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The correct citation for this paper is:

Javed, S, Nakos, H, Claesson, J, 2012. A method to evaluate thermal response tests on groundwater-filled boreholes. *ASHRAE Transactions*, vol. 118(1), pp. 540-549.

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A Method to Evaluate Thermal Response Tests on Groundwater-filled Boreholes

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ABSTRACT

The design of borehole heat exchangers for ground source heat pump system applications requires thermal properties, like ground thermal conductivity and borehole thermal resistance, as inputs. These properties are often determined from an in-situ thermal response test of a pilot borehole. For groundwater-filled boreholes, the ground thermal conductivity and borehole resistance estimations are affected by the heat-injection rates used during the test. Most existing methods for evaluating thermal response tests were not originally developed to analyze tests on groundwater-filled boreholes, and these methods can sometimes give erroneous results in such situations. This paper presents a new method for the evaluation of thermal response tests on groundwater-filled boreholes. The method is based on an analytical solution, which considers the thermal capacities, thermal response tests on groundwater-filled borehole and properties of all borehole elements. The proposed method simplifies the evaluation of thermal response tests on groundwater-filled boreholes and provides accurate estimations of ground thermal conductivity and borehole thermal resistance.

INTRODUCTION

A thermal response test (TRT) is often conducted when designing ground source heat pump (GSHP) systems. The *in-situ* test is performed to determine ground thermal conductivity and borehole thermal resistance. In a typical TRT, a known amount of heat is injected into a pilot borehole. Electric heaters are commonly used to inject heat into the ground by heating the circulating fluid. The heated fluid is circulated through the borehole for 2-3 days. Inlet and exit fluid temperatures and power input to the electric heater and the circulating pump are measured at regular intervals. The measurements are then analyzed using a mathematical heat transfer model to estimate ground conductivity and borehole resistance values. It is also becoming increasingly common to conduct multi-injection rate (MIR) tests when designing GSHP systems. Multi-injection rate tests are conducted using stepwise heat-injection rates to investigate the presence and influence of regional groundwater flow on grouted and groundwater-filled boreholes. Groundwater-filled boreholes are common in Scandinavian countries. The underground structure in these countries is mostly solid bedrock. The boreholes are generally not grouted and are allowed to fill naturally with groundwater. The convective flow in groundwater-filled boreholes has a positive influence on the heattransfer between the borehole and the surrounding ground. For groundwater-filled boreholes, an additional objective for conducting a MIR test is to study the variations of ground conductivity and borehole resistance estimations for the various injection rates expected for the borehole. The ground conductivity and borehole resistance estimations of ground-water filled boreholes are sensitive to heat-injection and extraction rates. A larger injection rate in a groundwater-filled borehole in solid bedrock enhances convective heat transport in the borehole, and the borehole resistance is consequently lowered. On the other hand, a larger injection rate in a groundwater-filled borehole in fractured bedrock results in convective flow from the borehole to the surrounding rock. The convective flow enhances the heat transport from the borehole to the surrounding ground, resulting in a higher ground conductivity estimation (Gustafsson and Westerlund, 2010).

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The evaluation of MIR tests on groundwater-filled boreholes poses challenges different to those of single-injection rate tests. The injection-rate dependence of ground conductivity and borehole resistance estimations in groundwater-filled boreholes complicates the evaluation process. Most existing TRT evaluation methods are not designed to analyze tests in which ground thermal conductivity and borehole thermal resistance estimations are dependent on heat injection rates. This paper presents a new method to evaluate TRTs, addressing the issue of MIR tests on groundwater-filled boreholes. The method can be used to evaluate both single and MIR tests on grouted or water-filled boreholes.

EXISTING METHODS

The existing methods for evaluating TRTs include both direct and parameter estimation methods. Direct methods assume constant-injection rates during a test. Direct methods can be used if the standard deviation and the maximum variation of the input power to its mean value is less than ± 1.5 and ± 10 %, respectively (ASHRAE, 2007). Direct methods to evaluate TRTs are based on approximations of classical line-source (Ingersoll et al., 1954) or cylindrical-source (Carslaw and Jaeger, 1959) solutions. The direct method that is based on line-source approximation (Gehlin, 2002) is simpler, and has thus gained greater acceptance. The method involves the plotting of experimentally measured mean fluid temperatures against logarithmic time, which results in a straight line. The ground conductivity is then estimated using the slope of the straight line for times larger than 20 r_b^2/a . This method does not provide a direct estimation of borehole resistance. However, an estimate of borehole resistance can be determined from the methods proposed by Mogensen (1983) and Beier and Smith (2002).

The line-source method can also be used with the parameter estimation technique to evaluate TRTs. This is done by fitting guess values of ground conductivity and borehole resistance to simulate the circulating fluid temperature. The guess values are then optimized by minimizing the error between simulated and experimentally measured fluid temperatures. Using the parameter estimation technique also allows the evaluation of tests with power variations higher than recommended by ASHRAE (2007). The variations in input power are accounted for by considering stepwise constant heat pulses rather than an overall constant injection rate. Evaluation methods based on the line-source model are generally implemented in a spreadsheet, or in mathematical analysis software, by individual users. However, commercial and research tools based on the line-source method are also available.

Other parameter estimation methods for analyzing TRTs include the numerical methods of Shonder and Beck (1999) and Austin et al. (2000). Shonder and Beck use a one-dimensional finite-difference approach to estimate ground conductivity and borehole resistance. The U-tube is modeled as a hollow cylinder of equivalent-diameter, and the borehole heat transfer is assumed to be radial only. A thin film layer surrounding the equivalent-diameter cylinder is introduced to account for the thermal properties of borehole elements. The ground conductivity and borehole resistance are estimated using the Gauss method, which minimizes the sum of the squared errors between the fluid temperatures measured experimentally and determined from the model. Austin et al. (2000) also use a parameter estimation technique to evaluate TRTs. However, instead of using an equivalent-diameter, Austin et al. use a pie-sector approximation to model the U-tube pipes. The model solves the resulting two-dimensional heat transfer problem by using a finite-volume approach. Austin et al. use the Nelder-Mead Simplex method (1965) to minimize errors between the modeled and experimental values in order to estimate the thermal conductivity values of the ground and the grout. The models of Austin et al. and Shonder and Beck have been implemented in high-level programming languages and are available as standalone computer programs.

NEW EVALUATION METHOD

The new method for evaluating TRTs is based on the analytical model of Javed and Claesson (2011). The model uses Laplace transformations to solve the radial heat transfer problem in the borehole. The model accounts for the thermal properties of circulating fluid, pipe, grout, and the surrounding ground. The model uses a single equivalent-diameter pipe as a substitute for the U-tube, as shown in Figure 1a. Therefore, the fluid temperatures entering and exiting the U-tube are modeled as mean fluid temperature $T_f(t)$. The thermal capacity C_p of the circulating fluid in the equivalent-diameter pipe is kept equal to that in the U-tube. A resistance value of R_p is used to account for fluid and pipe resistances. The model assumes that the equivalent-diameter pipe is surrounded by a grout region. The grout is modeled using thermal conductivity and thermal diffusivity values of λ_g and a_g , respectively. The borehole is assumed to be surrounded by infinite homogeneous ground (soil) of thermal conductivity λ_s and thermal diffusivity a_s . The model considers heat flux q_0 injected to the circulating fluid and assumes that the resulting heat flux from the fluid to the grout region through the pipe wall is $q_p(t)$. Similarly, the heat flux from the grout region to the surrounding ground, through the borehole radius, is $q_b(t)$. The model solves the radial heat transfer problem in the borehole and the ground using the thermal network of Figure 1b. The network, which involves a sequence of composite resistances and composite conductances, is used to determine the fluid temperature $\overline{T}_f(s)$ in the Laplace domain:

$$\bar{T}_{f}(s) = \frac{q_{0}}{s} \cdot \frac{1}{C_{p} \cdot s + \frac{1}{R_{p} + \frac{1}{\bar{K}_{p}(s) + \frac{1}{\bar{K}_{t}(s) + \frac{1}{\bar{K}_{b}(s) + \bar{K}_{s}(s)}}}.$$
(1)

The fluid temperatures in the time domain are obtained from Equation 1 using a standard inversion formula. The inputs to the model are the heat injection rate; the borehole geometry, including the borehole depth and the inner and outer diameters of the U-tube; the thermal conductivities of the pipe, grout, and ground; and the volumetric heat capacities of the grout and ground. The new method for evaluating TRTs uses the model of Javed and Claesson (2011) with a parameter estimation technique. The equivalent diameter of the borehole, the thermal capacities, and the resistances of the circulating fluid and the U-tube are determined from the input values. The ground and grout conductivities are assumed to be unknowns and their initial values are guessed. If the grout conductivity is known, then any of the above-mentioned input parameters can be estimated instead. The proposed method first simulates fluid temperature using guessed and input parameters. The simulated fluid temperature is then compared to the experimentally measured fluid temperature. Next, the initial guess values are iteratively refined to minimize the sum of the squared errors between the experimental and simulated fluid temperatures. The optimized guess values that are providing the minimum squared error sum are taken as the final estimated parameters.



Figure 1 (a) Heat transfer problem solved by the proposed method. (b) Thermal network for the heat transfer problem in the Laplace domain.

The borehole thermal resistance is estimated next. The borehole thermal resistance is the steady-state resistance between the circulating fluid and the borehole wall. An effective value of steady-state borehole resistance is estimated by taking the ratio of the temperature difference of the circulating fluid and the borehole wall to the specific heat-injection rate.

Equation 2, which is the finite line-source solution of Claesson and Javed (2011), is used to calculate the borehole wall temperature. The ground thermal conductivity value obtained from the parameter estimation approach described above is used when calculating the borehole wall temperature.

$$T(r_b,t) = \frac{q_0}{4\pi\lambda} \cdot \int_{1/\sqrt{4at}}^{\infty} du \cdot e^{-r_b^2 u^2} \cdot \frac{\left[2 \cdot \operatorname{ierf}(Hu) + 2 \cdot \operatorname{ierf}(Hu + 2Du) - \operatorname{ierf}(2Hu + 2Du) - \operatorname{ierf}(2Du)\right]}{Hu^2}$$
(2)

The proposed method estimates the steady-state borehole resistance by taking average resistance values for all times larger than $20 \cdot r_b^2/a_s$. The method has been implemented in such a way that the grout conductivity and the borehole resistance can be estimated for any given time range. This implies that ground conductivity and borehole resistance values can be estimated for a specific injection rate when evaluating MIR tests.

VALIDATION AND COMPARISON

The new evaluation method is validated against existing methods using a series of *in-situ* thermal response tests. The tests are performed on an 80 m (262 ft) deep borehole. The diameter of the borehole is 110 mm (4.3 in), and it has a single polyethylene U-tube inserted in it. The inner and outer diameters of the U-tube pipe are 35.4 and 40 mm (1.4 and 1.6 in), respectively. The pipe's thermal conductivity is 0.42 W/m·K (0.24 Btu/h·ft·°F). The borehole is drilled in igneous rock. The borehole is not grouted and is instead filled naturally with groundwater, which is within 0-1 m (0-3 ft) of the ground surface. The borehole geometry is shown in Figure 2. The circulating fluid is 29.5 % ethanol. The fluid thermal conductivity and specific heat capacity are 0.401 W/m·K (0.23 Btu/h·ft·°F) and 4180 J/kg·K (1.0 Btu/lb·°F), respectively. The setup of TRT includes an electric heater and a circulating pump, both of variable capacity. The electric heater can provide heat-injection rates between 30 and 180 W/m (31 and 187 Btu/h·ft). The circulating pump is designed specifically for GSHP applications and has negligible heat rejection to the circulating fluid.



Figure 2 Geometry of borehole used for in-situ testing.

The proposed method was tested and validated using four TRTs, including both single and MIR tests. The first and second tests were conducted with single injection rates. The mean injection rates during these tests were 68 and 140 W/m (71 and 146 Btu/h·ft), respectively. The duration of the first test (Figure 3a) was 50 hours, while the second test (Figure 3b) was conducted for 72 hours. The third and fourth tests were conducted with MIRs. For the third test (Figure 3c), a mean injection rate of 68 W/m (71 Btu/h·ft) was used for the first 52 hours, followed by a mean injection rate of 140 W/m (146 Btu/h·ft) for the next 67 hours. The fourth test (Figure 3d) used mean injection rates of 140 and 68 W/m (146 and

71 Btu/h·ft) for 51 and 65 hours, respectively. The TRTs were conducted over a period of ten months. The minimum time interval between successive tests was over six weeks. After each test the temperature of the surrounding ground was allowed to return to its undisturbed pre-test value. For all tests, the standard deviation and the maximum variation of the input power used for each injection rate were respectively less than ± 1.5 and ± 10 % of the mean input power. All tests were conducted with similar flow rates of circulating fluid. The flow rate was chosen to keep turbulent regime in the ground loop.



Figure 3 Thermal response test cases.

Both the direct and parameter estimation methods discussed in the "Existing models" section of this paper were used to evaluate single-injection rate tests 1 and 2. When evaluating these tests with line-source-based methods, the data for the first 15 hours was not considered. This is because the line-source model does not account for the local heat-transfer problem inside a borehole. For the other evaluation methods the complete data sets were considered. The evaluation of the first test, conducted with an injection rate of 68 W/m (71 Btu/h·ft), gives similar results for all evaluation methods. As seen from Table 1, the ground conductivity estimations from the existing methods vary between 2.99 and 3.24 W/m·K (1.73 and 1.87 Btu/h·ft·°F). The estimations of borehole resistance lie between 0.059 and 0.063 m·K/W (0.102 and 0.109 h·ft·°F/Btu). The new method estimates ground conductivity and borehole resistance values of 3.02 W/m·K (1.75 Btu/h·ft·°F) and 0.053 m·K/W (0.092 h·ft·°F/Btu), respectively.

For Test 2, which was conducted with a higher injection rate of 140 W/m (146 Btu/h·ft), the ground conductivity estimations from the existing methods are between 3.24 and 3.57 W/m·K (1.87 and 2.06 Btu/h·ft·°F). The borehole resistance estimations are in the 0.058 to 0.060 m·K/W (0.100 and 0.104 h·ft·°F/Btu) range. The new method estimates ground conductivity and borehole resistance values of 3.36 W/m·K (1.94 Btu/h·ft·°F) and 0.054 m·K/W (0.093 h·ft·°F/Btu), respectively. For the first two tests, the ground conductivity and borehole resistance estimations in the results from the new evaluation method are in agreement with those from the existing methods. The slight variations in the results from the different methods are within commonly assumed uncertainties for TRT evaluations (Witte et al., 2002). The fit of the models to the experimentally measured mean fluid temperatures for tests 1 and 2 are shown in Figure 4a and 4b, respectively. As seen, all methods estimate significantly higher values of ground conductivity for Test 2, which was conducted with a higher injection rate than Test 1. On the other hand, the borehole resistance estimations are analogous for the two tests. These results are consistent with the observations of Javed et al. (2011) and Gustafsson and Westerlund (2010) for groundwater-filled boreholes in fractured bedrock.

The evaluation of tests 3 and 4, conducted with MIRs, with the existing methods is relatively more complicated. The direct method that uses line-source approximation can only evaluate tests with constant injection rates. For tests 3 and 4, the direct method can only be used when taking the first injection rates into account, requiring us to neglect all the data corresponding to the second injection rates in these tests. The method of Shonder and Beck (1999) has similar limitations. The intended implementation of their method is to evaluate tests on grouted boreholes. The ground conductivity and borehole resistance values optimized for specific injection rates cannot be estimated with the method. Thus, the evaluation of tests 3 and 4, with the Shonder and Beck method, was also done only for the first injection rates. Evaluation of tests 3 and 4, with the direct method and the Shonder and Beck method, gives inaccurate results if both injection rates are included in the analysis. When evaluating tests 3 and 4 with the line-source-based parameter estimation method, the optimized values of ground conductivity and borehole resistance are obtained for times corresponding to specific heat-injection rates. As for single injection rate tests, the data corresponding to the first 15 hours of an injection rate are not considered when evaluating the tests from a line-source model. When using the parameter estimation method of Austin et al. (2000), the ground conductivity estimations are also obtained corresponding to specific injection rates. However, the method does not provide a direct estimation of borehole resistance. The evaluation of tests 3 and 4 with the new method provides estimations of both ground conductivity and borehole resistance values for specific injection rates. As the new method considers the local heat transfer inside a borehole, there is no need to disregard any data.

Test 3 was conducted with stepwise increasing injection rates of 68 and 140 W/m (71 and 146 Btu/h·ft). The new method's ground conductivity and borehole resistance estimations of 3.10 W/m·K (1.79 Btu/h·ft·°F) and 0.060 m·K/W (0.104 h·ft·°F/Btu), respectively, for the first injection rate of 68 W/m (71 Btu/h·ft), are in good agreement with those from other methods. Similarly, the ground conductivity and borehole resistance estimations of 3.48 W/m·K (2.01 Btu/h·ft·°F) and 0.055 m·K/W (0.095 h·ft·°F/Btu) for the second injection rate are comparable to results from the Austin et al. and line-source-based parameter estimation methods. Test 4 was conducted with stepwise decreasing injection rates of 140 and 68 W/m (146 and 71 Btu/h·ft). For the first injection rate of 140 W/m (146 Btu/h·ft), the ground conductivity and borehole resistance values of 3.16 W/m·K (1.83 Btu/h·ft·°F) and 0.044 m·K/W (0.076 h·ft·°F/Btu) are reasonably close to those estimated by other existing methods. For the second injection rate of 68 W/m (71 Btu/h·ft), the ground thermal conductivity and borehole thermal resistance estimations are 3.15 W/m·K (1.82 Btu/h·ft·°F) and 0.053 m·K/W (0.092 h·ft·°F/Btu), respectively. The ground conductivity estimation is comparable to the value of 3.26 W/m·K (1.88 Btu/h·ft·°F) estimated with the method of Austin et al. However, the line-source-based parameter estimation method gives comparatively lower values for both ground conductivity and borehole resistance. Figures 4c and 4d show the fit of the models to the experimentally measured mean fluid temperatures of tests 3 and 4, respectively.

New method 1st injection rate 2nd injection rate	Austin et al. 1st injection rate 2nd injection rate	Shonder and Beck 1st injection rate 2nd injection rate	2nd injection rate Line-source (parameter estimation) 1st injection rate 2nd injection rate	Line-source (direct) 1 st injection rate	Method	Tat
3.02 (1.75)	3.09 (1.79) -	2.99 (1.73) -	- 3.13 (1.81) -	3.24 (1.87)	Te λs W/m·K (Btu/h·ft.°F)	ole 1. Ground
0.053 (0.092) -		0.063 (0.109) -	- 0.060 (0.104) -	0.059 (0.102)	st 1 <i>R</i> _b m·K/W (h·ft°F/Btu)	d Conductivity
3.36 (1.94) -	3.42 (1.98) -	3.24 (1.87) -	- 3.41 (1.97) -	3.57 (2.06)	Te λ₅ W/m·K (Btu/h·ft·°F)	and Borehole
0.054 (0.093)		0.058 (0.100) -	- 0.060 (0.104) -	0.060 (0.104)	st 2 <i>R</i> ^b m·K/W (h·ft·°F/Btu)	Resistance E
3.10 (1.79) 3.48 (2.01)	3.15 (1.82) 3.61 (2.09)	3.01 (1.74) -	- 3.07 (1.77) 3.68 (2.13)	3.08 (1.78)	Te Å₅ W/m·K (Btu/h·ft·°F)	stimations fro
0.060 (0.104) 0.055 (0.095)		0.062 (0.107) -	- 0.059 (0.102) 0.060 (0.104)	0.060 (0.104)	st 3 <i>R</i> ^b m·K/W (h·ft·°F/Btu)	m Different Me
3.16 (1.83) 3.15 (1.82)	3.32 (1.92) 3.26 (1.88)	3.22 (1.86) -	- 3.46 (2.00) 2.81 (1.62)	3.08 (1.78)	Te Ås W/m·K (Btu/h·ft.°F))thods.
0.044 (0.076) 0.053 (0.092)		0.055 (0.095) -	- 0.052 (0.090) 0.035 (0.061)	0.041 (0.071)	st 4 <i>R</i> _b m·K/W (h·ft·°F/Btu)	

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Figure 4 Model fits to experimentally measured mean fluid temperature.

The first 52 hours of Test 3, and the last 65 hours of Test 4, were conducted with an injection rate of 68 W/m (71 Btu/h·ft). The ground conductivity and borehole resistance estimations for these periods in tests 3 and 4 are comparable to Test 1, which was also conducted with the same injection rate. The ground thermal conductivity estimations for tests 1, 3, and 4, when using an injection rate of 68 W/m (71 Btu/h·ft), are 3.02, 3.10, and 3.15 W/m·K (1.75, 1.79 and 1.82 Btu/h·ft·°F), respectively. The 0.053, 0.060, and 0.053 m·K/W (0.092, 0.104 and 0.092 h·ft·°F/Btu) estimations for borehole resistance are also very consistent across the three cases. Similarly, the last 67 hours of Test 3 and the first 51 hours of Test 4 and Test 2 are all comparable, as they are conducted with a mean injection rate of 140 W/m (146 Btu/h·ft). The ground conductivity estimations for these three cases, using the new method, are 3.48, 3.16, and 3.36 W/m·K (2.01, 1.83 and 1.94 Btu/h·ft·°F), respectively. The borehole resistance estimations of 0.055, 0.044, and 0.054 m·K/W (0.095, 0.076 and 0.093 h·ft·°F/Btu) are also in reasonably close agreement. The method of Austin et al. also gives consistent estimations of ground conductivity for all test lengths conducted with similar injection rates. The method, however, does not provide borehole thermal resistance estimations. On the other hand, the line-source-based parameter estimation method gives slightly different results for tests with stepwise decreasing injection rates.

The new method was also validated for TRTs on grouted boreholes. The estimated values of ground conductivity and borehole resistance with the new method were similar to results from the other parameter estimation methods.

CONCLUSION

A new method for evaluating thermal response tests has been presented. The proposed method estimates ground and grout thermal conductivities through a radial analytical solution developed by the authors using a parameter estimation approach. The estimated value of ground conductivity is used to simulate the borehole wall temperature. The borehole thermal resistance value is estimated as a ratio of the temperature difference between the mean temperature of the circulating fluid and borehole wall to the heat injection rate. The borehole resistance is determined for the steady-state conditions of times larger than $20 \cdot r_b^2 / a_s$. The proposed method was tested against various existing methods, using four *in-situ* tests on groundwater-filled boreholes. Two single-injection rate tests, conducted with 68 and 140 W/m (71 and 146 Btu/h·ft), respectively, were used for testing. In these tests, the new method's ground conductivity and borehole resistance estimations were comparable to those from other direct and parameter estimation methods. Two multi-injection rate tests, using 68 and 140 W/m (71 and 146 Btu/h·ft) in increasing and decreasing order, respectively, were also used for testing. The results derived from the single-injection rate tests were used as a reference when evaluating the multi-injection rate tests. Most existing methods are unable to evaluate multi-injection rate tests on groundwater-filled boreholes. The proposed method estimates ground accuracy.

ACKNOWLEDGMENTS

The authors would like to thank Professor Jeffrey Spitler and Mr. Palne Mogensen for providing us with the computer programs for the evaluation of the thermal response tests, as well as for many helpful discussions.

NOMENCLATURE

а	=	thermal diffusivity $(m^2/s \text{ or } ft^2/h)$
С	=	thermal capacity per unit length (J/m·K or Btu/ft·°F)
D	=	starting point of active borehole depth (m or ft)
Н	=	active borehole height (m or ft)
\overline{K}	=	thermal conductance in Laplace domain (W/m·K or Btu/h·ft·°F)
λ	=	thermal conductivity (W/m·K or Btu/h·ft·°F)
q	=	rate of heat transfer per unit length (W/m or Btu/h·ft)
R	=	thermal resistance (m·K/W or h·ft·°F/Btu)
$\overline{R}(s)$	=	thermal resistance in the Laplace domain (m·K/W or h·ft·°F/Btu)
r	=	radius (m or ft)
S	=	Laplace transform variable
Т	=	temperature (K or °F)
\overline{T}	=	mean temperature (K or °F)
$\overline{T}(s)$	=	Laplace transform of T (K·s or °F·h)
t	=	time (s or h)
и	=	integral parameter

Subscripts

- b = borehole
- f =fluid
- g = grout
- p = pipe
- s = ground (soil)
- t = transmittive

Abbreviations

- GSHP =ground source heat pump
- IR = injection rate
- *MIR* = multi-injection rate
- TRT = thermal response test

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