)} \ , \tag{2.8}$$

where $\omega_{(9-n)}$ is a non-degenerate 2-form on X in the directions transverse to w,

$$i_w \omega_{(9-n)} = 0 \ . \tag{2.9}$$

The integrability of the associated almost complex structure is equivalent to requiring that the cylinder $\mathbb{R} \times X^{10-n}$ is a complex manifold. Some of the integrability conditions are

$$dw^{2,0} = 0 \ , \quad i_w dw = 0 \ , \tag{2.10}$$

i.e. dw is (1,1)-form and dw is transverse to w. Note that by definition $|w|^2 = 1$.

Furthermore in the direct product case comparing the expression for the metric of $AdS_n \times X^{10-n}$ in (A.4) and (2.4), one has to take

$$\theta_\omega = -\frac{2}{\ell} Z^1 \ , \tag{2.11}$$

which restricts the choice of Hermitian structure on $\mathcal{S}$ which in turn restricts X^{10-n}. In addition to the integrability of the almost complex structure on $H_{n-2} \times X^{10-n}$ and (2.11), the two conditions in (2.1) must also be satisfied. These conditions will be investigated for each case separately.

In the warped product case, the geometric data are altered as follows. The metric on $\mathcal{S}$ is chosen as

$$ds^2(\mathcal{S}) = A^2[(Z^1)^2 + \sum_k (Z^k)^2] + ds^2(X^{10-n}) \ , \tag{2.12}$$

where now

$$ds^2(X^{10-n}) = A^2 w^2 + ds^2_{(9-n)} \ , \tag{2.13}$$

where A is the warp factor. The Hermitian form is chosen as

$$\omega = A^2[Z^1 \wedge w - \sum_k Z^{2k} \wedge Z^{2k+1}] + \omega_{(9-n)} \ . \tag{2.14}$$

The integrability conditions of the associated almost complex structure again imply (2.10). Again comparing the expression for the warped product metric in (A.11) and (2.4), one finds that the condition on the Lee form is now modified to

$$\theta_\omega = -\frac{2}{\ell} Z^1 - d \log A^2 \ . \tag{2.15}$$

As in the previous case, the two conditions (2.1) of the 2-SCYT structure should also be satisfied and will be investigated separately for each case.

3 Direct product AdS$_3$ gravitational duals

The metric on the direct product $AdS_3 \times X^7$ takes the form

$$ds^2_{(10)} = 2(dudr - \frac{2r}{\ell} du Z^1) + (Z^1)^2 + ds^2(X^7) \ , \quad dZ^1 = 0 \ , \tag{3.1}$$

and so $Z^1 = dx$. Note that AdS_3 is spanned by the coordinates x, u, r. Moreover $\mathcal{S}_{(8)} = \mathbb{R} \times X^7$. Thus to find solutions, one has to find those 7-dimensional manifolds X^7 such that the cylinder $\mathbb{R} \times X^7$ admits a 2-SCYT structure.

As in the general case described in the previous section, we set $ds^2(X^7) = w^2 + ds^2_{(6)}$, where $ds^2_{(6)}$ is a metric transverse to w. We also postulate the Hermitian form

$$\omega_{(8)} = Z^1 \wedge w + \omega_{(6)} \ , \tag{3.2}$$

with $i_w \omega_{(6)} = 0$, where the subscripts in ω denote the dimension of the associated space. The integrability of the almost complex structure implies (2.10).

The torsion 3-form is

$$H = w \wedge dw + Z^1 \wedge \xi - i_{I_{(4)}} \eta \ , \tag{3.3}$$

where we have decomposed

$$d\omega_{(6)} = w \wedge \xi + \eta \ , \quad i_w \eta = 0 \ . \tag{3.4}$$

Moreover, ξ is (1,1), and η is (2,1)+(1,2), with respect to $I_{(6)}$ as required by the integrability of the complex structure $I_{(8)}$.

Next consider the condition (2.11), $\theta_{\omega_{(8)}} = -\frac{2}{\ell} Z^1$, to find[2] that

$$dw \cdot \omega_{(6)} = \frac{4}{\ell} \ , \quad \xi \cdot \omega_{(6)} = 0 \ , \quad \eta \cdot \omega_{(6)} = 0 \ . \tag{3.5}$$

[2]Let ρ be a k-form, then $\rho \cdot \omega$ is the un-weighted contraction of the first two indices of ρ with the indices of ω.

Moreover, the 2-SKT condition (2.1), $d(\omega_{(8)}\wedge H)=0$, implies that

$$w\wedge di_{I_{(6)}}\eta+w\wedge\xi\wedge\xi+\eta\wedge\xi+\omega_{(6)}\wedge d\xi=0\ ,\quad dw\wedge\eta=0\ ,\quad \omega_{(6)}\wedge dw\wedge dw=0\ . \tag{3.6}$$

The first condition can be simplified a bit further by writing

$$\begin{aligned} &di_{I_{(6)}}\eta=w\wedge\alpha+\beta\ ,\quad i_w\beta=0\ ,\\ &d\xi=w\wedge\gamma+\delta\ ,\quad i_w\delta=0\ ,\end{aligned} \tag{3.7}$$

to find

$$\beta+\xi\wedge\xi+\omega_{(6)}\wedge\gamma=0\ ,\quad \eta\wedge\xi+\omega_{(6)}\wedge\delta=0\ . \tag{3.8}$$

It remains to solve the first equation in (2.1), $\hat\rho=0$. This has been expressed in (2.5) in terms of the determinant of the Hermitian metric and the Lee form of the 2-SCYT manifold $\mathcal{S}$. A straightforward computation reveals that this condition can be rewritten in terms of geometric data on X^7 as

$$\begin{aligned} &-\frac{w^i}{w^2}\partial_i\log\det g_{(7)}+\Lambda=0\ ,\\ &\frac{1}{4}di_{I_{(6)}}d\log\det g_{(7)}+\frac{2}{\ell}dw=0\ .\end{aligned} \tag{3.9}$$

where Λ is constant and $g_{(7)}$ is the Riemannian metric on X^7. The last equation requires that

$$\mathcal{L}_w i_{I_{(6)}}d\log\det g_{(7)}=0\ . \tag{3.10}$$

To summarize, the metric and the flux of the solutions can be written as

$$\begin{aligned} ds^2_{(10)} &= 2(dudr-\frac{2r}{\ell}duZ^1)+(Z^1)^2+w^2+ds^2_{(6)}\ ,\quad dZ^1=0\ ,\\ Y &= \frac{1}{4}\left(Z^1\wedge(-dw+\frac{2}{l}\omega_{(6)})+w\wedge\xi+\eta\right)\ ,\end{aligned} \tag{3.11}$$

where the 5-form flux F is given in terms of Y as in (2.2). The 2-SCYT structure on the horizon section $\mathcal{S}=\mathbb{R}\times X^7$ is associated with the Hermitian form (3.2) and the geometry is constrained by (3.5), (3.6) and (3.9).

3.1 $SO(2,2)$ invariant backgrounds

It is apparent from (3.11) that the 5-form flux breaks the $SO(2,2)$ isometry of AdS_3 space. This is unless

$$\xi=\eta=0\ . \tag{3.12}$$

In this case, the conditions on the geometry reduce to

$$d\omega_{(6)}=0\ ,\quad dw\cdot\omega_{(6)}=\frac{4}{\ell}\ ,\quad \omega_{(6)}\wedge dw\wedge dw=0\ , \tag{3.13}$$

and (3.9). As we shall demonstrate later for warped products, which includes the direct product case presented here, the $SO(2,2)$ symmetric backgrounds are the same as those presented in [4]. However, the way that the two different descriptions of the geometry of AdS_3 backgrounds are related is non-trivial.

3.2 Examples

Although the description of the geometry on $\mathbb{R} \times X^7$ is simple, the conditions on X^7 that arise from implementing the geometric restrictions are rather involved. Moreover, one can easily see that apparent geometries on X^7, like for example (Einstein) Sasakian, are not solutions. To give some examples, we shall focus on the solutions with $SO(2,2)$ isometry and take that w generates a holomorphic isometry[3] in $\mathbb{R} \times X^7$. Moreover, let us take X^6 to be a product of Kähler-Einstein spaces. Since X^6 is 6-dimensional it can be the product of up to 3 such spaces. Writing the Ricci form of these spaces as $\rho_i = -\lambda_i \omega_i$, where λ_i and ω_i is the cosmological constant and the Kähler form of each subspace, respectively, and taking $\omega_{(6)} = \sum_i \omega_i$, we have that

$$dw = \frac{\ell}{2} \sum_i \lambda_i \omega_i \; . \tag{3.14}$$

Then the second condition in (3.13) gives

$$\sum_i \lambda_i n_i = \frac{8}{\ell^2} \tag{3.15}$$

and the third condition in (3.13) implies either

$$\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3 = 0 \tag{3.16}$$

if $n_1 = n_2 = n_3 = 2$, or

$$2\lambda_1 \lambda_2 + \lambda_2^2 = 0 \tag{3.17}$$

if $n_1 = 2, n_2 = 4$; where n_i is the real dimension of the i-th subspace of X^6. There are solutions to these conditions by taking $X^6 = S^2 \times T^4$ and $X^6 = S^2 \times K_3$. Such examples of 2-SCYT geometries have been investigated in [9] and our result in section 6.1 relates them to those investigated in [15].

4 Direct product AdS$_5$ gravitational duals

The metric is

$$ds^2_{(10)} = 2du(dr - \frac{2r}{\ell} Z^1) + (Z^1)^2 + (Z^2)^2 + (Z^3)^2 + ds^2(X^5) \tag{4.1}$$

where Z^1, Z^2, Z^3 are given in (A.5) and satisfy the differential system (A.6). The AdS_5 subspace is spanned by the coordinates u, r, z, x^k.

Next writing the metric on X^5 as in (2.7), the Hermitian form is chosen as in (2.8), i.e.

$$\omega_{(8)} = Z^1 \wedge w - Z^2 \wedge Z^3 + \omega_{(4)} \; , \tag{4.2}$$

[3]This is not an assumption as we shall demonstrate in section 6.1.

where $\omega_{(4)}$ is a 2-form on X^5 such that $i_w \omega_{(4)} = 0$. The integrability of the complex structure implies (2.10).

The torsion 3-form of $\mathcal{S}_8$ is

$$H = w \wedge dw + \frac{2}{\ell} w \wedge Z^2 \wedge Z^3 + Z^1 \wedge \xi - i_{I_{(4)}} \eta \ , \tag{4.3}$$

where we have decomposed

$$d\omega_{(4)} = w \wedge \xi + \eta \ , \quad i_w \eta = 0 \ . \tag{4.4}$$

Moreover, ξ is (1,1) with respect to $I_{(4)}$ as required by the integrability of the complex structure $I_{(8)}$.

It remains to find the restrictions on the geometry of X^5 imposed by the condition on the Lee form (2.11) and the 2-SCYT condition (2.1) on $\mathcal{S}_8$. First, the Lee condition (2.11) $\theta_{\omega_{(8)}} = -\frac{2}{\ell} Z^1$ gives that

$$-\frac{1}{2} dw \cdot \omega_{(4)} + \frac{4}{\ell} = 0 \ , \quad \xi \cdot \omega_{(4)} = 0 \ , \quad \eta = 0 \ . \tag{4.5}$$

Then, the 2-SKT condition $d(\omega_{(8)} \wedge H) = 0$ gives

$$- w \wedge dw + \frac{\ell}{2} d\xi + \frac{2}{\ell} w \wedge \omega_{(4)} = 0 \ , \tag{4.6}$$

where to derive the above condition we have used the results obtained in (4.5).

It remains to solve the 2-SCYT condition $\hat{\rho} = 0$ (2.5). A straightforward computation reveals the conditions

$$\begin{aligned} & -\frac{w^i}{w^2} \partial_i \log \det g_{(5)} + \Lambda = 0 \ , \\ & \frac{1}{4} d i_{I_{(4)}} d \log \det g_{(5)} + \frac{3}{\ell} dw = 0 \ , \end{aligned} \tag{4.7}$$

where Λ is constant and $g_{(5)}$ is the Riemannian metric on X^5. The last equation requires that

$$\mathcal{L}_w i_{I_{(4)}} d \log \det g_{(5)} = 0 \ . \tag{4.8}$$

To summarize, the solutions can be written as

$$\begin{aligned} ds^2_{(10)} &= 2(du dr - \frac{2r}{\ell} du Z^1) + \sum_{k=1}^{3} (Z^k)^2 + w^2 + ds^2_{(4)} \ , \\ Y &= \frac{1}{4} \left(-\frac{4}{l} Z^1 \wedge Z^2 \wedge Z^3 + Z^1 \wedge (-dw + \frac{2}{l} \omega_{(4)}) + w \wedge \xi \right) \ , \end{aligned} \tag{4.9}$$

where again the 5-form flux is given in terms of Y as in (2.2). The 2-SCYT structure on the near horizon section $\mathcal{S} = H_3 \times X^5$ is with respect to the Hermitian form (4.2) and the restrictions on the geometry are given in (4.5), (4.6) and (4.7).

4.1 $SO(4,2)$ invariant backgrounds.

It is clear that the 5-form flux is not invariant under the full $SO(4,2)$ symmetry of the AdS_5 subspace unless

$$\xi = 0 \ , \quad \omega_{(4)} = \frac{\ell}{2} dw \ . \tag{4.10}$$

As a result X^5 is a Sasakian manifold. The remaining conditions imply that in addition X^5 is Einstein. Thus for $\xi = 0$, X^5 is a Sasaki-Einstein manifold. This class of solutions is well known [13] and include the $AdS_5 \times S^5$ solution of IIB supergravity.

5 AdS$_7$

The metric is

$$ds^2_{(10)} = 2(dudr - \frac{2r}{\ell} du Z^1) + (Z^1)^2 + (Z^2)^2 + (Z^3)^2 + (Z^4)^2 + (Z^5)^2 + ds^2(X^3) \tag{5.1}$$

where Z^1, Z^2, Z^3, Z^4, Z^5 are given in (A.5) and (A.6). The coordinates (u, r, z, x^k) span AdS_7.

Furthermore X^3 is an odd dimensional manifold and the metric can be written as in (2.7), $ds^2(X^3) = w^2 + ds^2_{(2)}$ where $ds^2_{(2)}$ is transverse to w. The Hermitian form is

$$\omega_{(8)} = Z^1 \wedge w - Z^2 \wedge Z^3 - Z^4 \wedge Z^5 + \omega_{(2)} \ , \tag{5.2}$$

where $\omega_{(2)}$ is a 2-form on X^3 such that $i_w \omega_{(2)} = 0$ and it is hermitian with respect to $ds^2_{(2)}$. The integrability of the complex structure implies (2.10). Though the condition $dw^{2,0} = 0$ is automatic in this case.

Next the torsion 3-form can be easily computed to find

$$H = w \wedge dw + \frac{2}{\ell} w \wedge (Z^2 \wedge Z^3 + Z^4 \wedge Z^5) + Z^1 \wedge \xi \tag{5.3}$$

where we have written

$$d\omega_{(2)} = w \wedge \xi \ , \quad i_w \xi = 0 \ . \tag{5.4}$$

Observe that ξ is (1,1), and that the 3-form, η, vanishes identically.

Next imposing the condition on the Lee form, $\theta_{\omega_{(8)}} = -\frac{2}{\ell} Z^1$, one gets

$$-\frac{1}{2} dw \cdot \omega_{(4)} + \frac{6}{\ell} = 0 \ , \quad \xi \cdot \omega_{(2)} = 0 \ . \tag{5.5}$$

The latter condition implies that

$$\xi = 0 \ . \tag{5.6}$$

Next take the 2-SKT condition $d(\omega_{(8)} \wedge H) = 0$ to find

$$w = 0 \ . \tag{5.7}$$

So there are no such solutions.

6 Warped AdS$_3$

Having explained the direct product case $AdS_n \times X^{10-n}$ in some detail, we shall only outline the key points that arise in the derivation of the geometric conditions for the warped products $AdS_n \times_w X^{10-n}$ to be solutions of IIB supergravity. In particular for the AdS_3 case, we have that the Hermitian form is

$$\omega = A^2[Z^1 \wedge w] + \omega_{(7)} \ , \quad dZ^1 = 0 \ . \tag{6.1}$$

To solve the conditions on the Lee form and those required by the 2-SCYT structure, we first compute the skew-symmetric torsion 3-form to find

$$H = -A^2 i_{I_{(6)}}\rho \wedge Z^1 \wedge w + A^2 w \wedge dw + Z^1 \wedge \xi - i_{I_{(6)}}\eta \ , \tag{6.2}$$

where we have used

$$d\omega_{(6)} = w \wedge \xi + \eta \ , \quad d\log A^2 = fw + \rho \ , \quad i_w\eta = i_w\rho = 0 \ . \tag{6.3}$$

The condition on the Lee form (2.15) gives

$$\frac{1}{2}\xi \cdot \omega_{(6)} = -f \ , \quad \frac{1}{2}A^2 dw \cdot \omega_{(6)} = \frac{2}{\ell} \ , \quad \frac{1}{2} i_{I_{(6)}}((i_{I_{(6)}}\eta) \cdot \omega_{(6)}) = 2\rho \ . \tag{6.4}$$

Next the 2-SKT condition $d(\omega_{(8)} \wedge H) = 0$ leads to

$$\begin{aligned}
&A^2 dw \wedge dw \wedge \omega_{(6)} + \lambda \wedge \omega_{(6)}^2 - 2i_{I_{(6)}}\rho \wedge \eta \wedge \omega_{(6)} = 0 \ , \\
&-A^2\rho \wedge dw \wedge \omega_{(6)} - A^2 dw \wedge \eta + \mu \wedge \omega_{(6)}^2 + 2i_{I_{(6)}}\rho \wedge \xi \wedge \omega_{(6)} = 0 \ , \\
&A^2 dw \wedge i_{I_{(6)}}\eta - \pi \wedge \omega_{(6)} - \xi \wedge \eta + A^2 i_{I_{(6)}}\rho \wedge dw \wedge \omega_{(6)} = 0 \ , \\
&-w \wedge d\big(i_{I_{(6)}}\eta A^2\big) - w \wedge d\big(i_{I_{(6)}}\rho A^2\big) \wedge \omega_{(6)} + A^2 w \wedge i_{I_{(6)}}\rho \wedge \eta \\
&-w \wedge \xi^2 - w \wedge \zeta \wedge \omega_{(6)} = 0 \ ,
\end{aligned} \tag{6.5}$$

where

$$d(i_{I_{(6)}}\rho) = w \wedge \mu + \lambda \ , \quad d\xi = w \wedge \zeta + \pi \ , \quad d\eta = -dw \wedge \xi + w \wedge \pi \ . \tag{6.6}$$

It remains to solve the other 2-SCYT condition in (2.1), $\hat{\rho} = 0$. A direct substitution reveals that (2.5) gives

$$\begin{aligned}
&-\frac{1}{4}\frac{w^i}{w^2}\partial_i \log\det g_{(7)} - \frac{5}{4}f + \Lambda = 0 \ , \\
&\frac{1}{4} d i_{I_{(6)}} d\log\det g_{(7)} + \frac{2}{\ell}dw + \frac{5}{4} d i_{I_{(6)}}\rho = 0 \ ,
\end{aligned} \tag{6.7}$$

where Λ is constant.

To summarize, the fields are given by

$$\begin{aligned}
ds^2 &= 2du(dr - \frac{2r}{\ell}Z^1 - r\, d\log A^2) + A^2(Z^1)^2 + ds^2(X^7) \\
Y &= \frac{1}{4}\left(Z^1 \wedge (-A^2 dw + 2A^2 w \wedge \rho + \frac{2}{l}\omega_{(6)}) + w \wedge \xi + \eta + (fw + \rho) \wedge \omega_{(6)}\right)
\end{aligned} \tag{6.8}$$

where the 5-form flux is given in terms of Y as in (2.2). The Hermitian form on $\mathcal{S} = \mathbb{R} \times X^7$ is given in (6.1) and the geometric conditions that restrict the geometry are given in (6.4), (6.5) and (6.7).

6.1 Backgrounds with $SO(2,2)$ symmetry

Requiring that the solution is invariant under the $SO(2,2)$ symmetry of AdS_3, we shall demonstrate that X^7 is a fibration over a 6-dimensional Kähler manifold. Clearly as in the direct product case, the 5-form flux is not invariant under $SO(2,2)$ isometries of the metric. Examining the expression for the 3-form Y as given in (6.8), one find that it is $SO(2,2)$ invariant provided that

$$\eta + \rho \wedge \omega_{(6)} = 0 \ , \tag{6.9}$$

and

$$\xi + f\omega_{(6)} = 0 \ . \tag{6.10}$$

The first condition in (6.4) together with (6.10) imply that

$$\xi = 0, \qquad f = 0 \ . \tag{6.11}$$

In addition, the third condition in (6.4) is implied by (6.9). It is also straightforward to show that the third and fourth conditions obtained from the 2-SCYT condition (6.5) hold automatically, leaving only the first two conditions. These can be written as

$$A^2 dw \wedge dw \wedge \omega_{(6)} + d\big(i_{I_{(6)}}\rho\big) \wedge \omega_{(6)} \wedge \omega_{(6)} + 2\big(i_{I_{(6)}}\rho\big) \wedge \rho \wedge \omega_{(6)} \wedge \omega_{(6)} = 0 \ . \tag{6.12}$$

Note in particular that

$$\mu = 0 \ . \tag{6.13}$$

Also, (6.12) can be rewritten as

$$\tilde{\nabla}^2 \log A = A^{-2}\Big(-\frac{1}{\ell^2} + \frac{1}{8} d(A^2 w).d(A^2 w)\Big) \ , \tag{6.14}$$

where $\tilde{\nabla}$ is the Levi-Civita connection on X^7.

To proceed, using the Einstein equation, we find that the Ricci tensor of X^7 is

$$\tilde{R}_{ab} = \frac{3}{2}\tilde{\nabla}_a \tilde{\nabla}_b \log A^2 + \frac{3}{4}\tilde{\nabla}_a \log A^2 \tilde{\nabla}_b \log A^2 - 4Y_{aij}Y_b{}^{ij} + \frac{2}{3} g_{ab} Y_{i_1 i_2 i_3} Y^{i_1 i_2 i_3} \tag{6.15}$$

where

$$Y = AZ^1 \wedge \left(w \wedge dA + \frac{1}{2\ell} A^{-1}\omega_{(6)} - \frac{1}{4} A dw \right) \ , \tag{6.16}$$

a, b are frame indices on X^7 and i, j are frame indices on $\mathcal{S}$. Note that

$$Y_{n_1 n_2 n_3} Y^{n_1 n_2 n_3} = 6A^{-2} dA.dA + \frac{3}{2\ell^2} A^{-2} + \frac{3}{16} A^2 (dw).(dw) \ . \tag{6.17}$$

Next, set

$$\kappa = A^2 w \tag{6.18}$$

and compute

$$\tilde{\nabla}^2 \kappa^2 = 2\tilde{\nabla}^{(a}\kappa^{b)}\tilde{\nabla}_{(a}\kappa_{b)} + \frac{1}{2}d\kappa.d\kappa + 2\kappa^b\tilde{\nabla}^a(d\kappa)_{ab} + 2\kappa^b\tilde{\nabla}^a\tilde{\nabla}_b\kappa_a \ . \tag{6.19}$$

The terms on the RHS of (6.19) can be simplified further on using

$$\begin{aligned}\kappa^b\tilde{\nabla}^a(d\kappa)_{ab} &= -\tilde{\nabla}^a(\kappa^b d\kappa_{ba}) - \frac{1}{2}d\kappa.d\kappa \\ &= \tilde{\nabla}^2 A^2 - \frac{1}{2}d\kappa.d\kappa \end{aligned} \tag{6.20}$$

and

$$2\kappa^b\tilde{\nabla}_a\tilde{\nabla}_b\kappa^a = 2\kappa^a\kappa^b\tilde{R}_{ab} = -2dA.dA + 2\ell^{-2} + \frac{1}{4}A^4 dw.dw \tag{6.21}$$

where we note that

$$\kappa^a\kappa^b\tilde{\nabla}_a\tilde{\nabla}_b \log A^2 = 2dA.dA \ . \tag{6.22}$$

The LHS of (6.19) can also be rewritten in terms of A, using

$$\kappa^2 = A^2 \ . \tag{6.23}$$

Then, on comparing (6.14) with (6.19), one finds that

$$\tilde{\nabla}^{(a}\kappa^{b)}\tilde{\nabla}_{(a}\kappa_{b)} = 0 \tag{6.24}$$

so κ is an isometry of X, which also satisfies

$$\mathcal{L}_\kappa A = 0, \qquad \mathcal{L}_\kappa w = 0 \ . \tag{6.25}$$

We rewrite the metric on X^7 as

$$ds^2(X^7) = A^2 w^2 + A^{-2} d\mathring{s}^2(B^6) \tag{6.26}$$

where $d\mathring{s}^2(B^6)$ does not depend on the coordinate along the isometry. It turns out that B^6 equipped with $d\mathring{s}^2(B^6)$ and Hermitian form

$$\mathring{\omega}_{(6)} = A^2 \omega_{(6)} \ , \tag{6.27}$$

is a Kähler manifold. This follows from (6.9), (6.10) and (6.11).

Furthermore, decomposing the Ricci tensor in (6.15) along the directions of B^6, one finds

$$\mathring{R}_{\alpha\beta} = \frac{2}{\ell}(I_{(6)})^\gamma{}_\alpha (dw)_{\gamma\beta} \tag{6.28}$$

where $\mathring{R}$ is the Ricci tensor of B^6 and α, β are (real) frame indices on B^6. Thus Ricci form of B^6 constructed from the Kähler metric is proportional to dw.

In addition, the second condition in (6.4) gives

$$(dw) \,.\, \mathring{\omega}_{(6)} = 4\ell^{-1}A^{-4} \ , \tag{6.29}$$

where now the contraction is taken with respect to the Kähler metric on B^6. This together with (6.28) implies that the Ricci scalar of B^6 is

$$\mathring{R} = \frac{8}{\ell^2}A^{-4} \ . \tag{6.30}$$

Next, we return to (6.14). This can be rewritten in terms of the curvature of B^6 as

$$\mathring{\nabla}^2 \mathring{R} = \frac{1}{2}\mathring{R}^2 - \mathring{R}_{\alpha\beta}\mathring{R}^{\alpha\beta} \ . \tag{6.31}$$

This equation has been found before in the context of AdS_3 solutions in IIB supergravity in [4]. Our result establishes a non-trivial relationship between some 2-SCYT manifolds and Kähler manifolds[4]. In particular, if $\mathcal{S}$ is an 8-dimensional 2-SCYT manifold with metric

$$ds^2(\mathcal{S}) = A^2[(Z^1)^2 + w^2] + A^{-2}d\mathring{s}^2(B^6) \ , \tag{6.32}$$

where Z^1, w generate commuting isometries, $dZ^1 = 0$, and with Hermitian form given in (6.1), then B^6 is a Kähler manifold with Kähler form given in (6.27) whose Ricci data satisfy (6.31). Furthermore the curvature dw of the fibration is proportional to the Ricci form of B^6, i.e. the fibration is the canonical fibration, and the warp factor is determined in terms of the Ricci scalar. It is remarkable that the 2-SCYT data quadratic in derivatives turn into an equation 4th order in derivatives on the metric of B^6. A similar equation arises also in 11 dimensions and in [10] the question was raised whether there always exists a solution on Kähler manifolds.

7 Warped AdS$_5$

As in the warped AdS_3 case, the Hermitian form is

$$\omega_{(8)} = A^2(Z^1 \wedge w - Z^2 \wedge Z^3) + \omega_{(4)} \ , \tag{7.1}$$

which gives rise to the torsion 3-form

$$\begin{aligned} H = &-A^2 f Z^1 \wedge Z^2 \wedge Z^3 - A^2 i_{I_{(4)}}\rho \wedge (Z^1 \wedge w - Z^2 \wedge Z^3) \\ &+A^2 w \wedge (dw + \frac{2}{\ell}Z^2 \wedge Z^3) + Z^1 \wedge \xi - i_{I_{(4)}}\eta \ . \end{aligned} \tag{7.2}$$

Using these, one finds that the Lee form condition (2.15) gives

$$-\frac{1}{2}A^2 dw \cdot \omega_{(4)} + \frac{4}{\ell} = 0 \ , \quad \frac{1}{2}\xi \cdot \omega_{(4)} = -2f \ , \quad \frac{1}{2}i_{I_{(4)}}(i_{I_{(4)}}\eta \cdot \omega_{(4)}) = 3\rho \ , \tag{7.3}$$

[4]Not all 2-SCYT manifolds are related to Kähler manifolds in this way.

where f and ρ are defined as in (6.3).

It remains to investigate the two 2-SCYT conditions in (2.1). In particular, $d(\omega_{(8)} \wedge H) = 0$ after some manipulation gives that

$$d\Big(2A^4 w \wedge i_{I_{(4)}}\rho - A^2\xi - fA^2\omega_{(4)}\Big) + \frac{2}{\ell}A^4 w \wedge dw$$
$$+\frac{4}{\ell}A^2\omega_{(4)} \wedge i_{I_{(4)}}\rho - \frac{4}{\ell^2}A^2\omega_{(4)} \wedge w = 0 \ , \qquad (7.4)$$

and

$$d\Big(\omega_{(4)} \wedge \big(-f\omega_{(4)} + A^2 w \wedge i_{I_{(4)}}\rho\big)\Big) = 0 \ , \qquad (7.5)$$

where ξ is defined as in the direct product case in (4.4). Using (2.5), the other 2-SCYT condition (2.1) gives

$$\begin{aligned} -\frac{1}{4}\frac{w^i}{w^2}\partial_i \log\det g_{(5)} - \frac{7}{4}f + \Lambda = 0 \ , \\ \frac{1}{4}di_{I_{(4)}}d\log\det g_{(5)} + \frac{3}{\ell}dw + \frac{7}{4}di_{I_{(4)}}\rho = 0 \ , \end{aligned} \qquad (7.6)$$

where Λ is constant.

To summarize, the fields for warped AdS_5 backgrounds are given by

$$\begin{aligned} ds^2 &= 2du(dr - \frac{2r}{\ell}Z^1 - r d\log A^2) + A^2[\sum_{k=1}^{3}(Z^k)^2] + ds^2(X^5) \ , \\ Y &= \frac{1}{4}\big(-\frac{4}{l}A^2 Z^1 \wedge Z^2 \wedge Z^3 + Z^1 \wedge (-A^2 dw + 2A^2 w \wedge \rho + \frac{2}{l}\omega_{(4)}) \\ &\quad -2A^2 Z^2 \wedge Z^3 \wedge (fw + \rho) + w \wedge \xi + \eta + (fw + \rho) \wedge \omega_{(4)}\big). \end{aligned} \qquad (7.7)$$

where again the 5-form field strength is given in terms of Y as in (2.2). The underlying geometry is 2-SCYT with hermitian form (7.1) and the geometry is restricted as in (7.3), (7.4), (7.5) and (7.6).

7.1 $SO(4,2)$ invariant solutions

As in the direct product case, the flux given in (7.7) is not invariant under the $SO(4,2)$ isometries of the metric. Enforcing $SO(4,2)$ symmetry for the whole background, we find that

$$f = \rho = \xi = \eta = 0 \ . \qquad (7.8)$$

This implies that A is constant. As a result, the warped product becomes direct and the analysis of the $SO(4,2)$ invariant solutions is identical to that given in the direct product case.

8 Warped AdS$_7$

As for direct product AdS_7 backgrounds, we shall show that there are no warped AdS_7 solutions. For this, the Hermitian form and torsion 3-form are

$$\omega_{(8)} = A^2\big(Z^1 \wedge w - Z^2 \wedge Z^3 - Z^3 \wedge Z^4\big) + \omega_{(2)} \ , \tag{8.1}$$

and

$$H = -A^2 f Z^1 \wedge Z^2 \wedge Z^3 - fA^2 Z^1 \wedge Z^4 \wedge Z^5 - A^2\, i_{I_{(2)}}\rho \wedge (Z^1 \wedge w - Z^2 \wedge Z^3 - Z^4 \wedge Z^5)$$
$$+A^2 w \wedge (dw + \frac{2}{\ell} Z^2 \wedge Z^3 + \frac{2}{\ell} Z^4 \wedge Z^5) + Z^1 \wedge \xi \ , \tag{8.2}$$

respectively.

Next the condition on the Lee form (2.15) gives

$$-\frac{1}{2} A^2 dw \cdot \omega_{(2)} + \frac{6}{\ell} = 0 \ ,$$
$$3f + \frac{1}{2}\xi \cdot \omega_{(2)} = 0 \ , \quad \rho = 0 \ . \tag{8.3}$$

This in turn implies that

$$d \log A^2 = f w \ , \tag{8.4}$$

and so

$$df w + f dw = 0 \ . \tag{8.5}$$

Now if $f = 0$, then A is constant and as a result we return to the direct product case. On the other hand if $f \neq 0$, taking the contraction of (8.5) with $\omega_{(2)}$, we find that $dw = 0$ which contradicts the first relation in (8.3). Thus f must be zero and so A is constant leading again to the direct product case. Combining this with our result for the direct product AdS_7 gravitational duals, we have shown that there are no AdS_7 solutions within this class.

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Appendix A From AdS spaces to black hole horizons

It has been shown in [7] under certain conditions that the near horizon geometry of extreme black holes can be written as

$$ds^2 = 2du(dr + rh - r^2\frac{\Delta}{2}du) + ds^2(\mathcal{S}) \ , \tag{A.1}$$

where the 1-form h and function Δ depend on the coordinates of the near horizon section $\mathcal{S}$. In the near horizon calculations, it is assumed that the horizon section $\mathcal{S}$ is compact without boundary. Here, we shall not make such an assumption and allow $\mathcal{S}$ to be non-compact. The metric (A.1) includes both the direct and warped product metrics of AdS or Minkowski spaces with a transverse "internal" space X, respectively. The change of coordinates that takes these metrics to the near horizon form (A.1) is an Eddington-Finkelstein transformation, for AdS_2 and AdS_3 see also [19].

A.1 Direct products

To see this, first consider the direct product metric $ds^2(AdS_n \times X) = ds^2(AdS_n) + ds^2(X)$ for $n > 2$ and focus on the metric of AdS_n. In particular, write

$$ds^2(AdS_n) = dz^2 + e^{\frac{2z}{\ell}}\left(2dudv + \sum_k (dx^k)^2\right) , \tag{A.2}$$

and perform the coordinate transformation

$$u = u, z = z, v = e^{-\frac{2z}{\ell}} r \tag{A.3}$$

to find that

$$\begin{aligned} ds^2(AdS_n \times X) &= ds^2(AdS_n) + ds^2(X) \\ &= 2du(dr - \frac{2}{\ell} r Z^1) + (Z^1)^2 + \sum_k (Z^k)^2 + ds^2(X) , \end{aligned} \tag{A.4}$$

where we have introduce the frame

$$Z^1 = dz , \quad Z^k = e^{\frac{z}{\ell}} dx^k . \tag{A.5}$$

Clearly (A.4) is of the form of near horizon geometry metrics for black holes (A.1) with $\Delta = 0$ and $h = -\frac{2}{\ell} Z^1$. Furthermore observe that the Z frame satisfies the differential relation

$$dZ^1 = 0 , \quad dZ^k = \frac{1}{\ell} Z^1 \wedge Z^k . \tag{A.6}$$

The AdS_2 case is special. For this write

$$ds^2(AdS_2) = \frac{\ell^2}{\rho^2}(-dt^2 + d\rho^2) \tag{A.7}$$

and perform the coordinate transformations

$$t = u + \ell^2 r^{-1} , \quad \rho = \ell^2 r^{-1} , \tag{A.8}$$

to find that the metric $ds^2(AdS_2 \times X) = ds^2(AdS_2) + ds^2(X)$ can be written as in (A.1) for $h = 0$ and $\Delta = \ell^{-2}$.

The direct product metric on $\mathbb{R}^{n-1,1} \times X$ is clearly a special case of (A.1) by taking $h = \Delta = 0$. Of course if $n > 2$, $\mathcal{S}$ should be invariant under the Euclidean group $SO(n-2) \ltimes \mathbb{R}^{n-2}$ so that the full metric on the product is invariant under the Poincare group of the $\mathbb{R}^{n-1,1}$ subspace.

A.2 Warped products

Consider the metric

$$ds^2 = A^2\big[e^{2z/\ell}\big(2dudv + \sum_{k>1}(dx^k)^2\big) + dz^2\big] + ds^2(X) , \tag{A.9}$$

on the warped product $AdS_n \times_w X$, where the function A depends only on the coordinates of X. This restriction is consistent with the requirement that the spacetime metric is invariant under the isometries of AdS_n.

First suppose that $n > 2$ and consider the coordinate transformation

$$v = A^{-2}e^{-2z/\ell}r , \quad u = u , \quad z = z , \quad x^k = x^k , \tag{A.10}$$

to find that

$$ds^2 = 2du(dr - \frac{2r}{\ell}Z^1 - rd\log A^2) + A^2\sum_{k\geq 1}(Z^k)^2 + ds^2(X) , \tag{A.11}$$

where the frame (Z^1, Z^k) is defined as in (A.5). Clearly the metrics on warped products of AdS_n, $n > 2$, spaces are also special cases of near horizon black hole geometries.

It is also easy to see that the warped product metrics on $AdS_2 \times_w X$ are special cases of near horizon metrics (A.1). Writing the metric on $AdS_2 \times_w X$ as

$$ds^2 = 2A^2du(dv - \ell^{-2}v^2du) + ds^2(X) , \tag{A.12}$$

and performing the the coordinate transformation $u = u, r = vA^2$, one can show that it can be re-expressed as the near horizon metric (A.1) with $\Delta = \ell^{-2}A^{-2}$ and $h = -d\log A^2$.

Warped product metrics of Minkowski spaces can also be written as in (A.1). The formulae of the AdS spaces described above can be adapted to the Minkowski spaces by taking the limit of large radius $\ell \to \infty$.

The inclusion of direct and warped products of AdS spaces in the near horizon geometries of black holes has the advantage that all these spaces can be understood in a unified way. In particular, the horizon section $\mathcal{S}$ is now spanned by the frame (Z^1, Z^k) and that of X. Therefore for a $AdS_n \times_w X$ spacetime,

$$\mathcal{S} = H_{n-2} \times_w X , \tag{A.13}$$

where H_{n-2} is the (n-2)-dimensional hyperbolic space. For all $AdS_n \times_w X$ the underlying geometry of $\mathcal{S}$, as specified by the KSEs, is the same irrespective of the AdS_n subspace but depending on the number of supersymmetries preserved. The only difference is that for each $AdS_n \times_w X$ space, one has to consider that $\mathcal{S}$ which is a warped product with the appropriate hyperbolic space.

It is clear that the classification of the local geometries of $AdS \times_w X$ spaces is a special case of that of black hole horizons. As a paradigm, we shall present the solution of both the field and KSEs for $AdS_n \times_w X$ spaces, for n=3,5,7, in IIB supergravity with only 5-form flux utilizing the classification of IIB horizons in [9]. In this context, all $\mathcal{S} = H_{n-2} \times_w X$ spaces are 2-strong Calabi-Yau manifolds with torsion.

Appendix B Heterotic AdS solutions

AdS backgrounds, and particularly AdS_3, arise naturally in the classification of supersymmetric heterotic backgrounds [17] and have been investigated extensively in [18]. Furthermore, it is known that heterotic horizons are either products $\mathbb{R}^{1,1} \times \mathcal{S}^8$, where $\mathcal{S}^8$ is a product of Berger manifolds, or certain fibrations of AdS_3 over 7-dimensional manifolds X^7 which admit at least a G_2-structure compatible with a connection with skew-symmetric torsion. The twisting of the fibration is with respect to a $U(1)$ connection. The G_2 structure appears for solutions preserving 2 supersymmetries and reduces to $SU(3)$, to $SU(2)$ and to $SU(2)$ for horizons preserving 4, 6 and 8 supersymmetries, respectively. Thus the geometry of heterotic AdS_3 solutions can be read off from that of black hole horizons in [8]. Because of this, we shall not proceed to give the details of the various geometries. However, as in the IIB case, we shall examine the symmetry of the heterotic AdS_3 backgrounds.

The fibration structure of AdS_3 over X^7 breaks the $SO(2,2)$ isometry group of AdS_3 to a subgroup which always contains $SL(2,\mathbb{R})$. This can be easily seen in [8]. In the heterotic case $SO(2,2)$ is broken by both the metric and the 3-form flux. To restore the full $SO(2,2)$ symmetry, one has to take the curvature of the fibration to vanish which implies

$$dh = 0 \ , \tag{B.1}$$

where we follow closely the terminology of [8]. In such a case, the solution becomes $AdS_3 \times X^7$. If the solution preserves 2 supersymmetries, then X^7 is a strong, conformally balanced manifold which is compatible with a connection with skew-symmetric torsion, i.e.

$$d\tilde{H}_{(7)} = 0 \ , \quad \theta_\varphi = 2d\Phi \ , \quad \mathrm{hol}(\hat{\nabla}_{(7)}) \subseteq G_2 \ , \tag{B.2}$$

where $H_{(7)}$ is the torsion 3-form on X^7, θ_φ is the Lee form of the fundamental G_2 form, and $\hat{\nabla}_{(7)}$ is the connection with skew-symmetric torsion $H_{(7)}$. In the remaining cases, the description of the geometry for AdS_3 backgrounds with $SO(2,2)$ symmetry follows from that for the black hole horizons of [8] by systematically inserting (B.1) into the equations.

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