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# On ARQ-based Fast-Fading Channels

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***Abstract*—Automatic repeat request (ARQ) protocols are normally studied under slow-fading or quasi-static channel assumption where the fading coefficients are assumed to remain fixed during the transmission of a codeword or for the duration of all ARQ retransmission rounds, respectively. This letter investigates the performance of basic ARQ and incremental redundancy hybrid ARQ protocols in fast-fading channels where a number of channel realizations are experienced in each retransmission round. Long-term throughput, delay-limited throughput and outage probability of the ARQ schemes are obtained. Compared to slow-fading and quasi-static channels, a fast-fading channel results in a higher performance for both basic and incremental redundancy ARQ. The fast-fading channel, however, can be mapped to an equivalent slow-fading model at low signal-to-noise ratios. Finally, we show that the efficiency of ARQ protocols is overestimated if the fast-fading variations during a codeword transmission are approximated by their average value.**

## I. Introduction

Automatic repeat request (ARQ) is an efficient approach for improving the data transmission efficiency of wireless communication systems [1]–[7]. Utilizing both forward error correction and error detection, the performance improvement is achieved by retransmitting the data which has experienced *bad* channel conditions. In basic ARQ protocols the same data is retransmitted and the receiver decodes the message based on the received signal in each round. Hybrid ARQ protocols, on the other hand, are more advanced methods where the receiver combines all received representations of a message.

Considering very slow moving users in, e.g., spectrum sharing [1] and single-user networks [2], ARQ schemes are normally studied under the assumption that the channel remains fixed during the transmission of a codeword and all of its retransmission rounds (quasi-static channel assumption). For medium speed users, on the other hand, the channel is supposed to change between two successive retransmission rounds, while it is fixed for the duration of each codeword (slow-fading channel assumption) [3]–[6]. However, for fast moving users or users with long codewords compared to the channel coherence time, this assumption should be further relaxed, as the channel may change during each retransmission round. For instance, the indoor ultra wideband (UWB) channels normally vary smoothly during a codeword transmission [8]. On the other hand, modern codes often use very long codewords, which may exceed the channel coherence time [2].

This paper studies the data transmission efficiency of ARQ protocols in fast-fading environments. Long-term throughput, delay-limited throughput and outage probability of different ARQ protocols are investigated in the case where a number of channel realizations are experienced during every data retransmission round. This is a new model which, to the best of our knowledge, has not been studied before. Considering basic ARQ and incremental redundancy (INR) hybrid ARQ protocols, it is shown how the system performance changes in different fading conditions. With the same fading distribution, higher throughput and lower outage probability are obtained when the channel variability increases in time. Using ARQ at low signal-to-noise ratios (SNRs), we show that a fast-fading channel can be mapped into an equivalent slow-fading model with fading distribution obtained by averaging the fast-fading behavior in a retransmission round. For higher SNRs, the throughput (the outage probability) of the ARQ protocols is upper (lower) bounded when the fast-fading channel realizations within a codeword duration are approximated by their average value, which changes the fading distribution correspondingly.

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## II. System model and definitions

Consider a communication setup where the input message $X$ multiplied by the fading coefficient $h$ is summed with an independent and identically distributed complex Gaussian noise $Z \sim \mathcal{CN}(0,1)$ resulting in the output

$$Y = hX + Z. \tag{1}$$

Let $g = |h|^2$ denote the *channel gain* random variable. The channel gain remains constant for a duration of $L$ channel uses, determined by the channel coherence time, and then changes according to the fading probability density function (pdf) $f_G(g)$.

The receiver is assumed to have perfect instantaneous channel state information (CSI). On the other hand, there is no CSI available at the transmitter, except the ARQ feedback bits. We consider a maximum of $M$ retransmission rounds, i.e., each codeword is (re)transmitted a maximum of $M+1$ times. Also, the results are presented in natural logarithm basis.

To model the fast-fading behavior, the length of each codeword is assumed to be $L_\text{c} = JL$, i.e., $J$ different channel gain realizations are experienced during each codeword (re)transmission. This is the key difference in our channel model compared to, e.g., [1]–[7], which, as seen in the following, leads to different analytical and numerical results for the ARQ protocols.

*Definitions:* We define a packet as the transmission of a codeword along with all its possible retransmission rounds. Also, the long-term throughput [5], the delay-limited throughput [1], [2], [7] and the average transmission power [9] are defined as

$$\eta_\text{LT} \doteq \frac{\mathbf{E}\{Q\}}{\mathbf{E}\{\tau\}}, \tag{2}$$

$$\eta_\text{DL} \doteq \mathbf{E}\{\mathcal{R}\}, \tag{3}$$

and

$$\phi \doteq \frac{\mathbf{E}\{\xi\}}{\mathbf{E}\{\tau\}}, \tag{4}$$

respectively. Here, $\mathbf{E}\{.\}$ represents the expectation operator and $\mathbf{E}\{Q\}$, $\mathbf{E}\{\tau\}$, $\mathbf{E}\{\mathcal{R}\}$ and $\mathbf{E}\{\xi\}$ denote the expected value of the successfully decoded information nats, the expected number of channel uses, the expected achievable rate, and the expected energy consumed within a packet transmission period, respectively. Finally, $\Pr(\text{outage})$ is the outage probability defined as the probability of the event that the data cannot be decoded by the receiver when all retransmission rounds are used.

## III. Theoretical analysis

In the following, first some closed-form expressions for (2)-(4) are derived which are valid for different ARQ protocols. Later, the results are particularized for the basic and INR hybrid ARQ.

Let $Q_{\text{c}}$ be information nats transmitted in each packet. If the data is successfully decoded in each retransmission round, all the information nats are received. Therefore, we have

$$\mathbf{E}\{Q\} = Q_{\text{c}} \sum\nolimits_{m=1}^{M+1} \Pr\left(\bar{S}_1, \dots, \bar{S}_{m-1}, S_m\right) \tag{5}$$

where $S_m$ is the event that the data is decodable at the $m$-th (re)transmission round, and $\Pr(\bar{S}_1, \dots, \bar{S}_{m-1}, S_m)$ is the probability that the data is successfully decoded at the $m$-th round while it was not decoded before.

As the length of the codewords is $L_{\text{c}} = JL$, the number of channel uses is $\tau_m = mJL$ if the data is successfully decoded at the end of the $m$-th round. Also, independent of the message decoding status, the number of channel uses is $\tau_{M+1} = (M+1)JL$ if all (re)transmission rounds are used. Therefore, the expected number of channel uses is found as

$$\begin{aligned}\mathbf{E}\{\tau\} &= JL \sum\nolimits_{m=1}^{M+1} m \Pr\left(\bar{S}_1, \dots, \bar{S}_{m-1}, S_m\right) \\ &\quad + (M+1)JL \Pr\left(\bar{S}_1, \dots, \bar{S}_{M+1}\right) \\ &\overset{(a)}{=} JL \left(1 + \sum\nolimits_{m=1}^{M} \Pr\left(\bar{S}_1, \dots, \bar{S}_m\right)\right)\end{aligned} \tag{6}$$

and the long-term throughput is obtained by

$$\eta_{\text{LT}} = r \frac{\sum_{m=1}^{M+1} \Pr\left(\bar{S}_1, \dots, \bar{S}_{m-1}, S_m\right)}{1 + \sum_{m=1}^{M} \Pr\left(\bar{S}_1, \dots, \bar{S}_m\right)}. \tag{7}$$

Here, $r = \frac{Q_{\text{c}}}{JL}$ denotes the initial codeword rate, $(a)$ is obtained by some manipulations on the probability terms and (7) follows from (2), (5) and (6).

Provided that the data is decoded at the $m$-th (re)transmission round, the achievable rate is $r_m = \frac{Q_{\text{c}}}{mJL} = \frac{r}{m}$. Hence, the delay-limited throughput (3), defined as the expected achievable rate in a packet period, is obtained by

$$\eta_{\text{DL}} = \sum\nolimits_{m=1}^{M+1} \frac{r}{m} \Pr\left(\bar{S}_1, \dots, \bar{S}_{m-1}, S_m\right). \tag{8}$$

Let the transmission power for the $m$-th (re)transmission round be $P_m$. Then, the transmission energy in the $m$-th round is $JLP_m$ and the sum energy up to the end of the $m$-th (re)transmission round is $JL\sum_{n=1}^{m} P_n$. Hence, the expected energy consumed in a packet period is

$$\begin{aligned}\mathbf{E}\{\xi\} &= JL \sum\nolimits_{m=1}^{M+1} \left(\sum\nolimits_{n=1}^{m} P_n\right) \Pr\left(\bar{S}_1, \dots, \bar{S}_{m-1}, S_m\right) \\ &\quad + JL \left(\sum\nolimits_{n=1}^{M+1} P_n\right) \Pr\left(\bar{S}_1, \dots, \bar{S}_{M+1}\right) \\ &= JL \left(P_1 + \sum\nolimits_{m=1}^{M} P_{m+1} \Pr\left(\bar{S}_1, \dots, \bar{S}_m\right)\right)\end{aligned} \tag{9}$$

which, along with (6), leads to the average transmission power

$$\phi = \frac{P_1 + \sum_{m=1}^{M} P_{m+1} \Pr\left(\bar{S}_1, \dots, \bar{S}_m\right)}{1 + \sum_{m=1}^{M} \Pr\left(\bar{S}_1, \dots, \bar{S}_m\right)}. \tag{10}$$

Finally, the outage probability is determined by $\Pr(\text{outage}) = \Pr(\bar{S}_1, \dots, \bar{S}_{M+1})$.

From (5)-(10), it follows that the only difference between different ARQ protocols is in the probability terms $\Pr(\bar{S}_1, \dots, \bar{S}_{m-1}, S_m)$ and $\Pr(\bar{S}_1, \dots, \bar{S}_m)$. In the following, these probabilities are obtained for the basic and INR hybrid ARQ protocols under fast-fading channel assumption.

### A. Basic ARQ protocol

Implementing basic ARQ, the receiver decodes the data in each round independent of the previously received codewords. Therefore, we have

$$\begin{aligned}&\Pr(\bar{S}_1, \dots, \bar{S}_m) = \prod\nolimits_{n=1}^{m} \Pr(\bar{S}_n) \\ &\Pr(\bar{S}_1, \dots, \bar{S}_{m-1}, S_m) = \Pr(S_m) \prod\nolimits_{n=1}^{m-1} \Pr(\bar{S}_n),\end{aligned} \tag{11}$$

where the probability $\Pr(S_n)$ is determined as follows. Let the channel gain realizations during the $n$-th (re)transmission round be $g_j$, $j = (n-1)J+1, \dots, nJ$, where each one happens in $L$ successive channel uses. In this case, the results of [9], [10, Chapter 15] can be used to show that the maximum decodable information nats with transmission power $P_n$ is

$$Q_{\text{basic},n}^{\max} = L \sum_{j=(n-1)J+1}^{nJ} \log\left(1 + g_j P_n\right) \tag{12}$$

which is obtained by Gaussian input distributions. Also, note that (12) is the same as the achievable nats in $J$ parallel Gaussian channels having gains $g_j$. Consequently, $\Pr(S_n)$ is found as

$$\begin{aligned}\Pr(S_n) &= \Pr\left(Q_c \le L \sum_{j=(n-1)J+1}^{nJ} \log(1 + g_j P_n)\right) \\ &= \Pr\left(r \le \frac{1}{J} \sum_{j=(n-1)J+1}^{nJ} \log(1 + g_j P_n)\right)\end{aligned} \tag{13}$$

where the last equality comes from $r = \frac{Q_{\text{c}}}{JL}$.

### B. INR hybrid ARQ protocol

Using INR hybrid ARQ, the transmitter sends a codeword with very aggressive rate in the first round. Then, if the receiver cannot decode the initial codeword, further parity bits are sent in the next retransmission rounds and in each round the receiver decodes the data based on all received signals. Consequently, with the same arguments as in (12), the maximum decodable information nats at the end of the $m$-th retransmission round is

$$Q_{\text{INR},m}^{\max} = L \sum_{n=1}^{m} \sum_{j=(n-1)J+1}^{nJ} \log\left(1 + g_j P_n\right) \tag{14}$$

and the probability terms $\Pr(\bar{S}_1, \dots, \bar{S}_m)$ and $\Pr(\bar{S}_1, \dots, \bar{S}_{m-1}, S_m)$ are respectively obtained by

$$\begin{aligned}&\Pr(\bar{S}_1, \dots, \bar{S}_m) = \Pr\left(Q_{\text{c}} > L \sum_{n=1}^{m} \sum_{j=(n-1)J+1}^{nJ} \log(1 + g_j P_n)\right) \\ &= \Pr\left(r > \frac{1}{J} \sum_{n=1}^{m} \sum_{j=(n-1)J+1}^{nJ} \log(1 + g_j P_n)\right)\end{aligned} \tag{15}$$

and

$$\begin{aligned}
&\Pr(\bar{S}_1,\dots,\bar{S}_{m-1},S_m)\\
&=\Pr\Big(L\textstyle\sum_{n=1}^{m-1}\sum_{j=(n-1)J+1}^{nJ}\log(1+g_jP_n)<Q_{\text{c}}\\
&\qquad\le L\textstyle\sum_{n=1}^{m}\sum_{j=(n-1)J+1}^{nJ}\log(1+g_jP_n)\Big)\\
&=\Pr\Big(\tfrac{1}{J}\textstyle\sum_{n=1}^{m-1}\sum_{j=(n-1)J+1}^{nJ}\log(1+g_jP_n)<r\\
&\qquad\le\tfrac{1}{J}\textstyle\sum_{n=1}^{m}\sum_{j=(n-1)J+1}^{nJ}\log(1+g_jP_n)\Big).
\end{aligned} \tag{16}$$

Here, it is interesting to note that setting $J=1$, i.e., assuming a fixed gain value in each round, the results are simplified to the ones obtained in slow-fading channels [4]–[6]. Also, with $g_j=g,\forall j$, the results match the ones presented in, e.g., [1], [2] where the channel is fixed during the whole packet transmission period (quasi-static channel).

The following theorem shows that at low SNRs the system performance under fast-fading channel assumption can be mapped into the one in a slow-fading channel with different fading pdf.

***Theorem 1:*** At low SNRs, the performance of the basic and INR hybrid ARQ protocols under fast-fading channel assumption tends towards the one in a slow-fading channel with the fading pdf obtained by averaging the fast-fading channel behavior in a retransmission round period.

*Proof:* As $\log(1+x)\simeq x$ for small values of $x$, the probability terms (13), (15) and (16) are changed to

$$\Pr(S_n)\simeq\Pr\big(r\le\frac{P_n}{J}\sum_{(n-1)J+1}^{nJ}g_j\big) \tag{17}$$

$$\Pr\big(\bar{S}_1,\dots,\bar{S}_m\big)\simeq\Pr\big(r>\frac{1}{J}\sum_{n=1}^{m}P_n\sum_{j=(n-1)J+1}^{nJ}g_j\big) \tag{18}$$

$$\begin{aligned}&\Pr\big(\bar{S}_1,\dots,\bar{S}_{m-1},S_m\big)\simeq\\&\Pr\left(\frac{1}{J}\sum_{n=1}^{m-1}P_n\sum_{j=(n-1)J+1}^{nJ}g_j<r\le\frac{1}{J}\sum_{n=1}^{m}P_n\sum_{j=(n-1)J+1}^{nJ}g_j\right)\end{aligned} \tag{19}$$

for low SNRs. Defining the random variable $U^{(J)}=\frac{1}{J}\sum_{j=1}^{J}g_j$, i.e., $U^{(J)}$ is the average of $J$ gain realizations $g_j$, (17)-(19) can be rewritten as

$$\Pr(S_n)\simeq\Pr(r\le P_nu_n) \tag{20}$$

$$\Pr(\bar{S}_1,\dots,\bar{S}_m)\simeq\Pr\left(r>\sum_{n=1}^{m}P_nu_n\right) \tag{21}$$

$$\Pr(\bar{S}_1,\dots,\bar{S}_{m-1},S_m)\simeq\Pr\left(\sum_{n=1}^{m-1}P_nu_n<r\le\sum_{n=1}^{m}P_nu_n\right) \tag{22}$$

which are the corresponding low-SNR probability terms in slow-fading channels, i.e., $J=1$ in (13), (15) and (16), with gain realization $u_n$ in the $n$-th retransmission round. ■

The intuition behind Theorem 1 is that less time diversity is provided by the fast-fading channel at low SNRs. Thus, the fast-fading channel will tend towards an equivalent slow-fading model when the SNR decreases. Note that (17)-(19) can provide very good approximations for the system performance at low SNRs, as (13), (15) and (16) do not have closed-form solutions.

*Remark 1:* For Rayleigh fading channels, $f_G(g)=\mu e^{-\mu g}$, on which we focus, the cumulative distribution function (cdf) of the auxiliary variable $U^{(J)}$ is obtained by

$$F_{U^{(J)}}(u)=\Pr\{\frac{1}{J}\sum_{j=1}^{J}g_i\le u\}\overset{(b)}{=}1-e^{-J\mu u}\sum_{j=0}^{J-1}\frac{(J\mu u)^j}{j!} \tag{23}$$

where "!" is the factorial operator, $\mu$ is the fading parameter determined by path loss and shadowing between the terminals and $(b)$ is found by iterative integration of the gain pdf.

Theorem 2 indicates that the throughput (the outage probability) of the ARQ protocols is upper (lower) bounded if the fast-fading realizations within a codeword duration are approximated by their average value, which changes the channel equivalent cdf as given in, e.g., (23).

***Theorem 2:*** Assume uniform power allocation, i.e., $P_m=P,\forall m$. The long-term and delay-limited throughput of the considered ARQ schemes in the fast-fading channel with $J$ gain realizations $g_j$, $j=1,\dots J$, in a retransmission round are respectively less than the long-term and delay-limited throughput in a slow-fading channel experiencing a single gain realization following the cdf $F_{U^{(J)}}$, $U^{(J)}=\frac{1}{J}\sum_{j=1}^{J}g_j$. Also, the outage probability in the fast-fading model is higher.

*Proof:* With some calculations in (7) and (8), we have

$$\eta_{\text{LT}}=r\frac{1-\Pr\left(\bar{S}_1,\dots,\bar{S}_{M+1}\right)}{1+\sum_{m=1}^{M}\Pr(\bar{S}_1,\dots,\bar{S}_m)} \tag{24}$$

$$\eta_{\text{DL}}=r\big(1-\sum_{m=1}^{M}\frac{\Pr(\bar{S}_1,\dots,\bar{S}_m)}{m(m+1)}-\frac{\Pr(\bar{S}_1,\dots,\bar{S}_{M+1})}{M+1}\big) \tag{25}$$

which are based on fact that $\Pr(\bar{S}_1,\dots,\bar{S}_{m-1},S_m)=\Pr(\bar{S}_1,\dots,\bar{S}_{m-1})-\Pr(\bar{S}_1,\dots,\bar{S}_m)$. Therefore, since from concavity of the function $\log(1+x)$ we have

$$\begin{aligned}&\Pr\left(\bar{S}_1,\dots,\bar{S}_m\right)^{\text{Fast}}=\\&\Pr\left(r>\tfrac{1}{J}\textstyle\sum_{n=1}^{m}\sum_{j=(n-1)J+1}^{nJ}\log(1+g_jP)\right)\\&\ge\Pr\left(r>\textstyle\sum_{n=1}^{m}\log(1+u_n^{(J)}P)\right)=\Pr\left(\bar{S}_1,\dots,\bar{S}_m\right)^{\text{Slow}}\end{aligned}$$

for the INR scheme, lower throughput and higher outage probability $\Pr(\text{outage})=\Pr(\bar{S}_1,\dots,\bar{S}_{M+1})$ are achieved in the fast-fading channel. This is particularly because the long-term and delay-limited throughput, i.e., (24) and (25), are decreasing functions of $\Pr(\bar{S}_1,\dots,\bar{S}_m),\forall m$. Finally, the same arguments are applicable in the basic ARQ as well. ■

The theorem is of interest when we remind that in practice the channel does not remain constant, even at low speeds, although it is approximated to be fixed. The theorem shows that the practical data transmission efficiency of ARQ protocols is worse than what is theoretically obtained by such approximations.

***Corollary 1:*** At high SNRs, upper bounds of the long-term and delay-limited throughput and a lower bound of the outage probability are obtained when (13), (15) and (16) are replaced by (17)-(19).

*Proof:* Since $\log(1+x)\le x,\forall x\ge 0$, the probability term obtained in (18) is less than the value found in (15). Hence, from (24) and (25), the long-term and the delay-limited throughput of the INR scheme increases when $\Pr(\bar{S}_1,\dots,\bar{S}_m)$ is calculated by (18), instead of (15). Also, the outage probability $\Pr(\text{outage})=$

$\Pr(\bar{S}_1, \dots, \bar{S}_{M+1})$ decreases. Finally, the same point is valid for the basic ARQ protocol. ■

## IV. Simulation results

Simulation results are obtained for Rayleigh fading channel $f_G(g) = \mu e^{-\mu g}$ where we set $\mu = 1$. Also, a maximum of $M = 1$ (re)transmission round and uniform power allocation is considered. Assuming $J = 2$, i.e., two channel realizations during each codeword transmission, Fig. 1 shows the performance of the ARQ protocols in a fast-fading channel. Also, the effect of different fading models on the data transmission efficiency of the INR ARQ scheme is studied in Fig. 2. From the figures it is deduced that 1) with a fast-fading channel, the difference between the throughput of basic and INR hybrid ARQ schemes decreases at high SNRs (Fig. 1a). 2) In harmony with (12) and (14), the INR ARQ scheme outperforms the basic ARQ method, in terms of throughput and outage probability (Fig. 1). 3) Compared to no feedback case, i.e., $M = 0$, considerable performance improvement is achieved by the basic ARQ in fast-fading channels (Fig. 1)[1]. This is different from the results in the quasi-static [1], [2] or slow-fading channels [3]–[6], where the basic ARQ has no or marginal effect on the system performance, respectively. 4) The difference between the system performance in fast- and, e.g., slow-fading channels increases with the initial codeword rate $r$ (Fig. 1b). 5) As expected from intuitions, with the same fading pdf, the highest long-term and delay-limited throughput is observed in the fast-fading channel (Fig. 2). The reason is that more time diversity is exploited by the ARQ in fast-fading channels. Also, 6) the ARQ shows better performance in the slow-fading model compared to the quasi-static model. Finally, due to space limitations, the numerical results associated with Theorems 1 and 2 and Corollary 1, in which we compared the fast-fading channel with a slow-fading model experiencing a different fading pdf, are not given in the figures. If desired, the numerical results for that case can be easily obtained via, e.g., (23).

## V. Conclusion

This letter studied the performance of different ARQ protocols in fast-fading channels. With the same fading distribution, the best performance of the ARQ models is obtained in the fast-fading channel. Using ARQ at low SNRs, the fast-fading channel can be mapped into an equivalent slow-fading channel. Finally, performance of the ARQ protocols in a fast-fading channel is worse, when compared to a slow-fading channel with fading distribution following the average characteristics of the fast-fading channel in a retransmission round.

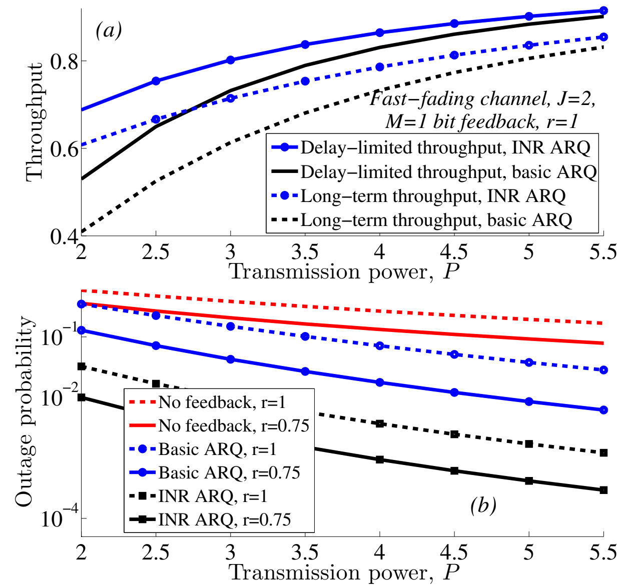


Figure 1. (a): Throughput and (b): outage probability vs the transmission power. Fast-fading channel, $J = 2$, uniform power allocation, $M = 1$ bit feedback.

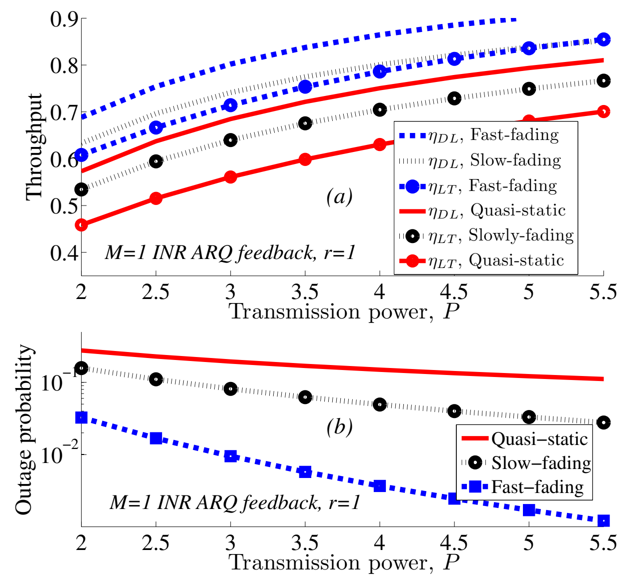


Figure 2. Throughput and outage probability in different fading models. $M = 1$ bit INR ARQ feedback. For fast-fading channel, we have $J = 2$. The results are obtained with the same fading distribution for the channels.

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[1]Note that, setting $M = 0$, the long-term and the delay-limited throughput of the no feedback case are the same, equal to the long-term throughput of the basic ARQ scheme with $M = 1$, i.e., the lowest curve in Fig. 1a.