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Sum rate analysis of ZF receivers in distributed MIMO systems with Rayleigh/Lognormal fading

Michail Matthaiou*, Caijun Zhong†, Matthew R. McKay‡, and Tharm Ratnarajah§
*Department of Signals and Systems, Chalmers University of Technology, Sweden
†Institute of Information and Communication Engineering, Zhejiang University, China
‡Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Hong Kong
§Institute of Electronics, Communications and Information Technology (ECIT), Queen's University Belfast, U.K.
E-mail: michail.matthaiou@chalmers.se, caijun.zhong@ieee.org, eemckay@ust.hk, t.ratnarajah@ecit.qub.ac.uk

***Abstract*—This paper presents a detailed sum rate characterization of distributed multiple-input multiple-output systems operating over composite fading channels and employing linear zero-forcing receivers. We consider the Rayleigh/Lognormal fading model and also take into account the effects of path-loss and spatial correlation at the transmit side. New closed-form upper and lower bounds on the achievable sum rate are proposed that apply for arbitrary numbers of antennas. Moreover, we investigate the concept of large-scale multiple-antenna systems when the number of receive antennas grow large. In this asymptotic regime, it is shown that the effects of Rayleigh fading are averaged out and the channel is dominated by the much more slowly varying shadowing. An interesting observation from our results is that in order to maximize capacity, the radio ports should be placed at unequal distances to the base station when they experience the same level of shadowing.**

I. Introduction

The capacity limits of point-to-point MIMO systems have been well investigated in the literature for more than a decade now [1]–[3]. The area of distributed multiple-input multiple-output (D-MIMO) systems, where multiple antennas at one end of the radio link are placed into multiple radio ports that are geographically separated, represents a promising technology for achieving macro-diversity gains [4]–[7]. A main difference between D-MIMO systems and conventional point-to-point MIMO configurations is that in the former case each radio link experiences different path-loss and large-scale fading (a.k.a. shadowing) effects, due to the different propagation paths. This makes their performance analysis a challenging problem and very few analytical works have been reported which investigate the impact of composite fading channels (i.e., with both small-scale and large-scale fading) on the performance of D-MIMO systems.

Some of the relevant works in this area include [8], in which a large-system capacity analysis was performed while closed-form expressions for the mean/outage spectral efficiency in the high signal to noise ratio (SNR) regime were derived. The ergodic capacity of D-MIMO systems was also explored in [9], [10], using tools of majorization theory to propose upper and lower capacity bounds. The common characteristic of [7]–[10] is that they consider optimal nonlinear receivers, whose complexity and implementation cost may be prohibitive particularly if the number of antennas is not small. In this light, linear receivers such as zero-forcing (ZF) [11] and minimum mean squared error receivers [12] are often considered as low complexity alternatives.

In the following, we pursue a detailed statistical analysis of ZF receivers over composite Rayleigh/Lognormal (RLN) fading MIMO channels, taking into account the effect of spatial correlation at the transmit side. This scenario can, for instance, occur when a number of small radio ports with small inter-element spacings (e.g., hand-held devices) transmit data to a base station (BS). To the best of the authors' knowledge, the only directly relevant works are [13] and [14]. In the former, the authors considered a point-to-point MIMO system under RLN fading and approximated the sum rate numerically via Gauss-Hermite polynomials. On the other hand, whilst [14] considered D-MIMO systems, the results were approximations based on replacing the lognormal shadowing model with the analytically friendly gamma distribution. Also, no large-system analysis, in order to exploit the advantages of large-scale multiple-antenna systems (LSMA), was performed therein either.

In the following, we derive closed-form upper and lower bounds on the sum rate of MIMO ZF receivers operating over RLN fading channels and experiencing correlation at the transmit side. The proposed bounds apply for any finite number of antennas and remain tight across the entire SNR range. We also investigate the performance of LSMA systems by assuming that the number of receive antennas grow large. In this case, it is explicitly demonstrated that the effect of small-scale Rayleigh fading is asymptotically averaged out and that the sum rate is affected only by the much more slowly varying large-scale fading. It is also shown that the proposed lower bound converges to a deterministic asymptote, which is analytically derived.

Notation: We use upper and lower case boldface to denote matrices and vectors, respectively. The $n \times n$ identity matrix is expressed as $\mathbf{I}_n$. A complex Gaussian variable with mean μ and variance σ^2 reads as $\mathcal{CN}(\mu, \sigma^2)$. The expectation of a random variable is denoted as $\mathcal{E}[\cdot]$, while the matrix determinant and trace by $\det(\cdot)$ and $\mathrm{tr}(\cdot)$. The (i,j)-th minor of a matrix is denoted by $\mathbf{A}_{ij}$, $\mathbf{A}_i$ is $\mathbf{A}$ with the i-th column removed while $[\mathbf{A}]_{mm}$ returns the m-th diagonal element of $\mathbf{A}$. The symbols $(\cdot)^{\dagger}$ and $(\cdot)^H$ represent the pseudo-inverse and Hermitian transpose of a matrix, respectively, while $\lceil\cdot\rceil$ is the ceiling operation to the nearest integer. Finally, $\psi(x)$ is Euler's digamma function [15, Eq. (8.360.1)].

II. MIMO Fading Model

We consider an uplink D-MIMO system with N_r receive antennas and L radio ports each connected to N_t transmit antennas and also require that $N_r \geq LN_t$. Assuming no channel state information at the transmitters, the available average power, P, is distributed uniformly amongst all data streams. In the case of composite fading, the input-output relationship is

$$\mathbf{y} = \sqrt{\frac{P}{LN_t}}\mathbf{H}\mathbf{\Xi}^{1/2}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$ is the received signal vector, $\mathbf{s} \in \mathbb{C}^{LN_t \times 1}$ is the vector containing the transmitted symbols which are drawn from a unit-power constellation and are also uncorrelated circularly symmetric zero-mean complex Gaussian variables. The complex noise term is zero-mean with covariance $\mathcal{E}\left[\mathbf{n}\mathbf{n}^H\right] = N_0\mathbf{I}_{N_r}$, where N_0 is the noise power.

The small-scale fading is captured by the random matrix $\mathbf{H} \in \mathbb{C}^{N_r \times LN_t}$ which is assumed to follow a complex zero-mean Gaussian distribution with correlation among every row; hence, we have $\mathbf{H} = \mathbf{Z}\mathbf{R}_{\mathrm{T}}^{1/2}$, where the entries of $\mathbf{Z}$ are modeled as i.i.d. $\mathcal{CN}(0,1)$ random variables while $\mathbf{R}_{\mathrm{T}}$ is the transmit positive definite covariance matrix. Note that small-scale fading correlation occurs only between the antennas of the same radio port since the L ports are, in general, geographically separated.

On the other hand, the entries of the diagonal matrix $\mathbf{\Xi} \in \mathbb{R}^{LN_t \times LN_t}$ represent the large-scale effects; thus $\mathbf{\Xi} = \mathrm{diag}\left\{\mathbf{I}_{N_t}\xi_m/D_m^{\upsilon}\right\}_{m=1}^{L}$ where D_m denotes the distance between the receiver and the m-th radio port, while υ is the path-loss exponent. As was previously mentioned, the shadowing manifestations are modeled via the classical log-normal distribution. As such, we can express the probability density function (PDF) of the large-scale fading coefficients $\xi_m, m = 1,\ldots,L$ according to

$$p(\xi_m) = \frac{\eta}{\xi_m\sqrt{2\pi\sigma_m^2}}e^{-\frac{(\eta\ln\xi_m - \mu_m)^2}{2\sigma_m^2}}, \quad \xi_m \geq 0 \tag{2}$$

where $\eta = 10/\ln 10$, while μ_m and σ_m are the mean and standard deviation (both in dB) of the variable's natural logarithm, respectively. At this point, it is worth mentioning that one intrinsic difficulty of the RLN model is that its PDF is not available in closed-form. This renders the performance analysis of D-MIMO systems operating in RLN fading channels cumbersome. Therefore, we first seek to derive some tractable bounds on the sum rate of MIMO ZF receivers.

III. Rate analysis of MIMO ZF Receivers

In the following, we will focus on the sum rate performance of MIMO ZF receivers. For the case under consideration, the ZF filter is expressed as $\mathbf{G} = (P/LN_t)^{-1/2}\mathbf{T}^{\dagger}$ where $\mathbf{T} \triangleq \mathbf{H}\mathbf{\Xi}^{1/2}$. The instantaneous received SNR at the m-th ZF output $(1 \leq m \leq LN_t)$ is equal to [11]

$$\gamma_m \triangleq \frac{\gamma}{LN_t\left[\left(\mathbf{T}^H\mathbf{T}\right)^{-1}\right]_{mm}} = \frac{\gamma\left[\mathbf{\Xi}\right]_{mm}}{LN_t\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right]_{mm}} \tag{3}$$

where $\gamma = P/N_0$ is the average SNR. Note that the second equality follows from the fact that $\mathbf{\Xi}$ is diagonal. The achievable sum rate, assuming independent decoding at the receiver, is determined as

$$R \triangleq \sum_{m=1}^{LN_t}\mathcal{E}\left[\log_2(1+\gamma_m)\right] \tag{4}$$

where the expectation is taken over all channel realizations of $\mathbf{H}$, $\mathbf{\Xi}$ and the channel is assumed to be ergodic.

A. Closed-form bounds

Capitalizing on the results of [16], we derive two novel generic bounds on the sum rate of MIMO ZF receivers that apply for a finite number of antennas and arbitrary SNRs:

Proposition 1: The sum rate of ZF receivers over correlated RLN MIMO channels is bounded by $R_L^{\mathtt{RLN}} \leq R \leq R_U^{\mathtt{RLN}}$ with

$$R_U^{\mathtt{RLN}} = LN_t\log_2\left(\frac{\mathrm{tr}\left(\mathbf{R}_{\mathrm{T}}^{-1}\right)}{LN_t(N_r - LN_t)} + \frac{\gamma}{N_tL^2}\sum_{m=1}^{L}\frac{\exp\left(\frac{\mu_m}{\eta}+\frac{\sigma_m^2}{2\eta^2}\right)}{D_m^{\upsilon}}\right) + \frac{LN_t}{\ln 2}\psi\left(N_r - LN_t + 1\right) - \sum_{m=1}^{LN_t}\log_2\left[\mathbf{R}_{\mathrm{T}}^{-1}\right]_{mm} \tag{5}$$

$$R_L^{\mathtt{RLN}} = \sum_{m=1}^{LN_t}\log_2\left(1+\frac{\gamma}{LN_t}\exp\left(\psi\left(N_r - LN_t + 1\right) + \frac{\mu_{m'}}{\eta} - \ln\left[\mathbf{R}_{\mathrm{T}}^{-1}\right]_{mm} - \upsilon\ln D_{m'}\right)\right) \tag{6}$$

where $m' = \lceil m/N_t\rceil$.

Proof: Applying the generic bounding techniques of [16, Th. 1] and [16, Th. 3], after some basic algebra and taking into account (3), we can easily get

$$R_U = LN_t\log_2\left(\mathcal{E}\left[\lambda^{-1}\right] + \frac{\gamma\mathcal{E}\left[\mathrm{tr}\left(\mathbf{\Xi}\right)\right]}{(LN_t)^2}\right) - \sum_{m=1}^{LN_t}\mathcal{E}\left[\log_2\left(\det\left(\mathbf{H}^H\mathbf{H}\right)\right) - \log_2\left(\det\left(\mathbf{H}_m^H\mathbf{H}_m\right)\right)\right] \tag{7}$$

$$R_L = \sum_{m=1}^{LN_t}\log_2\left(1+\frac{\gamma}{LN_t}\exp\left(\mathcal{E}\left[\ln\left(\mathbf{\Xi}_{mm}\right) + \ln\left(\det\left(\mathbf{H}^H\mathbf{H}\right)\right) - \ln\left(\det\left(\mathbf{H}_m^H\mathbf{H}_m\right)\right)\right]\right)\right) \tag{8}$$

where λ is an unordered eigenvalue of $\mathbf{H}^H\mathbf{H}$. We note that both bounds decouple the small and large-scale fading terms. The terms corresponding to the small-scale Rayleigh fading can be evaluated by successively using the property of square matrices $\det\left(\mathbf{AB}\right) = \det\left(\mathbf{A}\right)\det\left(\mathbf{B}\right)$, and then recalling the following result for a $N_r \times LN_t$ (with $N_r \geq LN_t$) central Wishart matrix [2, Eq. (A.8.1)]

$$\mathcal{E}\left[\ln\left(\det\left(\mathbf{Z}^H\mathbf{Z}\right)\right)\right] = \sum_{k=0}^{LN_t-1} \psi\left(N_r - k\right). \tag{9}$$

The first negative moment of λ is equal to $\mathcal{E}\left[\lambda^{-1}\right] = \mathrm{tr}\left(\mathbf{R}_{\mathrm{T}}^{-1}\right)/(LN_t(N_r - LN_t))$ [16, Th. 4]. For the shadowing terms, recall the fundamental properties of a lognormal variate $\xi_m \sim \mathrm{LN}(\mu_m/\eta, \sigma_m^2/\eta^2)$ [17],

$$\mathcal{E}\left[\xi_m^r\right] = \exp\left(\frac{r\mu_m}{\eta} + \frac{r^2\sigma_m^2}{2\eta^2}\right) \tag{10}$$

$$\mathcal{E}\left[\ln \xi_m\right] = \mu_m/\eta. \tag{11}$$

Combining these results and applying the key matrix property $\left[\mathbf{A}^{-1}\right]_{mm} = \det\left(\mathbf{A}_{mm}\right)/\det\left(\mathbf{A}\right)$, we conclude the proof. ∎

Note that for the special case $L = 1$ (i.e. point-to-point MIMO systems), we have an alternative upper bound as follows:

Corollary 1: For $L = 1$, the sum rate of ZF receivers over correlated RLN MIMO channels is upper bounded by $R \leq R_u^{\mathtt{RLN}}$ with

$$\begin{aligned} R_u^{\mathtt{RLN}} &= N_t \log_2\left(\frac{D_1^{\upsilon}\mathrm{tr}\left(\mathbf{R}_{\mathrm{T}}^{-1}\right)}{N_t(N_r - N_t)}\exp\left(\frac{\sigma_1^2 - 2\eta\mu_1}{2\eta^2}\right) + \frac{\gamma}{N_t}\right) \\ &+ \frac{N_t}{\ln 2}\psi\left(N_r - N_t + 1\right) - \sum_{m=1}^{N_t}\log_2\left[\mathbf{R}_{\mathrm{T}}^{-1}\right]_{mm} \\ &+ N_t\left(\frac{\mu_1}{\eta \ln 2} - \upsilon \log_2 D_1\right). \end{aligned} \tag{12}$$

Proof: The proof follows by noting that in the case $L = 1$, we have $\mathbf{\Xi} = (\xi_1/D_1^{\upsilon})\mathbf{I}_{N_t}$ and by simple rearrangement of (7). The first negative moment of ξ_1 can be evaluated by setting $r = -1$ in (10). ∎

As will be shown in the subsequent analysis, the main difference between the above bound and R_U, is that the former becomes exact at high SNRs. Interestingly, $R_L^{\mathtt{RLN}}$ is independent of the variance of the lognornal distribution, σ_m. We point out that R_U exists only for rectangular MIMO matrices with $N_r \geq LN_t + 1$, whereas R_L is more general and exists for square matrices (i.e. $N_r = LN_t$) also. Clearly, the sum rate tends to monotonically increase with the mean of the shadowing distribution μ_m, while in all cases under consideration, higher transmitter-receiver distances tend to effectively reduce the sum rate due to the increased path-loss.

In order to better assess the effects of shadowing parameters, we will now introduce two basic results from majorization theory:

Lemma 1: [18, p. 7] For any vector $\mathbf{x} \in \mathbb{R}^n$, let $\mathbf{a} \in \mathbb{R}^n$ denote the constant vector with the i-th element given by $\mathbf{a}_i = (1/n)\sum_{k=1}^n x_k$. Then, $\mathbf{x}$ majorizes $\mathbf{a}$, or $\mathbf{a} \prec \mathbf{x}$. This result indicates that the vector with identical entries is majorized by any vector with the same sum value.

Lemma 2: [18, 3.A.1] A real-valued function $f(\cdot)$ defined on a set $\mathcal{A}$ is said to be Schur-convex on $\mathcal{A}$ if

$$\mathbf{y} \prec \mathbf{x} \ \text{ on } \ \mathcal{A} \ \ \Rightarrow f(\mathbf{y}) \leq f(\mathbf{x}). \tag{13}$$

With these results in hand, we can now present the following key result for the case of uncorrelated RLN fading:

Corollary 2: When $\mu_m = \mu$, $m = 1, \ldots, L$, the lower bound $R_L^{\mathtt{RLN}}$ is a Schur-convex function with regard to D_m, $m = 1, \ldots, L$. Moreover, when $D_m = D$, $m = 1, \ldots, L$, the lower bound $R_L^{\mathtt{RLN}}$ is a Schur-convex function with regard to μ_m, $m = 1, \ldots, L$.

Proof: Let us start with the first claim. When $\mu_m = \mu$, the lower bound reduces to

$$R_L^{\mathtt{RLN}} = N_t \sum_{m=1}^{L} \log_2\left(1 + \frac{a}{D_m^{\upsilon}}\right) \tag{14}$$

where $a \triangleq \frac{\gamma}{LN_t}\exp\left(\psi\left(N_r - LN_t + 1\right) + \frac{\mu}{\eta}\right)$. Let us now define the function $f(x) \triangleq \log_2\left(1 + \frac{a}{x^{\upsilon}}\right)$. Then, it is not difficult to compute

$$\frac{d^2 f(x)}{dx^2} = \frac{a\upsilon}{\ln 2}\frac{((\upsilon+1)x^{\upsilon} + a)}{(x^{\upsilon+1} + ax)^2} > 0. \tag{15}$$

Hence, $f(x)$ is convex w.r.t. x. As such, the first claim can be proved by invoking a result from [9, Lemma 1].

Similarly, when $D_m = D$, the lower bound reduces to

$$R_L^{\mathtt{RLN}} = N_t \sum_{m=1}^{L} \log_2\left(1 + b\exp\left(\frac{\mu_m}{\eta}\right)\right) \tag{16}$$

where $b \triangleq \frac{\gamma}{LN_t}\exp\left(\psi\left(N_r - LN_t + 1\right) - \upsilon \ln D\right)$. Again, we can show that the function $g(x) \triangleq \log_2\left(1 + b\exp\left(\frac{x}{\eta}\right)\right)$ is a convex function w.r.t. x, which confirms the second claim. ∎

Combining Corollary 2 with the above Lemmas, we can infer that the radio ports should be placed at unequal distances to the BS when they experience the same level of shadowing; this implies that a symmetric deployment of radio ports (i.e. equal distances to the BS) can not in practice maximize the achievable sum rate of MIMO ZF receivers. This can be attributed to the decay rate of $D_m^{-\upsilon}$ being much faster for small values of D_m. As such, having some radio ports very close to the BS is highly beneficial since these ports contribute significantly to the total sum rate, thereby making the non-symmetric configuration superior.

We can now assess the performance of the proposed bounds against different model parameters. In Fig. 1, we examine the tightness of the upper bound $R_U^{\mathtt{RLN}}$ in (5), $R_u^{\mathtt{RLN}}$ in (12) and the lower bound $R_L^{\mathtt{RLN}}$ in (6) against the SNR. We keep $N_t = 4, L = 1$ and increase only N_r. Here and in subsequent figures, the transmit correlation matrix is constructed as $\mathbf{R}_{\mathrm{T}} = \mathrm{diag}\left\{\mathbf{R}_{\mathrm{T},m}\right\}_{m=1}^{L}$ where $\mathbf{R}_{\mathrm{T},m}$ is the correlation matrix between the antennas of the m-th port. The entries of the latter are modeled via the common exponential correlation model $\left\{\mathbf{R}_{\mathrm{T},m}\right\}_{i,j} = \rho_{t,m}^{|i-j|}$ with $\rho_{t,m} \in [0, 1)$ being the transmit correlation coefficient.

It is evident that having more receive antennas can systematically improve the performance of a MIMO ZF receiver, which is due to the mitigation of the noise enhancement effect. Clearly, both $R_u^{\mathtt{RLN}}$ and $R_L^{\mathtt{RLN}}$ tighten (i.e. they approach the exact sum rate curves) at high SNRs and when the number

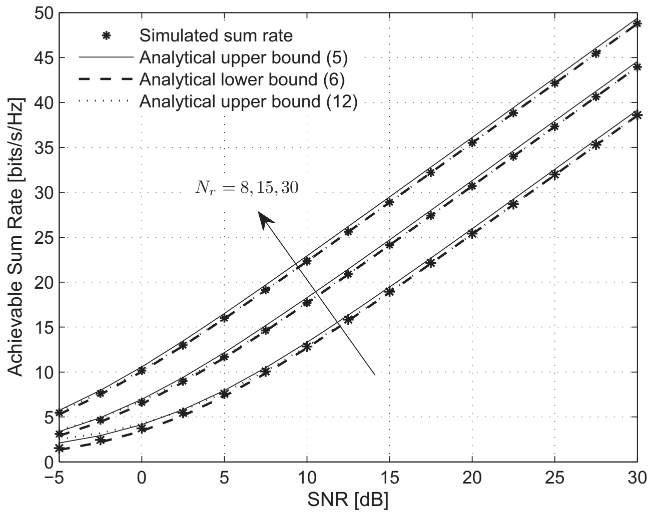

Fig. 1. Simulated sum rate and upper/lower bounds against the SNR γ for RLN fading ($N_t = 4, L = 1, \upsilon = 4, \rho_{t,m} = 0.8, \mu_m = 4$ dB, $\sigma_m = 2$ dB, $D_1 = 1000$ m).

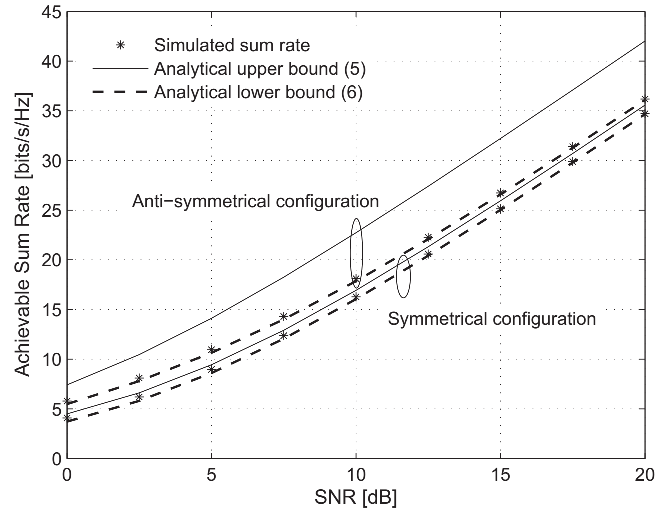

Fig. 2. Simulated sum rate and upper/lower bounds against the SNR γ for RLN fading and two different configurations ($N_r = 12, N_t = 2, L = 3, \upsilon = 4, \rho_{t,m} = 0.8, \mu_m = 4$ dB, $\sigma_m = 2$ dB with (a) anti-symmetrical configuration $D_1 = 1000$ m, $D_2 = 1500$ m, $D_3 = 2000$ m and (b) symmetrical configuration $D_1 = D_2 = D_3 = 1500$ m).

of antennas grows large. On the other hand, the more general upper bound $R_U^{\mathtt{RLN}}$ remains sufficiently tight across the entire SNR range; in fact, for small number of receive antennas, N_r, $R_U^{\mathtt{RLN}}$ is tighter than $R_u^{\mathtt{RLN}}$ in the low SNR regime. In order to get some additional insights into the tightness of $R_U^{\mathtt{RLN}}$, we can consider the associated bounding error at high SNRs, which is quantified as a fixed rate offset given by

$$\begin{aligned}\Delta R^{\mathtt{RLN}} &\triangleq R_U^{\mathtt{RLN}} - R^{\mathtt{RLN}} \\ &\overset{\gamma\to\infty}{=} LN_t \log_2\left(\frac{1}{L}\sum_{m=1}^{L}\frac{\exp\left(\frac{\mu_m}{\eta}+\frac{\sigma_m^2}{2\eta^2}\right)}{D_m^{\upsilon}}\right) \\ &\quad - N_t\sum_{m=1}^{L}\left(\frac{\mu_m}{\eta\ln 2}-\upsilon\log_2 D_m\right). \qquad (17)\end{aligned}$$

Interestingly, this offset is only a function of the shadowing parameters, the number of transmit antennas and radio ports, and is independent of the spatial correlation and numbers of receive antennas. This occurs due to the inherent structure of R_U, described in (7), which decouples the terms depending on small-scale fading from those depending jointly on SNR and large-scale fading. In general, $R_U^{\mathtt{RLN}}$ is tighter for small number of transmit antennas and radio ports and small transmitter-receiver distances. Note that for the special case of point-to-point MIMO systems ($L = 1$), the expression in (17) simplifies to

$$\Delta R^{\mathtt{RLN}} \overset{\gamma\to\infty}{=} \frac{N_t\sigma_1^2}{2\eta^2\ln 2}. \qquad (18)$$

The above result further indicates that for a fixed L, the upper bound is tighter for a small number of transmit antennas and smaller shadowing variance. In other words, the bound is tighter when the fading fluctuations are not very severe.

In Fig. 2, we investigate the impact of radio ports' deployment on the sum rate. To this end, we consider two different configurations with the same total distance constraint. For the symmetrical configuration, all ports are placed at the same distance to the BS ($D_1 = D_2 = D_3 = 1500$ m) while for the anti-symmetrical configuration we have that $D_1 = 1000$ m, $D_2 = 1500$ m, $D_3 = 2000$ m. As suggested by Corollary 2, the latter configuration yields higher sum rate due to weaker path-loss effects (i.e. it is beneficial to put the radio ports close to the BS to get a stronger signal). On the other hand, it turns out that both bounds, and especially the upper bound, are tighter for the former configuration.

B. Sum rate analysis of LSMA systems with ZF receivers

We can now investigate the emerging area of LSMA systems, which are anticipated to deploy very large number of low power antenna elements (e.g., hundreds of antennas at a BS with a power on the order of mW) [19]. In this case, we assume that the available transmit power is normalized by the large number of antennas at the BS. With the aid of this normalization, we guarantee that the total received power does not diverge as $N_r \to \infty$. Mathematically speaking, we set the transmit power, P, to be scaled down N_r times such that the effective transmit SNR can be defined as $\gamma_u \triangleq \gamma/N_r$, where γ is kept fixed and finite.

Corollary 3: For RLN MIMO channels as N_r grows to infinity and N_t is kept fixed, while the effective SNR is given by $\gamma_u \triangleq \gamma/N_r$, R_L approaches

$$R_L \overset{N_r\to\infty}{=} N_t\sum_{m=1}^{L}\log_2\left(1+\frac{\gamma_u N_r}{LN_t}\exp\left(\frac{\mu_m}{\eta}-\upsilon\ln D_m\right)\right) \qquad (19)$$

$$= N_t\sum_{m=1}^{L}\log_2\left(1+\frac{\gamma}{LN_t}\exp\left(\frac{\mu_m}{\eta}-\upsilon\ln D_m\right)\right). \qquad (20)$$

Proof: For the lower bound in (8) it suffices to recall that $\psi(x) = \ln(x) + 1/x + \mathcal{O}(1/x^2)$, if $x \to \infty$ and thereafter simplify. ■

From Corollary 3 a number of insightful observations can be drawn: first, when the number of receive antennas grow without bound, the effects of small-scale Rayleigh fading are asymptotically averaged out and the wireless channel is dominated by the much more slowly changing large-scale fading. This conclusion is consistent with [19], [20], although the authors therein considered shadowing as a deterministic manifestation. More importantly, even by scaling down the transmit power with the number of BS antennas, we can still average out the effects of fading and, at the same time, serve LN_t radio ports (users) with a simple linear ZF receiver. From (20), we can easily infer that in the large-antenna limit, we can still acquire a linear increase with the number of transmit antennas per radio port and a logarithmic increase with the SNR. Finally, (20) clearly demonstrates the impact of shadowing parameters on the sum rate that reflects the same observations made for the finite-N_r case.

IV. Conclusion

The impact of large-scale fading, path-loss and spatial correlation on the performance of D-MIMO systems is of fundamental importance since it can significantly limit the advantages of this promising technology. Despite this, performance results for D-MIMO systems are not yet well-known, mainly due to the difficulty in averaging the eigen-statistics over the shadowing distribution.

Motivated by this, we herein investigated in detail the sum rate analysis of D-MIMO ZF receivers over composite RLN fading channels. More specifically, we presented two new upper and lower bounds on the sum rate that apply for any arbitrary number of antennas. We also considered LSMA systems where the number of BS antennas grows without bound. A very important observation is that in the "large-system" regime, the effects of small-scale Rayleigh fading are averaged out and only the much more slowly varying shadowing effects remain. This is achieved with a simple ZF receiver and by scaling down the transmit power with the large number of receive antennas. Finally, our results explicitly showed that the radio ports should be placed at unequal distances to the base station when they experience the same level of shadowing,

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