A Revisit to Spatial Correlation in Terms of Input Network Parameters

PAPER B

A REVISIT TO SPATIAL CORRELATION IN TERMS OF INPUT NETWORK PARAMETERS

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Abstract

An alternative compact formula in terms of input network parameters for calculation of envelope correlations is provided. The main advantage of this formula is that it can be simply modified to include cases of lossy structures and general cases of correlated nonuniform multipath environments. This formula is based on the open-circuit covariance matrix of the multiport antennas. The latter can be either estimated or quickly measured in a reverberation chamber, which removes the need for costly measurements of the embedded far field patterns.

1 Introduction

In a multipath environment, spatial correlation is a measure of similarities between the received signals at different antenna ports. It plays a significant role in diversity performance of a multi-element antenna system, and its multiple-input-multiple-output (MIMO) multiplexing capacity. In general, as the separation between radiation elements shrinks, *e.g.*, in compact array antennas, coupling among them increases. Coupling is one of the main sources to cause spatial correlation which is a function of both embedded patterns and spatial properties of the incoming waves. The embedded pattern of a radiation element in the presence of coupling depends on the terminating impedances at the other ports as well. This causes some limitations in compact formulation of correlation.

In addition, in recent MIMO communication systems, the number of RF chains is not necessarily the same as the number of antenna ports. Sometimes, two or more ports are connected passively through a combiner/divider to share a single RF chain, which in turn changes the associated embedded pattern [1, 2]. By reciprocity, the latter case corresponds to exciting more antenna ports in transmit mode. Dependency of embedded pattern upon terminating impedances and excitation schemes can be consolidated into a single metric which is the associated current vector at different antenna ports. In this letter, we recast correlation in multiport antenna systems in terms of current vectors at their input ports. By this consideration, the proposed formula becomes flexible against terminating impedances in comparison with [3, Equation (11)], [4, Equation (31)], and robust against excitation schemes compared with [5, Equation (73)]. The open-circuit covariance matrix of embedded patterns plays a central role in our formulation rendering a symmetrical form to the presented formula. Another major feature of this novel formula is its robustness against any change in the properties of multipath environments. Indeed, with a slight modification the formula withstands against general cases of lossy structures in nonuniform correlated multipath environments.

To set the notations, matrices are denoted by bold capital letters, whereas the *column* vectors are shown by an overbar sign. The dagger sign shows Hermitian transpose and $\mathbf{I}$ denotes the identity matrix. $\mathbb{E}$ stands for expectation operator, $\Re$ for real part, and the superscript $\cdot^T$ goes for transpose.

2 Correlation in Isotropic Multipath Environments

It is well known that the embedded patterns can be recast in terms of their open-circuit counterparts at any arbitrary solid angle $\Omega(\theta, \psi)$ [4], [5]. For an n-port antenna system, let us denote the matrix of open-circuit embedded patterns by $\mathbf{G}_{2\times n}(\Omega)$, whose rows are the associated patterns' θ and ψ components. We assume that this multiport antenna with impedance matrix of $\mathbf{Z}_{n\times n}$ is excited by an arbitrary source of certain internal impedances given by $\mathbf{Z}_s$. Should we stack the source voltages at different ports in a vector, $\bar{v}_{n\times 1}$, the currents weights at the ports can also be given in a vector form governed by

$$\bar{i}_x = (\mathbf{Z} + \mathbf{Z}_s)^{-1}\ \bar{v}_x\ . \tag{1}$$

The embedded pattern associated to this current weights vector is obtained by $\bar{G}_x(\Omega) = \mathbf{G}(\Omega) \cdot \bar{i}_x$. When a single-port excitation occurs, the corresponding embedded pattern is referred to as *embedded element pattern*. The latter achieves considerable interest in receive-mode diversity antenna analysis.

On the other hand, random incoming waves in a multipath environment can be modelled by their amplitude and phase distributions, their angle of arrival (AoA) distribution and cross polarisation distinction (XPD) [5]. Let us denote an incident wave by a vector $\bar{E}_{2\times 1}$ whose entries are its E_θ and E_ψ components, respectively. For simplicity, we assume that the random incoming waves in each polarisation are zero-mean complex Gaussian random variables. If we denote the AoA in spherical coordinate system by $\Omega(\theta, \psi)$, the random incoming waves can be mathematically represented by *polarisation matrix* for incident waves

$$\mathbf{\Gamma}(\Omega, \Omega') = \mathbb{E}[\bar{E}(\Omega) \cdot \bar{E}(\Omega')^{\dagger}] \ , \tag{2}$$

wherein the expectation operates upon different realisations or time [5]. Note that the above expression is quite general including the case of correlated incoming waves of different AoAs and polarisations. Regarding the AoA distribution, the joint probability density function of different waves coming from Ω and Ω' directions is designated by $\mathsf{P}(\Omega, \Omega')$.

For the time being, we restrict ourselves to a special case of uncorrelated isotropic multipath environments and defer the more general cases to Section 3. Recall that multipath environments wherein incoming waves from different AoAs and polarisations are independent are referred to as uncorrelated multipath environments. If the AoA distribution in a multipath is uniform and XPD = 0 dB, it is called isotropic. Since it can be quickly created in a well-stirred reverberation chamber, the uncorrelated isotropic multipath environment is of considerable interests. The goal in this part is to achieve a compact formula for envelope correlation, ρ^{e}. For this particular multipath environment, after power normalisation we have

$$\begin{aligned} \mathbf{\Gamma}(\Omega, \Omega') &= \mathbf{I}_{2\times 2}\ \delta(\Omega' - \Omega) \ , \\ \mathsf{P}(\Omega, \Omega') &= \frac{1}{4\pi}\ \delta(\Omega' - \Omega) \ . \end{aligned} \tag{3}$$

The voltage at the, say, xth port of antenna is obtained through

$$v_x = Q \oint_{4\pi} \bar{E}(\Omega)^T \cdot \bar{G}_x(\Omega)\ d\Omega \ , \tag{4}$$

with Q being a constant complex factor given in [6, Equation (4-2)], and $\bar{G}_x$ the associated port's embedded element pattern. The incoming waves create voltage at the other ports too. The covariance between the signals at port x and y in this multipath environment becomes

$$\mathrm{cov}_{xy} = \mathbb{E}[v_y^* v_x] = |Q|^2 \oint_{4\pi} \bar{G}_y^{\dagger} \cdot \bar{G}_x\ d\Omega \ , \tag{5}$$

in which we exchanged the $\mathbb{E}$ and integral operators and used the expressions in (3). To achieve correlation coefficient ρ_{xy}, the expression in (5) needs to be normalised. Doing so, for the envelope correlation (*i.e.*, $\rho^{\mathsf{e}}_{xy} = |\rho_{xy}|^2$) we can write

$$\rho^{\mathsf{e}}_{xy} = \frac{|\bar{i}_x^{\dagger} \cdot \mathbf{C} \cdot \bar{i}_y|^2}{(\bar{i}_x^{\dagger} \cdot \mathbf{C} \cdot \bar{i}_x) \cdot (\bar{i}_y^{\dagger} \cdot \mathbf{C} \cdot \bar{i}_y)} \ , \tag{6}$$

in which the open-circuit covariance matrix of the antenna system is

$$\mathbf{C}_{n\times n} = \frac{1}{\eta} \oint_{4\pi} \mathbf{G}^{\dagger}(\Omega) \cdot \mathbf{G}(\Omega)\ d\Omega \ , \tag{7}$$

with η being the intrinsic impedance of the propagation medium. The latter is also known as pattern overlap matrix. Works of Wasylkiwskyj and Kahn [7], [8] as well as Vaughan and Andersen [5] showed that for certain type of multiport antennas known as minimum scattering antennas the open-circuit covariance matrix in watt represents the resistive part of Z-matrix of the system times 1 A^2. That is

$$\mathbf{C} = \Re[\mathbf{Z}] \qquad \text{(Minimum Scattering Antennas)}\ . \tag{8}$$

Lossless single-mode multiport antennas approximate minimum scattering antennas well [9]. Thus, exclusively for this group of antennas, the expression in (6) can be recast as

$$\rho^{\mathbf{e}}_{xy} = \frac{|\bar{i}^{\dagger}_x \cdot \Re[\mathbf{Z}] \cdot \bar{i}_y|^2}{(\bar{i}^{\dagger}_x \cdot \Re[\mathbf{Z}] \cdot \bar{i}_x) \cdot (\bar{i}^{\dagger}_y \cdot \Re[\mathbf{Z}] \cdot \bar{i}_y)}\ . \tag{9}$$

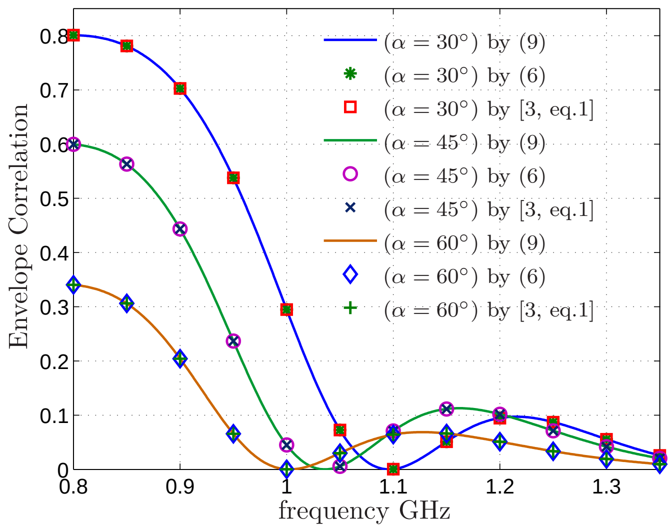
This is the only case in which correlation can completely be calculated by input network parameters. Indeed, the expression in (9) is an alternative expression for [3, Equation (11)], [4, Equation (31)], and [5, Equation (73)]. In Fig. 1 comparisons between correlations achieved in different ways are illustrated, which show agreement.

A few points about the presented formula merit further discussion. First of all, note that in case correlation for only 50 ohm terminations is desired, the formula presented in [3, Equation (11)] is the most suitable one. However, as soon as other terminating impedances are of concern, expressions in (9) and [5, Equation (73)] are handier. In addition, recall that all previously published compact formulas are based on single-port excitation scheme, *i.e.*, embedded element pattern. Nevertheless, in practice there are some cases (*e.g.*, beamforming) in which a number of ports in an array are excited simultaneously. In these cases, the concern is the spatial correlation between the two resultant embedded patterns created by different excitation schemes. In such a circumstance, using the above formula saves computation resources and is quite advantageous at the expense of two steps in calculation: first, evaluation of current weights by (1), and subsequently envelope correlation by (9). To clarify the point, the absolute values of complex correlations at the ports of an ideal cascaded Butler network to four horizontal dipoles at $0.15\lambda_\circ$ ($\lambda_\circ$ at 1 GHz) height above a perfect electric conductor (PEC) are plotted in Fig. 2. The element separation in this example is $d = 0.2\lambda_\circ$ and terminations are all 50 ohm . Let us assume that the current weights for different excitations are known through (1). Calculation of a correlation using (9) requires 50 multiplications plus 36 additions at each frequency point. In contrast, if we use [3, Equation (1)] for the same purpose, with known open-circuit embedded patterns ($5^\circ \times 5^\circ$) and known current weights given by (1), the necessary number of multiplications and additions are around 60000 and 50000, respectively. Expressions in [3, Equation (11)] and [5, Equation (73)] cannot be directly used in the aforementioned example.

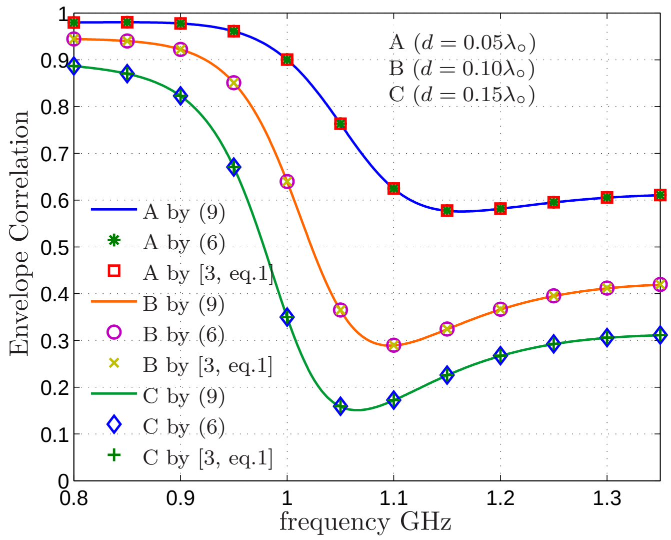
3 Correlation in Correlated Multipath Environments

Pursuing a similar path to Section 2, one can formulate correlation in a general case of non-uniform multipath environments of correlated incoming waves. Using (2), the general expression in (6) can still be held with slight changes in $\mathbf{C}$ as correlated pattern overlap matrix given by

$$\mathbf{C} = \frac{1}{\eta} \int_{4\pi} \int_{4\pi} \mathbf{G}^{\dagger}(\Omega') \cdot \mathbf{\Gamma}(\Omega, \Omega') \cdot \mathbf{G}(\Omega)\ \mathsf{P}(\Omega, \Omega')\ d\Omega' d\Omega\ . \tag{10}$$



(a)



(b)

Figure 1: Envelope correlations based on different formulas for (a) two lossless cross dipoles with an angle of α between them and (b) two lossless parallel horizontal dipoles above a PEC plane with element separation d.

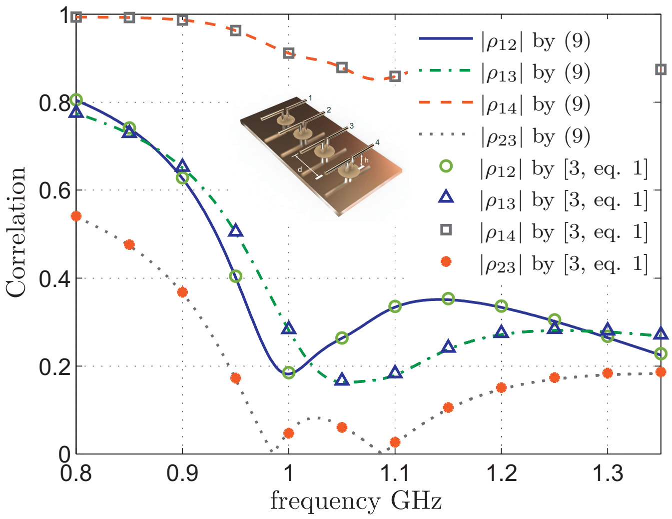


Figure 2: Spatial correlation in the presence of a cascaded ideal Butler network for four horizontal dipoles above a PEC plane. The element separation is $d = 0.2\lambda_\circ$ ($\lambda_\circ \approx 0.3$m).

As a point of caution, note that the expression in (10) is fixed and depends partly upon the open-circuit embedded patterns. Meanwhile, the input current weights in (1) are still dependent on the input network parameters. Therefore, in this general case, correlations can only partially be obtained based on the input network parameters. The optimum terminations rendering minimum correlations can be achieved through the expression in (6). Indeed, this expression is highly advantageous for optimisation purposes restricted to a certain set of multipath properties, *i.e.*, $\mathbf{\Gamma}$ and P.

Moreover, as an interesting point, if we can estimate the open-circuit embedded patterns properly within the desired range of AoA distribution, we are able to approximate correlation in nonuniform multipaths solely in terms of the input network parameters. This is rewarding since except elements' positions (or their phase centres), there would be no need to have precise knowledge of their far-field patterns. In this case, the expression in (10) can be obtained numerically. It is also worth mentioning that, in many practical cases, the open-circuit embedded patterns within a small AoA range can be well approximated by a uniform or sinusoidal function. This important observation cannot be made based on the previous formulas available in the literature. To further elaborate the point, Fig. 3 illustrates how we can estimate correlations between different ports of a four-port quarter-wavelength monopoles above a PEC plane without accessibility to the simulated embedded patterns. In this simulation, we arbitrarily assume an uncorrelated multipath of balanced polarisation with uniform AoA distribution in azimuth plane but truncated Gaussian AoA in elevation plane [10, Equation (10)]. The mean elevation AoA is $20°$ from horizon with standard deviation of $\sigma = 10°$. There are 50 ohm terminations at different ports. Note that in this example, we approximate the open-circuit pattern of a monopole by a sinusoidal function, *e.g.*, $G_\theta(\Omega) = \sin\theta$ $(0 \le \theta \le \pi/2)$. Based on this figure, around the resonance frequency (*i.e.*, $f_\circ = 1$ GHz), correlation between nearby elements decreases. This can best be attributed to the associated increase in coupling between them. Furthermore, the correlations are in general considerably higher than their counterparts in a uniform multipath environment. This is due to a reduced angle of spread for the incoming waves.

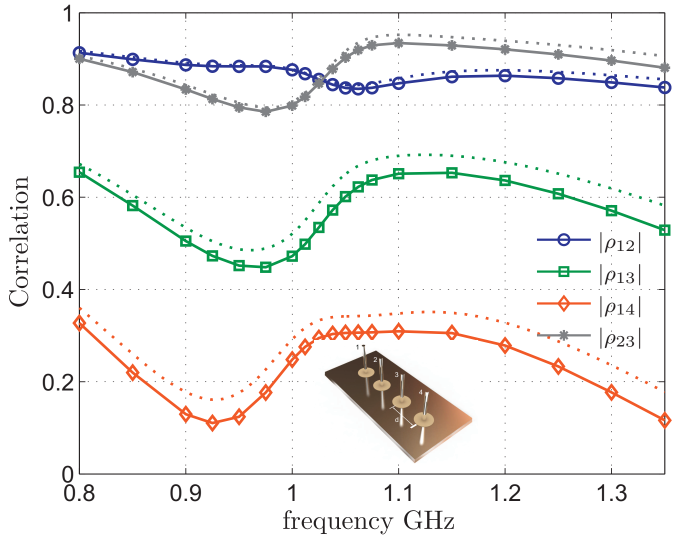


Figure 3: Estimation of correlation for a four-port antenna in an uncorrelated nonuniform multipath environment ($d = 0.2\lambda_\circ$ at $\lambda_\circ \approx 0.3$m). The solid curves show the simulated correlations, whereas the dotted ones illustrate the estimated ones.

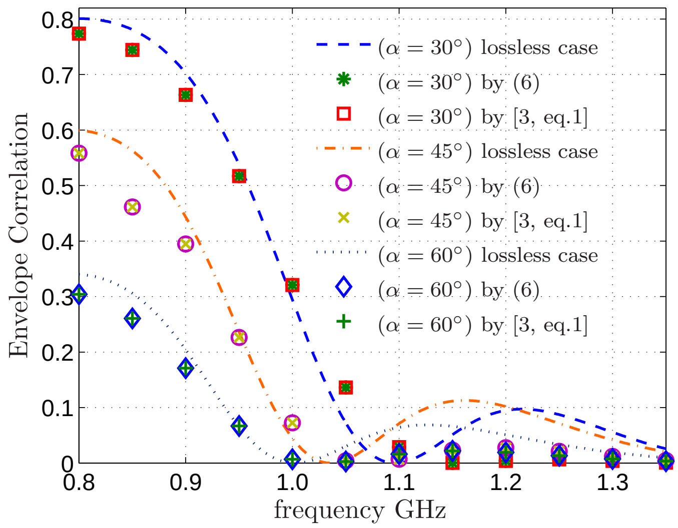
4 Correlation in Lossy Structures

Recalling the fact that losses in a multiport radiating structure affect both the resistive and reactive parts of its self-/mutual impedances, we stress that correlation cannot be achieved through the input network parameters. Yet, if by any chance the open-circuit covariance matrix can be measured, the corresponding correlation is given by (6) as a partial function of the input network parameters.

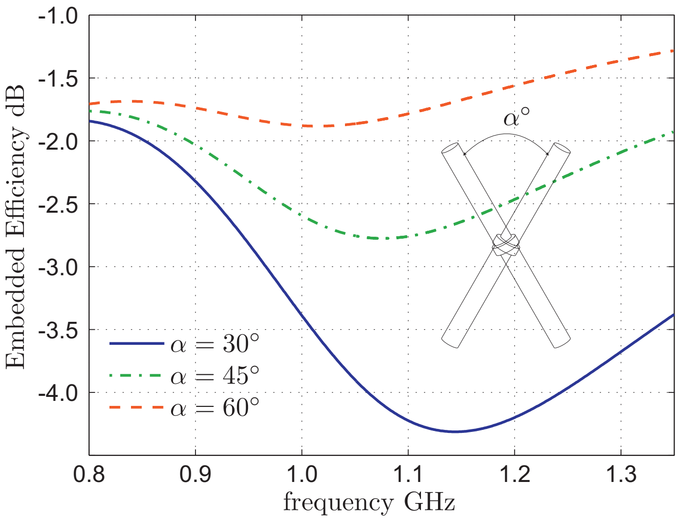
The open-circuit covariance matrix for uncorrelated isotropic multipath environments in (7) can be most quickly measured in a well-stirred reverberation chamber by virtue of two independent sets of known terminating impedances. The price paid for this purpose is that we need to measure the covariance matrix with the both sets of impedances and then derive the entries in $\mathbf{C}$ from them. Furthermore, fast measurement of antennas by reverberation chamber in non-isotropic Rayleigh multipath environments is shown to be possible [11]- [13]. This achievement, by all means, alleviates the burden of embedded pattern measurements for different terminating impedances. As an example, in Fig. 4(a), correlations for a two-port antenna in uncorrelated isotropic multipath have been achieved. These are obtained by use of (6) and the simulated embedded patterns based on the method of moments. Note that the losses are introduced arbitrarily over these wire antennas whose simulated embedded radiation efficiencies are given in Fig. 4(b). A quick comparison to those of the lossless case shows that the total radiation efficiencies do not affect the corresponding correlations considerably.

5 Conclusion

In this letter, an alternative compact formula is introduced to stand beside the available formulas from [3] and [4]. A modified version of the compact formula is discussed to hold in general correlated nonuniform multipath environments. The stress of this letter



(a)



(b)

Figure 4: (a) Envelope correlations for the case of two lossy cross-dipoles with variant angles, α, between them. The corresponding correlations for the lossless case are also plotted for comparison purpose. (b) Embedded radiation efficiencies of these antennas are also shown.

is on the open-circuit embedded covariance of the multiport antennas in isotropic or nonuniform multipath environments. The reverberation chamber is recommended for quick measurements of the latter parameter, which results in considerable alleviation in cumbersome measurements of embedded far-field patterns. Moreover, it is shown that by a proper estimation of open-circuit embedded patterns and knowledge of elements' phase centres, in a lossless case, one can approximate correlations solely based on the input network parameters.

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