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***Abstract*—The delay constraints imposed by future wireless applications require a suitable metric for assessing their impact on the overall system performance. Since the classical Shannon's ergodic capacity fails to do so, the so-called effective rate was recently established as a rigorous alternative. Yet, most prior relevant works have considered only the typical case of Rayleigh fading which allows for tractable manipulations. In this paper, we relax this assumption by considering the more general Rician fading model for multiple-input single-output (MISO) systems. A new, analytical expression for the exact effective rate is derived, along with tractable expressions for the key parameters dictating the effective rate performance in the high and low signal-to-noise (SNR) regimes.**

I. Introduction

It is well established that the most important emerging applications (e.g. voice over IP (VoIP), interactive and multimedia streaming, interactive gaming, mobile TV and computing) impose stringent quality of service (QoS) constraints; such constraints typically appear in the form of constraints on queuing delays or queue lengths. As such, a QoS metric that is able to capture the delay constraints of communication systems becomes of vital importance. Unfortunately, the conventional notion of Shannon capacity cannot account for the delay aspect. For this reason, it was suggested in [1] to use the effective capacity (or effective rate) as an appropriate metric to quantify the system performance under QoS limitations, such as data rate, delay and delay-violation probability.

The area of multiple-antenna delay-sensitive systems has attracted significant research interest. In this context, [2] investigated the effective capacity of Gaussian quasi-static block-fading multiple-input multiple-output (MIMO) systems with independent and identically distributed (i.i.d.) Rayleigh fading. Moreover, [3] derived the optimal precoding scheme with covariance feedback for correlated MISO systems. Recently, [4] examined in detail the MIMO effective capacity in the high- and low-SNR regimes and demonstrated the interactions between the queuing constraints and spatial dimensions over a wide range of SNR values. Finally, [5] considered the effective capacity of MISO systems by taking into account the effects of spatial correlation. By doing so, it was theoretically shown, using principles of majorization theory, that correlation always reduces effective capacity.

The common characteristic of the above mentioned works [2]–[5], however, is that they adopt the assumption of Rayleigh fading. Although the assumption of Rayleigh fading simplifies extensively the performance analysis of multiple-antenna systems, its validity is often violated in practical propagation scenarios [6]. In practice, the presence of a line-of-sight or specular component is highly likely, in which case the channel statistics can be more effectively modeled by the Rician distribution. To the best of the authors' knowledge, no prior work has investigated the effective rate performance of MISO Rician fading channels.[1]

In the following, we derive a new, analytical expression for the exact effective rate of MISO Rician fading channels. In order to get some additional insights into the impact of system parameters, such as delay constraints, fading parameters and number of antennas, we consider the asymptotically high- and low-SNR regimes. For example, in the latter case we investigate the notions of minimum normalized energy per information bit to reliably convey any positive rate and wideband slope. For these metrics, new tractable expressions are deduced that extend and complement previous results on Rayleigh fading channels.

Notation: We use upper and lower case boldface to denote matrices and vectors, respectively. The symbol $(\cdot)^\dagger$ represents the Hermitian transpose, while $\mathrm{tr}(\cdot)$ yields the matrix trace. The expectation of a random variable is denoted as $\mathtt{E}\{\cdot\}$ and $\Pr(\cdot)$ represents probability.

II. System Model

We consider a MISO system with N_t transmit antennas whose complex input-output relationship can be expressed as

$$y = \mathbf{h}\mathbf{x} + n \tag{1}$$

where $\mathbf{h} \in \mathbb{C}^{1\times N_t}$ represents the MISO channel fading vector, while $\mathbf{x} \in \mathbb{C}^{N_t\times 1}$ and n denote the transmitted vector and the complex additive white Gaussian noise (AWGN) term with zero-mean and variance N_0, respectively. According to [1], the effective capacity is defined as the maximum constant arrival rate that a given service process can support in order to guarantee a statistical QoS requirement, specified by the QoS exponent θ. Assuming block fading channels, the effective

[1]Note that in an extended journal version of this paper [7], the cases of Nakagami-m and generalized-$\mathcal{K}$ fading channels are also investigated.

capacity can be obtained as [2]

$$a(\theta) = -\frac{1}{\theta T} \ln \left\{ \mathtt{E} \left\{ \exp\left(-\theta T C\right) \right\} \right\}, \; \theta \neq 0 \tag{2}$$

where T is the block-length, C is the transmission rate which is a random variable (RV), and the expectation is taken over C. It is noted that the parameter θ determines to so-called asymptotic decay-rate of the buffer occupancy and is given by

$$\theta = -\lim_{x\to\infty} \frac{\ln \Pr[L > x]}{x} \tag{3}$$

where L is the equilibrium queue-length of the buffer at the transmitter [1]. Then, assuming that the transmitter sends uncorrelated circularly symmetric zero-mean complex Gaussian signals and uniform power allocation across the transmit antennas, the effective rate can be succinctly expressed as follows

$$\mathcal{R}(\rho,\theta) = -\frac{1}{A}\log_2\left(\mathtt{E}\left\{\left(1+\frac{\rho}{N_t}\mathbf{h}\mathbf{h}^\dagger\right)^{-A}\right\}\right) \quad \text{bits/s/Hz} \tag{4}$$

where $A \triangleq \theta T B/\ln 2$, with B denoting the bandwidth of the system, while ρ is the average transmit SNR.

III. Effective Rate Analysis

In this section, we present a detailed effective rate analysis of the Rician fading channel model. Under these circumstances, the entries of the channel vector $\mathbf{h}$ are assumed to be i.i.d. Rician RVs with parameters K and Ω, where K represents the Rician K-factor and Ω the average fading power. The probability density function of $x = |h_k|^2$ $(k = 1,\ldots,N_t)$ is given by [6, Eq. (2.16)]

$$\begin{aligned} p(x) = {} & \frac{(1+K)e^{-K}}{\Omega}\exp\left(-\frac{(K+1)x}{\Omega}\right) \\ & \times I_0\left(2\sqrt{\frac{K(K+1)x}{\Omega}}\right), \quad \Omega \geq 0 \end{aligned} \tag{5}$$

where $I_\nu(x)$ is the ν-th order modified Bessel function of the first kind [8, Eq. (8.405.1)].

1) Exact analysis: We first obtain the exact $\mathcal{R}(\rho,\theta)$ as follows:

Proposition 1: For Rician fading, the effective rate of MISO channels is equal to (6), given at the bottom of the page.

Proof: The proof relies on the properties of non-central chi-square distributions. For the case under consideration, we can use the following expression for the density of the sum of N_t squared i.i.d. Rician RVs, $z = \sum_{k=1}^{N_t} |h_k|^2$ [9, Eq. (5)],

$$\begin{aligned} p(z) = {} & \frac{(K+1)e^{-KN_t}}{\Omega}\left(\frac{(K+1)z}{KN_t\Omega}\right)^{\frac{N_t-1}{2}}\exp\left(-\frac{(K+1)z}{\Omega}\right) \\ & \times I_{N_t-1}\left(2\sqrt{\frac{K(K+1)N_t z}{\Omega}}\right). \end{aligned} \tag{7}$$

Substituting (7) into (4) and thereafter using the infinite series representation of $I_0(\cdot)$ from [8, Eq. (8.445.1)], we can obtain the desired result after invoking [8, Eq. (3.383.5)]. ■

In order to evaluate (6) we need to truncate the infinite series. We therefore seek to obtain the truncation error which also demonstrates the series' convergence. Assuming that $T_0 - 1$ terms are used, the associated truncation error E_0 can be expressed as

$$E_0 = \sum_{n=T_0}^{\infty} \frac{(KN_t)^n}{\Gamma(n+1)} U\left(A; A+1-N_t-n; \frac{(K+1)N_t}{\Omega\rho}\right) \tag{8}$$

$$< U\left(A; A+1-N_t-T_0; \frac{(K+1)N_t}{\Omega\rho}\right)\sum_{n=T_0}^{\infty}\frac{(KN_t)^n}{\Gamma(n+1)} \tag{9}$$

$$\begin{aligned} = {} & U\left(A; A+1-N_t-T_0; \frac{(K+1)N_t}{\Omega\rho}\right)\exp\left(KN_t\right) \\ & \times \left(1 - \frac{\Gamma(T_0, KN_t)}{\Gamma(T_0)}\right) \end{aligned} \tag{10}$$

where $\Gamma(x) = \int_0^\infty t^{x-1}\exp(-t)dt$ represents the Gamma function [8, Eq. (8.310.1)] and $\Gamma(p,x) = \int_x^\infty t^{p-1}e^{-t}dt$ the upper incomplete gamma function [8, Eq. (8.350.2)]. Note that from (8) to (9) we have used the fact that $U(a, b-n, z)$ is a monotonically decreasing function in n, while (10) is a result of [10, Eq. (6.5.29)].

2) High-SNR analysis: The presence of a Tricomi function in the effective rate expression (6) does not allow straightforward algebraic manipulations. Yet, by considering the initial expression (4) and keeping only the dominant term therein as $\rho \to \infty$, we can obtain the following tractable result.

Proposition 2: For Rician fading, the effective rate of MISO channels at high SNRs and for $A < N_t$ is approximated by

$$\begin{aligned} \mathcal{R}^\infty(\rho,\theta) \approx {} & \log_2\left(\frac{\rho\Omega}{(K+1)N_t}\right) + \frac{KN_t}{A\ln 2} \\ & - \frac{1}{A}\log_2\left(\frac{\Gamma(N_t - A)}{\Gamma(N_t)}\,{}_1F_1\left(N_t - A; N_t; KN_t\right)\right) \end{aligned} \tag{11}$$

where ${}_pF_q(\cdot)$ represents the generalized hypergeometric function with p, q non-negative integers [8, Eq. (9.14.1)].

Proof: By taking ρ large in (4), the proof boils down

$$\mathcal{R}(\rho,\theta) = \log_2\left(\frac{\Omega\rho}{(K+1)N_t}\right) + \frac{KN_t}{A\ln 2} - \frac{1}{A}\log_2\left(\sum_{n=0}^{\infty}\frac{(KN_t)^n}{\Gamma(n+1)}U\left(A; A+1-N_t-n; \frac{(K+1)N_t}{\Omega\rho}\right)\right). \tag{6}$$

to the computation of the A-th negative moment of z, $\mathtt{E}\{z^{-A}\}$. As a next step, we express the Bessel function in (7) via a hypergeometric function according to [11, Eq. (03.02.26.0002.01)]

$$I_\nu(x) = \frac{1}{\Gamma(\nu+1)}\left(\frac{x}{2}\right)^\nu {}_0F_1\left(\nu+1; \frac{x^2}{4}\right). \tag{12}$$

Combining (12) with (7), we can obtain the desired result by invoking the following integral identity [8, Eq. (7.522.5)]

$$\int_0^\infty e^{-x}x^{\nu-1}{}_pF_q\left(a_1,\dots,a_p;b_1,\dots,b_q;\alpha x\right) = \Gamma(\nu){}_{p+1}F_q\left(\nu,a_1,\dots,a_p;b_1,\dots,b_q;\alpha\right) \tag{13}$$

for $p < q$ and $\mathrm{Re}(\nu) > 0$, and simplifying. Note that the condition on the arguments of (13) is satisfied in our setting by taking $A < N_t$. ∎

The above result indicates that the high-SNR slope is 1 when $A < N_t$, which is consistent with the results of [5]. From Proposition 2, it can be also shown that the high-SNR effective rate is a monotonically increasing function in the Rician K-factor. This is anticipated, since larger values of K reduce the signal's envelope fluctuations, making fading manifestations more deterministic. We finally note that the effective rate grows logarithmically with the SNR, when $\rho \to \infty$.

In Fig. 1, the analytical expression for the effective rate in (6) is plotted (for $T_0 = 90$ terms) against the outputs of a Monte-Carlo simulator and the high-SNR approximation of Proposition 2.

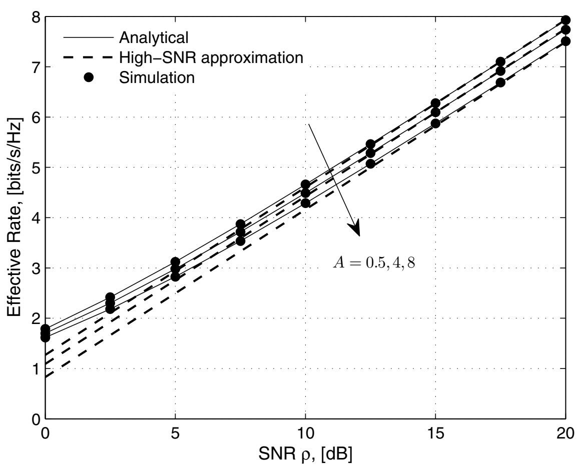

Fig. 1. Simulated effective rate, analytical expression and high-SNR approximation against the SNR ($N_t = 10, \Omega = 2.5, K = 3$ dB).

The match between theory and simulation is excellent in all cases under consideration. More importantly, the effective rate is systematically reduced as the QoS requirements become more stringent, i.e., A gets larger. This is consistent with the results reported in [2], [4], [5]. In addition, the high-SNR approximations are sufficiently tight and become exact even at moderate SNR values (e.g., around 15 dB). Note that for $A < N_t$, we can not increase the high-SNR slope by increasing N_t. Yet, a larger N_t will effectively reduce the power offset, thereby yielding higher effective rate [7].

3) Low-SNR analysis: Following the generic methodology of [4], we can assess the low-SNR performance via a second-order expansion of the effective rate around $\rho \to 0^+$ according to

$$\mathcal{R}(\rho,\theta) = \dot{\mathcal{R}}(0,\theta)\rho + \ddot{\mathcal{R}}(0,\theta)\frac{\rho^2}{2} + o(\rho^2) \tag{14}$$

where $\dot{\mathcal{R}}(\rho,\theta)$ and $\ddot{\mathcal{R}}(\rho,\theta)$ denote the first and second order derivatives of the effective rate (4) with respect to ρ. We point out that these derivative expressions are inherently related with the notions of the *minimum normalized energy per information bit to reliably convey any positive rate* and the *wideband slope* respectively, originally proposed in [12]. For the case of QoS constraints, the latter two metrics are respectively defined as,

$$\frac{E_b}{N_0}_{\min} \triangleq \frac{1}{\dot{\mathcal{R}}(0,\theta)}, \quad \mathcal{S}_0 \triangleq -\frac{2\ln 2\left[\dot{\mathcal{R}}(0,\theta)\right]^2}{\ddot{\mathcal{R}}(0,\theta)}. \tag{15}$$

Proposition 3: For Rician fading, the minimum $\frac{E_b}{N_0}$ and wideband slope $\mathcal{S}_0$ are respectively given by

$$\frac{E_b}{N_0}_{\min} = \frac{\ln 2}{\Omega} \tag{16}$$

$$\mathcal{S}_0 = \frac{2N_t(K+1)^2}{N_t(K+1)^2 + (A+1)(2K+1)}. \tag{17}$$

Proof: Omitting explicit details and following a similar line of reasoning as in [5, Appendix I], the first and second-order derivatives in (15) are given by

$$\dot{\mathcal{R}}(0,\theta) = \frac{1}{N_t \ln 2}\mathtt{E}\left\{\mathbf{h}\mathbf{h}^\dagger\right\} \tag{18}$$

$$\ddot{\mathcal{R}}(0,\theta) = -\frac{A+1}{N_t^2\ln 2}\mathtt{E}\left\{\left(\mathbf{h}\mathbf{h}^\dagger\right)^2\right\} + \frac{A}{N_t^2\ln 2}\left(\mathtt{E}\left\{\mathbf{h}\mathbf{h}^\dagger\right\}\right)^2. \tag{19}$$

Recalling that $\mathtt{E}\{|h_k|^2\} = \Omega, \forall k = 1,\dots,N_t$, we can easily infer that $\mathtt{E}\left\{\mathbf{h}\mathbf{h}^\dagger\right\} = N_t\Omega$. The fourth moment of $|h_k|$ can now be evaluated according to [6, Eq. (2.18)]

$$\mathtt{E}\{|h_k|^4\} = \frac{(2+4K+K^2)\Omega^2}{(K+1)^2}. \tag{20}$$

As a next step, (20) can be used in the following way:

$$\begin{aligned}\mathtt{E}\left\{\left(\mathbf{h}\mathbf{h}^\dagger\right)^2\right\} &= \mathtt{E}\left\{\left(\sum_{k=1}^{N_t}|h_k|^2\right)^2\right\} \\ &= \sum_{k=1}^{N_t}\mathtt{E}\left\{|h_k|^4\right\} + \sum_{k=1}^{N_t}\sum_{j=1,j\neq k}^{N_t}\mathtt{E}\left\{|h_k|^2|h_j|^2\right\} \qquad (21)\\ &\overset{(20)}{=} \frac{N_t\Omega^2}{(K+1)^2}(2+4K+K^2) + N_t(N_t-1)\Omega^2 \qquad (22)\\ &= N_t\Omega^2\left(N_t + \frac{1}{K+1} + \frac{K}{(K+1)^2}\right). \qquad (23)\end{aligned}$$

From (21) to (22) we have exploited the independence of $|h_k|^2$ and $|h_j|^2$. The proof then follows trivially by invoking (18)–

(19) and simplifying. ■

It can be easily noticed that $\frac{E_b}{N_0}_{\min}$ is independent of the K-factor and delay constraints, while the wideband slope is an increasing function in K, satisfying

$$\frac{2N_t}{A+1+N_t} \leq \mathcal{S}_0 \leq 2. \quad (24)$$

The lower bound in (24) is attained for $K=0$ (i.e. Rayleigh fading), while the upper bound is approached for $K \to \infty$ (i.e. AWGN channel). It is noteworthy that for the case of no delay constraints ($A=0$), the wideband slope reduces to

$$\mathcal{S}_0 = \frac{2N_t(K+1)^2}{(N_t+1)(2K+1)+N_tK^2} \quad (25)$$

which coincides with [13, Eq. (18)]. It is also worth mentioning that S_0 is a monotonically decreasing function in A, since we have that

$$\frac{d\mathcal{S}_0}{dA} = -\frac{2N_t(2K+1)(K+1)^2}{\left((2K+1)(A+1)+N_t(K+1)^2\right)^2} < 0. \quad (26)$$

This validates that strict delay constraints tend to reduce the effective rate.

Figure 2 investigates the low-SNR performance of Rician fading channels. Clearly, the linear approximations remain sufficiently tight across a wide range of SNR values. It is readily seen that a 50% increase in the average fading power Ω reduces the minimum energy per bit by 3 dB. Meanwhile, a higher K-factor leaves $\frac{E_b}{N_0}_{\min}$ unaffected but still increases the effective rate through an enhanced $\mathcal{S}_0$. This increase is more pronounced for smaller values of K. For example, an increase in K from 0 to 1 will increase the wideband slope by $1+(A+1)/(4N_t+3(A+1))$. Note that these results are in line with those originally reported in [13].

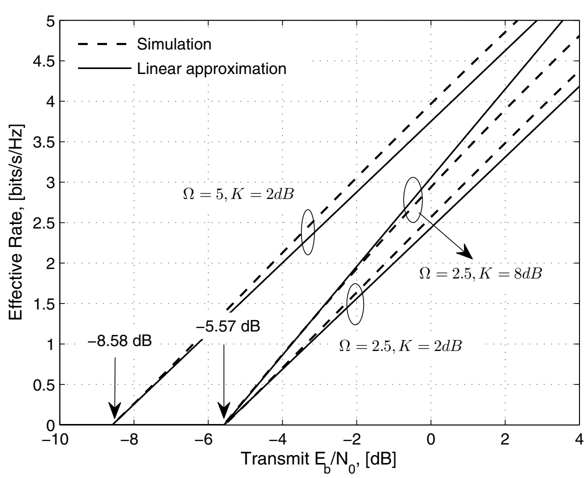

Fig. 2. Low-SNR effective rate and analytical linear approximation against the transmit energy per bit ($N_t = 6, A = 4$).

IV. Conclusion

The majority of future applications, like VoIP and mobile computing, incur strict delay constraints. Yet, the classical notion of Shannon's ergodic capacity cannot account for these constraints. For this reason, the concept of effective rate has been recently proposed, which can efficiently characterize communication systems in terms of data rate, delay and delay-violation probability. However, most studies reported in this context consider the tractable case of Rayleigh fading channels. In this paper, we have extended these previous results to the more general case of Rician fading. In particular, we derived a new analytical expression for the exact effective rate and also investigated the asymptotically high- and low-SNR regimes. In these two limiting cases, simple closed-form expressions were deduced that offer additional physical insights into the implications of several parameters (e.g. fading parameters, number of antennas, delay constraints) on the system performance.

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References

[1] D. Wu and R. Negi, "Effective capacity: A wireless link model for support of quality of service," *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 630–643, July 2003.

[2] L. Liu and J.-F. Chamberland, "On the effective capacities of multiple-antenna Gaussian channels," in *Proc. IEEE Int. Sym. Inf. Theory (ISIT)*, Toronto, ON, July 2008, pp. 2583–2587.

[3] E. A. Jorswieck, R. Mochaourab, and M. Mittelbach, "Effective capacity maximization in multi-antenna channels with covariance feedback," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Dresden, Germany, June 2009.

[4] M. C. Gursoy, "MIMO wireless communications under statistical queueing constrains," *IEEE Trans. Inf. Theory*, vol. 57, no. 9, pp. 5897–5917, Sept. 2011.

[5] C. Zhong, T. Ratnarajah, K.-K. Wong, and M.-S. Alouini, "Effective capacity of correlated MISO channels," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Kyoto, Japan, June 2011.

[6] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd edition. John Wiley & Sons, 2005.

[7] M. Matthaiou, G. C. Alexandropoulos, H. Q. Ngo, and E. G. Larsson, "Analytic framework for the effective rate of MISO fading channels," *in press IEEE Trans. Commun.*, vol. 60, 2012.

[8] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th edition. Academic Press, San Diego, 2007.

[9] J. Hu and N. C. Beaulieu, "Accurate closed-form approximations to Ricean sum distributions and densities," *IEEE Commun. Lett.*, vol. 9, no. 2, pp. 133–135, Feb. 2005.

[10] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th edition. Dover, 1970.

[11] Wolfram, "The Wolfram functions site," [Online], Available: http://functions.wolfram.com, 2012.

[12] S. Verdú, "Spectral efficiency in the wideband regime," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1319–1343, June 2001.

[13] A. Lozano, A. M. Tulino, and S. Verdú, "Multiple-antenna capacity in the low-power regime," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2527–2544, Oct. 2003.