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Advanced Electro Hydraulic Control of Forestry Machine

A state-space based optimal control solution

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Abstract

This project, designed and implemented at Parker Hannifin AB, was to design and implement an advanced control system for the swing function of a forestry machine, a forwarder. The objective behind the swing control function is to improve the accuracy of the forwarding procedure, i.e. to improve the machines loading performance by means of advanced closed-loop control design. The focus aim was on exploring the benefits of model-based and optimal control systems with specific attention on real time control applications.

Alternative control scenarios were developed and tested on a real forestry machine configuration. To start with, a steady-state, state-feedback and Linear Quadratic (LQ) control solution was chosen as a preliminary controller for the project. Integral action was implemented in order to remove steady-state errors. Since the inertia is time varying, LQ solution is not optimal for the entire operating window and therefore two other control solutions were also tested. Gain scheduled LQ controller, where a set of linear controllers are designed for different working points, and a Linear Parameter Varying (LPV) controller where some parameters are allowed to vary in specified intervals.

When estimating the unknown velocity of the crane, the estimation became noisy and filtering was needed. A Kalman filter was implemented for an optimal estimation of the states fed back to the controller.

To evaluate the control solutions an experienced forestry machine driver tested and evaluated the system. Compared to the old system, open-loop control solution, the machine behaved more harmonically, less shaky and better damped with the LPV or gain scheduled LQ controller. With these new systems the amount of lever shifts could become less than for the old system if the driver learns how to use it. The change in response to the driver is not remarkably impaired. The gain scheduled solution is slightly faster than the LPV solution, while the LPV solution behaves more harmonically.

Keywords: LQ, RLS, LPV, Kalman Filter, Control, Filtering

Sammanfattning

Det här projektet, designat och utformat av Parker Hannifin AB, syftade till att designa och implementera ett styrsystem för svängfunktionen på en skotare. Syftet var särskilt att undersöka och utreda om det är möjligt att uppnå en mjukare och effektivare styrning än med dagens system.

En LQ-regulator valdes som utgångspunkt i projektet. Integralverkan och ett Kalman Filter implementerades för att få bort kvarstående fel samt reducera bruspåverkan. Eftersom trögheten i systemet varierar med tiden så är LQ-regulatorn inte optimal för hela arbetsområdet. För att lösa detta togs två andra lösningar fram. Schemalagd LQ-reglering där ett set av LQ-regulatorer designas för olika arbetspunkter, och en LPV-regulator där vissa parametrar tillåts variera i förbestmda intervall.

Vid skattning av den okända hastigheten hos kranen, blev skattningen brusig och filtrering nödvändig. Ett Kalman filter implementerades för en optimal skattning av de tillstånd som återkopplas till regulatorn.

För att utvärdera de två olika lösningarna fick en testförare köra med systemen och utvärdera dem. Jämfört med det gamla systemet som används idag, så beter sig maskinen mer harmoniskt, mindre skakigt och har en bättre dämpning med LPV eller den schemalagda LQ-regulatorn. Med de nya systemen skulle troligen antalet joystick rörelser kunna minskas om föraren blir van vid systemet och inser att han inte behöver agera regulator själv. Responstiden är i stort sett oförändrad. Den schemalagda lösningen är något snabbare än LPVn, medan LPV lösningen är mer harmonisk.

Nyckelord: LQ, RLS, LPV, Kalman Filter, Reglering, Filtrering

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1 INTRODUCTION

The hydraulic industry is matured where the major efforts has been put into production quality and accurate dimensions. A critical area is to be as energy efficient as possible. In the traditional way of improving the fluid power systems, limited measures are available for energy efficiency break troughs. That is why a new perspective of the problem is needed, and an electro hydraulic control system could be a part of the solution. With such a system the performance of the system may also be improved, especially when considering damping of the system.

Earlier studies of electro hydraulic control have already been made on other applications for the proof of concept. Attention can now be directed to apply sophisticated and model-based control theory to explore these advanced theories in novel forestry machine configurations.

A hydraulic system with high inertia, as in the swing function for a forestry machine, which is controlled in a traditional way has problems with large oscillations when the commands of the driver are not smooth. Pressure feedback with guide functions and online identification with pressure feedback from an observer are two techniques for handling those problems (Krus 1989).

An earlier study made by Krus and Gunnarsson on a lorry crane describes how an Recursive Least Square, RLS, algorithm can be used to, online, estimate unmeasurable parameters from the system, which then are used to calculate the parameters for the controller. Difficulties of implementing such an algorithm into a real application, with for example a poor excitation and time varying parameters, are also described (Krus and Gunnarsson 1993).

An approach for controlling the system electrohydraulically could be by state feedback. Since all states are not directly measurable it brings up the problem of unknown states. One way of dealing with this is by implementing a Kalman Filter. An and Seperhi (2003) show, for a hydraulic system, a practical implementation of how an Extended Kalman Filter uses the control signal and the measurements of the pressures in the cylinder chambers, together with a mathematical model of the system, to estimate the pistons position and velocity, and noise rejected pressures from the chambers.

To design a state feedback gain, an optimal controller called Linear Quadratic Regulator, LQR, may be used. It is optimal in the sense that the cost function is minimized

$$J = \sum_{k=0}^{\infty}[\boldsymbol{x}^T(k)\boldsymbol{Q}\boldsymbol{x}(k) + \boldsymbol{u}^T(k)\boldsymbol{R}\boldsymbol{u}(k)] \tag{1.1}$$

where x is the state vector, $\boldsymbol{Q}$ represents the cost for each state, $\boldsymbol{u}$ is the control signal and $\boldsymbol{R}$ represents the cost of the control signal usage. A study made by Shao-jun and Sheng describes a practical implementation of this kind of control for a electro hydraulic system (Shao-Jun, Sheng and Jun-Feng 2000).

The project was carried out at Parker Hannifin, a world leading company in motion and control technologies. For them there is a need for exploring modern control theories and of interest to see if it is convenient to via Matlab/Simulink generate code straight into the controller. The implementation is done for the swing function on a forestry machine.

The goal of the project is to achieve a smooth, fast, and well-damped closed-loop system response to driver reference signals.

2 NOTATION

Abbreviations

DC	Direct Current
ECU	Electronic Control Unit
EKF	Extended Kalman Filter
LPV	Linear Parameter Varying
LQ	Linear Quadratic
LQI	Linear Quadratic Integrating
PWM	Pulse width modulation
RLS	Recursive Least Square

Capital Letters

A	effective area (m^2)
A_i	effective area, cylinder chamber i (m^2)
A_v	area opening valve (m^2)
A_{vi}	area opening valve i (m^2)
B_p	viscous friction coefficient
C_q	flow coefficient
F_f	force, friction (N)
F_i	force, cylinder chamber i (N)
F_l	external load (N)
J	cost function
V_i	volume cylinder chamber i (m^3)
V_{10}	initial volume cylinder chamber 1
V_{20}	initial volume cylinder chamber 2
$\boldsymbol{A}$	state matrix
$\boldsymbol{B}$	input matrix
$\boldsymbol{C}$	output matrix
$\boldsymbol{L}$	feedback gain
$\boldsymbol{Q}_1$	state cost matrix
$\boldsymbol{Q}_2$	control signal cost matrix
$\boldsymbol{Q}_{12}$	dependence of state and control signal cost matrix

Small Letters

a	acceleration (m/s^2)
e	measurement noise
m	equivalent mass, inertia (kgm^2)
p	pressure (N/m^2)
p_i	pressure cylinder chamber i (N/m^2)
p_t	pressure tank (N/m^2)
p_s	pressure supply (N/m^2)
q	volumetric flow rate (m^3/s)
q_{vi}	volumetric flow rate, valve i (m^3/s)
q_i	volumetric flow rate, cylinder chamber i (m^3/s)
u_i	input signal valve i
v	process noise
x	cylinder piston position
$\boldsymbol{r_k}$	reference vector
$\boldsymbol{u}$	input vector
$\boldsymbol{u}_0$	working point input vector
$\boldsymbol{x}$	state vector
$\boldsymbol{x_r}$	state vector of reference states
$\boldsymbol{x}_0$	working point state vector
$\boldsymbol{y}$	output vector

Greek Letters

β	bulk modulus (N/m^2)
δp	pressure difference (N/m^2)
ρ	density (kg/m^3)
$\mathbf{\Phi}$	discrete state matrix
$\mathbf{\Gamma}$	discrete input matrix

3 MODELING

The starting point of the modeling presented is a hydraulic cylinder controlled with four valves. The forestry crane is modeled as an external mass load applied on the cylinder. To decrease the model complexity leakage losses are not considered in the model.

3.1 Cylinder

Two cylinders working in parallel and operating in opposite direction create a movement of a wheel, which makes the crane turn, see Figure 3.1. To be able to control the cylinders models for the flows and forces have been derived.

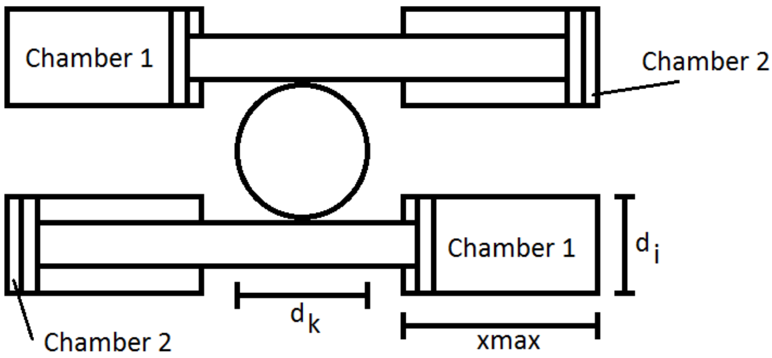

Figure 3.1. Cylinder movement.

3.1.1 Flow equations

The flow in the cylinder can be described by the general continuity equations for hydraulic flow.

$$Q = \dot{p}\frac{V}{\beta} + A\dot{x} \tag{3.1}$$

where Q is the flow entering the cylinder, V is the volume, p the pressure, x is the cylinder piston position, A is the effective area and β is the bulk modules of oil (An and Sepehri 2003). The volume for each cylinder chamber is given by

$$V_1 = A_1 x + V_{10} \tag{3.2}$$

and

$$V_2 = A_2(x_{max} - x) + V_{20} \tag{3.3}$$

where V_{10}, V_{20} and x_{max} are the initial volumes of the two cylinder chambers, and x_max is the maximum value of x see figure 3.1. The following two equations describe the flow entering cylinder chamber one and two.

$$Q_1 = Q_{v1} + Q_{v2} \tag{3.4}$$

$$Q_2 = Q_{v3} + Q_{v4} \tag{3.5}$$

where Q_{Vi} is the flow from each separate valve. From Equation (3.1) it is possible to derive how the pressures change in time, i.e.

$$\dot{p}_i = (Q_i - A_i\dot{x})\frac{\beta}{V_i} \quad \forall i = 1, 2 \tag{3.6}$$

3.1.2 Forces

The pressure in the cylinder creates forces in both cylinder chambers though with opposite directions, i.e.

$$F_i = p_i A_i \tag{3.7}$$

where p_i is the pressure in the cylinder chamber and A_i is the effective area. The total force that makes the piston move is described by Newtons second law,

$$F = ma = m\ddot{x} \tag{3.8}$$

where m is the equivalent mass of the crane and $a = \ddot{x}$ is the acceleration of the cylinder piston.

Depending on the positioning of the machine an external load force, F_l, will be created, which will act as a force on the cylinder. There is also a friction force, F_f in the cylinder, which can be given by

$$F_f = B_p\dot{x} \tag{3.9}$$

where B_p is the friction coefficient. Using Equations (3.7) - (3.9), the following force equilibrium for the cylinder can be derived:

$$m\ddot{x} = F_1 - F_2 - F_f - F_l = p_1A_1 - p_2A_2 - B_p\dot{x} - F_l \tag{3.10}$$

3.2 Valve

Four valves are used to control the flow through the cylinder. The valves are assumed to static, since the valves are approximately hundred times faster compared to the dynamics of the crane and therefore it can be neglected. The flow in each valve is given by

$$q_v = C_q A\sqrt{\frac{2}{\rho}\Delta p} \tag{3.11}$$

where C_q is a flow coefficient, A is the cross-sectional opening area for the valve, ρ is the oil density and Δp is the pressure drop over the valve (An and Sepehri 2003).

Each valve is connected to a pressure compensator, keeping the pressure over each valve constant. The pressure drop Δp over each valve, is set to 7 Bar. This implies that the flow can be modeled as a linear function. If k is defined as

$$k = C_q\sqrt{\frac{2}{\rho}\Delta p} = C_q\sqrt{\frac{14}{\rho}} \tag{3.12}$$

then the flow is (Merritt 1967).

$$q_v = kA_v \tag{3.13}$$

3.3 State-space representation of the overall system

According to the Equations (3.6), (3.10) and (3.13) a state space model of the crane system can be formulated as.

$$\begin{aligned} x_1 &= p_1 \\ x_2 &= p_2 \\ x_3 &= x \\ x_4 &= \dot{x} \\ u_1 &= A_{v1} \\ u_2 &= A_{v2} \end{aligned}$$

$$\begin{aligned} \dot{x_1} &= \frac{\beta}{A_1x_3 + V_{10}}(ku_1 - A_1x_4) \\ \dot{x_2} &= \frac{\beta}{A_2(x_{max} - x_3) + V_{20}}(ku_2 + A_2x_4) \\ \dot{x_3} &= x_4 \\ \dot{x_4} &= \frac{A_1x_1 - A_2x_2 - B_px_4 - F_l}{m} \end{aligned} \tag{3.14}$$

where the first two states are the pressures in cylinder chamber one and two. The third state is the cylinder piston position and the fourth state is the velocity of the piston. The control signals u_1 and u_2 are the valve area openings .

3.4 Nonlinear vs linear state-space models

The state space model 3.14 is non linear and can be written on the form

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= f(\boldsymbol{x}(t), \boldsymbol{u}(t)) \\ y(t) &= g(\boldsymbol{x}(t), \boldsymbol{u}(t)) \end{aligned} \tag{3.15}$$

Most control design methods requires linear models. The nonlinear model is therefor linearized around an operating point such that

$$\Delta\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\Delta\boldsymbol{x}(t) + \boldsymbol{B}\Delta\boldsymbol{u}(t) \tag{3.16}$$

$$\Delta\boldsymbol{y}(t) = \boldsymbol{C}\Delta\boldsymbol{x}(t) + \boldsymbol{D}\Delta\boldsymbol{u}(t)$$

where

$$\boldsymbol{A} = \left.\frac{\delta f}{\delta \boldsymbol{x}}\right|_{(\boldsymbol{x_0},\boldsymbol{u_0})} \qquad \boldsymbol{B} = \left.\frac{\delta f}{\delta \boldsymbol{u}}\right|_{(\boldsymbol{x_0},\boldsymbol{u_0})} \qquad \boldsymbol{C} = \left.\frac{\delta g}{\delta \boldsymbol{x}}\right|_{(\boldsymbol{x_0},\boldsymbol{u_0})} \qquad \boldsymbol{D} = \left.\frac{\delta g}{\delta \boldsymbol{u}}\right|_{(\boldsymbol{x_0},\boldsymbol{u_0})} \tag{3.17}$$

$$\Delta\boldsymbol{x}(t) = \boldsymbol{x}(t) - \boldsymbol{x}_0, \qquad \Delta\boldsymbol{u}(t) = \boldsymbol{u}(t) - \boldsymbol{u}_0, \qquad \Delta\boldsymbol{y}(t) = \boldsymbol{y}(t) - \boldsymbol{y}_0, \tag{3.18}$$

and the operating point follows from assuming steady-state ($\dot{\boldsymbol{x}} = 0$) i.e.

$$\begin{aligned} 0 &= f(\boldsymbol{x}_0, \boldsymbol{u}_0) \\ \boldsymbol{y}_0 &= g(\boldsymbol{x}_0, \boldsymbol{u}_0) \end{aligned} \tag{3.19}$$

(Lenartsson 2002).
The linearization of the crane system can be seen in Appendix A.

3.5 Measurements

Some states of the modeled system cannot be measured directly, namely the position and velocity of the piston in the cylinder. The angle of the crane though is measurable and by scaling the measurement with a scaling factor, the position of the piston becomes indirectly measurable.

4 CONTROL DESIGN

This chapter gives a brief description of the theory behind each type of controller that has been designed, implemented on the crane and then evaluated.

4.1 Optimal control

A continous time state space model including noise can be described as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{v}(t)$$

where $\boldsymbol{A}$ is the state matrix, $\boldsymbol{B}$ is the input matrix, $\boldsymbol{x}$ is the linearized state vector and $\boldsymbol{u}$ is the input signal and $\boldsymbol{v}$ is white gaussian process noise with zero mean value. Sampled into discrete time the model can be given by

$$\boldsymbol{x}(k+1) = \boldsymbol{\Phi}\boldsymbol{x}(k) + \boldsymbol{\Gamma}\boldsymbol{u}(k) + \boldsymbol{v}(k) \tag{4.1}$$

$$\boldsymbol{y}(k) = \boldsymbol{C}\boldsymbol{x}(k) + \boldsymbol{e}(k) \tag{4.2}$$

where $\boldsymbol{y}$ is the output vector, Φ is the discrete state system matrix, $\boldsymbol{\Gamma}$ is the discrete input matrix, $\boldsymbol{C}$ is the output matrix and $\boldsymbol{e}(k)$ is white gaussian measurement noise with zero mean value. The basic idea of regulation is to drive the system towards the origin starting from an arbitrary state

$$\boldsymbol{x}(0) \in \boldsymbol{X} \tag{4.3}$$

For an observable and controllable system, using a design criteria, which will be a weighting between the magnitude of the states and the control signals.

$$J = \sum_{i=0}^{\infty} [\boldsymbol{x}^T(k)\boldsymbol{Q_1}\boldsymbol{x}(k) + 2\boldsymbol{x}^T(k)\boldsymbol{Q_{12}}\boldsymbol{u}(k) + \boldsymbol{u}^T(k)\boldsymbol{Q_2}\boldsymbol{u}(k)] \tag{4.4}$$

is called the cost function and minimizing this results in an optimal control. The matrix $\boldsymbol{Q_1}$ represents the cost for each state. The matrix $\boldsymbol{Q_2}$ represents the cost of the control signal usage and the matrix $\boldsymbol{Q_{12}}$ represents the cost of the correlation between the states and the control signal (Astrom and Wittenmark 1997).

4.1.1 Steady-state Linear Quadratic control

Assuming the deterministic case, where $\boldsymbol{v}(k) = 0$ and $\boldsymbol{e}(k) = 0$ implies that the model (4.1) is

$$\boldsymbol{x}(k+1) = \boldsymbol{\Phi}\boldsymbol{x}(k) + \boldsymbol{\Gamma}\boldsymbol{u}(k) \tag{4.5}$$

A linear quadratic controller (LQ) which will minimize the cost function is defined as

$$\boldsymbol{u}(k) = -\boldsymbol{L}\boldsymbol{x}(k) \tag{4.6}$$

where

$$\boldsymbol{L} = (\boldsymbol{\Gamma}^T\boldsymbol{S}\boldsymbol{\Gamma} + \boldsymbol{Q_2})^{-1}(\boldsymbol{\Gamma}^T\boldsymbol{S}\boldsymbol{\Phi} + \boldsymbol{Q_{12}}^T) \tag{4.7}$$

with S being the solution for the discrete time Riccati equation

$$\boldsymbol{S} = \boldsymbol{\Phi}^T\boldsymbol{S}\boldsymbol{\Phi} + \boldsymbol{Q_1} - (\boldsymbol{\Phi}^T\boldsymbol{S}\boldsymbol{\Gamma} + \boldsymbol{Q_{12}})(\boldsymbol{\Gamma}^T\boldsymbol{S}\boldsymbol{\Gamma} + \boldsymbol{Q_2})^{-1}(\boldsymbol{\Gamma}^T\boldsymbol{S}\boldsymbol{\Phi} + \boldsymbol{Q_{12}}) \tag{4.8}$$

Thus the linear system with LQ control is described by

$$\boldsymbol{x}(k+1) = (\boldsymbol{\Phi} - \boldsymbol{\Gamma}\boldsymbol{L})\boldsymbol{x}(k) \tag{4.9}$$

(Astrom and Wittenmark 1997).

A pure LQ-controller is not able to handle hard constraints. The $\boldsymbol{Q_1}$ and $\boldsymbol{Q_2}$ matrices are only used to punish the states and the control signal activity. This means that each state is given different priority. In this system the pressure in the cylinder chambers must be positive. Since the LQ-controller cannot handle constraints a reference signal is set for the pressures. The smallest pressure is then punished hard enough to avoid the pressure becoming negative.

The $\boldsymbol{Q_1}$ and $\boldsymbol{Q_2}$ matrices may for a starting point be chosen by Brysons rule as

$$\boldsymbol{Q_{1ii}} = \frac{1}{(\text{maximum value of state i})^2} \tag{4.10}$$

$$\boldsymbol{Q_{2jj}} = \frac{1}{(\text{maximum value of input signal j})^2} \tag{4.11}$$

to get a reasonable relation between the state costs and the control signal usage costs. Thereafter tuning is needed to get desirable performance of the system (Franklin,Powell and Emami-Naeini 2002).

Assuming that the radius of the crane is fixed and that no external mass is being lifted by the crane. An LQ controller was then designed using Matlab and Simulink. In order to design the controller, the nonlinear state-space model was linearized according to Appendix A. The design process was divided into two different steps. First, the controller was designed to keep the velocity at zero and then to make a step and follow a given reference signal. To avoid the pressure in the cylinder chambers becoming negative, a reference pressure for the chambers was implemented as well as an additional controller with a switch to make sure that the lowest pressure is controlled. The switching controller can be seen in Figure 4.1.

Due to poor closed-loop performance this solution was dropped. Switching between two different controllers made the movement jumpy and the overall performance poor. Instead, an additional state was added to the model, which represents the lowest pressure when the pressure difference between the two cylinder chambers is

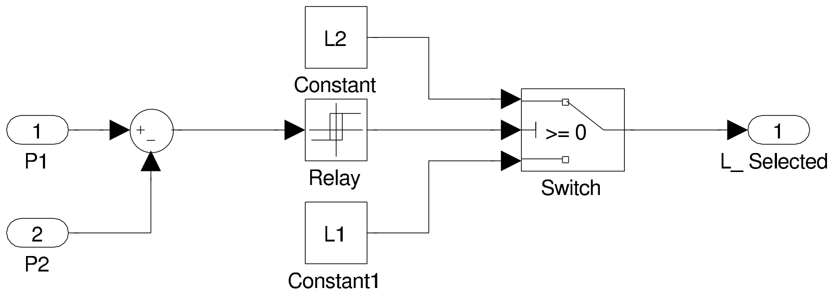

Figure 4.1. Selection of LQ controller, depending on pressure difference.

high. Punishing this state in the Q matrix makes it possible to keep the pressure in the chambers above zero and avoid cavitation with only one controller. The state x_5 that was added to the model is described as,

$$x_5 = \frac{p_1 p_2}{p_1 + p_2} \tag{4.12}$$

The state space vector is then augmented to

$$\boldsymbol{x_a} = \begin{bmatrix} \boldsymbol{x} \\ x_5 \end{bmatrix} \tag{4.13}$$

4.1.2 Linear quadratic controller with integral action

Normally a standard LQ controller cannot eliminate stationary errors if the reference control signal is different from zero or if load disturbance are acting on the system. In order to remove the steady-state error, the model was extended with an integral state according to

$$\boldsymbol{x_i} = \int_0^t (\boldsymbol{x_r} - \boldsymbol{r_x}) d\tau \tag{4.14}$$

where $\boldsymbol{x_r}$ is the state vector with reference tracking and $\boldsymbol{r_x}$ is the reference vector of the states with reference tracking (Haugen 2009). The state space vector is then augmented to

$$\boldsymbol{x_e} = \begin{bmatrix} \boldsymbol{x_a} \\ \boldsymbol{x_i} \end{bmatrix} \tag{4.15}$$

4.1.3 Gain scheduled linear quadratic control

Gain scheduling of a controller is a method to make a set of linear controllers, designed for different operating points of a nonlinear system, and interpolate among the controllers.

The LQ solution is optimal for a linear system, and the state space is linearized around a specific operating point, but the real system is expected to work for the entire region. An LQ solution can therefore not be optimal for the entire region, and may actually result in an unstable system in some parts of the region. By using the theory for gain scheduling, a controller could be achieved for the entire region. Even though the theory doesn't guarantee stability, the generated controllers in many cases are stable.

When scheduling gains, the control signal becomes discontinuous at the times the controller switches gain. Therefore interpolation can be used between the different gains to make the controller smooth for the complete region of the varying parameters. The varying parameter, for the swing function on a forestry machine, which has to be considered, is the equivalent mass, m (Bruzelius 2004).

4.2 Linear Parameter Varying control

Another option for designing the controller is by solving a Linear Matrix Inequality (LMI) problem where the parameters in the matrix are allowed to vary in given intervals. Apart from a gain scheduling solution it can be shown that such a solution is stable and robust for the entire region. The parameters which are varying are set up in a vector $\rho(k)$ (Bruzelius 2004). For this problem we set.

$$\rho(k) = 1/m(k) \tag{4.16}$$

The system can now be written as

$$\boldsymbol{x}(k+1) = \boldsymbol{A}(\rho(k))\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{u}(k) \tag{4.17}$$

By assuming that

$$\boldsymbol{x}(k) \quad \text{and} \quad \frac{1}{m(k)} \tag{4.18}$$

are available a state feedback scheduled solution for tracking can be obtained.

$$\boldsymbol{u}(k) = -\boldsymbol{K}(\rho(k))\boldsymbol{x}(k) + \boldsymbol{r}(k) \tag{4.19}$$

The closed loop system is then

$$\begin{aligned}
\boldsymbol{x}(k+1) &= \boldsymbol{A}(\rho(k))\boldsymbol{x}(k) + \boldsymbol{B}(-\boldsymbol{K}(\rho(k))\boldsymbol{x}(k) + \boldsymbol{r}(k)) \\
&= (\boldsymbol{A}(\rho(k)) - \boldsymbol{B}(\boldsymbol{K}(\rho(k)))\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{r}(k) \\
&= \tilde{\boldsymbol{A}}(\rho(k))\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{r}(k)
\end{aligned} \tag{4.20}$$

where

$$\boldsymbol{A}(\rho(k)) - \boldsymbol{B}(\boldsymbol{K}(\rho(k)) = \tilde{\boldsymbol{A}}(\rho(k)) \tag{4.21}$$

$$\boldsymbol{z}(k) = \boldsymbol{C}\boldsymbol{x}(k) - \boldsymbol{r}(k). \tag{4.22}$$

which gives the closed loop system

$$\boldsymbol{A}(\rho(k)) = \boldsymbol{A_0} + \boldsymbol{A_1}\rho(k) \tag{4.23}$$

$$\boldsymbol{K}(\rho(k)) = \boldsymbol{K_0} + \boldsymbol{K_1}\rho(k) \tag{4.24}$$

For quadratic stability, define a lyapunov function:

$$V(k) = \boldsymbol{x}(k)^T \boldsymbol{P}\boldsymbol{x}(k) \quad \text{, where} \quad \boldsymbol{P} = \boldsymbol{P}^T > 0 \tag{4.25}$$

With stability condition:

$$V(k+1) - V(k) < 0 => \boldsymbol{x}(k+1)^T \boldsymbol{P}\boldsymbol{x}(k+1) - \boldsymbol{x}(k)^T - \boldsymbol{P}\boldsymbol{x}(k) < 0 \tag{4.26}$$

Replacing $\boldsymbol{x}(k+1)$ by $\boldsymbol{A}(\rho(k))\boldsymbol{x}(k) + \boldsymbol{B}r(k)$ gives

$$V(k+1) - V(k) < 0, (\boldsymbol{A}(\rho(k))\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{r}(k))^T \boldsymbol{P}(\boldsymbol{A}(\rho(k))\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{r}(k)) < 0 \tag{4.27}$$

By solving this matrix inequality $K(\rho(k))$ is obtained. By introducing disturbance rejection, reference tracking is obtained:

$$V(k+1) - V(k) + J \leq 0 \tag{4.28}$$

$$T_{rt}(\boldsymbol{P}) = \frac{||\boldsymbol{z}||}{||\boldsymbol{r}||} \leq \gamma \tag{4.29}$$

$$J = \boldsymbol{z}(k)^T \boldsymbol{z}(k) - \gamma^2 \boldsymbol{r}(k)^T \boldsymbol{r}(k) <= 0 \tag{4.30}$$

The above inequality can be transformed to Linear Matrix Inequality which is parameter dependant. Because of scheduling parameter affinity, only a finite number of LMIs has to be solved to obtain $\boldsymbol{K}(p)$ (Bruzelius 2004).

5 FILTER DESIGN

This chapter presents different filter strategies used in order to reduce the noise impact and to make it possible to estimate non measureable states for state-feedback control design.

5.1 Kalman Filter

The Kalman Filter estimates the states for a given linear process in a way that minimizes the variance of the state estimation error. The Kalman filter is only optimal if the noise is white and Gaussian. For cases with given mean and standard deviation of noise the Kalman filter is considered to be the best linear estimator.

Depending on the available measurements different estimators can be designed. Given the data $\boldsymbol{Y}_k = \{\boldsymbol{y}(i), \boldsymbol{u}(i) | i \leq k\}$ an estimation of the state $\boldsymbol{x}(k+n)$ is desired. This implies three different types of estimators.

- Smoothing: $n < 0$
- Filtering: $n = 0$
- Prediction: $n > 0$

The two most common versions are the Kalman prediction and filter cases. For a control system implementation on a forestry a combination of these two can be used.(Astrom and Wittenmark 1997)

5.1.1 Discrete Time Kalman Filter

The discrete time plant model which is used for designing the filter is

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{\Phi}\boldsymbol{x}(k) + \boldsymbol{\Gamma}\boldsymbol{u}(k) + \boldsymbol{v}(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k) + \boldsymbol{e}(k) \end{aligned} \tag{5.1}$$

The noise covariances are given by

$$E\{\boldsymbol{v}(k)\boldsymbol{v}(k)^T\} = \boldsymbol{R}_1, E\{\boldsymbol{e}(k)\boldsymbol{e}(k)^T\} = \boldsymbol{R}_2 \quad \text{and} \quad E\{\boldsymbol{v}(k)\boldsymbol{e}(k)^T\} = \boldsymbol{R}_{12} \tag{5.2}$$

The design criterion for the filter is to minimize the variance of the estimation error

$$\begin{aligned} \boldsymbol{P}(k+1) &= \boldsymbol{\Phi}\boldsymbol{P}(k)\boldsymbol{\Phi}^T + \boldsymbol{R}_1 \\ &- (\boldsymbol{\Phi}\boldsymbol{P}(k)\boldsymbol{C}^T + \boldsymbol{R}_{12})(\boldsymbol{R}_2 + \boldsymbol{C}\boldsymbol{P}(k)\boldsymbol{C}^T)^{-1}(\boldsymbol{C}\boldsymbol{P}(k)\boldsymbol{\Phi}^T + \boldsymbol{R}_{12}^T) \end{aligned} \tag{5.3}$$

where

$$\boldsymbol{P}(0) = \boldsymbol{R}_0$$

The estimator itself is

$$\hat{\boldsymbol{x}}(k+1|k) = \boldsymbol{\Phi}\hat{\boldsymbol{x}}(k|k-1) + \boldsymbol{\Gamma}\boldsymbol{u}(k) + \boldsymbol{K}(k)(\boldsymbol{y}(k) - \boldsymbol{C}\hat{\boldsymbol{x}}(k|k-1)) \tag{5.4}$$

where the Kalman gain $\boldsymbol{K}$ can be calculated using

$$\boldsymbol{K}(k) = (\boldsymbol{\Phi}\boldsymbol{P}(k)\boldsymbol{C}^T + \boldsymbol{R}_{12})(\boldsymbol{R}_2 + \boldsymbol{C}\boldsymbol{P}(k)\boldsymbol{C}^T)^{-1} \tag{5.5}$$

and the $\hat{\boldsymbol{x}}$ denotes estimated state (Astrom and Wittenmark 1997).

5.2 Extended Kalman Filter

Since the operating region for a forestry machine varies in time a regular steady-state Kalman Filter based on a linear model is not enough to cover a range of operating points. However an Extended Kalman Filter (EKF) can handle systems with nonlinear dynamics, unlike the standard Kalman filter (Hameed 2010).

Consider a system with nonlinear dynamics

$$\boldsymbol{x}(k+1) = f((\boldsymbol{x}(k), \boldsymbol{u}(k)) + \boldsymbol{v}(k) \tag{5.6}$$

$$\boldsymbol{y}(k) = h((\boldsymbol{x}(k)) + \boldsymbol{e}(k)$$

where $\boldsymbol{v}(k)$ and $\boldsymbol{e}(k)$ are gaussian noise and the covariance data matrices are as in (5.2).

5.2.1 EKF algorithm

The EKF algorithm is divided into two different parts: the predict cycle (step 1-2 below) and the filter cycle (step 3-5 below).

1. Linearize the estimation of the system around the current estimate, $\hat{\boldsymbol{x}}(k|k)$:

$$\hat{\boldsymbol{x}}(k+1|k) = f(\hat{\boldsymbol{x}}(k|k)) \tag{5.7}$$

2. Calculate the state error variance matrix:

$$\boldsymbol{P}(k+1|k) = \boldsymbol{F}(k)\boldsymbol{P}(k|k)\boldsymbol{F}^T(k) + \boldsymbol{Q} \tag{5.8}$$

where $\boldsymbol{F}$ is the Jacobian matrix of $f(\cdot)$ and $\boldsymbol{Q}$ is state noise

3. Update the Kalman gain:

$$\boldsymbol{K}(k+1) = \boldsymbol{P}(k+1|k)\boldsymbol{H}^T(k+1)[\boldsymbol{H}(k+1)\boldsymbol{P}(k+1|k)\boldsymbol{H}^T(k+1)+\boldsymbol{R}(k+1)]^{-1} \tag{5.9}$$

where $\boldsymbol{H}$ is the Jacobian matrix of $h(\cdot)$.

4. Derive new state estimates:

$$\hat{\boldsymbol{x}}(k+1|k+1) = \hat{\boldsymbol{x}}(k+1|k) + \boldsymbol{K}(k+1)[\boldsymbol{y}(k+1) - \boldsymbol{H}(k+1)(\hat{\boldsymbol{x}}(k+1|k))] \quad (5.10)$$

5. Update the state error covariance matrix:

$$\boldsymbol{P}(k+1|k+1) = [\boldsymbol{I} - \boldsymbol{K}(k+1)\boldsymbol{H}(k+1)]\boldsymbol{P}(k+1|k) \quad (5.11)$$

An EKF is not an optimal filter since it is based on a set of approximations. Neither is it necessary convergent. This means that the matrices $\boldsymbol{P}(k|k)$ and $\boldsymbol{P}(k+1|k)$ do not represent the true covariance of the state estimates. However with good approximations an EKF gives satisfactory results for the hole working area (Hameed 2010).

5.3 Recursive Least Squares algorithm (RLS)

Recursive Least Squares, RLS, is a recursive filter for estimating unknown or uncertain parameters.

For a system described by

$$y(t) + a_1 y(t-1) + ... + a_n y(t-n) = b_1 u(t-1) + ... + b_n u(t-n) + v(t) \quad (5.12)$$

the RLS algorithms goal is to minimize the criteria

$$V_t(\theta) = \sum_{k=1}^{t} (y(k) - \boldsymbol{\varphi}^T(k)\boldsymbol{\theta})^2 \quad (5.13)$$

where

$$\boldsymbol{\varphi}(t) = [-y(t-1) \ ... \ -y(t-n) \ u(t-1) \ ... \ u(t-n)]^T \quad (5.14)$$

$$\boldsymbol{\theta} = [a_1 \ ... \ a_n \ b_1 \ ... \ b_n]^T \quad (5.15)$$

$$y(t) = \boldsymbol{\varphi}^T(t)\boldsymbol{\theta} + v(t) \quad (5.16)$$

and $v(t)$ is the measurement noise.

The minimizing parameter vector can be computed recursively as

$$\hat{\boldsymbol{\theta}}(t) = \hat{\boldsymbol{\theta}}(t-1) + \boldsymbol{P}(t)\boldsymbol{\varphi}(t)\epsilon(t) \quad (5.17)$$

where

$$\epsilon(t) = y(t) - \boldsymbol{\varphi}^T(t)\hat{\boldsymbol{\theta}}(t-1) \quad (5.18)$$

and

$$\boldsymbol{P}(t) = \boldsymbol{P}(t-1) - \frac{\boldsymbol{P}(t-1)\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^T(t)\boldsymbol{P}(t-1)}{1 + \boldsymbol{\varphi}^T(t)\boldsymbol{P}(t-1)\boldsymbol{\varphi}(t)} \quad (5.19)$$

If the estimated parameters are time varying a forgetting factor λ can be used to get a descending effect of old values (Krus and Gunnarsson 1993). i.e. by minimizing

$$V_t(\boldsymbol{\theta}) = \sum_{k=1}^{t} \lambda^{t-k}(y(k) - \boldsymbol{\varphi}^T(k)\boldsymbol{\theta})^2 \tag{5.20}$$

The equation then becomes

$$\boldsymbol{P}(t) = \frac{1}{\lambda(t)}\left[\boldsymbol{P}(t-1) - \frac{\boldsymbol{P}(t-1)\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^T(t)\boldsymbol{P}(t-1)}{\lambda(t) + \boldsymbol{\varphi}^T(t)\boldsymbol{P}(t-1)\boldsymbol{\varphi}(t)}\right] \tag{5.21}$$

For a good estimation of the parameters the system also has to be excited well. In a real application as for a forestry machine the system is often not excited at all for large intervals of the time. Therefore, the update needs to be switched off in some way during poor excitation. One way of solving these two problems is by adding a varying forgetting factor which depends on both the change of the parameters and the excitation (Krus and Gunnarsson 1993). The forgetting factor can be chosen as

$$\lambda(t) = 1 - (1 - \lambda_0)\left(1 - \frac{tr(\boldsymbol{P})}{\gamma}\right) \tag{5.22}$$

where λ_0 and γ are tuning parameters.

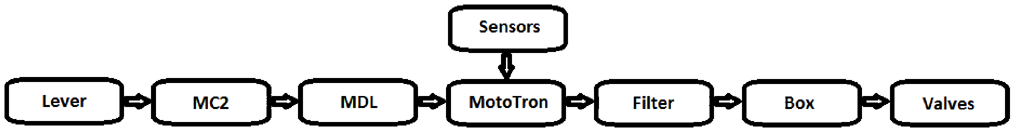

Figure 6.1. Simplified connection scheme, electrical components

6 IMPLEMENTATION

This chapter presents the hardware and software that is used in this project. Some of the equipment used in this project has been custom made by Parker.

6.1 Mechanical

The mechanical equipment used in the project is a valve block consisting of four DF+ valves from Parker Hannifin AB. To supply the DF+ valves, a K190 Valve delivers a constant supply pressure of 140 bar to the DF+ valve block. There are also chock valves connected between the cylinders and the DF+ valves to make sure the pressure doesn't get above 160 bar in the system.

6.2 Electrical

The control unit of the system is a Motoron Electronic Control Unit, ECU, it receives the reference signal from the lever and sends Pulse Width Modulation, PWM, control signals to the active lowpass filter which converts the signals to a Direct Current, DC, signal and then sends it to the signal conditioning box, which sends the signals to the valves. Custom made units are the active low pass filter and the signal conditioning box. The signal conditioning box receives the control signals, sends them to the valves and it also supplies power to the valves. Sensors that are used are four pressure sensors for measuring the pressures in the valves and one potentiometer for measuring the angle position of the crane. A simple connection scheme can been seen in Figure 6.1. A complete scheme is given in Appendix C.

In order to log data and analyze the performance of the system two main parts are used, a USB to CAN converter to read CAN signals into the computer, and a USB chassi together with a module from National Instruments to read signals into the computer.

6.3 Software

In order to develop the software for the Mototron ECU some other tools were used. The model of the system was implemented into Matlab/Simulink where simulations could be made. The controller was designed using the Matlab/ Control toolbox. Motohawk, which is an addon to Simulink, was used to make the model compatible with the Mototron ECU and to include input and output signals to receive and send signals with the Mototron. The real-time workshop toolbox and a c-compiler was used in order to convert the model into programmable files for the Mototron. Through the Mototune software the files were programmed to the Mototron.

6.3.1 Filtering

Since the DF+ valves needs a DC voltage, and the Mototron unit does not have analog outputs, a filter had to be implemented to make the PWM-output become a DC voltage. The PWM-output is a Low signal output, which means that when switched high, the ground is connected and the filtered signal becomes only about 60% of its desired value. Therefore an active filter had to be developed. Unfortunately the PWM-output of the Mototron unit was not sufficiently accurate for higher frequencies (the duty cycle became to small), and therefore a calibration of the output signals had to be made in the software as well.

6.3.2 Safety

The control system implemented is electrically supplied from an external socket. Therefore the emergency switch from the machine was connected to a hardware switch in the control system which cuts the voltage supply to the valves. On the IQAN module MDL, there is a switch for enabling the crane. That switch was also implemented into the software of the controller.

6.3.3 Measuring possibilities

To be able to evaluate the results of the system measurements are required. The USB measuring device from National Instrument was used to log data from all the sensors at a rate of 50 Hz. Mototune can only log data at a rate of 20 Hz, and was therefore only used for logging the internal data from the ECU, namely the reference velocity, and the differentiated and filtered angular velocity. The control signals were also logged via Mototron, but could also have been logged after the lowpassfilter via the USB measuring device.

The state space of the system is built on the position and the velocity of the piston in the cylinder, but those states cannot be measured directly. Therefore a ratio was calculated between the angle of the crane, α, and the piston position, x.

7 RESULTS

This chapter presents the results obtained in the project. The controllers that gave satisfactory results in simulations were tested in a real crane system. The final experimental results have been done by a test driver.

7.1 Controller for fixed, known inertia

The first controller designed was the standard LQ-controller designed for a fixed inertia. Problems occurred with cavitation as can be seen in Figure 7.2.

A controller controlling the output of the lowest pressure, LQpmin, as in Figure 4.1 and Equation (4.12), was implemented and in Figure 7.2 it is seen that cavitation is no longer present. However from the step response in Figure 7.3 it can be seen that there is a significant steady-state error for the LQpmin controller. Introducing integral action through Equations (4.15) and (4.14) error vanish.

An off-switching integral action, LQIreset, makes zero tracking faster as can be seen in Figure 7.3, implemented as in Figure 7.1. For the real tests of the LQIreset controller, a differentiation of the position was used to achieve an estimation of the velocity. The measurement noise of the position made the velocity extremely noisy and a lowpass filter with bandwith of 20 rad/s was used. It can be seen in Figure 7.4 that the output is still very noisy compared to the simulated one in Figure 7.3.

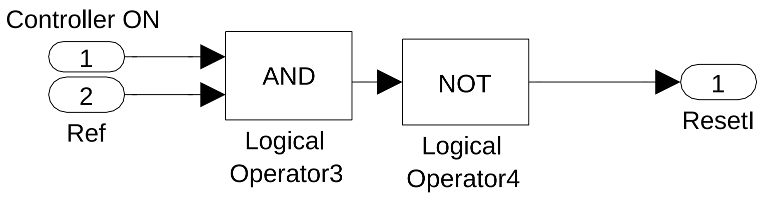

Figure 7.1. ResetOfIntegrator

7.2 Filtering

The differentiation and lowpass filtering (Derivationfilter 20) of the position is stable but noisy for the entire region of the inertia. A Kalman Filter (Kalman Filter) designed for filtering of all of the feedback states performs really well for a fixed and known inertia, but when the inertia changes, the estimation becomes bad as can be seen in Figure 7.5. An Extended Kalman Filter (EKF), with non-fixed, but known

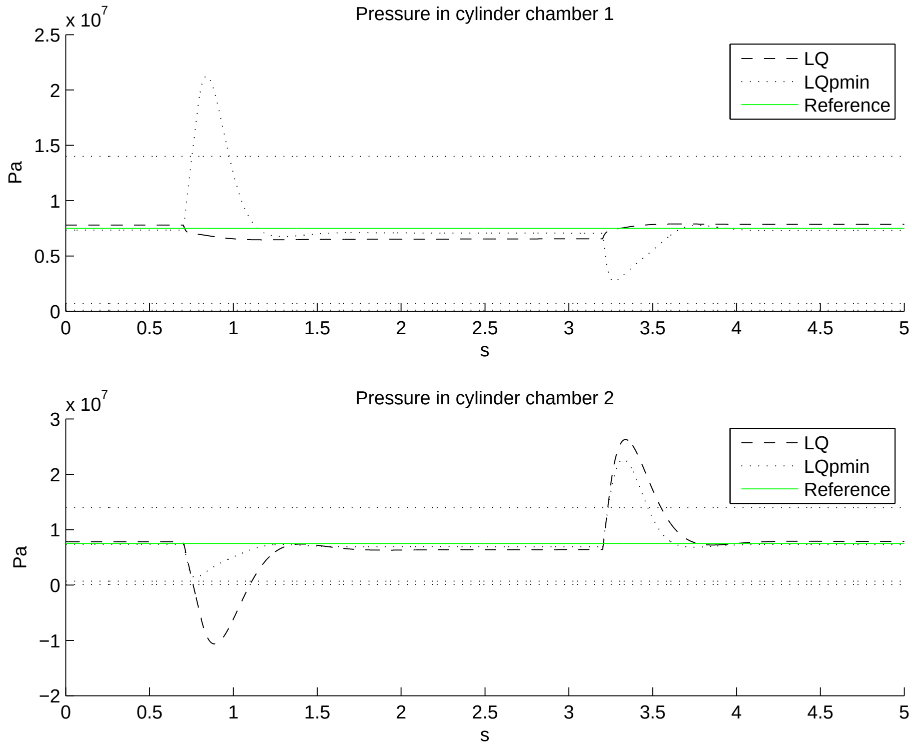

Figure 7.2. Simulation: Pressure behavior during a step response.

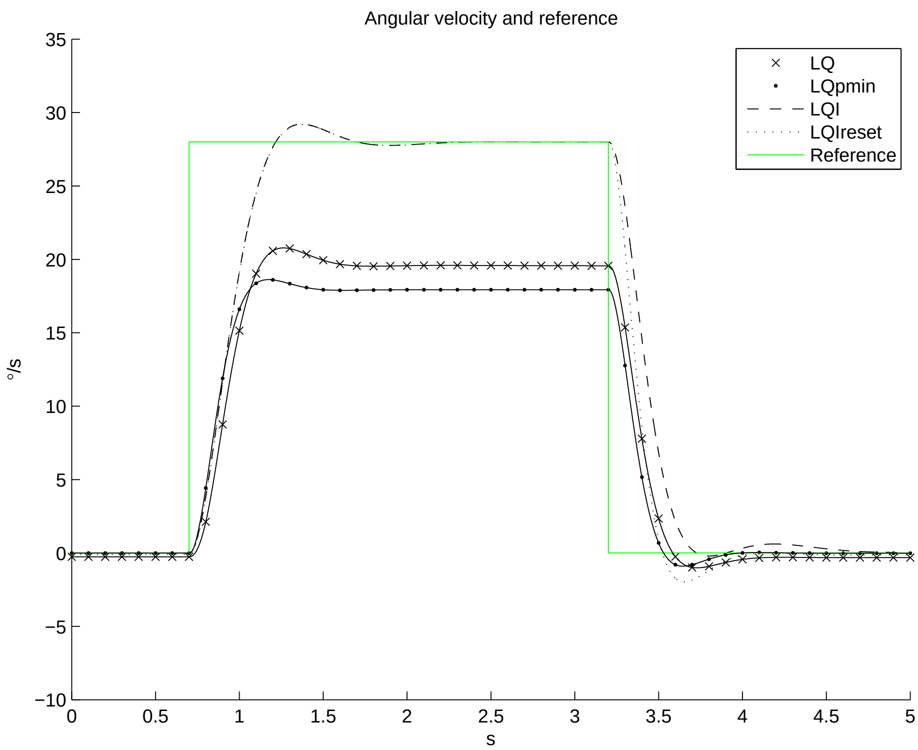

Figure 7.3. Simulation: Step responses in angular velocity.

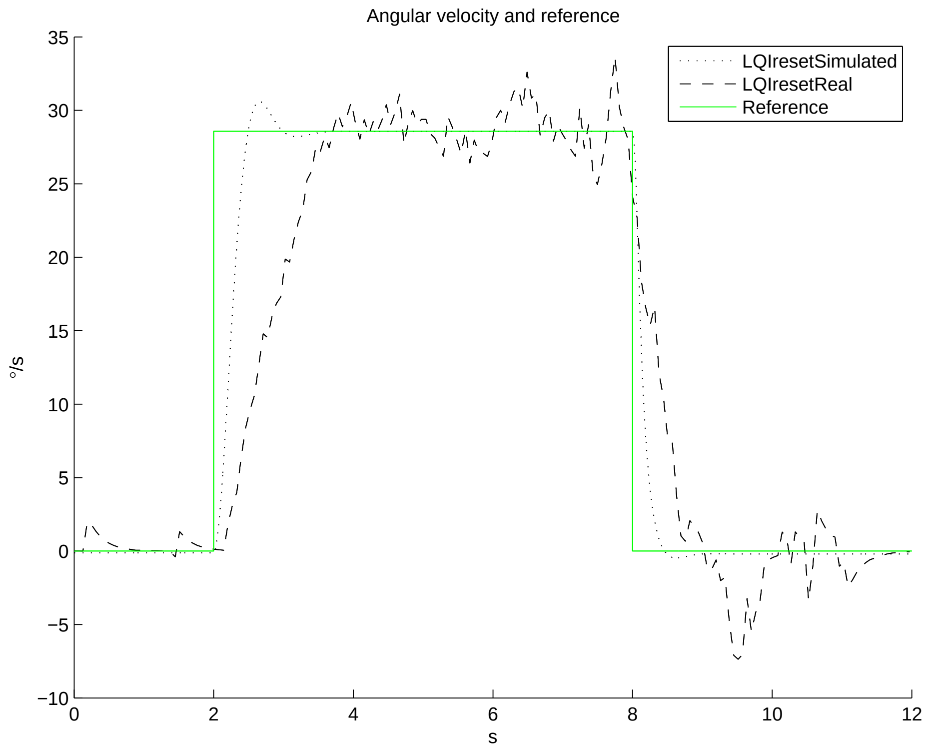

Figure 7.4. Reality: LQI-reset step response

inertia, performs really well for the entire region.

7.3 Mass estimation

A Recursive Least Square filter, RLS, used for estimating the mass was compared to the EKF with a dummy state representing the mass. It can be seen in Figure 7.6 that both of the filters perform quite good after they have got time to converge from the initial settings. The EKF is stable throughout the entire simulation while the RLS becomes unreliable after about 40 seconds.

7.4 Controller for non-fixed inertia

The performance of a LQIreset controller, a Linear Parameter Varying (LPV) controller and a Gain Scheduled LQIreset controller was tested. From simulations, with known inertia (see Figure 7.7) it can be seen that

- The LQIreset controller is a bit slow for higher inertias than designed for, optimal for the inertia it is designed for, and too powerful for a decreased inertia. It becomes unstable when reaching minimum inertia.

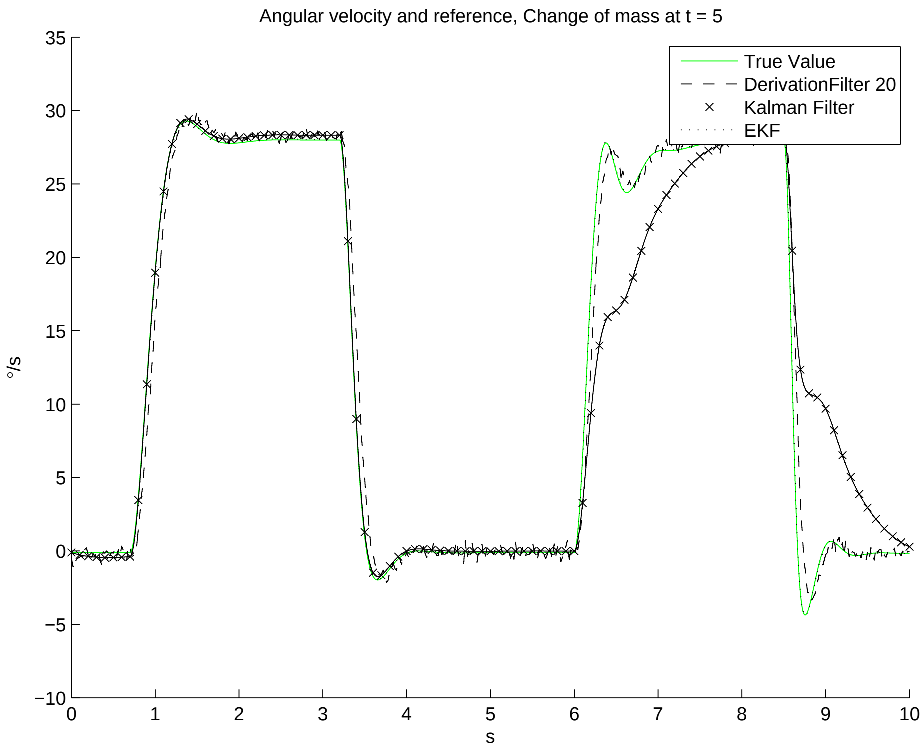

Figure 7.5. Experimental: Noise reduction

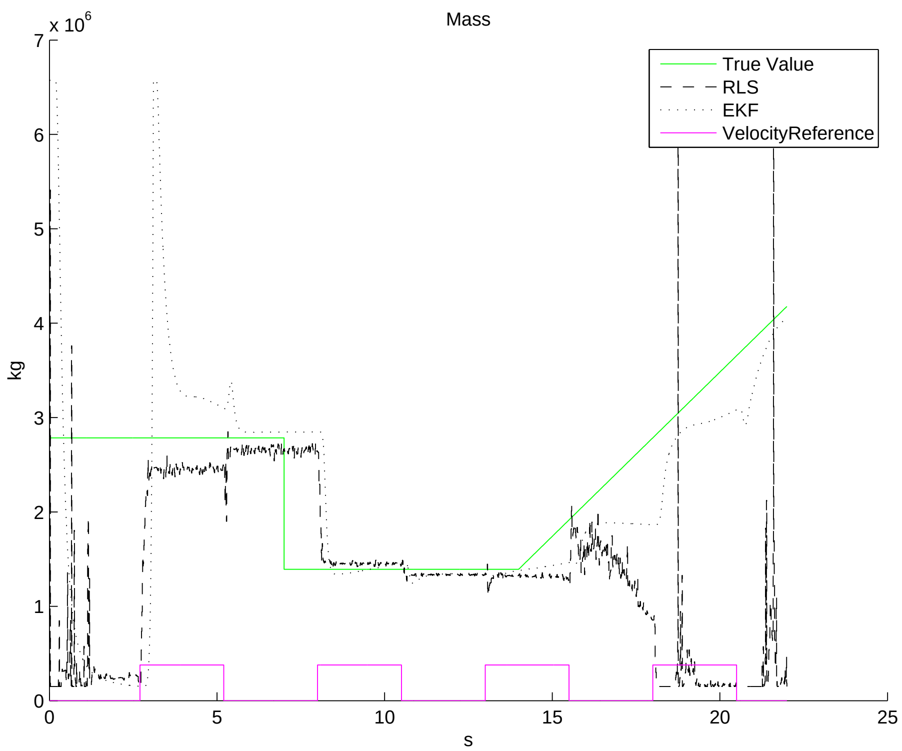

Figure 7.6. Simulation: Estimation of the inertia.

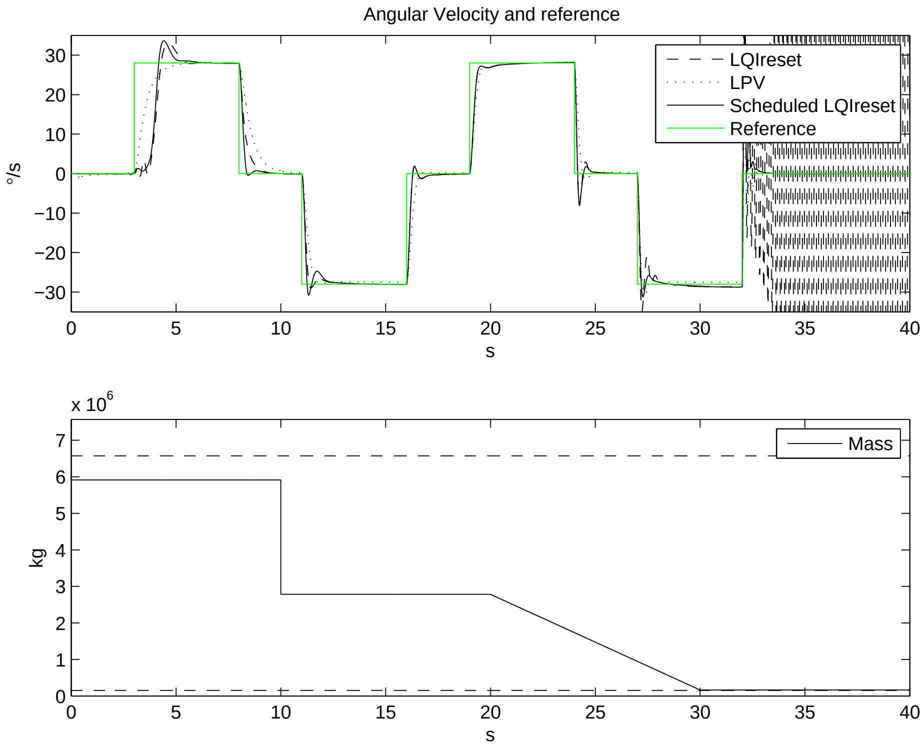

Figure 7.7. Simulation: Angular velocity, Controller Comparison between LQR, LPV and scheduled LQR.

- The LPV controller is stable for the entire region but slightly slow.
- The Gain Scheduled LQIreset controller is stable and performs well for the entire region.

From experimental results where the estimation of the mass is used (see Figure 7.8) it is hard to decide which controller has the best response to set point changes. Damping of the system is a little faster for the Gains Scheduled LQIreset controller than for the others. It is, though, clear that all the controllers damp the system much better than the original system L90, and that the response is not affected.

7.5 Test driver opinion

The test driver tested the LPV, gain scheduled LQIreset and the L90 system. His opinion was that with the controlled systems the machine behaved more harmonically and a lot less shaky and better damped than with the L90 system. He also believe that if the driver would use it for a longer time the amount of lever shifts could become less than for the L90 system. The change in response to the driver is not remarkably decreased. The gain scheduled solution is a bit faster than the LPV solution, while the LPV solution behaves more harmonically.

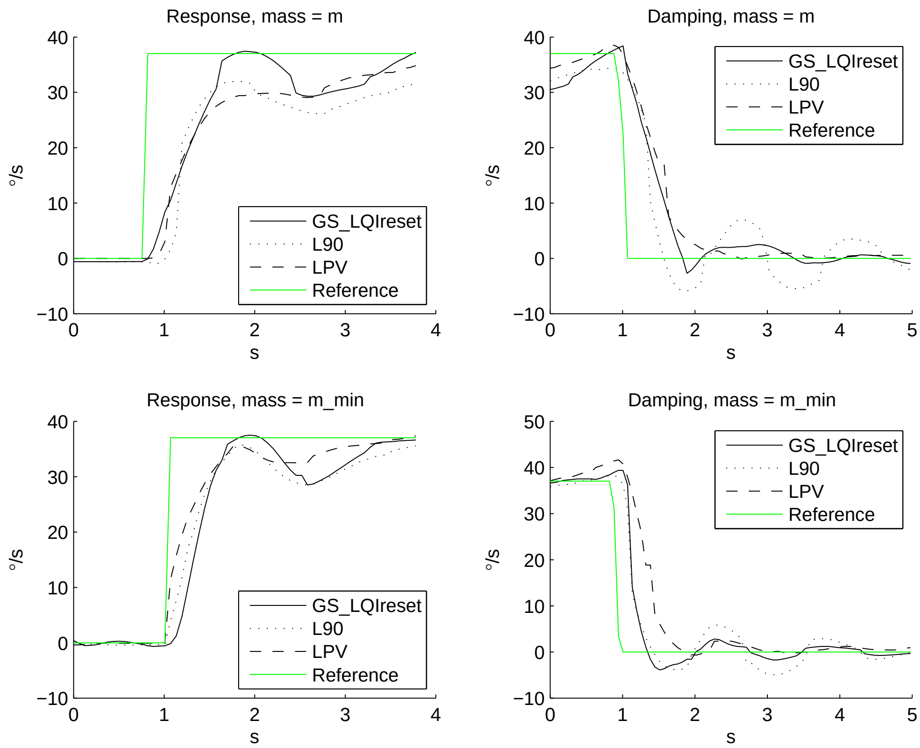

Figure 7.8. Experimental: Controller comparison of LPV, scheduled LQR and original system L90.

8 CONCLUSIONS

The project was carried out by applying control theories in a step by step way depending on the results and problems which occurred. Most of the problems were expected, like that differentiation of a noisy signal amplifies the noise, a steady-state error occurs when no integral action is implemented and that the LQ controller is only optimal around its operating point.

8.1 Compared to initial expectations

The final designed and implemented systems, LPV and Gain Scheduled LQIreset, has in many aspects reached the expectations which were stated from the beginning. It demonstrates that it is possible to, design and implement a controller for the crane which improves the accuracy of the system, especially concerning the damping of the system. It is possible to tune the controller in a Simulink environment and then implement it into the application for experimental results.

8.2 Tuning and weight selection

The tuning of the controller is based on the results from simulations of the nonlinear plant. Matrices are tuned in such a way that the constraints of the system are not violated. The tuning of the Extended Kalman Filter was based on data from experimental results. The covariance matrix for the measurement was estimated from data collected when the system was in a steady-state position. The process noise matrix was tuned based on simulation results and then verified to give satisfactory experimental results. Since the process noise matrix is not analytically derived by this may be one of the most important parts to tune to get a better overall performance of the closed loop system.

8.3 Challenges

The biggest issue considering advanced control theory has been to get a good estimation of the mass. Both the scheduled controller and the estimation of the velocity, depend on the estimation of the mass. A bad estimation will result in a non-optimal controller trying to control a badly estimated state. By implementing the mass estimation into the Extended Kalman Filter it became easier to tune the filter than by tuning an RLS solution and a Kalman filter solution separately. The RLS solution would probably work if more time were spent on tuning and testing.

8.4 Further improvements

The quality of the closed loop system response highly depends on the quality of the estimated states, of the machine. By improving the model and by putting more effort into the tuning of the Extended Kalman Filter, the accuracy could be further improved.

Before implementing such a control system into a real project, testing and verification has to be done to ensure the robustness of the system in an environment where the machine is used by customers. The controller designed in this project is for a crane standing on a flat ground. By adding an additional dimension to the model, where the horizontally angular position of the machine is nonzero, the model complexity would increase but then would the controller made be more robust compared to the controllers designed in this project.

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A Linearized Model

The linear system is described as

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}\\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u}\end{aligned}$$

and the states are

$$\begin{aligned}x_1 &= p_1\\ x_2 &= p_2\\ x_3 &= x\\ x_4 &= v\\ u_1 &= A_{v1}\\ u_2 &= A_{v2}\end{aligned}$$

The A, B, C and D matrices are

$$\boldsymbol{A} = \begin{pmatrix} A11 & A12 & A13 & A14\\ A21 & A22 & A23 & A24\\ A31 & A32 & A33 & A34\\ A41 & A42 & A43 & A44 \end{pmatrix}$$

$$\boldsymbol{B} = \begin{pmatrix} B11 & B12\\ B21 & B22\\ B31 & B32\\ B41 & B42 \end{pmatrix}$$

$$\boldsymbol{C} = \begin{pmatrix} 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{D} = 0$$

where the matrices are

$$
\begin{aligned}
A11 &= 0 \\
A12 &= 0 \\
A13 &= \frac{-\beta A_1(ku_{10} - A_1 x_{40})}{(A_1 x_{30} + V_{10})^2} \\
A14 &= \frac{-\beta A_1}{A_1 x_{30} + V_{10}} \\
A21 &= 0 \\
A22 &= 0 \\
A23 &= \frac{\beta A_2(ku_{20} + A_2 x_{40})}{((x_{max} - x_{30})A_2 + V_{20})^2} \\
A24 &= \frac{\beta A_2}{(x_{max} - x_{30})A_2 + V_{20}} \\
A31 &= 0 \\
A32 &= 0 \\
A33 &= 0 \\
A34 &= 1 \\
A41 &= \frac{A_1}{m} \\
A42 &= -\frac{A_2}{m} \\
A43 &= 0 \\
A44 &= -\frac{B_p}{m} \\
B11 &= \frac{\beta k}{A_1 x_{30} + V_{10}} \\
B12 &= 0 \\
B21 &= 0 \\
B22 &= \frac{\beta k}{(x_{max} - x_{30})A_2 + V_{20}} \\
B31 &= 0 \\
B32 &= 0 \\
B41 &= 0 \\
B42 &= 0
\end{aligned}
\tag{A.1}
$$

B Simulink Models

This appendix presents the simulink models for the different control solutions

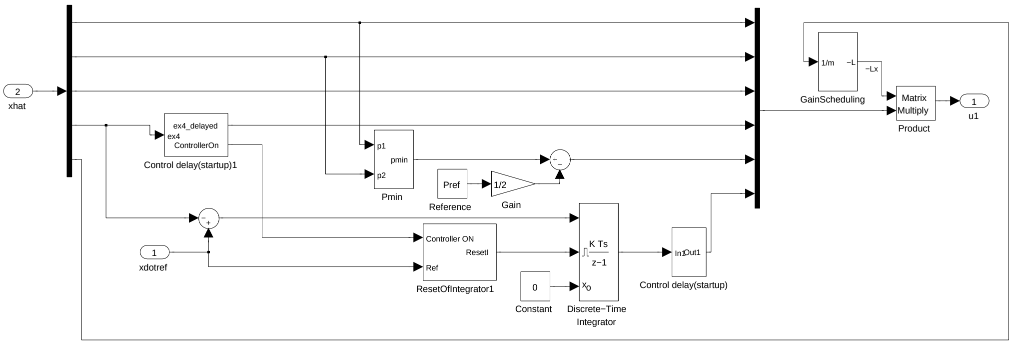

Figure B.1. ControllerGSLQIreset

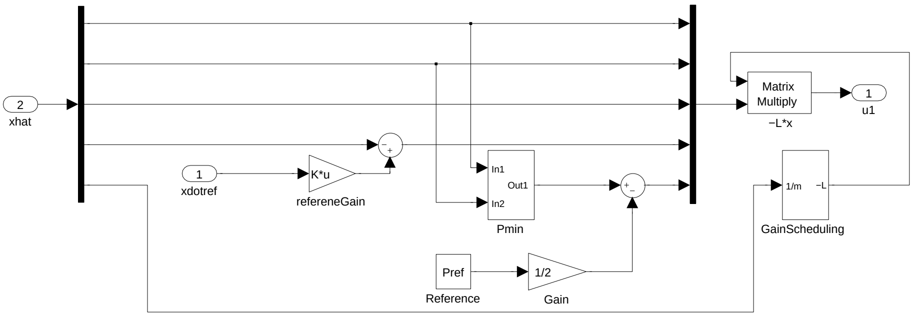

Figure B.2. ControllerLPV

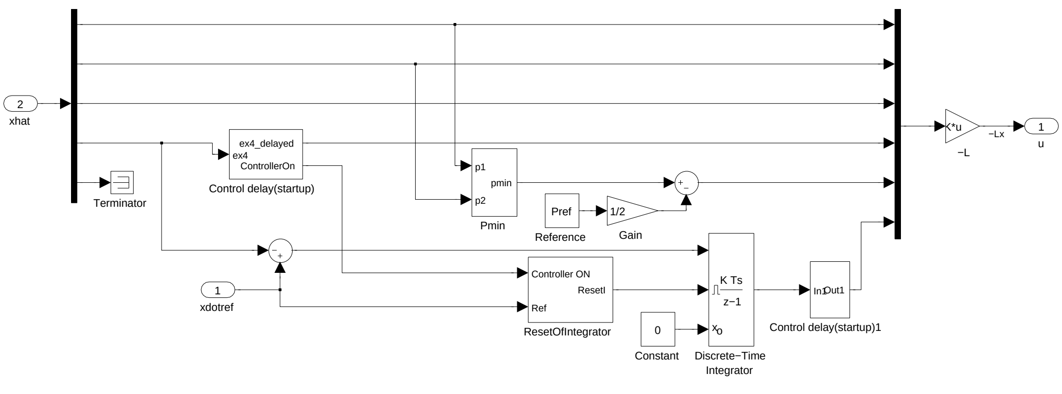

Figure B.3. ControllerLQIreset

C Connection scheme