The correct citation for this paper is:

Claesson, J, Javed, S, 2012. A load-aggregation method to calculate extraction temperatures of borehole heat exchangers. *ASHRAE Transactions*, vol. 118(1), pp. 530-539.

CH-12-C069

# A Load-Aggregation Method to Calculate Extraction Temperatures of Borehole Heat Exchangers

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## ABSTRACT

*Hourly simulations of extraction fluid temperatures from borehole heat exchangers tend to be very time consuming. A new load aggregation scheme to perform long-term simulations of borehole heat exchangers is presented. The starting point is the step-response function for the considered borehole heat exchanger and the corresponding long sequence of cells, each with a load and a weighting factor. On the first level, the original weighting factors are kept. On the following levels, 2, 4, 8, etc., the weighting factors are lumped together. The lumped weighting factors are obtained directly from the step-response function. The number of cells to be lumped together is chosen so that the extraction temperatures using lumped weighting factors give a sufficiently good approximation of the non-aggregated scheme. The new scheme is applied to a test case to simulate extraction fluid temperature over a 20-year time period. Comparison of the results from the new scheme with the non-aggregated setting shows that the new scheme can perform very accurate and fast simulations of borehole heat exchangers.*

## BACKGROUND

Modeling and simulation of borehole heat exchangers is a topic of active research in the field of ground source heat pump (GSHP) system applications. The main research interest is conducting multi-year simulations to accurately determine the extraction fluid temperature for a prescribed sequence of heating and cooling loads on a borehole heat exchanger. It is customary to run 20-year or longer simulations to study the effect of long-term heat injections and extraction on the fluid temperature exiting the borehole system. The performance of the heat pump and the overall system depends on the extraction fluid temperature from the borehole heat exchanger. The loads on the borehole heat exchanger depend on the heating and cooling demands of the building. The borehole heat exchanger loads and the heating and cooling demands of the building are both typically presented using annual hourly values. However, hourly simulations of a borehole heat exchanger performed over a number of years require a great deal of computational time. This has lead to the development of various load aggregation schemes to reduce the computational time requirements when performing multi-year simulations of borehole heat exchangers.

Yavuzturk and Spitler (1999) developed an aggregation scheme that lumps the hourly loads on a borehole heat exchanger into larger blocks of time. They used aggregated blocks of 730 hours (i.e., 1 month). Each block uses a single average value to represent the aggregated monthly loads. Yavuzturk and Spitler kept a minimum waiting period of 192 hours for which the loads are not aggregated. Murugappan (2002) later extended the model of Yavuzturk and Spitler to also include sub-hourly loads. Most existing commercial tools (Spitler, 2000; Hellström and Sanner, 1994) to design borehole heat exchangers also use monthly aggregated values of heating and cooling loads. When determining minimum or maximum

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extraction fluid temperatures for a particular month, these tools superimpose the peak loads of that month on the aggregated load values. Bernier proposed two load aggregation schemes for borehole heat exchangers. The first scheme (2001), called the simple load aggregation algorithm (SLAA), divides the borehole heat exchanger loads into two time periods. The loads in the first time period are aggregated and a single mean value for the borehole loads is used. The loads in the second period are not aggregated. Bernier et al. (2004) later revised the SLAA to a more comprehensive Multiple Load Aggregation Algorithm (MLAA). The MLAA categorizes the borehole heat exchanger loads into "immediate" and "past" time periods. The loads in the immediate time period ($X_h$) are not aggregated. The past time period is divided into blocks of daily aggregated ($X_d$), weekly aggregated ($X_w$), monthly aggregated ($X_m$), and yearly aggregated ($X_y$) loads. The duration of periods $X_h$, $X_d$, $X_w$, and $X_m$, as suggested by the authors, is 12, 48, 168, and 360 hours. Liu (2005) presented the so-called hierarchical load aggregation scheme. This scheme uses aggregation blocks at three levels for small, medium, and large time periods. A small block represents the aggregated loads for up to 24 hours. The waiting period for a small block is 12 hours. A medium block consists of 5 small blocks. The waiting period for a medium block is equal to 3 small blocks. A large block is made up of 73 medium blocks. The waiting time for a large block is equal to 40 medium blocks. More recently, Marcotte and Pasquier (2008) used a geometrical scheme for aggregation of borehole loads. The scheme uses a waiting period of 48 hours for which the loads are not aggregated. The remaining loads are aggregated using a geometrical pattern, i.e., loads are aggregated for hours 49–50, 51–54, 55–62, 63–78, 79–110, 111–174, 175–302, 303–558, and so on.

This paper presents a new aggregation scheme to perform rapid and accurate multi-year simulations. The new scheme uses the step-response function for the considered borehole heat exchanger and the corresponding long sequence of loads, each with a weighting factor. The loads are placed in a long sequence of "cells". The original weighting factors are kept without aggregation on the first level. On the following levels, 2, 4, 8, etc., the loads are aggregated. We get lumped loads in lumped or aggregated cells. The lumped weighting factors are obtained directly from the step-response function. The number of lumped cells on level $q$ is $P_q$. The number $P_q$ is chosen so that the extraction temperatures using lumped weighting factors give a sufficiently good approximation of the original full sum. A particular feature of the presented model is that it does not involve any natural periods (year with monthly averages, week, day, etc.) or any pulses to represent peak load conditions. The choice of lumped cells is only determined from the response function and its weighting factors. The method, proposed by Marcotte and Pasquier (2008), corresponds to the choice $P_1=48$ and $P_q=1$ for $q>1$.

## TEMPERATURE RESPONSE FOR A HEAT INJECTION STEP

Let $Q_{step}$ (W or Btu/h) be a constant heat injection rate starting at $t=0$ for a single vertical borehole or a system of multiple vertical boreholes. The required temperature of the heat carrier fluid in the pipes of the boreholes to sustain this injection rate is a basic tool in the analysis of the dynamic relations between heat injection/extraction and fluid temperatures. This step-response temperature $T_{step}(t)$ (K or °F), or so-called *g-function* (Eskilson, 1987), increases monotonously from zero at $t=0$ to a steady-state value at very large times.

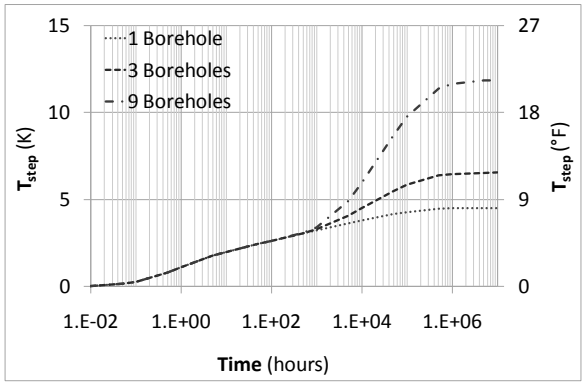


***Figure 1*** *Temperature response $T_{step}(t)$ for a heat injection step for 1, 3, and 9 boreholes.*

The step-response solution for temperature concerns the *excess* temperature above undisturbed ground and the borehole conditions. This means that the initial temperature of the ground and the borehole with the heat carrier fluid is zero for the step-response solution. The annual temperature variation at the ground surface influences the first few meters of the ground. Its influence on the borehole fluid temperatures is quite small. Neglecting this variation, there remains a constant temperature at the ground surface, and the excess temperature at the ground surface is zero for $t \geq 0$. Examples of step-response functions for 1, 3, and 9 boreholes are shown in Figure 1 (Claesson and Javed, 2011).

The steady-state temperature (minus the zero temperature at the ground surface) defines the thermal resistance $R_{ss}$ (K/W or h·°F/Btu) between the heat carrier fluid and the ground surface:

$$T_{\text{step}}(\infty) - 0 = R_{\text{ss}} \cdot Q_{\text{step}}, \qquad R_{\text{ss}} = T_{\text{step}}(\infty) / Q_{\text{step}}. \tag{1}$$

## TEMPERATURE RESPONSE FOR PIECE-WISE CONSTANT INJECTION RATES

In this study, the prescribed heat injection rate is treated as constant during each time step, $n$:

$$Q_{\text{in}}(t) = Q_n, \qquad nh - h < t \leq nh, \qquad n = 1, \ldots n_{\max}; \qquad t_{\max} = n_{\max} h \cdot \tag{2}$$

The value of $Q_{in}(t)$ is negative for heat extraction from the ground. The length of the time step $h$ (seconds or hours) may be chosen at will. The number of pulses $n_{max}$ is very large to cover a calculation period up to, say, $t_{max} = 20$ years.

The fluid temperature, $T_{\text{f}}(nh)$, at the end of pulse $n$ due to the preceding pulses may be obtained by superposition of the solution from each of the preceding pulses $Q_{n+1-v}$, $v=1, \ldots n$.

$$T_{\text{f}}(nh) = \sum_{v=1}^{n} \frac{Q_{n+1-v}}{Q_{\text{step}}} \cdot \left[ T_{\text{step}}(vh) - T_{\text{step}}(vh - h) \right] \cdot \tag{3}$$

Here, $v$ enumerates the pulses backwards in time. By superposition, pulse $v$ may be considered a step that starts at the time $vh$ before $t=nh$ minus a second step that starts at the time $vh-h$ before $t=nh$, as given by the expression within the brackets.

A second notation for the injection values is used:

$$Q_{n+1-v} = Q_v^{(n)} = Q_{\text{in}}\left((n+1-v)h\right); \qquad Q_v^{(n)} = 0, \qquad v > n. \tag{4}$$

The fluid temperature at time step $n$ is given by the sum (3) of the preceding injection rates times a factor that depends on $v$:

$$T_{\text{f}}(nh) = \sum_{v=1}^{n} Q_v^{(n)} \cdot R_v = R_{\text{ss}} \cdot \sum_{v=1}^{n} Q_v^{(n)} \cdot \kappa_v. \tag{5}$$

The thermal resistance factors $R_v$ (K/W or h·°F/Btu) and the dimensionless factors $\kappa_v$ are given by (1):

$$R_v = \frac{T_{\text{step}}(vh) - T_{\text{step}}(vh - h)}{Q_{\text{step}}}, \qquad \kappa_v = \frac{T_{\text{step}}(vh) - T_{\text{step}}(vh - h)}{T_{\text{step}}(\infty)} = \frac{R_v}{R_{\text{ss}}} \tag{6}$$

An advantage of using the thermal resistance factors $R_v$ (K/W or h·°F/Btu) is that the final steady-state value $T_{step}(\infty)$ is not needed. But the dimensionless weighting factors $\kappa_v$ directly give the relative influence of the preceding injection rates. The weighting factor is determined by the increase of the step-response function over the time from $vh-h$ to $vh$ divided by the total increase of $T_{step}(t)$ from zero to infinity. The sum of the weighting factors tends to 1 as $v$ tends to infinity, (16).

The calculations are performed for consecutive time steps. The loads are shifted one step from time $n-1$ to $n$:

$$Q_{v+1}^{(n)} = Q_v^{(n-1)}, \qquad v = 1, \ldots n-1, \qquad Q_1^{(n)} = Q_{\text{in}}(nh). \tag{7}$$

The required number of terms in the summation (5) increases with the number of time steps. After 10 years with $h=1$ hour, a summation of 87,600 preceding values is required. The weighting factors decrease strongly with $v$, but the factors for larger $v$ cannot be neglected since there are so many. For the single borehole of Figure 1, we have ($h=1$ hour):

$$\kappa_1 = 0.25, \quad \kappa_2 = 0.07, \quad \kappa_{24} = 0.003, \quad \kappa_{30\cdot 24} = 8\cdot 10^{-5}, \quad \kappa_{365\cdot 24\cdot 10} = 4\cdot 10^{-7}. \tag{8}$$

The idea of using some kind of aggregated values for preceding loads in suitable time intervals before the considered time lies near at hand.

## LOAD AGGREGATION

The long sequence of loads, or cells with a prescribed load in each cell, is aggregated into larger, lumped cells in the following way. The original cells from $v=1$ to the value $v=P_1$ are kept on the first level $q=1$. Then the cells are doubled to $2h$ for $P_2$ lumped cells. On the third level $q=3$, the cells are again doubled to the width $4h$. This doubling is continued up to the last level $q_{max}$. The number of lumped cells with the width $2^{q-1}$ on level $q$ is $P_q$. The numbers $P_q$ are chosen so that a suitable accuracy is obtained by comparing the sequence of fluid temperatures for the original and lumped-load sequences. The width of lumped cells on level $q$, the very last $v$-value, and the number of lumped cells become:

$$r_q = 2^{q-1}, \quad q = 1,\ldots q_{\max}; \qquad \nu_{\max} = \sum_{q=1}^{q_{\max}} P_q \cdot r_q \geq n_{\max}; \qquad N_{\text{lumped cells}} = \sum_{q=1}^{q_{\max}} P_q. \tag{9}$$

The value $q_{max}$ is chosen so that $v_{max}$ exceeds the required number of original loads, which typically is of the order 200,000. In the example below, we have used $P_q=5$ for all $q$ levels. This choice was made to provide good accuracy. Then $q_{max}$ becomes 16, and the number of lumped cells is $5\cdot 16=80$. The number of loads is reduced from 200,000 to 80. It may be noted that the choice (9), left, i.e., a doubling for each level, is not necessary. All formulas are valid for any choice of the number $r_q$. However, the doubling worked so well that we did not see a reason to test any other choices.

We need to keep track of the $v$-values for each lumped cell $p$ on any $q$-level. Let $v_{q,0}$ denote the very last $v$-value on level $q-1$, and $v_{q,p}$ the last $v$-value in lumped cell $p$ on level $q$. Then we have:

$$\begin{aligned} &\nu_{q+1,0} = \nu_{q,0} + r_q \cdot P_q, \qquad q = 1,\ldots q_{\max} - 1, \qquad \nu_{1,0} = 0, \\ &\nu_{q,p} = \nu_{q,0} + r_q \cdot p, \qquad p = 1,\ldots P_q, \qquad q = 1,\ldots q_{\max}. \end{aligned} \tag{10}$$

The $v$-values from 1 to $v_{max}$ may now be enumerated in the following way:

$$\nu = \nu(q,p,r) = \nu_{q,p} - r, \qquad q = 1,\ldots q_{\max}, \quad p = 1,\ldots P_q, \quad r = 0,\ldots r_q - 1. \tag{11}$$

The sum (5), which gives the fluid temperature at time step $n$, may now be written in the following way:

$$T_{\mathrm{f}}(nh) = R_{\mathrm{ss}} \cdot \sum_{q=1}^{q_{\max}} \sum_{p=1}^{P_q} \sum_{r=0}^{r_q-1} Q_\nu^{(n)} \cdot \kappa_\nu, \qquad \nu = \nu_{q,p} - r, \qquad r = 0,\ldots r_q - 1.. \tag{12}$$

Here, we take the sum up to $v_{max}$, based on the definition of zero loads for $v>n$, (4) right. We will use a suitable average load in each lumped cell $p$ on level $q$:

$$Q_\nu^{(n)} \cong \bar{Q}_{q,p}^{(n)} \quad \text{for} \quad \nu = \nu_{q,p} - r, \qquad r = 0,\ldots r_q - 1. \tag{13}$$

In the aggregated representation of the loads, we get the following approximation:

$$T_{\mathrm{f}}(nh) \cong R_{\mathrm{ss}} \cdot \sum_{q=1}^{q_{\max}} \sum_{p=1}^{P_q} \bar{Q}_{q,p}^{(n)} \cdot \bar{\kappa}_{q,p}. \tag{14}$$

The lumped weighting factor is equal to the sum of the corresponding original weighting factors, (5) right:

$$\bar{\kappa}_{q,p} = \sum_{r=0}^{r_q-1} \kappa_\nu = \left[\nu = \nu_{q,p} - r\right] = \frac{T_{\text{step}}(\nu_{q,p}h) - T_{\text{step}}(\nu_{q,p-1}h)}{T_{\text{step}}(\infty)}. \tag{15}$$

The original weighting factors, (6) right, are inserted in the sum. All intermediate terms cancel and the simple difference above is obtained. The lumped weighting factor is determined by the increase of the step-response function over the "time window" of cell $q, p$ divided by the total increase of $T_{step}(t)$ from zero to infinity.

It should be noted that the sum of all the lumped factors and the sum of all original weighting factors are equal, and given by step-response function over the time span from zero to the last time $\nu_{max}h$. This sum tends to 1 as the number of cells $\nu_{max}$ tends to infinity.

$$\sum_{q=1}^{q_{\max}} \sum_{p=1}^{P_q} \bar{\kappa}_{q,p} = \frac{T_{\text{step}}(\nu_{\max} h) - T_{\text{step}}(0)}{T_{\text{step}}(\infty)} = \sum_{\nu=1}^{\nu_{\max}} \kappa_\nu \to 1, \quad \nu_{\max} h \to \infty. \tag{16}$$

## TIME-SHIFT OF AGGREGATED LOADS

The original sequence of loads is shifted one cell position at each time step $n$, (7). This corresponds to a time displacement $h$. The problem is how to do this time displacement $h$ for the aggregated cells with the width $2h$, $4h$, etc. The immediate answer is to displace the lumped cells the length $h$ and conserve the energy.

This gives the following set of equations to calculate the aggregated loads at step $n$ from the values at step $n-1$:

$$\begin{aligned} &\bar{Q}_{1,0}^{(n)} = Q_{\text{in}}(nh), \qquad q = 2, \ldots q_{\max} : \quad \bar{Q}_{q,0}^{(n)} = \bar{Q}_{q-1,P_q}^{(n-1)}; \\ &q = 1, \ldots q_{\max}, \quad p = 1, \ldots P_q : \quad \bar{Q}_{q,p}^{(n)} = \bar{Q}_{q,p}^{(n-1)} + \frac{1}{r_q} \cdot \left[ \bar{Q}_{q,p-1}^{(n-1)} - \bar{Q}_{q,p}^{(n-1)} \right]. \end{aligned} \tag{17}$$

The shift for the aggregated cell $q,p$ is given on the lower line. One value from cell $p-1$ is shifted into the cell and one value from the cell is shifted out of the cell, as shown within the brackets. This difference divided by the width $r_q$ of the aggregated cell gives the change of the average value in the aggregated cell in the time shift. The first line ensures that the formulas are also valid for $p=1$. The new heat injection at time $n$ is put in cell $1,0$, and the old value in cell $q-1,P_q$ is put into cell $q,0$.

## TIME-SHIFT DISPERSION

Figure 2 illustrates the energy-conserving displacement $h$ for two cells with the width $h$ followed by lumped cells with the width $2h$. All loads are zero, except for a single value +1, which moves one step $h$ to the right for each time step, as shown by the dot. In the third displacement, the left-hand half of the first lumped cell has the value +1 and the right-hand part 0. Energy conservation requires that the lumped cell load is 1/2. In the next displacement, the left-hand part of the first lumped cell is 0 and the right-hand part 1/2, which gives the mean value 1/4. The value +1 of the original pulse is increasingly spread out for each time step.

| Time step *n* | h | h | 2h | 2h | 2h | 2h | 2h |
|---|---|---|---|---|---|---|---|
| 1 | 1 * | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 * | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | * 1/2 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 1/4 * | 1/4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 1/8 | * 2/8 | 1/8 | 0 | 0 |
| 6 | 0 | 0 | 1/16 | 3/16 * | 3/16 | 1/16 | 0 |
| 7 | 0 | 0 | 1/32 | 4/32 | * 6/32 | 4/32 | 1/32 |

***Figure 2*** *Aggregated loads using energy-conserving displacement h for lumped cells with the width 2h causing a time-shift dispersion.*

The same type of "time-shift dispersion" occurs for lumped cells on all higher levels ($q>2$). This error in the representation of the load sequence is rather insidious. Energy is conserved but differences between neighboring loads are smeared out.

A way to analyze this is to note that the problem is due to the mixing of loads to an average within the lumped cells. The remedy is to *minimize* the mixing. This is discussed in some detail in Claesson (2012). See also Appendix A in Wentzel (2005). Consider again the single value +1 that moves one cell to the right at each time step. The mixing of 0 and +1 to 1/2 is unavoidable when +1 is moved into the first lumped call with the width $2h$. In the displacement, when +1 is to be moved to the first cell on level $q=2$, we may instead add an extra cell so that the number of cells on the first level is $P_1+1$. In the following displacement 0 in cell $p=P_1$ and +1 in cell $p=P_1+1$ are added and put as the first value 1/2 on level $q=2$, while the number of cells on level $q=1$ switches back to $P_1$, and the number of cells on level $q=2$ is increased to $P_2+1$. The number of the cell *floats* between two consecutive values. After two more steps, 0+0 is put into the first cell on level $q=2$ and the value 1/2 is displaced to the second cell. This procedure means that the value 1/2 is maintained without further mixing until the lumped value meets the next level $q=3$. The procedure using a floating representation is used on all levels $q$. The effect is that the value +1 becomes 1/2 in the cells on level $q=2$, 1/4 on level $q=3$, etc.

This floating representation is somewhat intricate. A further problem is to get the lumped values (13) from the floating representation. It is actually possible to do this in an exact way, as described in Claesson (2012). The final formulas are surprisingly simple. There is a set of equations of the same complexity as (17) for the relations between the fixed and floating representation of the lumped loads, and another set of equations for the time shifts in the floating representation, essentially on the same level of (computational) complexity.

The above model, using a floating representation of the loads with minimized mixing, has been compared to the exact full sum (5) for the load sequence used below in the comparison of the energy-conservation method presented in this paper. The errors in the latter case are shown in Table 3 for different choices of $P_q$. The errors for the floating representation are between 30 to 70% smaller than those using the energy-conservation method. The differences between the two methods are moderate. One reason for this is probably that the weighting factors vary slowly with $v$ so that the spurious time-shift dispersion does not introduce a large error.

## SUMMARY OF REQUIRED CALCULATIONS

The method presented in this paper requires the step-response function and the heat loads as input:

$$T_{\text{step}}(t), \qquad 0 \le t < \infty \quad (\text{or } \ 0 \le t < t_{\max}); \qquad Q_{\text{in}}(nh), \qquad n = 1, \ldots n_{\max}. \tag{18}$$

The time step $h$ and the magnitude of the injection step, $Q_{step}$, may be chosen at will. The number of cells $P_q$ at all levels $q$, and the number of levels, $q_{max}$, must also be specified. We will use the *same* number of cells on all levels. The following quantities are calculated initially from (1), (9), (10), and (15):

$$R_{\text{ss}}, \qquad r_q, \qquad \nu_{\max}, \qquad \nu_{q,p}, \qquad \bar{\kappa}_{q,p}. \tag{19}$$

For each time step starting from $n=1$, the lumped loads are calculated from (17). The initial values for $n=0$ are zero for all aggregated cells $q,p$. Then, the current fluid temperature is given by the sum (14).

## AN EXAMPLE

The synthetic ground load profile suggested by Pinel (2003) is used to perform a multi-year simulation of a borehole heat exchanger. The synthetic load profile is shown in Figure 3. Multi-year simulation is performed for loads of Figure 3 repeated yearly for the simulation period. The load profile has been used by many researchers, including Bernier et al. (2007), Lamarche and Beauchamp (2007), and Lamarche (2009), when performing GSHP system simulations. Further details of the load profile can be found in the research of Pinel (2003).

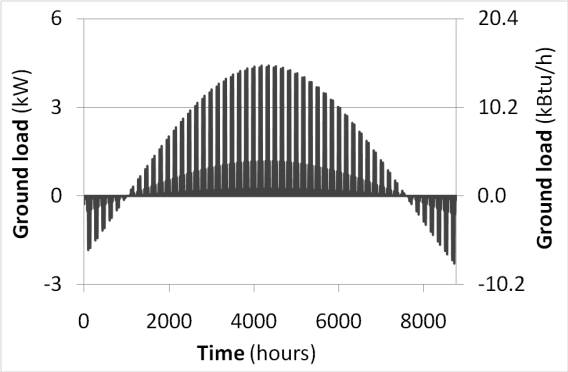


***Figure 3*** *Synthetic load profile of Pinel (2003) used to perform multi-year simulations.*

As a step-response function, we use the case of one borehole in Figure 1. The step-response function was developed by the authors for time scale from minutes to decades. The step-response is obtained by using an analytical radial solution (Javed and Claesson, 2011) for shorter time periods and a finite line-source solution for longer time periods. Further details of the solution can be found elsewhere (Claesson and Javed, 2011). First, we use non-aggregated loads to simulate the fluid temperatures from the step-response function of Figure 1 and the load profile of Figure 3. The computational times (using an Intel® dual core 2.10 Ghz processor) for a direct calculation of the original sum (5) are given in Table 2. Using non-aggregated loads, it takes 14 seconds for a 1-year simulation, 22 minutes for a 10-year simulation, and 88 minutes for a 20-year simulation. The simulated fluid temperatures for the 20th year using non-aggregated loads are shown in Figure 4. The temperatures lie in the range from -4 to +9 °C (25 to 48 °F).

Next, we use the proposed aggregation scheme to simulate the fluid temperatures. We use the same $P_q$ on all levels. For the first case, we take $P_q=5$. The calculations are performed for 20 years with the time step $h=1$ hour. The number of calculation steps becomes $n_{max}=20\cdot365\cdot24=175,200$. Then, we need to take $q_{max}=16$. The number of aggregated cells becomes $5\cdot16=80$. The right-hand limits $v_{q,p}$ of the aggregated cells are calculated from (9) and are given in Table 1, left. In the first line, $q=1$, the first 5 cells are given. In the second line, the right-hand values of the doubled cells, 7 to 15, are shown. The value 5 from the first line is shown for $p=0$. We see on the last line that 16 levels are needed to exceed $n_{max}$.

The lumped weighting factors are calculated from (15) and are shown in Table1, right. The values are quite instructive. The first value, 0.246, means the first cell has 25% of the influence on the extraction temperature. A lumped weighting factor of 0.01 (=10/1000) represents an influence of 1%. It should be kept in mind that the weighting factors are multiplied by the corresponding loads in the sum (14) for the extraction temperature.

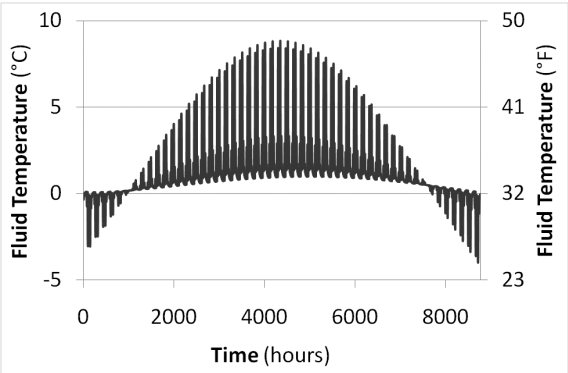


***Figure 4*** *20th year fluid temperatures using non-aggregated loads.*

**Table1. The Limits $v_{q,p}$ of the Aggregated Cells (Left) and the Lumped Weighting Factors $1000 \cdot \kappa_{q,p}$ (Right).**

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **1** | 0 | 1 | 2 | 3 | 4 | 5 |
| **2** | 5 | 7 | 9 | 11 | 13 | 15 |
| **3** | 15 | 19 | 23 | 27 | 31 | 35 |
| **4** | 35 | 43 | 51 | 59 | 67 | 75 |
| **5** | 75 | 91 | 107 | 123 | 139 | 155 |
| **6** | 155 | 187 | 219 | 251 | 283 | 315 |
| **7** | 315 | 379 | 443 | 507 | 571 | 635 |
| **8** | 635 | 763 | 891 | 1019 | 1147 | 1275 |
| **9** | 1275 | 1531 | 1787 | 2043 | 2299 | 2555 |
| **10** | 2555 | 3067 | 3579 | 4091 | 4603 | 5115 |
| **11** | 5115 | 6139 | 7163 | 8187 | 9211 | 10235 |
| **12** | 10235 | 12283 | 14331 | 16379 | 18427 | 20475 |
| **13** | 20475 | 24571 | 28667 | 32763 | 36859 | 40955 |
| **14** | 40955 | 49147 | 57339 | 65531 | 73723 | 81915 |
| **15** | 81915 | 98299 | 114683 | 131067 | 147451 | 163835 |
| **16** | 163835 | 196603 | 229371 | 262139 | 294907 | 327675 |

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **1** | 245.7 | 68.1 | 35.7 | 23.3 | 17.1 |
| **2** | 24.4 | 17.4 | 13.5 | 11.0 | 9.3 |
| **3** | 15.1 | 12.0 | 10.0 | 8.6 | 7.5 |
| **4** | 12.6 | 10.4 | 8.8 | 7.7 | 6.8 |
| **5** | 11.6 | 9.6 | 8.1 | 7.1 | 6.3 |
| **6** | 10.9 | 9.2 | 7.9 | 6.9 | 6.2 |
| **7** | 10.7 | 9.0 | 7.8 | 6.8 | 6.1 |
| **8** | 10.5 | 8.9 | 7.7 | 6.7 | 6.0 |
| **9** | 10.4 | 8.7 | 7.5 | 6.6 | 5.9 |
| **10** | 10.2 | 8.5 | 7.4 | 6.5 | 5.7 |
| **11** | 9.9 | 8.3 | 7.1 | 6.2 | 5.5 |
| **12** | 9.4 | 7.9 | 6.7 | 5.9 | 5.2 |
| **13** | 8.8 | 7.3 | 6.2 | 5.4 | 4.7 |
| **14** | 8.0 | 6.5 | 5.5 | 4.7 | 4.1 |
| **15** | 6.8 | 5.4 | 4.5 | 3.8 | 3.2 |
| **16** | 5.2 | 4.0 | 3.2 | 2.6 | 2.1 |

The computational times for the new aggregation scheme are given in Table 2 for simulation times from 1 to 20 years. These times depend on the computer, but the relative times of aggregated and non-aggregated schemes are what are of interest. The sum (5) for the non-aggregated scheme is calculated for $n$ from 1 to $n_{max}$, which means the number of operations increases as $(n_{max})^2$. This is in good agreement with the times in the right-hand column for the non-aggregated scheme. The number of operations in the aggregated scheme are proportional to $q_{max} \cdot P_q \cdot n_{max}$. The saving of calculation time becomes proportional to $n_{max}$. For a calculation period of 20 years, the aggregated scheme is 200 times faster.

**Table 2. Computational Times for the New Scheme ($P_q$=5) and for Non-aggregated Loads.**

| Simulation Time | **Computational Time,** seconds (minutes) | |
|---|---|---|
| | **New Aggregation Scheme** | **Non-aggregated Loads** |
| 1 year | 3 | 14 (< 1) |
| 2 years | 4 | 59 (1) |
| 3 years | 5 | 131 (2.2) |
| 5 years | 7 | 330 (5.5) |
| 10 years | 14 | 1321 (22) |
| 20 years | 25 | 5289 (88) |

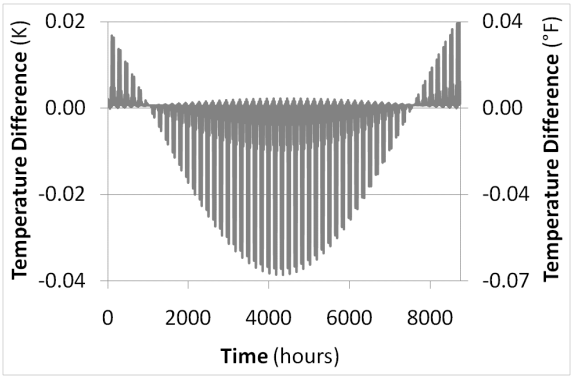


***Figure 5*** *Temperature differences in simulated fluid temperatures for the 20th year from the new aggregation scheme ($P_q$=5) and the non-aggregated scheme.*

**Table 3. Maximum Absolute Error in Fluid Temperatures Simulated for 20 Years Using the New Scheme and Non-aggregated Loads for Different Values of $P_q$**

| $P_q$ | **Maximum Absolute Error,** K (°F) |
|---|---|
| 1 | 0.300 (0.540) |
| 2 | 0.132 (0.238) |
| 3 | 0.077 (0.139) |
| 5 | 0.039 (0.070) |
| 10 | 0.018 (0.032) |
| 20 | 0.006 (0.011) |

The errors in using the aggregated scheme are given by the differences between the fluid temperatures calculated both ways. These differences are shown in Figure 5 for the last, 20th year. The maximum difference for all fluid temperatures during the 20-year period (175,200 values) was calculated as 0.039 K (0.070 °F). This should be compared to the fluid temperatures that lie in the range from -4 to +9 °C (25 to 48 °F). The accuracy of the aggregation scheme is quite good for the choice of $P_q$ equal to 5. Table 3 shows this maximum absolute error for all $P_q$ equal to 1, 2, 3, 5, 10, and 20. We see that $P_q=1$ perhaps gives too large an error. On the other hand, $P_q=20$ gives a very small error. The choice of $P_q$ has minor effect on the required computational time and any value of the above $P_q$ can be chosen to attain the desired accuracy level without significant influence on the computational time.

## CONCLUSIONS

Energy simulations of ground source heat pump systems are critical for design and operation optimization of these systems. However, hourly simulations of borehole heat exchangers performed for multiple years are very time consuming. The extraction fluid temperature depends on a long sequence, backwards in time, of heat extraction and injection rates. In this paper, we present a new load-aggregation scheme to perform multi-year simulations of borehole heat exchangers. The starting point is the step-response function for the considered borehole heat exchanger and the corresponding long sequence of cells, each with a load and a weighting factor. The aggregation is performed on different levels. At the first level, the original weighting factors are kept. At levels 2, 4, 8, etc., weighting factors are lumped together. The accuracy of the scheme depends on the number of lumped cells on each aggregation level. The number of cells to be lumped can be chosen freely to obtain the desired accuracy level. A choice of 5 lumped cells on each of 16 aggregation levels required for a 20-year simulation gives a maximum absolute error of 0.039 K (0.070 °F) compared to a non-aggregated scheme. Some 80 aggregated loads were used, and the new scheme proved 200 times faster than the non-aggregated case.

## NOMENCLATURE

| | | |
|---|---|---|
| $h$ | = | time step (s) |
| $p$ | = | cell numbers on level $q$ |
| $P_q$ | = | number of aggregated cells on level $q$ |
| $Q_{in}(t)$ | = | prescribed heat injection (W or Btu/h) |
| $Q_n$ | = | prescribed heat injection rate at time step $n$ (W or Btu/h) |
| $Q_{step}$ | = | amplitude of heat injection step (W or Btu/h) |
| $Q_v^{(n)}$ | = | heat injection rate pulse $v$ at time step $n$ (W or Btu/h) |
| $\bar{Q}_{q,p}^{(n)}$ | = | heat injection rate for aggregated cell $q,p$ (W or Btu/h) |
| $q$ | = | level of aggregation |
| $R_{ss}$ | = | thermal resistance between carrier fluid and ground surface (K/W or h·°F/Btu) |
| $R_v$ | = | thermal resistance factor for cell $v$ (K/W or h·°F/Btu) |
| $r_q$ | = | number of original cells in lumped cells of level $q$ |
| $T_f$ | = | injection fluid temperature (°C or °F) |

$T_{step}(t)$ = step-response temperature (K or °F)
$\kappa_v$ = original weighting factors
$\bar{\kappa}_{q,p}$ = weighting factor for aggregated cell $q,p$
$v$ = enumeration of loads or cells backwards from the current time step $n$
$v_{q,p}$ = last $v$-value in cell $q,p$

## REFERENCES


Bernier, M.A. 2001. Ground-coupled heat pump system simulation. A multiple load aggregation algorithm for annual hourly simulations of GCHP systems. ASHRAE Transactions, 106(1): 605–616.

Bernier, M.A., Kummert, M. and Bertagnolio, S. 2007. Development and application of test cases for comparing vertical ground heat exchanger models. In: Proceedings of Building Simulations Conference, Beijing, China.

Bernier, M.A., Labib, R., Pinel, P. and Paillot, R. 2004. A multiple load aggregation algorithm for annual hourly simulations of GCHP systems. HVAC&R Research, 10(4): 471–487.

Claesson, J. 2012. Calculation of extraction temperatures of ground heat exchangers for long load sequences using a time-saving load-aggregation method. Mathematical report, Chalmers University of Technology, Sweden. (In writing)

Claesson, J. and Javed, S. 2011. An analytical method to calculate borehole fluid temperatures for time-scales from minutes to decades. ASHRAE Transactions: 117(2).

Eskilson, P. 1987. Thermal analysis of heat extraction boreholes. PhD Thesis, Lund University, Sweden.

Hellström G. and Sanner B. 1994. Software for dimensioning of deep boreholes for heat extraction. In: Proceedings of Calorstock Conference, Helsinki, Finland.

Javed, S. and Claesson, J. 2011. New analytical and numerical solutions for the short-term analysis of vertical ground heat exchangers. ASHRAE Transactions, 117(1): 3–12.

Lamarche, L. 2009. A fast algorithm for the hourly simulation of ground-source heat pumps using arbitrary response factors. Renewable Energy, 34(10): 2252–2258.

Lamarche, L. and Beauchamp, B. 2007. A fast algorithm for the simulation of GCHP systems. ASHRAE Transactions, 113(1): 470–476.

Liu, X. 2005. Development and experimntal validation of simulation of hydronic snow melting systems for bridges. PhD Thesis, Oklahoma State University, USA.

Marcotte, D. and Pasquier, P. 2008. Fast fluid and ground temperature computation for geothermal ground-loop heat exchanger systems. Geothermics, 37(6): 651–665.

Murugappan, A. 2001. Implementing ground source heat pump and ground loop heat exchanger models in the EnergyPlus simulation environment. M.Sc. Thesis, Oklahoma State University, USA.

Pinel, P. 2003. Amélioration, validation et implantation d'un algorithme de calcul pour évaluer le transfert thermique dans les puits verticaux de systèmes de pompes à chaleur géothermiques. M.Sc. Thesis, École Polytechnique de Montréal, Canada.

Spitler, J.D. 2000. GLHEPRO – a design tool for commercial building ground loop heat exchangers. In: Proceedings of the Fourth Internaltional Heat Pump in Cold Climates Conference, Aylmer, Québec, Canada.

Wentzel, E.L. 2005. Thermal modeling of walls, foundations and whole buildings using dynamic thermal networks. PhD Thesis, Chalmers University of Technology, Sweden.

Yavuzturk, C. and Spitler, J.D.1999. A short time step response factor model for vertical ground loop heat exchangers. ASHRAE Transactions, 105(2): 475–485.