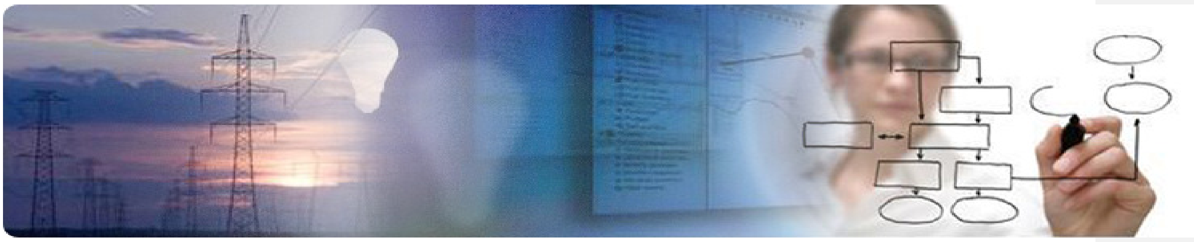

Data Driven Medium Term Electricity Price Forecasting in Ontario Electricity Market and Nord Pool

Thesis for the Degree of Master of Science

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"It began with hope and belief"

To all people I love

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Having accurate predictions on market price variations in the future is of great importance to participants in today's electricity market. Many studies have been done on Short Term Price Forecasting (STPF). However, few works can be found in the literature with their main focus on predictions of electricity price in medium term horizon. Generally speaking, Medium Term Price Forecasting (MTPF) has applications where there exist markets for electricity with medium term contracts (e.g., forward/future contracts); Risk management and derivative market pricing, balance sheet calculations, and inflow of "finance solutions" are a few examples of these applications.

The goal of this project is to predict the next 12 months monthly average electricity prices in the electricity market of Ontario and Nord Pool. To do so, mathematical models that are known to be capable of predicting series with acceptable accuracy using the limited number of samples available, such as Linear Regression Model (LR), Radial Basis Function Neural Network (RBF-NN), Support Vector Machine (SVM), and Weighted Nearest Neighbor (WNN) are employed. First, different attributes of each market have been studied and the most informative ones, those that can better address future behavior patterns of the price, have been identified. Then, different input parameters designs for each model within each market have been examined. For example, the effect of previous month's price, month indicator, Ontario demand, temperature and gas price is studied. For each market, different models' forecasting results are compared and the most accurate ones are ranked for each market. Following this approach, 12 months ahead electricity prices in both markets have been forecasted. The Mean Absolute Percentage Error (MAPE) for each model in each market is calculated by dividing the difference between forecasted and actual price of a month, by its actual price. In the case of Nord Pool different models have ended up to relatively similar results, with the WNN being the best with MAPE of 11.95% and LR the worst with MAPE of 17.23%. Due to more volatility characteristics of Ontario market, there is greater difference between different models results. Hence identification of appropriate model to predict the price in this market is of greater importance. In this market, the SVM with MAPE of 13.17% and WNN of 32.95% turn out to be the most and least accurate models, respectively.

It can be concluded from the study that, in contrary to STPF, models that are only based on price data are incapable of capturing the price trends in medium horizon. The study results also show that different features on each model's performance in each market (e.g., inclusion of temperature data to predict the price of market of Nord Pool using the SVM) play roles with different degrees of significance in the results of the models. Ontario demand, for instance, is recognized as an important factor to be included in models to achieve acceptable results, whereas inclusion of temperature data into input features set of the LR model, deteriorates this model's accuracy.

Keywords: Auto regressive Process, Forecasting, Nearest neighbor searches, Neural Networks, Support vector machines.

Contents

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Acronyms:

A	Actual value
AA	Annual Averaged value
ANN	Artificial Neural Networks
APE	Absolute Percentage Error
ARIMA	Auto Regression Integrated Moving Average
DV	Dummy Variables
Gs	Natural Gas monthly average Price
HOEP	Hourly Ontario Energy Price
HD	Hydro Content Derivative
IESO	Independent Electricity System Operator
ISO	Independent System Operator
LR	Linear Regression Model
MAPE	Mean Absolute Percentage Error
MCP	Market Clearing Price
MI	Month Indicator
MTPF	Medium Term electricity Price Forecasting
MTLF	Medium Term electricity Load Forecasting
NFD	Number of Free Days in a month
OD	Ontario Demand
PP	Previous month Price
SVM	Support Vector Machines
STPF	Short Term electricity Price Forecasting
STLF	Short Term electricity Load Forecasting
Temp	monthly average Temperature data
TSO	Transmission System Operator
WNN	Weighted k Nearest Neighbor

Chapter 1

Introduction

DUring the past two and half decades, reforms in the electricity sector of many countries around the globe have come into effect. As a result, liberalization in monopole electricity markets has become widespread. Hence, modeling and forecasting the electricity price is gaining concern, especially in alliance with the emergence of increasing trends towards the share of financial trades in these markets.

Electricity is inherently non storable; the balance between the supply and demand needs to be maintained at all times. Electricity demand is inelastic against variations in power supply, specifically in short periods of time. Moreover, price series are non-stationary, that is, its mean value varies over time. Therefore, financial models previously developed for other commodities (i.e., AR and ARMA) are no longer applicable to the problem of electricity price forecasting due to the distinctly different characteristics that the electricity price exhibits. It is a complex task to fulfill and becomes more difficult by incorporating exogenous factors like Market design, tariffs, bidding/pricing strategies, etc.

Kommentar [ss1]: Did you mean short periods?or short time consumptions

In fact, two decisions should be made effectively in forecasting the market price:
-Selection of the most informative price attributes to address the market price behavior,
-Selection of the appropriate forecasting model capable of predicting the price using the provided data.

There are three major approaches that can be observed in electricity price forecasting: time series models, simulation models, and game theory based models. In linear models, price prediction is accomplished by using historical data. Simulation one's are mostly an extension of traditional production cost-based optimization. Game theory based problem is the one in which they model gaming of market participants and find optimum solutions of those games. However the last two models are not applicable to the problem of Medium Term electricity Price Forecasting (MTPF), as they are costly to implement and require detailed system operation data which is not always easy to provide. Time series approaches consists of 3 main subgroups: Linear regression based models-including, GARCH, ARMA, ARIMA, etc.- Nonlinear heuristic models-including ANN, Fuzzy and Chaotic models- and Stochastic (jump diffusion) models. Aggrawal et al, have suggested 40 different factors that are reported to be affective for short-term price prediction, and categorized them into 5 main groups. Among these groups we found nonstrategic uncertainties (i.e., Forecasted load, temp), behavior indices (i.e., historical price), and temporal effects (i.e., holiday code, month indicator) to be applicable to the problem of medium term price forecasting [1], [2].

Authors in other studies [3], [4] and [5], using Wavelet, Extended Kalman filter and parallel input output hidden Markov model, respectively, decomposed the price data into well behaved series and ended up with interesting results. However, implementation of these methods to predict the price in long-term horizons is difficult, considering the minimal number of available samples, in addition to the inherent error and uncertainty of the results of these methods, [6]. Moreover, not many of these models can be adopted to predict the price, as they lack a simple factor, which is capable of explaining the price comprehensively using a limited number of samples available for young existing markets, like the one from Nord Pool.

According to the literature there exists a series of works concerning the problem of medium term load forecasting, but there has been no work done in terms of the price-forecasting problem. Chen et al. [7], succeeded in predicting the next month's daily peak load using SVM as their forecasting engine. Amjady et al. in[8], committed similar attempts using an ANN based model. There are other examples of load prediction with medium term horizons in the literature, however, despite the similarities between load and price, the latter has distinct characteristics that makes it different from the former. For instance in a same market, during same period, price series reveals significant volatile characteristics compared to the load []. Moreover, power infrastructure in most countries had been installed many years ago; hence their load profile, as well as other attributes has become mature. Their markets, on the other hand, are mostly a decay (?) old, which means they still are in their premature era. Therefore, there are less available samples for the system price comparing to the load. One important aspect of load prediction is to design a model, which is capable of identifying the most informative input features among numerous numbers available in price features. This is in contradiction to the case of medium price forecasting, where we lack the informative features and appropriate number of samples. Moreover, there are other factors that directly affect the price but not the load (i.e., Emission tariffs, transmission lines capacities and contingencies, Fuel price variations) and there are still no methods developed that are capable of estimating these factors with adequate accuracy in the long run, especially those which are related to financial indexes. Therefore, Medium Term electricity Price Forecasting is a completely distinct problem from that of the load and is becoming a major research field in electrical engineering.

According to (you can name the guy as well) [9], research is divided into statistical and fundamental models. Briefly saying, fundamental electricity price models are based on the equilibrium electricity models for the electricity market, while in the statistical ones, properties of process are being described based on a set of parameters. In statistical models electricity price is predicted directly by estimating the parameters of the price using historical data. Applications of statistical models are possible where plenty of historical data is available, i.e., financial markets. In the case of electricity price, as mentioned earlier, there are different statistical models that have been used in short-term price forecasting. In fundamental models however, electricity price is obtained from a model that is comprised of expected production cost and consumption of electricity. It can be concluded that it would be possible to use favorable parts of both models, that is, fundamentals that are affecting the price are modeled as stochastic factors that affect the price. In this way, climate data is estimated, possibly from a longer historical time period. Afterwards, using forecasted temperature data besides the historical load data, dependent parameters for demand and supply are estimated. Finally, market equilibrium parameters are estimated from the historical price data and the corresponding fundamental data given in the first two steps. Although this model is accurate, not all the information they used is public, especially in the new deregulated privatized market, either in supply or demand. Also, their model depends on climate factors, which are prone to being misleading due to the unpredictable behavior of weather variations, especially in the long run, which in turn increases the risk of error.

Pardo et al. introduced a different approach in forecasting load with short-term horizons [10]. In this article, the influences of temperature and seasonality on the Spanish system energy consumption in the short-term horizon are studied. What makes this method unique is their new approach to the problem that is, factors affecting the load directly, such as temperature, calendar factors, and number of free days, are combined to form the predictive model while their forecasting engine is a simple linear regression model.

(name the guys) [11] based on the model developed by the Spanish group, the Finnish group predicted the next month's daily load consumption. Later, by introducing simple changes to the model, they predicted the next year's monthly load. This model is simple, easy to implement, and accurate. Both aforementioned models have been reported to perform perfectly using a limited number of samples. Especially in the case of monthly forecasting in the Greek market [11], where historical data of the 10 previous years has been used to predict the next month's price with considerable accuracy. However, [10],[11] & [9] have recognized the temperature data as an influential factor. As the actual value of these parameters were not available at the prediction time, and there is no accurate forecasting of these values for such a long horizon; to accomplish predictions, several meteorological scenarios have been composed, according to which load prediction is accomplished. Although predictability of these models is proven to be high, due to the sensitivity of price series to climate factors, forecasting based on different scenarios will lead to a wide range of prices, which makes it useless.

Kommentar [ss2]: Did you mean "Finish" (people from finland?) if that's the case put it at the beginning of the sentence instead of the name of the guys, but leave the ref there

In [12], using several different linear time series models (i.e., ARIMA) the Spanish group succeeds in predicting next month's price in the electricity market of Spain. Although they've achieved promising results, their prediction horizon and so the input design is mostly similar to the problem of STPF. Moreover, they have examined their model during the period that the economy was stable and almost all affecting factors were following an unchanged pattern, which was the condition most markets were at, before the negative impact of the recent worldwide economic recession came into effect on the electricity markets.

In this article, predictability of four different forecasting models in 2 different markets is examined.

These two markets are selected mainly due to similarities that exist between them: both markets are still in their prematurity era and a large share of their generation capacity is based on hydro and nuclear based units.

In spite of similar characteristics that exist between these markets, there are differences that clearly cause distinction. Market price volatility, participants, share of coal and gas units, amount of energy consumed and pattern of climate variation's are a few of them.

Later in the context of this work, models that are employed to predict the price in these markets will be introduced. However, before finishing this chapter, it is good to bear in mind that "there are no good models, there are only useful models" as Nunes et al. quoted in [12].

The rest of this work is organized as follows, in section II, several applications of Medium Term electricity Price forecasting (MTLF) are discussed. In III, specifications of each market as well as its contracts are briefly reviewed. In section IV, models we have employed for forecasting are mathematically explained. Numerical results and different models' result comparison is brought in section V. Section VI concludes the work and future works are described.

Chapter 2

2. Applications of MTPF

2.1 Literature review

Where ever there is a market for electricity with medium term time of delivery (e.g., forward/ future contracts) there will be a need for forecasting the prices. Risk management and derivatives pricing, balance sheet calculations and inflow of "finance solutions", maintenance scheduling, coordination of limited generation units, cost efficient fuel purchasing policies and reducing financial risks by hedging both Gencos and retailers against market's inherent risks are a few applications of MTPF as mentioned in [8], [13], [14], [15]. Accurate MTPF is also used as the basis for risk management at both retailer and generation level.

In fact, future price in financial markets are related to the expected spot price in the future. This means that future prices are derived by applying the effects of risk premium on the expected spot price in the future [16]. In [17] Tanlapco et al., using given forward contracts and expected price and variance, have studied different risk minimizing hedging strategies (fully hedging, direct hedging and cross hedging). [18], Carrion et al. proposed a risk constraint stochastic programming framework for retailers to determine maximum profit considering pre-specified risk levels on profit volatility. By applying similar approaches later, the Spanish group in [19] studied the optimal involvement of generation units in a future market in a way that they will be hedged against volatile behavior of the market. In both cases, expected (forecasted) electricity price in forward and pool markets are assumed to be known and is used as an input feature of their proposed models.

Futures markets have several applications in restricted electricity markets. Market participants reduce the risk of their contracts by making long-term contracts on one hand, and introduction of risk management on the other. The price of these contracts is recognized as an indicator of investment in power system infrastructure. Therefore a better understanding of these markets is vital for both power companies and financial market players to be able to trade through, and make well-made markets, which are viable for the long run [20]. Therefore, the importance of having a better understanding of market behavior in the future, which implies more accurate forecasts on market price, seems inevitable.

System expansion is based on marked based resources in competitive energy markets. Among all factors, expected energy price is the main driver in all market based expansion projects. Thus, new generators are allocated in high price areas, and new transmission lines should be built across congested areas; in [20], the Portuguese group described a new approach for Generation Expansion Planning (GEP) which based on, generation companies can decide whether to invest in new assets. To do so, it is mentioned there that some-so called sources of uncertainty, determining future operation of market (i.e., predicted price), should be taken into consideration. As the result, inherent volatile behavior of the market is incorporated.

As there are delays in investment in markets with this amount of uncertainty, expectations take time to be updated and construction of a new generation unit takes a long time, power market cannot respond to immediate changes and new needs [21]. Therefore, it is of great importance to investors to have a better prediction of market variations in far future in different time steps, especially to compute the economic attractiveness of investing in each type of technology at each stage.

In [22], optimum generation schedule and volume of a bilateral contract to be signed by a hydro generation unit for pre-specified time period is determined so it can maximize its profit. Therefore, expected averaged spot price is assumed to be an exogenous variable. Different possible scenarios are studied which resulted in calculation of expected values and variances. It can be intuitively said that having more accurate predictions on the price, results in lower variance and so a smaller risk in decision making.

In addition to generation, MTPF has applications in transmission management. The main goal in classical transmission planning was to ensure the reliable supply of power to demand. However, introduction of deregulation and emergence of new market based electricity systems, made changes in objectives of TP so the economical consideration became one of the main factors, hence transmission investors will benefit from accurate MTPF and LTPF[23], [24], [20].

In [23] Latorre et al. have presented a complete literature review on transmission expansion planning. Theory and software that have been developed so far on this topic are stated to be far below the practical needs. This is mainly due to the difficulty that exists in studying different aspects of planning, such as uncertain characteristics of a competitive market, necessity of taking different possible scenarios for each factor (i.e., price) into account and existence of different agents with different point of views. Thus, it is important to have a more accurate prediction on market price and load in the long run.

2.2 Nord Pool:

More than 60% of contracts in Nord Pool are traded with settling period of a quarter (**Fel! Hittar inte referenskälla.**). This illustrates important of having a better understanding of market behavior in future especially in medium term horizons (i.e., 1 to 12 months ahead) which was one of our incentives for conducting this research.

Fig. 1. Share of different contracts in Nord Pool, 2007

2.3 Regulated Price Plan in the Market of Ontario

In addition to the Nord Pool, MTPF has applications in the Market of Ontario. There are two kinds of contracts for residential (i.e., home owners) and certain designated consumers (including municipalities, universities, hospitals, farmers and charitable organizations) in the electricity market of Ontario. In retail contracts, the electricity price is paid based on bilateral agreement between consumers and the retailer, based on per KWh electricity price which is usually guaranteed for couple of years. However, most Ontario's electricity consumer's payments are calculated based on Regulated Price Plan (RPP). RPP is an electricity price plan, which ensures stable electricity pricing in the electricity market of Ontario [25]. The main objective of setting this plan is to encourage energy conservation. Customers' payment transparency; that is to ensure consumers' payments be a better reflector of GENCOS delivery.

Every six months, the Ontario Energy Board (OEB) regulates the base price: once in summer (May 1st to October 31st) and once in winter (November 1st to April 30th). For each period within a year; a threshold price, which is the amount of electricity, which, based on consumers will be charged according to the lower price (in summer period, it is set to be 600 kWh per month and while in winter it is 1000 kWh per month in 2010). If a customer consumes beyond this amount (e.g. more than 600 kWh in June), the price for the extra usage will be calculated based on higher rates [25], [26], [27]. If a difference between actual price that is paid to the generation utilities and the forecasted price realized during the period, it would be calculated and reflected in the future RPP, hence all consumers will pay or receive this difference. Therefore, it is important to have a more accurate prediction on market price: The more accurate price predicted, the more stable and risk averts the market would be.

In the market of Ontario, RPP is set according to forecasted price that is provided by Navigant consulting (NCI). To do so, a statistical approach is employed to forecast hourly-based Ontario electricity price (HOEP). Later we will use Navigant Co.'s forecasting results published in April 2009, and compare it with proposed model's forecasting results to evaluate different models' merit. [25].

Chapter 3

3. Markets and Contacts

3.1 Nord Pool

From 1991 to 2000, Norway, Sweden, Finland, and Denmark began to reform their electricity sectors and opened their electricity market as a response to the emerging need for introduction of competition in the industry. In parallel, Nordic power exchange has been developing. At the beginning, it only served the Norwegian market (1993). Three years later, Swedish and Norwegian markets merged and formed a single market. Finland, West and East Denmark joint this market in following years. It is the first multinational electricity market in the world, which is owned by Swedish and Norwegian transmission system operators and a great close cooperation exists with the owners and other Scandinavian TSO's. So far, Nord Pool is evaluated as successful a market, according to industry representatives and electricity market analysis [1], [28], [29].

There are three kinds of contracts in Nord Pool: Elspot, Eltermin and Elbas. On Elspot, physical power contracts are traded on an hourly basis, one day prior to the day of delivery. Price is calculated in auction trading. Observe that transmission lines capacity limitations are considered in this auction and so, this market implicitly is a capacity auction on the interconnectors between bidding areas. That is, whenever transmission lines become congested, each country's market separated to several pricing areas. Note that unconstraint whole area wide price is called the system price and is the basis for financial trades in the market.

Elbas, is a continuous cross border intra-day market. Trading in Elbas is available from the first hour the Elspot is closed till one hour prior to the hour of delivery. This market is the first cross border intra-day market in Europe, which includes Germany besides the Scandinavian countries. It is an alternative for balancing market; hence it reduces risks impact on the market.

Eltermin is the market of financial contracts. These contracts are being held for predefined amount of power, at an agreed price to be traded during an agreed interval. Financial contracts are used for hedging purposes. There is mutual insurance in alliance to obligations that both parties have taken out. These contracts are categorized in four groups: Forward, Future, Contract for Difference and option contracts, which are all organized by Nord Pool ASA.

Forward and future contracts are of our interest in this article. Future contracts are settled daily during trading and delivery period, with daily (Only Nordic countries) and weekly time of delivery while forward contracts settled only during delivery period. They are traded on monthly (744h), quarterly (2209h) and yearly (8760h) time basis. They both are available for "Base Load" and "Peak Load" with minimum trading volume of 1 MW. System price is the contract reference for all financial contracts in the market. The price difference between contracted price and actual system price at the time of delivery times the amount of power to be transferred (e.g., assume system price being larger than the future contract) is the loss that would have been imposed to one side (retailer) and now is hedged by compensation that is covered by the other side (supply) [30].

3.2. Ontario

Align to global acceptance of deregulation in the electricity sector around the globe, the Ontario electricity act of 1998, opened the Ontario wholesale electricity market on May 1st, 2002. This market consists of physical markets for energy and operating reserve, and a financial transmission rights market.

All market entities that have direct connection to transmission network must participate in this market. Market participants are categorized as dispatchable (?) and non-dispatchable. Those loads with capacity of more than 1 MW in addition to the generation units that are capable of following the Independent Electricity System Operator (IESO) instructions, are categorized as dispatchables and the rest are non-dispatchables.

The IESO is a non-profit organization regulated by Ontario Energy board and is responsible for operation of the market. For financial settlements, every 5 minutes a uniform province wide Market Clearing Price (MCP) is determined. This price is applicable only to dispatchable participants, while non-dispatchables operate base on the hourly average of these MCPs (called the Hourly Ontario Energy Price, HOEP). As Zareipour et al. mentioned in [31], Ontario electricity market is highly volatile. Thus, to increase stability of market price, eligible consumers at retail level should obey Regulated Price Plan (RPP). As mentioned earlier, up to a certain power consumption threshold for each month, RPP consumers are charged by a lower (6.5 Cent/kWh, by fall of 2010) rate and beyond that, by the upper one (7.5 Cent/kWh, by fall of 2010). Note that the wholesale market price is applied to consumers with more than 250 MWh/year power consumption [32], [25].

Chapter 4

4. METHODS

There is no superior forecasting model due to large differences which exists in methods of developing the price in different markets and characteristics of each of these markets [33]. To conduct the MTPF effectively, there are difficulties, which one should overcome, such as finding the most appropriate informative exogenous input factors and efficient methods that are capable of conducting the forecasting using limited available data. As mentioned earlier, electricity price series are highly volatile. Thus finding a model that is capable of capturing their nonlinear, non-stationary behavior is always not that easy. In this section, we introduce mathematical models we used to achieve this goal. All these models are known to be capable of dealing with non-stationary time series, using least number of samples that are available and still result in promising forecasting [7], [10], [11], [34], [35], [36], [37]. It will be shown here that these models are still useful even if such volatile series as electricity market price is applied to them to predict the magnitude and trends of this variable in the future.

4.1. Linear Regression Model

In our first article [37], a hybrid Stochastic- Auto regressive model was introduced; similar model is employed here. As we will explain later, for each market, different input feature designs are examined and the most accurate practical model is approved.
Simple linear regression model is selected due to its accuracy and capability in estimating time series. Its fast convergence (approach to results) using a limited number of available samples validates this choice.
Linear regression forecasting models are generally in the following format:

$y = \beta 0 + \beta 1 * x1 + \beta 2 * x2 + \cdots + \beta k * xk + \varepsilon \quad (1)$

In which, y is the variable to be predicted (prediction variable), and x_i, i=1,2,…,k are predictor variables (A.K. A. explanatory variables or features).
Here, y is modeled by linear combination of predictor variables. Note that the difference between the forecasted and actual price value is considered by the residual term ($\boldsymbol{\varepsilon}$) in the model.
In cases where a functional relationship between the variable to be predicted and the independent variable is unknown, using linear regression, adequate approximations of the independent variable resulted over a certain range. Parameter Estimation is done by reducing the error calculated using the famous linear least square method.
Assume $n > k$ observation are available, let X_{ij} denotes the i^{th} observation of the j^{th} feature.

The observations are:

$$x_{i,1}, x_{i,2}, \ldots, x_{i,k}, y_i, i = 1, 2, \ldots, n \text{ and } n > k$$

Each observation satisfies equation (1) above. The Data matrix can be written as follow:

$$y = \beta 0 + \sum_{i=1}^{i=k} \beta i * xi + \varepsilon \qquad (2)$$

Residuals ε are supposed to be uncorrelated with normal distribution of zero mean and constant-unknown variance. “n” independent observation (X1,y1), (X2,y2), …, (Xn,yn) of the predictor x and response variable y as given in table 1 build the linear regression model as n-by-p system of equation as follow:

$$\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} f_1(x_1) & \cdots & f_p(x_1) \\ \vdots & \ddots & \vdots \\ f_1(x_n) & \cdots & f_p(x_n) \end{pmatrix} \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_p \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_p \end{pmatrix}$$

Error term defines as:

$$e_i^2 = \left(y_i - \beta_i - \sum_{i=1}^{k} (\beta_j * x_{ij}) \right)^2 \qquad (3)$$

for each of the n observations.

Using the least square model, we want to minimize L with respect to $\beta 0, \beta 1, \dots, \beta k$ .The least square must satisfy the following condition with respect to each of the coefficients:

$$\frac{\partial L}{\partial \beta_0}|_{\beta_0,\beta_1,\dots,\beta_k} = -2 * \sum_{i=1}^{n}\left(y_i - \beta_0 - \sum_{j=1}^{k}\beta_j * x_{ij}\right) = 0$$

$$\frac{\partial L}{\partial \beta_j}|_{\beta_0,\beta_1,\dots,\beta_k} = -2 * \sum_{i=1}^{n}\left(y_i - \beta_0 - \sum_{j=1}^{k}\beta_j * x_{ij}\right)_{x_{ij}} = 0$$

Where L (least square function) is:

$$L = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n}\left(y_i - \beta_i - \sum_{j=1}^{k}\beta_j * x_{ij}\right)^2$$

The Linear Regression Models are based on certain assumptions, like; observed error terms are normally distributed. If the distribution of errors is asymmetric or prone to outliers, these assumptions do not stand anymore and estimated coefficients are not applicable for the forecasting. In these cases, another fitting method called the robust regression model can be used, which is less sensitive than ordinary least squares to large changes in small parts of the data. It automatically assigns a weight to each data point using a process called iterative reweighted least squares. All points ought to be assigned equal weights at first iteration. At subsequent iterations weights are recomputed in the way that points located further from model’s predictions in the previous iteration are given lower weights [38].

To avoid residuals to be dependent, any trends should be justified before the data can be applied to the linear model. As a linear increasing trend observed in the Nord Pool price series, in proposed model, a constant term is considered which represents this trend. . In Ontario on the other hand, as no such trend exists in this market´s electricity price, this factor is not considered in this model. This will be discussed later in Chapter 5.

4.2. Support Vector Machine (SVM):

SVM is a strong technique for data classification and regression. Its extended version (Support Vector Regression) has been applied to time series predictions applications. Given training data in the following format:

$$\{x_i, y_i\} \text{ where } i = 1, 2, \dots, L, y_i \in R, x \in R^D \qquad (4)$$

SVM solves the optimization problem:

$$\min \frac{1}{2} \omega^T \omega + C \sum_{i=1}^{L} (\xi_i^* + \xi i) \qquad (5)$$

$$\text{subject to } y_i - (\omega^T \Phi(x_i) + b) \le \varepsilon + \xi_i^*,$$

$$(\omega^T \Phi(x_i) + b) \le \varepsilon + \xi_i,$$

$$\xi_i^*, \xi_i \ge 0, i = 1, 2, \dots L$$

"D" is the dimension of input set (i.e., number of features), L is the number of samples, ξ and ξ^* are slack variables which are outside the ε insensitive tube $|y_i - (\omega^T \Phi(x_i) + b) \le \varepsilon|$, and ω is the normal vector of hyper plan that is solved for minimizing (4).

Function, which maps x to higher space (Φ), Cost of error (C) and width of the insensitive tube (ε) are parameters to be used to control SVM performance to minimize the error (i.e., $\xi_i$or ξ_i^*) in the objective function (5). This means most data would be fitted inside the ε insensitive tube.

Both training error $(C \sum_{i=1}^{L} (\xi_i^* + \xi i))$ and regulation term $\left(\frac{1}{2} \omega^T \omega\right)$ ought to be minimized to avoid the training data to be over or under fitted.

As input data sets are not usually linearly separable in the main forecasting space input data x_i should mapped to higher dimension space so the data become separable/ repressible and the SVM became applicable. To make it easier, the dual equivalent of the main minimization problem usually is solved:

$$min_{\alpha,\alpha^*} (\alpha - \alpha^*)^T Q(\alpha - \alpha^*) + \varepsilon \sum_{i=1}^{L} (\alpha + \alpha^*) + \varepsilon \sum_{i=1}^{L} y_i (\alpha - \alpha^*) \quad (6)$$

$$\text{subject to } \varepsilon \sum_{i=1}^{L} (\alpha - \alpha^*) = 0,$$

$$0 \le \alpha_i, \qquad \alpha_i \le C, \qquad i = 1, 2, \dots, L$$

Since $Q_{ij} = \Phi(x_i)^T \Phi(x_j)$ is usually too costly to solve, kernel trick is employed in the mapping. That is to employ special forms so inner products of the functions in higher space can easily be calculated in the original space. There are different kinds of Kernels each is appropriate for a set of problems, however in this article we use the RBF kernel defined as $\Phi(x_i)^T \Phi(x_j) = e^{\gamma |x_1 - x_2|^2}$. This kernel maps the original data into infinite dimensioned space, which makes it an appropriate asset to deal with highly volatile nonlinear data. A prepared library for support vector machine (LIBSVM) is used to implement the SVR to conduct forecasting using this method [7], [36], [39].

4.3.Radial Basis Function Neural Network (RBF NN)

RBF NN is generally composed of three layers, one input, one hidden, and one output layer. There is one node for each predictor variable (i.e., input feature) in the input layer. Hidden layer constitutes of hidden nodes with Gaussian transfer function that transform features and extract informative data. The output layer derives the linear combination of outputs of hidden layer's nodes (A.K.A., influence of each neuron). In (5), (X,Y) is an N dimensional learning sample where $X = (X1, X_2, \dots, X_N)$ and $Y = (Y_1, Y_2, \dots, Y_N)$ are input and output vectors, respectively and $X_i = (x_1, x_2, \dots, x_n)$. N is the number of observaitions. Having input vector X, the output of the network is calculated by:

$$\hat{y} = \sum_{i=1}^{m} \omega_i \phi_i(X, \sigma_i, C_i) \qquad (7)$$

Where $\phi_i(X, \sigma_i, C_i) = \exp(-\|x - c_i^2\| / \sigma_i^2\ )$ denotes radial basis function of the i th hidden node with center of $Ci \in R^n$ and width of $\sigma_i \in R$; ǁ ǁ denotes Euclidean norm. Observe ω_i is the linear output weight. The RBF network places neurons in the space described by predictor variables. Then influence of each neuron is calculated by implementing the RBF kernel function to the distance between the point to be evaluated and the center of each neuron. Adjustable parameters hence are Ci, σ_i and $\omega_i$for each of the m nodes of the hidden layer. For an input Xi, the error is defined as:

$$e_i = y_i - \hat{y}_i \qquad (8)$$

The objective function is to find C_i, σ_i in the way that error is minimized [34], [35], [40] . In this paper error is calculated using least square method. Hence the RBF NN objective is to minimize the total error, that is:

$$\min E = \frac{1}{2}\sum_{i=1}^{N} e_i^2 \qquad (9)$$

RBF neural network is a forward network and recognized by remarkable properties such as being free of local minima and capable of approximating globally. Note that it has so far being used widely for multi-input, single-output networks.

4.4. Weighted K nearest Neighbor Technique

KNN is categorized as lazy learning method in which no process regarding to training the model is done prior to implementing the main forecasting. That is, there are no separated training and test stages in application of this model [40]. In [41] and[42], Weighted K Nearest Neighbor techniques is proven to be capable of predicting day-ahead hourly prices with acceptable accuracy. In these articles, for any arbitrary day “i”, a matrix representing of 24 hourly prices of “m” consecutive days is composed:

$$PP_i = [P_{i-m+1}, P_{i-m+2}, \dots, P_{i-1}, P_i] \quad (10)$$

Where the size of PPi is $(24 * m)$.

Using Euclidean norm, distance between any two days “i” and “j” is defined as:

$$Dist(i,j) = |PP_i - PP_j| \qquad (11)$$

Based on (11), a set of k nearest neighbors of day “d” are identified: $NS = \{q_1, q_2, \dots, q_k\}$, in which q_1 and q_v refer to the closest and furthest neighbors, respectively. Using (12), weighted averages of k days following those in NS are calculated which equals 24 hourly predicted price of the forecasting day:

$$P_{m+1} = \frac{1}{\sum_{i \in NS} \alpha_i} \cdot \sum_{i \in NS} \alpha_i * P_{i+1} \qquad (12)$$

Where weighting factor are obtained from

$$\alpha_i = \frac{dist(q_k, m) - dist(i, m)}{dist(q_k, m) - dist(q_1, m)} \qquad (13)$$

Note that the idea here is, if PP_i is close to PP_m, then PP_{i+1} already known is similar to PP_{m+1}.

However, as it will be shown later in this work, this model (i.e., defining neighborhood merely based on previous months prices) is not strong enough to be used for volatile price series with small number of samples available-which are the constraints of the problem we are facing. Hence we introduce a modified WNN model in which the definition of neighborhood is modified to some extent.

For any arbitrary month “m”, a vector $A_m$composed of most informative attributes regarding that markets monthly price is considered.: $A_m = [f_1, f_2, \dots, f_n]$ where “f_i” are attributes other than previous month’s price. Then, to forecast the price of each month, the distance between forecasting month and its preceding months is calculated” using (14):

$$Dist(i,j) = \|A_i - A_j\| \qquad (14)$$

Similar to the previous model, according to (14), a set of k nearest neighbors of month “m” are identified: $NS = \{q_1, q_2, \dots, q_k\}$, in which q_1 and q_v refer to the closest and furthest neighbors, respectively.

Weighted average price of the closest months to the forecasting month is calculated using:

$$P_{m+1} = \frac{1}{\sum_{i \in NS} \alpha_i} \cdot \sum_{i \in NS} \alpha_i * P_i \qquad (14)$$

In which the weighting factor is obtained by (13). Observe that the idea behind this model is slightly different from the conventional WNN. Here we assumed that in order to predict the price of the month "m", if A_i is close to A_m, then P_i (instead of P_{i+1}) contributes in predicting the price of month P_m (instead of P_{m+1}).

Note, to avoid the effect of noises on forecasted price, it is better to set $k \geq 10$. Therefore, as will be shown later, once we predict the price using the value of k determined from validation set analysis results. Once again, price is forecasted by assuming k=10, regardless of the validation period analysis outcomes. Finally, prediction results of both models will be compared.

Chapter 5

5. Numerical Results

Generally saying, factors that mostly affect the price series are categorized as structural, behavioral, operational, historical and external classes. As each market has its own characteristics, the most affecting factors should be determined based on the market´s specifications. In this section, among different factors that may have impact on the price, most informative ones are identified. Hence, different parameter setting and input feature designs are ought to be examined to derive the most accurate prediction that is possible via each model. That is, it is very likely that some informative attributes in a model turn out to be redundant or even misleading in the others Moreover, there are several features that contain overlapped data and hence should be excluded from the input vector so only the most informative independent features are selected., Therefore, according to each market´s characteristics the most influential explanatory factors on the market's price is determined. Then the most informative ones are selected. Hence, each input feature design should be applied to different possible parameters configurations of the model to determine the optimum design. At the end, a comparison between different models´ prediction results is conducted and the best method for each market is recognized.

The available monthly prices (prior to the first forecasting months) are divided in two sets called: training set and validation set. Each model is trained by the training data. Validation set is kept hidden to the model during its training period and is being used to examine its predictability. Hence this period should be as much similar as possible to the main forecasting period so the models prediction on validation period would be a good measure of its predictability on the main forecasting period.

As it is mentioned earlier, each model within each market, with different feature and different parameters design ought to be trained. Then each model´s predictability is evaluated during prediction period and based on that, the optimum design for each model within each market is determined. Moreover, for nonlinear models, a constant multiplier is employed to improve the models´ performance. These constants are derived from validation period analysis. Constant are set to scale down predicted values of each model. As variations in electricity markets´ financial conditions are slow enough, we can assume that all constant remain unchanged all through validation period [26]. Its mainly due to the conservation exists in electricity market which in turn decreases the probability of abrupt variations in the price, in contrary to other economic sectors.

To evaluate our results, Absolute Percentage Error (APE) for each month is calculated by (15). For each model, the Mean Absolute Percentage Error (MAPE) is also calculated by taking the average of the APEs on the whole forecasting period which is used as a comparison between two different models performance (16). Finally, standard deviation of APEs is calculated as an indicator of correlation between MAPE calculated for the period and APE of each month within that period. This indicates to how extent the MAPE is a good representative for the model's predictability in each forecasting months. Moreover, it's a measure for stability of the results: The more stable the results are, the more reliable the prediction model is. Standard deviation is determined by (17):

$$APE_i = |P_{(i,f)} - P_{(i,a)}| / |P_{(i,a)}| \qquad (15)$$

$$MAPE = \frac{1}{n}\sum_{i=1}^{n} APE_i \qquad (16)$$

$$StD = \left(\frac{1}{n} * \sum_{i=1}^{n}(APE_i - MAPE)^2\right)^{\frac{1}{2}} \qquad (17)$$

Where $P_{i,a}$ **and** $P_{i,f}$ are actual and forecasted market prices, respectively.

In Nord Pool, the year prior to the forecasting year (2008) is considered as the validation period. In Ontario, as we are in lack of adequate number of samples, besides the volatile price behavior especially in vicinity of the forecasting months as a consequence of the worldwide financial recession occurred by mid 2008, less correlation exists between prices of consecutive months. Hence, instead of 12, the last 6 months prior to the first forecasting months are selected as the validation months.

Eventually, observe that as our goal is to predict next 12 months price, for each month, previously forecasted price of preceding months is considered as the actual price and is incorporate to the model to predict that month's price.

[As mentioned earlier, in the rest of this section, we report each models performance using different input factors. Then, the most informative features for each model are determined and the results are is analyzed.

In the diagram below (Fig. 2), general scheme of the proposed forecasting method is depicted. For each month to be forecasted, the forecasting engine is trained using training data. Then, its performance is evaluated by the validation set. In fact validation period has two applications; to identify optimum input feature and to find the optimum forecasting engine's parameter design.

Due to importance of validation period, it is added to the training period to train the model derived from the validation analysis to conduct the final forecasting.

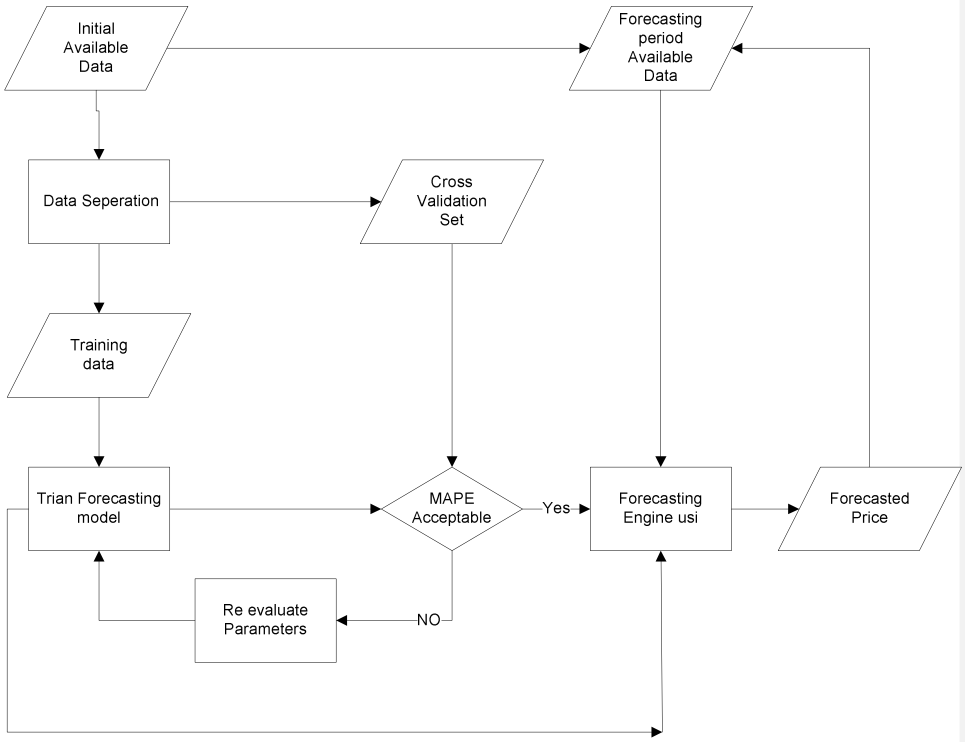

Fig. 2. Forecasting Engine Block Diagram

5.1.Nord Pool

The most informative factor in predicting the price in almost any market is the price of the previous month (PP) to be forecasted [16]. It contains information on market characteristics, especially regarding to those that vary slowly from one month to the next, such as financial condition. Moreover, inclusion of this factor simplifies the problem that the model should solve; that is, it should estimate price variations from one month to the next based on input data, instead of predicting the absolute price value. Thus, this factor is considered as one the input features in all of our models. In order our model to be practical, previous months' prices that are previously forecasted considered as they are actual value of those month to forecast next month's price. That is, to predict P_{m+1}, $\widehat{P_m}$ replace P_m where the former is forecasted and the later is the actual price of the month m, respectively.

Over a half of the generation capacity in Nord Pool market is based on hydro units. Therefore, introduction of a factor representing hydro reservoir in the area is essential. However, as mentioned in [36], electricity price of a month in this market is more correlated with variations of hydro reservoir content variations from one month to the next, than the absolute value of this variable. Thus, the first derivative of hydro reservoir over time is applied. To be practical, actual hydro content during forecasted period should not be used. Unfortunately, no prediction on this value in such horizon has been provided in the market. As forecasting this parameter with acceptable accuracy is not in the scope of this work, for each month, the average hydro content value of that month from 2000 to 2008 is calculated and replaced its actual value in 2009. For instance, Hydro reservoir content in March 2009 is replaced by the average value of hydro contents of March in 9 consecutive years (i.e.,2000 to 2008). This value is called Annual Averaged Hydro Derivative data (AAHD).

As it can be seen in Fig. 3, price abnormaliy increases in months 36 (Dec 2002) and 80 (Sep 2007). In the former spike occurs due to the sudden fall of hydro reservoir, and in the later, lower hydro reservior level- comparing to year before and the one after- causes abnormal rises in price. Note hydro reservoir derivative can also be used to detect months with abnoraml level of reservoir comparing to nearby years (i.e., outlier months).

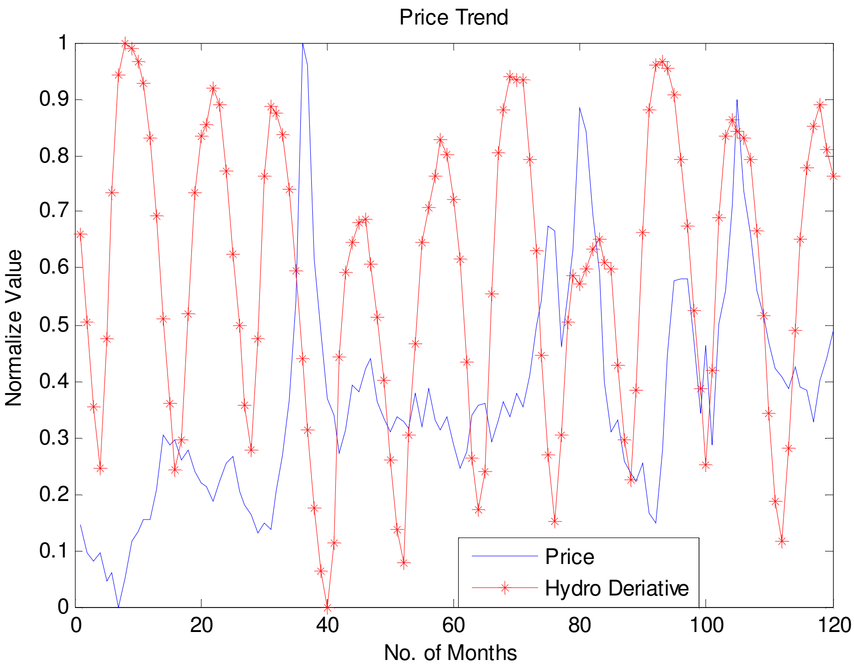

Fig. 3. Monthly Price against Hydro Content Variations in Nord Pool

Electricity price in Nord Pool shows uniform seasonal property with cold (high) and hot (low) seasons (consumption) which illustrates the need for indicator(s) representing this property. Considering the fact that weather variations (Effect of seasonality) has a grear impact on the market price, 11 dummy variables we've used in [36], are employed here once more due to their capability in addressing seasonality to the model. Hence, during each month, its respective variable becomes one, and will be set zero otherwise. As mentioned in [10], [11], to make regression models more stable, (s-1) variables are used to denote (s) periods. All variables will be zero for the 12th month (Here January is selected as this month).

Since energy consumed in non working days is less than regular days [43], a factor indicating number of non-working days in each month should be considered as well. A side from common free days, there are occasions which a certain day is a non/working day in a country (e.g., independent day) but not in the others. As we need one variable representing number of nonworking days in the whole market, weighted average of number of nonworking days in the 4 countries for each month is calculated. Weights are defined as the ratio of gross power consumed in one country to the total power consumed in the whole region in a year. Therefore, for each country, a number representing its share of consumed power in each year is derived. Our analysis is based on data provided by [45].

As it can be seen in Fig. 4, there is long-term increasing trend exists in the price series, which is represented with a “t” factor in the LR model.

To improve each model´s performance, two modifications are needed to be done on the input attributes. In the LR model, to reduce the impact of heteroskedasticity that exists due to high seasonal frequency, natural logarithm of price series is applied to the model instead of the actual value. As this normalizing method deteriorates other models performance, instead of taking natural logarithm, all variables are scaled to the range between zero and one inclusive ([0,1]) in all other models.

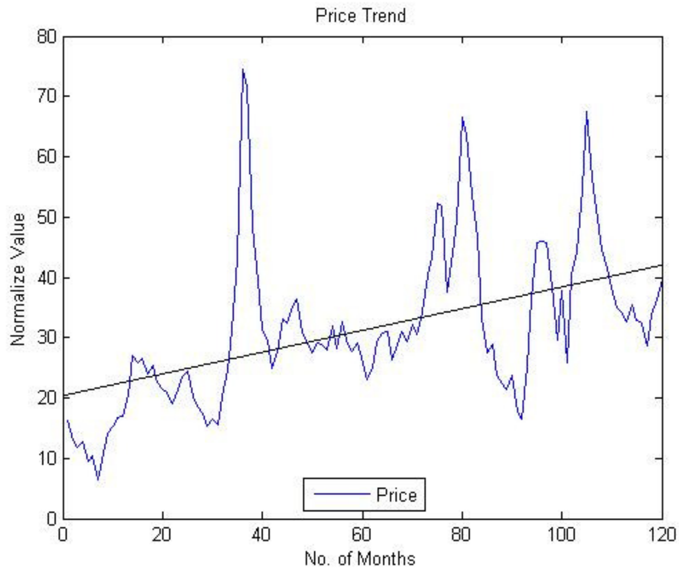

Fig. 4. Monthly averaged price trend

5.1.1. Linear Regression Model

In [36], the merit of this model has been reported. Here, once again we employ this model and compare its performance with the others. In addition to that, the effect of temperature data on this model’s prediction accuracy is investigated.

TABLE I. Prediction Results of Linear regression model in Nord Pool

Features	C1	C2	MAPE
PP, AHD, NFD, DV, t, ATemp	1	0.9	17.18
PP, AAHD, NFD, DV, t, ATemp	1	0.9	21.25
PP, AAHD, NFD, DV, t, AATemp	1	0.9	17.56
PP, AHD, NFD, DV, t, ATemp	0.99	1	21.05
PP, AAHD, NFD, DV, t, AATemp	0.99	1	21.05
PP, AAHD, NFD, DV, t	1	0.8	8.46
PP, AHD, NFD, DV, t	1	0.8	13.15
PP, F, t, D	1	1	30.58
PP, F, t, D, ATemp	1	1	44.18
PP, F, t, D, AATemp	1	1	35.04

TABLE II presents different input designs prediction results using the LR model. As can be seen, the model that has Previous month price, Hydro derivative, Number of free days, Dummy variables as well as t factor incorporated, results in most accurate predictions. However, this is not the design that being introduced as the main forecasting periods design. The reason is, as mentioned earlier, the model´s predictability of the validation period is the criteria for selecting the design to be applied for the main forecasting period. Hence, in order our model to be practical we have to select the design which result in most accurate prediction on the validation period, which not necessarily result in most accurate prediction. Therefore, the model with having PP, AAHD, NFD, DV, t and AA Temp as its input features with MAPE= 17.56 is selected. Once again, note that this model has resulted in most accurate prediction when it was applied to the validation period. Also note that the most accurate design with MAPE=8% is derived accidently and could be identified in practice as it is ad-hoc. Note that these impractical models ‘results are being used to study the effect of inclusion of different features on that model’s performance.

As can be seen in table below, inclusion of temperature data deteriorate the performance of the model by 9%.Therefore, once again what we have concluded in our previous work is observed here, that is, inclusion of temperature data as an input to LR model not only does not provide more information to it, but also increase complexity of the system which causes decrease in the performance.

The main reason, in contrary to other markets that have two load peaks in a year (in Summer and winter), this market has only one winter peak and so the model can exclude effect of seasonality from price pattern. Therefore, inclusion of temperature data only sophisticates the problem the model should solve.

Note that, C 1 and C2 are multipliers of the constant term and the models total output, respectively. They are derived according to models prediction on the validation period. Details of this analysis can be found in Appendix A-1.

TABLE II. HYBRID MODEL ERROR,

WITH AND WITHOUT TEMPERATURE DATA, HM: HYBRID LINEAR MODEL

	LR- No. Temp.	LR with Temp.
MAPE %)	*8.46*	17.18
C 1	*1*	.9
C 2	*0.8*	1

5.1.2. Support Vector Machines

In this part, predictability of the SVM model using different input feature configurations are tested and compared. Note that, for each input design, different SVM parameters (i.e., C and γ) are set and implemented to achieve the most accurate prediction. TABLE III below, present different input features selection with their most accurate designs. The following abbreviations are considered: PP: Previous month's Price, HD: Hydro Data, A-: Actual -, AA-: Annual Averaged-, T: Temperature and NFD: Number of Free Days

TABLE III. DIFFERENT INPUT FEATURES DESIGN
IN PREDICTING 12 MONTHS AHEAD USING SVM DESIGN, STUDY THE EFFECT OF TEMP. DATA

Input feature	Design	MAPE
PP	'-s 3 -t 2 -g .25 -c 3000 -e 0.01 -p 0.025'	8.00
PP, NFD, AAHD, Temp	'-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.025'	12.01
PP, Temp	-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.05'	13.15
PP, Temp, NFD	'-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.05'	14.15
PP, NFD, AAHD	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.025'	11.63
PP, NFD, AHD	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.025'	10.26
PP,NFD	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.05'	6.62

Each model's predictability can be evaluated according to its MAPE and its capability in capturing price variation trends. According to TABLE III, the most accurate model is the one that has Previous months price and NFD as its input features. However, as it can be seen in Appendix B-1, selecting PP, NFD and HD as input features will result in the model that has largest capability in capturing price trends variations. That is, beside the MAPE, it is important to compare different models capability in predicting price variations trends from one month to the next (i.e., sign and magnitude of the slope of the line connecting to consecutive forecasted prices).

It can be seen here, similar to the LR model; exclusion of Temperature data increases the accuracy of the SVM model. This occurs in contrary to this model's prediction results on validation set, since the model is trained by the actual temperature data which is available for the validation period but not during the main forecasting period. Application of annual averaged temperature data has not improved the models accuracy and so, this factor should be discarded. However, to be practical, the model that results in most accurate predictions on the validation period is adopted- second row of **TABLE III**- as the main forecasting design.

The role of NFD in providing information to this model is not observable in this table. Note that incorporating this feature, slightly increases the MAPE from (1%) (Comparing the third and the fourth model).

Importance of having more accurate estimation on hydro content variations is also realized by replacing actual vales of this factor by annually averaged ones which increases the error by 1.5%. Thus it can be concluded that the SVM model is more sensitive to hydro content data comparing to the LR model. Moreover, as it can be seen in diagrams in Appendix, all designs are incapable of capturing sudden price jump during last three months. However inclusion of Actual Hydro Data (AHD) instead of Annual Averaged (AAHD) ones, results in more accurate prediction in preceding months-and so PP will be estimated more accurately to be applied for the 4th quarter- and hence this model introduces smaller error in this period.

Moreover, AHD enhance the model with the capability of predicting the price variations from one month to the next in middle months of the forecasting period (i.e., March-August).

Note that 11 dummy variables have not been considered in this table, as they deteriorate prediction results of this model to a great extent.

5.1.3. RBF NN

As can be seen in TABLE IV below, RBF Neural Network is less capable of predicting the price comparing to the other two. However, as it will be mentioned later, this models is shows its merit in predicting the price of the last 2 quarters. For this period, the RBF NN results in the most accurate prediction comparing to all other models.

Importance of having more accurate estimation on hydro variations once again realized here: Replacing of actual hydro data by Annual Averaged ones decreases the RBF accuracy up to 2.5%. Moreover, it can be seen that after previous month's price, HD is the most informative feature for this model in Nord Pool market. Finally, similar discussion on the effect inclusion of dummy variables on performance of the SVM model is applied here.

TABLE IV. EFFECT OF HAVING HD AND TEMPERATURE DATA ON RBF MODEL PERFORMANCE IN NORD POOL

RBF in Nord Pool			
	Goal	Spread	MAPE
PP, AAHD, NFD	0.01	0.5675	11.71
PP, AHD, NFD	0.01	0.5675	9.84
PP, AAHD	0.5	0.5	*9.90*
P, AHD	0.5	0.5	*7.53*
PP,NFD	0.5	0.5	14.86
PP,NFD	0.01	0.5675	33.16
PP, HD, NFD,Temp	0.01	0.75	15.28
PP, HD, NFD,Temp	0.05	1	15.59

Moreover, once again it is observed that inclusion of temperature data decreases the model's performance. The reason is, although this variable provides information to our models however, these information must have been provided by other variables, and so inclusion of temperature increases complexity of the problem and dependence of input features which in turn decreases models performance, and so inclusion of this factor is not recommended for forecasting the Nord Pool market price.

5.1.4. WNN

As the final model, modified Weighted k Nearest Neighbors is employed. . Attributes that may be incorporated are Hydro derivatives, Month indicator, Number of Free days and Temperature. Assuming these factors to be known for each month to be forecasted, among all available months, closest months (i.e., most similar) to the month for which its price to be forecasted are identified. Then using (12) and (13), forecasting month's price is predicted. Note that this iteration is repeated for 11 consecutive months following the first forecasting month. In each iteration, price of the months previously forecasted is considered as actual available price and used for determination the distance between next forecasting months and its preceding ones, to find the closest month to the forecasting month.

In TABLE V, WNN forecasting results in Nord Pool is reported:

TABLE V. WNN PREDICTION RESULTS IN NORD POOL

Design	MAPE, K=10	MAPE, K according to CV Set	K
NFD, AHD, ATemp	*8.27*	*11.98*	7
NFD, AAHD, AATemp	11.95	16.81	7
NFD, AAHD, MI	12.94	13.00	11
NFD, AAHD	12.37	14.57	6
NFD, AAHD, MI, ATemp	*8.84*	*11.98*	8
NFD, AAHD, MI,AATemp	*10.48*	12.88	8

As can be seen here, this model ends up with promising results. As mentioned earlier, optimum number of neighbors to be considered to predict the price is the factor we derive from cross validation analysis. However, in all cases, k=10 result in more accurate predictions comparing to of those for which the value of k is derived from validation analysis. Never the less, to ensure consistency between all models, the optimum model is selected according to the best value of k derived from cross validation analysis.

It can be seen that inclusion of temperature data increases the model's predictability.. Moreover, application of annual averaged temperature data instead of actual ones causes a decline in models performance by 1%!

It can be seen, in the design for which NFD in addition to annual averaged values of hydro derivative and temperature data are employed, the model is less capable in predicting the price as it lacks a factor addressing the effect of seasonality to the model, and so inclusion of MI is beneficial to this model.

In addition to modified WNN, conventional price base WNN is also examined for this market. It is observed that this model is not practical for predicting the price in this market, due to its poor performance and so its results are discarded.

The reason is, the very basic idea behind this model is to simply calculate the average of several historical monthly price data which were selected previously; In markets with low volatility, this model result in acceptable prediction accuracy, even if most qualified addressing factors have not being used in defining the distance and neighborhood. It will be mentioned later in this work, that WNN with unqualified factors or definition of neighborhood is less capable in predicting the price especially when it comes down to more volatile markets such as Electricity market of Ontario.

5.1.5. Different Models comparison

In Fig. 5 different models' prediction results are plotted against actual price. As can be seen, SVM and WNN are more capable in predicting price variation trends. The LR and RBF on the other hand, have shown that are only capable of predicting total seasonal variations of the price instead of its variations from one month to the next especially in during the second quarter.

However, the RBF has shown its merit in predicting both price trends and magnitude during the last 2 quarters. This becomes more important when different models forecasting results are compared during the last 2 quarters. For this period, the RBF is the superior to all other models. The LR on the other hand, predicts the price accurately during the first two quarters, but it has lost the track for following months.

In TABLE VI, forecasting results of different models are presented. Note that here we have reported each model's most possible accurate prediction, regardless of its performance on the validation period:

TABLE VI.
DIFFERENT MODELS PREDICTION RESULTS IN NORD POOL

	MAPE(%)	INPUT DESIGN
SVM	6.62	PP+NFD
LR	8.46	PP, AAHD, NFD, DV, t
RBF	9.90	PP+AAHD
WNN	10.48	NFD, AAHD, MI, AATemp
Price WNN	25.91	PP
LM ANN	31.46	PP+HD+NFD

In order for our models to be practical, the design for which each model predict the validation period prices most accurately, should be adopted as the final design. Hence, here we once again compare different models prediction results this time the most accurate practical prediction design and their respective MAPE are presented:

TABLE VII. Forecasting results based on MAPE derived from Cross validation period

WNN	F, AAHD, AAT	11.95
SVM	PP, NFD, AAH, AATemp	14.58
RBF	PP, NFD, AAH, AATemp	14.86
LR	PP, AAHD, NFD, DV, t, AATemp	17.23
PB-WNN	PP	34.502

By comparing different models optimum design forecasting accuracy during the validation and main period, its turned out that there may be a slightly difference between the outcomes of these models when they are applied to the different period. For instance the optimum prediction results of the SVM_{opt}=18.75%, and the RBF_{opt}=16.29% during the forecasting period, which brought up the expectation that the RBF results in the most accurate prediction during the main forecasting period. However, as can be seen in table above, the SVM is more accurate. This difference between different models performance during different period may arise from several different reasons. Employing the actual value of different factors during the validation period in contrary to their Annual averaged during the main forecasting period, which in turn introduces unanticipated error to the final results. Besides, both markets are significantly affected by worldwide economic recession occurred in the middle of 2008 which causes an unusual behavior of the price series -price drop in both markets- during validation and main forecasting period. Besides it increases the volatility of the market which imposes additional error to models prediction results.

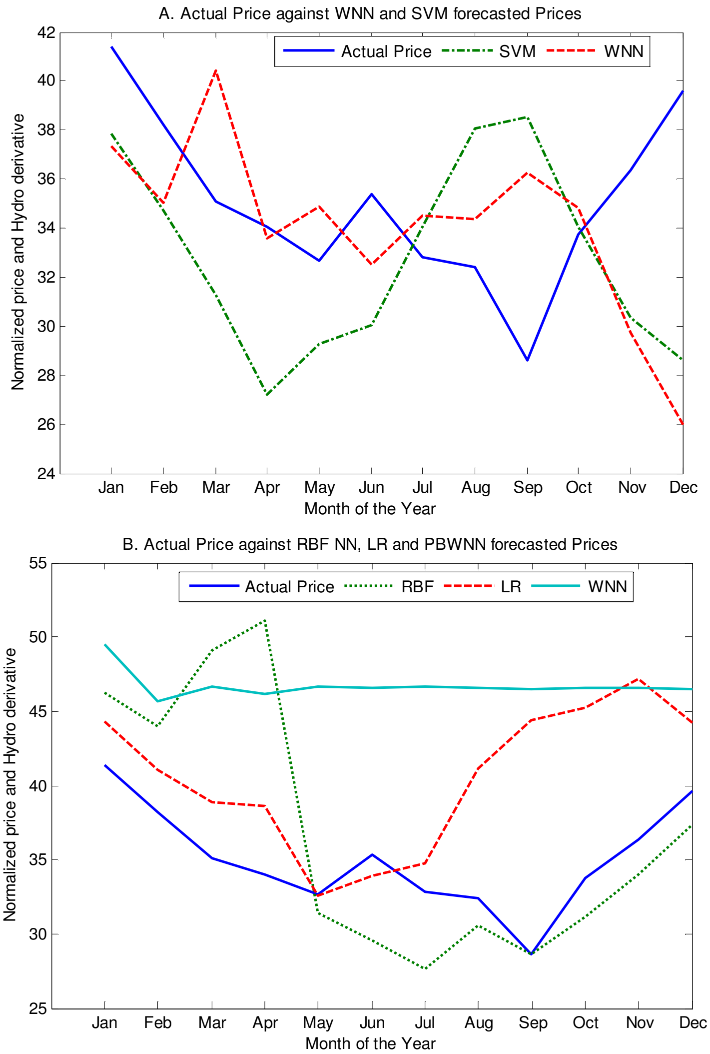

Fig. 5. DIFFERENT MODELS PERFORMANCE AGAINST ACTUAL PRICE

Every year hydro reservoir reaches its low peak value by April. Its maximum occurs at the end of summer, or the beginning of the fall. That is, somewhere between August to October. As there a large share of hydro generation exists in the Nordic region, reservoir content that exists in the reservoir is one of the factors that affect the price. We already know that the other factor is the energy consumption rate, which to great extent depends on temperature. Thus, it can intuitively be concluded that the electricity price will be low at months with low consumption and large water content stored in the reservoir, and vice versa. Hydro reservoir exhibits a periodic behavior, [that to some extent resembles sinusoidal wave]. The mean value for this wave is approximately 60+/- 10. According to this value, the month in which hydro reservoir content is larger than yearly mean value are called wet, and those that are lower are called dry. Thus, July to December are expected to be wet months. Temperature is observed becomes cold by October or December and so the energy consumption increases in this months int will be in this way till the end of winter. Increase in consumption usually ends up to increases in price. Therefore, each month can be classified regarding to two following aspects:

1. Wet and dry
2. Cold and warm (i.e., high and low consumption respectively)

Respectively, we'll have 4 different regions for prices:

I. In wet months with warm weather (i.e., June-Sep) we expect the price to be low.
II. Dry cold months, drives to price to its peak in corresponding year. As can be seen in figure below.
III. Dry month with warm weather (April (4)-June(6)), as well as
IV. Wet months with cold weather (Oct (10)- Jan(1)) have similar characteristics.

Note forecasting the price in intervals III and IV is a more complicated than the other two. Price is highly dependent on weather data, so there is a great lack of informative data in these months, which means models will be less accurate in these periods.

In Fig. 6, all models' prediction results during forecasting period against different factors prediction error are plotted. Here the influence of these factors on performance of the two most accurate models (i.e., WNN and SVM in Nord Pool) can be notified. It is observed that prediction errors of more accurate models are more strongly correlated to the influential factors estimations error. That is, these models introduce their largest error in those months when largest estimation errors on influential factors are met. This fact indicates that those models that can better capture the relation between the price and hydro content in this market result in more accurate forecasting predictions. Moreover, it illustrates the importance of having more accurate prediction of future values of this variable. It has already stated those load and fuel prices are two main drivers of the market price. It is also pointed out earlier that Nordic market consumption is strongly correlated to the temperature. However, incorporation of this factor in this market, reduce accuracy. It is mainly because of regular unilateral consumption (temperature) pattern with one peak occurred in winters that make it possible for the model to extract respective data from HD and NFD. Therefore, inclusion of temperature increases the complexity of the problem for the models and so have negative impact on final prediction results. Finally observed that employing actual hydro data instead of Annual averaged ones, increase the SVM model predictability to that extent that it would have been more accurate than the WNN model. Hence, it is concluded that SVM has the greatest merit among all models, as it has the capability to predict the price more accurate than all other models only if more reliable input data were been provided..

AS mentioned earlier, Except for the RBF, all other models have difficulty in predicting the price in the last 4 months. The main reason for this deficiency is, after two rather warm winter, the region experienced a snowy cold winter during 2009 which outbreak the seasonal pattern it was following and decreases the accuracy of all models 'performance in this period. The author has no explanation for the RBF great performance during this period.

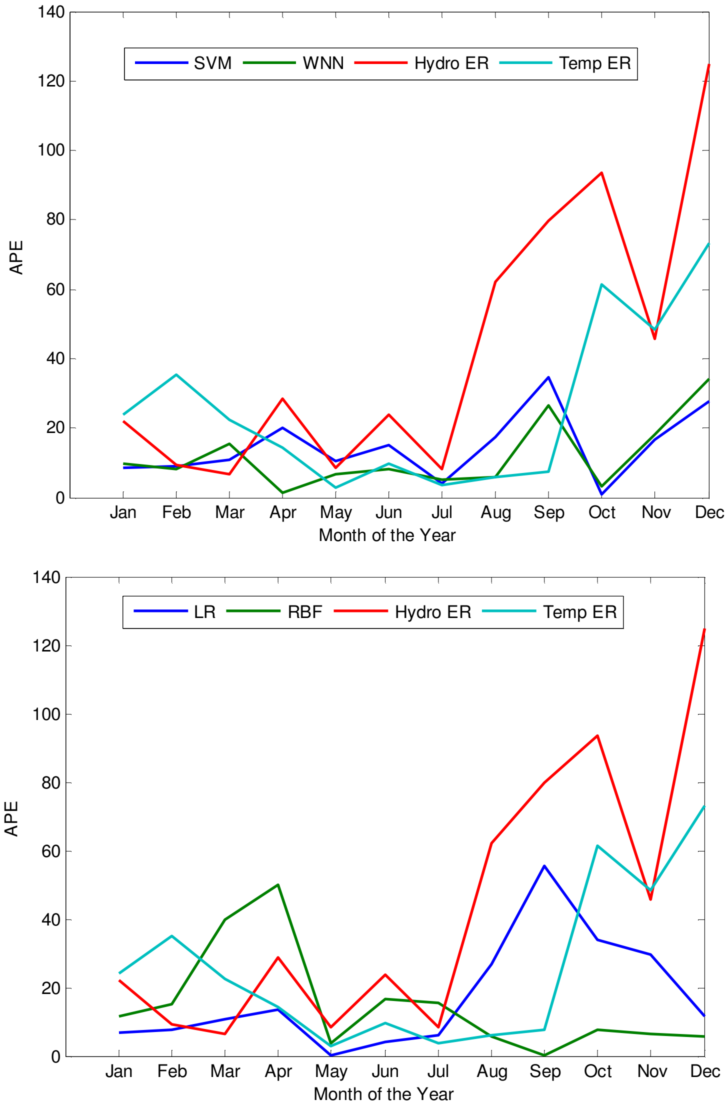

Fig. 6. DIFFERENT MODELS APE COMPARISON IN NORD POOL

5.2.Electricity Market of Ontario

In this section, 12 months a-head electricity market prices in the market of Ontario from November 2009 to October 2010 are predicted. Price series inherently are volatile. However, this defect is more sever in the market of Ontario, as there is one single price representing the price of electricity for the whole province.

In addition to Previous month's price and Number of free days in each month, there are other factors that can affect the price in this market. These factors are identified by studying the price variations behavior with respect to each factor:

As the results, following characteristics are observed:

-Seasonal pattern with two peaks within a year

-large increase in prices during summer 2005.

-Huge drop in price and consumption occurred in fall 2008 and the more volatile price behavior observed sine then that lasts till the end of the studying period.

As it can be seen in Fig. 7. a, there is a strong correlation between market price and the load demand. In fact seasonal characteristics of the price series can be explained by load and temperature. That is, there are two price peaks that realized in winter and summer, when temperature go beyond or drop below certain values which in turn causes the demand increase. Shoulder seasons on the other hand experience milder temperature and so the energy consumption in these period is smaller and so it the price. Thus, energy demand and the temperature should essentially be incorporated in the forecasting models. Hence, monthly averaged temperature of each month besides the Ontario's hourly demand are considered. Note that last factor (Ontario demand) is calculated by dividing total energy consumed each month in the area by number of hours that month constitutes of.

In this market gas fired units have the second largest share of generation. Similar to hydro reservoir content in Nord Pool, variations in gas price affect the market price. This effect can be significant in some cases, for instance, in summer 2005, on one hand humidity and temperature were too high, on the other hand reliance on gas units increase due to decrease in hydro electric outputs and shut down of coal fired stations. Due to increase in natural gas price, some gas fired units decided to sell their contracts in gas spot market rather than producing electricity. This causes an increase in electricity price and volatility of HOEP [32]. Moreover, according to IEA report, almost half of Canadian energy will be provided by Gas systems, the fact that increases the importance of this factor in setting the energy price in the whole country.

Note that as no linear trend does exist in this market's price series, it is no benefit in incorporating the increasing "t" factor; hence this factor is excluded from our model.

Finally a drop is observed in both price and consumption in autumn 2008. This is mainly due to economic recession occurred worldwide in during period that shuts many businesses down and causes the drop in both series, Fig. 7. C. . This illustrates the need for introduction of a variable representing financial property of the market. In Fig Below, different economic indicators that are suspected to carry information regarding price variations in future is depicted. Our investigations showed, among all candidates (e.g., NYMEX and Ontario's stocks index, Financial Condition Index), monthly averaged GDP is the most informative factor to be applied. . Note that the economy affects other factors such as gas price as well, and so co relation between these factors and their mutual impacts on one another should be considered as well. However, as there no forecast on GDP exists that is publicly available for free; this factor cannot be considered among other input features. However, due to importance of this factor, actual GDP data are considered in validation period. As latest official monthly GDP data regarding to period of August to October 2010 (Inclusive) has not been published by the time this work is being written, study the effects of this factor on different models performance should be postponed to some later times. As there is no other factor that can address the price collapse, all models are expected to forecast the price in the post-recession period (Which initiated by the month at which the GDP collapsed and its proceeding months), less accurate than Pre-recession period. Navigant Co.'s Forecasting results especially for the period includes May to July 2009 prove this conclusion as the quarterly averaged MAPE of their forecasted HOEP hits 80%..

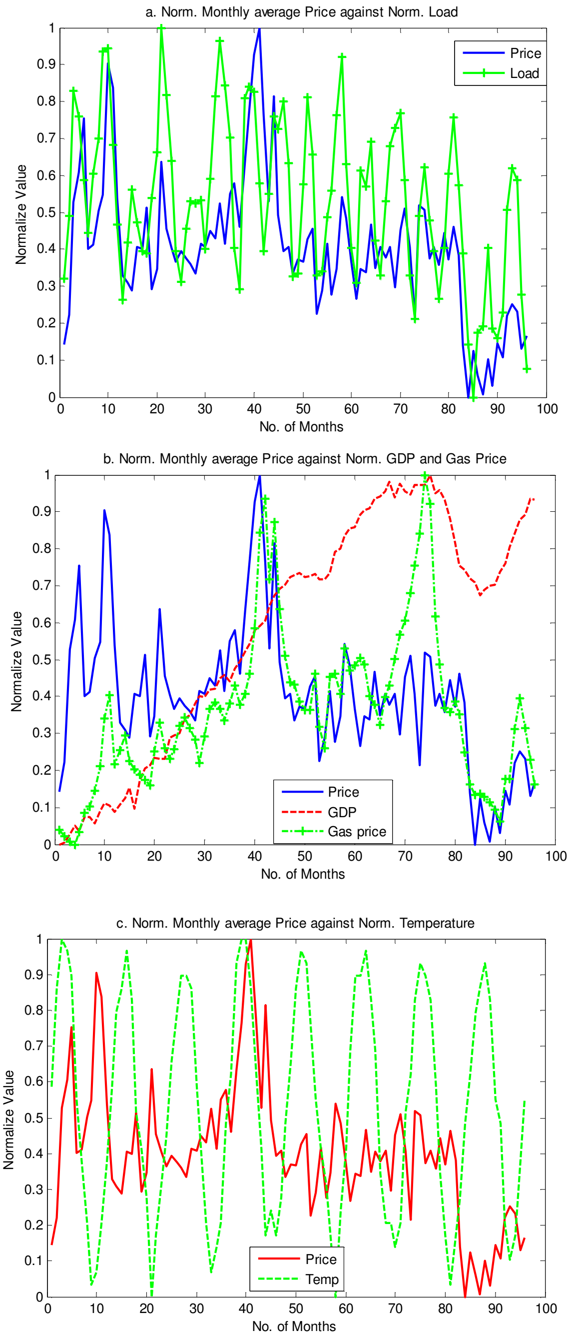

Fig. 7. Ontario's monthly price against (a) Load, (b) GDP and Gas Price, and (C) Temperature

Note that economic condition affect the market price in two ways: on one hand it affect the demand for electricity (i.e., load), as many industrial and financial loads went off due the crisis. On the other hand it causes a collapse on the price of other commodities such as Gas, which in turn itself cause a decrease in cost of electricity generation and so the final electricity price. Hence forecasting engines are not expected to extract information regarding to economical condition out of other input features, as the relation between the economic condition and the market price on one hand, and other factors that impact the price on the other is too complex.

As mentioned earlier, Navigant Co. is the company responsible for price prediction that is used for RPP. Thus to make our result comparable, we used similar actual and forecasted data that have been used by the Navigant Co. report [44]. Note that these data are provided by the IESO in *18Month Outlook: An Assessment of the Reliability of the Ontario Electricity System From April 2009 to September 2010* (March 16, 2009) [43].

Table below present Navigant Co.'s 18 months ahead HOEP forecasting from May 2009 through October 31st, 2010 released in March 2009.

TABLE VIII below presents Navigant's prediction results:

TABLE VIII.
Navigant Co. Prediction Results May 2009- October 2010

	Navigant	Actual	APE
May-July09	41.98	23.2	80.94
Aug-Oct 09	44.25	25.35	74.55
Nov09-Jan10	46.11	32.99	39.76
Feb-Apr 10	47.27	31.65	49.35
May- July10	48.91	43.32	12.90
Aug- Oct 10	50.32	38.66	30.16
		MAPE (Nov 09- Oct 10)	33.04
		STD	26.11

As it can be seen here, the Price variation especially from May to October 2009 (which is the validation period) was so volatile due to financial crisis. It is very unlikely for a model to be capable of predicting the price in such an unstable condition. In order to reduce this effect, the predicted prices of the nonlinear models during cross validation period are scaled down by a coefficient which equals the ratio of the average of the actual price regarding to January and February 2009 (Pre financial collapse-) to the average price of March and April 2009 (post financial collapse period). However because of the market volatility, this ratio is inconstant and so the coefficient is not applicable to the main forecasting period. It only alleviates the error introduced due to the crisis during the validation period to make prediction results regarding to this period more accurate and the proceeding decisions which are made based on more viable.

5.2.1. Linear Model

In TABLE IX below, forecasting results of the Linear model for different input designs is presented. Observe that, in all tables in this paper, those input architectures that are not reported, essentially have led to less accurate results and so did not have the merit to be presented in the following tables.

As mentioned earlier, two constants should be determined. The values of these constants are derived according to the validation period analysis. The down scaling factor already mentioned for nonlinear models in Nord Pool, should not be considered in this model as the criteria for implementing such a constant is not valid any more. The reason is, this factor affects the model's forecasting result driven for validation period. Hence the C1 and C2 coefficients would be different from what they are now and so the model would be different. Finally that configuration would not be valid to be applied to conduct the forecasting for the main period.

TABLE IX. HYBRID MODEL PERFORMANCEUSING DIFFERENT INPUT DESIGN

Features	C1	C2	MAPE
PP, OD, Gs, F	1	1	22.34
PP, OD, Gs, F	0.65	1	24.80
PP, OD, Gs, F	0.65	0.85	17.60
PP, OD, Gs	1	1	22.90
PP, OD, Gs	0.65	0.75	22.56
PP, OD, Gs, MI	1	1	23.39
PP, OD, Gs, MI	0.55	0.65	18.53
PP, OD, Gs, NFD, Temp	*1*	*0.985*	*14.75*
PP, OD, Gs, NFD, Temp	1	0.99	14.92

Inclusion of temperature data increase models accuracy up to 3%. Although Incorporation of the GDP data increase models performance to a great extent, at least in validation set, forecasting results based on GDP data ought to be discarded, as data regarding the final quarter of the forecasting period is not still available..

In contrary to what one may intuitively assume, inclusion of all features in the model does not essentially increase models predictability! For instance, Month indicator and dummy variables (seasonality related exogenous factors) mostly mislead the model and deteriorate its performance! In this table, from top to bottom, the effect of inclusion of different factors is presented. Note that our experiments indicate that Previous months price as well as the Ontario demand are the most important factors. Moreover, by comparing the last two rows of the table, the importance of having more accurate estimation on temperature variations in future becomes obvious.. The MAPE regarding to Ontario Demand which is provided by the IESO is 4%. However, this error is not distributed uniformly, for instance, largest error is observed in June 2009 with

MAPE=11%. Largest error in prediction the price is also observed in this month (although this month is in validation period). This large error in prediction the demand is mainly occurred due to instability that market experienced during post economic recession period which leads to unpredicted decline in power consumption. Later in the context of this work, the impact of Ontario demand error on each model's prediction accuracy will be studied.

Note that except for months 7 to 9, linear auto regressive model has presented its merit in predicting both price magnitude and trend. Hence it can be expected to perform more accurately as market become more stable and the more number of samples become available.

5.2.2. SVM

Table below presents different SVM designs forecasting results:

TABLE X. SVM RESULTS USING DIFFERENT INPUT DESIGN

Features	Design	MAPE
PP, MI, OD, Gs	-s 3 -t 2 -g .025 -c 7500 -e 0.1 -p 0.05	13.90
PP, MI, OD, Gs	*-s 3 -t 2 -g .0375 -c 7500 -e 0.1 -p 0.05*	*13.54*
PP, OD, GS, Temp	-s 3 -t 2 -g .0375 -c 7750 -e 0.1 -p 0.05	27.64
PP, OD, GS, ATemp	-s 3 -t 2 -g .0375 -c 7500 -e 0.1 -p 0.05	24.37
PP, OD, GS, Temp	-s 3 -t 2 -g .025 -c 7500 -e 0.1 -p 0.05	25.25
PP, OD, Gs, Temp, MI	-s 3 -t 2 -g .025 -c 7750 -e 0.1 -p 0.05	32.54
PP, OD, Gs, Temp, MI	-s 3 -t 2 -g .025 -c 7800 -e 0.1 -p 0.05	34.43
PP, OD, Gs	-s 3 -t 2 -g .0425 -c 7350 -e 0.1 -p 0.05	20.82

The SVM can extract information from almost all available features in order to conduct a better prediction. However, it is observed that inclusion of temperature data and exclusion of Month Indicator (MI) deteriorate the model's accuracy. The former is probably due to the reason that information that can be provided by temperature data, is extracted from other factors and so inclusion of this factor provide some overlapped data which in turn misleads the model. The later is mainly due to the effect of seasonality than month indicators can address to the model.

In overall view, this model is superior to all other models due to its capability in predicting the price with promising accuracy except for months 7 to 9. Even for this period, this model has captured price variation trends. This illustrates the fact that large error introduced in this period is mainly arose due to inaccuracy exists in estimation of input features values.

Inclusion of temperature data increase forecasting results error. It is mainly because temperature data imposes a seasonal variation pattern for the price. However, market has been significantly unstable with no clear seasonal pattern to follow especially in last 2 years, hence inclusion of this factor mostly decreases the model's predictability. The SVM prediction results are observed to be more significantly affected by inaccrate temperature data. Hence this model's sensitivity to misleading data is more severe comparing to the other three.

5.2.3. RBF

In **TABLE XI**, prediction results of the RBF model are presented. It is turned out that this model is less capable in predicting the price comparing to the SVM and the linear model but more than the WNN.

TABLE XI. 12 MONTHS AHEAD ELECTRICITY PRICE FORECASTING IN MARKET OF ONTARIO USING RBF NN

GDP: Gross Domestic Product, Temp: Forecasted Temperature, OD: Forecasted Ontario Demand, GS: Natural Gas Price, AAOD: Annual Averaged Ontario Demand, AOD: Actual Ontario Demand, AA Temp: Annual Averaged Temperature

Parameters	Design(Goal, Spread)	MAPE	Cross Validation MAPE	MAPE by implementing C factor derive from cross validation in to main model	α
PP, OD, GS, Temp	0.01, 0.4275	24.56	14.83	23.08	0.9063
PP, OD, GS, Temp	0.01, 0.425	24.45	16.29	22.80	0.8837
PP, OD, GS	0.01, 0.45	25.96	17.66	22.72	0.952
PP, OD, GS, Temp, NFD	0.005,0.5	36.67	22.70	32.57	0.9161
PP, OD, GS	0.01,0.5	23.13	28.66	*15.94*	0.8405
PP, NFD, Gs	0.015,0.5	24.6	55.25	26.01	0.6674
PP, OD, GS, Temp, NFD	0.01,0.5	21.12	42.62	27.26	0.7452
PP, NFD, Gs	0.01,0.625	15	207	Not really practical!	

Note there are three parameters should be set for the RBF model: Coefficient, Spread of radial basis functions (Spread) and desired mean squared goal (goal). Spread should be set according to characteristics of the input data to set the Goal value; a lower marginal value can be defined for which the value of this parameter should be larger than. If the Goal parameter is set so it is smaller than the lower bound, the model will become incapable of reducing prediction error below this value (error cannot reaches such a low value). Hence the RBF will add an extra input layer to the network and will continue this procedure till it can reduce the error below the Goal value or maximum number of iterations is met. Our investigations show that increasing the number of hidden layers complicates the problem which in turn reduces models capability in predicting the price. In our case, the expected error should be set equal to or larger than 0.01.

Due to poor performance of this model in this market, an extra factor is needed to be introduced as follow. Fist for the cross validation period, the down scaling factor which is discussed earlier is put in to effect and the prices of 6 consecutive months (May to October 2009) are predicted.

Then, for each input configuration design,

$$\alpha = \frac{Average(P_{A,May} : P_{A,Oct})}{\square} Average(P_{f,May} : P_{f,Oct})$$

is calculated and employed to scale down the forecasted price of each iteration during forecasting period. This simple modification increase the models performance. Although this model ends up with better results, it still needs more investigation as this model is expected to be able to predict the price with better accuracy than what it has done so far.

It should be noted that except for the last two months, the RBBF model has shown the lowest capability in predicting the market price trends, comparing to the LR and SVM (except the WNN).

However, our investigations showed this model can catch up the price trends and magnitude with much more accurately only if a factor addressing the economic condition of the market included in the input design. As mentioned earlier, GDP data regarding to the last quarter of the forecasting period is still not available, hence further research is needed to be done as soon as these data become available. Moreover, in spite of validation period analysis results, temperature data is concluded to better be discarded from the input design of the main forecasting period forecaster, as inclusion of this factor deteriorate the models prediction accuracy.

Eventually, by comparing the RBF results in Nord Pool and Ontario, it can be concluded that neither number of samples nor market price volatility has significant impact on RBF accuracy as it will be discussed later.

5.2.4. WNN

In the TABLE XII below, forecasting results of the proposed WNN as well as Price based WNN is presented. It is observed that this models is the least accurate model among all proposed models. Never the less, one advantage of using WNN is that it can be applied to market with any sample size, as no learning takes place prior to main forecasting. However, as learning for this model simply based on averaging the price of the most similar months to the forecasting months, it cannot extent any information from input data and hence there is no such a picture illustrating the relation between input features and the output price. Hence, although it is capable of capturing the general decline in the market price, this model is incapable of foreseeing the odd behavior of the price that have never been realized in the training period. Hence this model is not applicable to volatile, unstable markets such as Ontario and unanticipated era such as post recession period.

Hence forecasting the market price of Ontario with the WNN model results in prices with values being larger than actual ones.

Note that it is not possible to derive any constant from cross validation period analysis, as this model is a linear model and hence similar discussion for the LR model applies here. For instance, predicted price during validation period mostly exceed actual values and so the coefficient would be smaller than 1. However, during main forecasting period, WNN results in significantly smaller prices comparing to the actual ones (Observe volatile behavior of market price within this period) and so implementation of the scaling factor derived from validation period, not only does not improve models accuracy, but also deteriorate it. (Appendix G)

TABLE XII. WNN PREDICTION RESULTS FOR MARKET OF ONTARIO

WNN in Ontario		
Input Features	K	MAPE
OD, MI, Temp, F	10	35.75
OD, MI, Temp	10	35.64
F,OD, GS, MI, Temp	4	34.98
Temp, MI	7	32.99

In contrary to the market of Nord Pool, Price based WNN performs with similar accuracy to the WNN. That is both models are unacceptable:

TABLE XIII Price base WNN in Ontario

Price Based WNN		
m	k	MAPE
4	3	32.26
4	5	33.43
4	7	34.81
4	10	31.59
4	15	27.34
2	3	39.74
2	10	30.97
2	15	*25.91*

5.2.5. Different Models Comparison

In **TABLE XIV** presents a comparison between different models optimum forecasting results. As mentioned earlier, WNN is not capable of predicting the price with acceptable accuracy while, the other 3 have shown capabilities in finding the relation between input data for each month and their respective price. Once again the author wants to emphasize on the fact that these results are derived without respect to the economy condition. Inclusion of this data is presumed to affect these results to some extent.

TABLE XIV.
DIFFERENT MODELS PREDICTION RESULTS IN MARKET OF ONTARIO

	MAPE	INPUT DESIGN
SVM	13.54	PP, MI, OD, Gs
LR	14.92	PP, OD, Gs, NFD, Temp
RBF	15.94	PP, OD, GS
Price WNN	25.91	PP
WNN	32.99	Temp, MI
Navigant Co.	33.04	------
LM ANN	38.23	PP+NFD+ GS+OD

As can be seen here, all proposed models predict the price more accurately than the Navigant Co. To indicate capabilities of proposed models, LM ANN which is a common model to be used in short term applications is presented in the table as well.

In **Fig. 8**, forecasting price of the SVM and LR as well as WNN and the RBF are plotted. It is obviously can be seen that the SVM and LR model are capable in capturing price trends. WNN indicates poorest trend extraction as it was expected due to the nature of the model and could be observed in its MAPE in comparison with other models in **TABLE XIV**. It is also concluded that the SVM is the most efficient model in extracting information and as a better understanding on relation between different input features and the desired output. It is also important to bear in mind that different model's designs are derived from each models prediction results on validation period which is severely affected by financial crisis. Hence, more promising forecasting is expected to be achieved by executing these models for rather a more stable period or a more stable market.

Eventually, note that due to lack of information, economy condition has not yet been considered in any of these models. However, our initial analysis indicates that inclusion of a factor representing economy (e.g., GDP) will increase nonlinear models (i.e., RBF and the SVM) accuracy to a large extent, as these models are more capable in depicting the image presenting the relation between the input and the output factors and hence can benefit a lot from this kind of information.

As can be seen in these figures, the LR and the SVM have captured the price variations trends as well as price magnitudes. The LR has predicted the variations trends more severely than they really are. Moreover, it has predicted the monthly prices with one month delay; each severe variation in the price is estimated one month later than the month it took place during the actual time.

The SVM on the other hand, is incapable of capturing price variations as sharp as they were in the real world. However, it could estimate the variations in time, so this model resulted in the most accurate predictions.

The RBF has shown more or less similar capability in predicting the price variation trends to the LR. However, none of the input designs or parameters configurations could address the price variations to the model so it can predict the price with having no overestimation on the price values during the first two quarters. Moreover, similar delay as to the LR model can be observed in this models prediction results.

There is nothing especial to be mentioned on the WNN results.

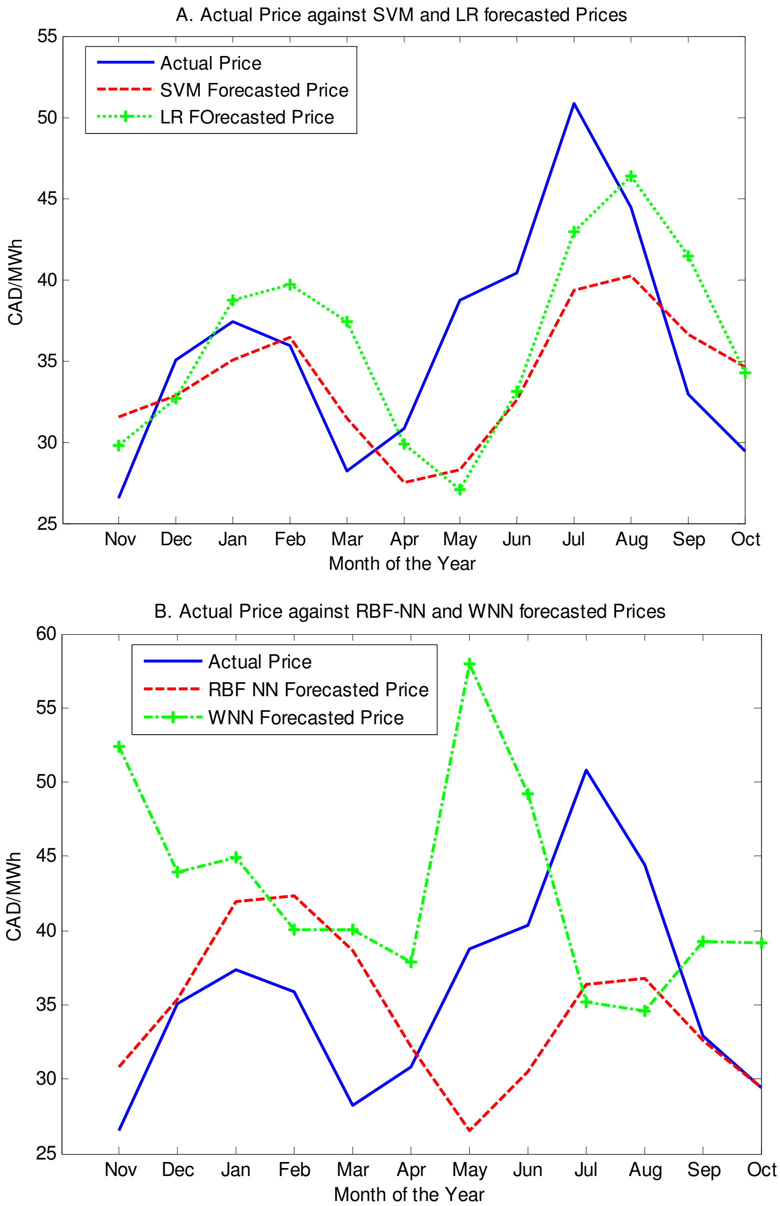

Fig. 8. DIFFERENT MODELS PERFORMANCE AGAINST ACTUAL PRICE IN MARKET OF ONTARIO

In figure below, effect of different parameters forecasting error on each models forecasting is plotted. Note that LM NN and Price based WNNs are discarded from following plots due to their poor performance.

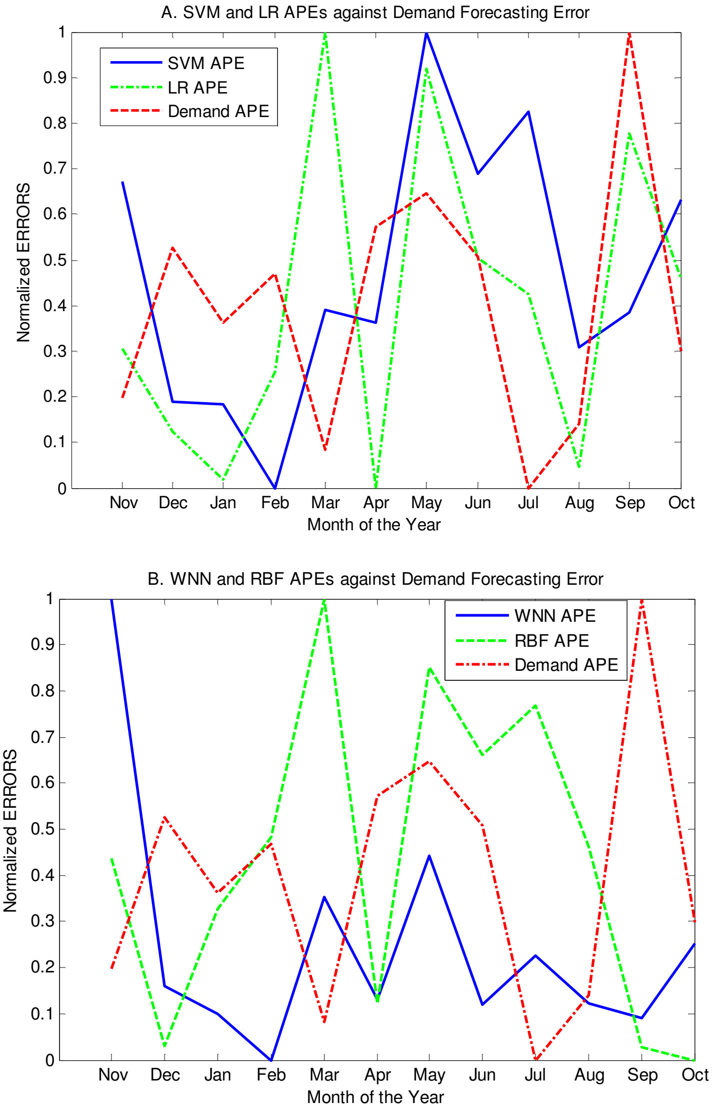

Fig. 9. Effect of Ontario demand prediction error on each model's performance: A.. Effect of Demand prediction error on SVM and LR prediction results, B. Similar factor impacts on RBF and WNN prediction results

It can be seen that more accurate models are those that can better capture the relation between the price and the most important input factor-Ontario demand in the case of Ontario. In **Fig. 9**. A it can be seen that the 2 most accurate models, perform less accurately in those month for which demand prediction is provided less accurately. That is the SVM in May and LR in September has recorded their largest error in alliance to large error met in demand prediction. Note that this behavior is not observed for two least accurate models (i.e., RBF NN and WNN). For instance they both result in accurate prediction on the price, in September, the month with largest error in demand prediction.

This observation indicates strong correlation between the price and the load, and so the essence of having more accurate prediction on this factor.

The SVM indicates its merit in predicting the price trends. In fact, except for 3th quarter, this model results in promising predictions in both the price value and variation trends. Although the LR results in acceptable prediction, it seems to be incapable of extracting information regarding to non-seasonal abrupt variations of the price. That is, when an unusual event occurred in the market in that affect the price and make it behaves irregularly, this model will be less capable in predicting the price, as no relation between the price data on one hand, and the input features on the other has been depicted . In contrary to the poor performance of the RBF model, it has shown its capability in predicting the price as aforementioned condition take place. Moreover, as mentioned earlier, inclusion of feature that can explain economic aspect of the market will increase this model's accuracy to a great extent. WNN has achieved the lowest merit among all proposed models in this market. However, it's most accurate results have obtained while important feature such as Ontario demand has been excluded from the model. Hence, its prediction result is independent from Ontario demand. Hence this model can be useful for markets where the demand is not available or is available with high degree of uncertainty on this factor.

Chapter 6

Chapter 6

6. Conclusions

Medium term price forecasting has not been the main concern of researchers so far. However it is becoming more important as the need for a more accurate prediction on the electricity price variations in future arises as the share of financial contracts within deregulated power markets increase.

Due to differences exist between price characteristics in short and medium term price , different approaches should be taken regarding each of these problems. On one hand, factors that affect the price in the medium term differ from those in the short term. On the other, as there is a lack in number of samples available, most conventional models that are being used in the problem of STPF (e.g., MLP-NN, Bayesian models, ARMA, ARMAX), are not applicable to the problem of MTPF. Hence only those which are more capable of extracting information from different features using few available samples should be employed. According to aforementioned results, non-linear models (i.e., SVM, RBF-NN) are proven to be more capable in dealing with non-stationary, volatile unstable price series in both markets. It is also illustrated that linear models have the merit of dealing with more stable less volatile markets, such as Nord Pool.

As mentioned earlier, among all factors that may affect the price, those that on one hand can better address the price variations to the model, and on the other hand are predictable should be selected. There are difficulties in identifications of these factors, as it is not valid to apply linear mathematical tools (i.e., Auto/Cross Correlation Function) to analyze these nonlinear series. It becomes more difficult when the numbers of samples that are available are fewer than normal problems.

Although the optimum input design are different for different models within each market, among all available factors in Nord Pool, Previous month price, Hydro content first derivative and Number of free days are identified to be the most informative ones. Hence, almost for all models these factors should be incorporated.

The situation is slightly different in Ontario, as this market is more volatile than Nord Pool while fewer samples are available. As the result, more features should be considered to derive the most informative ones. Our study shows that, Previous month price, Ontario demand, natural gas price are the most informative features to be incorporated in any models for this market. Moreover, inclusion of GDP as an extra exogenous factor, improves the performance of the RBF and the SVM to a great extent. More investigation is needed to be done on this factor. In contrary to the GDP, inclusion of temperature data surprisingly deteriorates different models performance and so this factor is discarded from input vector of most of our models.

It is observed that beside absolute predicted price, a model's capability in predicting the price variation trends (i.e., sign and magnitude of price variations from one month to the next) is of great importance. This capability indicates to how extent a model can identify the relation between input data and the value to be forecasted. That is, a model may be discarded due to relatively large MAPE it has resulted in on validation period, hence have merit to be employed to conduct the forecasting in the main forecasting period, due to its capability in capturing price variation trends. This miss-selection is more observable in more volatile markets. The reason is, in these markets, those models that have a clearer image of relation between the input data and the desired output value, can better understand unanticipated variations in the market price from provided input data and so result in more accurate predictions.

As mentioned earlier, the effect of incorporating the GDP and/or other factors representing the financial condition of a market (e.g., FCI, Stock index) are needed to be further investigated. Besides, align with emergence of renewable intermittent sources of energy in many markets around the world, more research is needed to identify factor(s) that can better address the impacts of these new sources on the price to the models.

Moreover, it is of our interest to develop a mathematical method to evaluate a model's predictability in capturing price trends variations to be incorporated in comparing different models forecasting merit and investigate the independence of different factors more mathematically.

Moreover, as more samples will be available in future, different hybrid models shall be designed to make it possible to benefit from each of incorporated models advantageous to result in more accurate predictions.

Eventually, further research is needed to study the effects of forecasting errors on decisions that are being made by either parties (i.e., supply or demand), and so the actual spot price at the time of delivery.

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Appendix

7.1 Linear Regression Model in Nord Pool:

7.1.1 Validation Results:

Input Design	Constant-Downscale	Constant Coefficient of the first factor	MAPE
PP, HD,NFD, DV, t, Temp	1	1	16.1696
	1	0.95	14.9471
	1	0.9	*14.0641*
	1	0.85	14.2614
	1	0.8	14.7524
	0.975	1	18.6222
	0.985	1	14.0001
	0.995	1	14.4511
	0.99	1	*13.5831*
	0.98	1	15.0691
	0.99	0.85	21.6286
	0.99	0.95	17.1768
PP, HD, NFD, DV, t	1	1	21.1647
	0.98	1	19.0089
	0.985	1	17.3834
	0.99	1	18.9087
	0.995	1	17.4727
	1	0.95	18.9376
	1	0.9	17.1043
	1	0.85	16.6993
	1	0.8	*16.5899*
	1	0.75	17.0347
PLM,F,t,D	1	1	18.3146
PLM,hd,F,t,D	1	1	21.1647
	0.99	1	17.4727
	0.995	1	18.9087
	0.985	1	17.3834
	0.98	1	19.0089
	0.98	0.9	21.5413
		0.95	20.2245
	0.98	0.995	19.1428
PLM,F,t,D	1	1	16.7549

PLM,hd,F,D,MI	1	1	20.0761
PLM,hd,F,D	1	1	20.0761
PLM,hd,F,t,D	1	1	21.1647
PLM,hd,t,D	1	1	21.4126
PLM,hd,F,t,MI	1	1	22.6907

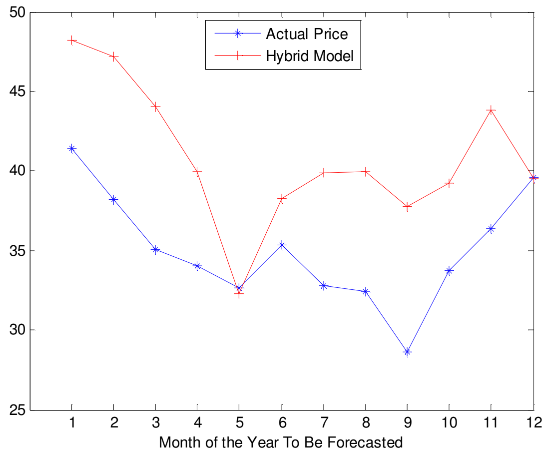
All three approved models are going to be employed using both actual and forecasted values of respective input features, such as Temperature and hydro data:

7.1.2 Main Forecasting Period

7.1.2.1 PP, AHD, NFD, DV, t, ATemp

C1= 1 , C2= 0.9:

MAPE: 17.1838

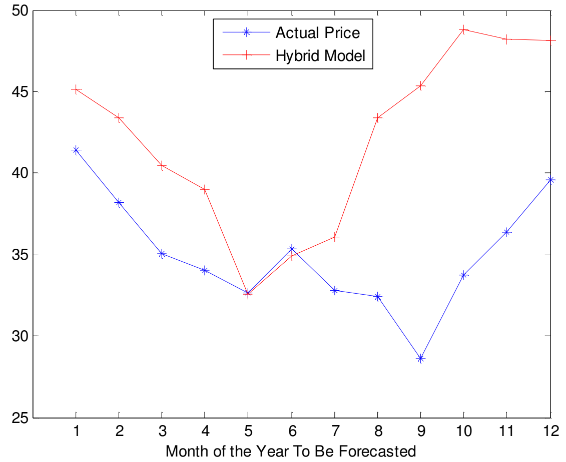


Appendix

7.1.2.2 PP, AAHD, NFD, DV, t, ATemp

C1= 1 , C2= 0.9:

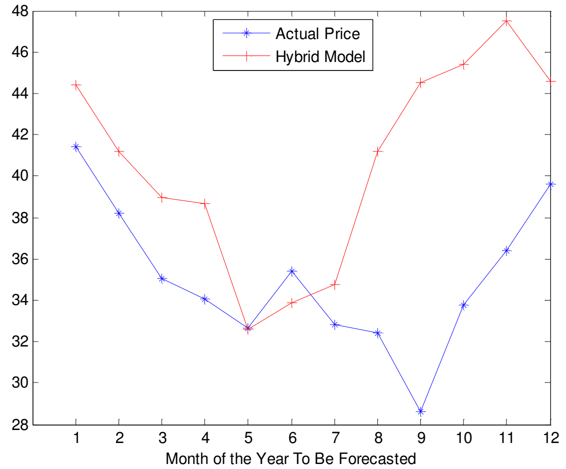
MAPE: 21.2519



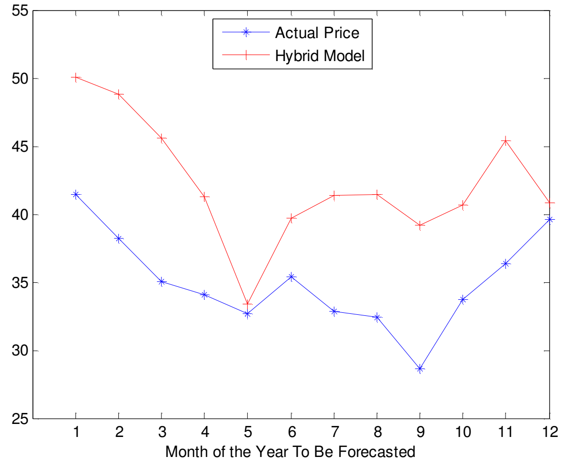
7.1.2.3 PP, AAHD, NFD, DV, t, AATemp

C1= 1 , C2= 0.9:

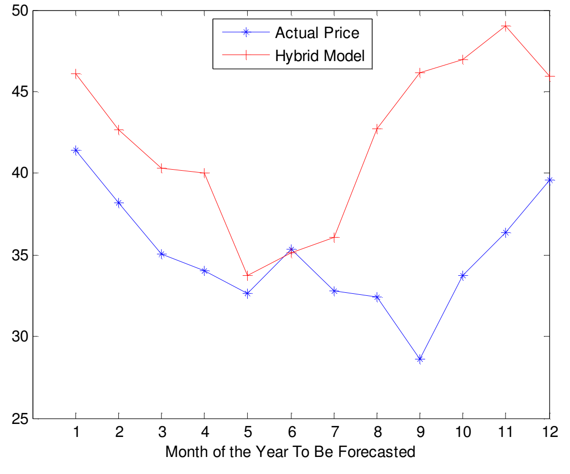
MAPE: 17.5621



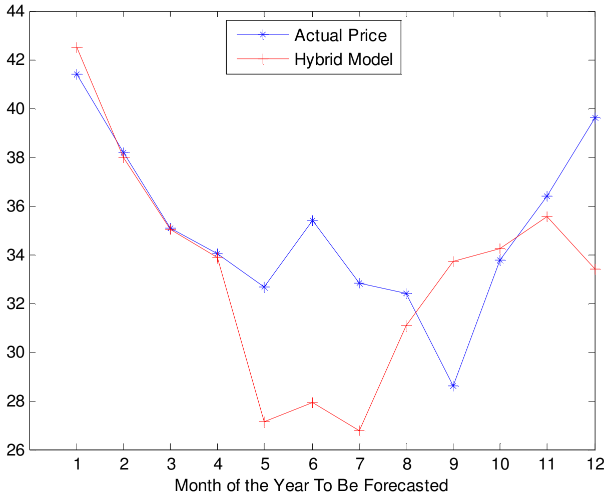
7.1.2.4 PP, AHD, NFD, DV, t, ATemp
C1= 0.99, C2= 1
MAPE: 21.0510



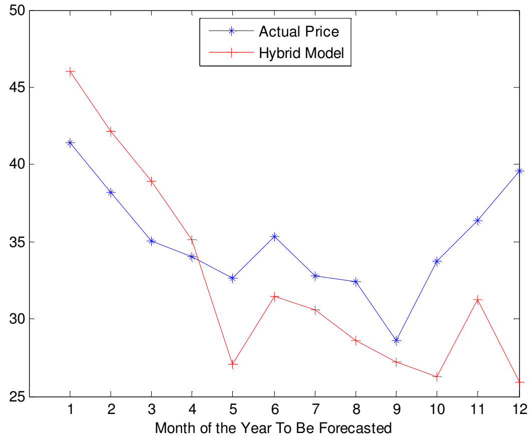
7.1.2.5 PP, AAHD, NFD, DV, t, AATemp
C1= 0.99, C2= 1
MAPE: 21.0541



7.1.2.6 PP, AAHD, NFD, DV, t,
C1= 1, C2= 0.8
MAPE: 8.4606

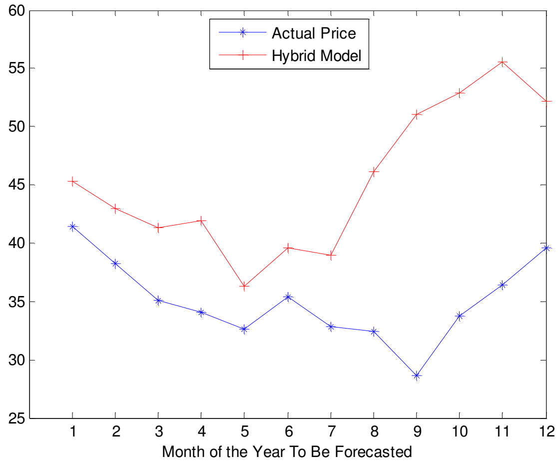


7.1.2.7 PP, AHD, NFD, DV, t,
C1= 1, C2= 0.8
MAPE: 13.1528

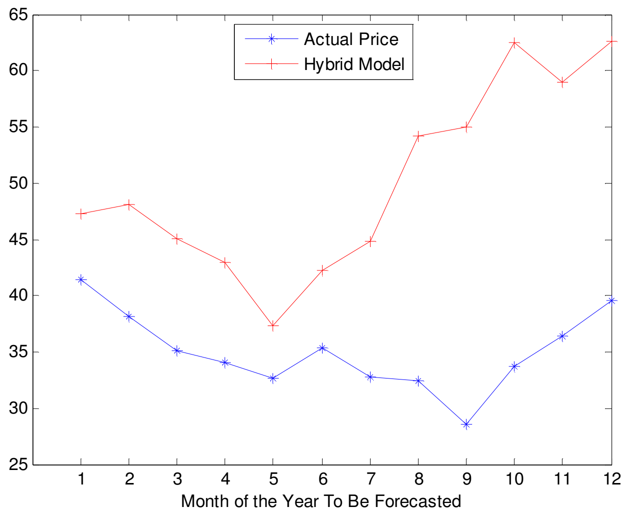


The three following models, indicates the importance of considering Hydro data in the models performance:

7.1.2.8 PLM,F,t,D, C1=C2=1
MAPE: 30.5842



7.1.2.9 PLM,F,t,D, ATemp, C1=C2=1
MAPE: 44.1829



7.1.2.10PLM,F,t,D, AATemp, C1=C2=1
MAPE: 35.0469

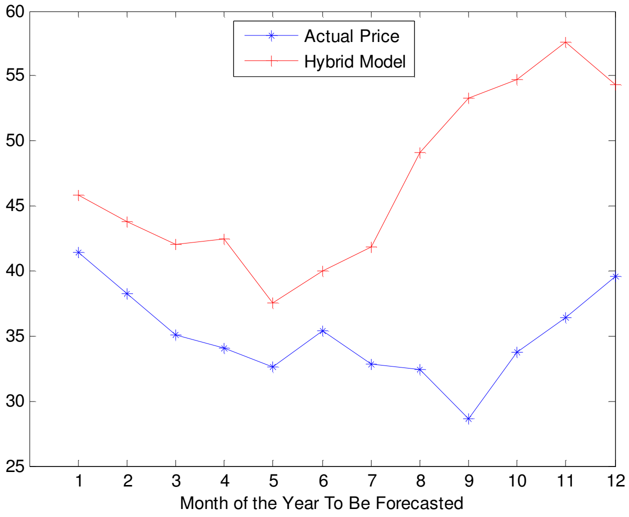


TABLE XV. Linear Regression Model Results in Nord Pool, C1 and C2 Derived from Cross Validation Analysis

Features	C1	C2	MAPE
PP, AHD, NFD, DV, t, ATemp	1	0.9	17.1838
PP, AAHD, NFD, DV, t, Atemp	1	0.9	21.2519
PP, AAHD, NFD, DV, t, AATemp	1	0.9	17.5621
PP, AHD, NFD, DV, t, ATemp	0.99	1	21.051
PP, AAHD, NFD, DV, t, AATemp	0.99	1	21.0541
PP, AAHD, NFD, DV, t	1	0.8	8.4606
PP, AHD, NFD, DV, t	1	0.8	13.1528
PLM,F,t,D	1	1	30.5842
PLM,F,t,D, ATemp	1	1	44.1829
PLM,F,t,D, AATemp	1	1	35.0469

7.2 SVM in Nord Pool

7.2.1 SVM Cross Validations results:

Note, due to large volatile behavior of the market within this interval, actual values of input features are implemented so the results will be a better indicator of that designs merit, regardless of the error the model may introduce in its previous iterations.

Input Features	Parameters Value	MAPE
P,F	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.025'	19.8104
	'-s 3 -t 2 -g .5 -c 10000 -e 0.01 -p 0.025'	21.3888
	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.5'	28.1876
	'-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.05'	19.9275
	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.05'	19.3564
	'-s 3 -t 2 -g .25 -c 3000 -e 0.01 -p 0.05'	19.3858
P,F,H,T	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.025'	19.4065
	'-s 3 -t 2 -g .5 -c 10000 -e 0.01 -p 0.025'	33.1155
	'-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.025'	18.7512
P,F,H	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.025'	20.0942
	'-s 3 -t 2 -g .25 -c 7500 -e 0.01 -p 0.025'	24.6138
	'-s 3 -t 2 -g .025 -c 7500 -e 0.01 -p 0.25'	21.4753
	'-s 3 -t 2 -g .5 -c 3500 -e 0.01 -p 0.25'	33.9525
P,T	'-s 3 -t 2 -g .5 -c 3500 -e 0.01 -p 0.25'	31.0669
	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.025'	20.0407
	'-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.025'	19.964
	'-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.05'	19.8023
P,T,F	'-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.05'	19.9829
	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.05'	20.1007
	'-s 3 -t 2 -g .25 -c 3000 -e 0.01 -p 0.05'	25.1358
P,H	'-s 3 -t 2 -g .25 -c 3000 -e 0.01 -p 0.05'	23.2144
	'-s 3 -t 2 -g .025 -c 3000 -e 0.01 -p 0.025'	21.398
	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.05'	21.0535
P	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.05'	20.938
	'-s 3 -t 2 -g .025 -c 3000 -e 0.01 -p 0.05'	20.6996
	'-s 3 -t 2 -g .25 -c 3000 -e 0.01 -p 0.025'	19.6973

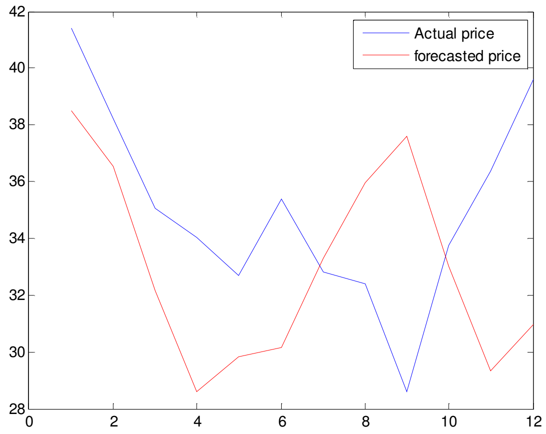
	'-s 3 -t 2 -g .25 -c 3000 -e 0.01 -p 0.05'	19.9853

7.2.2 SVM Final Prediction results:

In the following of this section, forecasting results of those design for which cross validation MAPE was low enough are drawn and compared. These designs are brightened by yellow color the table above:

7.2.2.1 P, F, H, T:

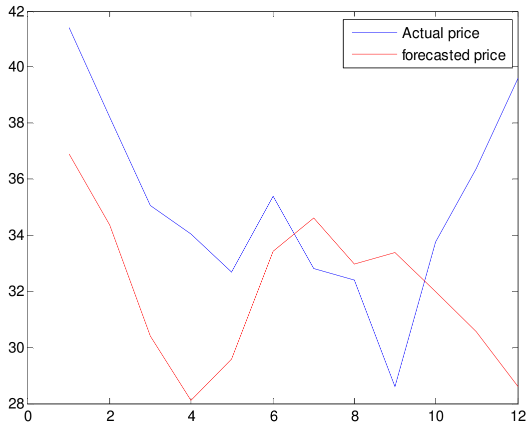
'-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.025'
MAPE= 12.1816, Standard_DV = 8.8609



7.2.2.2 P, F, H

'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.025'

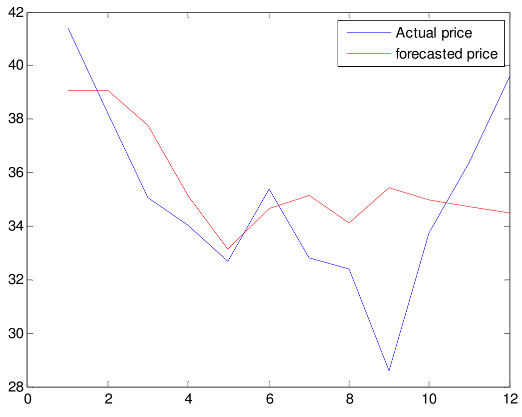
MAPE = 11.6320, Standard_DV = 7.1562



7.2.2.3 P, F

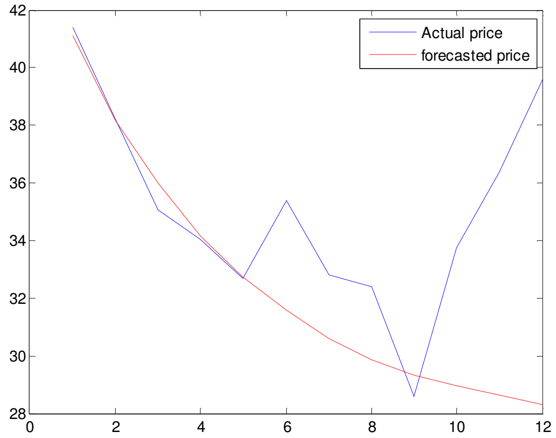
MAPE = 6.6243, Standard_DV = 6.2565

'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.05'

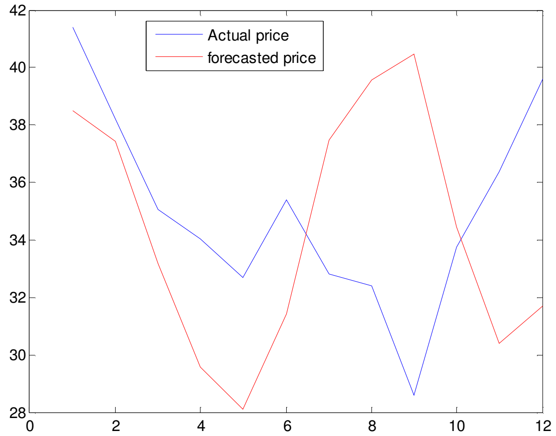


Appendix

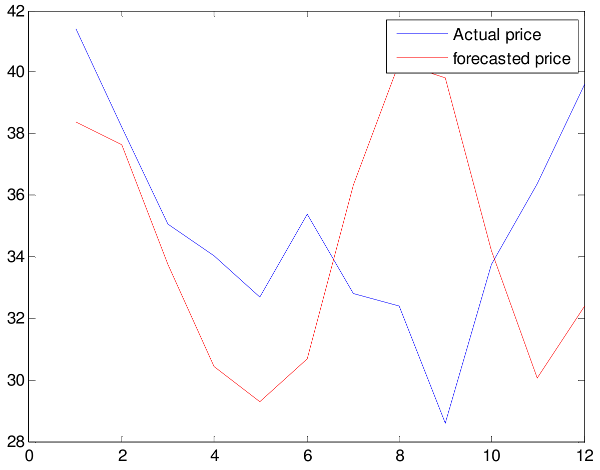
7.2.2.4 P:
'-s 3 -t 2 -g .25 -c 3000 -e 0.01 -p 0.025'
MAPE =8.0045, Standard_DV = 9.2537



7.2.2.5 P, T, F
'-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.05'
MAPE = 14.1513, Standard_DV = 10.7955



7.2.2.6 P-T:
MAPE =13.1577, Standard_DV =10.7243
'-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.05'



In table IV, different models forecasting results is summarized.

TABLE XVI. Different SVM Designs Forecasting Results

Input feature	Design	MAPE
P	'-s 3 -t 2 -g .25 -c 3000 -e 0.01 -p 0.025'	8.0045
P, F, AAHD, T	'-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.025'	12.01
P, T	-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.05'	13.1577
P, T, F	'-s 3 -t 2 -g .025 -c 3500 -e 0.01 -p 0.05'	14.1513
P, F, H	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.025'	11.632
P, F, AH	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.025'	10.2699
P,F	'-s 3 -t 2 -g .025 -c 10000 -e 0.01 -p 0.05'	6.6243

7.3 RBF in Nord Pool

7.3.1 Cross Validation Set results for RBF:

TABLE XVII. RBF Cross Validation Results

	RBF Design			
Input Features	Goal	Spread	MAPE	Real MAPE
P, H, F	0.05	0.5675	18.7708	30.8905
P, H, F	0.005	0.5675	28.7547	34.7234
P, H, F	0.005	0.1	37.1657	41.5407
P, H, F	0.005	1	25.9824	36.2
P, H, F	0.005	0.5	26.4352	36.9677
P, H, F	0.05	0.5	17.9073	28.6674
P, H, F	0.01	0.5	19.129	40.7682
P, H, F	0.1	0.5	17.9073	28.6674
P, H, F	0.1	1	19.7045	31.0479
P, H, F	0.5	0.5	17.9073	28.6674
P, H, F	0.5	0.25	24.8553	28.4226
P, H, F	0.5	1	19.7045	31.0479
P, H, F	0.005	0.75	29.406	33.7252
P, H, F	0.01	0.5675	17.6378	27.9429
P,H	0.1	0.75	20.5177	
P,H	0.1	0.5	19.9608	
P,H	0.05	0.5	19.9608	
P,H	0.01	0.5	21.0191	
P,H	0.01	0.1	28.0168	
P,H	0.01	0.05	33.018	
P,H	0.5	0.5	19.9608	
P,H	0.01	0.5	21.0191	
P,H	0.5	0.1	35.4057	
P,H	0.5	0.5	19.9608	40.7155
P,H	0.5	0.25	18.8118	17.5119
P,H	0.25	0.25	18.8118	
P,H	0.25	0.125	31.7943	
P,H	0.5	0.125	31.7943	
P, F	0.5	0.5	19.9323	19.6748
P, F	0.5	0.25	25.796	22.8224

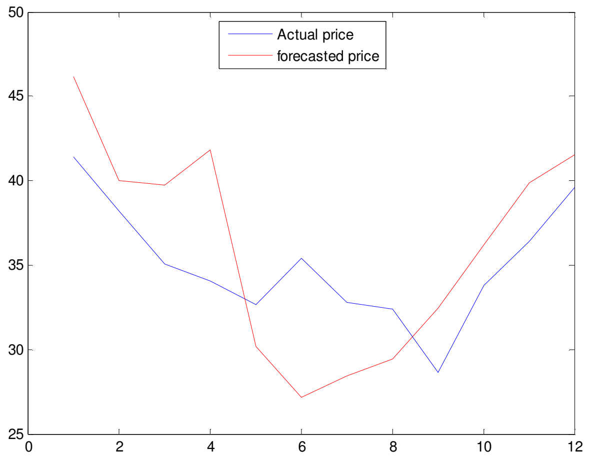
P, F	0.1	0.25	25.796	
P, F	0.1	0.75	18.702	
P, F	0.5	0.75	18.702	
P, F	0.5	1	18.2229	20.108
P, F	0.75	1	18.2229	
P, F	0.01	0.5675	18.9127	18.9218
P	0.5	1	19.2895	
P	0.1	0.1	32.1778	
P	0.05	0.05	26.2579	
P	0.1	0.05	26.2579	
P	0.05	0.1	22.4418	
P	0.1	0.1	22.4418	
P, T	0.1	0.1	32.4569	
P, T	0.5	0.5	25.1177	
P, T	0.5	0.25	21.857	
P, T	0.1	0.25	21.857	
P, F, H, T	0.5	0.75	18.6759	
P, F, H, T	0.5	0.5	23	30.0858
P, F, H, T	0.75	0.5	23.0095	
P, F, H, T	0.75	0.75	18.6759	
P, F, H, T	0.1	0.75	18.6759	27.3536
P, F, H, T	0.05	0.75	18.6759	
P, F, H, T	0.01	0.75	16.844	
P, F, H, T	0.005	0.75	16.2921	23.9039
P, F, H, T	0.0025	0.75	21.0796	
P, F, H, T	0.005	0.1	45.5047	
P, F, H, T	0.005	0.25	22.7377	
P, F, H, T	0.005	0.5	21.31	
P, F, H, T	0.005	1	17.8969	

Note that in some design, although the MAPE seems to be satisfactory, by looking at the forecasted price plot, it can be seen that the model is mostly acting as a memory, which replace previous months price for the forecasting month. In fact, these models has not been able to capture price trends! Later this can be seen for Price based WNN and others. Hence, in spite of acceptable accuracy they achieved on the validation period, they are not merit to be applied for main forecasting period.

7.3.2 Final Forecasting

In the following, those models that are yellow labeled or wrote in red are implemented for actual forecasting period:

7.3.2.1 P, H, F, 0.01,0.5675



MAPE: 11.7198

STD: 6.0806

7.3.2.2 The same model as 1, where this time actual hydro data is used instead of annual averaged values:

MAPE = 9.8486, Standard_DV = 5.6684

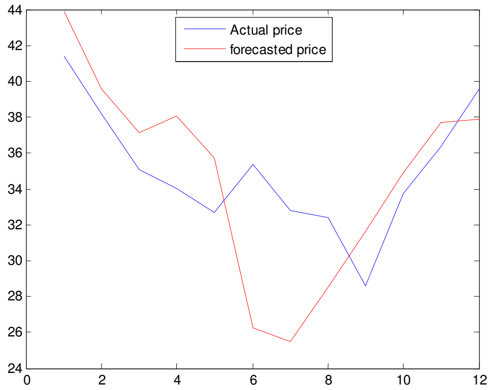
7.3.2.3 P, H, RBF: 0.5,0.5

MAPE = 9.9071, Standard_DV = 7.3815

Same model as 3, Actual hydro data used:

MAPE = 7.5344, Standard_DV = 7.0189

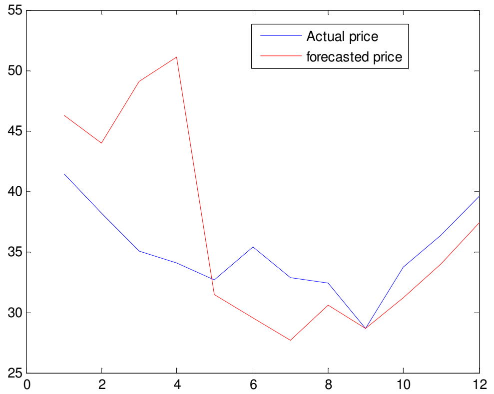
Note that neither 3 or 4 were going to be chosen as this model was not going to b cross validation set:



7.3.2.4 P,F:

MAPE = 14.8604, Standard_DV =15.0998

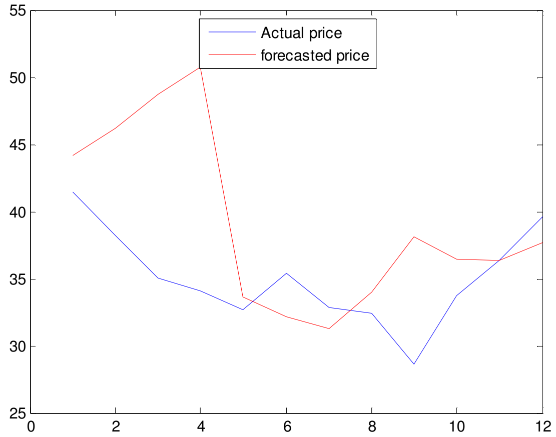
RBF Design: newrb(x,y,0.5,.5,50);



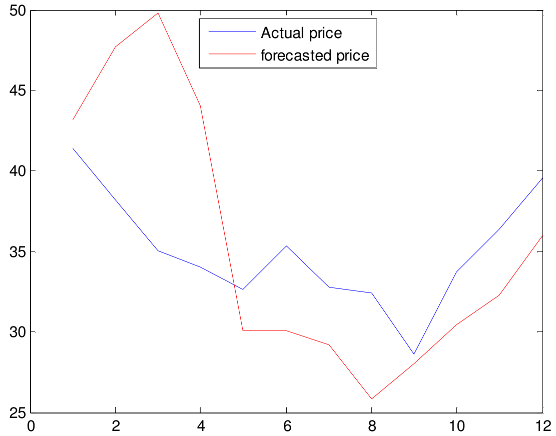
Appendix

7.3.2.5 P,F, newrb(x,y,0.01,.5675,50)
MAPE = 33.1625, Standard_DV = 16.9969

7.3.2.6 P, H, F,T
newrb(x,y,0.01,.75,50)
MAPE = 15.2869, Standard_DV = 16.3567



7.3.2.7 P, T, F, H, [net,tr]=newrb(x,y,0.05,1,50);



MAPE =
15.5999
Standard_DV =
11.6493

Among all PRATICAL models, the third design results in the most accurate predictions. However, this model would not be selected as its MAPE derived from validation forecasting was not the lowest However, as it is noted earlier, this model has shown its ability in capturing price trends in predicating price of cross validation set. Hence it should be considered among final designs.

Also, as can be seen, even the most accurate design with temperature data included, is not superior to models in which temperature data are excluded.

TABLE XVIII. RBF Prediction results in Nord Pool

RBF in Nord Pool			
	Goal	Spread	MAPE
P,AAH, F	0.01	0.5675	11.7198
P, AH, F	0.01	0.5675	9.8486
P, AAH	0.5	0.5	9.9071
P, AH	0.5	0.5	7.5344
P,F	0.5	0.5	14.8604
P,F	0.01	0.5675	33.1625
P, H, F,T	0.01	0.75	15.2869
P, H, F,T	0.05	1	15.5999

7.4 WNN in Nord Pool

7.4.1 Validation Forecasting:

Point 1: In both markets, and all WNN models, we set m= Number of all previous months of which data is available.

TABLE XIX. WNN Cross Validation Results for different values of K and different input design

	MAPE			
k \ Features	F, HD, MI	F, HD	F, HD, MI, AT	F, HD, AT
2	42.2344	40.7892	37.0617	43.1194
3	32.622	34.4687	33.1358	31.283
4	30.3531	28.7117	26.6797	28.415
5	29.9469	27.5765	27.4438	26.6478
6	29.3669	26.4224	26.0179	23.699
7	28.6571	27.4803	24.8006	22.8723
8	26.4739	27.0311	24.1259	23.5589
9	25.5946	27.1611	24.2777	23.9049
10	25.9878	27.1704	24.4261	24.129
11	25.0207	27.2256	23.9079	24.1031
12	25.3451	27.272	23.7112	24.0398
13	25.469	27.4497	23.9501	24.0932
14	25.6558	27.7618	24.5219	23.9987
15	26.0677	28.0349	24.4837	23.8235

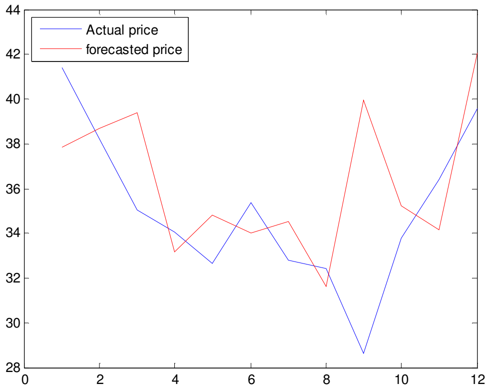
7.4.2 Main forecasting results

In the same order, I will draw all models results for their respective optimum value of k and input design.

However, as can be seen here, better result will be given for k=10, so we set this value for all models:

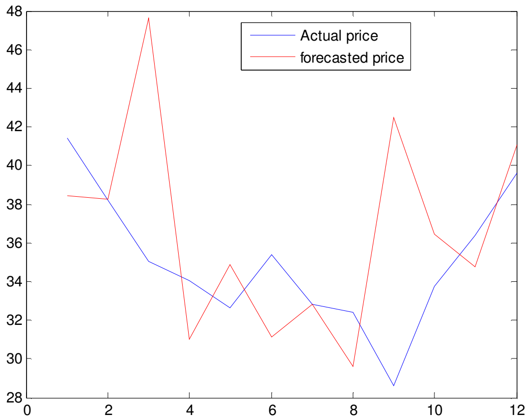
In this case, we expect the F,HD, T model be the most accurate one:

7.4.2.1 A- F, HD, T, K=10



MAPE: 8.2749,
STD: 10.3148

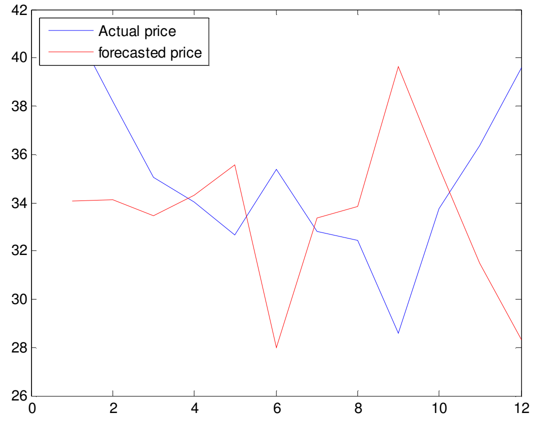
7.4.2.2 K=7, MAPE : 11.9820, Standard_DV: 14.7569



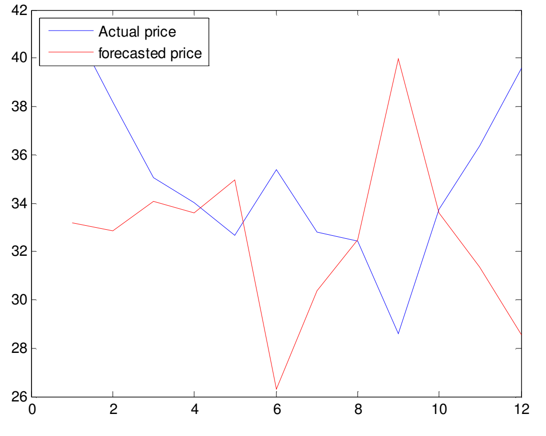
MAPE: 16.8165, STD_DV: 13.4572

Appendix

7.4.2.3 A: F, HD, MI, k=10
MAPE =12.9427
Standard_DV = 11.6104

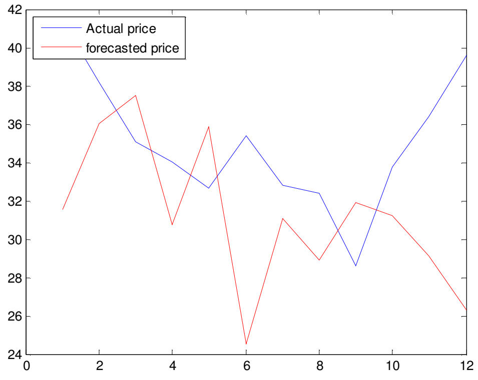


7.4.2.4 B: k=8, MAPE =13.3388, Standard_DV: 12.6956



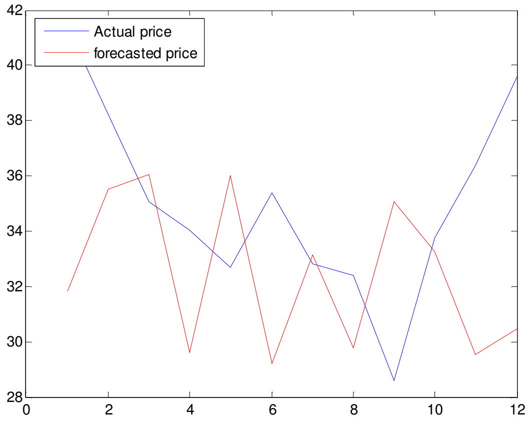
7.4.2.5 A: F, HD
A: k=6

MAPE = 14.5769
Standard_DV = 9.8981



7.4.2.6 k=10

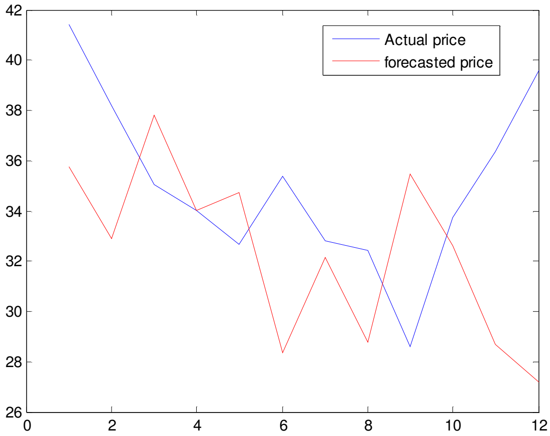
MAPE =
12.3749
Standard_DV =
8.4964



7.4.2.7 A: F, HD, MI, T, k=8

a) Annual averaged Temperature & Hydro :

MAPE = 12.8839, Standard_DV=9.6646



7.4.2.8 Actual Temperature & Hydro :

MAPE = 11.9867, Standard_DV = 13.2255

7.4.2.9 k=10

MAPE = 8.8477, Standard_DV = 11.6123

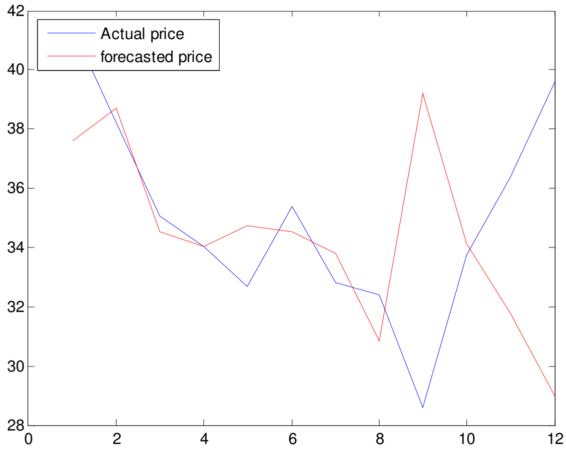


TABLE XX. WNN Results

Design	MAPE, K=10	MAPE, K according to CV Set	K
F, H, T	8.2749	11.98	7
F, H, AAT	11.9505	16.8165	7

F, AAH, MI	12.9427	13.0062	11
F, AAH	12.3749	14.5769	6
F, AAH, MI, T	8.8477	11.98	8
F, AAH, MI,AAT	10.4829	12.8839	8

The most accurate practical model to be used is the one based on F, Hd, MI and T with actual values of these parameters being replaced by their annual averaged values. Note that this finding is true for any value of k-factor: The one that is derived from validation analysis or for k=10.

7.5 Price based WNN in Nord Pool:

According to WNN paper, mostly m=1 leads to forecasting errors that are close to that of the optimal value. However, in your paper, you can run False Nearest Neighbor process. Note that increasing m, increases the cost of training. Moreover, it causes the number of candidate neighbors gets significantly reduced!

Also note that, increseing number of k not always affect the price, because, for furthest neighbors, wighted coefficient because so small that the effect of those prices on final price is low.Here is the cross Validation period MAPE for different values of m and k:

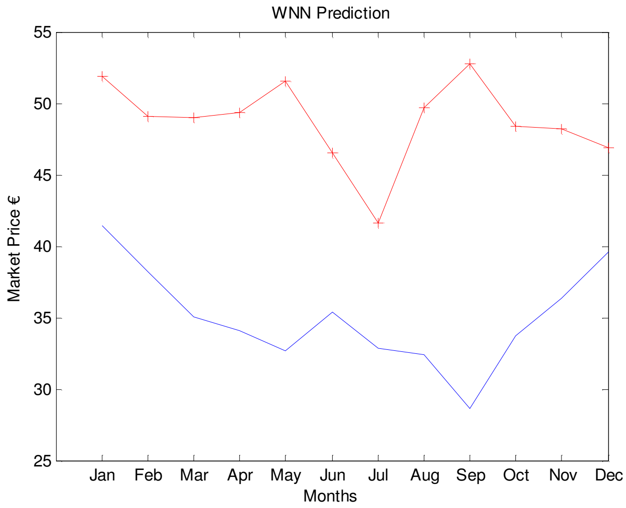
7.5.1 WNN Cross Validation in Nord Pool

k,m	1	2	3	4	5
3	21.3956	24.6259	21.9018	27.8306	27.5786
4	32.4198	25.2464	22.0341	25.7721	24.6468
5	27.7685	25.2464	21.6089	24.85	25.774
6	24.6415	24.3955	21.4295	24.0664	25.6574
7	24.5439	24.2059	21.913	23.5812	26.0382
8	23.0302	constant electricity price is predicted	21.7028	23.5025	26.0087
9	23.3169	24.2586	21.7457	22.9906	26.6678
10	24.6415	23.7114	21.5811	22.8779	27.1169
11	23.3314	23.3132	21.7959	22.676	27.5838
12	24.2842	22.7578	21.7768	23.1348	27.7651
13	24.4529	22.5122	21.7834	23.2114	28.1027
14	24.916	22.2049	21.7158	23.9491	28.4692
15	25.5397	21.9387	21.7163	24.1855	28.7868

7.5.2 WNN results on main forecasting period:

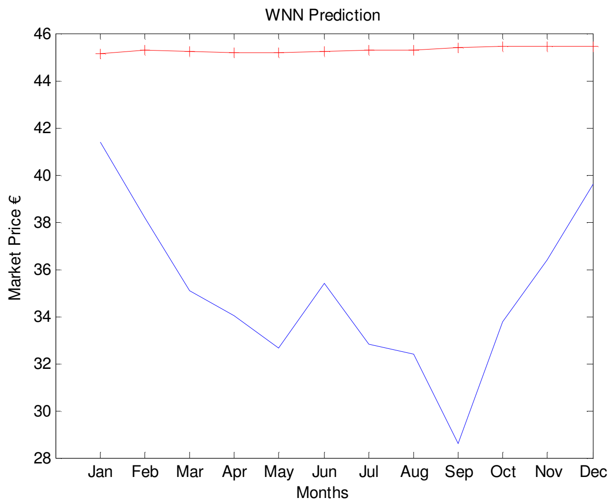
7.5.2.1 M=1, k=8

MAPE: 40.5049



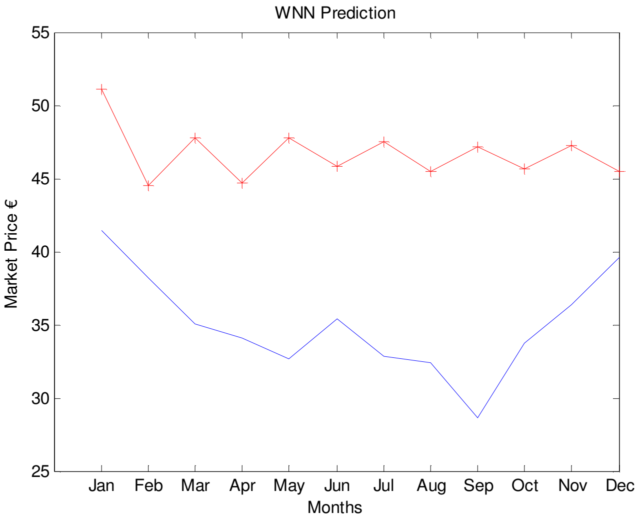
7.5.2.2 M=2, k=15

MAPE: 30.5259



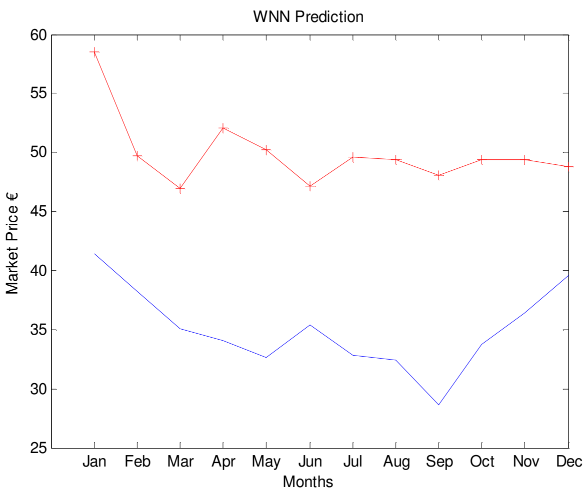
7.5.2.3 M=3, k= 6

MAPE: 34.4316



7.5.2.4 M=4, k=11

MAPE: 43.5274



7.5.2.5 M=5, k=4
MAPE: 54.8216

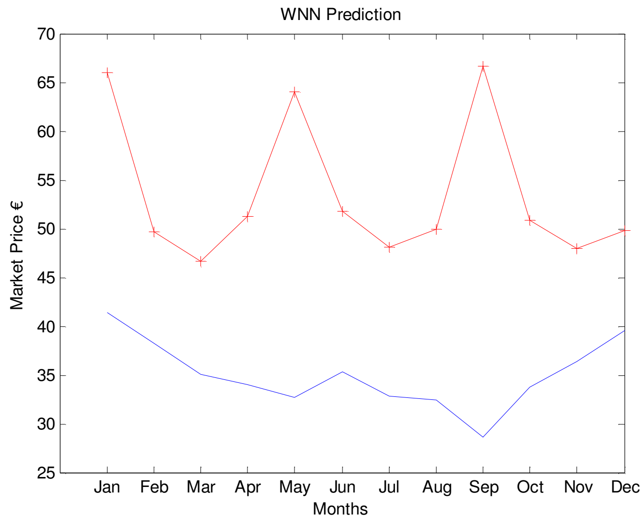


TABLE XXI. WNN Forecasting Results

Price Base WNN		
M	K	MAPE
1	8	40.5049
2	15	30.5259
3	6	34.4316
4	11	43.5274
5	4	54.8216

This model is not capable of sensing the recession occurred during the forecasting period and so capturing the price trends variations during this period. Its forecasting is drastically poor in trend extraction! It mostly propose a constant line as whole period price! As it was expected, it is not a proper model for this kind of forecasting at all!

7.6 Linear Regression Model in Ontario:

You can say, due to volatile behavior of market price in this market, especially during forecasting period due to post recession impacts, it can be said that price of consecutive months in this period are less correlated and so, we cannot conclude the constant coefficients from out cross Validation period. Hence we do not change any of them, and simply, we decide on the optimum input feature design according to cross validation accuracy. Or you can say, we just change the constant coefficient and cannot conclude anything on Down Scaling factor.

7.6.1 Validation price

TABLE XXII. Linear Regression Model Cross Validation results in Ontario

Input Desing	Constant-Downscaler	Constant Coefficient of the first factor	MAPE
PP, NFD, DV, t, Temp	1	1	16.1696
PP, NFD, DV, t, Temp	1	0.95	14.9471
PP, NFD, DV, t, Temp	1	0.9	14.0641
PP, NFD, DV, t, Temp	1	0.85	14.2614
PP, NFD, DV, t, Temp	1	0.8	14.7524
PP, NFD, DV, t, Temp	0.975	1	18.6222
PP, NFD, DV, t, Temp	0.985	1	14.0001
PP, NFD, DV, t, Temp	0.995	1	14.4511
PP, NFD, DV, t, Temp	0.99	1	13.5831
PP, NFD, DV, t, Temp	0.98	1	15.0691
PP, NFD, DV, t, Temp	0.99	0.85	21.6286
PP, NFD, DV, t, Temp	0.99	0.95	17.1768
PP, NFD, DV, t	1	1	21.1647
PP, NFD, DV, t	0.98	1	19.0089
PP, NFD, DV, t	0.985	1	17.3834
PP, NFD, DV, t	0.99	1	18.9087
PP, NFD, DV, t	0.995	1	17.4727
PP, NFD, DV, t	1	0.95	18.9376
PP, NFD, DV, t	1	0.9	17.1043
PP, NFD, DV, t	1	0.85	16.6993
PP, NFD, DV, t	1	0.8	16.5899
PP, NFD, DV, t	1	0.75	17.0347
PLM,hd,F,t,D	1	1	21.1647
PLM,hd,F,t,D	0.99	1	17.4727
PLM,hd,F,t,D	0.995	1	18.9087

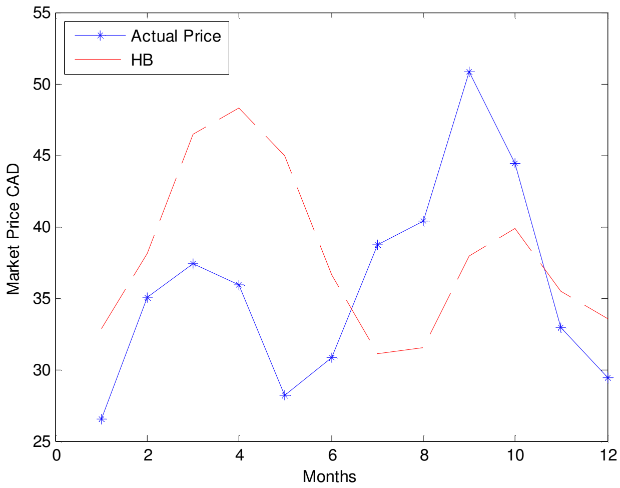
PLM,hd,F,t,D	0.985	1	17.3834
PLM,hd,F,t,D	0.98	1	19.0089
PLM,hd,F,t,D	0.98	0.9	21.5413
PLM,hd,F,t,D	0.98	0.95	20.2245
PLM,hd,F,t,D	0.98	0.995	19.1428
PLM,hd,F,D,MI	1	1	20.0761
PLM,hd,F,D	1	1	20.0761
PLM,hd,F,t,D	1	1	21.1647
PLM,hd,t,D	1	1	21.4126
PLM,hd,F,t,MI	1	1	22.6907

7.6.2 Main forecasting

7.6.2.1 PLM,OD,gs,F

C1=C2=1

MAPE: 22.3469

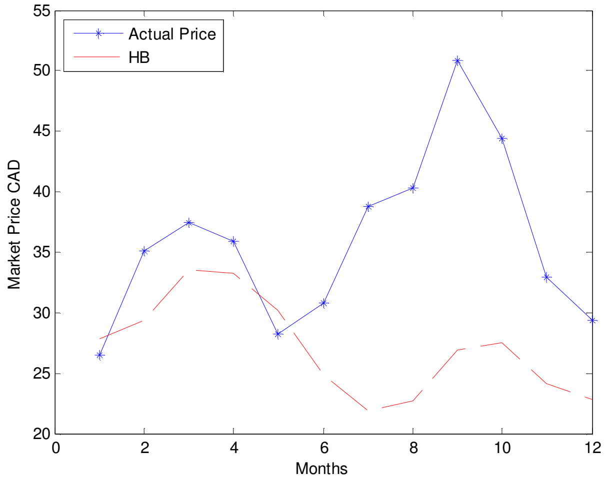


Appendix

7.6.2.2 PLM,OD,gs,F

C1=.65

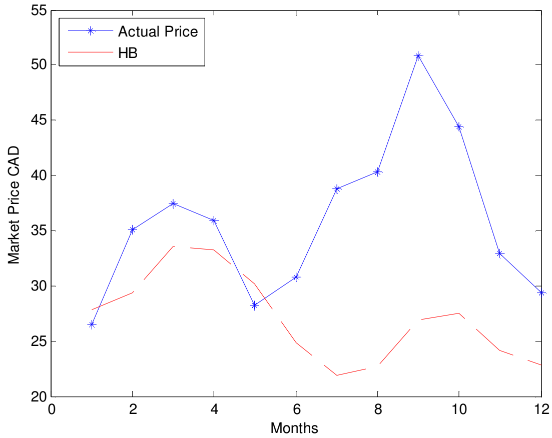
C2=1



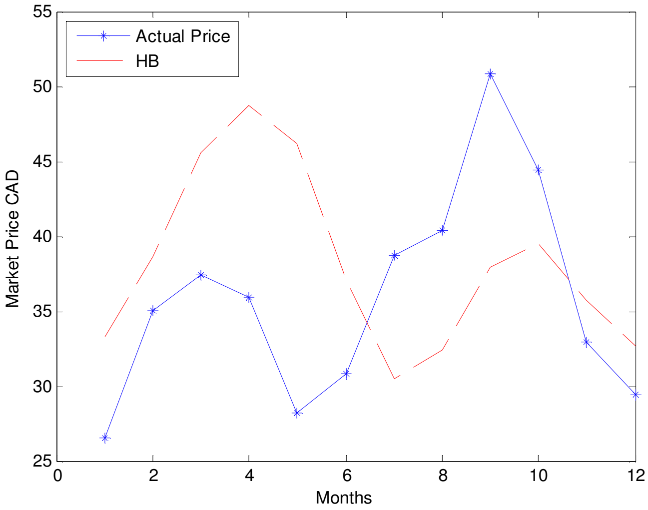
7.6.2.3 PLM,OD,gs,F

C1=.65

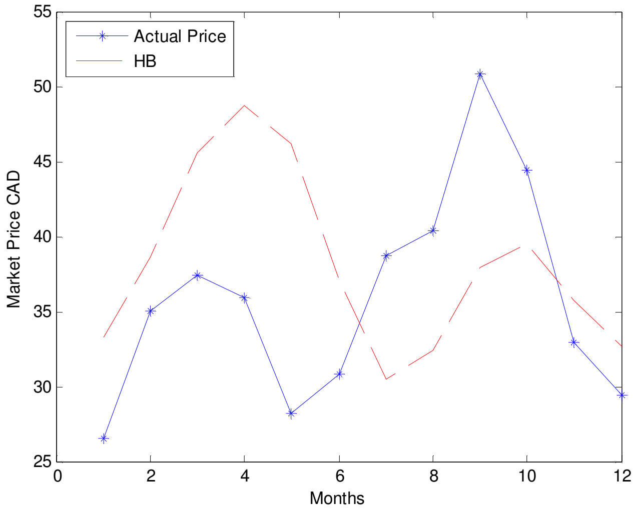
C2=.85



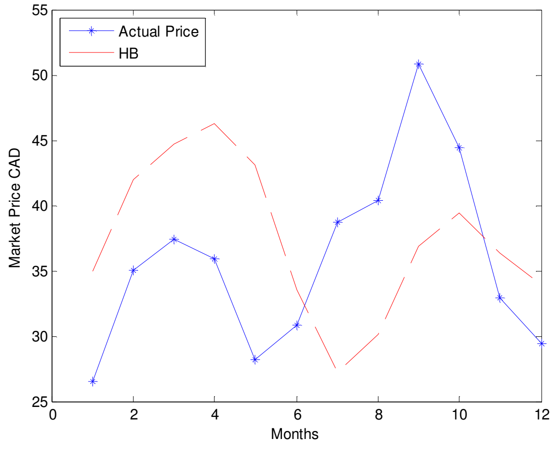
7.6.2.4 PLM,OD,gs
C1=C2=1
MAPE: 22.9022



7.6.2.5 PLM, OD, gs
C1= 0.65, C2= 0.75
MAPE: 22.5685

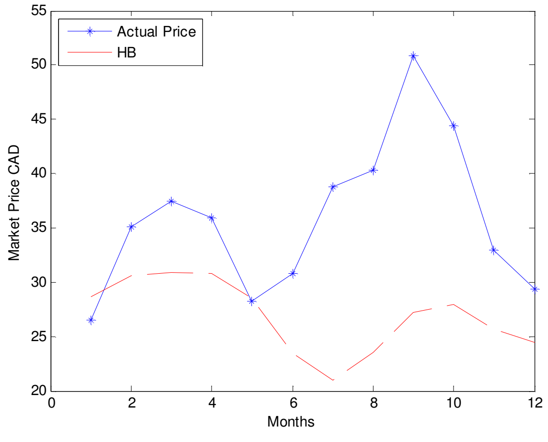


7.6.2.6 PLM, OD, Gs, MI
C1=C2=1



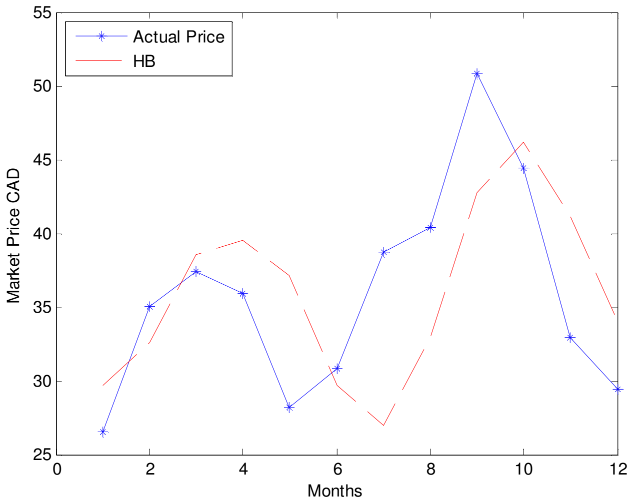
MAPE: 23.3990

7.6.2.7 PLM, OD, Gs, MI
C1=0.55
C2=0.65



7.6.2.8 `PLM,OD,gs,F,tmp`

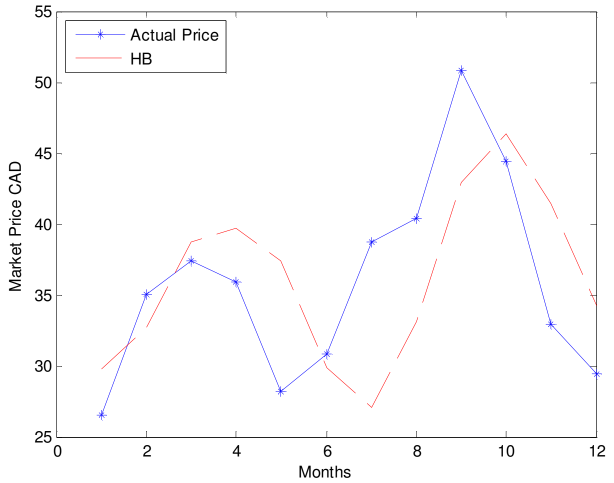
MAPE=14.7530 , C1=1, C2=0.985



7.6.2.9 `PLM,OD,gs,F,tmp`

C1=1, C2= 0.99

MAPE: 14.9267



Appendix

TABLE XXIII. Linear Regression Main Forecasting Results in Ontario

Features	C1	C2	MAPE
PLM,OD,gs,F	1	1	22.3469
PLM,OD,gs,F	0.65	1	24.8095
PLM,OD,gs,F	0.65	0.85	17.6023
PLM,OD,gs	1	1	22.9022
PLM, OD, gs	0.65	0.75	22.5685
PLM, OD, Gs, MI	1	1	23.399
PLM, OD, Gs, MI	0.55	0.65	18.5367
PLM, OD, Gs, NFD, Temp	1	0.985	14.7530
PLM, OD, Gs, NFD, Temp	1	0.99	14.9267

Neither models ended up with better results. Superiority of non linear model to linear ones can be observed here especially in such volatile nonlinear market as Ontario.

7.7 RBF in Market of Ontario:

Note 1:
For those months that result in acceptable results according to their MAPE, but not the graph, (low tendency in foreseeing price variation trends) we believe that these models outcomes are adhoc. One way to examine this is to use actual value of previous month price instead of previously forecasted one. This data replacement will result in no or small difference between the two cases, which illustrates the fact that these models have not realized the relation between price to be forecasted and the most informative factor in addressing the price to the model (i.e., previous month price) and so their initial forecasting results is not reliable.

7.7.1 RBF Validation results:

Table 24. RBF CROSS VALIDATION RESULTS

Parameters	Design(Goal, Spread)	MAPE
P, F, Gs	0.015,0.5	55.2583
	0.01,0.5	88.2776
	0.015,0.1	100.9788
	0.015,0.75	106.7305
P, OD, GS	0.015,0.5	56.271
	0.01,0.5	28.6631
	0.005,0.5	146.917
	0.01,0.25	30.6743
	0.01,0.75	36.1766
	0.01,0.55	35.0925
	0.01, 0.45	17.6635
	0.01, 0.475	37.06
	0.01,0.625	33.0053
P, OD, GS, T	0.01, 0.425	16.2925
	0.01, 0.455	45.1289
	0.01, 0.5	42.6223
	0.01, 0.415	23.1111
	0.01, 0.435	31.282
	0.01, 0.4275	14.8387
P, OD, T	0.01, 0.4275	59.3377
	0.01, 0.45	35.4383
	0.01, 0.5	44.431

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	0.01, 0.475	34.8402
	0.01, 0.4625	32.188
	0.01,0.47	34.0253
P, Gs, T	0.01, 0.425	119.3541
	0.015,0.425	121.3434
	0.005,0.425	67.1185
P, OD, GS, T, MI	0.01,0.425	75.2161
	0.005,0.425	42.4768
	0.001,0.425	78.055
	0.0025,0.425	42.7204
	0.005,0.5	22.7021
P, OD, GS, T, F	0.005,0.475	26.9842
	0.01,0.475	69.8945
	0.005,0.5	19.8868
	0.005,0.525	11.0018
	0.005,0.5325	37.0854
	0.005,0.5375	44.3366
	0.005,0.5175	27.9436
P, OD, GS, F	0.005,0.5175	65.3557
	0.0075,0.5	20.409
	0.005,0.5	81.9979
	0.0075,0.475	27.9861
Effect of GDP		

Based on cross validation MAPE, most accurate models those which indicate enough capability in detecting the price trend are selected. In following table most accurate models according to the cross validation set analysis is presented, then for most accurate ones, forecasted price are depicted:

7.7.2 RBF main forecasting results

TABLE 25. RBF MAIN FORECASTING PREDICTION RESULTS

Parameters	Design(Goal, Spread)	MAPE	Cross Validation MAPE	MAPE by implementing C factor derive from cross validation in to main model	C
P, OD, GS, T	0.01, 0.4275	24.5624	14.8387	23.0857	0.9063
P, OD, GS, T	0.01, 0.425	24.4504	16.2925	22.8092	0.8837
P, OD, GS	0.01, 0.45	25.9675	17.6635	22.7205	0.952
P, OD, GS, T, F	0.005,0.5	36.6708	22.7021	32.5766	0.9161
P, OD, GS	0.01,0.5	23.1354	28.6631	17.3959	0.8405
P, OD, GS	0.01,0.625	20.1985	33.0053	139.299	1.514
P, F, Gs	0.015,0.5	24.617	55.2583	26.0162	0.6674
P, OD, GS, T, F	0.01,0.5	21.12	42.6223	27.2605	0.7452
P, F, Gs	0.01,0.625	15	207	Not really practical!	

Note that in table above, once MAPE is calculated based on forecasting the price using most accurate design derived from cross validation analysis. Once more, similar attempt has been done, but this time ratio of forecasted price to actual price is derived and multiplied to the forecast price in each iteration and then multiplied to the forecasted price to scale it down. The results is reported in the second column from right side.

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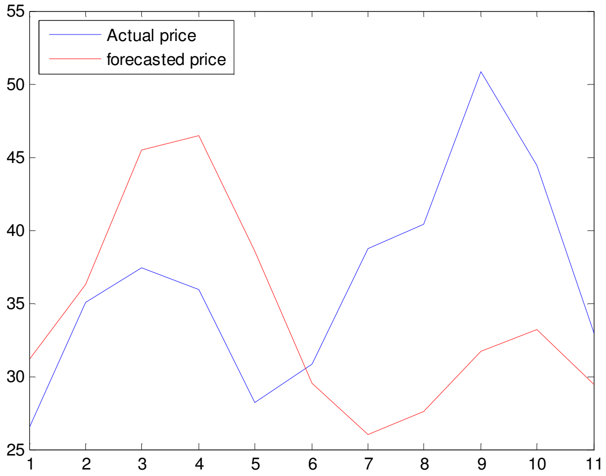
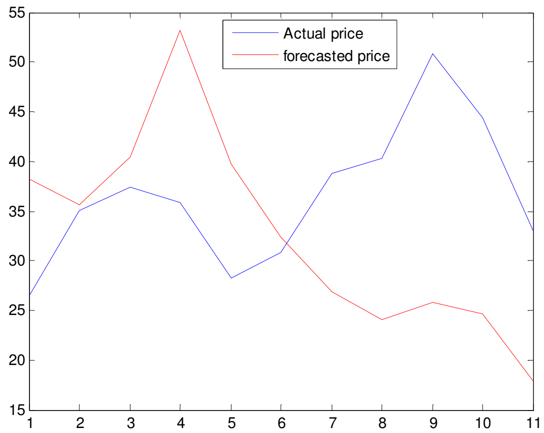
7.7.2.1 Factors:

P, OD, GS, T



7.7.2.2 P, OD, T ,F, GS



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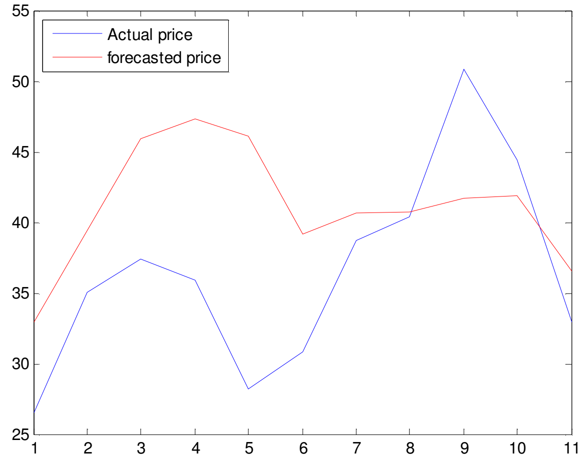
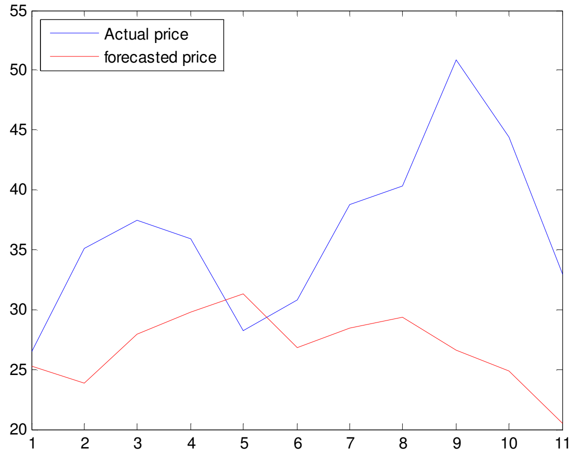
7.7.2.3 P, OD, GS



7.7.2.4 P, F, GS



Among all possible configurations, the best model is the one with Previous months price, Temperature and gas price as input feature. Later we will study the effect of GDP on this model as well.

7.8 SVM in Ontario:

For Cross Validation set, you can say, in order to make to calculated price in order, each month forecasted price is multiplied by a of c=mean(P(83:84),1)/P((81:82),1). This is done just to reduce the error arose due to large price decline right after economic recession. However, this factors is not considered in the main model as we were not aware of future price behavior.

That is:

```
c1=mean(P0(83:84))/mean(P0(81:82));
for i=0:5
     x=[P(1:83+i,1),MI(2:84+i,1),OD(2:84+i,1),F(2:84+i,1)];
     make
     model=svmtrain(P(2:84+i,1),x,'-s 3 -t 2 -g .025 -c 7500 -e 0.1 -p
0.05');
     y=[P(84+i,1),MI(85+i,1),OD(85+i,1),F(85+i,1)];
     [predict_label,accuracy,decision_values]=svmpredict(0,y,model,'-b
     0');
     P(85+i,1)=predict_label*c1;
     P_fore(i+1,1)=predict_label*c1;
end
```

7.8.1 SVM Cross Validation Set results:

Input features	Desing	CV MAPE
P, MI, OD, Gs	'-s 3 -t 2 -g .25 -c 2500 -e 0.1 -p 0.05'	28.68
	-s 3 -t 2 -g .025 -c 7500 -e 0.1 -p 0.05'	15.4371
	'-s 3 -t 2 -g .05 -c 7500 -e 0.1 -p 0.05'	18.7278
	'-s 3 -t 2 -g .025 -c 10000 -e 0.1 -p 0.05'	17.4136
	'-s 3 -t 2 -g .025 -c 7750 -e 0.1 -p 0.05'	15.6371
	-s 3 -t 2 -g .025 -c 7250 -e 0.1 -p 0.05'	18.3292
	'-s 3 -t 2 -g .025 -c 7625 -e 0.1 -p 0.05'	16.748
	'-s 3 -t 2 -g .0125 -c 7500 -e 0.1 -p 0.05'	16.2188
	'-s 3 -t 2 -g .0375 -c 7500 -e 0.1 -p 0.05'	15.748
	'-s 3 -t 2 -g .05 -c 7500 -e 0.1 -p 0.05'	18.7278
	'-s 3 -t 2 -g .03 -c 7500 -e 0.1 -p 0.05'	16.4505
	'-s 3 -t 2 -g .04 -c 7500 -e 0.1 -p 0.05'	16.1969
	'-s 3 -t 2 -g .0375 -c 7750 -e 0.1 -p 0.05'	16.9246
P, MI, OD, F	-s 3 -t 2 -g .25 -c 7500 -e 0.1 -p 0.05'	34.8477
	'-s 3 -t 2 -g .025 -c 7500 -e 0.1 -p 0.05'	23.0514
	-s 3 -t 2 -g .05 -c 7500 -e 0.1 -p 0.05'	19.3948
	'-s 3 -t 2 -g .0125 -c 7500 -e 0.1 -p 0.05'	23.0326
	'-s 3 -t 2 -g .0625 -c 7500 -e 0.1 -p 0.05'	24.8392
	'-s 3 -t 2 -g .05 -c 7000 -e 0.1 -p 0.05'	23.4515
	-s 3 -t 2 -g .05 -c 8000 -e 0.1 -p 0.05'	23.4684
	'-s 3 -t 2 -g .05 -c 7650 -e 0.1 -p 0.05'	24.4867
	'-s 3 -t 2 -g .05 -c 7350 -e 0.1 -p 0.05'	22.8864
	'-s 3 -t 2 -g .05 -c 7500 -e 0.1 -p 0.025'	28.9406
P, OD, Gs, T	'-s 3 -t 2 -g .0375 -c 7750 -e 0.1 -p 0.05'	14.6814
	-s 3 -t 2 -g .0375 -c 7500 -e 0.1 -p 0.05'	15.4193
	'-s 3 -t 2 -g .05 -c 7750 -e 0.1 -p 0.05'	16.644
	'-s 3 -t 2 -g .025 -c 7750 -e 0.1 -p 0.05'	15.0636
	'-s 3 -t 2 -g .025 -c 7500 -e 0.1 -p 0.05'	14.761
	'-s 3 -t 2 -g .0375 -c 7750 -e 0.1 -p 0.05'	14.6814
	'-s 3 -t 2 -g .025 -c 7750 -e 0.1 -p 0.05'	15.0636
P, OD, Gs, T, F	'-s 3 -t 2 -g .025 -c 7750 -e 0.1 -p 0.05'	19.0508
	'-s 3 -t 2 -g .025 -c 7500 -e 0.1 -p 0.05'	19.0381
	'-s 3 -t 2 -g .05 -c 7500 -e 0.1 -p 0.05'	20.472

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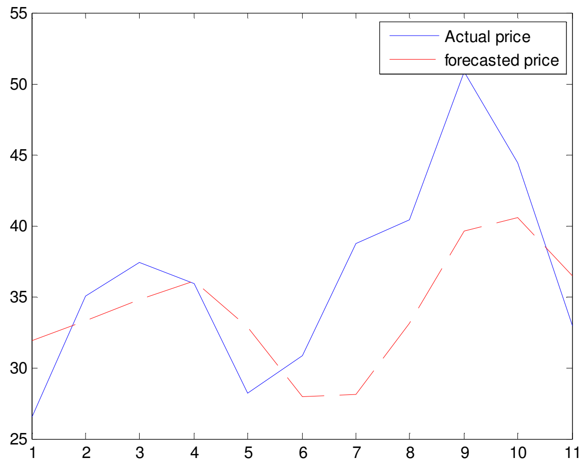
	'-s 3 -t 2 -g .025 -c 7250 -e 0.1 -p 0.05'	19.7604
	'-s 3 -t 2 -g .025 -c 10000 -e 0.1 -p 0.05'	20.3018
	'-s 3 -t 2 -g .05 -c 10000 -e 0.1 -p 0.05'	19.0773
	'-s 3 -t 2 -g .05 -c 7750 -e 0.1 -p 0.05'	21.5462
P, OD,Gs, T,MI	'-s 3 -t 2 -g .05 -c 7750 -e 0.1 -p 0.05'	16.4987
	'-s 3 -t 2 -g .025 -c 7750 -e 0.1 -p 0.05'	15.0587
	'-s 3 -t 2 -g .025 -c 7500 -e 0.1 -p 0.05'	16.0684
	'-s 3 -t 2 -g .025 -c 7800 -e 0.1 -p 0.05'	14.0452
	'-s 3 -t 2 -g .025 -c 7850 -e 0.1 -p 0.05'	15.926
	'-s 3 -t 2 -g .0375 -c 7800 -e 0.1 -p 0.05	17.2912
	'-s 3 -t 2 -g .0125 -c 7800 -e 0.1 -p 0.05'	16.0945
P, OD, GS	'-s 3 -t 2 -g .25 -c 7500 -e 0.1 -p 0.05'	38.3707
	-s 3 -t 2 -g .0375 -c 7350 -e 0.1 -p 0.05'	14.408
	'-s 3 -t 2 -g .0425 -c 7350 -e 0.1 -p 0.05'	14.7177
	'-s 3 -t 2 -g .0375 -c 7250 -e 0.1 -p 0.05'	15.2625

7.8.2 Main forecasting results

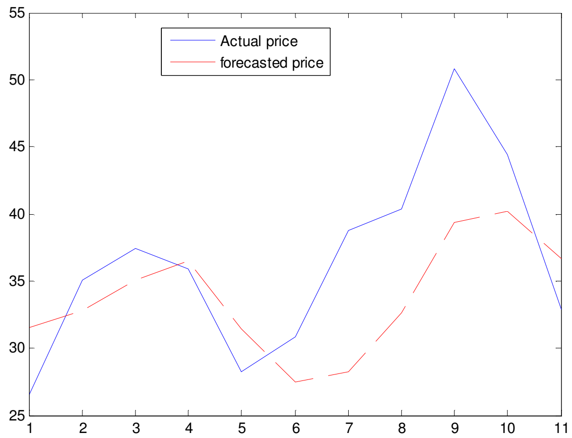
7.8.2.1 Features: P, MI, OD, Gs

Design: '-s 3 -t 2 -g .025 -c 7500 -e 0.1 -p 0.05'



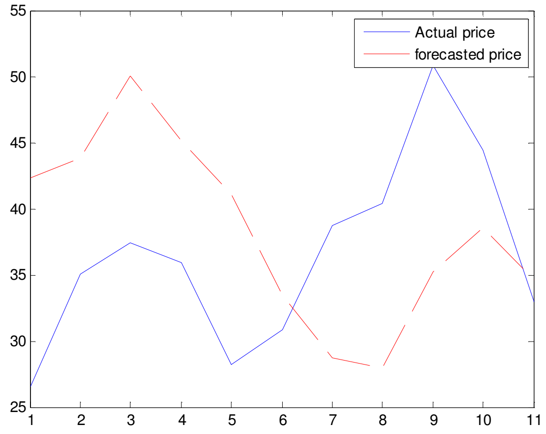
7.8.2.2 Features: P, MI, OD, Gs

Design: '-s 3 -t 2 -g .0375 -c 7500 -e 0.1 -p 0.05'



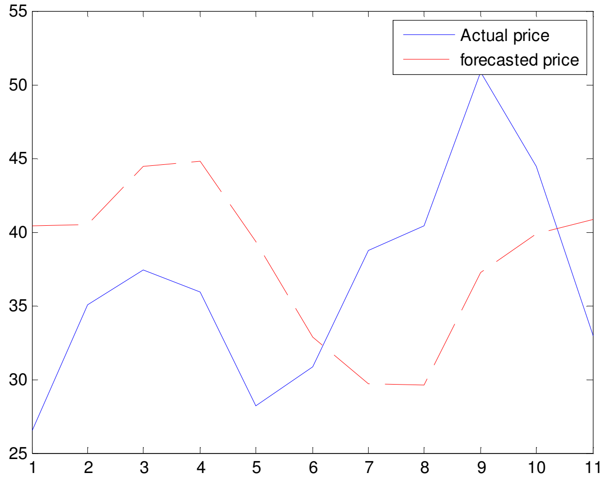
7.8.2.3 Features: P, OD, GS, T

Design: '-s 3 -t 2 -g .0375 -c 7750 -e 0.1 -p 0.05'



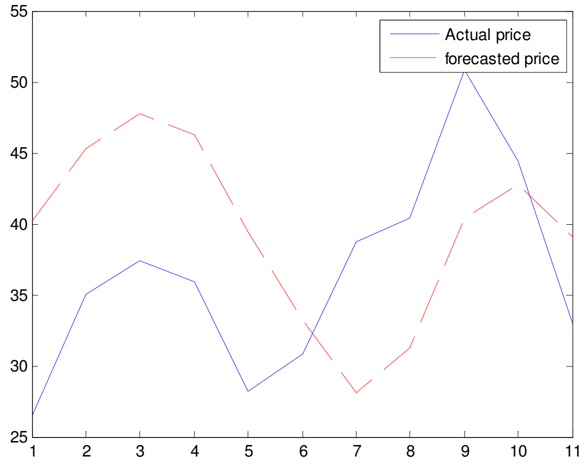
7.8.2.4 Features: P, OD, GS, T

Design: '-s 3 -t 2 -g .0375 -c 7500 -e 0.1 -p 0.05'



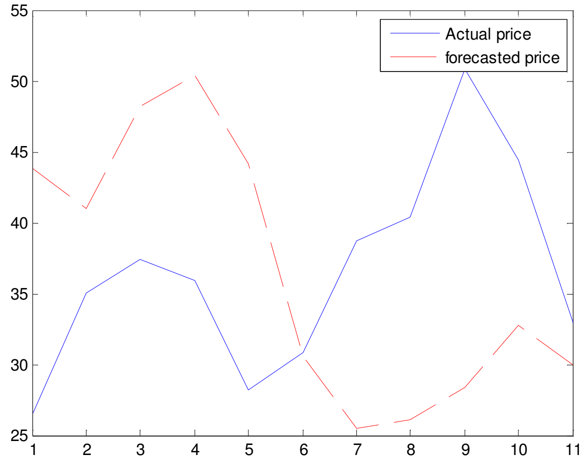
7.8.2.5 Features: P, OD, GS, T

Design: '-s 3 -t 2 -g .025 -c 7500 -e 0.1 -p 0.05'



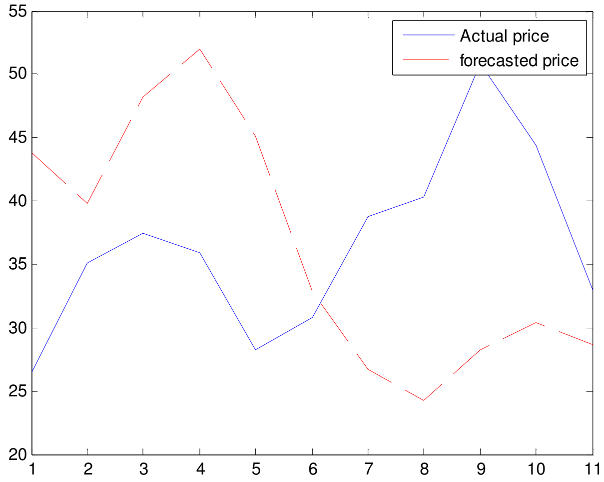
7.8.2.6 Features P, OD, Gs, T, MI

Design: '-s 3 -t 2 -g .025 -c 7750 -e 0.1 -p 0.05'



7.8.2.7 Features P, OD, Gs, T, MI

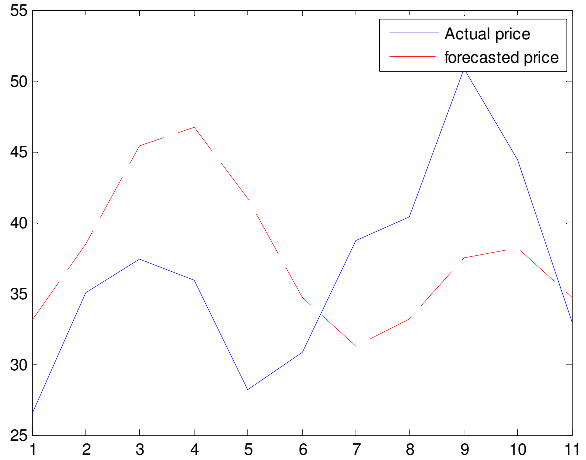
Design: '-s 3 -t 2 -g .025 -c 7800 -e 0.1 -p 0.05'



7.8.2.8 P, OD, Gs

'-s 3 -t 2 -g .0425 -c 7350 -e 0.1 -p 0.05'

MAPE: 20.8262



Appendix

TABLE XXVI. SVM main forecasting in Ontario

Features	Design	MAPE
P, MI, OD, Gs	-s 3 -t 2 -g .025 -c 7500 -e 0.1 -p 0.05	13.9092
P, MI, OD, Gs	-s 3 -t 2 -g .0375 -c 7500 -e 0.1 -p 0.05	13.1729
P, OD, GS, T	-s 3 -t 2 -g .0375 -c 7750 -e 0.1 -p 0.05	27.6477
P, OD, GS, T	-s 3 -t 2 -g .0375 -c 7500 -e 0.1 -p 0.05	24.3792
P, OD, GS, T	-s 3 -t 2 -g .025 -c 7500 -e 0.1 -p 0.05	25.2588
P, OD, Gs, T, MI	-s 3 -t 2 -g .025 -c 7750 -e 0.1 -p 0.05	32.5478
P, OD, Gs, T, MI	-s 3 -t 2 -g .025 -c 7800 -e 0.1 -p 0.05	34.4395
P, OD, Gs	-s 3 -t 2 -g .0425 -c 7350 -e 0.1 -p 0.05	20.8262

As can be seen here, there is an up rise which is controlled in first two models, those which made of P, OD, Gs as well as MI. Exclusion of Temperature is recommended. Therefore, the most accurate model for the SVM is number 1.

7.9 WNN in market of Ontario:

WNN Cross Validation set results:

I will write everything in Archive file.

By considering `c= mean(P0(83:84,1))/mean(P0(81:82,1));`

I can derive help the model to cope better with the decline in the price that has taken place.

Those factors that mostly carry the concept of seasonality should be considered in this model: MI, T, OD.

For k>10, it is not likely that models prediction on price trends varies that much. Even the MAPE slightly changes and so we could limit our search for optimum design to those values that are less than 10.

K=8 for in the case OD, MI, T, GDP where all factors are considered in the model. That is, the 9^{th} closest month enters error the model. Anyhow, according to GDP data, consideration of this data has not improved the forecasting model to that extent! Hence still we need to take a look at its effect in actual usage.

7.9.1 WNN cross validation error in Ontario:

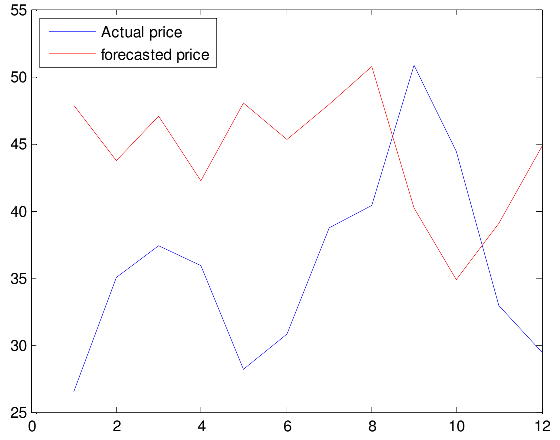
	MAPE							
k\Features	F,OD,GS,MI,T,GP	F,OD,GS,MI,T	F,OD,GS,MI	OD,GS,MI,T	OD,GS,T,F	OD,GS,T	GS,T,F,MI	OD, T, MI, F
2	11.4073	10.3624	28.7913	11.4073	19.2879	24.976	10.3624	15.6279
3	9.1351	9.9584	21.4413	20.0385	18.0095	18.8704	12.1055	9.9353
4	9.5849	12.9869	21.4449	22.9974	16.8083	19.6456	15.4208	9.6566
5	13.9405	16.3599	19.2686	23.5197	17.7438	20.3715	18.2848	10.5555
6	16.2295	18.1874	20.1666	23.8511	15.3275	20.3685	18.2153	12.573
7	16.4363	17.9606	20.6325	22.2064	15.4248	18.4721	17.9505	13.6698
8	16.3611	17.7849	21.0174	21.4379	16.2016	17.9115	17.8952	13.7166
9	15.1138	16.5083	22.3406	19.6389	16.6499	18.4555	17.3825	13.987
10	15.1836	16.2484	22.112	18.985	16.8371	17.8485	17.3321	13.3311
11	15.0009	16.0447	21.9211	18.1333	16.7353	17.5951	17.2469	13.4703
12	14.9673	16.0322	21.6622	17.873	16.6306	17.5077	17.0693	13.6795
13	14.822	15.8773	20.9891	17.3052	16.6606	17.2025	16.5577	13.6572
14	14.9719	15.8567	20.5257	17.1288	16.3175	17.0815	16.2342	13.7097
15	15.0736	16.0241	19.9915	17.1584	15.7622	17.0808	16.092	13.4703

k\Features	OD, T, MI, F	OD,MI,T	OD, MI, GS	OD, T	OD, MI	T, MI	GS, MI	OD	OD, MI, T, GDP
2	15.6279	11.3067	30.7587	27.6642	14.3227	16.1651	55.2467	22.8602	9.9132
3	9.9353	8.584	27.493	16.6603	12.2727		48.5058	13.4295	9.5775
4	9.6566	9.4971	28.0008	16.6451	12.609		42.0364	13.6388	7.1777
5	10.5555	8.7732	28.3568	17.1814	11.3814	16.1651	39.5302	14.2366	8.3703
6	12.573	10.9143	28.1177	16.8192	11.5376	13.5392	38.0646	15.1782	9.4935
7	13.6698	12.4363	28.2991	17.4585	11.871	14.1667	37.042	16.5547	11.3869
8	13.7166	13.0271	28.0803	18.1843	12.8	14.0246	34.0828	16.509	12.9093
9	13.987	13.0024	27.3798	18.0437	14.1734	14.1725	31.8816	16.4343	14.2529
10	13.3311	12.8379	26.2729	18.1361	14.4046	13.7275	30.919	16.3528	14.489
11	13.4703	13.7483	25.6289	18.1281	14.9071	14.551	30.4179	16.5178	14.2878
12	13.6795	12.8189	24.9036	18.0951	15.4496	14.2014	30.1031	16.7475	13.3588
13	13.6572	12.8454	24.044	18.1313	16.5983	14.7158	29.6301	16.762	13.2322
14	13.7097	13.1069	23.792	18.0281	16.8496	14.5752	29.3329	16.5435	13.053
15	13.4703	13.3807	23.361	17.4614	16.8749	14.6816	28.3811	16.2799	13.0667

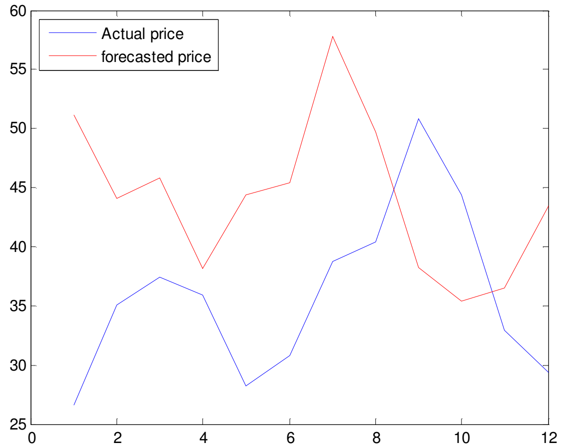
Due to poor performance of this model in Ontario, below for each design, the best input design is brought.

7.9.2 Different input design results for main forecasting period is presented in table below:

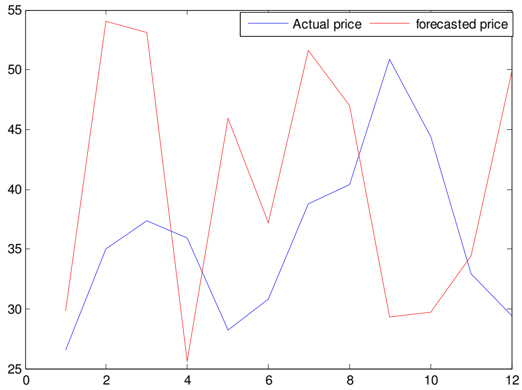
7.9.2.1 OD,MI,T, F-k=10:



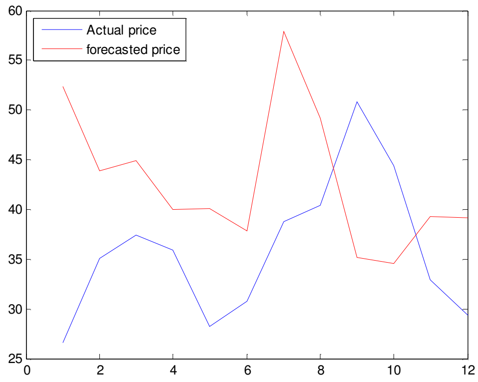
7.9.2.2 OD,MI,T-k=10:



7.9.2.3 F,OD,GS,MI,T-4



7.9.2.4 T,MI-7



As can be seen here, WNN is in capable of capturing the price trends in market of Ontario!

TABLE XXVII. WNN in Ontario

WNN in Ontario		
Input Features	K	MAPE
OD, MI, T, F	10	35.7536
OD,MI,T	10	35.6443
F,OD,GS,MI,T	4	34.9818
T, MI	7	32.995

7.10

7.11 Price based WNN:

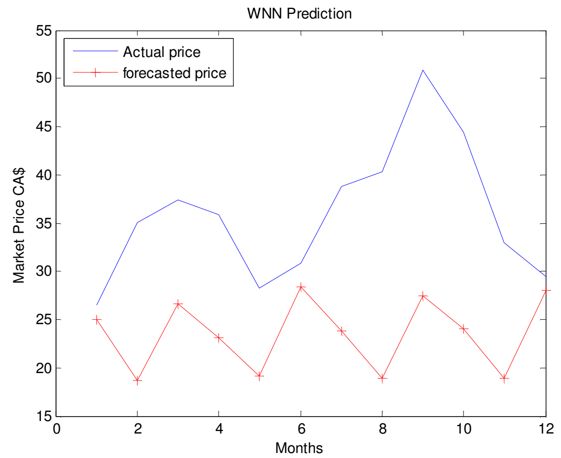
We cannot use the constant coefficient here, cause there is no clue that price behavior would be different from what it is during learning procedure, and hence by consideration of this factor, we are manipulating models learning capability! It was not this way for RBF or other nonlinear models as they were trained by training set, using actual data, and we took the constant coefficient into account so we can decide better on each design's merit!

7.11.1 Validation Results:

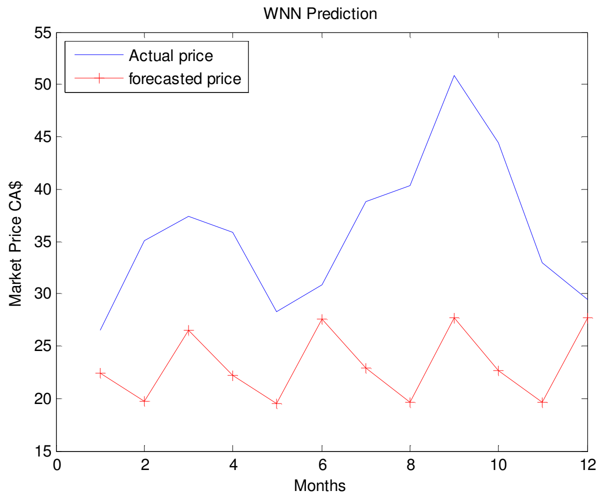
k\m	2	3	4	
2	62.8	75.1396	76.2146	Very good trend extraction for m+4 and small values of k, like 2 to 6
3	55.6691	71.7819	75.7836	
4	53.472	73.5615	74.8087	
5	54.9366	73.3091	77.4661	
6	56.5027	73.7828	78.3721	
7	59.9096	74.8875	81.0028	
8	66.4721	75.3137	81.1431	
9	67.5012	75.9108	82.6534	
10	69.0459	75.5446	83.2258	
11	71.3006	75.8865	83.8467	
12	72.4493	75.9737	84.3596	
13	73.4627	76.3359	85.6178	
14	74.245	77.9454	86.583	
15	74.6139	78.5745	87.1154	
	NO trend detection for large values of K!			

7.11.2 Main forecasting results:

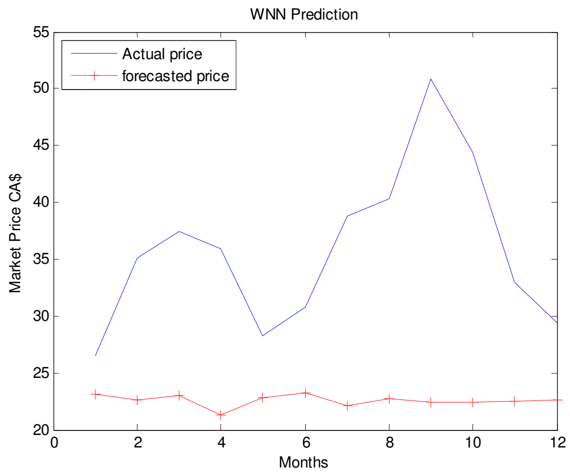
7.11.2.1M=4, k= 3, MAPE= 32.2649



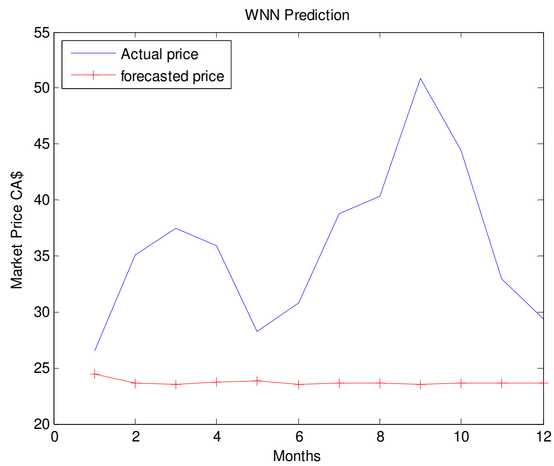
7.11.2.2M=4, k= 5, MAPE= 33.4376



7.11.2.3M=4, k= 7, MAPE= 34.8168

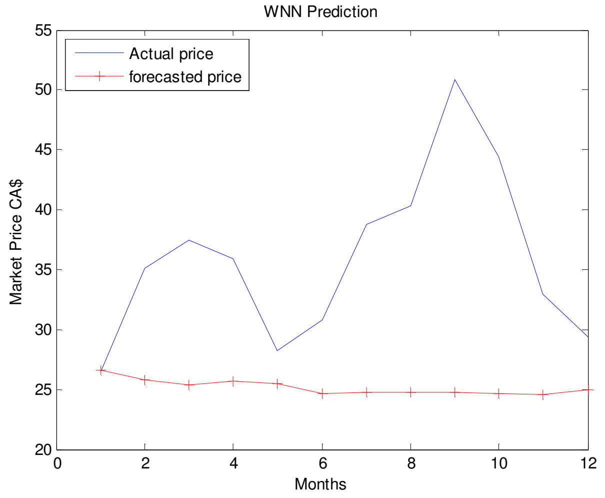


7.11.2.4M=4, k= 10, MAPE= 31.5916

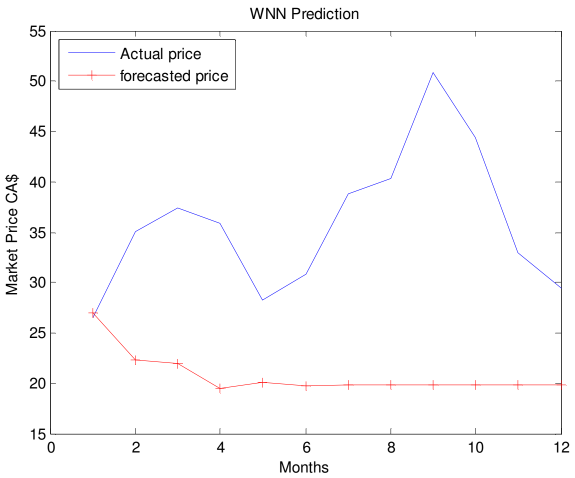


Appendix

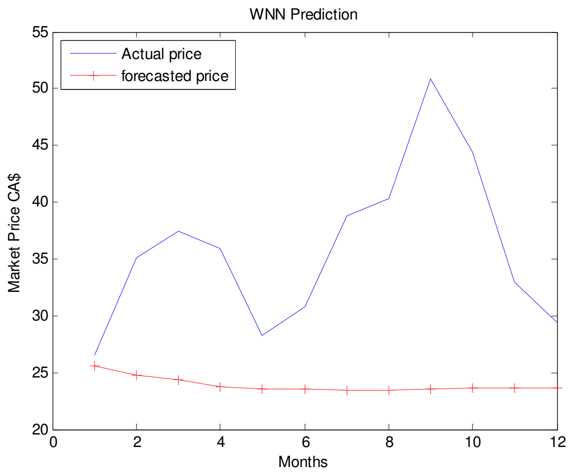
7.11.2.5M=4, k= 15, MAPE= 27.3465



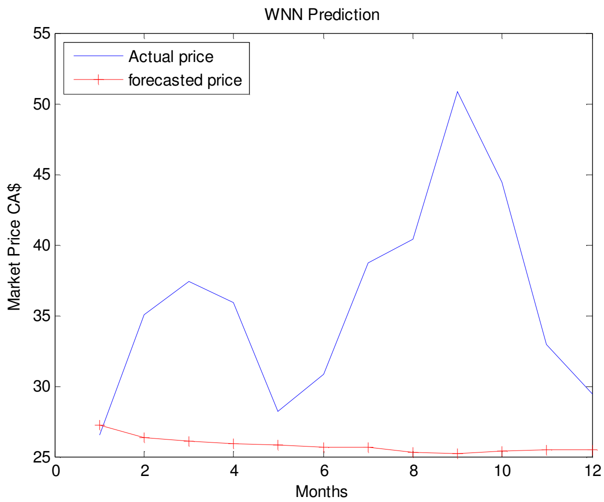
7.11.2.6M=2, k= 3, MAPE= 39.7470



7.11.2.7M=2, k= 10, MAPE= 30.9760



7.11.2.8M=2, k= 15, MAPE= 25.9151



Trying other designs has not lead to better results! As we expected, this model is not capable of predicting the price.

TABLE XXVIII. Price Base WNN in Market of Ontario

Price Based WNN		
m	k	MAPE
4	3	32.2649
4	5	33.4376
4	7	34.8168
4	10	31.5916
4	15	27.3465
2	3	39.747
2	10	30.976
2	15	25.9151