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***Abstract*—In this paper the covariance matrix of received voltage signals across ports of a multiport antenna in correlated multipath environments is formulated. This eventually leads to some expressions for mean effective gain and directivity in these environments, which to our best knowledge has not been addressed before. The notion of pattern overlap matrix is utilized to coin correlated pattern overlap matrix. We show the crucial role of the latter parameter in general correlated multipath environments.**

***Index Terms*—correlated multipath; covariance matrix; mean effective gain; mean effective directivity; spatial correlation; correlated pattern overlap matrix.**

I. Introduction

Multiport antennas are indispensable building blocks of the contemporary wireless communication systems and their characterization is the main concern of this paper. The covariance matrix of the received signals across ports of a multiport antenna system in multipath environments plays a significant role in its characterization. For instance, when normalized properly, the diagonal entries of this matrix are the corresponding mean effective gains (MEGs) which are undoubtedly very decisive metrics [1]. To elaborate more, it suffices to say that in case other sources of noise are negligible compared to the receiver noise, the received signal-to-noise ratios at different ports become directly proportional to their MEGs. Furthermore, the off-diagonal entries provide the corresponding spatial covariances between different ports which eventually yield the spatial correlation coefficients. These metrics, in turn, play a crucial role in multiplexing efficiency and diversity gains in a multipath environment.

The covariance matrix relies on different system parameters as well as properties of the associated multipath environment. Concerning system parameters, the covariance depends upon coupling between different radiation elements, array configuration, the structure of different radiation elements and their radiation efficiencies etc. In parallel, limited to uncorrelated multipath environments, the covariance reckons on probability density function of angle of arrival (AoA) for the incoming electromagnetic (EM) waves for both polarizations, and the cross polarization ratio (XPR) [2]. In [1], a compact formula for a single-port antenna's MEG in uncorrelated environments was introduced. Later, the authors in [2] and [3] studied the MEG formula for different statistical models, recast it in other ways, and subsequently derived some useful bounds for it. However, there are a few concerns about the aforemetioned formulas for MEG. First of all, the role of terminating impedances in the presence of coupling has not been properly emphasized. Furthermore, the impact of total radiation efficiencies in a compact multiport system was not shown analytically. Finally, to our best knowledge, all available formulas are solely restricted to uncorrelated multipath environments (e.g., see [4], [5]).

In this paper, we generalize the definition of MEGs to hold also in correlated multipath environments. The roles of the input network parameters, terminating impedances, and total radiation efficiencies are clarified. Note that the former parameter contains overall information about coupling in a radiation structure. In an uncorrelated multipath environment, the crucial role of pattern overlap matrix in received signals' covariance matrix is well known [6, Section 6-4], [7]. In harmony with that, we show that, by slight modifications, a revised version of the pattern overlap matrix still builds up the central core for the covariance matrix in correlated multipath environments.

To set the notations, bold letters denote matrices. Column vectors are shown by an overbar sign. The transpose, conjugate and Hermitian transpose operations are designated by $\cdot^T$, asterisk, and dagger superscripts, respectively. $\mathbb{E}$ signifies the expectation over time or realization. The symbol $\mathbf{I}$ denotes the identity matrix and η stands for the intrinsic impedance of the medium. $\Re$ returns the real part of its argument.

II. Received Voltage Signals in Multipath Environments

Let us denote the input impedance matrix of a multiport antenna by $\mathbf{Z}_{n\times n}$, in which n designates the number of ports. Throughout this paper, we assume reciprocal radiation networks, i.e., $\mathbf{Z} = \mathbf{Z}^T$. The terminating impedances at the antenna ports can be written in a diagonal matrix shown by $\mathbf{Z}_r$. We further denote the open-circuit embedded pattern matrix by $\mathbf{G}_{2\times n}$, whose rows are the corresponding vertical θ polarization and horizontal ψ polarization components associated with different ports. $\mathbf{G}$ is function of angular direction denoted by $\Omega(\theta, \psi)$. Here, θ is the latitude and ψ is the longitude coordinates in the spherical coordinate system. The embedded pattern matrix $\mathbf{G}_r$ in volts is thus

$$\mathbf{G}_r = \mathbf{G} \cdot \mathbf{i} \ , \tag{1}$$

where the columns of $\mathbf{i}$ are the current weight vectors at the ports associated with the corresponding embedded element pattern. If the embedded element patterns are obtained by exciting each port with a 1 volt voltage source, we have

$$\mathbf{i} = (\mathbf{Z} + \mathbf{Z}_r)^{-1} \ . \tag{2}$$

The incident wave from Ω direction is given by $\bar{E}_{2\times1}$, whose components are again $E_\theta(\Omega)$ and $E_\psi(\Omega)$. Using the chosen notations, we can recast the received voltage at the ports of a multiport antenna in an uncorrelated multipath as [8, Equation (3-16)]

$$\bar{v}_r = \frac{2\lambda}{j\eta} \ \mathbf{Z}_r \ \oint_{4\pi} \mathbf{G}_r^T(\Omega) \cdot \bar{E}(\Omega) \ \mathsf{P}(\Omega) \ d\Omega \ , \tag{3}$$

wherein P is the AoA probability density function of the random incoming EM waves. If it is preferable to rely on open-circuit embedded patterns, we may use (1)-(2) to rewrite (3) as

$$\bar{v}_r = \frac{2\lambda}{j\eta} \ \mathbf{Z}_r \ (\mathbf{Z} + \mathbf{Z}_r)^{-1} \ \oint_{4\pi} \mathbf{G}^T(\Omega) \cdot \bar{E}(\Omega) \ \mathsf{P}(\Omega) \ d\Omega \ . \tag{4}$$

Identifying the expression for open-circuit received voltage $\bar{v}_\circ$ in (4), we can write

$$\bar{v}_\circ = \frac{2\lambda}{j\eta} \ \oint_{4\pi} \mathbf{G}^T(\Omega) \cdot \bar{E}(\Omega) \ \mathsf{P}(\Omega) \ d\Omega \ . \tag{5}$$

It should be evident that the relation between the received terminated signals and their open-circuit counterparts is simply $\bar{v}_r = \mathbf{Z}_r \ (\mathbf{Z} + \mathbf{Z}_r)^{-1} \bar{v}_\circ$, which is already known [9].

III. Received Signals' Covariance in Uncorrelated Multipath Environments

Limiting ourselves to zero-mean complex Gaussian random incoming EM waves, we use the expression in (5) for calculation of open-circuit covariance matrix $\mathbf{C}_\circ = \mathbb{E}[\bar{v}_\circ \bar{v}_\circ^\dagger]$ in volts squared. Doing so and after some manipulations we arrive at

$$\mathbf{C}_\circ = \frac{8\lambda^2}{\eta} \oint_{4\pi} \mathbf{G}^T \cdot \mathbf{\Gamma} \cdot \mathbf{G}^* \ \mathsf{P} \ d\Omega \tag{6}$$

wherein $\mathbf{\Gamma}_{2\times2}$ is the polarization matrix of the uncorrelated incoming EM waves. Recall that the polarization matrix has been originally defined in [9, Equation (2)-(3)], [10]. Here, with a slight modification to its initial form, we redefine it as

$$\mathbf{\Gamma}(\Omega', \Omega) = \frac{1}{2\eta} \ \mathbb{E}[\bar{E}(\Omega') \cdot \bar{E}^\dagger(\Omega)] \ , \tag{7}$$

in $\mathrm{watts/m^2}$. In a special case of uncorrelated multipath environment, the incoming EM waves from Ω' and Ω directions are uncorrelated. Thus, $\mathbf{\Gamma} = \mathbf{\Gamma}(\Omega', \Omega)\delta(\Omega' - \Omega)$. Although in (6) we only derived the open-circuit covariance matrix, the relation between the covariance matrix of an arbitrary terminated multiport antenna and its open-circuit counterpart has already been given in [11, Equation (1)].

IV. Received Signals' Covariance in Correlated Multipath Environments

In a tantamount way to the preceding Section, we can generalize the covariance expression in (6) to hold for correlated multipath environments too. By virtue of (7) one can write

$$\mathbf{C}_\circ = \frac{8\lambda^2}{\eta} \iint_{4\pi} \mathbf{G}^T(\Omega') \cdot \mathbf{\Gamma}(\Omega', \Omega) \cdot \mathbf{G}^*(\Omega) \ \mathsf{P}(\Omega', \Omega) \ d\Omega' d\Omega \tag{8}$$

in which P is the joint probability density function for the incoming waves from Ω' and Ω directions. It is further interesting to recast the covariance matrix in terms of the associated directivities. Without loss of generality, let us consider the case of $\mathbf{Z}_r = \mathbf{Z}_\circ = Z_\circ \ \mathbf{I}_{n\times n}$ with $Z_\circ$ being the characteristic impedance of the system, which is commonly of resistive nature. Now, using (3) and expressing the embedded pattern matrix in terms of its associated embedded directivity matrix $\mathbf{D}_r$ which is dimensionless, we can recast the terminated covariance matrix $\mathbf{C}_r$ in watts as

$$\mathbf{C}_r = \sqrt{\mathbf{e}_{\mathrm{tot}}} \ \mathbf{C}_{\mathrm{D}_r} \ \sqrt{\mathbf{e}_{\mathrm{tot}}} \ , \tag{9}$$

wherein

$$\mathbf{C}_{\mathrm{D}_r} = \frac{\lambda^2}{4\pi} \ \iint_{4\pi} \mathbf{D}_r^T(\Omega')\mathbf{\Gamma}(\Omega', \Omega)\mathbf{D}_r^*(\Omega) \ \mathsf{P}(\Omega, \Omega') \ d\Omega' d\Omega \tag{10}$$

and $\mathbf{e}_{\mathrm{tot}}$ is a diagonal matrix of ports' total embedded element radiation efficiencies. For lossless multiport antenna systems, the embedded radiation efficiency equals the multiport matching efficiency. This latter metric was introduced in [12] providing a compact formula for its calculation based on the input network parameters. The expression in (9) is an important one separating out the impacts of radiation efficiencies and the shape of the embedded patterns in conjunction with properties of the multipath environment. Under the restrictions imposed by (1)-(2), the embedded directivity matrix can be further recast in a way to factor out the effects of terminating impedances in $\mathbf{C}_{\mathrm{D}_r}$.

V. Dual-port Dual-polarized Ideal Reference Antenna

As introduced by the author in [1], for normalization of the covariance matrix we need to opt for a suitable reference antenna. For this purpose, a dual-port, dual-polarized ideal isotropic antenna does more than suffice. To achieve maximum available power in a multipath environment, the terminating impedances at the port of this ideal two-port reference antenna should be conjugate matched to its input impedance matrix. For sake of simplicity, let us assume that the input impedance matrix of this ideal reference antenna is $\mathbf{Z}_{\mathrm{ref}} = Z_\circ \ \mathbf{I}_{2\times2}$. With some straightforward manipulations, it follows that the open-circuit embedded pattern matrix for this ideal dual-polarized reference antenna is

$$\mathbf{G}_{\mathrm{ref}} = \sqrt{\frac{\eta Z_\circ}{4\pi}} \ \mathbf{I}_{2\times2} \ . \tag{11}$$

Using the expressions in (1)-(3) and (7), and under presumption of zero-mean complex Gaussian random incoming EM waves, we can derive the total average received power at the ports of this ideal reference antenna as

$$P_{\text{ref}} = \frac{\lambda^2}{4\pi} \left\| \iint_{4\pi} \mathbf{\Gamma}(\Omega',\Omega)\ \mathsf{P}(\Omega',\Omega)\ d\Omega' d\Omega \ \right\|_{\text{F}} , \qquad (12)$$

where subscript F indicates the Frobenius norm.

VI. Normalization of Covariance Matrix

A proper normalization is essential for a fair study of received signals' covariance matrix. In this Section we first present an expression for the covariance matrix in a general multipath environment. Later, we offer equivalent expressions for two particular multipath environments which are of practical interest.

For a general multipath environment, using (8), [11, Equation (1)] and (12), the normalized arbitrary terminated covariance matrix $\mathbf{C}_{r_n}$ becomes

$$\mathbf{C}_{r_n} = \frac{1}{2P_{\text{ref}}} \times$$
$$\Re[\mathbf{Z}_r]^{-\frac{1}{2}} \mathbf{Z}_r (\mathbf{Z}+\mathbf{Z}_r)^{-1}\ \mathbf{C}_\circ\ (\mathbf{Z}+\mathbf{Z}_r)^{-1^\dagger} \mathbf{Z}_r^\dagger\ \Re[\mathbf{Z}_r]^{-\frac{1}{2}} \qquad (13)$$

Here the subscript n only signifies the normalized metric. Since the number of ports has also been shown by n, we may not allow this subindex to create any source of confusion. The diagonal entries of (13) are the corresponding MEGs at different ports. That is

$$\mathbf{M}_{\text{G}} = \text{diag}[\mathbf{C}_{r_n}] \qquad (14)$$

where $\mathbf{M}_{\text{G}}$ is the diagonal matrix of MEGs associated with different ports. This is a quite general expression. The off-diagonal entries in (13) represent the spatial correlation between different ports which after further normalization yield the corresponding renowned spatial correlation coefficients [7].

A. Case of Uncorrelated Multipath

Uncorrelated multipath environments are commonly referred to those multipath environments wherein incoming waves of different AoAs and those of the same AoAs but different polarizations are uncorrelated. In these environments the polarization matrix in (7) is diagonal. The cross-polarization ratio (XPR), denoted by χ, has been defined for these environments signifying the amount of polarization power imbalance of the incoming EM waves. In other words,

$$\mathbf{\Gamma} = \begin{bmatrix} \Gamma_{\theta\theta} & 0 \\ 0 & \Gamma_{\psi\psi} \end{bmatrix} = \Gamma_{\psi\psi}\ \begin{bmatrix} \chi & 0 \\ 0 & 1 \end{bmatrix} \quad \text{(Uncorrelated Multipath)} \qquad (15)$$

with $\Gamma_{\theta\theta}$ and $\Gamma_{\psi\psi}$ being the steradic power densities of the incoming EM waves in θ and ψ polarizations, whose ratio renders the XPR. In the frame of this paper, we assume that XPR is independent of angular direction Ω. Using the aforementioned covariance matrix in (8) and the reference power in (12), and by virtue of (1) and (2), we can derive the corresponding arbitrary terminated normalized covariance matrix as

$$\mathbf{C}_{r_n} = \frac{16\pi}{\eta}\ \Re[\mathbf{Z}_r]^{-\frac{1}{2}} \times$$
$$\mathbf{Z}_r \oint_{4\pi} \mathbf{G}_r^T \begin{bmatrix} \frac{\chi}{1+\chi} & 0 \\ 0 & \frac{1}{1+\chi} \end{bmatrix} \mathbf{G}_r^*\ \mathsf{P}\ d\Omega\ \mathbf{Z}_r^\dagger\ \Re[\mathbf{Z}_r]^{-\frac{1}{2}} \qquad (16)$$

It should be evident that the MEGs in uncorrelated environments are the diagonal entries in (16). Furthermore, expressing the corresponding embedded element patterns in terms of the associated directivities reveals a compact expression for mean effective directivity (MED) matrix $\mathbf{M}_{\text{D}}$ and its relation with $\mathbf{M}_{\text{G}}$ in (14). To find out more about this relation, let us restrict ourselves to $\mathbf{Z}_r = \mathbf{Z}_\circ$. This leads to

$$\mathbf{M}_{\text{G}} = \sqrt{\mathbf{e}_{\text{tot}}} \cdot \mathbf{M}_{\text{D}} \cdot \sqrt{\mathbf{e}_{\text{tot}}}\ , \qquad (17)$$

in which

$$\mathbf{M}_{\text{D}} = \text{diag}\left[\oint_{4\pi} \mathbf{D}_r^T \begin{bmatrix} \frac{\chi}{1+\chi} & 0 \\ 0 & \frac{1}{1+\chi} \end{bmatrix} \mathbf{D}_r^*\ \mathsf{P}\ d\Omega \right]\ . \qquad (18)$$

Until now the MED definition has been limited to uncorrelated multipath environments [13]. Yet, based on the expressions provided in the frame of this paper, we are able to redefine it for general cases of correlated multipath environments too. For this purpose, one only needs to take the diagonal entries (i.e., diag) in (10) when normalized by (12).

B. Case of Isotropic Multipath

An uncorrelated multipath environment of uniform AoA (i.e., $\mathsf{P} = 1/4\pi$) and balanced polarization (i.e., $\chi = 0$ dB) is referred to as isotropic multipath environment [13]. An isotropic multipath environment is important more due to the fact that it is the only type of multipath environment that can be created simply in a well stirred reverberation chamber. The measurement results in this case are not subject to considerable variations due to repetition and hence are most reliable ones. Inserting the necessary parameters (e.g., $\chi = 0$ dB and $\mathsf{P} = 1/4\pi$) into (13), we arrive at

$$\mathbf{C}_{r_n} =$$
$$2\ \Re[\mathbf{Z}_r]^{-\frac{1}{2}} \mathbf{Z}_r (\mathbf{Z}+\mathbf{Z}_r)^{-1}\ \mathbf{C}\ (\mathbf{Z}+\mathbf{Z}_r)^{-1^\dagger} \mathbf{Z}_r^\dagger\ \Re[\mathbf{Z}_r]^{-\frac{1}{2}} \qquad (19)$$

where the *pattern overlap matrix* $\mathbf{C}$ is [14]

$$\mathbf{C} = \frac{1}{\eta} \oint_{4\pi} \mathbf{G}^T \cdot \mathbf{G}^*\ d\Omega\ . \qquad (20)$$

Recall that in (20), $\mathbf{G}$ represents the open-circuit embedded pattern matrix. It is not difficult to show that the diagonal entries in (19) equal $\frac{1}{2}\ \mathbf{e}_{\text{tot}}$. This fact was anticipated in [13] and subsequently demonstrated in [2]. Table 1 in [15] indirectly shows the links between MEGs and the total embedded element efficiencies with regards to the selected reference antenna.

One point about the pattern overlap matrix in (20) merits further regards. Indeed, it has been said that for single-mode lossless multiport antennas, the pattern overlap matrix is a real matrix [9]. In this way, it can approximate the real part of the corresponding input impedance matrix [7, Equation (8)]. That is, $\mathbf{C} = \Re[\mathbf{Z}]$. This is of considerable help when one needs to calculate the covariance matrix in isotropic environments solely in terms of the input network parameters.

VII. Correlated and Uncorrelated Pattern Overlap Matrices

Referring back to the normalized terminated covariance matrix in (19) which is dimensionless, we stress on the central role of the pattern overlap matrix in this important expression. Note that although this expression has been derived for isotropic multipath environments, with slight modifications, it can be applied for both uncorrelated and correlated multipath environments too. For this purpose, only the pattern overlap matrix $\mathbf{C}$ has to be replaced by its properly modified versions referred to as *uncorrelated pattern overlap matrix*, $\mathbf{C}_{\mathrm{u}}$, and *correlated pattern overlap matrix*, $\mathbf{C}_{\mathrm{c}}$. The uncorrelated pattern overlap matrix is obtained by

$$\mathbf{C}_{\mathrm{u}} = \frac{8\pi}{\eta} \oint_{4\pi} \mathbf{G}^T \begin{bmatrix} \frac{\chi}{1+\chi} & 0 \\ 0 & \frac{1}{1+\chi} \end{bmatrix} \mathbf{G}^* \; \mathsf{P} \; d\Omega \; . \tag{21}$$

Remember that in (21), χ is a constant independent of angular direction Ω. Similarly, the correlated pattern overlap matrix is achieved through

$$\mathbf{C}_{\mathrm{c}} = \frac{8\pi}{\eta} \frac{\iint_{4\pi} \mathbf{G}^T(\Omega') \; \mathbf{\Gamma} \; \mathbf{G}^*(\Omega) \; \mathsf{P} \; d\Omega' d\Omega}{\left\| \iint_{4\pi} \mathbf{\Gamma}(\Omega', \Omega) \; \mathsf{P}(\Omega', \Omega) \; d\Omega' d\Omega \right\|_{\mathrm{F}}} \; . \tag{22}$$

In (20)-(22) some arguments were intentionally dropped for sake of conciseness. It follows that the correlated pattern overlap matrix in (22) reduces to (21) in uncorrelated multipath environments and to (20) in isotropic multipath environments.

Let us spend a few moments reviewing some important points. Recall the dimensionless normalized covariance matrix in (19). The diagonal entries in this matrix are the corresponding ports' MEGs while its off-diagonal entries are the spatial correlations between the received signals at different ports. The latter when further normalized yields the corresponding spatial correlation coefficients which are more known in the literature. It is of importance to observe that the pattern overlap matrix is the core of the aforementioned expression containing also the necessary information about the properties of multipath environments. The normalized covariance matrix in a general correlated multipath environment can be obtained by replacing $\mathbf{C}$ in (19) with $\mathbf{C}_{\mathrm{c}}$ in (22). Note also that the impact of terminating impedances upon MEGs and spatial correlations is entirely described by (19). As a quick reminder, different pattern overlap matrices are credible under zero-mean complex Gaussian random incoming EM waves restriction. In a similar way to Section VI-A, one can recast the pattern overlap matrices (20)-(22) in terms of the corresponding embedded directivities. This is left for the interested reader.

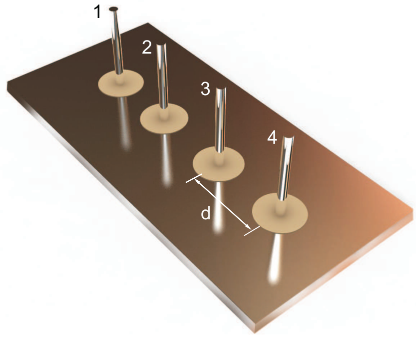
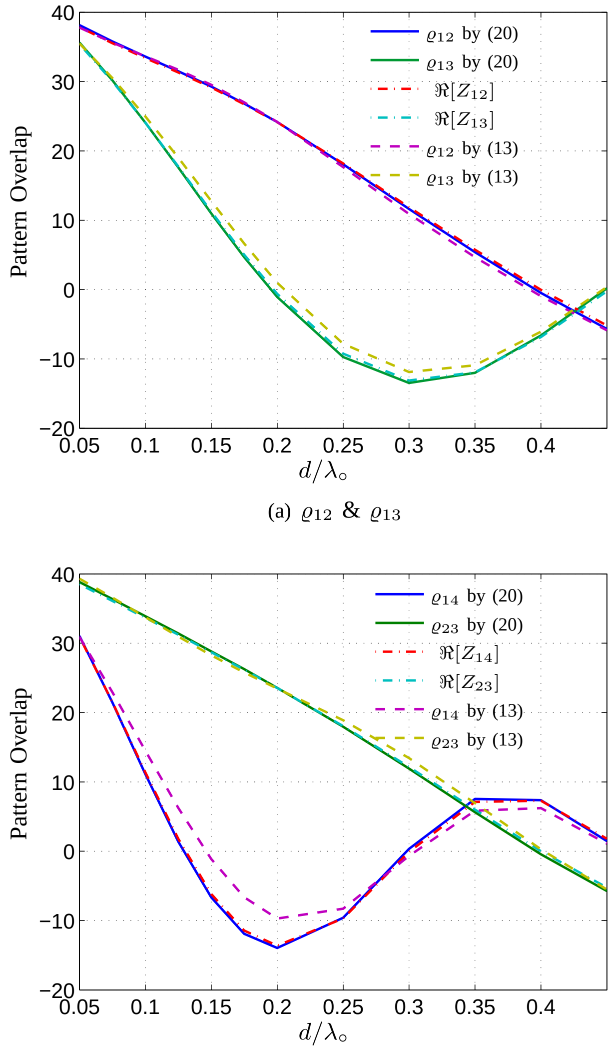


Fig. 1. Four equidistant lossless monopoles above a perfect electric conductor (PEC) plane. Parameter d denotes the element separation in this structure.

VIII. Simulation

The main goal in this Section is to choose an arbitrary multiport antenna and verify the formulas presented in the frame of this paper. For this purpose, we chose four quarter wavelength equidistant lossless thin monopoles above a perfect electric conductor (PEC). The resonance frequency of these identical monopoles slightly exceeded $f_\circ = 1$ GHz ($\lambda_\circ = 0.3$m). The element separation is denoted by d. This multiport radiation structure, which is illustrated in Fig. 1, is an actual example of lossless single-mode antennas. For sake of simplicity we selected $\mathbf{Z}_r = \mathbf{Z}_\circ$ for these simulations. The embedded element patterns $\mathbf{G}_r$ and the associated input network parameters (e.g., $\mathbf{Z}$) were all obtained by the well known full-wave method of moments (MoM) [16, Section 8.4]. We first emulated a random multipath scenario and exposed the embedded patterns of this multiport antenna to the incoming EM waves. Subsequently, we calculated the received voltage signals. This process of realization is repeated to achieve sufficient number of random voltage samples across the antenna ports. To leave one more option for verification of the results, which shall be cleared in a moment, we restricted ourselves to an isotropic multipath environment. In each realization, the number of random zero-mean complex Gaussian incoming waves incident on this antenna structure was 300 being sufficient for convergence in covariance [15]. The total number of realizations (or random voltage samples at the ports) was 10^4 rendering the desired accuracy. Further elaboration and details of this simulation method can be found in [8, Chapter 4].

Upon calculation of the received random signals, the pattern overlap matrix $\mathbf{C}$ could be achieved by virtue of equation (19). In parallel, it could also be directly obtained by the expression provided in (20) and the open-circuit embedded patterns by the full-wave MoM simulation. In addition, as already pointed out, since this structure is a single-mode lossless structure, $\mathbf{C}$ should approximate the resistive part of input impedance matrix. This stands as an



(a) ϱ_{12} & ϱ_{13}

(b) ϱ_{14} & ϱ_{23}

Fig. 2. Pattern overlap entries ϱ versus element separation d for four lossless monopoles above a PEC plane at $f = 1$ GHz.

extra option for verification of our analysis. Note that due to the symmetry in the structure, only four independent entries represent the covariance matrix. This considerably simplifies the presentation of the results. The outcome of these simulations are illustrated in Fig. 2 where the entries of the normalized covariance matrix versus the element separation at the frequency $f = f_\circ$ are plotted. As evident, the agreements between different curves are remarkable to a negligible error caused most likely by slight inaccuracy inherent in our numerical computations.

IX. Conclusion

In this paper, we provided a general compact expression for the normalized covariance matrix of an arbitrary multiport antenna system in correlated multipath environments. The formulas have been cast in a way to separate out the impact of terminating impedances and the total embedded radiation efficiencies. More for practical interests, two compact expressions for covariance matrices in uncorrelated and isotropic multipath environments were presented. The notion of pattern overlap matrix, which plays a central role in the normalized covariance matrix, was extended to general correlated multipath environments. The latter has led to definitions of two novel parameters referred to as correlated and uncorrelated pattern overlap matrices. Based on the aforementioned achievements, we could also generalize the definitions of MEGs and MEDs to correlated multipath environments. Some of the presented expressions were verified by a simulation tool which had already been well developed in the literature [8].

References

[1] T. Taga, "Analysis for mean effective gain of mobile antennas in land mobile radio environments," *IEEE Transactions on Vehicular Technology*, vol. 39, no. 2, pp. 117–131, 1990.

[2] A. Alayon Glazunov, A. Molisch, and F. Tufvesson, "Mean effective gain of antennas in a wireless channel," *IET Microwaves, Antennas & Propagation*, vol. 3, no. 2, pp. 214–227, March 2009.

[3] A. Alayon Glazunov, "Theoretical analysis of mean effective gain of mobile terminal antennas in ricean channels," in *in Proceedings of 56th IEEE Vehicular Technology Conference*, vol. 3, 2002, pp. 1796–1800.

[4] K. Kalliola, K. Sulonen, H. Laitinen, O. Kivekas, J. Krogerus, and P. Vainikainen, "Angular power distribution and mean effective gain of mobile antenna in different propagation environments," *IEEE Transactions on Vehicular Technology*, vol. 51, no. 5, pp. 823–838, September 2002.

[5] J. Krogerus, C. Icheln, and P. Vainikainen, "Dependence of mean effective gain of mobile terminal antennas on side of head," in *European Microwave Conference*, vol. 3, October 2005.

[6] K. F. Warnick and M. Jensen, "Antennas and Propagation for Wireless Communications," *course materials in Brigham Young University (available online)*, 2011.

[7] N. Jamaly, A. Derneryd, and Y. Rahmat-Samii, "A revisit to spatial correlation in terms of input network parameters," *IEEE Antennas and Wireless Propagation Letters*, vol. 11, 2012.

[8] N. Jamaly, "Spatial Characterization of Multi-element Antennas," Chalmers University of Technology, available online at http://publications.lib.chalmers.se/records/fulltext/137635.pdf, Tech. Rep., 2011.

[9] R. Vaughan and J. Andersen, "Antenna diversity in mobile communications," *IEEE Transactions on Vehicular Technology*, vol. 36, no. 4, pp. 149–172, November 1987.

[10] R. G. Vaughan, "Signals in mobile communications, A review,," *IEEE Transactions on Vehicular Technologies*, vol. 35, no. 4, pp. 133–145, 1986.

[11] R. Vaughan and N. Scott, "Closely spaced terminated monopoles for vehicular diversity antennas," in *IEEE Antennas and Propagation Society International Symposium, 1992*, vol. 2, July 1992, pp. 1093–1096.

[12] N. Jamaly and A. Derneryd, "Efficiency characterization of multi-port antennas," *Electronics Letters*, vol. 48, no. 4, February 2012.

[13] P.-S. Kildal and K. Rosengren, "Correlation and capacity of MIMO systems and mutual coupling, radiation efficiency, and diversity gain of their antennas: simulations and measurements in a reverberation chamber," *IEEE Communications Magazine*, vol. 42, no. 12, pp. 104–112, December 2004.

[14] S. Stein, "On cross coupling in multiple-beam antennas," *IRE Transactions on Antennas and Propagation*, vol. 10, no. 5, pp. 548–557, September 1962.

[15] N. Jamaly, M. Iftikhar, and Y. Rahmat-Samii, "Performance evaluation of diversity antennas in multipath environments of finite richness," in *2012 Proceedings of the 6th European Conference on Antennas and Propagation (EUCAP)*, March 2012, pp. 368–372.

[16] C. A. Balanis, *Antenna Theory: Analysis and Design, 2nd Edition*. John Wiley & Sons, 1997.