Thesis for the Degree of Licentiate of Engineering

# On Control Strategy and Safety Verification of Automated Vehicles

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To my family

# Abstract

Over the last few decades, congested traffic network have become a serious problem in many countries. Congestions result in time losses, increase of fuel consumption, increase of $CO_2$ emissions and also raise the risk of accidents. While developing the road networks is not a feasible solution in many countries, intelligent transportation systems (ITS) may contribute to mitigate such problems. It is known that human errors or the delay in human's reactions is the main cause of many of the problem in current transportation systems. Hence, cooperative driving or in particular vehicle platooning is an example of an ITS which exploits advanced technology like, on-board vehicle sensors, wireless communication and control engineering to improve the traffic situation. However, development of such complex system requires a reliable control algorithm which can guarantee passenger safety and comfort while satisfying certain specifications. This thesis deals with the development of a distributed control strategy for a vehicle platoon. The aim of the control strategy is to enable platooning with a short inter-vehicle distance while fulfilling the so called *string stability* criterion and maintaining the safety and comfort. The control design is divided into longitudinal and lateral control of vehicle. Simulation and experimental results indicate that string stability in longitudinal and lateral direction can be achieved using the proposed control strategy. Furthermore, a safety verification method based on reachability analysis technique and invariant set theory is proposed for safety analysis of such autonomous systems for a given cooperative controller. The safety verification method is extended to account for model uncertainty and measurement noises. The findings in this thesis are verified through simulations and field tests.

**Keywords:** Intelligent Transportation, Platooning, String Stability, Distributed Control, Reachability Analysis.

# Acknowledgments

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Roozbeh Kianfar
Göteborg, January 2013

# List of publications

This thesis is based on the following appended papers:

## Paper 1

R. Kianfar, B. Augusto, A. Ebadighajari, U. Hakeem, J. Nilsson, A. Raza, R. S. Tabar, N. Irukulapati, C. Englund, P. Falcone, S. Papanastasiou, L. Svensson, and H. Wymeersch, Design and Experimental Validation of a Cooperative Driving System in the Grand Cooperative Driving Challenge, *Intelligent Transportation Systems, IEEE Transactions on*, vol. 13, no. 3, pp 994-1007, 2012.

## Paper 2

R. Kianfar, P. Falcone and J. Fredriksson, On Safety Verification of Cooperative Driving Systems with Application to Cooperative Adaptive Cruise Controller, *Submitted to IEEE Magazine on Intelligent Transportation Systems (under revision).*

## Paper 3

R. Kianfar, P. Falcone and J. Fredriksson, A Distributed Model Predictive Control (MPC) Approach to Active Steering Control of String Stable Cooperative Vehicle Platoon, *Submitted to IFAC Symposium on Advances in Automotive Control*, September 2013, Tokyo, Japan.

## Other publications

In addition to the appended papers, the following papers are also written by the author of the thesis:

R. Kianfar, P. Falcone and J. Fredriksson, A Receding Horizon Approach for Designing String Stable Cooperative Adaptive

Cruise Control, *14th International IEEE Conference on Intelligent Transportation Systems* October 2011, Washington DC, USA.

R. Kianfar, P. Falcone and J. Fredriksson, Reachability Analysis of Cooperative Adaptive Cruise Controller, *15th International IEEE Conference on Intelligent Transportation Systems*, September 2012, Anchorage, Alaska, USA.

R. Kianfar and J. Fredriksson, Towards Integrated Design of Plant/Controller with Application in Mechatronics Systems, *Proceedings of the ASME 2011 International Mechanical Engineering Congress and Exposition IMECE2011*, November 2011, Denver, Colorado, USA.

R. Kianfar and T. Wik, Automated Controller Design using Linear Quantitative Feedback Theory for Nonlinear systems, *7th IEEE International Conference on Control and Automation*, December 2009, Christchurch, New Zealand.

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# Part I

# Introductory chapters

# Chapter 1

# Overview

The increase of number of vehicles in daily traffic as well as the transportation growth lead to traffic flow problems, like congestions. Over the last few decades congested traffic network have become a serious problem for the society. Congestions result in time losses, increase the air pollution by increasing the $CO_2$ level, increase the fuel consumption and increase the possibility of accidents. According to the European Union Road Federation 2010, the tonne-kilometer on the EU-27 road network has grown by 45.5% over the period 1995-2008, at a rate of 2.9% per year. Similarly, the passenger-kilometer has grown by 21.4%, at a rate of 1.5% per year. In 2008, this growth led to shares of 72.5% and 72.4% of the total inland EU-27 transportation of goods and passengers, respectively, to take place on the road network. In a separate report from the *European Union Commission* it is stated that 40000 people die in road accidents every year. This report also states that road accident are the main cause of death for people under the age of 45 in Europe. Considering congestions problem and the problems associated to that, motivates questioning whether the existing road network has, at the current growth rate, the capacity to meet the future demands for safe road transportation of both goods and passengers. The non-stopping demand for more transportation, requires developing more road and road infrastructure. However, in the mega-cities which are mainly subjected to congestions problem, developing the road network is not a feasible solution anymore. Fortunately, thanks to advances in vehicular, communication and information technologies traffic congestion can be alleviated, by enabling cooperation among vehicles to better exploit the usage of existing roads capacity.

Intelligent Transportation Systems (ITS) in a wide scenes refers to advances in the infrastructure unit, roadside units and also intelligent vehicle, see Figure 1.1. Cooperative driving is an example of intelligent transportation system, which consists of hardware and software mechanisms enabling

Figure 1.1: Intelligent transportation system (ITS), courtesy of the U.S. Department of Transportation

autonomous driving of multiple vehicles (vehicle platoons).

## 1.1 Platooning state of the art

Driver's reaction time is subjected to delay and error, i.e. it takes some time until the driver reacts to changes in the environment. This delay can have a great impact on the traffic flow, which can result in collisions or other undesired phenomena in traffic. Automated driving or in particular vehicle platooning can instead, help to mitigate congestions problem. In a vehicle platoon, a chain of vehicles follow each other in an automated way. The first vehicle in the platoon is called the leader and the rest of vehicles are called followers. In the simplest case, every vehicle in the platoon measures its position with respect to its preceding vehicle using on-board sensors e.g. radar, lidar or camera and maintain a safe distance to its preceding vehicle by controlling its velocity. An example of platooning in a real scenario can be seen in Figure 1.2. The idea of platooning can be traced back to the eighties when the California Partner for Advanced Transportation System (PATH) was established to develop and investigate the impact of vehicle-highway cooperation and communication systems [1]. Since then, this idea has been further studied and developed by many researchers, see e.g. [2–5]. The main advantages of platooning are:

- increased traffic throughput: automated driving or platooning can reduce the inter-vehicle distance between vehicles which results in a better usage of road capacity. The result of a recent study, [6], shows

Figure 1.2: Intelligent transportation system (ITS)

that the highway capacity can be increased up to 43% if all the vehicles in the highway enable platooning using on-board sensors (camera and radar). The same study shows that this figures can boost up to 273% providing that the vehicles also use wireless communications.

- reduced fuel consumption: By reducing the inter-vehicle distance between the vehicle the aerodynamic drag is also decreased. Reduction in the aerodynamic drag result in reduction of the fuel consumption. The result of a study, [7], shows that the fuel consumption can be reduced up to 7% for a vehicle platoon with only two trucks. In addition to decreasing the inter-vehicle distance, platooning can also contributes to reduction in fuel consumption by avoiding unnecessary acceleration and deceleration.

- reduced air pollution: apparently reducing the fuel consumption leads to reduction of air pollutant as well.

- increased safety: human reaction is naturally subjected to delay and the delay depends on the cognitive status of the driver. Statistics shows that human error is the main source of accident in almost 90% of the car accident . Hence, platooning can also mitigate accidents by enabling a safe inter-vehicle distance between the vehicles.

- increased comfort: driving a car can be as unpleasant as it is sometimes enjoyable. Every driver waste a significant amount of time while driving in a congested road. Platooning provides the opportunity to the driver to be relaxed or even let the driver to spend the time on more desirable task, e.g. reading news, surfing.

## 1.2 Enable platooning

To drive with a short inter-vehicle distance and without jeopardizing the safety demands a delicate engineering design. To accomplish platooning, interactions between three modules, i.e. *communication*, *sensing* and *control* are required. Thanks to advances in wireless communication, vehicles can send/receive information to/from other vehicles, i.e. vehicle-to-vehicle (V2V) communication and also the infrastructure, i.e. vehicle-to-infrastructure (V2I) communication. The communication messages can be transmitted in real-time, fail-safe and reliable way based on the communication standard protocol, IEEE.11p . Wireless communication can be used to enhance the sensing module. In other words, wireless communication can provide additional useful information to the control modules which cannot easily be measured using on board sensors. The sensing module consists of on-board sensors and positioning devices such as GPS and compass. The information delivered by the communication module and the measured information from the sensing module are fused to obtain a good estimate of desired quantities. Then, the fused information are sent to the control module which is responsible for decision making. The control unit is responsible for maintaining a desired safe distance to the preceding vehicle. The control action is sent to the vehicle actuators which are *i*) engine throttle and brake in case of longitudinal control and *ii*) steering wheel in case of lateral control. An overall system architecture of an automated vehicle in a platoon is depicted in Figure 1.3.

### Challenges in control problem

As mentioned earlier, driving in a close distance put a high demand on the controller. The controller should be able to guarantee safety and comfort of the passengers while respecting the limitations in the actuators. Furthermore when vehicles moves as a chain (vehicle platoon) not only the stability of individual vehicle should be considered in the control design but also the stability of the whole platoon together plays a key role on the overall performance of the platoon. The platoon stability is referred as *string stability* and defined as the capability of a platoon in attenuating disturbances in position error, velocity error and acceleration as the disturbances propagate towards the tail of the platoon [8]. Concept of string stability will be discussed further in detail in Chapter 3. String instability can introduce

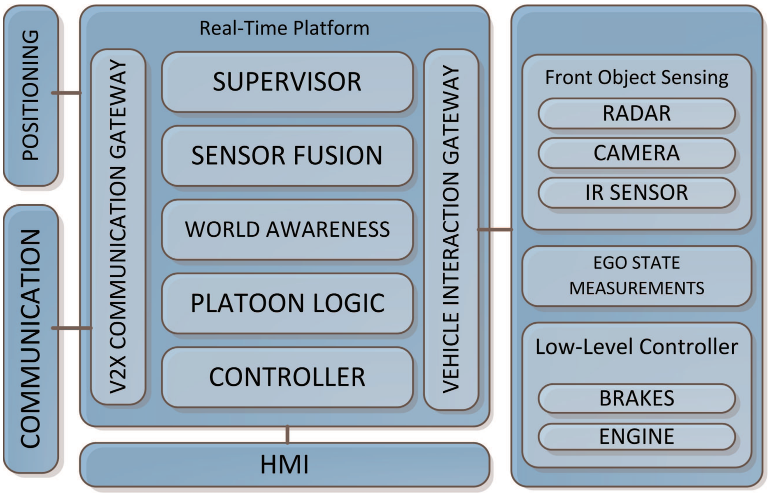


Figure 1.3: System architecture

shock waves in traffic flow which may result in collisions.

As the inter-vehicle distance reduces safety becomes a critical issue to consider. Considering different sources of uncertainty in a cooperative driving system, e.g. delay in communication, measurement noise and uncertainty in the actuator model, the need for a verification method becomes vital. Behaviour of vehicle should be verified for a given controller in presence of the aforementioned uncertainties.

## 1.3 Thesis contributions

The aim of this thesis is *i*) to develop a distributed control strategy which can enforce string stability of a vehicle platoon in longitudinal and lateral direction and simultaneously accounts for different constraints arising from specifications and limitations in the control problem, *ii*) to propose a safety verification method which can be used to safety verification of a given cooperative controller in presence of model uncertainty and measurement noise. The main contributions of the thesis are as follows,

- a control scheme to enable longitudinal vehicle platooning is developed. The developed controller is able to handle different constraints arising from different sources, e.g. actuator limitation and safety while preserving string stability.
- the traditional frequency domain longitudinal string stability specification is translated to a time domain specification. The specification

is then considered in the control design.

- the works mentioned in the first two items are experimentally validated.
- a method for safety verification of cooperative linear controllers, e.g. CACC and ACC is proposed and the results is experimentally validated.
- as an example, the proposed verification method is applied to study the impact of providing additional information e.g. acceleration of preceding vehicle to the controller on the inter-vehicle distance.
- the proposed safety verification method can be used to avoid extensive simulations and expensive experiments.
- a distributed control scheme is developed for lateral control of vehicle platooning.
- string stability in the lateral direction is enforced by proposing a control scheme which utilizes communication between vehicles.

## 1.4 Thesis outline

This thesis consists of two parts. Part I, provides context and a brief background for the second part. Part II includes three papers which serve as the core for the thesis. Part I comprises seven chapters. Chapter 1 provides an introduction to platooning. In chapter 2 an overview of advanced driver assistance systems with emphasize on autonomous and cooperative systems is given. In chapter 3, important properties of vehicle platoon, i.e. cooperation topology and string stability are introduced. Chapter 4 gives a brief background on vehicle modeling. Chapter 5 gives an overview of mathematical tools used in this thesis. A summary of the appended papers is in Chapter 6. At the end, Chapter 7 finalizes Part I with concluding remarks and future works.

# Chapter 2

# Advanced driver assistance systems

Over the last two decades advances in the vehicular technology, communication and control systems have led to introduction of several advanced functionalities by automotive industry, e.g. Anti Blocking Systsm (ABS), Vehicle Stability Control (VSC) and Lane Departure Warning (LDW), Figure 2.1. In automotive industry these new systems are called Advanced Driver Assistance Systems (ADAS). The primary objectives of ADAS are to assist the driver either when the safety is endangered or when the driver demands more comfort. However, recent research and rapid development of sensing technology have made it possible to have autonomous vehicles with a level of autonomy which goes far beyond the capability of standard ADAS systems.

(a)

(b)

Figure 2.1: (a) Lane departure warning system (b) Camera mounted on the back of mirror.

## 2.1 Autonomous driving

The dream of having fully autonomous vehicles has been initiated a few decades ago. However, this dream did not come true until almost ten years ago when the first DARPA Grand Challenge (2004) was held in the Mojave Desert in the USA. In that challenge several teams from leading universities and companies competed against each other. The idea was to develop fully autonomous vehicles which can travel a certain route in the desert without any interaction with humans. Even though none of the teams managed to finish the competition in that event, the first serious attempt for having fully autonomous vehicle was made. In DARPA Grand Challenge (2007, Urban Challenge) several teams competed over a course of 96 km in urban area. Vehicles were supposed to obey all traffic rules while negotiating with other traffics and avoiding any possible obstacles. One of the successful project which participated in DARPA is the Google driverless car. In Figure 2.2 the Google car is shown.

However, having such fully autonomous vehicles in every day transportation systems seems a bit unrealistic at the moment. The main reason is the high cost of having so many expensive sensors on normal passenger vehicles. Furthermore, safety verification of such autonomous vehicles requires tremendous amounts of work which seems unreasonable at the moment.

Figure 2.2: Google autonomous car

## 2.2 Cooperative driving (vehicle platooning)

Cooperative driving or in particular vehicle platooning can be consider as an intermediate step to fill the gap between manual driving and fully autonomous driving. In vehicle platooning, vehicles can operate fully or semi-autonomously in some part of the route, e.g. highways and return the control back to the driver when needed. The idea is to exploit the already available sensing module and actuators in production to have a cost efficient product.

A well known example of semi-autonomous systems is adaptive cruise control (ACC) which was launched in 1995's by the car maker Mitsubishi. ACC is an enhanced version of cruise control (CC). In an ACC system, the relative distance and velocity between two adjacent vehicles are measured using a radar or lidar and are sent to the control unit, Figure 2.3. Then, the control unit maintains a safe distance between vehicles by controlling the acceleration and brake pedal. ACC has a hierarchical control architecture, meaning that, the controller consists of an upper and a lower level as depicted in Figure 2.4. The upper level controller is responsible to determine the desired acceleration while the lower level controller is responsible to provide the commanded acceleration by controlling throttle and brake.

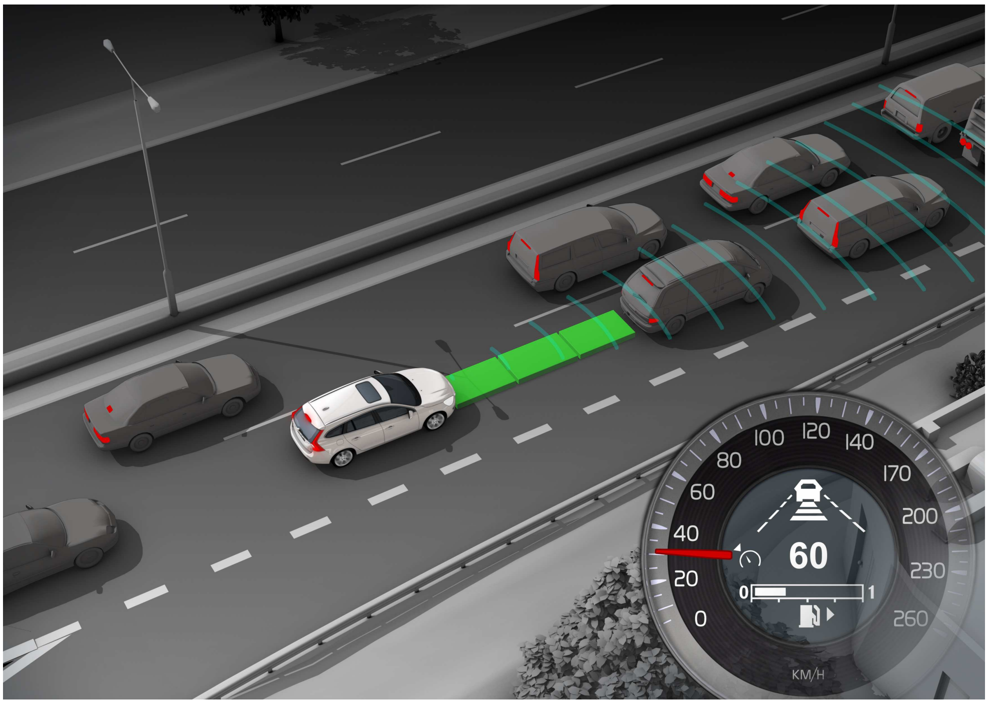


Figure 2.3: A radar based ACC system

Rapid advances in fast and reliable wireless communication, resulted in introduction of a new system so called Cooperative Adaptive Cruise Control (CACC). CACC functionality can be seen as an add-on for the ACC which, exploit wireless communication. Through wireless communication

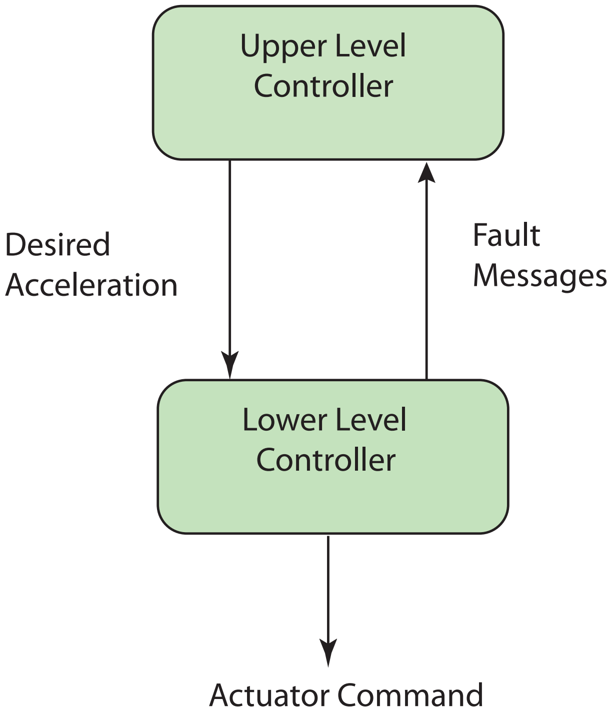


Figure 2.4: Hierarchical structure of ACC

vehicles can exchange information such as maximum braking capability, intended acceleration and commanded control signal. Furthermore, wireless communication can provide information about the status and topology of the road ahead which can have a great impact on the fuel economy of heavy duty vehicles. The overall information provided by communication can be exploited to enhance controller which is responsible for decision making.

### 2.2.1 Impact of ACC and CACC on traffic throughput

ACC and CACC may be used to enable platooning, [9, 10]. In Grand Cooperative Driving Challenge (GCDC) 2011, nine international teams competed against each other. The competition scenarios were made such that the performance of CACC systems proposed by different teams were evaluated in both urban and highway scenario, [11]. GCDC was one of the first event in which the efficiency of CACC was evaluated in a heterogeneous environment. Furthermore, over the last decade, the impact of (semi) autonomous systems such as ACC and CACC on the traffic throughput and also their capability in alleviating congestion problem studied by several researchers, [12–14]. The results in [14] reveal that CACC systems can lead to drastic improvement in the traffic efficiency. This study is limited to passenger cars and is not considering any overtaking manoeuvre. However, it indicates that the traffic flow can increase from 2100 veh/hr/ln (vehicle per hour per lane) on a 100% manual highway to 2900 veh/hr/ln on a 20% manual, 20% ACC and 60% CACC equipped highway.

As stated CACC shows a good potential to improve traffic flow by reducing the inter-vehicle distance between vehicles. However, we should note that reducing the inter-vehicle distance can introduce undesired effects, e.g. jeopardizing safety and string instability. Even though, most of the aforementioned studies indicate a good potential to improve traffic flow using CACC, there are still challenges such as string stability, robustness and safety verification to be addressed.

# Chapter 3

# Cooperation topology and stability of vehicle platoon

In this Chapter, two important concepts related to vehicle platooning are introduced. First, an overview of the cooperation topology in a vehicle platoon is given. Different cooperation topologies may result in different choices of control structure. Hence it can have a great impact on overall performance of a vehicle platoon. Secondly, string stability as an important concept in vehicle platooning is introduced and defined formally. String instability can severely affect the performance and safety of a platoon.

## 3.1 Cooperation topology

As mentioned in Chapter 1, every vehicle in a platoon may be equipped to wireless communication link which via that can send and receive information to/from other vehicles in the platoon. In this thesis, there is no intention to give an extensive survey of different cooperation topologies, instead, among different types of cooperation topology, the most common two cooperation topologies in the literature are introduced. As can be seen from Figure 3.1, The simplest communication topology is when every vehicle in the platoon only receives information from its immediate preceding vehicle and only send information to its immediate follower. This topology is preferable from

Figure 3.1: Predecessor-follower communication

an implementation point of view and is called CACC [15, 16]. The second common assumed communication topology for a platoon requires that all the vehicles in a platoon receive information from the leader of the platoon in addition to their neighbour vehicles. This configuration can be seen in Figure 3.2. The advantage of this topology is that the extra information provided to the controller may result in a better control performance [2]. However, this may require a more complicated control algorithm and also a wireless communication system with higher bandwidth.

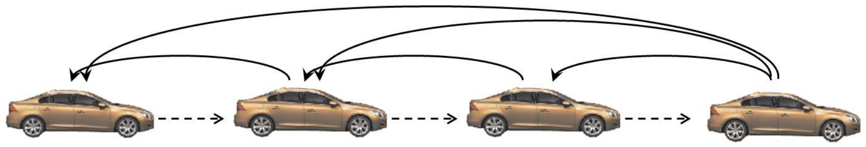


Figure 3.2: Predecessor-follower and leader followers communication

## 3.2 String stability

String stability is an important property of a vehicle platoon which refers to the capability of a platoon in attenuating the oscillation which is introduced by the leader or any other vehicle in the platoon. The oscillation can be considered with respect to different signals like position error between the vehicles [2] or with respect to acceleration [17]. Hence, a platoon is string stable if any oscillation with respect to the desired signal damps out as it propagated toward the tail of platoon. An example of string stable platoon with respect to acceleration signal is depicted in Figure 3.2. The red dashed signal is the acceleration of lead vehicle and as can be seen from the figure the acceleration signal is attenuating form the leader to the last follower (blue dashed signal).

### String stability (longitudinal direction)

String stability can be defined mathematically as a norm condition in the frequency domain. Denoting $e_{p,i} \quad i = 1, ..., N$ as the position error between two adjacent vehicles in a platoon with $N$ vehicles, the string stability is defined as,

**Definition 1** *String stability in frequency domain (predecessor-follower string stability),*

- *a vehicle platoon is predecessor-follower string stable if*

$$\|\frac{e_{p,i}(j\omega)}{e_{p,i-1}(j\omega)}\|_{\infty} \leq 1 \quad \forall i = 2, ...N, \forall \omega \tag{3.1}$$

- *a vehicle platoon is leader-followers string stable if*

$$\|\frac{e_{p,i}(j\omega)}{e_{p,1}(j\omega)}\|_{\infty} \leq 1 \quad \forall i = 2, ...N, \forall \omega \tag{3.2}$$

To guarantee string stability, a controller should be designed such that the condition (3.1) or (3.2) be satisfied. We should note that condition (3.1) is more stringent compared to (3.2).

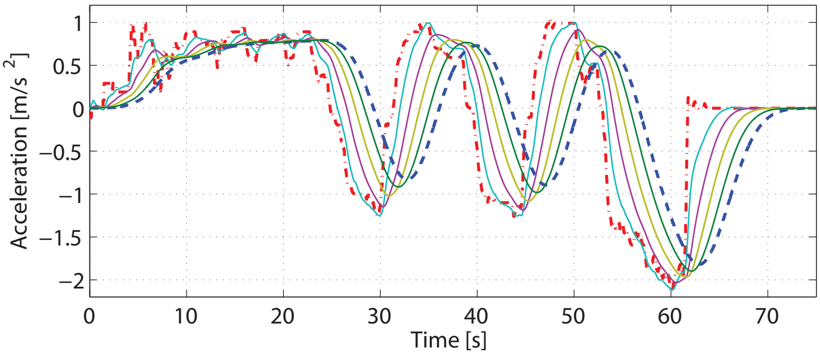


Figure 3.3: String stable platoon with respect to acceleration signal. The red dashed signal is the acceleration of the lead vehicle and the acceleration of the last follower is shown in blue dashed signal.

### String stability (lateral direction)

While string instability in the longitudinal direction may result in shockwaves and consequently accidents, string instability in the lateral direction may result in vehicles ending in the wrong lane after, e.g, a lane change manuever. Hence, in a vehicle-following setup with automatic steering, string stability in the lateral direction should be considered in the control design as well. Similar to the longitudinal case, string stability in the lateral direction can be defined as norm condition. However, the string stability is defined w.r.t lateral offset between vehicles, [18].

# Chapter 4

# Vehicle dynamics modeling

In this chapter, the basic of vehicle dynamics and its fundamental limitation is presented. The aim of this chapter is to provide a basic understanding of vehicle dynamics and the modeling assumptions that are used in this thesis. From a control perspective it is more desirable to work with linear model as long as they are a good approximation of real process. Hence, in this chapter the main source of nonlinearity in the vehicle dynamics are described. First, we start by modeling the longitudinal motion of a vehicle. Then, we extend our model to capture the lateral vehicle dynamics. Finally, we discuss fundamental limitation of vehicle capability which, mainly arise from the limitation in the tire friction. For further studies on vehicle dynamic we refer to [19, 20].

## 4.1 Longitudinal dynamics

The longitudinal dynamics of a vehicle can be modeled as a point mass using a force balance (Newton's second law), Figure 4.1

$$m\frac{dv}{dt} = F_{xf} + F_{xr} - F_d - F_{roll} - F_g \tag{4.1}$$

where $m$ and $v$ are the mass and velocity of the vehicle, respectively. $F_{xf}$ and $F_{xr}$, $F_d$, $F_{roll}$ and $F_g$ are the longitudinal tire forces at the front and rear tires, the aerodynamic drag, the rolling resistance and gravity forces, respectively. The longitudinal tire forces depend on *i*) the so called *slip ratio*, *ii*) normal load on the tire and *iii*) friction coefficient between the tires and the road $C_\alpha$. The slip ratio is defined as,

$$\kappa = -\frac{v_{xw} - \omega r_w}{v_{xw}}, \tag{4.2}$$

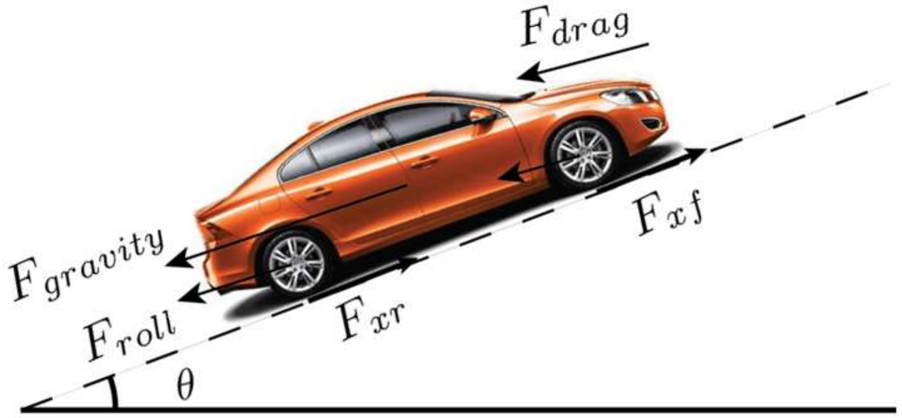
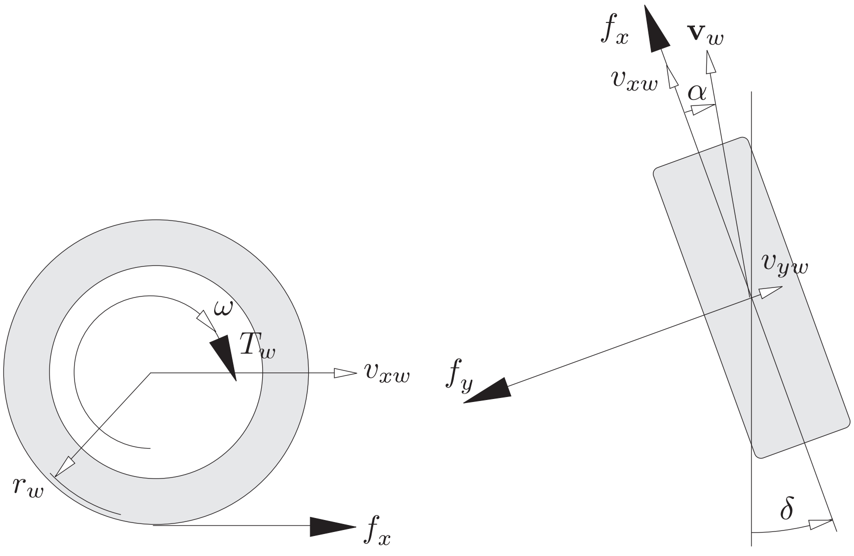


Figure 4.1: Vehicle model

where $v_{xw}$, $\omega$ and $r_\omega$ are the longitudinal velocity at the axle of the wheel, rotational velocity of the wheel and effective tire radius, respectively, according to Figure 4.2. The typical characteristic of longitudinal tire force



(a) (b)

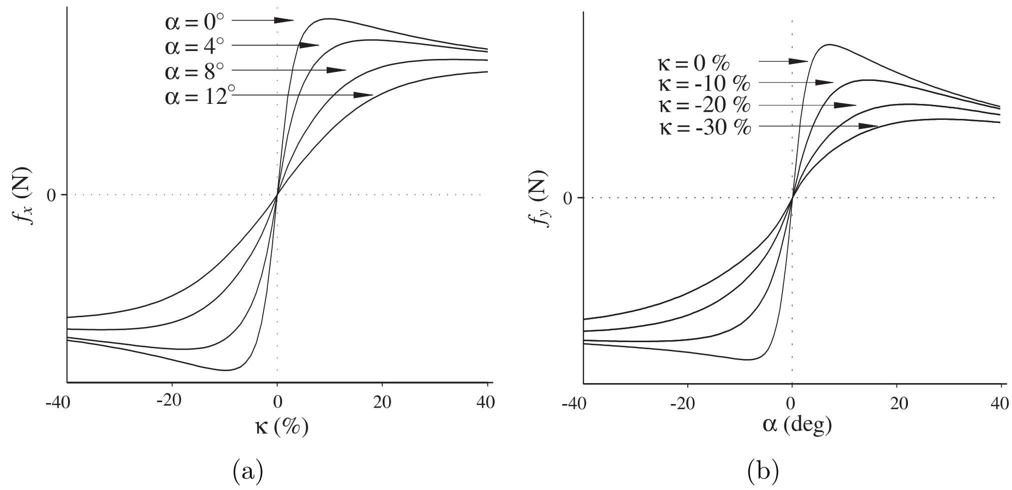
Figure 4.2: Tyre modeling notation

versus the slip ratio is depicted in Figure 4.3(a). As can be seen from the figure, for small slip ratio, the longitudinal tire force is proportional to the slip ratio,

$$F_{xf} \simeq C_{\alpha_f}\kappa_f \tag{4.3}$$

$$F_{xr} \simeq C_{\alpha_r}\kappa_r \tag{4.4}$$

where $C_{\alpha_f}$ and $C_{\alpha_r}$ are the tire stiffness for the front and rear tire, respectively. However, for the larger slip ratio the relation between longitudinal tire force and slip ratio becomes nonlinear, which requires a more sophisticated model, to further study of tire characteristic we refer to [20].



(a) (b)

Figure 4.3: Tyre forces (a) Longitudinal force as a function of longitudinal slip $\kappa$, for different slip angles $\alpha$ (b) Lateral force as a function of side slip angle $\alpha$, for different slip values $\kappa$.

The aerodynamic drag force is denoted by $F_d$ and is represented as

$$F_d = \frac{1}{2}\rho C_d A_F (v + v_w)^2 \tag{4.5}$$

where $\rho$, $C_d$, $A_F$ and $v_w$ are the mass density, drag coefficient, frontal area of the vehicle and wind velocity, respectively. The rolling resistance in the tire is denoted by $F_{roll}$ and is proportional to the normal forces, i.e.,

$$F_{roll} = f(F_{Nf} + F_{Nr}) \tag{4.6}$$

where $F_{Nf}$ and $F_{Nr}$ and $f$ are the normal forces at the place of front and rear tires and rolling resistance coefficient, respectively. Finally the gravitational force can be written as,

$$F_g = mg\sin(\theta) \tag{4.7}$$

where $\theta$ is the slope of the road.

## 4.2 Lateral vehicle dynamics

In the previous section, the longitudinal dynamics of a vehicle along its longitudinal axis was modeled. In this section, we present a model which describe the so called lateral dynamics of a vehicle. Here, lateral dynamics refers to both the dynamics of vehicle along the axis perpendicular to the longitudinal axis of vehicle and also the yaw dynamics. Here, we introduce the basic of the so called *bicycle model*, which is well accepted model to capture the lateral vehicle motion. A bicycle model assumes identical slip angles for the left and right wheel on each axis. However, this assumption is reasonable for negotiating curves of moderate radius at a normal driving velocity. Hence, the bicycle model sometimes is referred to as single track model as well. Applying Newton's second law along the $y$ axis depicted in Figure 4.4,

$$ma_y \quad = \quad F_{yr} + F_{yf} \tag{4.8}$$

where $a_y$, $F_{yr}$ and $F_{yf}$ are the inertial acceleration of the vehicle along the $y$ axis at the central of gravity (c.g), lateral tire force of rear and front tires, respectively. The inertial acceleration $a_y$ consists of lateral acceleration $\ddot{y}$ in the vehicle body frame and also the centripetal acceleration $v_x\dot{\psi}$. Hence, the inertial acceleration can be written as,

$$a_y = \ddot{y} + v_x\dot{\psi} \tag{4.9}$$

where $\ddot{y}$, $v_x$ and $\dot{\psi}$ are the acceleration in the body frame, longitudinal velocity in the body frame and the angular velocity of body frame coordinate in inertial frame. By replacing (4.9) in (4.8), the vehicle motion along the $y$ axis can be written as,

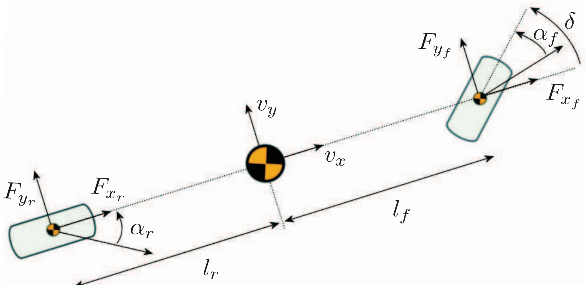


Figure 4.4: Vehicle model

$$m(\ddot{y} + v_x\dot{\psi}) = F_{yf} + F_{yr} \tag{4.10}$$

The lateral forces $F_{yf}$ and $F_{yr}$ are proportional to the so called *slip angle* of front and rear tires, respectively. As can be seen in Figure 4.3, the slip angle of tire $\alpha$ is defined as the angle between the orientation and velocity of tire.

$$\alpha = \arctan(\frac{v_c}{v_l}) \tag{4.11}$$

where $v_c$ and $v_l$ are the lateral and longitudinal velocity of tire. Slip angle $\alpha_f$ of the front wheel can be written as,

$$\alpha_f = \delta - \theta_f \tag{4.12}$$

where, $\delta$ and $\theta_f$ are the steering angle and the angle between the velocity vector of front wheel and longitudinal axis of the vehicle. Similarly, the slip angle of the rear tire can be described as,

$$\alpha_f = -\theta_r \tag{4.13}$$

where $\theta_r$ is angle between the velocity vector of rear wheel and longitudinal axis of the vehicle. Therefore, the lateral forces of front and rear tires can be written as,

$$F_{yf} = C_{\alpha f}(\delta - \theta_f) \tag{4.14}$$

$$F_{yr} = C_{\alpha r}(-\theta_f) \tag{4.15}$$

where, $C_{\alpha f}$ and $C_{\alpha r}$ are the cornering stiffness of the front and rear tires, respectively. However, the proportionality relation between lateral force and slip angle only holds for small slip angle. For larger slip angle, a more sophisticated formula is required. Figure 4.3(b) shows the lateral force of tire versus the slip angle.

Moment equation around the $z$ axis in Figure 4.4 describes the yaw dynamic,

$$I_z\ddot{\psi} = F_{yf}l_f - Fyrl_r \tag{4.16}$$

where $I_z$, $l_r$ and $l_f$ are the the moment of inertia around $z$ axis and the distances between the rear and front tires and C.g of vehicle, respectively. Under assumption of $i$) small slip angles and $ii$) constant longitudinal velocity $v_x$, the lateral vehicle dynamics can be described using (4.10) and (4.16).

### 4.2.1 Global frame

The vehicle model developed in the previous section was based on a coordinate frame which was fixed to the vehicle. However, it might be also of interest to describe the vehicle motion with respect to the global coordinate frame as well. The vehicle's equation of motion in the inertial frame are

$$\dot{Y} = v_x \sin(\psi) + v_y \cos(\psi) \tag{4.17}$$

$$\dot{X} = v_x \cos(\psi) - v_y \sin(\psi) \tag{4.18}$$

where $v_x$ and $v_y$ are the longitudinal and lateral velocity of vehicle's c.g. As can be seen from (4.17) and (4.18), the vehicle motion's dynamics in the inertial frame is nonlinear, see Figure 4.5.

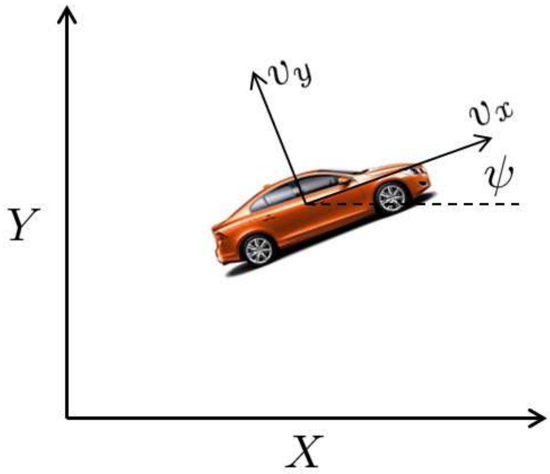


Figure 4.5: Vehicle in inertial frame

# Chapter 5

# Tools

In chapter 1, an overview of cooperative driving in particular platooning was given. In chapter 4 an introduction to vehicle dynamics was given. To accomplish platooning a control strategy is required to control the vehicle and fulfils certain requirements and specifications. Furthermore, to ensure that such automated system can operate safely without endangering the passenger safety, a safety verification method is required. This chapter is dedicated to an overview of the tools and techniques which are used for the controller design and for further safety analysis of automated vehicles, i.e. vehicle platooning.

## 5.1 Receding horizon control

Control of a vehicle platoon can be formulated as a constrained optimal control problem. The control objective is to minimize the control signal and the error states. Minimizing the control signal reduces the fuel consumption while minimizing the errors apparently improves the performance. There are also several requirements and specifications to be fulfilled by the controller while accounting for limitations in the actuators. Hence, Model predictive control (MPC) as a powerful tool to handle constraints on control input and states is considered as the control scheme to develop our automated driving system. In most cases, MPC controller can be formulated as a quadratic programming (QP), i.e. a quadratic objective function subject to linear constraints,

$$\min_{U} x(N)^{\mathrm{T}} P x(N) + \sum_{k=0}^{N-1} x(k)^{\mathrm{T}} Q x(k) + u(k)^{\mathrm{T}} R u(k) \tag{5.1}$$

**subject to**

$$x(k+1) = f(x(k), u(k)) \tag{5.2}$$

$$x(k) \in \mathcal{X}, u(k) \in \mathcal{U}, \tag{5.3}$$

$$g(x(k), u(k)) = 0 \tag{5.4}$$

where $U = [u_{t|t}, u_{t+1|t}, ..., u_{t+N|t}]$ and $x$ are control input and the state vectors, respectively. $P$, $Q$ and $R$ are the weighting matrices with appropriate dimensions to penalize the final state, state and input signal, $N$ is the prediction horizon, $\mathcal{X}$, $\mathcal{U}$ are admissible set of states and control signals, respectively. At every time step $k$ the optimization problem (5.1) is solved using the new measurement. As can be seen from Figure 5.1, the first optimal move $u^*_{t|t}$ is applied to the plant. Then, at the next time step $k+1$ the same procedure is repeated. Solving the optimization problem online using new measurements provides feedback effect in the controller. For further study of the subject we refer to [21].

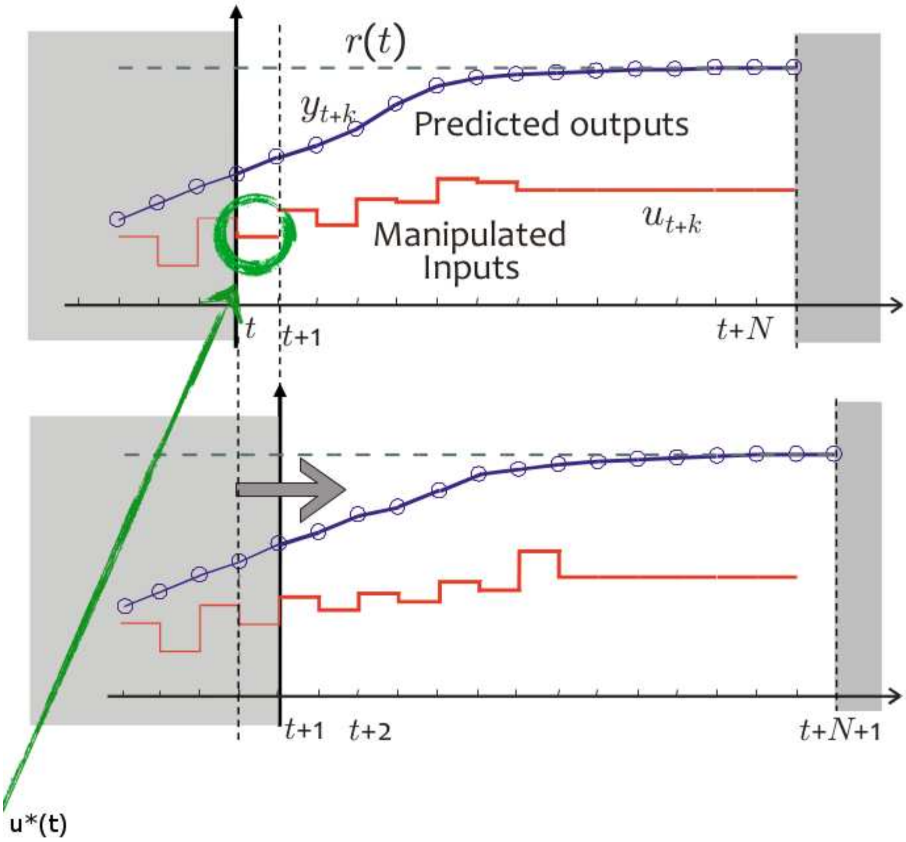


Figure 5.1: Model predictive control scheme

## 5.2 Reachability analysis and invariant set theory

To increase the traffic throughput, it is desirable to reduce the inter-vehicle distance between vehicles in a platoon. Decreasing the inter-vehicle distance can also result in reduction of fuel consumption. However, reducing the inter-vehicle distance apparently can increase the risk of rear end collisions. As described earlier, every vehicle in a platoon is equipped with a sensing module, e.g. radar and camera, to measure the relative distance and velocity between the vehicles. In addition to that, each vehicle can also be equipped with wireless communication which via that can receive information like acceleration of preceding vehicle. The information sensed by the sensing module and received by communication are fused and sent to the control unit. Then, using the received measurement and based on a motion model, the control unit calculate a commanded acceleration which will be sent to either throttle or brake system. However, the overall performance of such complex system which requires interaction between different subsystems, largely depends on the accuracy of sensor measurement, communication delay, packet drops, model mismatch and delay in the throttle and brake actuator. Although, extensive simulation can be used to verify system reliability and performance in different situation, any analysis based on simulation may not capture all the phenomena and may require an enormous amount of time. To verify such complex system mathematical tools are required. As an alternative for extensive simulations and expensive experiments, reachability analysis technique and invariant set theory can be used to safety verification of autonomous systems, [22], [23]. In this section, a brief overview on the aforementioned methods is presented which can serve a background for the second part of this thesis.

In this section, a few definitions are introduced and basic results on reachability analysis, are presented. For further information regarding invariant set theory and reachability analysis, refer to [24] and [25].

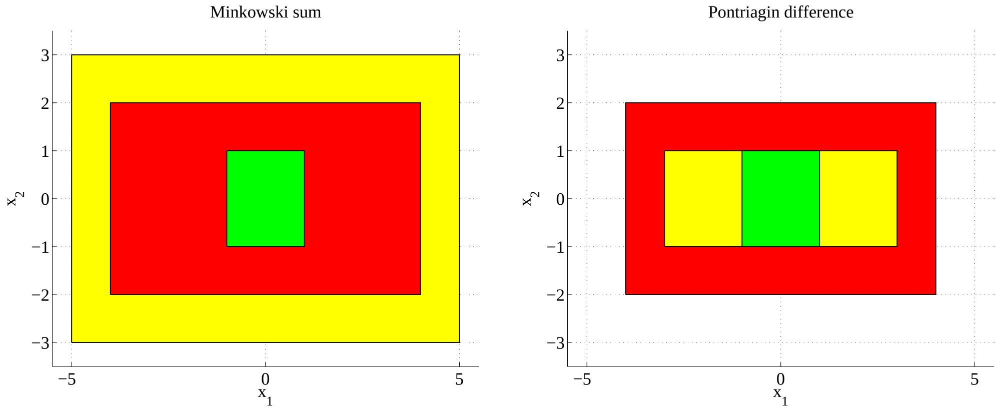
**Definition 2** *A polyhedron $\mathcal{P} \in \mathbb{R}^n$ is the intersection of finite number of closed halfspaces in $\mathbb{R}^n$*

$$\mathcal{P} = \{x \in \mathbb{R}^n | Hx \leq h\} \tag{5.5}$$

**Remark:** A closed polyhedron is called a polytope.

**Definition 3** *The Minkowski sum of two polytopes $\mathcal{R}$ and $\mathcal{Q}$ is a polytope defined as,*

$$\mathcal{R} \oplus \mathcal{Q} = \{x + y \in \mathbb{R}^n | x \in \mathcal{R}, y \in \mathcal{Q}\} \tag{5.6}$$



(a) Minkowski sum (b) Pontryiagin difference

Figure 5.2: Minkowski sum and Pontryagin difference of two polytopes

an example of Minkowski sum of two polytopes $\mathcal{R}$ (red box) and $\mathcal{Q}$ (green box) is depicted in Figure 2.3(a).

**Definition 4** *The Pontryagin difference of two polytopes* $\mathcal{R}$ *and* $\mathcal{Q}$ *is a polytope defined as,*

$$\mathcal{R} \ominus \mathcal{Q} = \{x \in \mathbb{R}^n | x + q \in \mathcal{R}, \forall q \in \mathcal{Q}\} \tag{5.7}$$

An example of Pontryagin difference of two polytopes $\mathcal{R}$ (red box) and $\mathcal{Q}$ (green box) is depicted in Figure 2.3(a).

**Definition 5** *The convex hull of a set of points* $X = \{X^i\}_{i=1}^{N_x}$ *is the smallest convex set which contains* $X$*.*

$$hull(X) = \{x \in \mathbb{R}^n : x = \sum_{i=1}^{N_x} \lambda_i X^i, 0 \leq \lambda_i \leq 1, \sum_{i=1}^{N_x} \lambda_i = 1\} \tag{5.8}$$

**Definition 6** *Composition of an affine mapping* $f$ *and a polyhedron* $\mathcal{P}$*, with* $f$ *as,*

$$f : z \in \mathbb{R}^m \mapsto Az + b, \quad A \in \mathbb{R}^{m_A \times m}, \quad b \in \mathbb{R}^m \tag{5.9}$$

*is defined as,*

$$f \circ \mathcal{P} = \{y \in \mathbb{R}^m | y = Ax + b \quad \forall x \in \mathbb{R}^n, \quad Hx \leq h\} \tag{5.10}$$

Denote by $f_a$ the state update function of an autonomous system,

$$x(k+1) = f_a(x(k), \omega(k)), \tag{5.11}$$

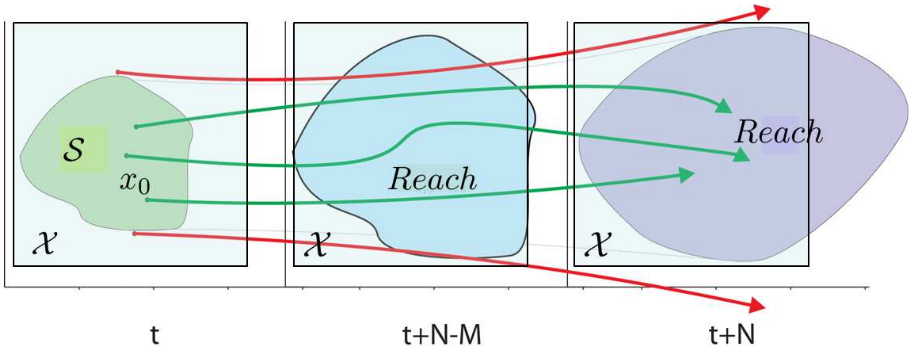


Figure 5.3: N-step reachable set. $\mathcal{S}$ represents the set of initial states. The blue and purple sets are the reachable set for two time instances. $\mathcal{X}$ is the admissible set

where $x(k)$ and $\omega(k)$ are the state and disturbance vector, respectively. The system (5.11) is subject to the following constraint,

$$x \in \mathcal{X}, \quad \omega \in \mathcal{W}, \tag{5.12}$$

where $\mathcal{X}$ and $\mathcal{W}$ are polytopes in $\mathbb{R}^n$ and $\mathbb{R}^d$, respectively.

**Definition 7** *For the autonomous system (5.11), we denote the robust one-step reachable set for initial states $x(0)$ contained in the set $\mathcal{S}$ as,*

$$Reach^{f_a}(\mathcal{S}, \mathcal{W}) = \{x \in \mathbb{R}^n : \exists x(0) \in \mathcal{S}, \exists \omega \in \mathcal{W} | x = f_a(x(0), \omega)\}$$

In Figure 5.2, the reachable set of initial states $\mathcal{S}$ of the dynamical system (5.11) is represented. The reachable set is represented for two time instances with blue and purple sets. The green arrows show how the initial states $x_0$ evolve into the reachable sets.

In words, the one step robust reachable set of dynamical systems (5.11) is defined as the set of states which can be reached from the set of initial states $x_0 \in \mathcal{S}$ for a disturbance $\omega \in \mathcal{W}$ in one time step.

**Definition 8** *For the autonomous system (5.11), the robust Pre set is defined as the dual of one-step reachable set,*

$$Pre^{f_a}(\mathcal{T}, \mathcal{W}) = \{x \in \mathbb{R}^n : f_a(x(k), \omega(k)) \in \mathcal{T}, \forall \omega \in \mathcal{W}\},$$

*where $\mathcal{T}$ is the target set.*

Backward reachable sets from a desired target set $\mathcal{T}$ are shown in Figure 5.2.

In words, the one step robust backward reachable set of dynamical systems (5.11) subject to constraint is defined as the set of states which can evolve to a target set $\mathcal{T}$ for any disturbance $\omega \in \mathcal{W}$ in one time step.

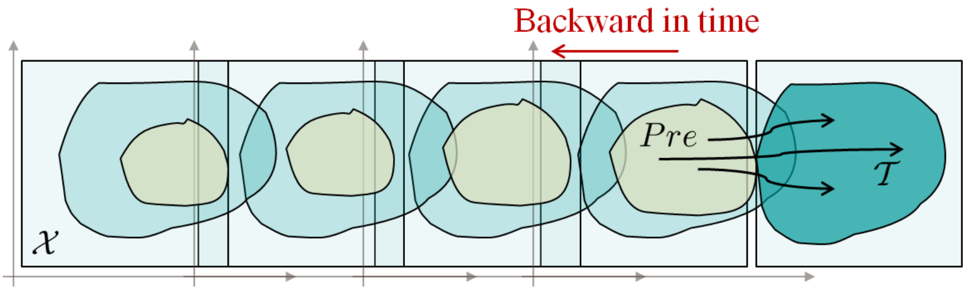


Figure 5.4: N-step backward reachable set. $\mathcal{T}$ represents the Target (desired) set. The yellowish set are the backward reachable sets for some time instances. $\mathcal{X}$ is the admissible set

# Chapter 6

# Summary of included papers

This chapter provides a brief summary of the papers that constitute the base for this thesis. Full versions of the papers are included in Part II. The papers have been reformatted to increase readability and to comply with the layout of the rest of the thesis.

## Paper 1

> R. Kianfar, B. Augusto, A. Ebadighajari, U. Hakeem, J. Nilsson, A. Raza, R. S. Tabar, N. V. Irukulapati, C. Englund, P. Falcone, S. Papanastasiou, L. Svensson and H. Wymeersch, Design and Experimental Validation of a Cooperative Driving System in the Grand Cooperative Driving Challenge, *Intelligent Transportation Systems, IEEE Transactions on*, vol. 13, no. 3, pp 994-1007, 2012.

This paper presents a simulation and experimental study of a string stable cooperative adaptive cruise controller (CACC). The first part of the paper briefly describe the implementation of a communication module based on IEEE protocol. This follows by a brief explanation about the sensor fusion module which was used to filter the data in a real time scenario. The rest of the paper is dedicated to evaluation of proposed control strategy. The proposed control strategy is a Model predictive control (MPC) approach which can handle different constraints and specifications while stabilizing the vehicle behaviour. Constraints and specifications are resulted from safety constraints, actuator limitations and performance requirements. Finally traditional frequency domain definition of string stability is translated into the time domain and is accounted for in the control design. The simulation and experimental results indicate the effectiveness of the proposed method.

The author of this thesis is responsible for designing the MPC controller and collecting the experimental data related to the controller, conducting the comparison between the two proposed control strategy. Furthermore, the author of the thesis is responsible for editing the overall paper.

# Paper 2

> R. Kianfar, P. Falcone and J. Fredriksson, On Safety Verification of Cooperative Driving Systems with Application to Cooperative Adaptive Cruise Controller, *submitted for publication in IEEE Magazine on Intelligent Transportation Systems (under revision).*

In this paper a method based on reachability analysis technique and invariant set theory is presented for safety verification of cooperative controllers. The main idea is to develop a mathematical framework for safety verification of cooperative control systems to avoid extensive simulations and expensive experiments. As an example, the method is applied to study the minimum required safe distance between two adjacent vehicles equipped with two given linear controllers. The method is further extended to account for model uncertainty and possible measurement noise. The proposed method can be used to calculate the maximal admissible safe set. In words, maximal admissible safe set is a set which vehicle safety is guaranteed for all the future time. Simulation results are validated with field experiment.

# Paper 3

> R. Kianfar, P. Falcone and J. Fredriksson, A Distributed Model Predictive Control (MPC) Approach to Active Steering Control of String Stable Cooperative Vehicle Platoon *Submitted to IFAC Symposium on Advances in Automotive Control*, September 2013, Tokyo, Japan.

A distributed receding horizon control strategy to active steering control of vehicle platoon is presented in this paper. It is assumed that every vehicle in the platoon can receive information form its preceding vehicle and send information to its follower. The information is sent via communication links between vehicles. The control problem is formulated as an optimization problem (MPC-based approach) and is solved locally by each vehicle. Each vehicle broadcasts an intention over a future prediction horizon. The intention is the optimal open loop trajectory calculated by the the local

MPC controllers. Furthermore, lateral string stability is formulated as a time domain constraint which is accounted for in the control design. The simulation results show the effectiveness of proposed approach.

# Chapter 7

# Concluding remarks and future challenges

This thesis preliminary investigates how cooperation between vehicles can help to mitigate congestions problem. Alleviating congestions and other problem associated to that have a clear benefit in traffic flow, fuel economy and air pollution. The focus of the Paper 1 in this thesis is to develop a model based control strategy to enable such cooperation. The main challenge along this way is to account for constraints and string stability simultaneously in the control design. There are several limitations and specifications to be considered in control design. The limitations usually arise from the limitations in the vehicle's actuators. However, specifications are mainly the safety and performance. The proposed control strategy in this thesis can formulate these limitations and specifications into a constrained optimal control problem which is solved in a receding horizon. String stability criterion is an important property of a platoon to be fulfilled by designing a proper controller. One of the main objective in Paper 1 is to include string stability condition as a constraint in the proposed MPC-based controller. In Paper 2, the vehicle dynamics and given controllers are combined to form the closed loop system. Then reachability analysis technique and invariant set theory are used for safety verification of closed loop system. Unlike the method proposed in Paper 1, there are many cooperative linear controllers which do not account for constraints on the control input and states explicitly. Hence, a verification method is required to study the behaviour of such controllers.

As an example the proposed method can be applied to study the minimum required safe inter-vehicle distance between vehicles. The reachability analysis can be used to calculate a set which can guarantee safety over a finite or even infinite horizon. The method is extended to account for uncertainty in the vehicle model as well.

To have fully autonomous vehicle, controllers must be developed which can enforce string stability in longitudinal and lateral direction. The lateral string stability is addressed in Paper 3. A distributed receding horizon approach is adopted for active steering control of vehicle platoon. The proposed method requires communication between two adjacent vehicles for enforcing string stability in the lateral direction. Each vehicle calculates and sends an intention of its movement for a finite steps ahead to its follower. A combination of old and future information (intention) is used as a constraint to enforce string stability.

### Future work

As a future work, the proposed safety analysis method must be extended to account for time delay and packet losses. Furthermore, the proposed method should be extended to verify string stability in presence of model mismatch and measurement noise. To extend Paper 3, the performance of proposed control strategy can be evaluated using a more detailed model and possibly with field experiment. Another direction to further proceed this work is to establish fundamental mathematical basis for the proposed strategy to enforce string stability.

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