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On Independent Platform Sample Number for Reverberation Chamber Measurements

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***Abstract*—The turn-table platform stirring technique is used to improve the measurement accuracy of a reverberation chamber (RC). It is usually assumed that the half-wavelength platform spatial sampling results in independent samples, which can be used to approximate the independent platform sample number. But the approximation is rather coarse. This paper presents a measurement-based method to determine the independent platform sample number. This method is simple and accurate. In contrast to the half-wavelength coarse approximation that depends only on frequency, this method in addition illustrates the dependence of the independent platform sample number on RC loading and mode stirring.**

***Index Terms*—Independent samples, platform stirring, reverberation chamber (RC).**

I. Introduction

THE reverberation chamber (RC) has traditionally been used for electromagnetic compatibility tests [1]–[9]. Over the past decade, it has found application for over-the-air measurements of wireless devices [10]–[14]. Due to the complicated test conditions (e.g., inhomogeneous test objects, irregular mode stirrers, changing boundary conditions, etc.) RC measurements are usually studied from a statistical point of view [1]. Therefore, it is of importance to gather enough independent samples in order to make an accurate statistical analysis of the measured data. Great effort has been exerted in determining and increasing the independent sample number N_{ind} in the literature. For example, N_{ind} was estimated from the Q-factor in [2]; it was studied using the (modified) autocorrelation function method in [3], and using the autoregressive model and the central limit theorem in [4]. Effort has also been exerted to increase N_{ind} by improving the effectiveness of the mode stirrers [5] or introducing two independent mode stirrers [6] (where a 2-D correlation was introduced to estimate N_{ind} in the case of two independent moving systems), or using source stirring [7], [8], or electronic (frequency) stirring [9]. Source stirring requires moving the transmit antennas to different positions and orientations during a measurement. A practical way of implementing it is to introduce a turn-table platform, on which the transmit antenna (or equivalently receive antenna due to reciprocity) is mounted. The antenna on the platform is then rotated circularly (controlled by a central computer) with a fixed orientation. This is referred to as platform stirring [14]. Although being vastly deployed in RC measurements, e.g. [12]–[14], the independent platform sample number $N_{\mathrm{ind,pf}}$ has not been studied thoroughly; and previous studies simply approximate it by dividing the summation of all the piece-wise linear distances between consecutive platform positions by the half-wavelength λ/2 correlation length [15]. However, this approximation is coarse; it does not reflect the fact that $N_{\mathrm{ind,pf}}$ depends also on the effectiveness of the mode stirrers and the level of RC loading. Note that $N_{\mathrm{ind,pf}}$ also depends on the radius of the platform stirring circle and that $N_{\mathrm{ind,pf}}$ is larger for a larger radius. For the conciseness of this paper, we keep the platform radius (i.e., the distance between the antenna and the center of the platform) fixed throughout this paper.

Manuscript received July 27, 2012; accepted September 29, 2012. Date of publication October 18, 2012; date of current version December 14, 2012.

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Digital Object Identifier 10.1109/TEMC.2012.2222417

In this paper, we present a measurement-based method for estimating $N_{\mathrm{ind,pf}}$. Simple as it is, the presented method is accurate (as will be shown by simulations) and shows the dependence of $N_{\mathrm{ind,pf}}$ on mode stirring and RC loading. Based on an accurately estimated $N_{\mathrm{ind,pf}}$ for a measurement setup, the optimal platform sampling is to choose the number of platform stirring positions N_{pf} equals to $N_{\mathrm{ind,pf}}$, avoiding time-consuming oversampling while minimizing measurement uncertainty.

II. Independent Platform Sample Number

During an RC measurement sequence, the turn-table platform rotates (stepwise); and the antenna mounted on the platform samples the field following a circular trajectory. It is intuitive that the optimal sampling positions should be uniformly distributed along the circle (with a radius R that is the distance between the antenna and the center of the circle). The question is how frequently should we (spatially) sample the field? A rule of thumb that is used ubiquitously in the RC literature is the correlation length λ/2 derived in [15]. Based on the correlation length, the independent platform sample number can be approximated by [14]

$$N_{\mathrm{pf,ind}} = \min\left\{N_{\mathrm{pf}},\ \frac{2R\sin(\pi/N_{\mathrm{pf}})N_{\mathrm{pf}}}{\lambda/2}\right\} \tag{1}$$

where N_{pf} denotes the number of platform stirring positions and min represents the minimum operator. The derivation of (1) is quite intuitive: the second term in the curly bracket is the number of half-wavelengths obtained by dividing the summation of all the piece-wise linear distances between consecutive platform positions by λ/2; since $N_{\mathrm{ind,pf}}$ cannot be larger than N_{pf}, only the minimum value of the two terms in the curly bracket should be chosen.

The model (1) is inaccurate in that the half-wavelength correlation length is derived based on the spatial correlation of the total electric field assuming a perfectly stirred RC. Thus, the actual $N_{\mathrm{ind,pf}}$ experienced by a realistic linear-polarized antenna with anisotropic radiation pattern will differ from that predicted by (1). Moreover, the model (1) cannot take into account the imperfections of a practical RC (e.g., less effective mode stirrers or inherent lossy objects). Therefore, a more accurate method for $N_{\mathrm{ind,pf}}$ estimation is needed.

Knowing that the actual correlation length depends on the antenna in use, which probably differs in different RC laboratories, it is of little help to improve the model (1) by taking into account a specific antenna. Instead, we propose a measurement-based method that can accurately predict $N_{\mathrm{ind,pf}}$ based on *a priori* measurement. Based on this method, once $N_{\mathrm{ind,pf}}$ is accurately estimated, different laboratories can decide their optimal platform string positions and use these for future measurements.

The proposed method is formulated as follows:

1) Denote the field samples (one corresponding to a different mode-stirrer position) at the mth platform position as a column vector $\mathbf{x}_m$, $m = 1 \ldots N_{\mathrm{pf}}$.
2) Concatenate $\mathbf{x}_m$ into a matrix $\mathbf{X} = [\mathbf{x}_1 \ \ldots \ \mathbf{x}_{N\,\mathrm{pf}}]$;
3) Estimate the correlation matrix of $\mathbf{X}$ as $\mathbf{R} = \mathbf{X}\mathbf{X}^H$, where the superscript ^{H} denotes conjugate transpose.
4) The independent platform sample number can be estimated as

$$N_{\mathrm{ind,pf}} = \frac{\mathrm{tr}(\mathbf{R})^2}{\mathrm{tr}(\mathbf{R}^2)} \tag{2}$$

where tr represents the trace operator. Equation (2) has its root from majorization theory [16], which deals with correlation and, therefore, is particularly useful for determining the number of independent platform samples at hand. It determines the dimensionality of the column space of $\mathbf{X}$ [17] and, therefore, the independent platform sample number. The accuracy of this method depends on the number of samples per platform position (or realizations). For large realization numbers, (2) approaches asymptotically to the true value.

III. Simulation

We resort to simulations to study the performance of the proposed method. For the sake of easy exposition and without loss of generality, we assume three platform stirring positions, i.e., $N_{\mathrm{pf}} = 3$. We first numerically generated a $3 \times N$ random matrix $\mathbf{X}_w$ consisting of independent and identically distributed Gaussian elements, where N ranges from 10 to 10 000. For simplicity and without loss of generality, we assume a real-valued correlation coefficient ρ. Since these three platform stirring positions are uniformly distributed over one circle, their correlation matrix is

$$\mathbf{R}_0 = \begin{bmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{bmatrix}. \tag{3}$$

Correlations can be introduced to the platform stirring positions by [13]

$$\mathbf{X} = \mathbf{R}_0^{1/2}\mathbf{X}_w. \tag{4}$$

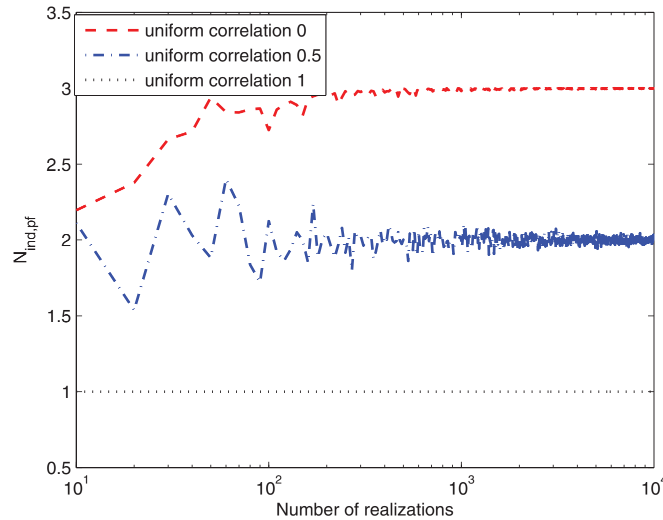

Fig. 1. Estimated $N_{\mathrm{ind,pf}}$ using (2) as a function of number of realizations.

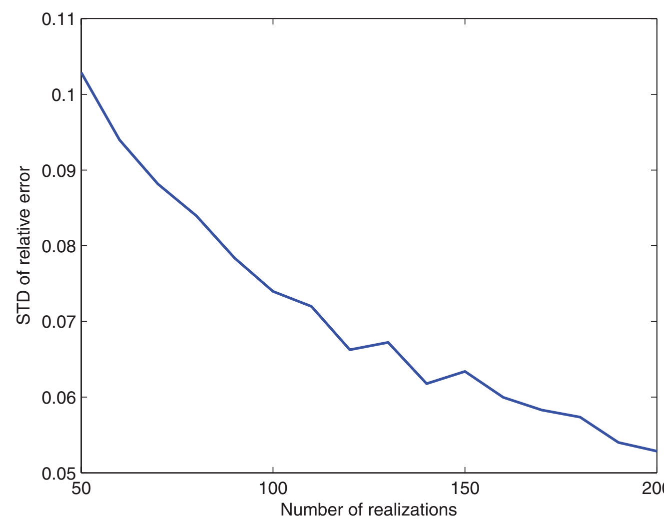

Fig. 2. STD of the relative error of the estimated $N_{\mathrm{ind,pf}}$ using (2) as a function of number of realizations for $\rho = 0.5$.

The estimated correlation matrix is $\mathbf{R} = \mathbf{X}\mathbf{X}^H$, based on which $N_{\mathrm{ind,pf}}$ is calculated using (2). Note that due to finite realizations, $\mathbf{R}$ almost surely differs from $\mathbf{R}_0$, but as N increases the estimated $N_{\mathrm{ind,pf}}$ approaches its true value. This is illustrated in Fig. 1 for three correlation coefficients, i.e., $\rho = 0$, 0.5, 1. For $\rho = 0$, the three platform positions are independent (to be exact, the random fields seen by the antenna at the three platform positions are uncorrelated, and since the field in an RC is Gaussian distributed [1], they are independent as well); therefore, the asymptotic $N_{\mathrm{ind,pf}} = 3$; for $\rho = 1$, the three platform positions are totally dependent; therefore, the asymptotic $N_{\mathrm{ind,pf}} = 1$. In order to study the accuracy of the estimator (2), we repeat the simulation 2000 times and calculated the standard deviation (STD) of its relative error for $\rho = 0.5$ and plotted it in Fig. 2. (Note that it was numerically checked that the STD for other ρ values is smaller than that for $\rho = 0.5$.) It can be seen from Fig. 2 that (2) gives accurate estimates (with less than

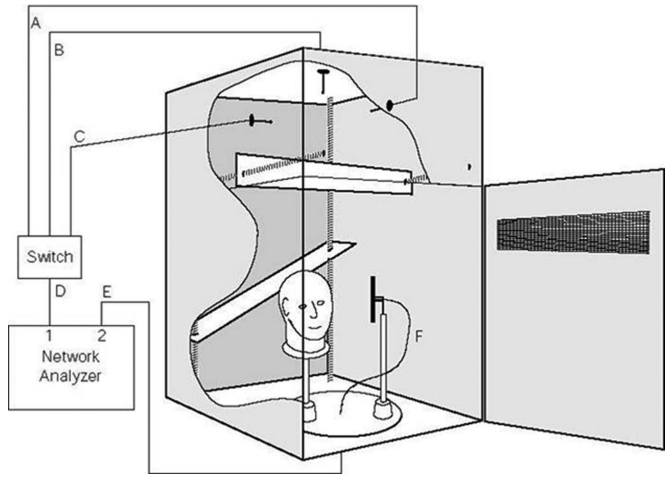

Fig. 3. Drawing of Bluetest RC with two mechanical plate stirrers, a platform, and three wall antennas.

0.07 relative STD) once the realization number (i.e., sample number per platform stirring position) is larger than 100, which is practical to achieve during measurements.

IV. Measurement

Measurements were performed from 500 to 3000 MHz in the Bluetest HP RC [14] with a size of $1.80 \times 1.75 \times 1.25$ m^3 (a drawing of which is shown in Fig. 3). It has two plate mode stirrers (or diffusers), a turn-table platform (on which a wideband discone antenna is mounted), and three antennas mounted on three orthogonal walls (referred to as wall antennas hereafter). The wall antennas are actually wideband half-bow-tie antennas. The measurement setup (or stirring sequence) of the RC is chosen such that the turn-table platform was stepwise moved to 20 platform stirring positions evenly distributed over one complete platform rotation; at each platform position the two plates were simultaneously and stepwise moved to 50 positions (equally spanned on the total distances that they can travel). At each stirrer position and for each wall antenna, a full frequency sweep was performed by a vector network analyzer with a frequency step of 1 MHz, during which the scattering parameter is sampled as a function of frequency and stirrer position. Hence, for this measurement setup, $N_{\rm pf} = 20$ and $N = 150$ (i.e., for each platform stirring position, we have 50 plate positions and 3 wall antennas). This large N ensures accurate estimation of $N_{\rm ind,pf}$ using the proposed method (cf. Section III).

The same measurement procedure was repeated for three loading conditions: *load 0* (unloaded RC), *load 1* (head phantom that is equivalent to a human head in terms of microwave absorption), and *load 2* (the head phantom plus three Polyvinyl Chloride cylinders filled with electromagnetic absorbers cut in small pieces), whose quality factors are around 1000, 550, and 300 at 1.5 GHz, respectively. Fig. 4 shows a photograph of the *load 2* configuration. Hereafter, measured data from these different loading configurations are simply referred to as *load 0*, *load 1*, or *load 2* data.

Applying the proposed method to the measured data, $N_{\rm ind,pf}$ is calculated and shown in Fig. 5. It is shown that $N_{\rm ind,pf}$

Fig. 4. Photo of the *load 2* configuration in the RC.

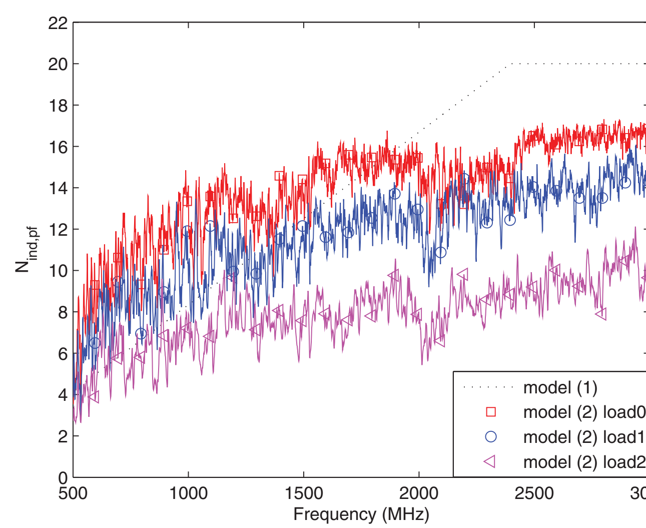

Fig. 5. Estimated $N_{\rm ind,pf}$ using (1) and (2) based on measurement in the RC.

(referred to as model (2) in Fig. 5) increases with increasing frequency and decreases with increasing loading. This is intuitive: at higher frequencies, the platform stirring positions are less correlated due to the increasing electrical distance; by increasing the RC loading, the angular spread reduces (the field is less isotropic), and therefore, the correlation length (which is inversely proportional to the angular spread) increases [18], rending more correlated platform stirring positions. Note that $N_{\rm ind,pf}$ is strictly smaller than 20 over the frequency range of interest; this agrees with our experience that for this RC increasing $N_{\rm pf}$ (while keeping the other setting unchanged) does not improve the measurement uncertainty. For comparison, the estimator (1) (referred to as model (1) in Fig. 5) is also plotted in the same figure. It can be seen that the estimator (1) tends to underestimate $N_{\rm ind,pf}$ for low frequencies and overestimate it for high frequencies; and it is independent of the RC loading. Thus, model (1) is a rather coarse approximation.

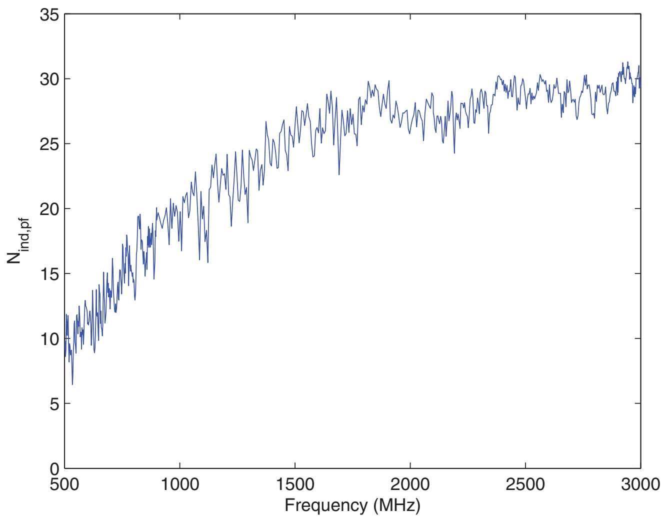

Fig. 6. Estimated $N_{\mathrm{ind,pf}}$ using (2) based on measurement in the upgraded RC.

In order to study the mode-stirring effects on $N_{\mathrm{ind,pf}}$, additional measurements were performed in an upgraded RC where the plate width is doubled, and the wall antennas are moved in one corner of the RC with a metallic shield blocking the line-of-sight path between them and a discone antenna on the platform. It was shown in [14] that this upgraded RC results in a reduced K-factor and, therefore, is more effective in mode stirring [19]. The measurement setup is almost the same as the previous one except that the platform stirring number is doubled, i.e., $N_{\mathrm{pf}} = 40$ and $N = 150$. Since the loading effect on $N_{\mathrm{ind,pf}}$ is known already, this measurement was only performed for the load 0 case. The estimated $N_{\mathrm{ind,pf}}$ is calculated using the proposed method and plotted in Fig. 6. Comparing Figs. 5 and 6, it is easy to conclude that $N_{\mathrm{ind,pf}}$ increases with more effective mode stirring. Note that $N_{\mathrm{ind,pf}}$ of the upgraded RC is larger than 20 above 1 GHz; this agrees with our experience that by increasing N_{pf} from 20 to 40 (while keeping the other setting unchanged) for this upgraded RC, the measurement uncertainty above 1 GHz can be improved (e.g., see [20]).

V. Conclusion

In this paper, a measurement-based method for determining $N_{\mathrm{ind,pf}}$ is presented. The accuracy of the method was studied by simulations. It is shown that the method has an STD of the relative error that is smaller than 0.07 when the sample number per platform stirring position is larger than 100. This method is applied to measurements in two RCs. It is shown that the independent platform sample number depends not only on frequency but also on the level of RC loading and the effectiveness of mode stirring.

References

[1] J. G. Kostas and B. Boverie, "Statistical model for a mode-stirred chamber," *IEEE Trans. Electromagn. Compat.*, vol. 33, no. 4, pp. 366–370, Nov. 1991.

[2] K. Madsen, P. Hallbjörner, and C. Orlenius, "Models for the number of independent samples in reverberation chamber measurements with mechanical, frequency, and combined stirring," *IEEE Antennas Wireless Propag. Lett.*, vol. 3, no. 1, pp. 48–51, Dec. 2004.

[3] H. G. Krauthäuser, T. Winzerling, J. Nitsch, N. Eulig, and A. Enders, "Statistical interpretation of autocorrelation coefficients for fields in mode-stirred chambers," in *Proc. IEEE Int. Symp. Electromagn. Compat.*, Chicago, FL, Aug. 2005, vol. 2, pp. 550–555.

[4] C. Lemoine, P. Besnier, and M. Drissi, "Estimating the effective sample size to select independent measurements in a reverberation chamber," *IEEE Trans. Electromagn. Compat.*, vol. 50, no. 2, pp. 227–236, May 2008.

[5] N. Wellander, O. Lunden, and M. Bäckström, "Experimental investigation and mathematical modeling of design parameters for efficient stirrers in mode-stirred reverberation chambers," *IEEE Trans. Electromagn. Compat.*, vol. 49, no. 1, pp. 94–103, Feb. 2007.

[6] F. Moglie and V. M. Primiani, "Analysis of the independent positions of reverberation chamber stirrers as a function of their operating conditions," *IEEE Trans. Electromagn. Compat.*, vol. 53, no. 2, pp. 288–295, May 2011.

[7] Y. Huang and D. J. Edwards, "A novel reverberating chamber: Source-stirred chamber," in *Proc. Int. Conf. Electromagn.Compat.*, Edinburgh, U.K., Sep. 1992, pp. 120–124.

[8] G. Cerri, V. M. Primiani, S. Pennesi, and P. Russo, "Source stirring mode for reverberation chambers," *IEEE Trans. Electromagn. Compat.*, vol. 47, no. 4, pp. 815–823, Nov. 2005.

[9] D. A. Hill, "Electronic mode stirring for reverberation chamber," *IEEE Trans. Electromagn. Compat.*, vol. 36, no. 4, pp. 294–299, Nov. 1994.

[10] G. Ferrara, M. Migliaccio, and A. Sorrentino, "Characterization of GSM non-line-of-sight propagation channels generated in a reverberating chamber by using bit error rates," *IEEE Trans. Electromagn. Compat.*, vol. 49, no. 3, pp. 467–473, Aug. 2007.

[11] E. Genender, C. L. Holloway, K. A. Remley, J. M. Ladbury, G. Koepke, and H. Garbe, "Simulating the multipath channel with a reverberation chamber: Application to bit error rate measurements," *IEEE Trans. Electromagn. Compat.*, vol. 52, no. 4, pp. 766–777, Nov. 2010.

[12] J. F. Valdes, M. A. Fernandez, A. M Gonzalez, and D. A. Hernandez, "The influence of efficiency on receive diversity and MIMO capacity for Rayleigh-fading channels," *IEEE Trans. Antennas Propag.*, vol. 56, no. 5, pp. 1444–1450, May 2008.

[13] X. Chen, P.-S. Kildal, and J. Carlsson, "Fast converging measurement of MRC diversity gain in reverberation chamber using covariance-eigenvalue approach," *IEICE Trans. Electron.*, vol. E94-C, no. 10, pp. 1657–1660, Oct. 2011.

[14] P.-S. Kildal, X. Chen, C. Orlenius, M. Franzén, and C. Lötbäck Patané, "Characterization of reverberation chambers for OTA measurements of wireless devices: Physical formulations of channel matrix and new uncertainty formula,," *IEEE Trans. Antennas Propag.*, vol. 60, no. 8, pp. 3875–3891, Aug. 2012.

[15] D. A. Hill, "Spatial correlation function for fields in a reverberation chamber," *IEEE Trans. Electromagn. Compat.*, vol. 37, no. 1, p. 138, Feb. 1995.

[16] A. W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Tts Application*. London, U.K: Academic, 1979.

[17] A. J. Laub, *Matrix Analysis for Scientists and Engineers*. Philadelphia, PA: SIAM, 2005.

[18] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-time Wireless Communication*. Cambridge Univ. Press, Cambridge, U.K, 2003.

[19] C. Lemoine, E. Amador, and P. Besnier, "Mode-stirring efficiency of reverberation chambers based on Rician K-factor," *Electron. Lett.*, vol. 47, pp. 1114–1115, 2011.

[20] E. Engvall *Uncertainty Models for Reverberation Chambers*, Master thesis, Chalmers Univ. Technol., Gothenburg, Sweden, 2012.