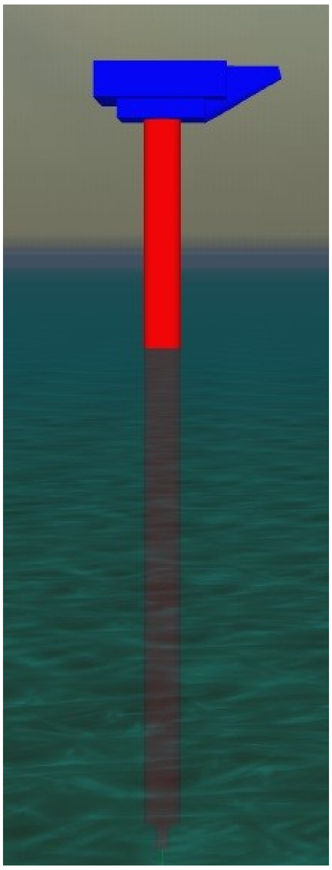

Decommissioning of SPM buoy

Master of Science Thesis

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Department of Shipping and Marine Technology
Division of Marine Design
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden, 2012

Report No. X-12/284

A THESIS FOR THE DEGREE OF MASTER OF SCIENCE

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Abstract

Over the following years many of the installations in the North Sea, installed in the 1980s, will have to be decommissioned. Due to rules and regulations most of them will be taken onshore for recycling. In this report a study of the decommissioning process of a SPM buoy in the Statfjord field is perfomed. The column of the buoy is containing 18 compartments and weighs about 7150 tonnes. The topside is weighing about 519 tonnes. The height of the buoy is about 182 m measured from the unijoint.

In the decommissioning process the hydrostatics, structural and hydrodynamics should be investigated. In the hydrostatic analysis hand calculations are performed to find equilibrium and ballast condition for given sequences during the operation. In the structural analysis the same sequences are analyzed. The global stress on the buoy is calculated by hand to see of the longitudinal strength of the buoy is satisfying. The hydrodynamic part of the report investigates the difference of linear frequency domain analysis and non- linear time domain analysis, to see if the simpler linear analysis is accurate enough.

The hand calculations used in the hydrostatic and structural parts are performed in MathCAD. The calculations for the hydrostatic part are performed by integrating over the geometry to find the hydrostats. The hydrostats was then compared with results obtained in AutoHydro, a program for simulating hydrostatic analysis. To calculate the global stresses the shear force diagram was set up, and later the bending moment and stress diagrams. In the hydrodynamic analysis two programs were used to run the simulations. Wadam was used for the frequency domain analysis and OrcaFlex for the time domain. In both programs Morison theory was used.

The hand calculations put up in MathCAD are good enough for rough estimations. The VCB, GM_T and LCF are not satisfying for small trim angles. The calculations are satisfying when the buoy is positioned vertically. The bending moment is not zero at bulkhead 19, due to approximation of the lever arm. In the hydrodynamic analysis the damping near the natural period is important. The non- linear Morison theory has only impact near the natural period.

Keywords: Decommissioning; hydrostatic analysis; global stress; frequency domain analysis; time domain analysis; Morison theory;

Preface

This thesis is a part of the requirements for the master's degree at Chalmers University of Technology, Gothenburg, and has been carried out at the Division of Marine Design, Department of Shipping and Marine Technology, Chalmers University of Technology.

I would like to acknowledge and thank my examiner and supervisor, Professor Rickard Bensow at the Department of Shipping and Marine Technology. I would also like to thank my co-supervisors at Semar AS in Oslo, Norway, Mathieu Kreyer, Terje Nistad and Southinanh Oudomphanh for great support.

Gothenburg, May, 2012
Christina Sjöbris

List of abbreviations

$\lvert V_{rxy} \rvert$	Absolute magnitude of relative velocity x- y- plane
$\lvert V_{rz} \rvert$	Absolute magnitude of relative velocity z- plane
$\lvert W_{rxy} \rvert$	Absolute magnitude of relative acceleration x- y- plane
$\lvert W_{rz} \rvert$	Absolute magnitude of relative acceleration z- plane
∇_L	Longitudinal displaced volume
∇_T	Transverse submerged volume
M_{∇}	Moment of displaced volume
$M_{\nabla T}$	Moment of the displaced volume about the keel
$\lvert r \rvert$	Absolute distance from axis of rotation
∇	Mass of fluid displaced by the body
A	Element of drag area
a	Waterplane length
A	Drag area
$\mathbf{A}(\omega)$	Frequency dependent added mass
A_a	Axial drag area
A_{column}	Area of column
AI_a	Axial added moments of inertia
AI_n	Normal added moments of inertia
A_L	Area in longitudinal direction
A_n	Normal drag area
a_r	Fluid acceleration relative to the body
A_{rx}	x- component of local water particle acceleration
A_{ry}	y- component of local water particle acceleration
A_{rz}	z- component of local water particle acceleration
A_w	Area waterplane
a_w	Fluid acceleration relative to the earth
A_z	Area in traverse direction
b	Buoyancy per unit length
B	Linear viscous damping matrix
b	Waterplane bredth
BM_L	Longitudinal metacentric radius
BM_T	Transverse metacentic radius
C	Hydrostatic restoring matrix
$\mathbf{C}(p,v)$	System damping load
$\mathbf{C}_a$	Diagonal added mass coefficient matrix
C_A	Added mass coefficient
C_{aa}	Axial added mass coefficient
C_{an}	Normal added mass coefficient
$\mathbf{C}_D$	Diagonal drag coefficient matrix
C_D	Drag coefficient
C_{da}	Axial drag coefficient
C_{dn}	Normal drag coefficient
$\mathbf{C}_e$	External restoring matrix
C_{MM}	Munk moment coefficient
CoG	Center of gravity
DI_a	Axial moment of inertia of displaced mass

DI_n	Normal moment of inertia of displaced mass
DM_a	Axial instantaneous displaced mass
DM_n	Normal instantaneous displaced mass
DNV	Det Norske Veritas
$\mathbf{F}$	Detail force
$\mathbf{F}(p,v,t)$	External load
$\mathbf{F}(\omega,\beta)$	Complex exiting force vector
$\mathbf{f}_b$	Fluctuating body force
$\mathbf{f}_c$	Fluctuating hydrostatic restoring force
F_c	Load of compartment
F_d	Design load
$\mathbf{F}_D$	General viscous drag force
FE	Finite element
$\mathbf{f}_g$	Fluctuating gravity force
F_k	Characteristic load
F_W	Fluid force
g	Acceleration of gravity
GM_L	Longitudinal metacentric height
GM_T	Transverse metacentric height
GoM	Gulf of Mexico
H_s	Significant wave height
I	Moment of inertia of the cross- section of the beam about its neutral axis
$\mathbf{I}$	Inertia matrix
I_c	Moment of inertia corrected due to corrosion
I_L	Longitudinal moment of inertia about the waterplane
IMO	International Maritime Organization
I_T	Transverse moment of inertia about the waterplane
k	Wave number
$\mathbf{K}(p)$	System stiffness load
L_c	Length of compartment
LCB	Longitudinal center of buoyancy
LCF	Longitudinal center of flotation
LCG	Longitudinal center of gravity
M	Bending moment
$\mathbf{M}$	Mass inertial matrix
$\mathbf{M}(p,a)$	System inertia load
M_c	Moment of compartment
M_{da}	Moment drag area
M_M	Munk moment
M_w	First moment of the waterplane
M_W	Mass of water currently displaced
$\mathbf{n}$	Unit vector
OSPAR	Oslo- Paris Environmental Ministers Organization
p	Load per unit length
PW	Proportion wet
RAO	Response amplitude operator
R_d	Design resistance
R_k	Characteristic resistance
SF	Safety factor
SPM	Single point moring

t	Time
T	Natural period
t_c	Thickness of wall corrected due to corrosion
TCB	Transverse center of buoyancy
TCG	Transverse center of gravity
T_p	Wave period
t_w	Thickness of wall
T_z	Zero crossing period
UDF_a	Axial unit damping forces
UDF_n	Normal unit damping forces
UDM_a	Axial unit damping moment
UDM_n	Normal unit damping moment
UNCLOS	The United Nations Convention of the Law of the Seas
V	Shear force
V_c	Shear of compartment
V_{column}	Volume of column
V_M	Displaced volume of the Morison element
V_{max}	Linearized velocity amplitude
V_r	Fluid velocity relative to the body
V_{rx}	x- component of the water velocity
V_{ry}	y- component of the water velocity
V_{rz}	z- component of the water velocity
V_W	Flow velocity relative to body
w	Weight per unit length
W_{rx}	x- component of angular acceleration
W_{ry}	y- component of angular acceleration
W_{rz}	z- component of angular acceleration
W_x	x- component of angular acceleration of local water isobar relative to the body
W_y	y- component of angular acceleration of local water isobar relative to the body
W_z	z- component of angular acceleration of local water isobar relative to the body
$\mathbf{x}$	Complex amplitude of the incident wave
x_{area}	Longitudinal length of actual waterplane area
x_B	Longitudinal center of buoyancy
x_G	Longitudinal center of gravity
z	Distance from neutral axis to the fiber under consideration
z_B	Vertical center of buoyancy
z_G	Vertical center of gravity
α	Angle between relative flow velocity and buoy
β	Angle between the direction of propagation of the incident wave and positive x- axis
γ_f	Load factor
γ_m	Resistance factor
λ	Eigenvalues
ξ	Complex amplitude of motion
ξ_j	Complex amplitude
ρ	Density of water
ρ_∞	Frequency dissipation

σ_c	Stress corrected due to DNV rules
σ_{top}	Stress at top of cylinder
σ_z	Longitudinal stress
Φ	Velocity potential
Φ	Eigenvectors
ϕ	Complex velocity potential
ϕ_7	Disturbance of the incident wave
ϕ_D	Total diffraction potentials
ϕ_j	Corresponding unit amplitude radiation potentials
ω	Frequency

Contents

1. Introduction

This chapter specifies the problem and presents some background information, aim and delimitations of the chosen problem.

1.1 Background

Today, there are a number of oil platforms and buoys built in the 1980s in the North Sea. Over the following years these platforms and buoys have to be and will be decommissioned. Due to regulations and environmental aspects, this must be done in a proper way and all parts have to be either recycled or treated as dangerous goods.

The buoy studied in this project is a buoy of SPM (Single Point Mooring) type that consists of a cylinder shaped column and topside, connected to the seabed structure by means of a unijoint. The weight of the column of buoy is estimated to 7150 tonnes, including solid ballast. The topside weighs about 519 tonnes. The height of the column is about 182 m from the unijoint. The weight and buoyancy distribution and larger drawings of the buoy are presented in Appendix A. It is designed to transfer crude oil from a subsea pipeline to a tanker moored to the column. The buoy was taken out of service in 2005. The general arrangement of the buoy is shown in Fig. 1.

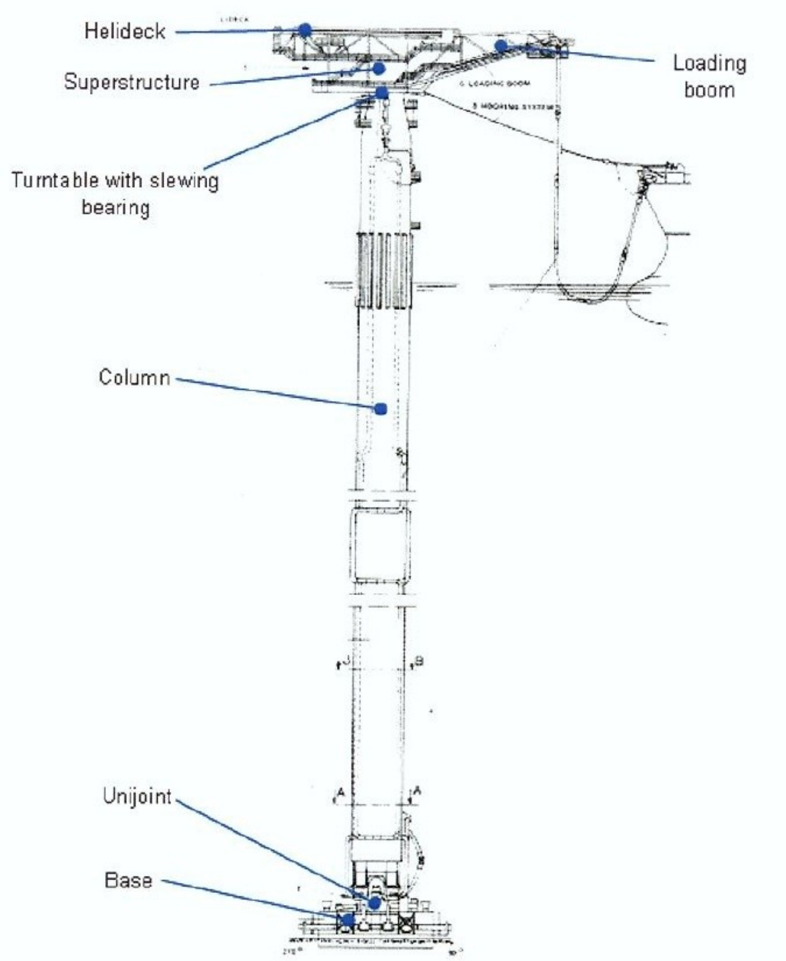

Fig. 1. General Arrangement of SPM buoy

In the process of decommissioning, the buoy will first be detached from the unijoint and towed vertically towards the fjord where the buoy will be handled. In the fjord, the buoy will be moored at a temporary location about 500m from the quay. From this point the buoy is towed to the yard area and the decommissioning process starts. The decommissioning process is finished when the buoy is recycled.

According to the as-built documentation and the design basis, the column part of the buoy consist of 17 compartments, numbered from the surface and down. At site, compartment 17 is filled with heavy ballast consisting of iron ore. Compartment 15, 16 and probably also a part of 14 are filled with permanent ballast water. This differs from what is shown in Fig. 2.

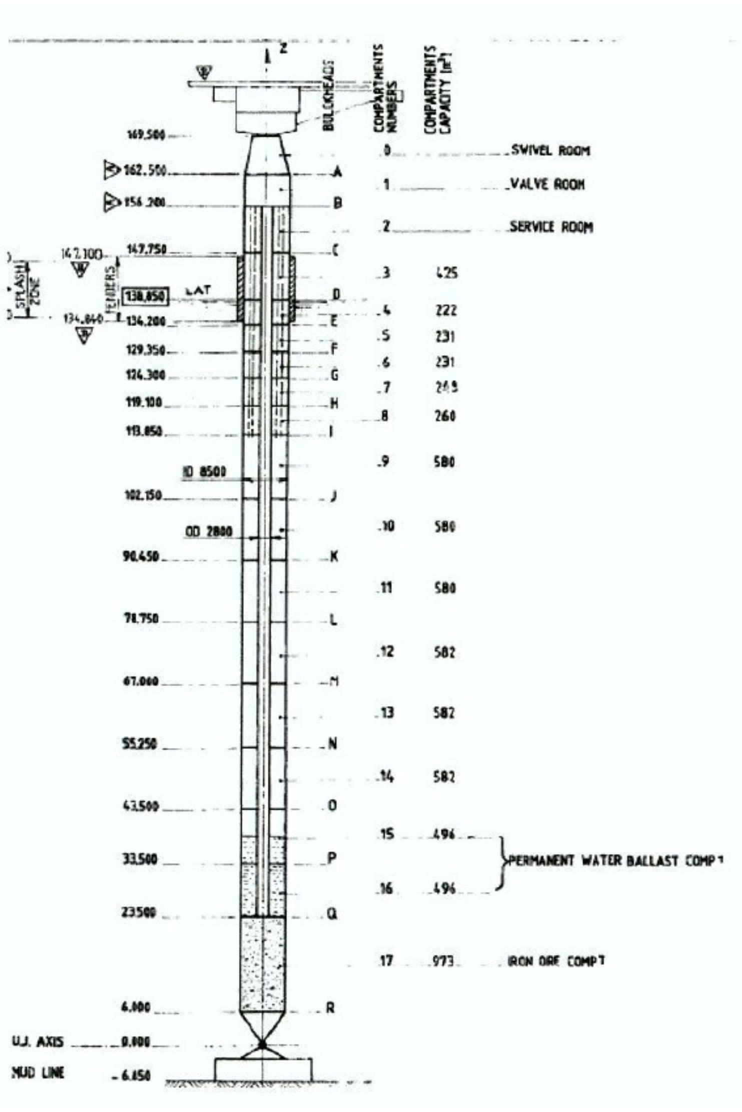

Fig. 2. Column tank subdivision

There is no as-installed weight available for the SPM buoy, the weights is therefore compiled from theoretical weight reports. In the cases were the information differed the weight specified in the as-built document was chosen as the correct ones. These uncertainties are important to take into account and require a larger safety margin.

1.2 Aim

The objective of the project is to investigate the permanent and environmental loads acting on the buoy. The investigation of the buoy consists of studies of the,

- Hydrostatic condition
- Structural strength, and
- Hydrodynamic behavior.

In the hydrostatic analysis the aim is to investigate different cases during the decommissioning phase according to a base case given in the design basis from the costumer. The reason is to investigate if there are suitable ballast conditions that satisfy the costumers' requirements. The hydrostatic analysis is performed by hand calculations in MathCAD and later compared to calculations run in AutoHydro to evaluate the hand calculations. In the structural analysis the global stresses should be investigated and the aim is to check if the longitudinal capacity of the buoy is enough to bear the different loading conditions. The aim for the hydrodynamic analysis is to investigate the motion of the SPM buoy in waves and to compare the results from Wadam (frequency domain, linear analysis of buoy motion in waves) and Orcaflex (time domain, non-linear analysis of slender elements) at a given loading condition. The study is made to see if it is sufficient enough to use linear potential theory or if non-linear theory is better suited.

1.3 Delimitations

Due to time limits the theory behind FE method is not taken into account. Mainly the high frequency motion of the SPM buoy in waves will be investigated, restoring forces from the mooring lines and the low frequency motion from wind and wave drift will not be looked into.

2. History of Decommissioning

The first offshore platform in Gulf of Mexico was installed in 1947 by Kerr- McGee. The water depth was 5.5 m. When this generation of platforms was to be decommissioned the natural choice was to remove and scrap them onshore. But as the water depths grew larger the trend of partial removal grew at the same speed [1]. According to Bostock [1] already in Geneva Convention of 1958 it was stated that “Any installations which is abandoned or disused must be entirely removed”. In 1958 most of the platforms where small and positioned in shallow waters, which made complete removal relatively easy and cheap compared to the platforms that are installed today.

The phrase “abandonment” was often used in the early days, but today decommissioning is a more accurate term to use [2] [3]. In beginning of 1970’s a number of regional conventions were put up, i.e. the Oslo convention, Brazil convention and the Paris convention. The London dumping convention was also created as a global convention. The Oslo convention and London dumping convention were both concluded in 1972. London dumping convention received the necessary 15 ratifications and entered into force in 30 of august 1975. Oslo convention had 7 ratifications and entered into force the 7 of April 1974. The conventions are administrated through annual meetings [4]. The London dumping convention created three lists of materials, one black, grey and white. The black consist of the most harmful materials such as mercury, cadmium and high- level radioactive materials. These are stated in Annex I of the London dumping convention and were not allowed to be dumped. The grey list consists of materials such as arsenic, lead and organosilicon and was stated in Annex II. These materials may be dumped, but required “special care” and a special license. All other materials were allowed to be dumped [5]. The London dumping convention was renamed to London convention in 1992 [6].

The early offshore installations were not designed to be removed even though all offshore installations must be decommissioned at the end of production [7]. But since 1 January 1998 no installations are allowed to be installed on the continental shelf or in the Exclusive Economic Zone, if they are not designed and constructed so it is possible to remove the entire installation [3]. Many of the oil- producing developed countries have specific laws and legislations concerning platform decommissioning [3]. There are three main international conventions, the first is the United Nations Convention on the Law of the Seas, UNCLOS, in 1982, the second one is London Convention in 1972 and the third is the International Maritime Organization, IMO, in 1989 [8]. Almost all countries that have offshore oil and gas installations have regional laws and regulations concerning installation and decommissioning. The main global authority is the IMO who sets the standards and guidelines for the removal of offshore installations. According to IMO regulations all installations standing in 75 meters or less of water depth and weighing under 4 000 tonnes should be completely removed. Installations in deeper waters should be partially removed so that there is 55 meters of free water above them to secure safe navigation. In some countries this depth is extended to 100 meters [9].

The decommissioning process is where the removal of an offshore installation is planed, gained approval of and removed, disposed or reused when it lifetime is over. This operation is costly and needs to be financed, because the platform does not generate any money at this stage. According to Jahn et al. [9] there are five major considerations to take into account during the decommissioning process. They are “the potential impact on the environment, the potential impact on human health and safety, the technical feasibility, the cost of the plan and the public acceptability”. These considerations are due to the risk and complexity of a decommissioning operation. Therefore is the operation performed in different ways depending on the type and location of the installation. The decommissioning process starts when the economic lifetime of an installation is over, i.e. when the cost of the installation is greater than the income.

Today there are 6 500 oil and gas installations located on the continental shelves of 53 countries, 4 000 in Gulf of Mexico, GoM, 950 in Asia, 700 in the Middle East and 400 in Europe [9]. The water depth where an offshore structure is situated is divided into shallow, medium, deep and ultra-deep water. Shallow water structures are comparable to 20- story building weighing less than 4 000 tonnes. Medium water structures are higher than the Eiffel tower and deep and ultra- deep waters are larger than a number of football- fields. The complexity of decommissioning gets higher when the water gets deeper [7]. In deep and ultra-deep waters it is more common with floating than fixed platforms. These have the advantage of just being moored on site. The mooring lines could be released and the platform could be towed to shore, this is a relatively cheap and attractive way to perform the removal. On fixed platforms only the topside modules are removed and taken to shore by a barge. Gravity base structures can in theory be ballasted and floated away to be used in another field or sunk in a deep ocean, and steel jackets can be cut and removed at an agreed depth. Some jackets can be cleaned and left at site or moved to another place to work as artificial reefs. The largest “rigs-to- reef” programme involving 90 decommissioned installations has been implemented outside Louisiana in the GoM. Subsea facilities are relatively easy to decommission because they are small and easy to lift [9].

The incident that started a major discussion about decommissioning all over the world was the Brent spar in the North Sea, in 1995. Shell, who owned the spar, got permission to dump Brent in deep water outside the UK, but Greenpeace was able to stop them. The spar was then taken onshore and was recycled [1] [3]. According to Ekins et al. [10] Greenpeace “established international trend against dumping” when they managed to stop the dumping of the Brent spar [10]. This is a fact confirmed by Pulsipher and Daniel [11] , which lead to UK and Norwegian government signing the 1998 OSPAR convention. OSPAR is the Oslo- Paris Environment Ministers Organization that deals with maritime pollution in the North East Atlantic. To accept onshore- only disposal was a rational change in political attitudes, values and outlook in Western Europe both by government and petroleum industry strategists. The most interesting thing about the Brent spar is that it was a spar, not a platform or a rig. The spar was only a floating storage unit moored to the seabed. It was used before pipelines were installed to the field [11]. The Brent spar was taken out of service in 1991, and due to Shells investigations a deep water dumping operation would have negliable impacts on the environment. This fact was confirmed by independent scientists. The UK government therefore accepted the plan of deep water disposal. Due to the massive protests by public and environmental organizations Shell decided not to go through with the plans, due to reputational consideration [12]. Because Shell took the decision not to dispose Brent in deep sea their reputation was reserved which nowadays is a strong concern to companies and also affect their decision of which method to use when decommissioning [7].

There are a number of ways to decommission a platform, but the five most common ways for the substructure are leaving in situ, recycling/disposal onshore, deep- sea disposal, toppling on site or turn it in to an artificial reef. For the topside there are two different ways onshore disposals recycling or refurbishment/reuse, shown in Fig. 3[9].

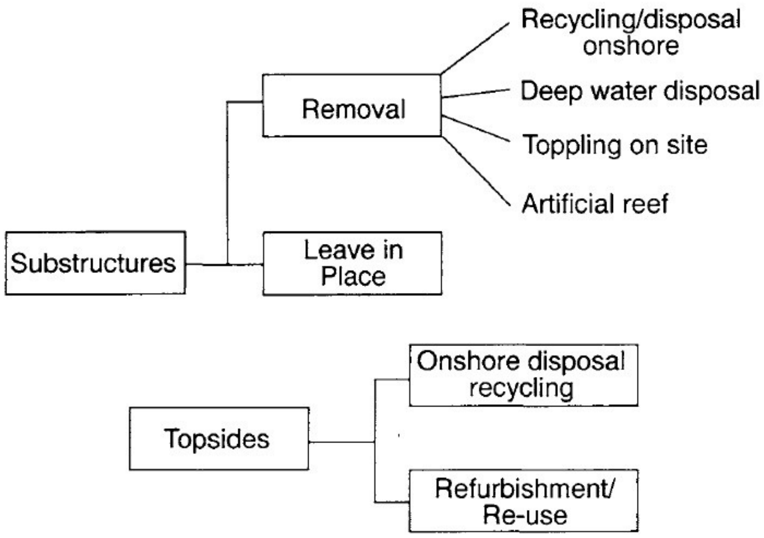

Fig. 3. Different methods for decommissioning of substructure and topside [9]

Leaving in situ is when only the dangerous materials need to be removed, the rest of the platform is cleaned and remains on site. This is the easiest and cheapest way in short term, but there is an ongoing cost that often is not taken into account. The facility needs to be maintained so it does not become dangerous to shipping or the environment. Over time, this cost will be significant. Partial removal is when a part of the structure is removed this is only practical when the water depth over the remaining structure is deep enough to allow for safe navigation. Toppling is when only the topside and processing facilities are removed. In this case it is also important that the water depth is great enough for ships to pass over the remaining structure. Complete removal is when the whole structure is removed. This is by far the most expensive method, but also the one that has been used the most. The reason for this is that most of the structures that have been decommissioned structures have been placed in shallow waters. The technical complexity and cost of these operations has not been very high. The structures that need to be decommissioned today and in the future are both larger and more complex. This makes complete removal very difficult [2]. Deep- sea disposal is when the structure is removed for disposal in deep waters. After the disposal there would be no further human interaction with the platform. This is almost never done since the OSPAR regulations made it illegal [10]. The structures in artificial reefs should be well away from any sea-lanes [13].

The OSPAR Convention decision 98/3 in 1998 requires that "all topsides of all structures are to be removed and brought to shore for reuse, recycling or disposal. All sub- structures or jackets weighing less than 10 000 tonnes must be totally removed and brought to shore for reuse, recycling or disposal. For sub- structures weighing over 10 000 tonnes there is a presumption to remove totally but with potential of derogation being agreed on whether the footings might be left in place. Derogation may be considered for the heavy concrete gravity based structures as well as for floating concrete installations and any concrete anchor- base." [10]

Jackets are often not reusable, because they are designed for a certain depth. The topside and deck on the other hand may be reused on another jacket at another site [7]. The materials in the structure are often re-useable [10]. When it is possible to use recycled parts in a new platform, this may accelerate the schedule to the first production. This means that money may be earned faster and the component is bought at a lower price. As the cost of new offshore facilities rise, so does the price/tonnes of used platform components [14]. There is a general accepted disposal hierarchy used to maximize the value of the waste stream. The hierarchy order is first try to restore and reuse, then scrap and recycle and last dispose in designated landfills. Often all these three steps are used in a decommissioning process. The age, supply/demand conditions, regulatory restrictions cost of restoring, vintage and technical specification decides which method to use [15].

Today Norway has 35% of the world's decommissioning costs, but only 7% of the installations. This is due to the higher technical complexity and weight of the platforms. The need for more complexity and weight is because of the severe weather conditions and high environmental standards forced by Norwegian authorities [7].

The cost of removal is larger for platforms in deep and ultra- deep waters. There are two main factors that make decommissioning of deep water structures more debated than other decommissioning processes. There are uncertainties regarding environmental consequences of disposal/reefing in deep water and these alternatives would greatly decrease the cost to the oil industry in the decommissioning process [1]. The biggest challenge is plugging and disposal of the wells. Well- plugging and disposal is the two most expensive activities in the decommissioning process. In Norway the government covers the largest part of the cost of platform removals [7].

According to Parente et al. [7] an ideal decommissioning assessment report should contain energy use, biological and technological impact of discharges, secondary air emissions, physical and habitat matters, fisheries waste management, littering, drill cutting deposits, free passage, personnel safety, national contents, employment, cost feasibility and impacts on local communities including visual interference, noise, odor and traffic [7].

3. Theory

The theory behind the methods used in the project is presented in the following sub- chapters, starting with the definition of the coordinate system and ending with the theory behind hydrodynamic analysis. The order the theory is presented is the order analyzes has been performed.

3.1 Definition of coordinate system

In the calculations two different coordinate systems are used, one earth fixed and one body fixed. The earth fixed is defined with x,y plane in the waterline and z is positive upwards. The body fixed coordinate system is defined with x in the aft in the center of the cross section, y is defined positive in starboard direction and z is positive upwards. The two coordinate systems are shown in Fig. 4.

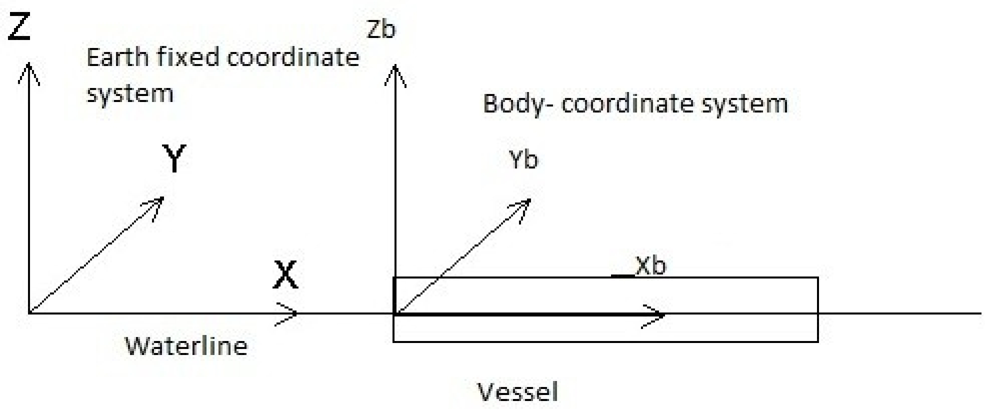

Fig. 4. The earth fixed and body fixed coordinate systems used in the calculations

3.2 Hydrostatic Analysis

The theory behind hydrostatics is due to the geometry of the body which is defined by curves or curved surfaces. This means that the hydrostatic properties can be represented and calculated by integration over the geometry. This is a number of infinite small rectangular elements summed up to describe the shape of the geometry between two limits. The area is often calculated from amidships, the limits are the L/2 and –L/2 when integrating in longitudinal direction.

$$A_L = \int_{-L/2}^{L/2} y dx \tag{1}$$

In transverse direction the limits are from the keel, 0, to the waterline, T, the integral is also multiplied by two due to symmetry around the centerline.

$$A_z = 2\int_0^T ydz \tag{2}$$

The waterplane area is calculated from amidships with the limits of L/2 and –L/2 and is also multiplied by 2 due to symmetry around the centerline.

$$A_W = 2\int_{-L/2}^{L/2} ydx \tag{3}$$

3.2.1 Longitudinal center of flotation

The longitudinal center of flotation, LCF, is the distance from the center of the waterplane area and a given reference plane, often amidships. To get the center of area the first moment of the waterplane needs to calculated and then divide it by the area of the waterplane.

$$M_W = 2\int_{-L/2}^{L/2} xy dx \tag{4}$$

$$LCF = \frac{M_W}{A_W} \tag{5}$$

3.2.2 Longitudinal center of buoyancy

The longitudinal center of buoyancy, LCB, is the distance from the center of buoyancy and a given reference plane, often amidships. LCB is calculated by dividing the moment of the displaced volume in longitudinal direction with the total displaced volume.

$$\nabla_{\mathrm{L}} = \int_{-L/2}^{L/2} A_z dx \tag{6}$$

$$M_{\nabla} = \int_{-L/2}^{L/2} x A_z dx \tag{7}$$

$$LCB = \frac{M_{\nabla}}{\nabla_{\mathrm{L}}} \tag{8}$$

3.2.3 Transverse center of buoyancy

The transverse center of buoyancy is the distance from the center of buoyancy and a given plane, often the keel. It is calculated by dividing the moment of the submerged volume in transverse direction about the keel with the total submerged volume.

$$\nabla_T = \int_0^T A_w dz \tag{9}$$

$$M_{\nabla T} = \int_0^T z A_W dz \tag{10}$$

$$TCB = \frac{M_{\nabla T}}{\nabla_T} \tag{11}$$

3.2.4 Longitudinal center of gravity

The longitudinal center of gravity, LCG, is the distance from the center of gravity and a given plane, often amidships. LCG is calculated as the sum of all weights times their distance from their center of gravity to amidships, divided by the total weight.

$$LCG = \frac{\sum w_i lcg_i}{\sum w_i} \tag{12}$$

3.2.5 Transverse center of gravity

The transverse center of gravity, TCG, is the distance from the center of gravity and a given plane, often the keel. TCG is calculated as the sum of all weights times their distance from their center of gravity to the keel, divided by the total weight.

$$TCG = \frac{\sum w_i tcg_i}{\sum w_i} \tag{13}$$

3.2.6 Calculation of metacentric height

The metacentric height, GM, determine the magnitude of the righting arm, GZ, which determines the stability of the vessel. GM can be specified in either transverse or longitudinal direction.

$$GM_T = TCB + BM_T - TCG \tag{14}$$

$$GM_L = LCB + BM_L - LCG \tag{15}$$

Here, TCB and LCB is the distance from the keel to the center of buoyancy, BM is the metacentric radius and TCG and LCG is the distance from the keel to the center of gravity, all in transverse and longitudinal direction respectively.

$$BM_L = \frac{I_L}{\nabla_L} \tag{16}$$

$$BM_T = \frac{I_T}{\nabla_T} \tag{17}$$

Here, I is the transverse and longitudinal moment of inertia of the waterplane respectively [16].

$$I_L = 2\int_{-L/2}^{L/2} x^2 A_z dx - A_w (LCF)^2 \tag{18}$$

$$I_T = \frac{2}{3}\int_{-L/2}^{L/2} y^3 dx \tag{19}$$

3.3 Structural Analysis

The structural analysis is made to determine the global load capacity of the buoy.

3.3.1 Load case

The load is distributed over the whole buoy, and can be uniform or non-uniform. The support may also be uniform or non-uniform. To get the load per length unit the weight and forces acting on the buoy is subtracted from the buoyancy at every length unit.

$$p = b - w \tag{20}$$

Where, b is the buoyancy per unit length, w is the weight per unit length and p is the load per unit length.

3.3.2 Shear force

When the load case is known, the shear stress can be calculated. The shear force is the integral of the load case.

$$V = \int p dx \tag{21}$$

The integral is taken from the stern to the point where the shear force is to be calculated. The shear force at any point along the buoy is equal to the area enclosed by load diagram from the end of the beam to the point in question. If the buoyancy exceeds the weight in a given point the shear stress is positive and if it is not, the shear stress is negative. The shear force is calculated at every bulkhead and should be zero at the first and last bulkhead.

3.3.3 Bending moment

The bending moment is the integral of the shear force and therefore also the double integral of the load case.

$$M = \int V dx = \iint p dx dx \tag{22}$$

The bending moment is given at any point in the buoy. As for the shear force, the bending moment is at any point in the buoy equal to the area of the shear diagram from the end of the buoy to the given point. The bending moment is calculated at every bulkhead and should be zero at the first and last one.

3.3.4 Longitudinal stress

The longitudinal stresses are caused by bending in the fibers of the buoy due to the banding moment acting on it.

$$\sigma_z = \frac{Mz}{I} \tag{23}$$

Here, M is the bending moment, z is the distance from the neutral axis to the fiber under consideration and I is the moment of inertia of the cross section of the beam about its neutral axis. The largest stresses are often at the top or the bottom of the structure. If the cross section is of a shape that is not symmetric around the neutral axis, different stress are obtained at the top and bottom. This is often the case for a ships cross section [16].

3.3.5 Corrections due to DNV rules for scrapping

The DNV rules DNV-RP-H102 consists of rules and recommendations for marine operations during removal of offshore installations. This document refers to the DNV-OS-C101 rules for design of metal structures. According ch 2.5.9 in DNV-RP-H102 the buoy is allowed to deform plastically and fail in some parts.

Regulation DNV-RP-H102 states that to get the design load, F_d, a load factor, γ_f, has to be multiplied to the characteristic load, F_k. The factor is given for different cases according to a table, in this case 1.2.

$$F_d = \gamma_f F_k \tag{24}$$

To investigate if the buoy will deform plastically the rule refer to regulation DNV-OS-C101 ch D207 that gives a formula for the design resistance of a material, which is based on the characteristic resistance of the material used, divided by a resistance factor. The resistance factor is stated in the regulations.

$$R_d = \frac{R_k}{\gamma_m} \tag{25}$$

If the design resistance is larger than the calculated stress, the buoy will not deform plastically. If the stress is larger than the design resistance but smaller than the characteristic resistance the buoy may deform plastically. And in the last case, if the stress exceeds the characteristic resistance the buoy will deform plastically.

3.4 Hydrodynamic Analysis

In this report two different programs are used for hydrodynamic analyses. The first used is Wadam that uses potential theory in frequency domain, and the second used is OrcaFlex that uses non-linear theory in time domain to calculate the motions. Theory behind each program is given in following chapters.

3.4.1 Wadam

Wadam uses potential theory to calculate the first order radiation and diffraction effects on large volume structure and a 3D panel model to evaluate velocity potentials and hydrodynamic coefficients. This method can be used for both finite and infinite water depths. The flow in Wadam is assumed to be ideal and time- harmonic. The free surface condition is linearized for the first order potential theory while the non- linear free surface condition is used in the second order potential theory calculation. The calculations are performed only on the wet part of the buoy which means that the part above the water is not included. The radiation and diffraction velocity potentials are determined from the solution of an integral equation found by using Green's theorem with the free surface source potentials as Green's functions. The source strengths are calculated based on the source distribution method using the same source potentials as in the velocity potentials. Discretization of the integral equation to a set of algebraic equations approximating the body surface with a number of panels is performed. The source strengths are assumed to be constant over each panel. In this case no symmetry planes are used. The solution of the algebraic equation system provides the strength of the sources on the panels. The equation system, which is complex and indefinite, is solved either by a direct LU factorization or by an iterative method.

Due to the assumption of potential flow the velocity flow can be described as the gradient of the velocity potential, Φ, which fulfills the Laplace equation

$$\nabla^2 \Phi = 0 \tag{26}$$

in the fluid domain. The harmonic time dependence gives the definition of the complex velocity potential, ϕ, in relation to the velocity potential as

$$\Phi = \mathrm{Re}(\phi e^{i\omega t}) \tag{27}$$

Here ω is the frequency of the incident wave and t is the time. The connected boundary value problem will be expressed in terms of the complex velocity potential given that the product of all complex quantities with the factor $e^{i\omega t}$ applies. The linearized form of the free- surface condition is

$$\phi_z - K\phi = 0 \qquad z = 0 \tag{28}$$

Here $K = \omega^2/g$ and g is the acceleration of gravity. The velocity potential of the incident wave is then given as

$$\phi_0 = \frac{igA}{\omega}\frac{\cosh(kz+H)}{\cosh(kH)}e^{-k(x\cos\beta + y\sin\beta)} \tag{29}$$

Here the wave number k is the real root of the dispersion relation and β is the angle between the direction of propagation of the incident wave and the positive x- axis. Linearization of the problem allows breaking down the complex velocity potential, ϕ, into the radiation and diffraction components

$$\phi = \phi_R + \phi_D \tag{30}$$

$$\phi_R = i\omega \textstyle\sum_{j=1,6} \xi_j \phi_j \tag{31}$$

$$\phi_D = \phi_0 + \phi_7 \tag{32}$$

The constants ξ_j indicate the complex amplitudes of the body oscillatory motion in its six rigid- body degrees of freedom and ϕ_j denote the corresponding unit- amplitude radiation potentials. The velocity potential ϕ_7 represents the disturbance of the incident wave by the body, fixed at the undisturbed position of the body. The total diffraction potential ϕ_D represents the sum of ϕ_7 and the incident wave potential. The radiation and diffraction potentials is subjected to following conditions, on the undisturbed position of the body boundary

$$\phi_{jn} = n_j \tag{33}$$

$$\phi_{Dn} = 0 \tag{34}$$

Here (n_1, n_2, n_3) = **n** and (n_4, n_5, n_6) = **n** × **r**, **r** = (x, y, z). The unit vector **n** is normal to the body boundary and its direction is out of the fluid domain. The boundary value problem must be complemented by a condition of the outgoing waves applied to the velocity potentials,ϕ_j, j = 1,…7.

Wadam calculates the sum- and difference- frequency components of the second order forces, moments and rigid body motions in presence of bi- chromatic and bi- directional waves for the second order potential theory. The rigid body motions are here presented by quadratic transfer functions.

To account for viscous effects, Morison’s theory is applied. It is used to calculate the contributions to the equation of motion, later described, and to calculate the detailed forces, **F**, acting on the 2D Morison elements. The formulation of Morison’s equation used in Wadam is described as follows

$$\mathbf{F} = \omega^2(\mathbf{M} + \rho V_M \mathbf{C}_a)\xi - \omega^2 \rho V_M(\mathbf{C}_a + \mathbf{I})\mathbf{x} + i\omega\mathbf{B}(\mathbf{x} - \xi) + \mathbf{f}_c + \mathbf{f}_g + \mathbf{f}_b \qquad (35)$$

Here ω is the incident wave frequency, **M** is the 3 by 3 diagonal mass inertia matrix, $\mathbf{C}_a$ is the 3 by 3 diagonal added mass coefficient matrix, **I** is the 3 by 3 identity matrix, ρ is the density of water, V_M is the displaced volume of the Morison element and **B** is the linearized viscous damping matrix expressed by

$$\mathbf{B} = \frac{1}{2}\rho\sigma\mathbf{C}_D \cdot \frac{8}{3\pi}V_{max} \qquad (36)$$

Here $\mathbf{C}_D$ is the 3 by 3 diagonal drag coefficient matrix, σ is the projected area of the Morison element, **x** is the complex amplitude of the incident wave field, ξ is the complex amplitude of the motion, $\mathbf{f}_c$ is the fluctuating hydrostatic restoring force representing the first order restoring contributions integrated in the equation of motion, $\mathbf{f}_g$ is the fluctuating gravity force representing the acceleration of gravity calculated in a coordinate system fixed with the Morison model, $\mathbf{f}_b$ is the fluctuation body force calculated in a coordinate system fixed within the Morison model.

The linearized viscous damping matrix, **B**, in Morison's equation is found by linearization of the general viscous drag force, $\mathbf{F}_D$, stated as

$$\mathbf{F}_D = \frac{1}{2}\rho\sigma\mathbf{C}_D(\mathbf{v} - \dot{\mathbf{x}})|\mathbf{v} - \dot{\mathbf{x}}| = \frac{1}{2}\rho\sigma\mathbf{C}_D \cdot \frac{8}{3\pi}V_{max}(\mathbf{v} - \dot{\mathbf{x}}) = \mathbf{B}(\mathbf{v} - \dot{\mathbf{x}}) \qquad (37)$$

The term

$$\frac{8}{3\pi}V_{max} \qquad (38)$$

is a standard result obtained by the assumption that equal work is preformed at resonance by the non- linearized and equivalent linear damping term. V_{max} is the linearizing velocity amplitude given as input to Wadam. V_{max} is also applied in the linearized drag force calculation for all motion modes and all incident wave frequencies.

The contributions from Morison elements are calculated in the local coordinate systems given for the certain element, and are later transformed into the body coordinate system prior to the assembling of rigid body quantities.

The equation of motion is established for harmonic motion of rigid body systems expressed in the global coordinate system. The complex 6 by 1 motion vector, X(ω,β), can be found from the equation of motion by applying Newtons second law and including the added mass, damping and exciting force contributions acting on the panel and Morison elements of a hydro model. The equation of motions is given by

$$\left[-\omega^2\left(\mathbf{M} + \mathbf{A}(\omega)\right) + i\omega(\mathbf{B}(\omega)_p + \mathbf{B}_v) + \mathbf{C} + \mathbf{C}_e\right]\mathbf{X}(\omega, \beta) = \mathbf{F}(\omega, \beta) \qquad (39)$$

here **M** is the 6 by 6 body inertia matrix, **A**(ω) is the 6 by 6 frequency dependent added mass matrix, $\mathbf{B}(\omega)_p$ is the 6 by 6 frequency dependent potential damping matrix, $\mathbf{B}_v$ is the 6 by 6 linearized viscous damping matrix, **C** is the 6 by 6 hydrostatic restoring matrix, $\mathbf{C}_e$ is the 6 by

6 external restoring matrix and $\mathbf{F}(\omega,\beta)$ is the 6 by 1 complex exciting force vector for a certain frequency, ω, and incident wave heading angle, β.

The eigenvalues, λ, and eigenvectors, Φ, of the rigid body system is obtained for a given incident wave frequency by solving the eigenvalue problem

$$[-\lambda(\mathbf{M}+\mathbf{A}(\omega))+\mathbf{C}]\Phi=\mathbf{0} \tag{40}$$

The natural periods of the rigid body system at a given incident wave frequency is given as

$$T=\frac{2\pi}{\sqrt{\lambda}} \tag{41}$$

The wave theory used in Wadam for first order potential theory and Morison equation is planar and linear harmonic waves described by Airy theory [17].

3.4.2 OrcaFlex

The dynamic analysis in OrcaFlex is a time simulation of the motions of a model over a specified period of time, starting at the position determined by the static analysis. OrcaFlex uses two complementary methods to perform dynamic integration, one explicit and one implicit. The equation of motion is given by

$$\mathbf{M}(\mathrm{p},\mathrm{a})+\mathbf{C}(\mathrm{p},\mathrm{v})+\mathbf{K}(\mathrm{p})=\mathbf{F}(\mathrm{p},\mathrm{v},\mathrm{t}) \tag{42}$$

Here $\mathbf{M}$(p,a) is the system inertia load, $\mathbf{C}$(p,v) is the system damping load, K(p) is the system stiffness load and $\mathbf{F}$(p,v,t) is the external load. p, v and a are the position, velocity and acceleration vectors respectively and t is the simulation time.

The explicit integration scheme is the forward Euler method with a constant time step. The integration is used when the forces and moments acting on the free body and node are calculated. The equation of motion, which is derived from Newton's second law, is then obtained for each free body and each line

$$\mathbf{M}(\mathrm{p},\mathrm{a})=\mathbf{F}(\mathrm{p},\mathrm{v},\mathrm{t})-\mathbf{C}(\mathrm{p},\mathrm{v})-\mathbf{K}(\mathrm{p}) \tag{43}$$

The equation given above is not the system- wide equation of motion, but a local equation of motion for each free body and each line node. This means that solving these equations of motion only requires the inversion of 3 by 3 or 6 by 6 mass matrices, depending on the number of degrees of freedom. This equation is solved for the acceleration vector at the beginning of the time step, for each free body and each line node, and integrated using forward Euler integration. The time step required for stable integration is very short. Hydrodynamic and aerodynamic forces change little over a short time step, to save some computation time these may use a longer time step.

The implicit integration scheme uses the Generalized-α integration method. The forces, moments, mass etc. are calculated in the same way as for the explicit scheme. In the implicit scheme the system of equations of motions are solved at the end of each time step. The unknowns p, v and a are not known at the end of each time step, therefore an iterative solution method is required. Consequently each implicit time step consumes significantly more computational time than an explicit time step. Nevertheless, the implicit scheme is more stable for longer time steps than the explicit, which means that the implicit method is often faster for a whole calculation. The generalized-α integration has a controllable numerical damping which gives a more stable convergence, which allows longer time steps and faster calculations. The numerical damping is determined by specifying the level of high frequency dissipation, ρ_∞.

The added mass is calculated for each degree of freedom

$$F_{x,y} = DM_n \cdot A_{x,y} + C_{an} \cdot DM_n \cdot A_{rx,y} \tag{44}$$

$$F_z = DM_a \cdot A_z + C_{aa} \cdot DM_a \cdot A_{rz} \tag{45}$$

$$M_{x,y} = DI_n \cdot W_{x,y} + PW \cdot AI_n \cdot W_{rx,y} \tag{46}$$

$$M_z = DI_a \cdot W_z + PW \cdot AI_a \cdot W_{rz} \tag{47}$$

Here the first term is known as the Froude Krylov force or moment and the second term is the added mass force or moment. DM_n and DM_a are the instantaneous displaced mass for flow normal and axial to the cylinder respectively. DI_n and DI_a are the normal and axial moments of inertia of the instantaneous displaced mass of the cylinder. C_{an} and C_{aa} are the normal and axial added mass coefficients. PW is the proportion wet of the cylinder. AI_n and AI_a are the normal and axial added moments of inertia. A_{rx}, A_{ry} and A_{rz} are the components of local water particle acceleration relative to the body. W_x, W_y and W_z are the components of the angular acceleration of the local water isobar relative to global axis. W_{rx}, W_{ry} and W_{rz} are components of the angular acceleration of the local water isobar relative to the body.

The components relative to the body axes of the damping force and moment applied to a cylinder are as follows

$$F_{Dx,y} = PW \cdot UDF_n \cdot V_{rx,y} \tag{48}$$

$$F_{Dz} = PW \cdot UDF_a \cdot V_{rz} \tag{49}$$

$$M_{Dx,y} = PW \cdot UDM_n \cdot W_{rx,y} \tag{50}$$

$$M_{Dz} = PW \cdot UDM_a \cdot W_{rz} \tag{51}$$

Here UDF_n and UDF_a are the unit damping forces for the normal and axial directions. UDM_n and UDM_a are the unit damping moments for the normal and axial directions. V_{rx}, V_{ry} and V_{rz} are the components of the water velocity relative to the body.

To calculate the drag forces an assumption of cross- flow is used. The local x- and y-directions i.e. normal to the cylinder axis, the drag forces are given by

$$F_{Drx,y} = PW \frac{1}{2} \rho C_{dn} A_n V_{rx,y} \left| V_{rxy} \right| \tag{52}$$

$$F_{Drz} = PW \frac{1}{2} \rho C_{da} A_a V_{rz} |V_{rz}| \tag{53}$$

Here A_n and A_a are the drag area for the normal and axial direction, C_{dn} and C_{da} are the drag coefficient for the normal and axial direction and $|V_{rxy}|$ and $|V_{rz}|$ are the absolute magnitude of the relative velocity in the x-y plane and z plane respectively.

The drag moments are obtained by following equations

$$M_{Drx,y} = PW\frac{1}{2}\rho C_{dn} A_n W_{rx,y} |W_{rxy}| \tag{54}$$

$$M_{Drz} = PW\frac{1}{2}\rho C_{da} A_a W_{rz} |W_{rz}| \tag{55}$$

Here $|W_{rxy}|$ and $|W_{rz}|$ are the absolute magnitude of the component in the x- y plane or z plane of the angular velocity of the local water isobar relative to the buoy.

The drag area moments in the above equations are the 3rd moments of drag area about the axis of rotation. The drag area moment is therefore

$$M_{da} = \sum A|r|^3 \tag{56}$$

Here A is an element of drag area at an (absolute) distance $|r|$ from the axis of rotation. The modulus $|r|$ arises from the drag term in Morison's equation.

Slender bodies in near- axial flow experience a destabilizing moment called the Munk moment. This comes from potential flow and is separate from any moments associated with viscous drag. It is only well defined for a fully submerged body. The Munk moment can be modeled in OrcaFlex given by following equation

$$M_M = C_{MM} M_W \frac{1}{2}\sin(2\alpha) {V_W}^2 \tag{57}$$

Here C_{MM} is the Munk moment coefficient, M_W is the mass of water currently displaced. If the buoy is surface- piercing then this allows for the proportion of the body that is in the water. However, note that C_{MM} is still defined for a partially submerges body. V_W is the flow velocity relative to the buoy, at the point on the stack axis that is half way between the ends of the stack. α is the angle between the relative flow velocity and the buoy axis. The moment is applied about the line that is normal to the plane of buoy axis and the relative flow vector, in the direction that tries to increase the angle α.

OrcaFlex uses an extended form of the Morison equation to calculate hydrodynamic loads on slender elements. Morison's equation was originally put up to calculate the wave loads on a fixed vertical cylinder. This means that it is two components calculating the force, one relative to the water particle acceleration i.e. the inertia force, and one related to the water particle velocity i.e. the drag force. For a moving object the force equations is modified to take the movement of the body into account. This gives following equation

$$F_w = (\Delta \cdot a_w + C_a \cdot \Delta \cdot a_r) + \frac{1}{2}\rho C_D A V_r |V_r| \tag{58}$$

Here F_W is the fluid force, Δ is the mass of the fluid displaced by the body, a_w is the fluid acceleration relative to the earth, a_r is the fluid acceleration relative to the body, A is the drag area and V_r is the fluid velocity relative to the body. The first term is the inertia force and the second one is the drag force. The inertia force consists of two components, the Froude-Krylov and the added mass. The Froude- Krylov represents the proportional fluid acceleration relative to the earth and the added mass represents the proportional fluid acceleration relative to the body.

In OrcaFlex either use regular or irregular waves can be used to describe the wave profile. For the regular waves linear Airy theory or non-linear Dean Steam, Stokes' fifth or Cnoidal theory can be used. For irregular waves JONSWAP, ISSC (also known as Bretschneider or modified Pierson- Moskowitz), Ochi-Hubble, Torsethaugen and Gaussian Swell can be used [18].

4. Method

The methods used in his project are described in this chapter. In the hydrostatic and structural part hand calculations are performed, and in the hydrodynamic part simulations are performed in two different software. The hydrostatic and structural calculations are describing different sequences in the decommissioning phase, near the quay. While the hydrodynamic analysis occurs when the buoy is free floating at the site or in the fjord.

4.1 Hydrostatic Analysis

To get the equilibrium of the different ballast conditions some hand calculations in MathCAD was performed. These were later compared to the results calculated in AutoHydro. The results from AutoHydro need to be recalculated into the coordinate system used in this project.

First, the radius of each section was determined, due to change in diameter along the column. The different radius was later used to define the limits in the integrals.

The longitudinal center of gravity was calculated due to the weight of the column, water ballast, solid ballast and the topside, if this was connected.

$$LCG = \frac{\sum w_i lcg_i}{\sum w_i} \tag{12}$$

The first thing to investigate was how large part of the buoy that was submerged. To be able to do this, the draft of the buoy had to be projected to the water surface, this to find the intersection between the water surface and the buoy. The intersection was used as one integration limit when integrating over the cross section to get the area of the column.

$$A_{column} = 2\int_{-r}^{r} \sqrt{r^2 - z^2}dz \tag{59}$$

The area is later used to get the volume of the submerged part of the column.

$$V_{column} = \int_0^L A_{column}dx \tag{60}$$

To obtain the correct draft and trim the volume of the cylinder was set to the displaced volume due to the buoyancy. To be able to get the vertical and longitudinal center of buoyancy an equation for the area of the cross section at a given draft is needed.

$$A_z = 2\left(\frac{z}{tan\alpha} - draft\right)\sqrt{r^2 - z^2} \tag{61}$$

Now, the vertical and longitudinal center of buoyancy can be calculated.

$$z_B = \frac{\int_{-r}^{r} zA_z dz}{V_{column}} \tag{62}$$

$$x_B = \frac{\int_0^L xA_{column}dx}{V_{column}} \tag{63}$$

To obtain equilibrium the center of buoyancy and gravity must be at the same longitudinal position in the coordinate system of the waterline. With the equations typed above and the constraint of longitudinal center of buoyancy and gravity the trim angle and draft at the aft could be found using an iteration process, built into MathCAD.

When the trim angle and draft at the aft is known, the center of flotation could be calculated. The center of flotation is calculated according to the definition. The first thing to find out is where, in longitudinal direction, the waterline and the column intersects. This is where the distance from the waterline to the center of the buoy is equal to the radius. The next step is to investigate if the forward part of the buoy is submerged or not. This is done by calculating at which point the distance between the center of the buoy and the waterline is equal to minus the radius. If this point lies outside the range of the buoy the forward part is fully submerges, otherwise it is not. When knowing this, the center of flotation may be calculated. When the buoy is tilted, the area of the waterplane has the geometry of an ellipse. If the forward part is fully submerged, the water plane area is a part of an ellipse, and if the forward part leaves the water the area is a full ellipse. When the waterplane area is a full ellipse the center of flotation lies at half the waterline due to symmetry. In the other case a correction needs to be done because the waterplane area is not symmetric. This is performed by integrating over the area to get the center of the waterplane area.

$$x_{area} = \frac{2\int_{-a}^{a}\frac{b}{a}\sqrt{a^2-b^2}\,x\,dx}{2\int_{-a}^{a}\frac{b}{a}\sqrt{a^2-b^2}\,dx} \tag{64}$$

Here, a, is the waterplane length, and b is the waterplane breadth i.e. the radius. X_{area} is equal to zero the case of a full ellipse. The longitudinal center of flotation is then known.

$$LCF = x_1 + a + x_{area} \tag{65}$$

The last thing to do was to calculate the metacentric height, GM. To be able to do this the transverse and longitudinal center of gravity, the transverse and longitudinal center of buoyancy and the metacentric radius, BM, is needed. The first two things have already been calculated, remaining is the BM in transverse and longitudinal direction.

4.2 Structural Analysis

The calculations are performed in the same MathCAD sheet as the hydrostatic analysis. The calculations were performed for each given case.

To get the distributed load all loads had to be vectorized. The buoy was divided in 19 pieces, one for each compartment of the column, and one extra for the part between 0 m and 6 m. The steel weight of each compartment was taken from the weight report and distributed over the length of the compartment. The weight of water and solid ballast were distributed over the compartments that their weight was located. The buoyancy was also distributed over each submerged compartment.

$$\overrightarrow{F_{load}} = \sum \overrightarrow{F_{ci}L_{ci}} \tag{66}$$

When calculating the shear forces, V, a loop was put up to calculate the area under the load case graph. The force was calculated at each bulkhead, 0- 19. To get the shear force at each bulkhead the bulkhead named 1 and 19 had to be calculated manually and put into the vector, this because at bulkhead 1 no area needs to be added, and for 19 the area of compartment 18 needs to be added. The others were calculated in a new loop.

$$\vec{V_c} = \sum \overrightarrow{F_{load\iota} L_{c\iota}} \tag{67}$$

The bending moments were calculated by the same method as the shear forces. Bulkhead 1 was put in manually, and the other ones were calculated in a loop. The lever arm was approximated to half the length of each compartment.

$$\overrightarrow{M_c} = \sum \frac{\overrightarrow{V_{c\iota} L_{c\iota}}}{2} \tag{68}$$

Knowing the bending moment, the stress can be calculated. The stress was calculated for each bulkhead.

$$\overrightarrow{\sigma_{top}} = \frac{\vec{M_c} z}{I} \tag{69}$$

Here z is equal to the radius and I is the moment of inertia for a thin-walled tube about its neutral axis.

$$I = \pi r^3 t_w \tag{70}$$

Here r is the radius and t_w is the thickness of the wall of the tube.

According to DNV rules DNV-RP-H102, the buoy is allowed to deform and fail in some parts during the operation. The rule refers to the rule DNV-OS-C101 which gives a safety factor of 1.2 for this case. In the design basis it is stated that the buoy has not corroded, but to be on the safe side the calculations is performed with a corrosion of 5 mm. The characteristic resistance of the material is 335 MPa.

$$\sigma_c = \frac{\vec{M}_c z}{I_c} \tag{71}$$

with I_c as

$$I_c = \pi r^3 t_c \tag{72}$$

To be sure if the buoy deforms plastically or not, the characteristic resistance is compared with the calculated stress. The characteristic resistance is divided by a resistance factor, γ_m, which in this case is equal to 1.15. The design resistance, R_d, is then obtained.

$$R_d = \frac{R_k}{\gamma_m} \tag{73}$$

4.3 Hydrodynamic Analysis

The hydrodynamic analysis is made in two steps, first a frequency domain analysis in Wadam and later a time domain analysis in OrcaFlex. In Wadam the motions of the buoy cannot be visualized, therefore are the results imported to OrcaFlex, case HD1 and HD2. The case, HS5, investigated is presented in chapter 5, but no trim or heel is taken into account.

4.3.1 Wadam

The panel model used in the Wadam simulations is created in Autodesk Inventor, and via Abaqus, imported to Wadam. The model consists of a small cylinder reaching from 0 to 6 m, to account for the lower buoyancy at the bottom, and a larger one from 6 m and upwards. A mass model was also created to combine with the panel model give the model the correct properties. The mass model was created according to the ballast condition given in case HS5 in section 5. The model is presented in Fig. 5 and the mass model is shown in Fig. 6. The mass model contained a topside, that is not shown in Fig. 5.

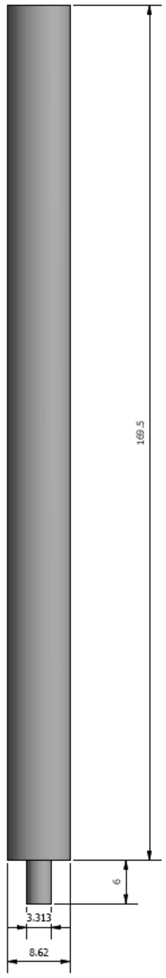

Fig. 5. Model used in hydrodynamic analysis

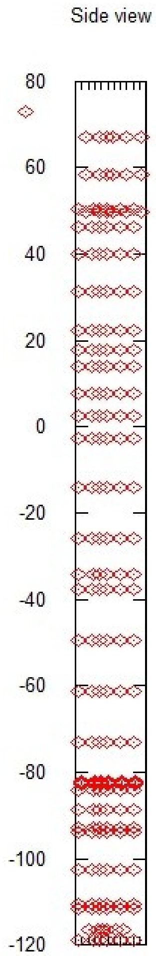

Fig. 6 The distribution of the mass in the mass model used in Wadam

In addition to the panel and mass model a Morison model is created, to account for viscous effects. To the Morison model drag properties are defined. The drag coefficient is put to 0.95 from the DNV-RP-C205 fig 6-6. The three models are connected in a Fortran- program and the input files to Wadam are generated.

The first thing is to define the properties to be calculated and which environment parameters that will be used. For the first order potential theory and Morison theory Wadam uses Airy theory as wave spectrum for surface waves.

After the first simulation the hydrostatic properties are compared with the ones in case HS5, and a hand calculations of the coupled natural periods according to the equation given in the chapter 4.3.1 was made to confirm the obtained eigen periods from the simulation. When the periods and the hydrostatic values were confirmed, the mass model was updated so that the TCG and LCG was set to 0, this to avoid the trim of the buoy, and for easier comparison with the results from OrcaFlex.

A damping matrix was created in HydroD, based on information from similar buoys. The matrix was later used in the calculations. The calculations where run three times, to created different output files which later were used as necessary input files to OrcaFlex.

After the simulations were performed, the data was post processed in Postresp, to obtain the Response Amplitude Operators, RAO's, from the model. In Postresp the results from the simulation can be easily shown.

4.3.2 OrcaFlex

Two models were set up; one to use with imported RAO's from Wadam and one for simulation according to Morison theory in OrcaFlex. The same geometry as for the Wadam model was used. The geometric model used in the simulations is shown in Fig. 7.

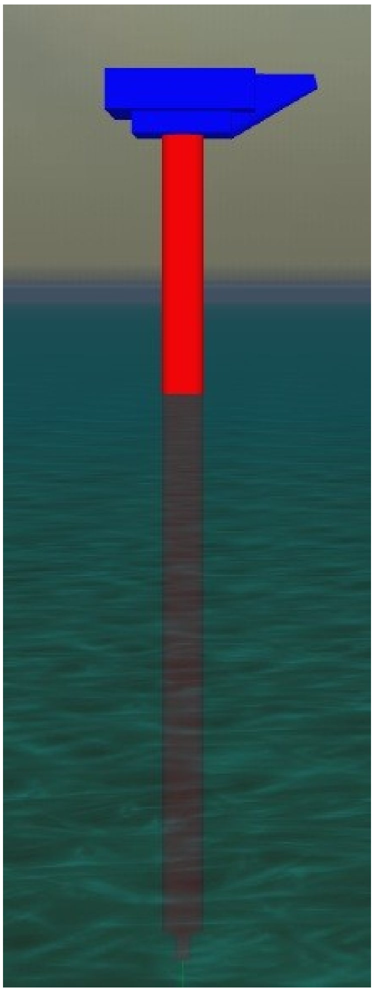

Fig. 7 Geometric model used in OrcaFlex

Four simulation types were performed in OrcaFlex the first one with imported displacement RAO's from Wadam, the second one with imported load RAO's from Wadam, the third one was with imported load RAO's from Wadam and Morison theory in OrcaFlex, the fourth and final one was with only Morison theory in OrcaFlex.

Simulation 1 was with imported displacement RAOS's was designed to get the same values as in Wadam. No added mass, damping or drag were imported or typed in OrcaFlex. This case uses the RAO's from the simulation performed in Wadam to calculate the motions.

Simulation 2 with imported load and displacement RAO's from Wadam consist of the added mass, hydrostatic stiffness and hydrodynamic damping from potential theory. No Morison theory is taken into account in this case, and therefore no viscous damping. For the added mass and damping it is only the first order motions taken into account.

Simulation 3 is as simulation 2 but in addition drag force according to Morison theory was added in OrcaFlex. The drag force coefficients, C_D, were set to .0.95 according to DNV-RP-C205 fig. 6-6, and the drag area was calculated in normal and axial direction.

In the last simulation, simulation 4, the whole model was set up in OrcaFlex. The coefficients for added mass, C_A, were set to 0.7 according to DNV-RP-C205 appendix D and the drag coefficients were set to 0.95 according to DNV-RP-C205 fig. 6-6, with the assumption of corrosion and marine growth. The drag was applied on the small column and at the bottom of the larger one. The upper part of the column was considered having a skin friction of 0.02 as drag coefficient. The mass moment of inertia is defined through the center of gravity in the local coordinate system for each part, and was therefore calculated for each part in the model i.e. hull, topside, solid ballast etc. The drag area was calculated for the hull in normal and axial direction. To keep the buoy in the same place, a small link is anchored at the seabed, with really low stiffness.

The simulations were first performed with regular waves according to Airy theory with a wave amplitude of 1 m. Airy theory was chosen from DNV-RP-C205 fig. 3-2, as the most proper one describing the case. It is also the one used in Wadam which is convenient when comparing the results. The simulations were performed for periods of 5, 10, 15, 20 and 120 s. For case 4 the simulation was performed with a wave amplitude of 10 too, with the same periods.

The dynamic simulation was performed with implicit integration with a time step of 0.1 s. The simulation time, for the regular waves, was 200 s, divided into one sequence from 0 to 20 s when the waves were building up and one from 20 to 200 s when the simulation was performed. To see if the models have a settling time a longer simulation of 3600 s were performed.

5. Results

During the hydrostatic and structural analysis case HS 1 to 4 where calculated, and in the hydrostatic and hydrodynamic part case HS5 was calculated. The first case, HS1, was made with low complexity to get a short calculation time and to be able to check the equations used. These cases were given from the costumer in the design basis. For case HS1, HS2 and HS4 the requirements is that the bottom part, containing solid ballast shouldn't be submerged more than a couple of meters so that the iron ore could be removed. The requirements for case HS3 is that the buoy should be tilted so that the topside reaches over the quay and the bottom part is still floating. The topography of the bottom and the height of the quay is the actual one at the yard.

Case HS1 – without topside with water ballast in the top and solid ballast in the bottom, uniform geometry

The first case is when compartment 3 to 9 is fully filled with water ballast and compartment 17 is filled with solid ballast. Compartments 1 and 2 are considered flooded. The topside is considered removed. To simplify the problem the column is considered uniform, with a constant cross section, the diameter is set to 8.59 m according to the design basis. In this case the whole column is considered buoyant. A drawing of the case could be seen in Fig. 8.

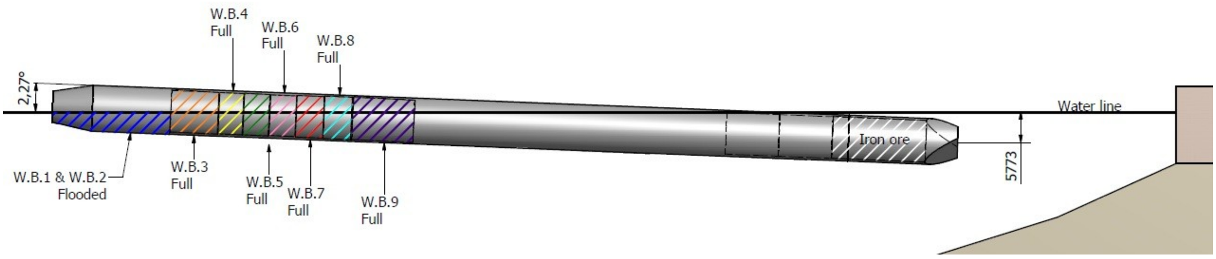

Fig. 8. The ballast condition investigated in case 1

Case HS2 – without topside with water ballast in the top and solid ballast in the bottom with varying diameter

The second case is similar to the first one. Compartment 3 to 8 is fully filled with water ballast, and compartment 17 is filled with solid ballast. In this case the column is not considered uniform; the diameter is varying over the column. In this case the whole buoy is not considered buoyant, the aft tank, 18, is considered to have 40 tonnes of buoyancy. Compartments 0 and 1 are considered flooded and do not have any buoyancy. Compartment 2 is considered to have tube with a diameter of 2.8 m that is buoyant, the rest is considered flooded. The drawing of case HS2 is shown in Fig. 9.

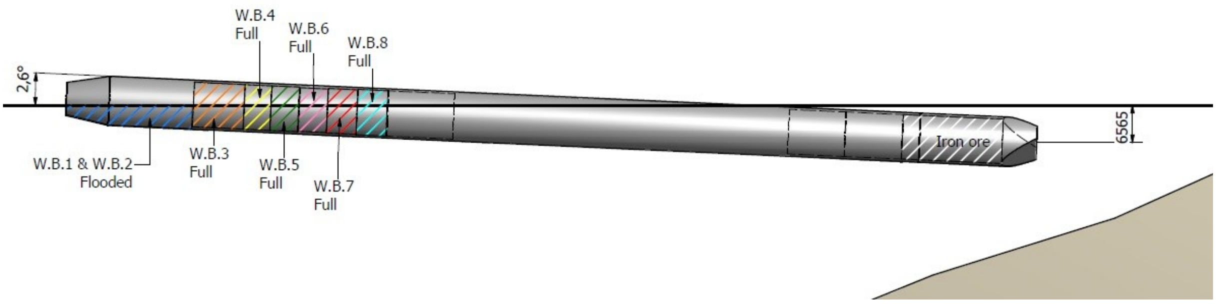

Fig. 9 Ballast condition investigated in case 2

Case HS3 – with topside with water ballast and solid ballast in the bottom

In the third case the buoy is considered tilting over the quay, before the topside is removed. Compartment 17 is filled with solid ballast and compartment 16 is partly filled with water ballast. The diameter is varying over the column. The topography shown in Fig. 10 is the actual topography at the yard where the operation is going to be performed.

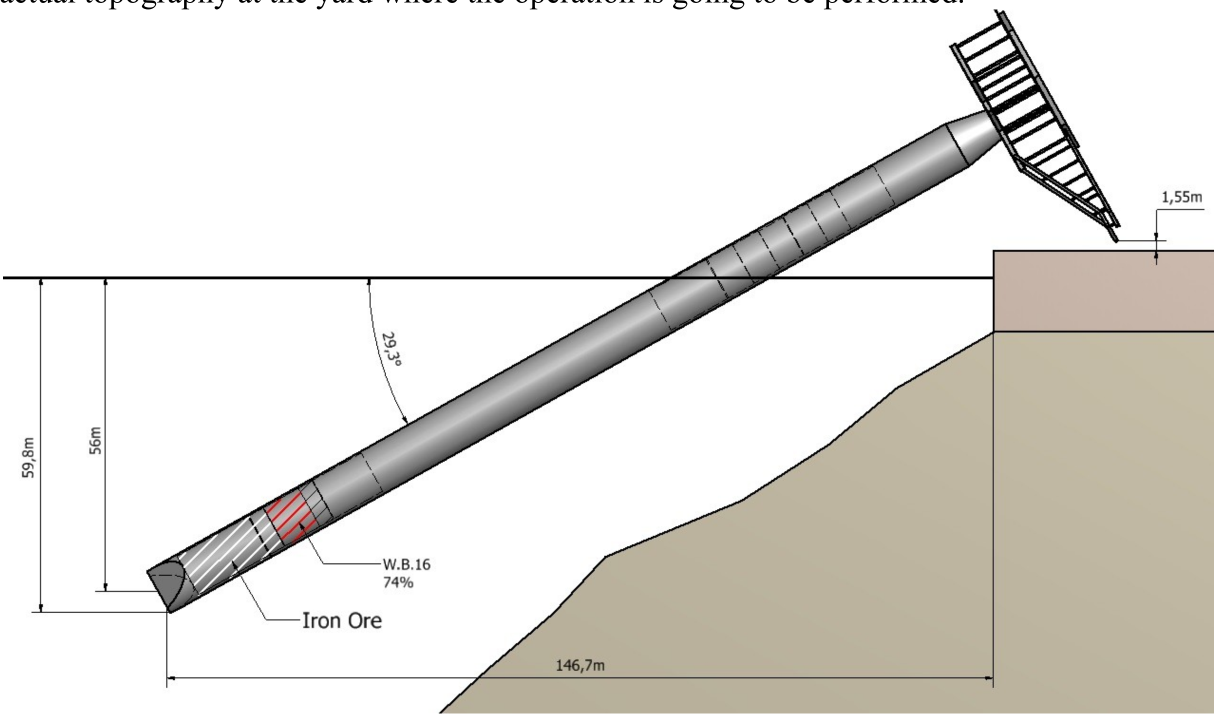

Fig. 10. The buoy is located over the quay just before topside removal

Case HS4 – without topside water ballast in the top, solid ballast in the bottom and external force lifting the aft part

In the fourth case the topside is removed and compartment 3 to 9 is fully filled with water ballast. Compartment 17 is filled with solid ballast. To be able to remove the solid ballast the aft needs to be only a couple meters below the waterline. In this case this is investigated if this is possible when a crane is lifting the aft part. In this case the whole buoy is not considered buoyant, the aft tank, 18, is considered to have 40 tonnes of buoyancy. Compartments 0 and 1 are considered flooded and do not have any buoyancy. Compartment 2 is considered to have tube with a diameter of 2.8 m that is buoyant, the rest is considered flooded. A drawing of case HS4 is shown in Fig. 11.

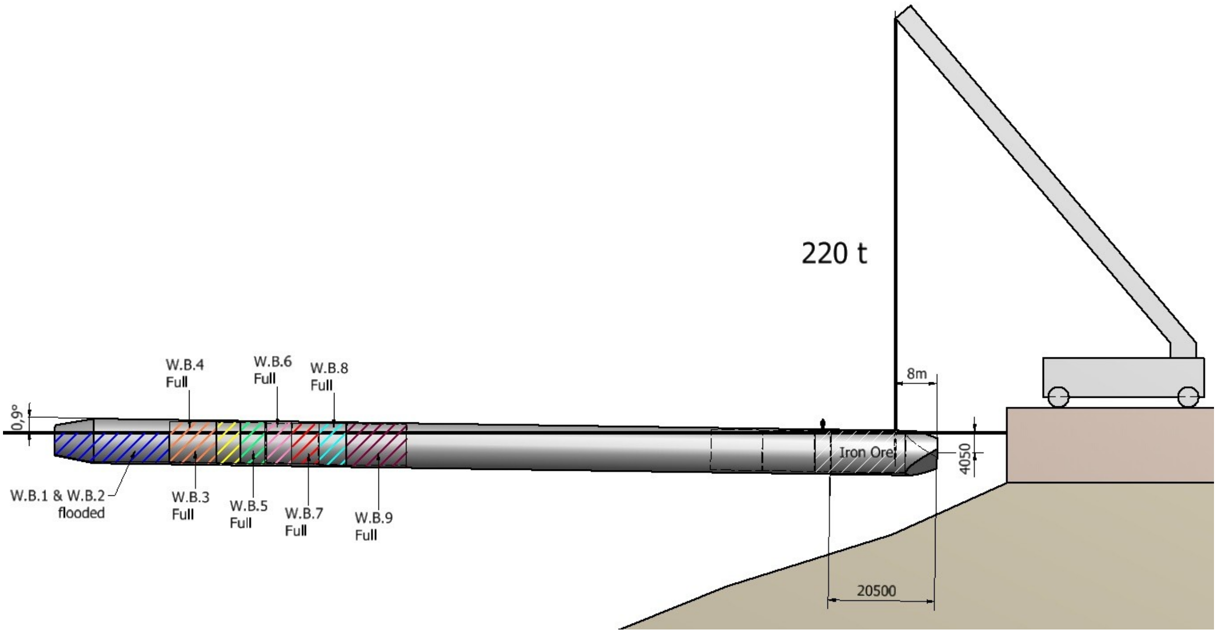

Fig. 11. Removal of solid ballast, one end is supported by a crane

Case HS5 – buoy disconnected from seabed standing in vertical direction

In case five the buoy is disconnected from the seabed, just before tow to shore. This situation also occurs after the tow, before the mooring at the fjord. Compartment 16 is completely filled with water ballast, and compartment 15 is partly filled. In compartment 17 the solid ballast is located. The diameter is considered varying over the column and the topside is connected to the column. This case is only used in the hydrodynamic part. The case is show in Fig. 12.

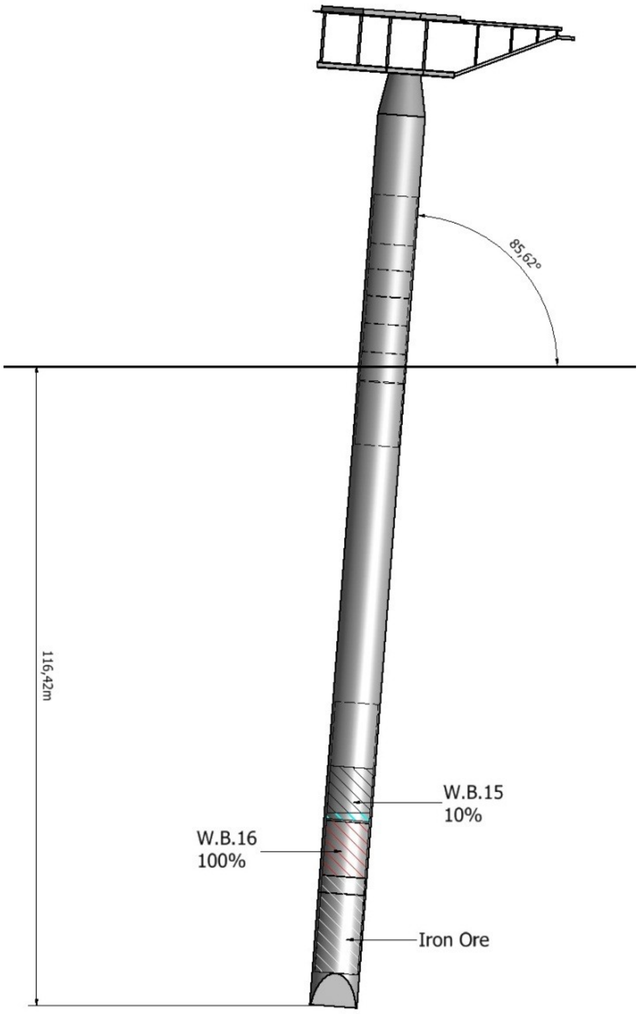

Fig. 12. Buoy in equilibrium before and after wet tow

5.1 Hydrostatic Analysis

The calculations of the hydrostatic part were calculated in MathCAD and later the results were compared with the results from AutoHydro. The results match in most of the variables, but there is a difference in VCB and GM at almost all cases. The draft from AutoHydro has been re-calculated so they are defined in the same way as the MathCAD calculations and therefore comparable. All distances are defined from the aft i.e. the bottom. The draft, T, is defined in the aft of the column as the vertical distance from the waterline to the center of the column. This is not the case in AutoHydro so the drafts are recalculated to fit to this coordinate system. The trim angle, α, is positive when trimming forward and negative when trimming aft. The full calculations in MathCAD are shown in Appendix D, and the results from AutoHydro are presented in Appendix B. The recalculated drafts from AutoHydro are presented in Appendix C.

5.1.1 Case HS1

In this case the calculations were performed first with a uniform geometry i.e. the diameter did not differ over the column. Later the calculations were performed with a changing diameter.

When the geometry was uniform following results were obtained from the MathCAD sheet and AutoHydro:

Table 1 Comparison of the results from case 1

	MathCAD	**AutoHydro**
T	5.779 m	5.773 m
α	-2.272˚	-2.27˚
LCB	72.265 m	72.278 m
VCB	-0.874 m	-0.513 m
LCG	72.3 m	72.3 m
VCG	-0.039 m	-0.039 m
LCF	112.469 m	112.328 m
GM_T	-0.239 m	0.044 m

The results are similar in most if the hydrostats, the largest difference is in VCB and GM_T. The GM_T is calculated from the VCB and the differences are the same.

5.1.2 Case HS2

In this case the diameter varies over the column which makes the case a bit more complex.

Following results were obtained from the MathCAD sheet and AutoHydro:

Table 2 Comparison of the results from case 2

	MathCAD	**AutoHydro**
T	6.732 m	6.606 m
α	-2.648˚	-2.59˚
LCB	69.36 m	69.378 m
VCB	-0.811 m	-0.39 m
LCG	69.395 m	69.395 m
VCG	-0.043 m	-0.043 m
LCF	112.175 m	108.259 m
GM_T	-0.282 m	0.039 m

In this case the draft and trim angles differs a bit more than in the previous case. The difference in trim angle affects the LCF rather much in this case. As in the previous case the VCB and GM_T are not similar.

5.1.3 Case HS3

In this case the calculations were performed with a variable diameter. The trim angle in this case is larger than for the other cases. In this case a topside is added to the calculations.

Following results were obtained in the MathCAD sheet and AutoHydro:

Table 3 Comparison of the results from case 3

	MathCAD	AutoHydro
T	58.75 m	56.027 m
α	-30.86°	-29.29°
LCB	59.828 m	59.879 m
VCB	-0.07 m	-0.068 m
LCG	59.828 m	59.828 m
VCG	-0.149 m	-0.149 m
LCF	114.536 m	114.525 m
GM_T	0.139 m	0.189 m

In this case the trim angle and draft differs a bit, but the VCB and GM_T is almost the same. This is the opposite of the two cases presented above. The LCG is almost the same.

5.1.4 Case HS4

In this case the calculations are performed with a variable diameter and with a vector between the center of gravity and buoyancy to take the heel into account.

Results obtained from MathCAD and AutoHydro:

Table 4 Comparison of the results from case 4

	MathCAD	AutoHydro
T	4.116 m	4.087 m
α	-0.862°	-0.85°
LCB	74.106 m	74.134 m
VCB	-2.369 m	-0.353 m
LCG	74.141 m	74.141 m
VCG	-0.04 m	-0.04 m
LCF	87.115 m	85.117 m
GM_T	- 1.902 m	0.045 m

As in the two fist cases the VCB and GM_T differs a lot. The LCF is larger due to the larger trim angle.

5.1.5 Case HS5

In case HS5 the column is located in a vertical direction, and the calculations are performed according to this. The equations used in the previous cases are not valid when the trim angle is about 90 degrees. In this case the equations are due to a cylinder in vertical position. The diameter is varying over the length of the column.

The results from MathCAD are compared with the ones from AutoHydro as follows in Table 5:

Table 5 Comparison of results from MathCAD and AutoHydro for case 5

	MathCAD	**AutoHydro**
T	115.839 m	116.03 m
α	-4.265˚	-4.38˚
LCB	0.003 m	0.003 m
VCB	60.806 m	60.767 m
LCG	0.113 m	0.113 m
VCG	59.332 m	59.332 m
LCF	0 m	0 m
GM_L	1.516 m	1.482 m

Longitudinal center flotation is always 0 in the vertical case it is always in the body coordinate system. In this case all hydrostats in MathCAD is similar to the ones obtained in AutoHydro.

5.2 Structural Analysis

The structural analysis was put up for the first four cases. In case HS1 and HS4 the maximum stress, calculated from DNV rules, does not exceed the yield strength of the material. In case 2 and 3 the calculated stress exceeds the yield stress of the material and the buoy will therefore deform plastically. The maximum yield stress allowed before plastic deformation, is 335 MPa. The full calculations can be seen in Appendix E.

5.2.1 Case HS1

The shear force and bending moment are calculated for each bulkhead, shown in Fig. 13. As seen in the figure the bending moment on bulkhead 19 is not equal to zero. This is due to the approximation that the lever arm is equal to half the length of each compartment, because the shear force is zero at bulkhead 19.

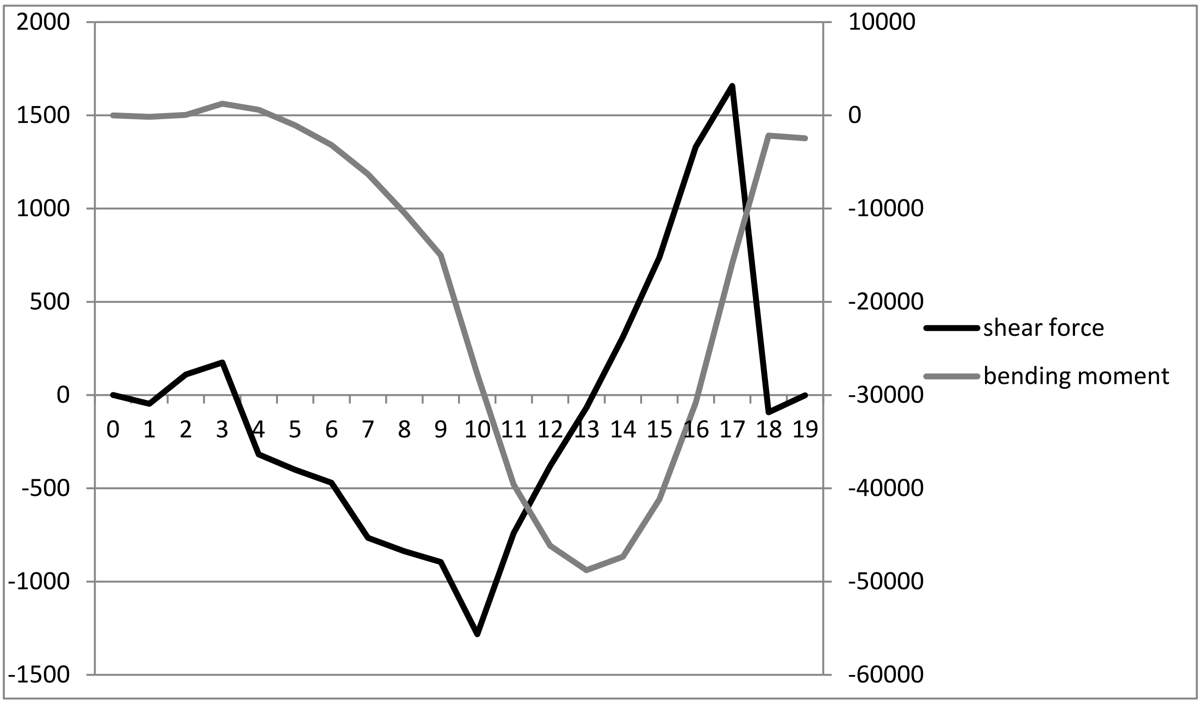

Fig. 13. Shear and bending moment diagram for case 1

The different stresses, the real and the one calculated with a load factor according to DNV rule, DNV-RP-H102, and corrosion of 5 mm. The maximum stress calculated does not exceed the yield stress given for the material, shown in Fig. 14 and Table 6.

Table 6 Maximum stresses calculated for case 1

	σ_{min}
Not corroded	-183.42 MPa
With corrosion and SF	-247.617 MPa
Design resistance DNV	-291.30 MPa

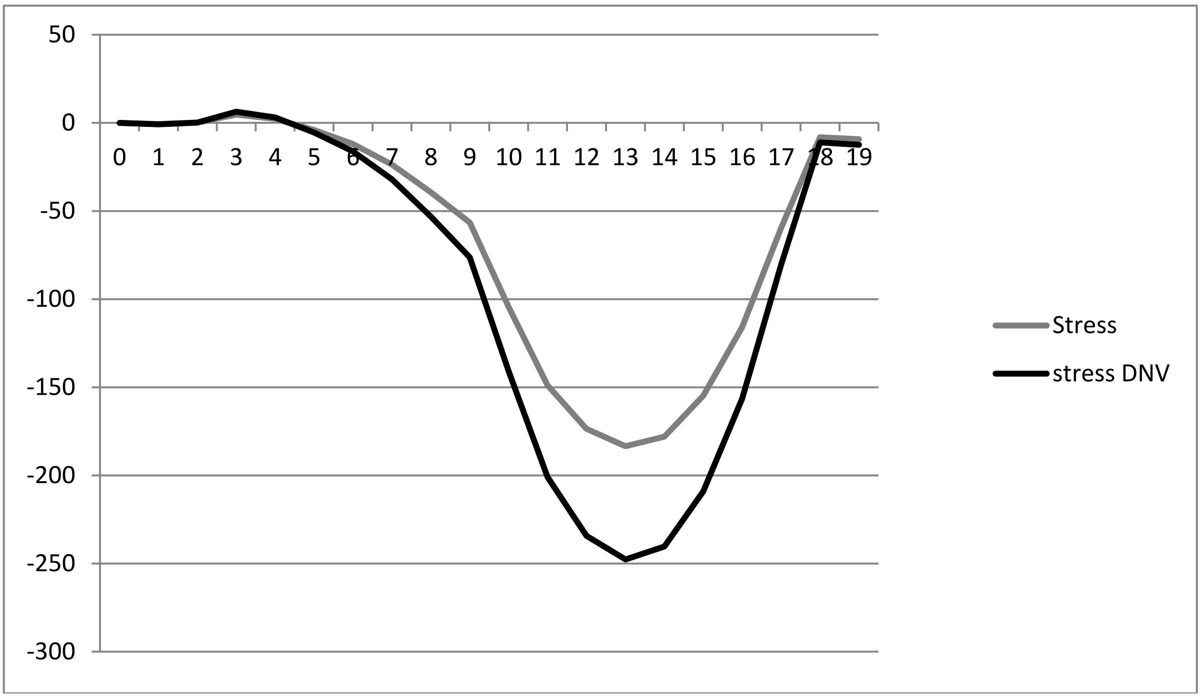

Fig. 14. Stress diagram over the real stress and the stress with load factor according to DNV and corrosion

5.2.2 Case HS2

The shear force and bending moment is calculated for each bulkhead, shown in Fig. 15. As seen in the figure the bending moment on bulkhead 19 is not equal to zero, the same observation as in the previous case. This is due to the approximation that the lever arm is equal to half the length of each compartment, because the shear force is zero at bulkhead 19.

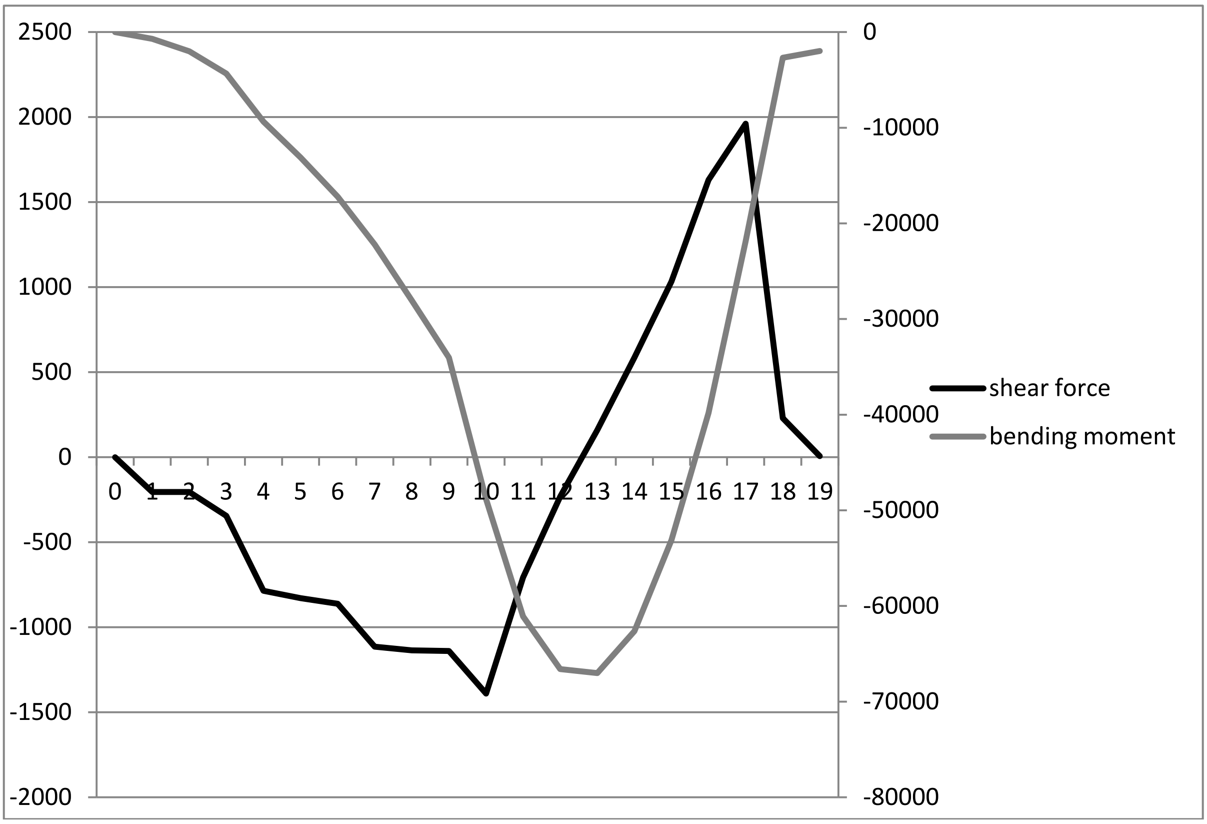

Fig. 15. Shear and bending moment diagram for case 2

The different stresses, the real and the one calculated with a load factor according to DNV rule, DNV-RP-H102, and corrosion of 5 mm. The maximum stress calculated exceeds the yield stress given for the material, shown in Fig. 16 and Table 7. The real stress does not exceed the yield stress, this means that the buoy may deform plastically.

Table 7 Maximum stresses calculated for case 2

	σ_{min}
Not corroded	-252.43 MPa
With corrosion an SF	-340.79 MPa
Design resistance DNV	-291.30 MPa

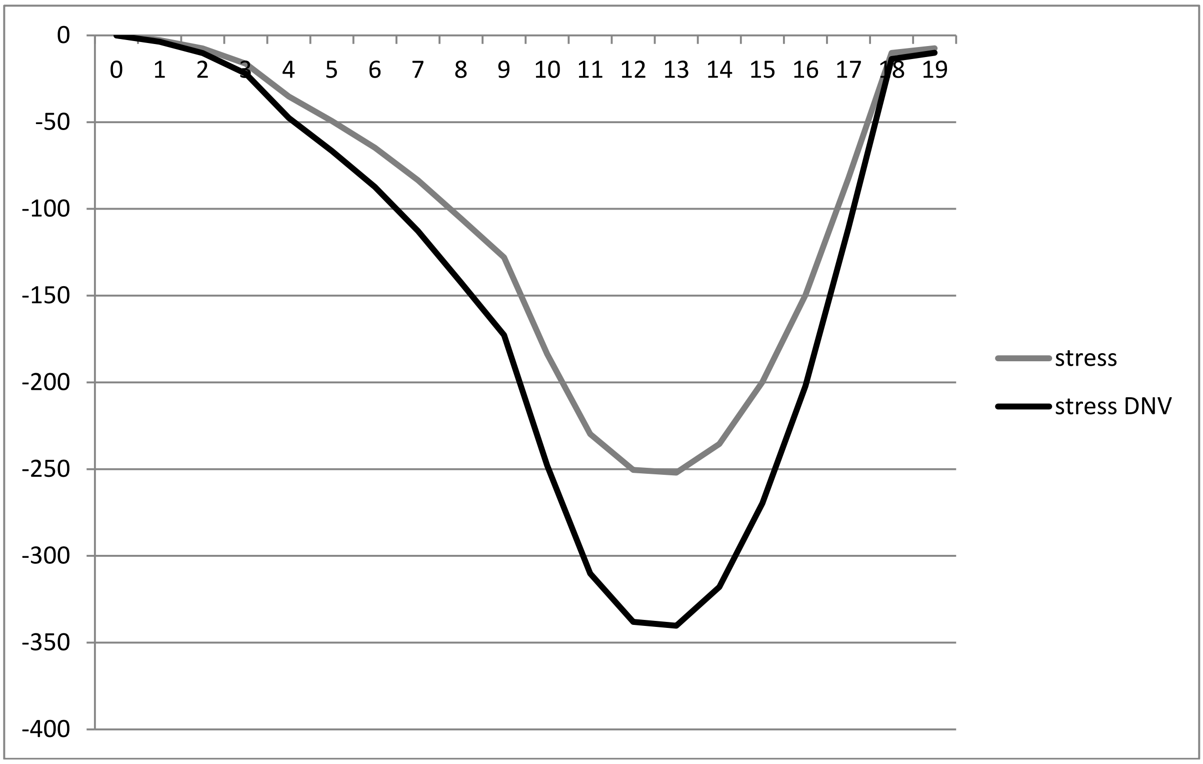

Fig. 16. Stress diagram over the real stress and the stress with a load factor according to DNV and corrosion

5.2.3 Case HS3

The shear force and bending moment is calculated for each bulkhead, shown in Fig. 17. As for the previous cases the bending moments are not equal to 0 at bulkhead 19.

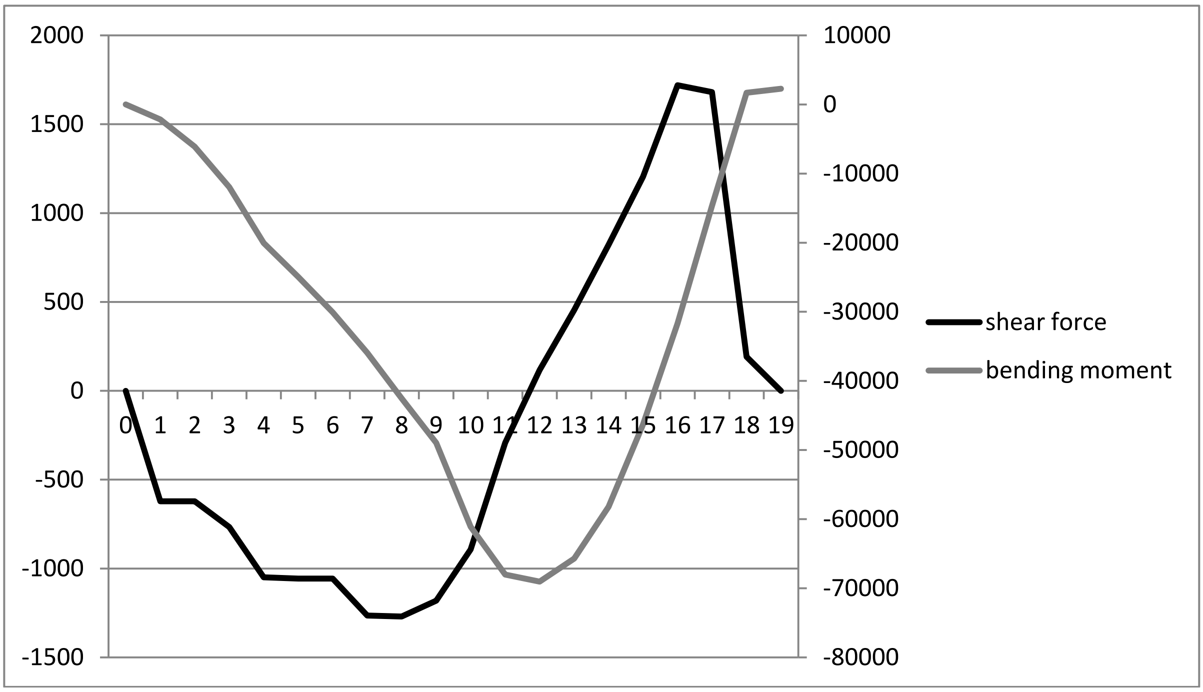

Fig. 17. Shear and bending moment diagram for case 3

The maximum stress calculated does not exceed the yield stress given for the material, shown in Fig. 18 and Table 8. The stress calculated with respect to DNV rules and corrosion does exceed the yield stress, this means that the buoy may deform plastically.

Table 8 Maximum stresses calculated for case 3

	σ_{min}
Not corroded	-259.66 MPa
With corrosion and SF	-350.53 MPa
Design resistance DNV	291.30 MPa

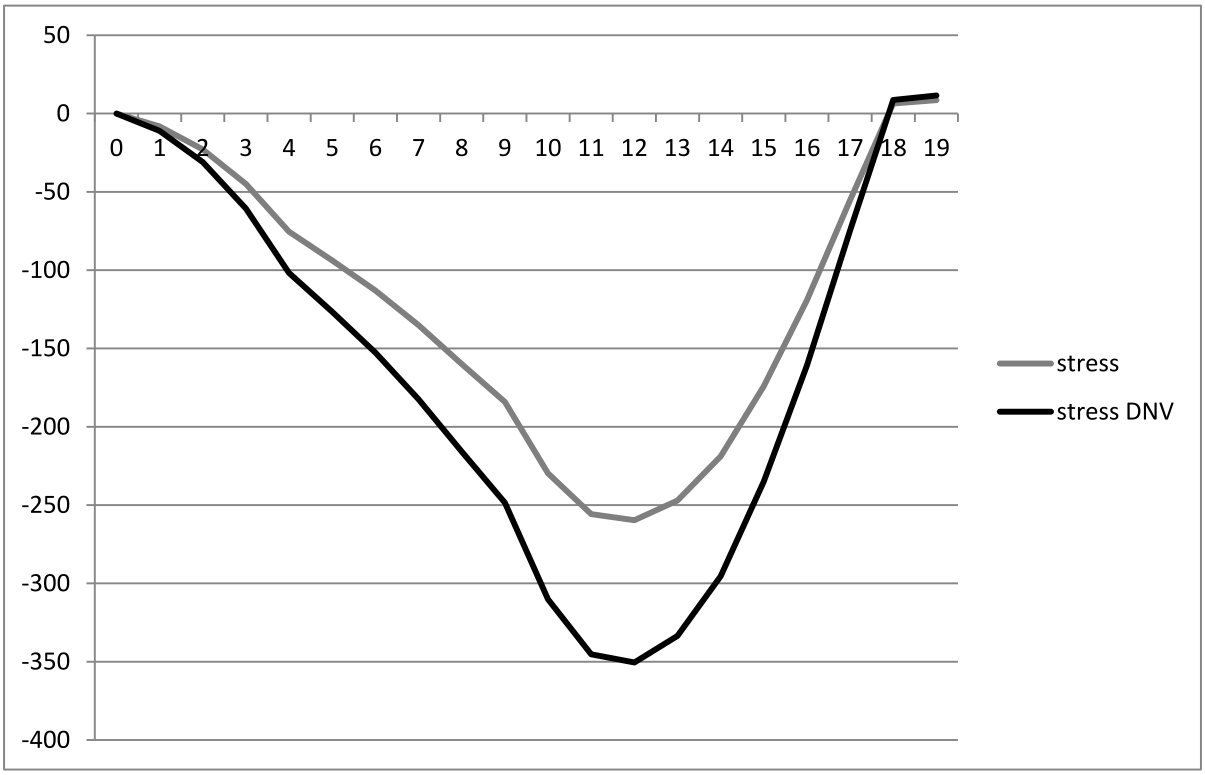

Fig. 18. Stress diagram over the real stress and the stress with a load factor according to DNV and corrosion

5.2.4 Case HS4

The shear force and bending moment is calculated for each bulkhead, shown in Fig. 19. As seen in the figure the bending moment on bulkhead 19 is not equal to zero.

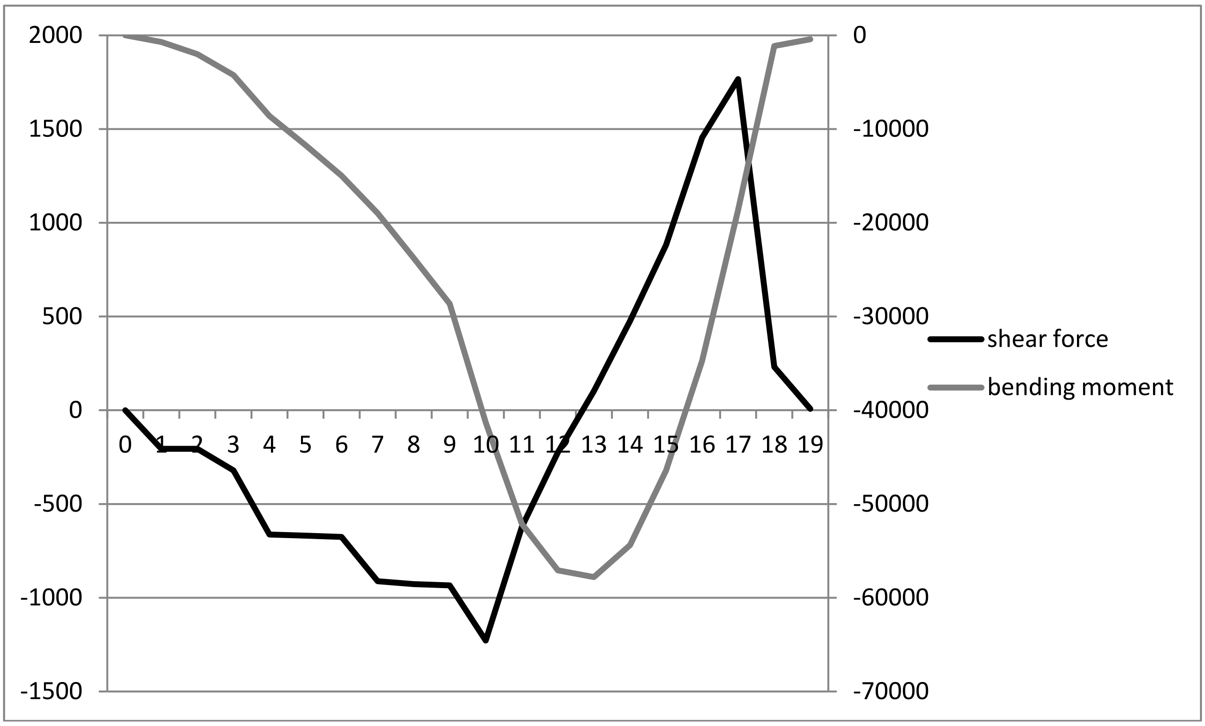

Fig. 19. Shear and bending moment diagram for case 4

The different stresses, the real and the one calculated with a load factor according to DNV rule, DNV-RP-H102, and corrosion of 5 mm. The maximum stress calculated does exceed the yield stress given for the material, shown in Fig. 20 and Table 9. This small difference will is too small to have any impact on the material.

Table 9 Maximum stresses calculated for case 4

	σ_{min}
Not corroded	-217.31 MPa
With corrosion and SF	-293.37 MPa
Design resistance DNV	-291.30 MPa

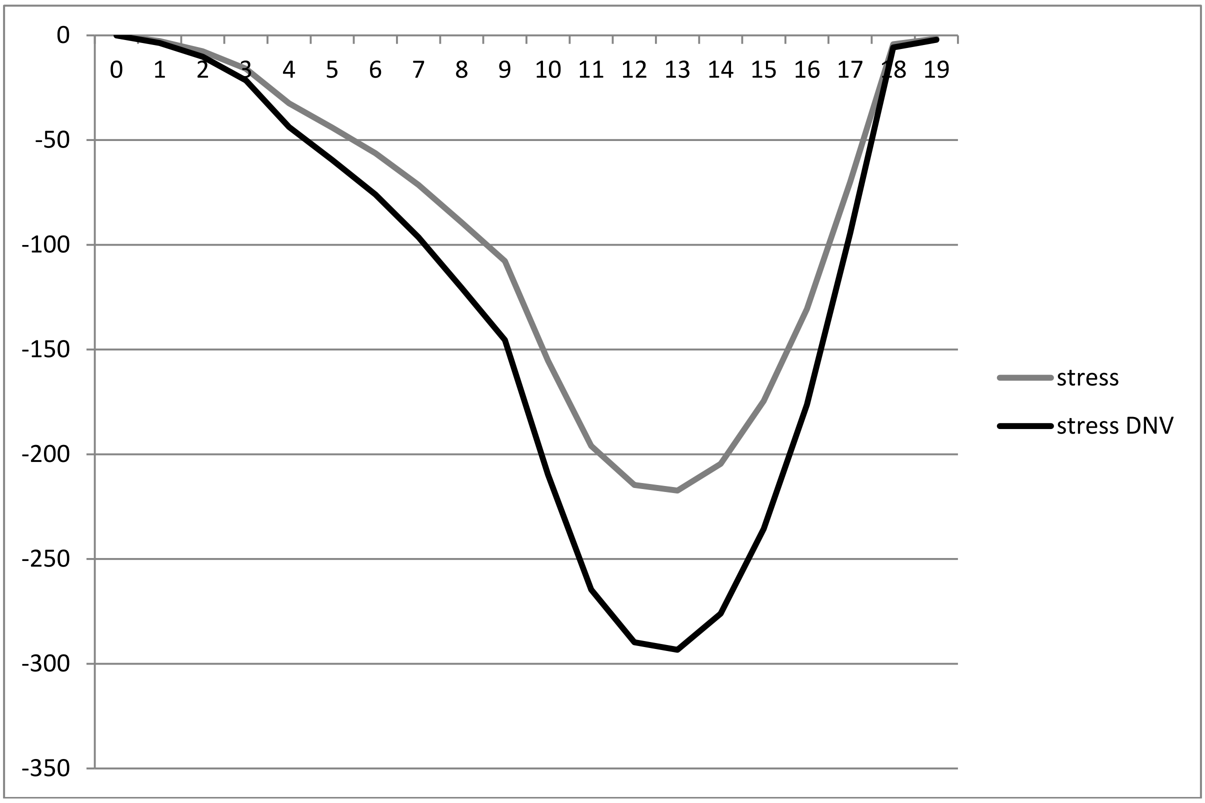

Fig. 20. Stress diagram over the real stress and the stress with a load factor according to DNV and corrosion

5.3 Hydrodynamic Analysis

The hydrodynamic analyses were performed in two steps, first a frequency domain analysis in Wadam and later a time domain analysis in OrcaFlex.

5.3.1 Wadam

The Wadam analysis was performed, case HD1, and the first comparison was to the hydrostatic case HS5. The CoG is calculated in the global coordinate system, given in Wadam, but in the results they are transformed into the local coordinate system to enable comparison to the AutoHydro results. The results are similar and shown in Table 10.

Table 10 Comparison of results from AutoHydro and Wadam

	AutoHydro	**Wadam**
LCG	0 m	-4.41E-7 m
TCG	0 m	-2.68E-7 m
VCG	59.33 m	59.33 m
LCB	0 m	-5.70E-8 m
TCB	0 m	-2.04E-8 m
VCB	60.77 m	60.78 m
GM	1.48 m	1.48 m

The obtained RAO’s, Response Amplitude Operator’s, are plotted in Fig. 21 and Fig. 22. The maximum heave amplitude is about 10 m in period 21.4 s, and the roll and pitch amplitude is

4 m in period about 121 s in 90 degrees heading for roll and 180 degrees heading for pitch. To obtain the periods for roll and pitch in reality is most unlikely.

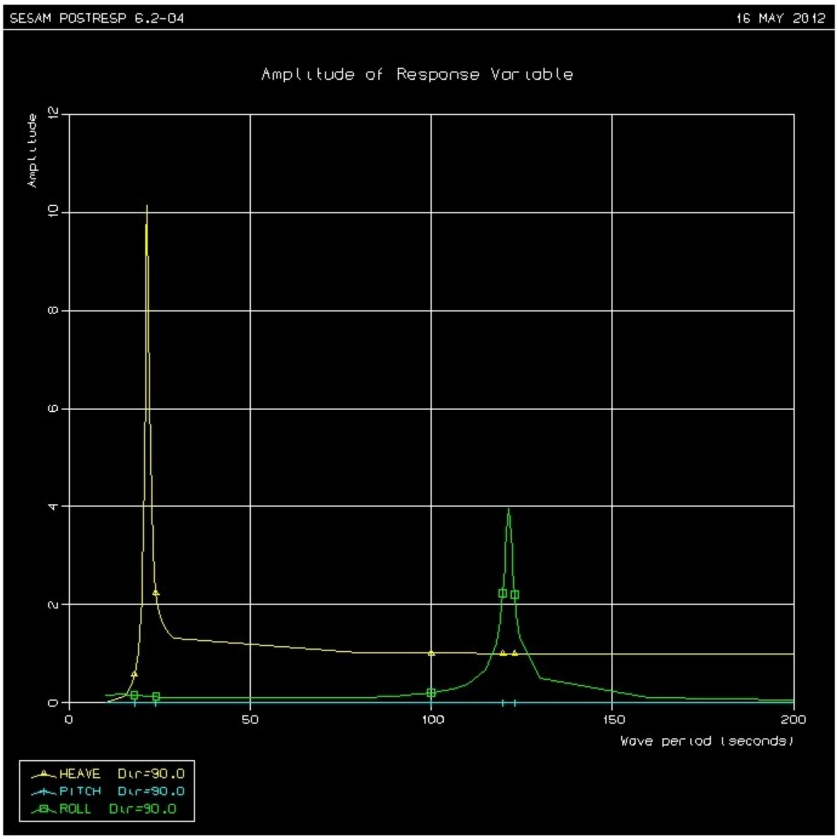

Fig. 21 RAO's for heading 90 degrees

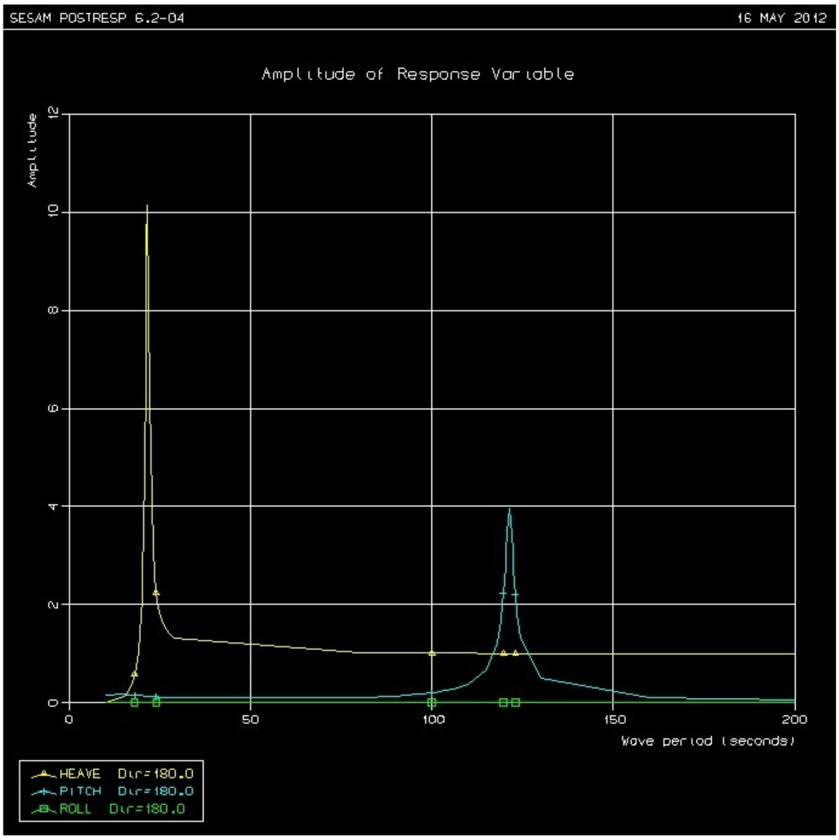

Fig. 22 RAO's for heading 180 degrees

The coupled natural periods in heave, roll and pitch are obtained in Wadam and calculated in MathCAD. The results are presented in Table 11.

Table 11 Coupled natural periods in Wadam and MathCAD

	Wadam	MathCAD
Heave	21.4 s	21.4 s
Roll	121.85 s	121.83 s
Pitch	121.85 s	121.72 s

The natural periods should be the same, because the matrices from Wadam are used and equation stated in the Wadam manual and in equation (40) in this report. The periods should also be the same for roll and pitch due to symmetry, the calculations are shown in Appendix F and the output from Wadam where the matrices is taken from is shown in Appendix G. The natural periods show that the governing motion is the heave motion, because the probability of obtaining wave periods about 120 s is almost 0. The most interesting periods that occur in the North Sea are 5 to 20 s. The wave period of 20 s is close to the natural period for heave and is therefore extra interesting. The RAO for heave in 20 s and wave amplitude of 1 m is for that reason shown in Fig. 23.

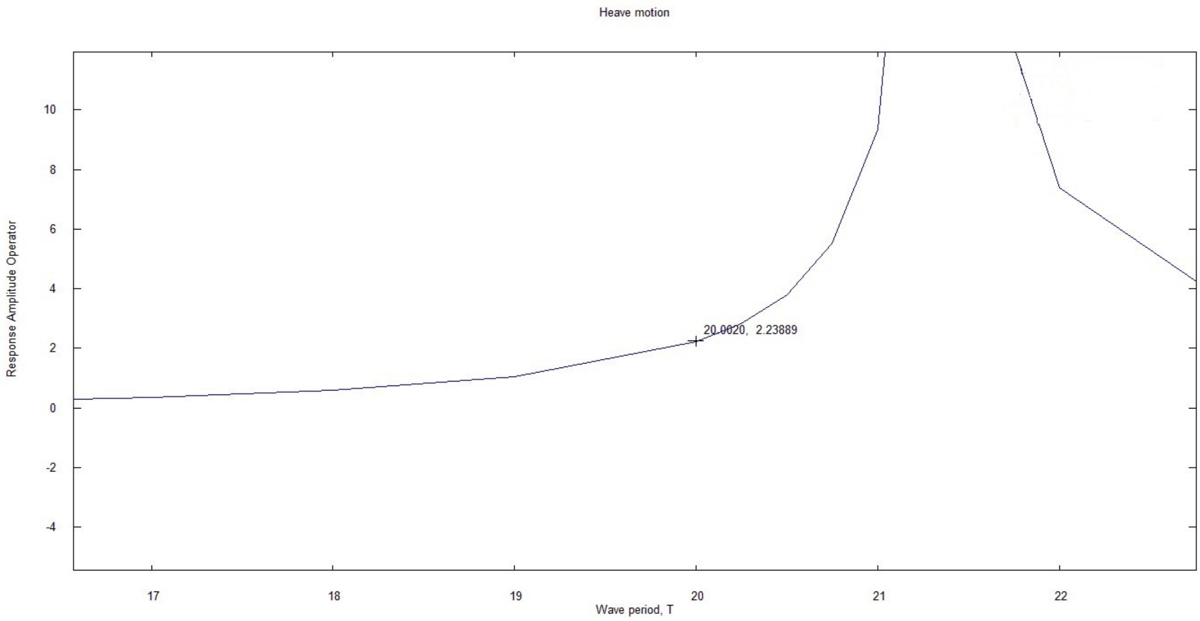

Fig. 23 RAO for heave motion in 90 degrees

5.3.2 OrcaFlex

Four cases were simulated in OrcaFlex, case HD2- case HD5. Two simulations were performed for the three first cases, and three simulations for the last case.

Case HD2

In the first case simulated in OrcaFlex, case HD2, the displacement RAO's from Wadam was imported into OrcaFlex. The RAO's obtained after a simulation of regular waves, according to Airy theory, with wave amplitude of 1 m, and the same wave periods as in Wadam, are the same as seen in Wadam. The RAO's are shown in Fig. 24 and Fig. 25. As stated for the previous case it is most unlikely to obtain the periods for roll and pitch.

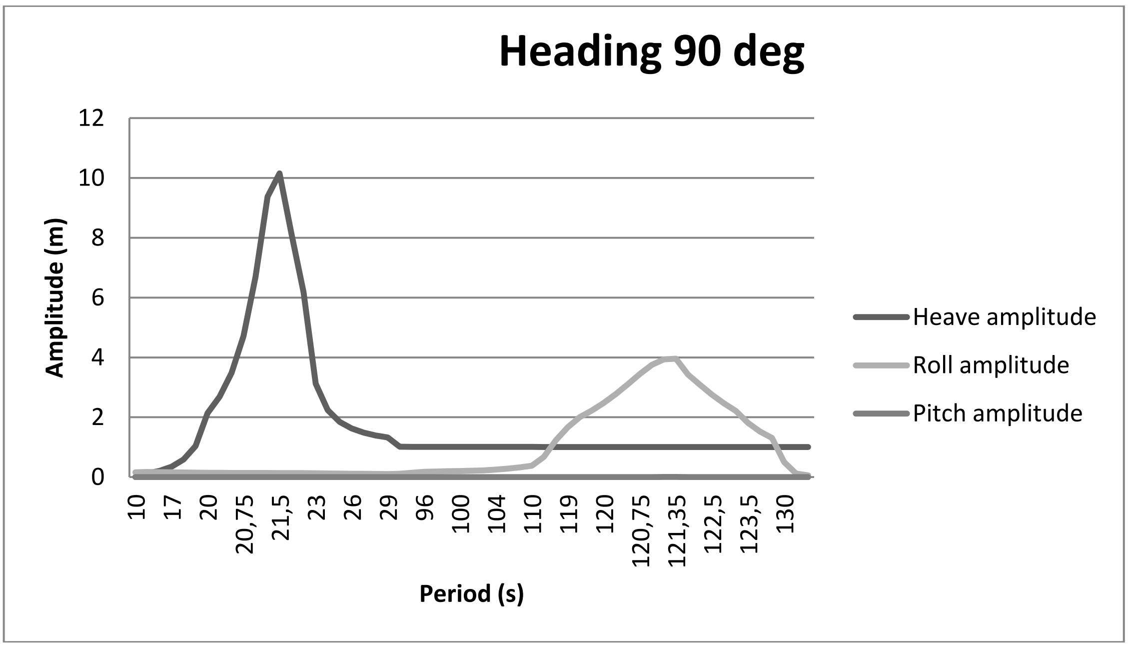

Fig. 24 RAO's for heading 90 deg for case HD2

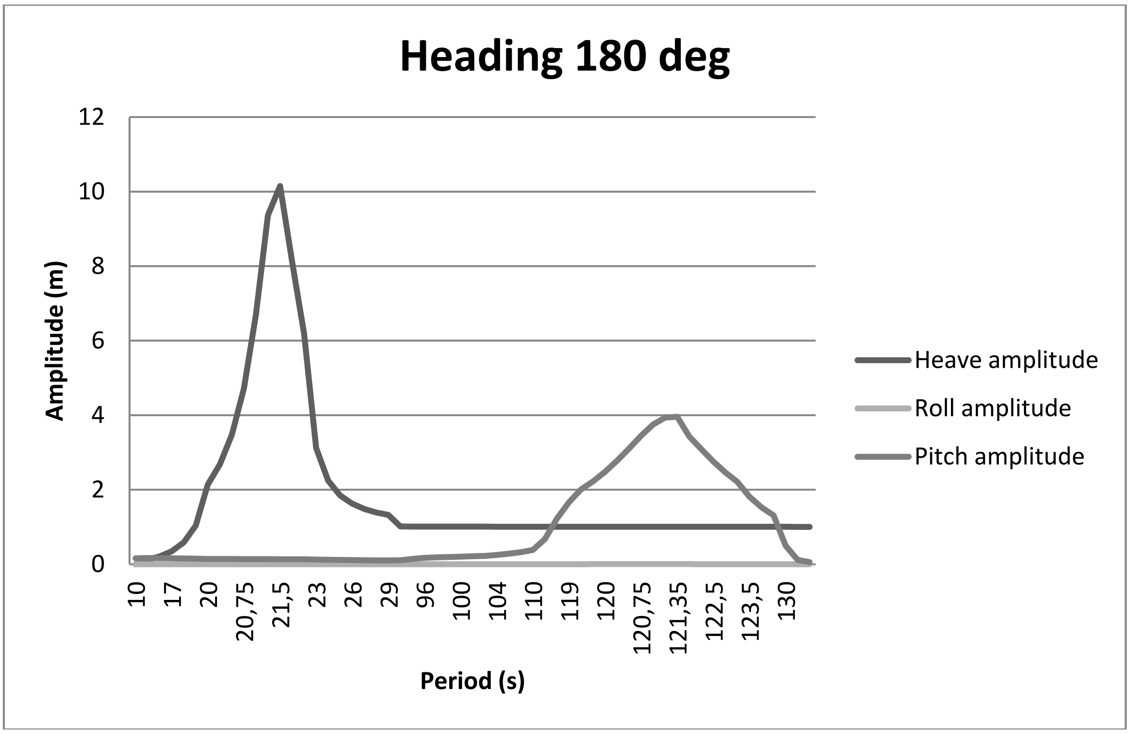

Fig. 25 RAO's for heading 180 deg for case HD2

In the tables below the maximum and minimum motions for the heave, roll and pitch motion for heading 90 and 180 degrees are presented. In Table 12 and Table 13 the results from regular waves are shown where the wave height is 1 m for wave period 5, 10, 15, 20 and 120 s. As shown in Fig. 24 and Fig. 25 the motions are really small for both cases. It is also shown that the highest motions in regular waves occur for heave at 20 s, and for roll and pitch at 120 s in heading 90 degrees and 180 degrees respectively.

Table 12 Heading 90 deg, wave height 1 m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	0.060	-0.060	0.073	-0.073	1.80E-5	-1.80E-5
10	0.0014	-0.0014	0.079	-0.079	2.00E-5	-2.00E-5
15	0.062	-0.062	0.085	-0.085	2.14E-5	-2.14E-5
20	1.11	-1.11	0.072	-0.072	1.90E-5	-1.90E-5
120	0.50	-0.50	1.72	-1.72	0.035	-0.035

Table 13 Heading 180 deg, wave height 1 m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	0.060	-0.060	1.62E-5	-1.62E-5	0.073	-0.073
10	0.0014	-0.0014	1.95E-5	-1.95E-5	0.079	-0.079
15	0.062	-0.062	2.28E-5	-2.28E-5	0.085	-0.085
20	1.11	-1.11	2.47E-5	-2.47E-5	0.072	-0.072
120	0.50	-0.50	0.022	-0.022	1.61	-1.61

As seen in the tables above the values for heading 90 degrees and 180 degrees are similar, as they are expected to be. The simulation for wave amplitude of 1m is therefore only performed for heading 90 degrees shown in Table 14. For the heave motion in period 20 s, the values are the same as shown in Fig. 23.

Table 14 Maximum and minimum amplitudes for heave, roll and pitch for wave amplitude of 1m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	0.0014	-0.0014	0.079	-0.079	0	0
10	0.0028	-0.0028	0.16	-0.16	0	0
15	0.13	-0.13	0.17	-0.17	0	0
20	2.22	-2.22	0.14	-0.14	0	0
120	1.01	-1.01	3.45	-3.45	0.070	-0.070

Case HD3

In case HD3 the displacement and load RAO's are imported from Wadam as input to OrcaFlex. The natural period in heave for case HD3 is given in Fig. 26. As seen the natural period for heave is 21.5 s, for roll is about 121 s and for pitch is about 121 s. It is most unlikely that the periods for roll and pitch will occur.

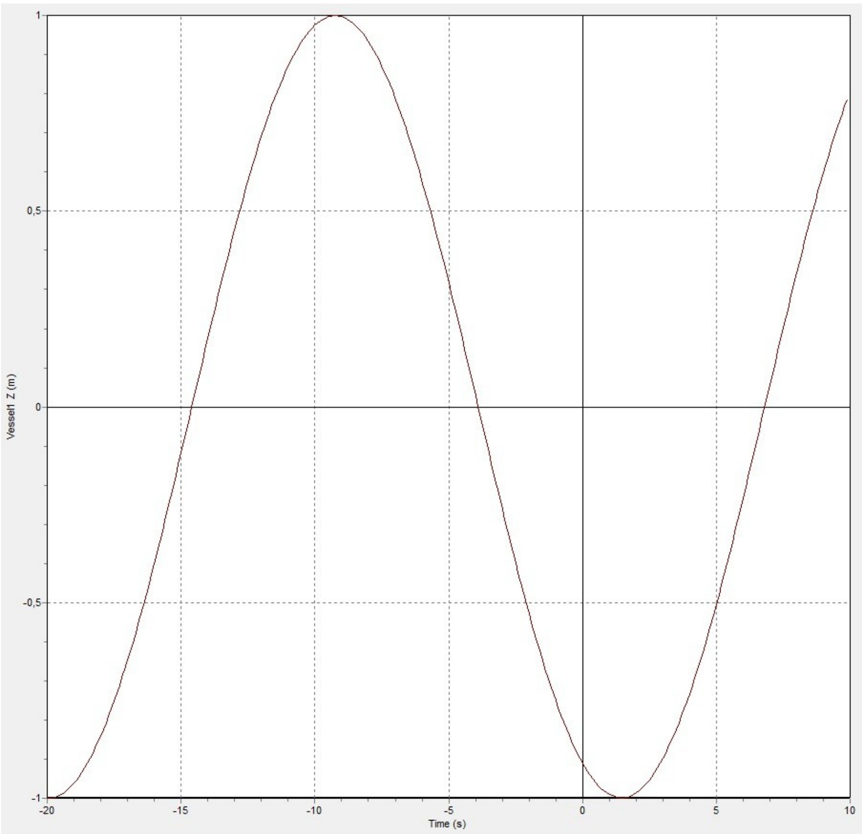

Fig. 26 Natural period heave for case HD3

In case HD3 the same simulations as in case HD4 are performed, and as shown in Table 15 and Table 16. The motions are still small, but the heave amplitude is higher than for case HD2.

Table 15 Heading 90 deg, wave height 1 m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	0.0030	-0.0036	0.023	-0.022	7.06E-7	-1.01E-6
10	0.0023	-0.0024	0.069	-0.071	3.21E-6	-3.85E-6
15	0.13	-0.13	0.13	-0.14	7.56E-6	-8.20E-6
20	2.28	-2.28	0.18	-0.21	2.80E-5	-2.32E-5
120	0.74	-0.73	0.10	-0.084	7.87E-5	-6.81E-5

Table 16 Heading 180 deg, wave height 1 m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	0.0030	-0.0036	0.00061	-0.0011	0.022	-0.022
10	0.0023	-0.0024	0.00069	-0.0018	0.068	-0.071
15	0.13	-0.13	0.0019	-0.0035	0.13	-0.14
20	2.28	-2.28	0.0045	-0.0084	0.18	-0.21
120	0.74	-0.73	0.0025	-0.0061	0.12	-0.090

As already mentioned for the previous case the results are, as expected, similar in for heading 90 degrees and 180 degrees. Due to this fact, the simulation for wave amplitude of 1 m is only performed for heading 90 degrees. As seen the result for the heave motion for period 20 s is twice as high as in case HD1 and HD2. This is because there is no damping in this case.

Table 17 Maximum and minimum motion in heave, roll and pitch for wave amplitude 1m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	0.0003	-0.0003	0.014	-0.014	0	0
10	0.0044	-0.0044	0.14	-0.15	0.0059	-0.0059
15	0.24	-0.24	0.28	-0.28	0.026	-0.025
20	4.42	-4.43	0.43	-0.43	0.045	-0.051
120	1.42	-1.43	0.22	-0.22	0.029	-0.029

Case HD4

In case HD4 load RAO’s are imported from Wadam and with drag from OrcaFlex. The drag coefficient is set to 0.95 from DNV-RP-C205 fig 6-6, with the assumption of corrosion and marine growth. The natural periods for case HD4, is about 21.5 s for heave and about 121 s for roll and pitch. It is most unlikely that the natural periods for roll and pitch will be obtained.

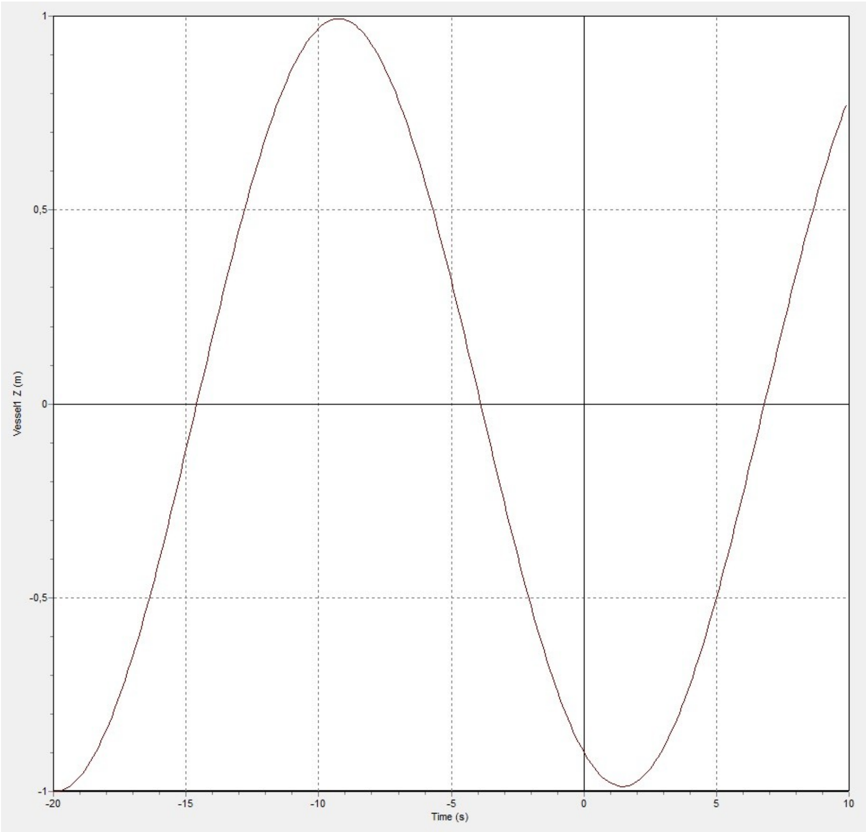

Fig. 27 Natural period in heave for case HD4

Due to the long natural periods for roll and pitch the heave motion will be the governing one.

As in the previous two cases Table 18 and Table 19 shows the results from the simulation of regular waves. When adding some drag the motions get smaller in regular waves.

Table 18 Heading 90 deg, wave height 1 m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	0.0029	-0.0036	0.019	-0.023	0.00015	-0.0021
10	0.0022	-0.0024	0.056	-0.074	0.0010	-0.0051
15	0.13	-0.13	0.11	-0.15	0.0035	-0.010
20	2.07	-2.06	0.098	-0.36	0.023	-0.027
120	0.74	-0.73	0.29	-0.11	0.013	-0.055

Table 19 Heading 180 deg, wave height 1 m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	0.0029	-0.0036	0.00093	-0.0020	0.021	-0.022
10	0.0022	-0.0024	0.0015	-0.0037	0.060	-0.073
15	0.13	-0.13	0.0043	-0.0090	0.11	-0.15
20	2.07	-2.06	0.11	-0.013	0.11	-0.36
120	0.74	-0.73	0.00050	-0.027	0.29	-0.10

The simulation of wave amplitude of 1 m is only performed in heading 90 degrees. In this case damping is added from the Morison theory and the heave motion for the wave period of 20 s is 2.2 m as in case HD1 and HD2. If the damping is removed the same results are obtained as for case HD3.

Table 20 Maximum and minimum amplitude in heave, roll and pitch for wave amplitude of 1 m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	0.0003	-0.0006	0.013	-0.013	0	0
10	0.0037	-0.0038	0.094	-0.10	0.0054	-0.0050
15	0.17	-0.16	0.11	-0.10	0.014	-0.014
20	2.20	-2.20	0.080	-0.093	0.020	-0.017
120	1.22	-1.23	0.078	-0.077	0.079	-0.081

To investigate the value of the heave motion, in period of 20 s further the motion is simulated over a longer time. The heave amplitude is shown in Fig. 28. The figure shows that the amplitude has a settling time and gets stable with amplitude of about 2.2 meters.

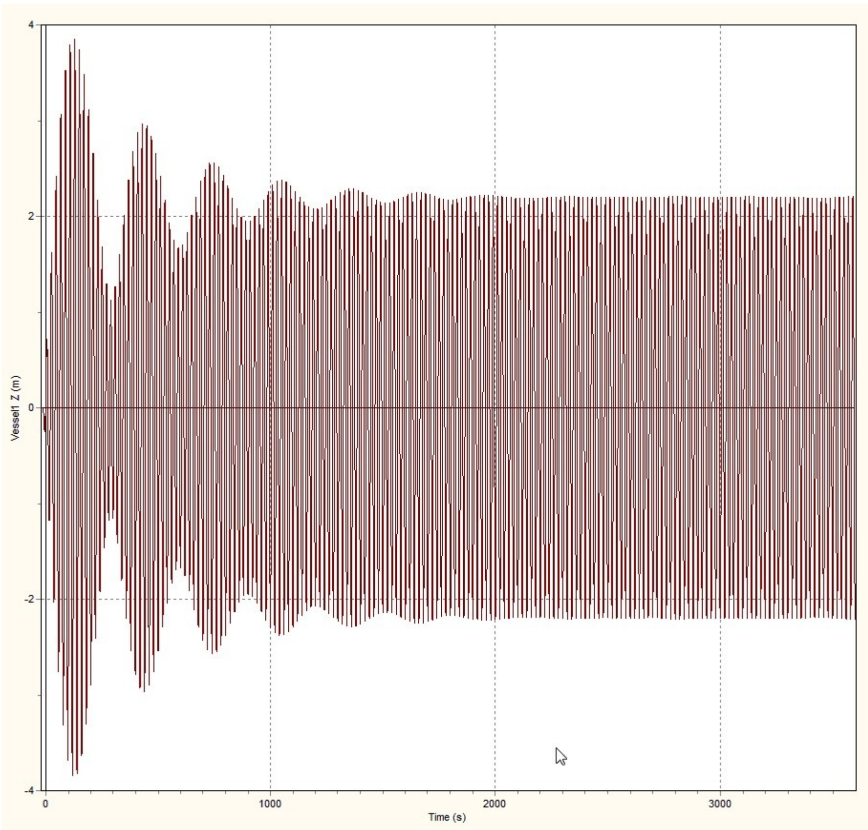

Fig. 28 Settling time of case HD4 for wave amplitude of 1 m

Case HD5

For case HD5 the whole model is created in OrcaFlex, without importing anything from Wadam. The added mass coefficients are set to 0.7 in both normal and axial direction according to DNV-RP-C205 appendix D and the drag coefficients were set to 0.95 in both axial and normal direction also according to DNV-RP-C205 fig 6-6. The natural periods for case HD5, is about 27.5 s for heave and about 111.5 s for roll and pitch. This means that the natural period in heave is longer and in roll and pitch is shorter than for the previous cases. The differences in natural periods are due to the different models the difference is small but will impact the results especially around 20 s and 120 s. With the previous investigated periods, the natural period for roll and pitch will not be investigated. The values for roll and pitch are therefore expected to be lower than in the previous cases.

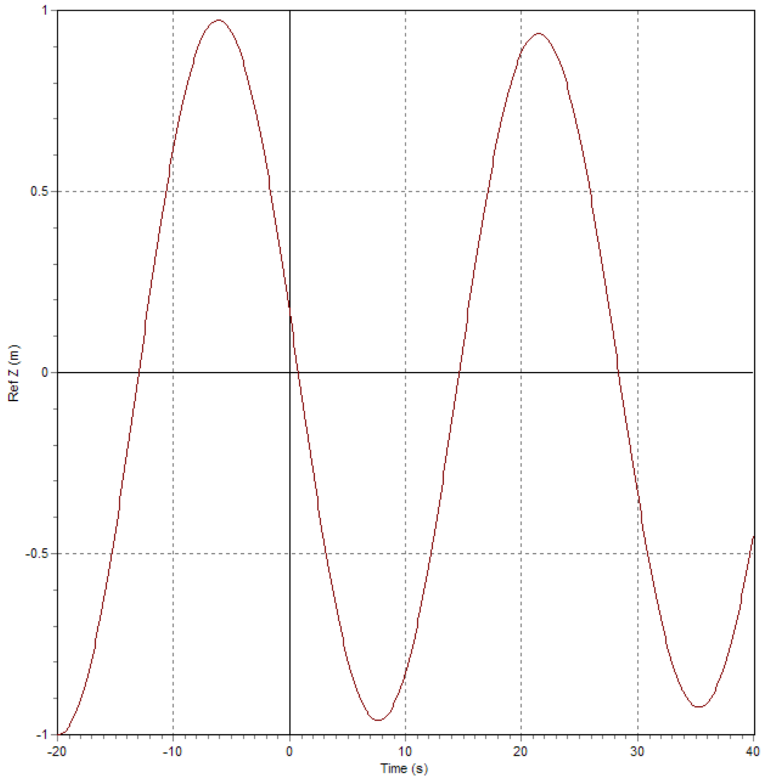

Fig. 29 Natural period in heave for case HD5

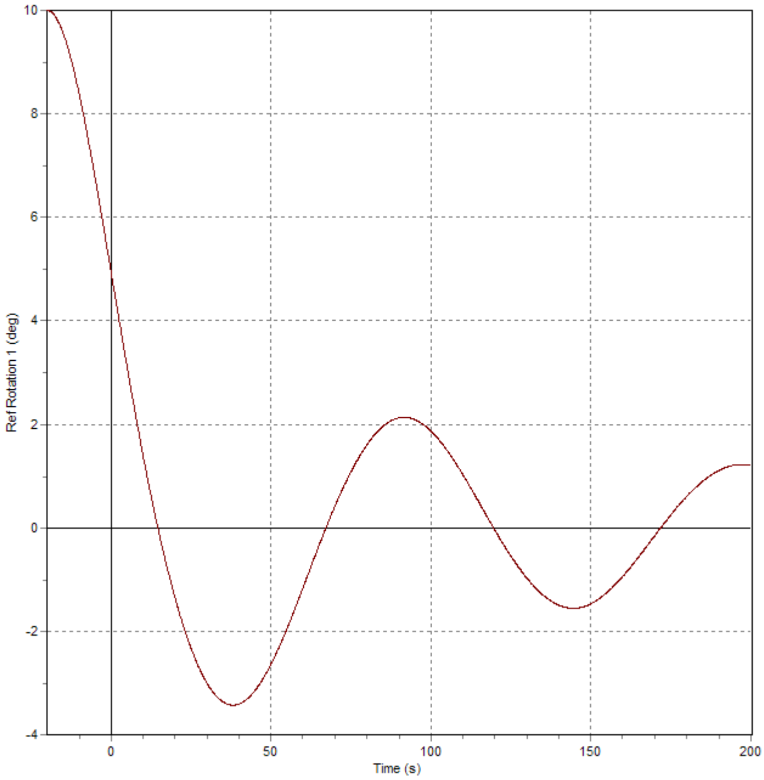

Fig. 30 Natural period in roll and pitch for case HD5

With the added mass and drag the motions of the buoy gets even lower, especially for heave at 20 s. In this case the simulation of regular waves was run for a wave amplitude of 10 m. The results of this simulation are almost 10 times larger than the results from the simulation with wave amplitude of 1 m, which prove the linear theory.

Table 21 Heading 90 deg, wave height 1 m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	0.013	-0.018	0.00037	-0,00037	2.59E-20	-2.51E-20
10	-6.87E10-5	-0.011	0.0028	-0.0028	1.71E-19	-1.70E-19
15	0.12	-0.17	0.015	-0.015	9.45E-19	-9.18E-18
20	0.23	-0.20	0.032	-0.029	1.78E-18	-1.93E-18
120	0.81	-0.79	0.12	-0.11	6.52E-18	-7.52E-18

Table 22 Heading 180 deg, wave height 1 m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	0.013	-0.018	4.59E-20	-4.62E-20	0.00037	-0.00037
10	-6.87E10-5	-0.011	3.40E-19	-3.43E-19	0.0028	-0.0028
15	0.12	-0.17	1.84-18	-1.89E-18	0.015	-0.015
20	0.23	-0.20	3.86E-18	-3.55E-18	0.032	-0.029
120	0.81	-0.79	1.50-17	-1.30E-17	0.12	-0.11

Table 23 Heading 90 deg, wave height 10 m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	0.13	-0.26	0.0038	-0.0038	2.40E-19	-2.52E-19
10	-0.0021	-0.65	0.032	-0.035	2.12E-18	-1.98E-18
15	1.01	-2.19	0.20	-0.24	1.44E-17	-1.20E-17
20	2.45	-2.52	0.95	-0.87	5.35E-17	-5.83E-17
120	8.12	-7.85	1.09	-1.26	7.73E-17	-6.66E-17

Table 24 Heading 180 deg, wave height 10 m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	0.13	-0.26	4.66E-19	-4.68E-19	0.0038	-0.0038
10	-0.0021	-0.65	3.99E-18	-4.24E-18	0.032	-0.035
15	1.01	-2.19	2.39E-17	-2.88E-17	0.20	-0.24
20	2.45	-2.52	1.17E-16	-1.07E-16	0.95	-0.87
120	8.12	-7.85	1.33E-16	-1.55E-16	1.09	-1.26

When performing a simulation of the model in this case, a lower value of the heave motion in wave period of 20 s is expected, due to the longer natural period. As the results show the heave motion is lower than for the other cases. If one look at the results for the natural period, of 28 s, a value of about 2.4 m is obtained. This confirms that the models are alike.

Table 25 The maximum and minimum motion in heave, roll and pitch in wave amplitude of 1m

Tp	Max heave amplitude	Min heave amplitude	Max roll amplitude	Min roll amplitude	Max pitch amplitude	Min pitch amplitude
5	-0.085	-0.14	-0.061	-0.17	0	0
10	0.064	-0.14	0.12	-0.23	0	0
15	0.19	-0.22	0.18	-0.21	0	0
20	0.15	-0.16	0.16	-0.17	0	0
120	1.15	-1.16	0.29	-0.29	0	0

6. Discussion

The discussion is divided into four sub- chapters, first one for each analysis and ending with a discussion of the project in general and feedback of the aims of the project.

6.1 Hydrostatics Analysis

The main uncertainty in the hydrostatic analysis of the buoy is the position of the CoG. A weight log has not been updated when renovations etc. has been performed during the buoys life time. The CoG has been estimated from the weight documents and as- installed documents from early 1980's. The actual CoG will not be known until the buoy is released from the site and free floating.

The results differ when the calculations become more complex. The two programs, AutoHydro and MathCAD solve the integrals in two different ways AutoHydro uses a summation method while MathCAD solves the integrals direct. For the iteration procedure to find equilibrium the programs have different tolerances. In MathCAD the tolerance accepted is defined. If the tolerances in the two programs are investigated, AutoHydro has lower tolerance. If the tolerance is set as low in MathCAD the calculation time gets very long and it does not always find equilibrium within the given tolerance. It is not known exactly how AutoHydro calculates, because there is no theory manual available. The largest difference in results is detected in case HS3, when the diameter is varying and the heel is set to zero and an external load is applied 8 m from the bottom. Here the VCB is off by about 2 m, which is not reasonable. The results show that the VCB gets more incorrect with a lower trim angle. The LCF differs in low trim angle too, which is due to small change in trim angle changes the water plane length a significant amount. When the calculations get more complex the calculation time in MathCAD gets long. When the trim angle gets larger the trim angle and the draft differ a bit but the VCB and LCF is almost the same in MathCAD and AutoHydro. And for the vertical case all hydrostats are similar.

When the trim angle is small, the waterline is large; a small change in angle makes a large difference in center of flotation. In case 4 is the vertical center of buoyancy different for the two calculation methods. The difference in GM is due to the calculation of I_T, where an assumption of the radius of 4.295 m. Due to the low GM in AutoHydro, small difference in VCB and VCG may give negative value of GM.

In all cases the GM is really low, and due to the uncertainties in center of gravity, it is not recommended to perform the operations without using cranes, barges, wings or something similar to help increase the stability of the buoy. In some regulations, ABS for example, the only requirement is that the GM should be positive, but it is good if it is a couple of dm. In this case where there is an uncertainty of where the CoG is positioned it is especially important to take extra precaution in case of stability. The calculations have been preformed with the assumption that all compartments are completely filled with water. During the operation of removing the solid ballast from the bottom part, the water ballast in the top will be removed. When the water and solid ballast is removed, free surface effects will occur, and decrease the stability further.

All cases investigated fulfill the requirement from the costumer, in case HS2 and HS4 the buoy should be floating in equilibrium with a draft of the top of about 2 m. In case HS3 the requirement was that the buoy should float over the quay without touching the quay or the bottom.

6.2 Structural Analysis

The bending moment is not zero at bulkhead 19, this could be due to the approximation of the lever arm at half the length of each compartment. The actual lever arm is at CoG of each compartment.

In the calculations due to DNV rules, corrosion is considered even though the inspection described in the design basis stated that the buoy had not corroded. This gives a thinner wall of the buoy and therefore a higher stress. The buoy is allowed to deform plastically and break in some parts, as long as the buoy does not collapse. Case HS2 and Case HS3 are the most critical ones, which needs more investigation. A FE models is recommended to be put up to investigate the stresses further, and to see more exact where the stress concentrations occur. The plastically behavior of the material should be investigated to choose an accurate method for the analysis. When the investigation of the plastically behavior has been performed the risk of the operation is known.

If the results still point in the direction of a high risk to perform the operations, the recommendation is to do this operation in another way, for example use cranes to lift of the topside when it is placed in the fjord. This operation is a bit more expensive, due to the vessels and cranes needed. But with the low GM and high loads this is a safer way to perform the operation, without risking a failure of the buoy.

6.3 Hydrodynamic Analysis

The expected result for case HD1- HD3 should be the same. Case HD4 is with Morison theory, to see what influence the Morison theory has on the results. Case HD5 has a different model with only Morison theory, which is expected to give a difference in results especially in heave for wave period 20 s, due to the longer natural period. The natural periods for roll and pitch are really high, and the motions are bad, but the probability of obtaining periods of that range is almost 0. However low frequency forces could have a period like that.

The requirements for small volume structures are given by DNV-RP-H103 chapter 2.4.1.1. For a certain geometry to be treated as a small volume structure it must fulfil the requirement below

$$D < \lambda/5 \tag{75}$$

In the equation the wave length is defined by DNV-RP-C205 chapter 3.2.2.6

$$\lambda = 1.56T^2 \tag{76}$$

This gives a minimum wave period of 5.2 s, which is fulfilled all the periods in all cases, except the period for 5 s, presented in this report. The calculation is presented in Appendix H. When the requirements of small volume structures are fulfilled it is relevant to use Morison theory to assume drag and inertia loads on three dimensional objects in waves. When assuming small volume structure the waves generated by the body will be negliable. The period of 5 seconds is on the limit between small volume and large volume, which means that the waves generated by the buoy may have an effect of the motions. Large volume structures are inertia- dominated which means that the diffraction forces are dominating. This means that the results may be inaccurate for the wave period of 5 s.

For the period of 10 s the small volume structure assumption is valid, and the values are similar, as expected.

When the period is reaching 20 s, the damping gets important. This is shown when comparing the results from case HD3 and HD4. The heave motion is twice as big in case HD3 without damping as in case HD4 with damping from Morison theory. And if the damping is removed from case HD4 the same result of the heave motion is obtained.

As seen when comparing the results from case HD1- HD4 with case HD5 the impact of the natural period is shown. When the natural period is offset a few seconds, the motions change considerably. The motions reach their peak at the natural periods. Some of the difference is also due to non-linear viscous damping used in the Morison theory. In case HD5 the peak in natural period for roll and pitch is not investigated, which means that the values should be lower for these motions in wave period 120 s. For the natural periods calculated in MathCAD and compared with the results from Wadam, one interesting thing was obtained. As seen in Table 11 the results do not match for roll and pitch, which they should, but when performing the same calculations with the matrices from the same case but with LCG and TCG divided from zero, the same natural periods in both Wadam and MathCAD are obtained.

It is expected that the motions are low for roll and pitch in the wave periods given in this report. Normal wave periods in the North Sea are between 5 to 20 s. The low motions are due to the high natural period and low GM.

For case HD5 a simulation was performed with 1 m wave height and later 10 m, when comparing the results it is shown that the results for 10 m are 10 times larger than the values for 1 m. This proves the linear theory.

6.4 General discussion

In general the project has been both interesting and demanding. All aims were not fulfilled.

For the hydrostatic part the aim was fulfilled. Suitable ballast conditions were found for the different cases. When comparing the results with AutoHydro there is some difference, but the values is not of any major difference, in most of the hydrostats.

In the structural analysis the aim was fulfilled. The global stresses were calculated for the different cases with the ballast conditions given from the hydrostatic analysis. The stresses were not satisfying in all cases because the buoy may deform plastically. The buoy is allowed to deform plastically, but not break.

The aim for the hydrodynamic part has not been completely fulfilled because it was too comprehensive for the thesis. Only one model was investigated in Wadam and OrcaFlex, the vertical one. The ambition was to investigate a horizontal case too, but this was not possible within the time frames of the master thesis.

7. Future work

Further studies of the stability of the buoy should be performed. The influence of difference in the CoG should be investigated to see what effects the different positions would have. During the operation of removing the solid ballast free surface effects will occur, the effects of these should be investigated and taken into account when planning the operation.

The MathCAD sheet could be developed further the calculations for VCB in small angles should be investigated, to see if some modifications could lead to a more correct value. A calculation for heel and wind loads could be added to improve the sheet.

As discussed in the previous chapter a FE model should be put up to investigate the stress concentrations and the plastically behavior of the buoy. The most important thing to be certain of is that the buoy does not collapse during the operation. In the FE model the bending moments can be calculated and compared to the results from MathCAD. This would give the accuracy of the hand calculations. The hand calculations are then a good tool for rough estimates of the stress over a certain volume.

In the MathCAD sheet the lever arms should be put to the actual ones and see if the bending moment is 0 at bulkhead 19. A method that describes the plastically behavior could be included in the sheet to make it more detailed.

The next step for the hydrodynamic analysis would be to investigate the motions in irregular waves, in the sea states obtained at the site where the buoy is located. The actual towing procedure will take place in irregular waves, current and wind. To perform the wet tow the buoy need some towing lines and these should be analyzed too.

It would be interesting to do a similar analysis as the one performed in this project but with the buoy located in a horizontal position. The buoy would then have covered multiple wave lengths and the influence of Morison theory may have been different.

8. Concluding remarks

The sheets created in MathCAD are good for rough estimates of different geometries, because they are built from the definitions of each hydrostatic property and the structural behavior. For larger angles the VCB and GM_T are satisfying, but not for small angles. LCF changes a lot with a small difference in trim angle when the angle is small. This means that the LCF is not accurate for small trim angles. The model for vertical position of the buoy is satisfying.

The motions obtained are small when the periods are not close to the natural period. Near the natural period the damping gets important. The Morison theory does not influence the results that much, only when damping is needed near the natural period.

For the hydrostatic and structural parts the aims were fulfilled. In the hydrodynamic part only one model was investigated, when the buoy was located in a vertical position. The aim was to investigate it in a horizontal position too. Due to the time frames of the master thesis, this was too comprehensive.

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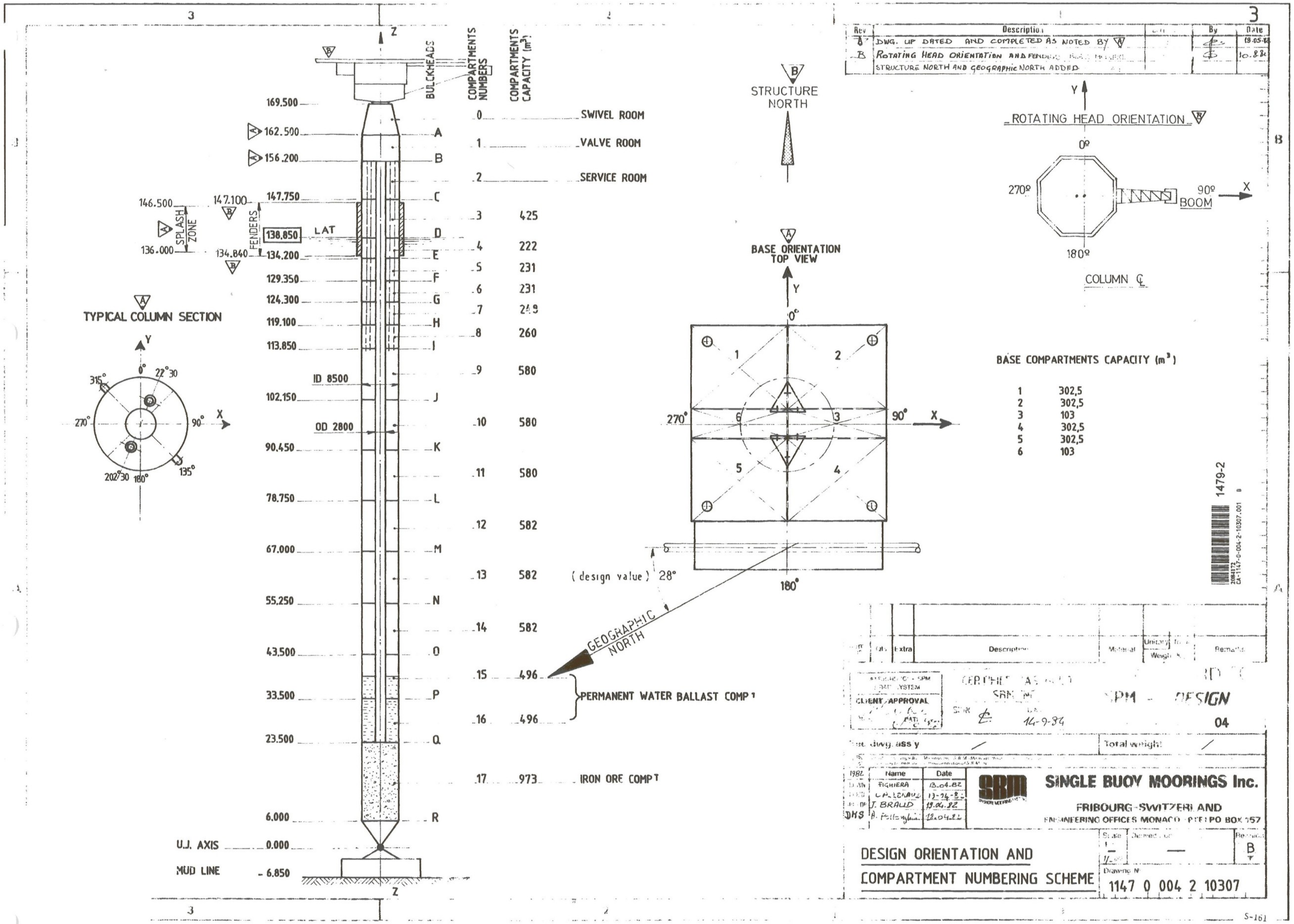
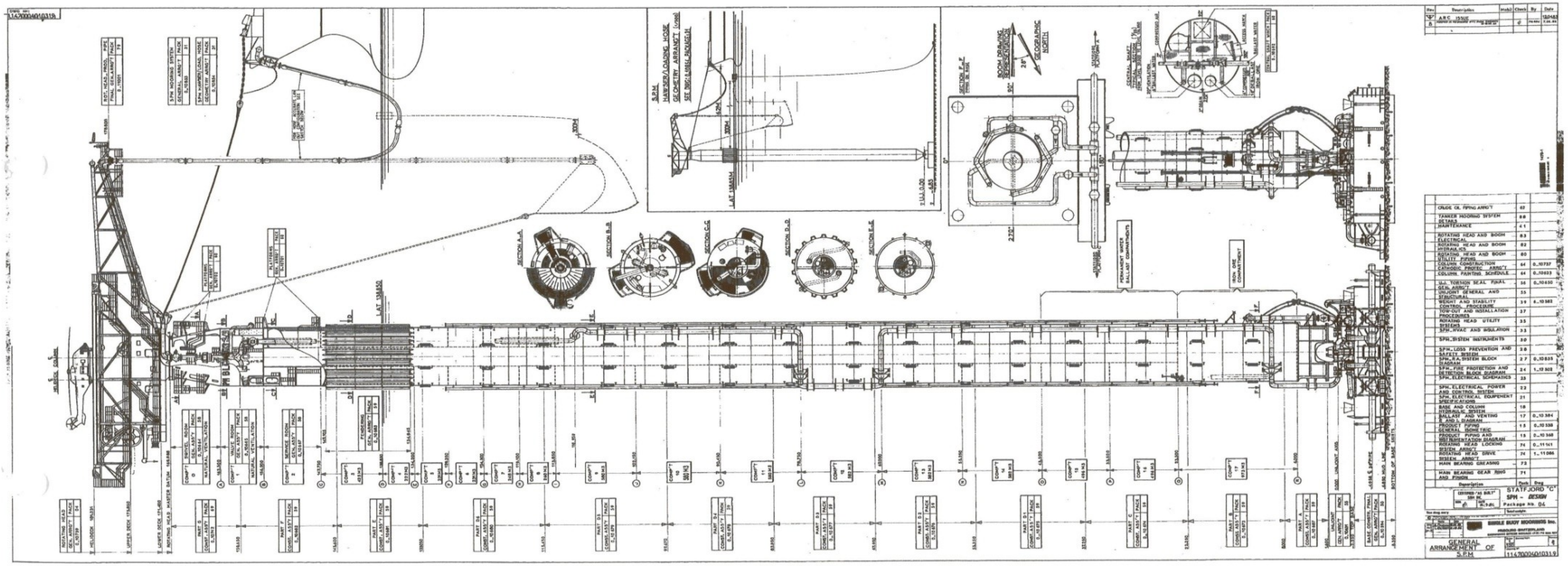
Appendix A

Weight and buoyancy distribution of the different compartments in the column, including ballast.

Compartment no	Elevation from Unijoint [m]		Weight [t]	Buoyancy [t]
	From	To		
-	0,00	1,60	120	16
-	1,60	6,00	180	26
17	6,00	11,00	980	308
17	11,00	23,50	1970	748
16	23,50	26,00	210	150
16	26,00	33,50	520	449
15	33,50	43,50	700	599
14	43,50	55,25	220	703
13	55,25	67,00	220	704
12	67,00	78,75	210	703
11	78,75	83,70	90	296
11	83,70	90,45	120	404
10	90,45	102,15	200	700
9	102,15	113,85	200	700
8	113,85	119,10	109	311
7	119,10	124,30	108	***301****
6	124,30	129,35	105	299
5	129,35	134,20	101	287
4	134,20	138,85	97	292
3	138,85	147,75	310	
2	147,75	156,50	110	
1	156,50	162,50	110	
0	163,50	169,50	160	
Sum:			**7150**	**7996**



TYPICAL COLUMN SECTION
SWIVEL ROOM
VALVE ROOM
SERVICE ROOM
BULKHEADS
COMPARTMENTS NUMBERS
COMPARTMENTS CAPACITY (m³)
SPLASH ZONE
FENDERS
LAT
ID 8500
OD 2800
PERMANENT WATER BALLAST COMP'T
IRON ORE COMP'T
U.J. AXIS
MUD LINE
STRUCTURE NORTH
BASE ORIENTATION TOP VIEW
GEOGRAPHIC NORTH
(design value) 28°
ROTATING HEAD ORIENTATION
BOOM
COLUMN ℄
BASE COMPARTMENTS CAPACITY (m³)
1 302,5
2 302,5
3 103
4 302,5
5 302,5
6 103
DWG. UP DATED AND COMPLETED AS NOTED BY
ROTATING HEAD ORIENTATION AND FENDERS
STRUCTURE NORTH AND GEOGRAPHIC NORTH ADDED
CLIENT APPROVAL
14-9-84
SPM - DESIGN
04
Total weight
SINGLE BUOY MOORINGS Inc.
FRIBOURG-SWITZERLAND
DESIGN ORIENTATION AND
COMPARTMENT NUMBERING SCHEME
1147 0 004 2 10307
1479-2


STATFJORD "C"
SPM - DESIGN
GENERAL
ARRANGEMENT OF
S.P.M.

Appendix B

Case HS1
Hull Data (with appendages)

Baseline Draft: 6.477 at Origin
Trim: aft 2.27 deg.
Heel: port 26.87 deg.

DIMENSIONS
Length Overall: 169.500 m LWL: 169.500 m Beam: 8.597 m BWL: 8.590 m
Volume: 7709.350 m^3 Displacement: 7902.125 MT

COEFFICIENTS
Prismatic: 0.785 Block: 0.525 Midship: 0.669 Waterplane: 0.641

RATIOS
Length/Beam: 19.716 Displacement/length: 45.223 Beam/Depth: 0.853
MT/ cm Immersion: 9.571

AREAS
Waterplane: 933.714 m^2 Wetted Surface: 5650.798 m^2
Under Water Lateral Plane: 1110.981 m^2 Above Water Lateral Plane: 345.808 m^2

CENTROIDS (Meters)
Buoyancy: LCB = 72.278 fwd TCB =0.269 port VCB = -0.531
Flotation: LCF = 112.318 fwd
Under Water LP: 72.055 fwd of Origin, 3.714 below waterline.
Above Water LP: 125.256 fwd of Origin, 1.743 above waterline.

Note: Coefficients calculated based on waterline length at given draft

Floating Status

Draft FP	-1.050 m	Heel	port 26.87 deg.	GM(Solid)	0.044 m
Draft MS	2.713 m	Equil	Yes	F/S Corr.	0.000 m
Draft AP	6.477 m	Wind	Off	GM(Fluid)	0.044 m
Trim	aft 2.27 deg.	Wave	No	KMT	0.000 m
LCG	72.300f m	VCG	-0.039 m	TPcm	9.57
Displaceme nt	7,902.13 MT	WaterSpgr	1.025		

Stability and Strength

Max VCG margin	<und>	Bend Mom (sea)	0.00%	Shear Force (sea)	0.00%

Loading Summary

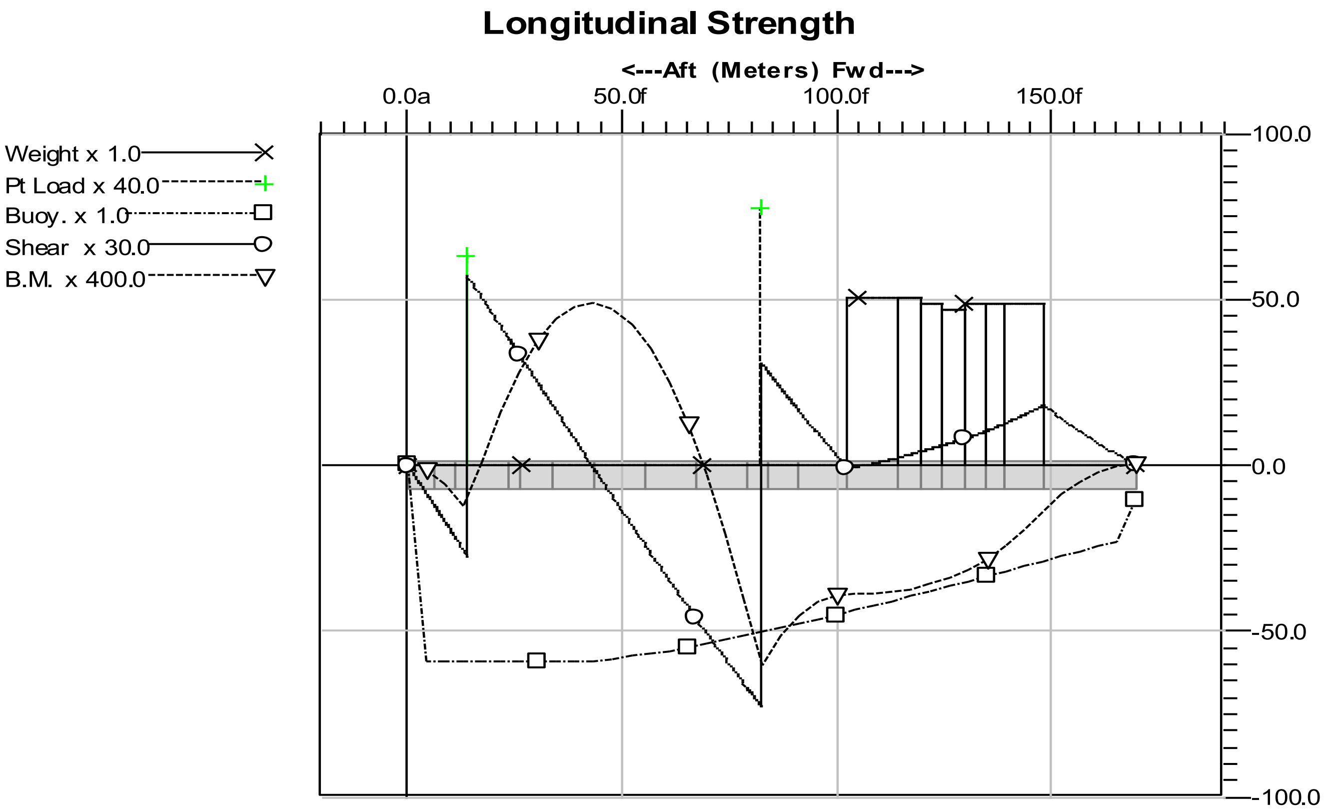
Item	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
Light Ship	3,110.19	82.180f	0.050p	-0.100
Deadweight	4,791.94	65.887f	0.000	0.000
Displacement	7,902.13	72.300f	0.020p	-0.039

Fixed Weight Status

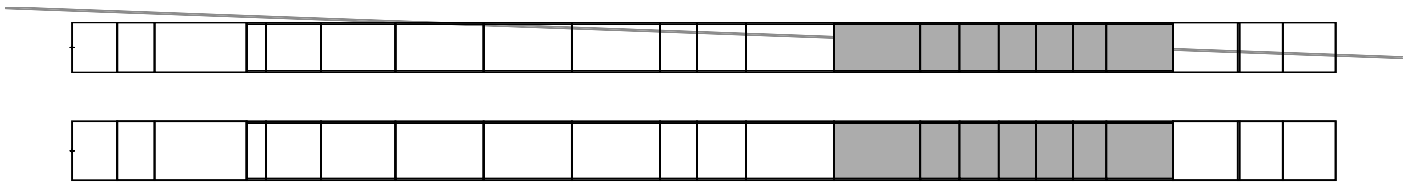
Item	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
LIGHT SHIP	3,110.19	82.180f	0.050p	0.100d
SOLID BALLAST	2,540.00	13.710f	0.000	0.000
Total Fixed:	5,650.19	51.400f	0.028p	0.055d

Displacer Status

Item	Status	Spgr	Displ (MT)	LCB (m)	TCB (m)	VCB (m)	Eff /Perm
hull	Intact	1.025	7,902.13	72.278f	0.269p	-0.531	1.000
SubTotals:			7,902.13	72.278f	0.269p	-0.531	



Max. Shear	-2178.30 MT	at	82.180f
Max. Bending Moment	-24479 MT-m	at	82.180f (Sagging)



Fluid Legend

Fluid Name	Legend	Weight (MT)	Load%
SALT WATER		2,251.94	26.33%

Tank Status

SALT WATER

Tank Name	Spgr	Load (%)	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
comp17.c	1.025	<empty>				
comp16.c	1.025	<empty>				
comp15.c	1.025	<empty>				
comp14.c	1.025	<empty>				
comp13.c	1.025	<empty>				
comp12.c	1.025	<empty>				
comp11.c	1.025	<empty>				
comp10.c	1.025	<empty>				
comp9.c	1.025	100.00%	594.54	108.000f	0.000	0.000
comp8.c	1.025	100.00%	266.51	116.475f	0.000	0.000
comp7.c	1.025	100.00%	254.22	121.700f	0.000	0.000
comp6.c	1.025	100.00%	236.79	126.825f	0.000	0.000
comp5.c	1.025	100.00%	236.74	131.775f	0.000	0.000
comp4.c	1.025	100.00%	227.57	136.525f	0.000	0.000
comp3.c	1.025	100.00%	435.56	143.300f	0.000	0.000
comp2.c	1.025	<empty>				
comp1.c	1.025	<empty>				
comp0.c	1.025	<empty>				
Subtotals:		26.33%	2,251.94	124.739f	0.000	0.000

All Tanks

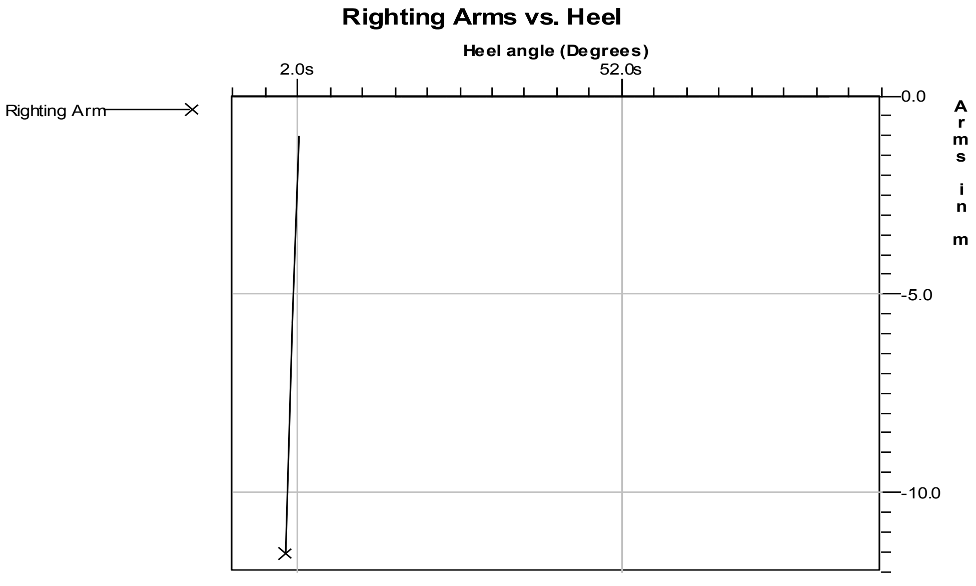
	Spgr	Load (%)	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
Totals:		26.33%	2,251.94	124.739f	0.000	0.000

Righting Arms vs. Heel

Righting Arms vs Heel Angle

Heel Angle (deg)	Trim Angle (deg)	Origin Depth (m)	Righting Arm (m)	Notes
0.00	2.27a	5.773	-0.020	
1.00p	2.27a	5.773	-0.019	
2.00p	2.27a	5.773	-0.018	
3.00p	2.27a	5.773	-0.018	
4.00p	2.27a	5.773	-0.017	
5.00p	2.27a	5.773	-0.016	
6.00p	2.27a	5.773	-0.015	
7.00p	2.27a	5.773	-0.015	
8.00p	2.27a	5.773	-0.014	
9.00p	2.27a	5.773	-0.013	
10.00p	2.27a	5.773	-0.013	
11.00p	2.27a	5.773	-0.012	
12.00p	2.27a	5.773	-0.011	
13.00p	2.27a	5.773	-0.010	
14.00p	2.27a	5.773	-0.010	
15.00p	2.27a	5.773	-0.009	
16.00p	2.27a	5.773	-0.008	
17.00p	2.27a	5.773	-0.007	
18.00p	2.27a	5.773	-0.007	
19.00p	2.27a	5.773	-0.006	
20.00p	2.27a	5.773	-0.005	
21.00p	2.27a	5.773	-0.004	
22.00p	2.27a	5.773	-0.003	
23.00p	2.27a	5.773	-0.003	
24.00p	2.27a	5.773	-0.002	
25.00p	2.27a	5.773	-0.001	
26.00p	2.27a	5.773	0.000	
27.00p	2.27a	5.773	0.000	
28.00p	2.27a	5.773	0.001	
29.00p	2.27a	5.773	0.002	
30.00p	2.27a	5.773	0.003	
31.00p	2.27a	5.773	0.003	
32.00p	2.27a	5.773	0.004	
33.00p	2.27a	5.773	0.005	
34.00p	2.27a	5.773	0.006	
35.00p	2.27a	5.773	0.006	
36.00p	2.27a	5.773	0.007	
37.00p	2.27a	5.773	0.008	
38.00p	2.27a	5.773	0.009	
39.00p	2.27a	5.773	0.009	

40.00p	2.27a	5.773	0.010	
41.00p	2.27a	5.773	0.011	
42.00p	2.27a	5.773	0.012	
43.00p	2.27a	5.773	0.012	
44.00p	2.27a	5.773	0.013	
45.00p	2.27a	5.773	0.014	
46.00p	2.27a	5.773	0.015	
47.00p	2.27a	5.773	0.015	
48.00p	2.27a	5.773	0.016	
49.00p	2.27a	5.773	0.017	
50.00p	2.27a	5.773	0.017	
51.00p	2.27a	5.773	0.018	
52.00p	2.27a	5.773	0.019	
53.00p	2.27a	5.773	0.020	
54.00p	2.27a	5.773	0.020	
55.00p	2.27a	5.773	0.021	
56.00p	2.27a	5.773	0.022	
57.00p	2.27a	5.773	0.022	
58.00p	2.27a	5.773	0.023	
59.00p	2.27a	5.773	0.024	
60.00p	2.27a	5.773	0.024	MaxRa



Case HS2
Hull Data (with appendages)

Baseline Draft: 7.413 at Origin
Trim: aft 2.59 deg.
Heel: stbd 26.87 deg.

DIMENSIONS
Length Overall: 169.500 m LWL: 156.500 m Beam: 8.837 m BWL: 8.777 m
Volume: 7129.313 m^3 Displacement: 7307.585 MT

COEFFICIENTS
Prismatic: 0.758 Block: 0.486 Midship: 0.642 Waterplane: 0.498

RATIOS
Length/Beam: 19.180 Displacement/length: 53.131 Beam/Depth: 0.833
MT/ cm Immersion: 7.012

AREAS
Waterplane: 684.094 m^2 Wetted Surface: 5168.802 m^2
Under Water Lateral Plane: 1080.864 m^2 Above Water Lateral Plane: 288.122 m^2

CENTROIDS (Meters)
Buoyancy: LCB = 69.378 fwd TCB =0.197 port VCB = -0.390
Flotation: LCF = 108.259 fwd
Under Water LP: 72.011 fwd of Origin, 4.010 below waterline.
Above Water LP: 127.187 fwd of Origin, 1.616 above waterline.

Note: Coefficients calculated based on waterline length at given draft

Floating Status

Draft FP	-1.176 m	Heel	stbd 26.87 deg.	GM(Solid)	0.048 m
Draft MS	3.119 m	Equil	Yes	F/S Corr.	0.000 m
Draft AP	7.413 m	Wind	Off	GM(Fluid)	0.048 m
Trim	aft 2.59 deg.	Wave	No	KMT	0.000 m
LCG	69.395f m	VCG	-0.043 m	TPcm	7.01
Displacement	7,307.59 MT	WaterSpgr	1.025		

Loading Summary

Item	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
Light Ship	3,110.19	82.180f	0.050s	-0.100
Deadweight	4,197.40	59.922f	0.000	0.000
Displacement	7,307.59	69.395f	0.021s	-0.043

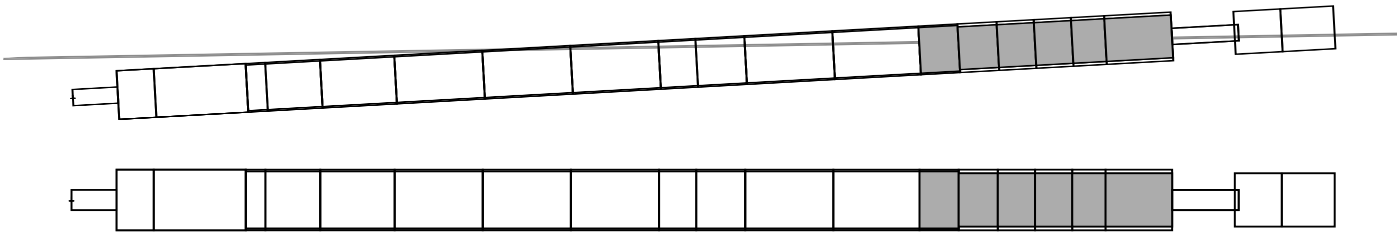
Fixed Weight Status

Item	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
LIGHT SHIP	3,110.19	82.180f	0.050s	0.100d
SOLID BALLAST	2,540.00	13.710f	0.000	0.000
Total Fixed:	5,650.19	51.400f	0.028s	0.055d

Displacer Status

Item	Status	Spgr	Displ	LCB	TCB	VCB	Eff

			(MT)	(m)	(m)	(m)	/Perm
hull	Intact	1.025	7,307.59	69.378f	0.197s	-0.390	1.000
SubTotals:			7,307.59	69.378f	0.197s	-0.390	



Fluid Legend

Fluid Name	Legend	Weight (MT)	Load%
SALT WATER		1,657.40	22.71%

Tank Status

SALT WATER

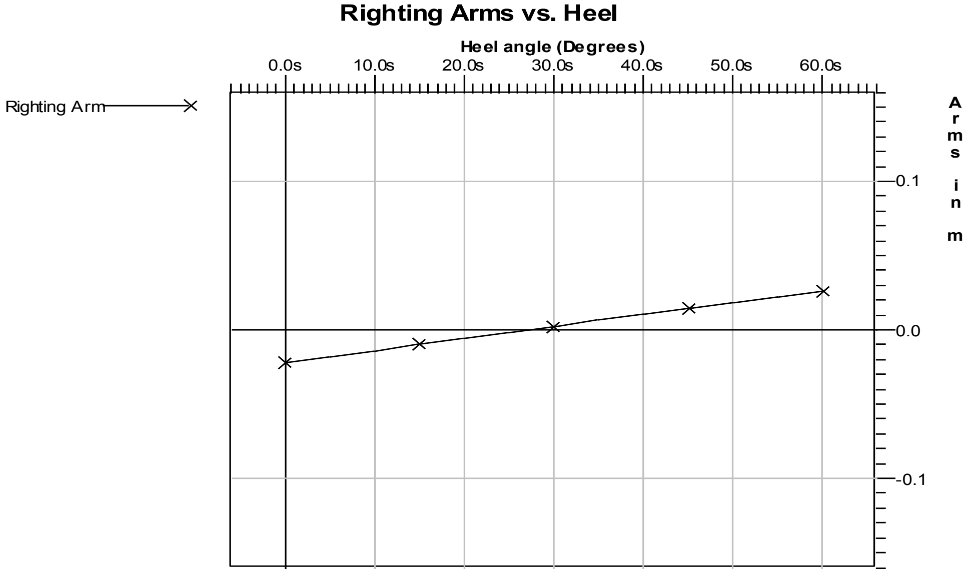
Tank Name	Spgr	Load (%)	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
WB17.c	1.025	<empty>				
WB16.c	1.025	<empty>				
WB15.c	1.025	<empty>				
WB14.c	1.025	<empty>				
WB13.c	1.025	<empty>				
WB12.c	1.025	<empty>				
WB11.c	1.025	<empty>				
WB10.c	1.025	<empty>				
WB9.c	1.025	<empty>				
WB8.c	1.025	100.00%	266.51	116.475f	0.000	0.000
WB7.c	1.025	100.00%	254.22	121.700f	0.000	0.000
WB6.c	1.025	100.00%	236.79	126.825f	0.000	0.000
WB5.c	1.025	100.00%	236.74	131.775f	0.000	0.000
WB4.c	1.025	100.00%	227.57	136.525f	0.000	0.000
WB3.c	1.025	100.00%	435.56	143.300f	0.000	0.000
WB2.c	1.025	<empty>				
Subtotals:		22.71%	1,657.40	130.743f	0.000	0.000

All Tanks

	Spgr	Load (%)	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
Totals:		22.71%	1,657.40	130.743f	0.000	0.000

Righting Arms vs Heel Angle

Heel Angle (deg)	Trim Angle (deg)	Origin Depth (m)	Righting Arm (m)	Notes
0.00	2.59a	6.606	-0.021	
5.00s	2.59a	6.606	-0.017	
10.00s	2.59a	6.606	-0.014	
15.00s	2.59a	6.606	-0.010	
20.00s	2.59a	6.606	-0.005	
25.00s	2.59a	6.606	-0.001	
30.00s	2.59a	6.606	0.003	
35.00s	2.59a	6.606	0.007	
40.00s	2.59a	6.606	0.011	
45.00s	2.59a	6.606	0.015	
50.00s	2.59a	6.606	0.019	
55.00s	2.59a	6.606	0.023	
60.00s	2.59a	6.606	0.026	MaxRa



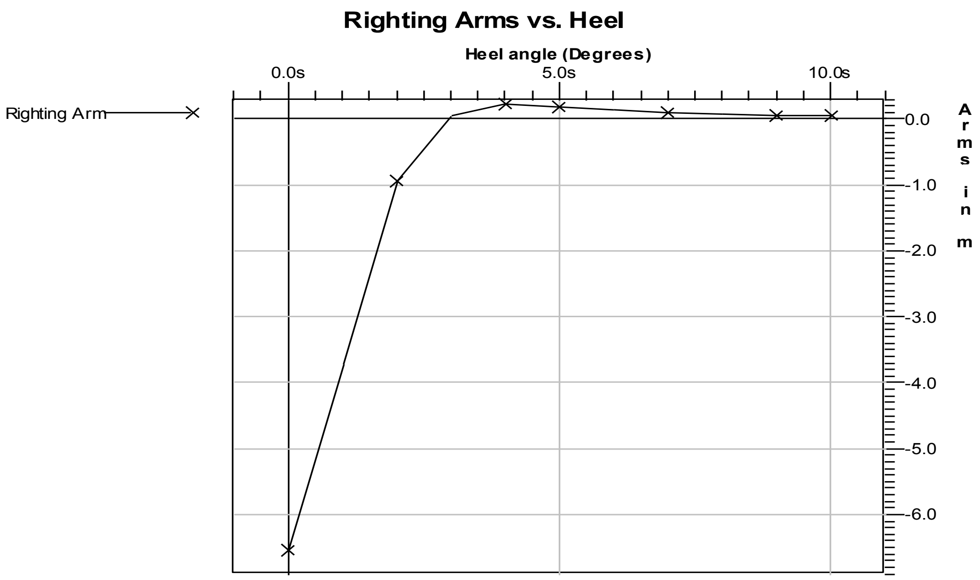
Righting Arms vs. Heel

Righting Arms vs Heel Angle

Heel axis rotated Aft 90.00 degrees

Heel Angle (deg)	Trim Angle (deg)	Origin Depth (m)	Righting Arm (m)	Notes
0.00	31.54s	2.506	-6.532	
1.00a	12.39s	3.870	-3.690	
2.00a	13.39s	5.431	-0.923	
3.00a	27.90s	6.725	0.064	
4.00a	43.80s	7.174	0.231	
4.00a	43.95s	7.160	0.225	MaxRa
5.00a	54.42s	7.214	0.198	
6.00a	61.40s	7.140	0.145	

7.00a	66.02s	7.083	0.111	
8.00a	69.34s	7.036	0.088	
9.00a	71.85s	6.996	0.071	
10.00a	73.92s	6.927	0.053	



Case HS3
Hull Data (with appendages)

Baseline Draft: 71.395 at Origin
Trim: aft 29.29 deg.
Heel: port 25.87 deg.

<u>DIMENSIONS</u>
Length Overall: 181.231 m LWL: 181.231 m Beam: 20.000 m BWL: 8.499 m
Volume: 6385.757 m^3 Displacement: 6545.436 MT

<u>COEFFICIENTS</u>
Prismatic: 0.586 Block: 0.016 Midship: 0.028 Waterplane: 0.077

<u>RATIOS</u>
Length/Beam: 9.062 Displacement/length: 30.645 Beam/Depth: 0.304
MT/ cm Immersion: 1.218

<u>AREAS</u>
Waterplane: 118.862 m^2 Wetted Surface: 4860.815 m^2
Under Water Lateral Plane: 954.050 m^2 Above Water Lateral Plane: 946.065 m^2

<u>CENTROIDS (Meters)</u>
Buoyancy: LCB = 59.879 fwd TCB =0.033 port VCB = -0.068
Flotation: LCF = 114.525 fwd
Under Water LP: 51.725 fwd of Origin, 27.133 below waterline.
Above Water LP: 138.404 fwd of Origin, 21.876 above waterline.

Note: Coefficients calculated based on waterline length at given draft

Floating Status

Draft FP	-41.571 m	Heel	port 25.87 deg.	GM(Solid)	0.189 m
Draft MS	14.912 m	Equil	Yes	F/S Corr.	0.047 m
Draft AP	71.395 m	Wind	Off	GM(Fluid)	0.142 m
Trim	aft 29.29 deg.	Wave	No	KMT	0.000 m
LCG	59.828f m	VCG	-0.149 m	TPcm	1.22
Displacement	6,545.43 MT	WaterSpgr	1.025		

Loading Summary

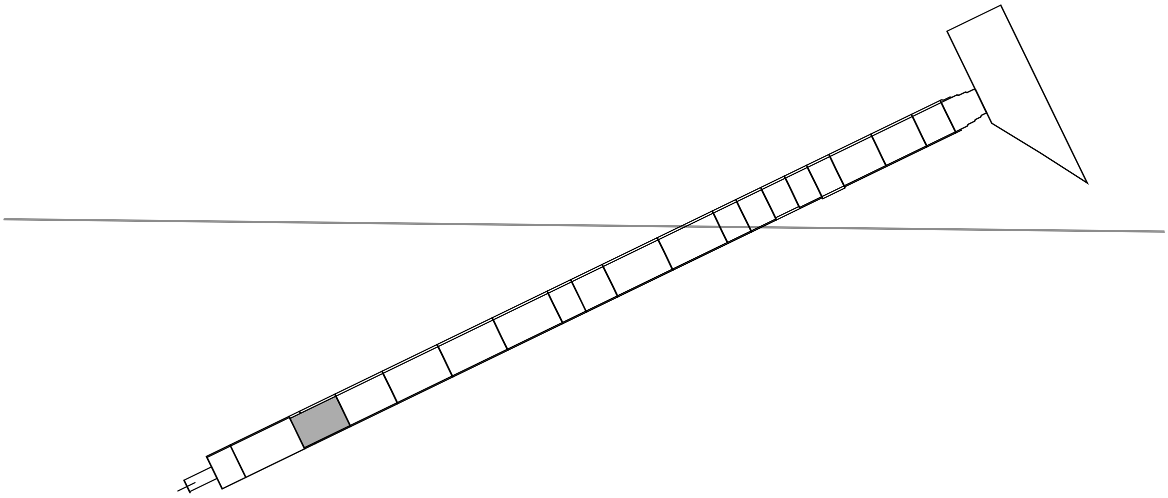
Item	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
Light Ship	3,110.19	82.180f	0.100p	-0.050
Deadweight	3,435.24	39.591f	0.046p	-0.238
Displacement	6,545.43	59.828f	0.072p	-0.149

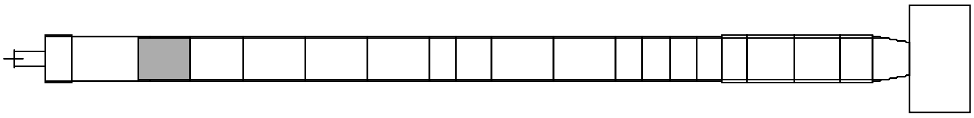
Fixed Weight Status

Item	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
LIGHT SHIP	3,110.19	82.180f	0.100p	0.050d
SOLID BALLAST	2,540.00	13.710f	0.000	0.000
TOPSIDE	519.02	174.860f	0.070p	1.090d
Total Fixed:	6,169.21	61.787f	0.056p	0.117d

Displacer Status

Item	Status	Spgr	Displ (MT)	LCB (m)	TCB (m)	VCB (m)	Eff /Perm
hull	Intact	1.025	6,545.44	59.879f	0.033p	-0.068	1.000
SubTotals:			6,545.44	59.879f	0.033p	-0.068	





Fluid Legend

Fluid Name	Legend	Weight (MT)	Load%
SALT WATER		376.22	5.06%

Tank Status

SALT WATER

Tank Name	Spgr	Load (%)	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
WB17.c	1.025	<empty>				
WB16.c	1.025	74.00%	376.22	27.712f	0.324p	-0.667
WB15.c	1.025	<empty>				
WB14.c	1.025	<empty>				
WB13.c	1.025	<empty>				
WB12.c	1.025	<empty>				
WB11.c	1.025	<empty>				
WB10.c	1.025	<empty>				
WB9.c	1.025	<empty>				
WB8.c	1.025	<empty>				
WB7.c	1.025	<empty>				

WB6.c	1.025	<empty>				
WB5.c	1.025	<empty>				
WB4.c	1.025	<empty>				
WB3.c	1.025	<empty>				
WB2.c	1.025	<empty>				
WB1.c	1.025	<empty>				
WB0.c	1.025	<empty>				
Subtotals:		5.06%	376.22	27.712f	0.324p	-0.667

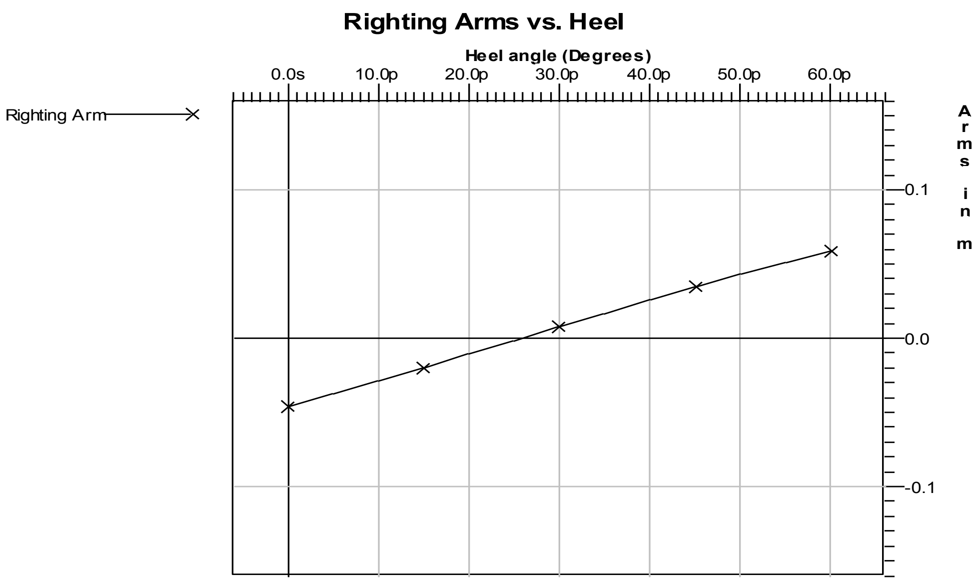
All Tanks

	Spgr	Load (%)	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
Totals:		5.06%	376.22	27.712f	0.324p	-0.667

Righting Arms vs. Heel

Righting Arms vs Heel Angle

Heel Angle (deg)	Trim Angle (deg)	Origin Depth (m)	Righting Arm (m)	Notes
0.00	30.10a	57.451	-0.046	
5.00p	30.10a	57.451	-0.037	
10.00p	30.10a	57.451	-0.029	
15.00p	30.10a	57.451	-0.020	
20.00p	30.10a	57.451	-0.011	
25.00p	29.48a	56.373	-0.001	
30.00p	29.48a	56.373	0.008	
35.00p	29.48a	56.373	0.017	
40.00p	29.48a	56.373	0.026	
45.00p	29.48a	56.373	0.035	
50.00p	30.07a	57.383	0.044	
55.00p	30.07a	57.383	0.052	
60.00p	30.90a	58.818	0.059	MaxRa



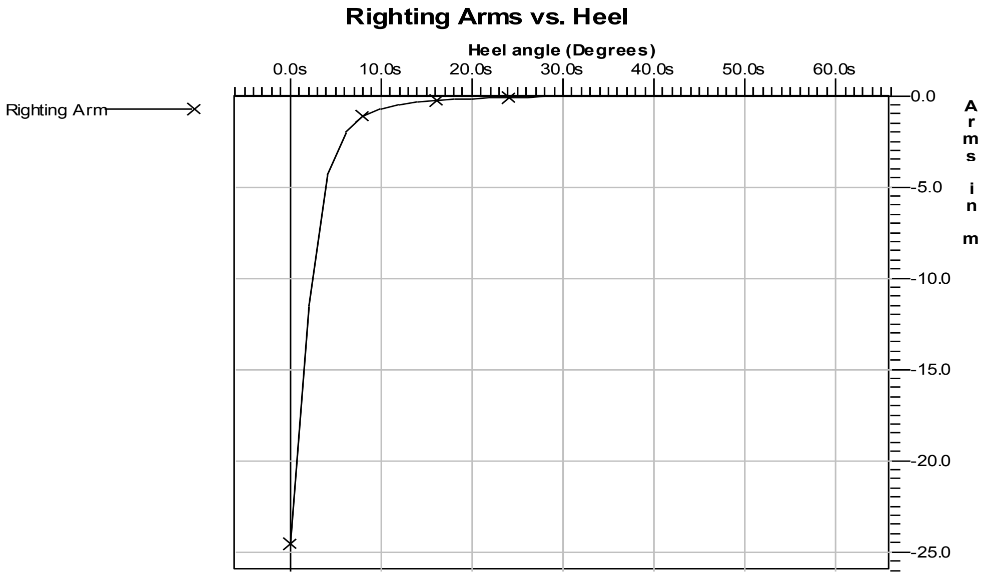
Righting Arms vs. Trim

Righting Arms vs Trim Angle

Heel axis rotated Aft 90.00 degrees

Heel Angle (deg)	Trim Angle (deg)	Origin Depth (m)	Righting Arm (m)	Notes
0.00	23.87p	1.155	-24.478	
2.00a	5.87p	4.292	-11.463	
4.00a	7.34p	7.928	-4.286	
6.00a	9.51p	11.797	-1.943	
8.00a	11.37p	15.623	-1.070	
10.00a	12.91p	19.387	-0.664	
12.00a	14.25p	23.087	-0.441	
14.00a	14.58p	26.823	-0.307	
16.00a	16.36p	30.297	-0.222	
18.00a	16.69p	33.906	-0.162	
20.00a	17.03p	37.460	-0.118	
22.00a	18.84p	40.611	-0.087	
24.00a	19.17p	44.005	-0.061	
26.00a	19.50p	47.331	-0.040	
28.00a	19.84p	50.583	-0.024	
30.00a	21.61p	53.246	-0.011	
32.00a	20.61p	56.813	0.001	
34.00a	22.92p	58.993	0.010	
36.00a	21.92p	62.457	0.020	
38.00a	24.33p	64.255	0.026	
40.00a	23.33p	67.604	0.034	
42.00a	25.83p	68.980	0.038	
44.00a	24.83p	72.205	0.045	
46.00a	27.49p	73.089	0.048	

48.00a	26.49p	76.182	0.054	
50.00a	29.37p	76.459	0.056	
52.00a	28.37p	79.412	0.062	
54.00a	31.44p	79.054	0.062	
56.00a	30.44p	81.862	0.067	
58.00a	33.72p	80.792	0.067	
60.00a	34.05p	82.182	0.069	MaxRa



Case HS4
Hull Data (with appendages)

Baseline Draft: 4.582 at Origin
Trim: aft 0.85 deg.
Heel: stbd 26.87 deg.

DIMENSIONS
Length Overall: 169.500 m LWL: 156.500 m Beam: 8.837 m BWL: 7.803 m
Volume: 7494.719 m^3 Displacement: 7682.128 MT

COEFFICIENTS
Prismatic: 0.810 Block: 0.650 Midship: 0.802 Waterplane: 0.705

RATIOS
Length/Beam: 19.180 Displacement/length: 55.855 Beam/Depth: 1.060
MT/ cm Immersion: 8.820

AREAS
Waterplane: 860.440 m^2 Wetted Surface: 5165.107 m^2
Under Water Lateral Plane: 1144.920 m^2 Above Water Lateral Plane: 224.066 m^2

CENTROIDS (Meters)
Buoyancy: LCB = 74.134 fwd TCB =0.179 port VCB = -0.353

Flotation: LCF = 85.117 fwd
Under Water LP: 79.756 fwd of Origin, 3.568 below waterline.
Above Water LP: 103.850 fwd of Origin, 0.861 above waterline.

Note: Coefficients calculated based on waterline length at given draft

Floating Status

Draft FP	1.759 m	Heel	stbd 26.87 deg.	GM(Solid)	0.045 m
Draft MS	3.170 m	Equil	Yes	F/S Corr.	0.000 m
Draft AP	4.582 m	Wind	Off	GM(Fluid)	0.045 m
Trim	aft 0.85 deg.	Wave	No	KMT	0.000 m
LCG	74.141f m	VCG	-0.040 m	TPcm	8.82
Displacement	7,682.13 MT	WaterSpgr	1.025		

Loading Summary

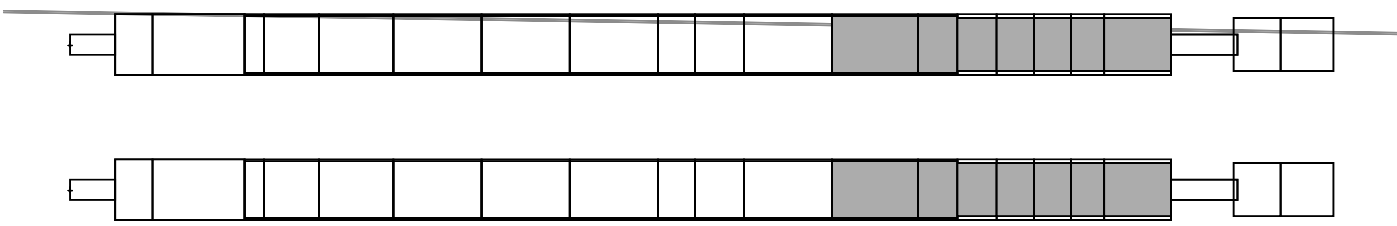
Item	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
Light Ship	3,110.19	82.180f	0.050s	-0.100
Deadweight	4,571.94	68.673f	0.000	0.000
Displacement	7,682.13	74.141f	0.020s	-0.040

Fixed Weight Status

Item	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
LIGHT SHIP	3,110.19	82.180f	0.050s	0.100d
CRANE LIFT	-220.00	8.000f	0.000	0.000
SOLID BALLAST	2,540.00	13.710f	0.000	0.000
Total Fixed:	5,430.19	53.158f	0.029s	0.057d

Displacer Status

Item	Status	Spgr	Displ (MT)	LCB (m)	TCB (m)	VCB (m)	Eff /Perm
hull	Intact	1.025	7,682.13	74.134f	0.179s	-0.353	1.000
SubTotals:			7,682.13	74.134f	0.179s	-0.353	



Fluid Legend

Fluid Name	Legend	Weight (MT)	Load%
SALT WATER		2,251.94	30.85%

Tank Status

SALT WATER

Tank Name	Spgr	Load (%)	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
WB17.c	1.025	<empty>				
WB16.c	1.025	<empty>				
WB15.c	1.025	<empty>				
WB14.c	1.025	<empty>				
WB13.c	1.025	<empty>				
WB12.c	1.025	<empty>				
WB11.c	1.025	<empty>				
WB10.c	1.025	<empty>				
WB9.c	1.025	100.00%	594.54	108.000f	0.000	0.000
WB8.c	1.025	100.00%	266.51	116.475f	0.000	0.000
WB7.c	1.025	100.00%	254.22	121.700f	0.000	0.000
WB6.c	1.025	100.00%	236.79	126.825f	0.000	0.000
WB5.c	1.025	100.00%	236.74	131.775f	0.000	0.000
WB4.c	1.025	100.00%	227.57	136.525f	0.000	0.000
WB3.c	1.025	100.00%	435.56	143.300f	0.000	0.000
WB2.c	1.025	<empty>				
Subtotals:		30.85%	2,251.94	124.739f	0.000	0.000

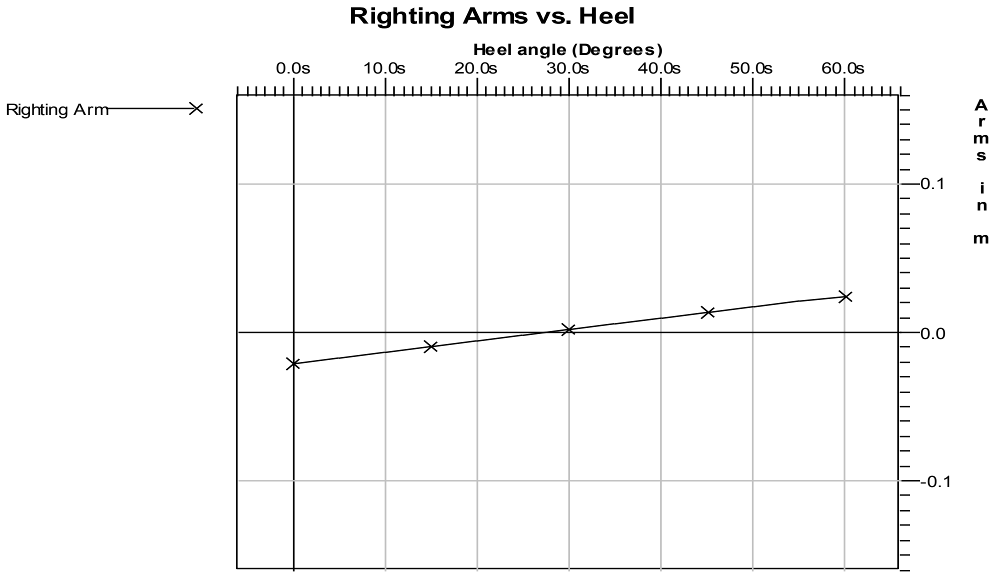
All Tanks

	Spgr	Load (%)	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
Totals:		30.85%	2,251.94	124.739f	0.000	0.000

Righting Arms vs Heel Angle

Heel Angle (deg)	Trim Angle (deg)	Origin Depth (m)	Righting Arm (m)	Notes
0.00	0.85a	4.086	-0.020	
5.00s	0.85a	4.086	-0.017	
10.00s	0.85a	4.086	-0.013	
15.00s	0.85a	4.086	-0.009	
20.00s	0.85a	4.086	-0.005	
25.00s	0.85a	4.086	-0.001	
30.00s	0.85a	4.086	0.003	
35.00s	0.85a	4.086	0.007	
40.00s	0.85a	4.086	0.011	
45.00s	0.85a	4.086	0.014	

50.00s	0.85a	4.086	0.018	
55.00s	0.85a	4.086	0.022	
60.00s	0.85a	4.086	0.025	MaxRa



Righting Arms vs. Heel

Righting Arms vs Heel Angle

Heel axis rotated Aft 90.00 degrees

Heel Angle (deg)	Trim Angle (deg)	Origin Depth (m)	Righting Arm (m)	Notes
0.00	31.54s	2.889	-2.481	
1.00a	31.87s	4.084	-0.003	
1.60a	47.94s	4.421	0.436	MaxRa
2.00a	55.98s	4.495	0.434	
3.00a	68.81s	4.440	0.247	
4.00a	75.20s	4.343	0.131	
5.00a	78.60s	4.291	0.082	
6.00a	80.71s	4.260	0.057	
7.00a	82.21s	4.230	0.040	
8.00a	83.21s	4.224	0.034	
9.00a	84.10s	4.193	0.023	
10.00a	84.68s	4.192	0.021	

Case HS5

Hull Data (with appendages)

Baseline Draft: 116.449 at Origin
Trim: fwd 4.38 deg.

Heel: port 2.11 deg.

DIMENSIONS
Length Overall: 47.800 m   LWL: 33.378 m   Beam: 20.000 m   BWL: 8.561 m
Volume: 6497.364 m^3 Displacement: 6659.834 MT

COEFFICIENTS
Prismatic: 0.201   Block: 0.184   Midship: 0.916   Waterplane: 0.203

RATIOS
Length/Beam: 2.390   Displacement/length: 4990.965   Beam/Depth: 0.172
MT/ cm Immersion: 0.595

AREAS
Waterplane: 58.027 m^2 Wetted Surface: 5308.264 m^2
Under Water Lateral Plane: 997.839 m^2 Above Water Lateral Plane: 1056.578 m^2

CENTROIDS (Meters)
Buoyancy: LCB = 0.003 fwd   TCB =0.002 port   VCB = 60.767
Flotation: LCF = 0.000
Under Water LP: 4.590 fwd of Origin, 56.189 below waterline.
Above Water LP: 15.093 fwd of Origin, 43.923 above waterline.

Note: Coefficients calculated based on waterline length at given draft

Floating Status

Draft FP	118.673 m	Heel	port 2.11 deg.	GM(Solid)	1.482 m
Draft MS	116.840 m	Equil	Yes	F/S Corr.	0.061 m
Draft AP	115.008 m	Wind	Off	GM(Fluid)	1.421 m
Trim	fwd 4.38 deg.	Wave	No	KMT	60.808 m
LCG	0.113f m	VCG	59.332 m	TPcm	0.59
Displacement	6,659.83 MT	WaterSpgr	1.025		

Loading Summary

Item	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
Light Ship	3,110.19	0.050f	0.100p	82.180
Deadweight	3,549.64	0.168f	0.014p	39.313
Displacement	6,659.83	0.113f	0.054p	59.332

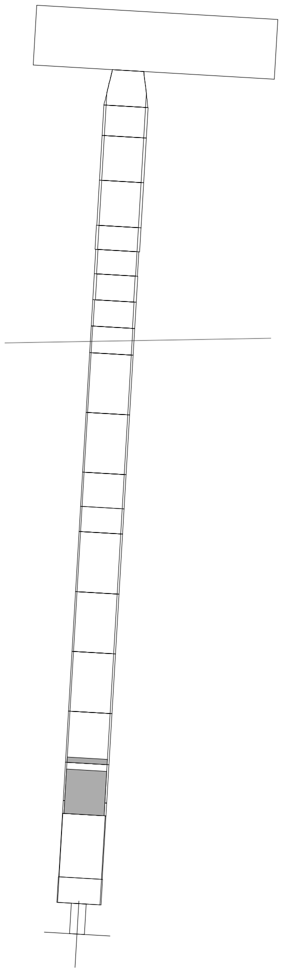
Fixed Weight Status

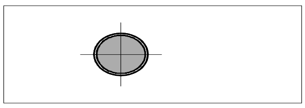
Item	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
LIGHT SHIP	3,110.19	0.050f	0.100p	82.180u
SOLID BALLAST	2,540.00	0.000	0.000	13.710u

TOPSIDE	519.02	1.090f	0.070p	174.860u
Total Fixed:	6,169.21	0.117f	0.056p	61.787u

Displacer Status

Item	Status	Spgr	Displ (MT)	LCB (m)	TCB (m)	VCB (m)	Eff /Perm
hull	Intact	1.025	6,659.83	0.003f	0.002p	60.767	1.000
SubTotals:			6,659.83	0.003f	0.002p	60.767	





Fluid Legend

Fluid Name	Legend	Weight (MT)	Load%
SALT WATER		490.62	6.60%

Tank Status

SALT WATER

Tank Name	Spgr	Load (%)	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
WB17.c	1.025	<empty>				
WB16.c	1.025	86.50%	439.78	0.035f	0.017p	27.827
WB15.c	1.025	10.00%	50.84	0.302f	0.145p	34.014
WB14.c	1.025	<empty>				
WB13.c	1.025	<empty>				
WB12.c	1.025	<empty>				
WB11.c	1.025	<empty>				
WB10.c	1.025	<empty>				
WB9.c	1.025	<empty>				
WB8.c	1.025	<empty>				
WB7.c	1.025	<empty>				
WB6.c	1.025	<empty>				
WB5.c	1.025	<empty>				
WB4.c	1.025	<empty>				
WB3.c	1.025	<empty>				
WB2.c	1.025	<empty>				
WB1.c	1.025	<empty>				
WB0.c	1.025	<empty>				
Subtotals:		6.60%	490.62	0.063f	0.030p	28.468

All Tanks

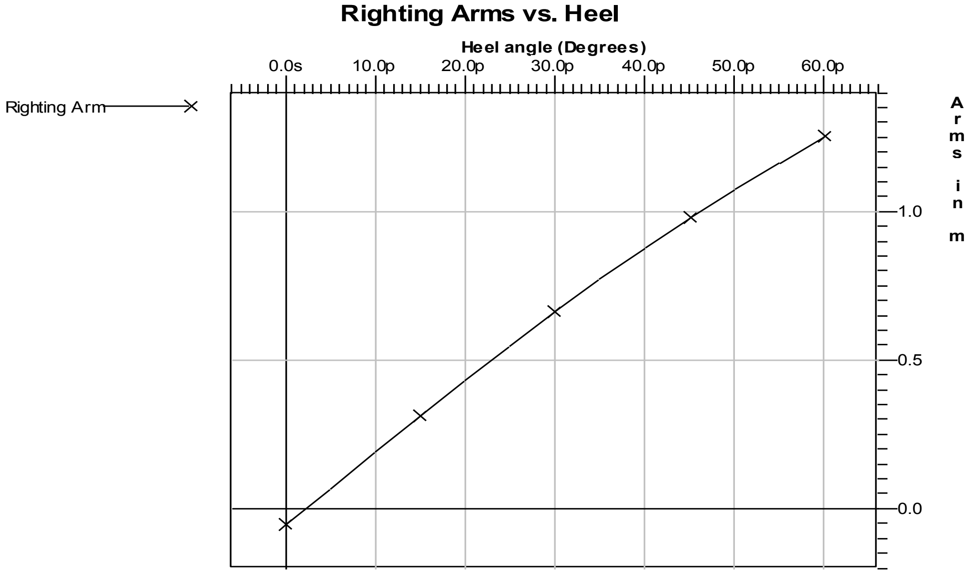
	Spgr	Load (%)	Weight (MT)	LCG (m)	TCG (m)	VCG (m)
Totals:		6.60%	490.62	0.063f	0.030p	28.468

Righting Arms vs. Heel

Righting Arms vs Heel Angle

Heel Angle (deg)	Trim Angle (deg)	Origin Depth (m)	Righting Arm (m)	Notes
0.00	4.38f	116.109	-0.052	
5.00p	4.38f	115.667	0.071	
10.00p	4.38f	114.345	0.194	
15.00p	4.48f	112.138	0.315	
20.00p	4.58f	109.076	0.434	
25.00p	4.72f	105.181	0.551	
30.00p	4.90f	100.479	0.665	
35.00p	5.14f	95.007	0.775	
40.00p	5.44f	88.805	0.880	
45.00p	5.82f	81.919	0.981	
50.00p	6.30f	74.401	1.076	
55.00p	6.93f	66.307	1.167	

60.00p	7.74f	57.697	1.255	MaxRa



Righting Arms vs. Trim

Righting Arms vs Trim Angle

Heel axis rotated Aft 90.00 degrees

Heel Angle (deg)	Trim Angle (deg)	Origin Depth (m)	Righting Arm (m)	Notes
0.00	2.11p	116.370	-0.108	
5.00f	2.11p	115.927	0.016	
10.00f	2.11p	114.602	0.139	
15.00f	2.11p	112.405	0.262	
20.00f	2.18p	109.347	0.382	
25.00f	2.24p	105.458	0.501	
30.00f	2.32p	100.765	0.618	
35.00f	2.42p	95.305	0.731	
40.00f	2.55p	89.118	0.840	
45.00f	2.72p	82.251	0.945	
50.00f	2.93p	74.756	1.045	
55.00f	3.20p	66.691	1.141	
60.00f	3.55p	58.116	1.235	MaxRa

Appendix C

Calculations to obtain correct draft from AutoHydro

Case 1

$heel_1 := 26.87deg$

$trim_1 := 2.27deg$

$draft_1 := 6.477m$

$draft_{corr1} := draft_1 \cdot \left(\cos\left(heel_1\right) \cdot \cos\left(trim_1\right)\right) = 5.773m$

Case 2

$heel_2 := 26.87deg$

$trim_2 := 2.59deg$

$draft_2 := 7.413m$

$draft_{corr2} := draft_2 \cdot \cos\left(heel_2\right) = 6.613m$

Case 3

$heel_3 := 25.87deg$

$trim_3 := 29.29deg$

$draft_3 := 71.395m$

$draft_{corr3} := draft_3 \cdot \cos\left(heel_3\right) = 64.24m$

Case 4

$heel_4 := 26.87deg$

$trim_4 := 0.85deg$

$draft_4 := 4.582m$

$draft_{corr4} := draft_4 \cdot \cos\left(heel_4\right) = 4.087m$

Case 5

$heel_5 := 2.11deg$

$trim_5 := 4.38deg$

$draft_5 := 116.449m$

$draft_{corr5} := draft_5 \cdot \cos\left(heel_5\right) = 116.37m$

Appendix D

Case HS1

Data

$F_{sb} := 25400\,tonne$

$d_{sb} := 13.71m$

$F_{col} := 3110.19\,tonne$

$d_{col} := 82.18m$

$F_{wb} := 2251.94\,tonne$

$d_{wb} := 124.739m$

$F_b := F_{sb} + F_{col} + F_{wb}$

$L_v := 169.5m$

$\rho := 1025\,kg \cdot m^{-3}$

Set the diameter over the column

$$D(x) := \begin{cases} 8.59m & \text{if } 0m \le x \le 169.5m \\ (0m) & \text{otherwise} \end{cases}$$

Define the radius due to the diameter

$$R(x) := \frac{D(x)}{2}$$

Guess values for α and draft

$\alpha := -10deg$

$draft := 10m$

Define the displaced volume, the center of gravity in x- and z- direction

$$W_v := \frac{F_b}{\rho} = 7709.395m^3$$

$$x_G := \frac{\left(F_{sb} \cdot d_{sb} + F_{wb} \cdot d_{wb} + F_{col} \cdot d_{col}\right)}{\left(F_{sb} + F_{wb} + F_{col}\right)} = 72.3m$$

$z_G := -0.039m$

Set tolerance for convergation and time step for iteration

$TOL := 0.1$

$CTOL := 0.1$

Put up equations that define the draft and the submerged area

Given

$d(x, draft, \alpha) := \tan(\alpha) \cdot x + draft$

$$a(x, draft, \alpha) := \begin{cases} R(x) & \text{if } d(x, draft, \alpha) \ge R(x) \\ d(x, draft, \alpha) & \text{if } |d(x, draft, \alpha)| \le R(x) \\ (-R(x)) & \text{if } d(x, draft, \alpha) < -R(x) \end{cases}$$

Area of submerged part

$$A_{cyl}(x, draft, \alpha) := 2 \cdot \int_{-a(x, draft, \alpha)}^{R(x)} \sqrt{R(x)^2 - t^2}\, dt$$

Displaced volume

$$V_v(draft, \alpha) := \int_{0m}^{L_v} A_{cyl}(x, draft, \alpha)\, dx$$

Displaced volume should be equal to the earlier defined by buoyancy

$$V_v(draft, \alpha) = W_v$$

Area of submerged part defined in z- direction

$$A_z(z, draft, \alpha) := 2\left(\frac{z}{\tan(\alpha)} + draft\right)\sqrt{(4.295m)^2 - z^2}$$

Center of buoyancy in z- and x- direction

$$z_B(draft, \alpha) := \frac{\int_{-4.295m}^{4.295m} z \cdot A_z(z, draft, \alpha)\, dz}{V_v(draft, \alpha)}$$

$$x_B(draft, \alpha) := \frac{\int_{0m}^{L_v} x \cdot A_{cyl}(x, draft, \alpha)\, dx}{V_v(draft, \alpha)}$$

Equilibrium when center of gravity and buoyancy is in the same longitudinal position

$$\begin{pmatrix} x_G - x_B(draft, \alpha) \\ z_G - z_B(draft, \alpha) \end{pmatrix} \cdot \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix} = 0$$

Find the draft and α that fulfills the requirements

$$\begin{pmatrix} draft \\ \alpha \end{pmatrix} := Find(draft, \alpha)$$

$draft = 5.784m$

$\alpha = -2.272deg$

$draft_{real} := draft \cdot \cos(\alpha) = 5.779m$

Center of buoyancy in x- and z- direction for calculated draft and trim angle

$x_B(draft, \alpha) = 72.267m$

$z_B(draft, \alpha) = -0.874m$

Find where the column hits the water

$x := 50m$

$d(x) := \tan(\alpha) \cdot x + draft$

Given

$d(x) = 4.295m$

$x_1 := Find(x)$

$x_1 = 37.525m$

Find out if and where the top leaves the water

$d(L_v) = -0.941m$

Given

$d(x) = -4.295m$

$x_3 := Find(x)$

$x_3 = 254.055m$

Length of waterline

$a_3 := (x_3 - x_1)\cos(\alpha) = 216.36m$

$$a_4 := \frac{a_3}{2} = 108.18\,m$$

Distance the center of waterline changes if the top dont leave the water

$$L_e := \begin{cases} \dfrac{(L_v - x_1)}{\cos(\alpha)} - a_4 & \text{if } d(L_v) > -4.295\,m \\ a_4 & \text{otherwise} \end{cases}$$

$$LCF := \frac{2\left[\displaystyle\int_{-a_4}^{L_e} t\left[\frac{4.295\,m}{a_4}\sqrt{(a_4)^2 - t^2}\right] dt\right]}{\left[2\cdot\displaystyle\int_{-a_4}^{L_e} \frac{4.295\,m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right] dt\right]}$$

$$LCF = -33.303\,m$$

The center of floatation for the buoy

$$CF := \frac{x_1\cdot\cos(\alpha) + a_4 + LCF}{\cos(\alpha)} = 112.461\,m$$

Waterline area of the buoy

$$A_w := 2\cdot\left[\int_{-a_4}^{L_e} \frac{4.295\,m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right] dt\right] = 933.461\,m^2$$

Calculation of longitudinal moment of intertia of the waterplane

$$I_L := 2\left[\int_{-a_4}^{L_e} t^2\left[\frac{4.295\,m}{a_4}\sqrt{(a_4)^2 - t^2}\right] dt\right] - 2\cdot\int_{-a_4}^{L_e} \frac{4.295\,m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right] dt\cdot LCF^2$$

Calculation of the longitudinal metacentric radius

$$BM_L := \frac{I_L}{W_v} = 147.692\,m$$

Calculation of the metacentric height

$$GM_L := z_B(draft, \alpha) + BM_L - z_G = 146.858\,m$$

Calculation of transverse moment of intertia of the waterplane

$$I_T := \frac{2}{3}\cdot\int_{-a_4}^{L_e} \left[\frac{4.295\,m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right]\right]^3 dt$$

Calculation of the transvese metacentric radius

$$BM_T := \frac{I_T}{W_v} = 0.596\,m$$

Calculation of the tranverse metacentric height

$$GM_T := z_B(draft, \alpha) + BM_T - z_G = -0.239\,m$$

Case HS2

Data

$$F_{sb} := 2540\,tonne$$

$d_{sb} := 13.71m$

$F_{col} := 3110.19tonne$

$d_{col} := 82.18m$

$F_{wb} := 1657.4tonne$

$d_{wb} := 130.743m$

$F_b := F_{sb} + F_{col} + F_{wb}$

$L_v := 156.5m$

$\rho := 1025kg \cdot m^{-3}$

Set the diameter over the column

$$D(x) := \begin{vmatrix} (0m) \text{ if } x \geq 156.2m \\ (2.95m) \text{ if } 0m \leq x < 6m \\ (8.75m) \text{ if } 6m \leq x < 11m \\ (8.62m) \text{ if } 11m \leq x < 23.5m \\ (8.63m) \text{ if } 23.5m \leq x < 26m \\ (8.62m) \text{ if } 26m \leq x < 33.5m \\ (8.63m) \text{ if } 33.5m \leq x < 43.5m \\ (8.62m) \text{ if } 43.5m \leq x < 55.25m \\ (8.63m) \text{ if } 55.25m \leq x < 67m \\ (8.62m) \text{ if } 67m \leq x < 113.85m \\ (8.58m) \text{ if } 113.85m \leq x < 119.1m \\ (8.6m) \text{ if } 119.1m \leq x < 124.3m \\ (8.58m) \text{ if } 124.3m \leq x < 129.35m \\ (8.57m) \text{ if } 129.35m \leq x < 134.2m \\ (8.83m) \text{ if } 134.2m \leq x < 138.85m \\ (8.72m) \text{ if } 138.85m \leq x < 147.75m \\ (2.8m) \text{ if } 147.75m \leq x < 156.2m \end{vmatrix}$$

Define the radius due to the diameter

$$R(x) := \frac{D(x)}{2}$$

Guess values for α and draft

$\alpha := -4deg$

$draft := 10m$

Define the displaced volume, the center of gravity in x- and z- direction

$$W_v := \frac{F_b}{\rho} = 7129.356m^3$$

$$x_G := \frac{(F_{sb} \cdot d_{sb} + F_{wb} \cdot d_{wb} + F_{col} \cdot d_{col})}{(F_{sb} + F_{wb} + F_{col})} = 69.395m$$

$z_G := -0.043m$

Set the tolerance of convertion and the iteration step

$CTOL := 0.01$

$TOL := 0.01$

Put up equations that define the draft and the submerged area

Given

$d(x, draft, \alpha) := \tan(\alpha) \cdot x + draft$

$$a(x, draft, \alpha) := \begin{cases} R(x) & \text{if } d(x, draft, \alpha) \geq R(x) \\ d(x, draft, \alpha) & \text{if } |d(x, draft, \alpha)| \leq R(x) \\ (-R(x)) & \text{if } d(x, draft, \alpha) < -R(x) \end{cases}$$

Area of submerged part

$$A_{cyl}(x, draft, \alpha) := 2 \cdot \int_{-a(x, draft, \alpha)}^{R(x)} \sqrt{R(x)^2 - t^2}\, dt$$

Displaced volume

$$V_v(draft, \alpha) := \int_{0m}^{L_v} A_{cyl}(x, draft, \alpha)\, dx$$

Displaced volume should be equal to the earlier defined by buoyancy

$$V_v(draft, \alpha) = W_v$$

Area of submerged part defined in z- direction

$$A_z(z, draft, \alpha) := 2\left(\frac{z}{\tan(\alpha)} + draft\right)\sqrt{(4.295m)^2 - z^2}$$

Center of buoyancy in z- and x- direction

$$z_B(draft, \alpha) := \frac{\int_{-4.295m}^{4.295m} z \cdot A_z(z, draft, \alpha)\, dz}{V_v(draft, \alpha)}$$

$$x_B(draft, \alpha) := \frac{\int_{0m}^{L_v} x \cdot A_{cyl}(x, draft, \alpha)\, dx}{V_v(draft, \alpha)}$$

Equilibrium when center of gravity and buoyancy is in the same longitudinal position

$$\begin{pmatrix} x_G - x_B(draft, \alpha) \\ z_G - z_B(draft, \alpha) \end{pmatrix} \cdot \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix} = 0$$

Find the draft and trim angle that fulfills the requirements

$$\begin{pmatrix} draft \\ \alpha \end{pmatrix} := \text{Find}(draft, \alpha)$$

$$draft = 6.739m$$

$$\alpha = -2.648deg$$

$$draft_{real} := draft \cdot \cos(\alpha) = 6.732m$$

Center of buoyancy in z- and x- direction for calculated draft and trim angle

$$z_B(draft, \alpha) = -0.811m$$

$$x_B(draft, \alpha) = 69.36m$$

Find where the column hits the water

$$x := 10m$$

$$d(x) := \tan(\alpha) \cdot x + draft$$

Given

$$d(x) = 4.295m$$

$$x_1 := \text{Find}(x)$$

$$x_1 = 52.842m$$

Find out if and where the top leaves the water

$$d(L_v) = -0.499m$$

Given

$d(x) = -4.295m$

$x_3 := \mathrm{Find}(x)$

$x_3 = 238.566m$

Length of waterline

$a_3 := (x_3 - x_1)\cos(\alpha) = 185.526m$

$a_4 := \frac{a_3}{2} = 92.763m$

Distance the center of waterline changes if the top do nott leave the water

$$L_e := \begin{vmatrix} \frac{(L_V - x_1)}{\cos(\alpha)} - a_4 \text{ if } d(L_V) > -4.295m \\ a_4 \text{ otherwise} \end{vmatrix}$$

$$LCF := \frac{2\left[\int_{-a_4}^{L_e} t\left[\frac{4.295m}{a_4}\sqrt{(a_4)^2 - t^2}\right] dt\right]}{\left[2\cdot\int_{-a_4}^{L_e} \frac{4.295m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right] dt\right]}$$

$LCF = -33.494m$

The center of floatation of the buoy

$$CF := \frac{x_1\cdot\cos(\alpha) + a_4 + LCF}{\cos(\alpha)} = 112.175m$$

Waterline area of the buoy

$$A_W := 2\cdot\left[\int_{-a_4}^{L_e} \frac{4.295m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right] dt\right] = 720.152m^2$$

Calculation of longitudinal moment of intertia of the waterplane

$$I_L := 2\left[\int_{-a_4}^{L_e} t^2\left[\frac{4.295m}{a_4}\sqrt{(a_4)^2 - t^2}\right] dt\right] - 2\cdot\int_{-a_4}^{L_e} \frac{4.295m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right] dt\cdot LCF^2$$

Calculation of the longitudinal metacentric radius

$$BM_L := \frac{I_L}{W_v} = 76.057m$$

Calculation of the longitudinal metacentric height

$GM_L := z_B(\mathrm{draft}, \alpha) + BM_L - z_G = 75.289m$

Calculation of transverse moment of intertia of the waterplane

$$I_T := \frac{2}{3}\cdot\int_{-a_4}^{L_e} \left[\frac{4.295m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right]\right]^3 dt$$

Calculation of the transvese metacentric radius

$$BM_T := \frac{I_T}{W_v} = 0.486\,m$$

Calculation of the tranverse metacentric height

$$GM_T := z_B(draft, \alpha) + BM_T - z_G = -0.282\,m$$

Case HS3

Data

$F_{sb} := 2540\,tonne$

$d_{sb} := 13.71\,m$

$F_{col} := 3110.19\,tonne$

$d_{col} := 82.18\,m$

$F_{wb} := 376.22\,tonne$

$d_{wb} := 27.712\,m$

$F_{top} := 519.02\,tonne$

$d_{top} := 174.86\,m$

$F_b := F_{sb} + F_{col} + F_{wb} + F_{top}$

$L_v := 169.5\,m$

$\rho := 1025\,kg \cdot m^{-3}$

Set the diameter over the column

$$D(x) := \left| \begin{array}{l} (0m) \text{ if } x \geq 156.2m \\ (2.95m) \text{ if } 0m \leq x < 6m \\ (8.75m) \text{ if } 6m \leq x < 11m \\ (8.62m) \text{ if } 11m \leq x < 23.5m \\ (8.63m) \text{ if } 23.5m \leq x < 26m \\ (8.62m) \text{ if } 26m \leq x < 33.5m \\ (8.63m) \text{ if } 33.5m \leq x < 43.5m \\ (8.62m) \text{ if } 43.5m \leq x < 55.25m \\ (8.63m) \text{ if } 55.25m \leq x < 67m \\ (8.62m) \text{ if } 67m \leq x < 113.85m \\ (8.58m) \text{ if } 113.85m \leq x < 119.1m \\ (8.6m) \text{ if } 119.1m \leq x < 124.3m \\ (8.58m) \text{ if } 124.3m \leq x < 129.35m \\ (8.57m) \text{ if } 129.35m \leq x < 134.2m \\ (8.83m) \text{ if } 134.2m \leq x < 138.85m \\ (8.72m) \text{ if } 138.85m \leq x < 147.75m \\ (2.8m) \text{ if } 147.75m \leq x < 156.2m \end{array} \right.$$

Define the radius due to the diameter

$$R(x) := \frac{D(x)}{2}$$

Guess values for α and draft

$\alpha := -40\,deg$

$draft := 70\,m$

Define the displaced volume, the center of gravity in x- and z- direction

$$W_v := \frac{F_b}{\rho} = 6385.785\,m^3$$

$$x_G := \frac{\left(F_{sb} \cdot d_{sb} + F_{wb} \cdot d_{wb} + F_{col} \cdot d_{col} + F_{top} \cdot d_{top}\right)}{\left(F_{sb} + F_{wb} + F_{col} + F_{top}\right)} = 59.828\,m$$

$$z_G := -0.149\,m$$

$$y_G := -0.072\,m$$

$$zy_G := \sqrt{\left(0 - y_G\right)^2 + \left(0 - z_G\right)^2}$$

Set the tolerance of convertion and the iteration step

$$TOL := 0.0$$

$$CTOL := 0.1$$

Put up equations that define the draft and the submerged area

Given

$$d(x, draft, \alpha) := \tan(\alpha) \cdot x + draft$$

$$a(x, draft, \alpha) := \begin{cases} R(x) & \text{if } d(x, draft, \alpha) \geq R(x) \\ d(x, draft, \alpha) & \text{if } |d(x, draft, \alpha)| \leq R(x) \\ (-R(x)) & \text{if } d(x, draft, \alpha) < -R(x) \end{cases}$$

Area of submerged part

$$A_{cyl}(x, draft, \alpha) := 2 \cdot \int_{-a(x, draft, \alpha)}^{R(x)} \sqrt{R(x)^2 - t^2}\, dt$$

Displaced volume

$$V_v(draft, \alpha) := \int_{0m}^{L_v} A_{cyl}(x, draft, \alpha)\, dx$$

Displaced volume should be equal to the earlier defined by buoyancy

$$V_v(draft, \alpha) = W_v$$

Area of submerged part defined in z- direction

$$A_z(z, draft, \alpha) := 2\left(\frac{z}{\tan(\alpha)} + draft\right)\sqrt{(4.295m)^2 - z^2}$$

Center of buoyancy in z- and x- direction

$$z_B(draft, \alpha) := \frac{\int_{-4.295m}^{4.295m} z \cdot A_z(z, draft, \alpha)\, dz}{V_v(draft, \alpha)}$$

$$x_B(draft, \alpha) := \frac{\int_{0m}^{L_v} x \cdot A_{cyl}(x, draft, \alpha)\, dx}{V_v(draft, \alpha)}$$

Equilibrium when center of gravity and buoyancy is in the same longitudinal position

$$\begin{pmatrix} x_G - x_B(draft, \alpha) \\ -zy_G - z_B(draft, \alpha) \end{pmatrix} \cdot \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix} = 0$$

Find the draft and trim angle that fulfills the requirements

$$\begin{pmatrix} draft \\ \alpha \end{pmatrix} := \mathrm{Find}(draft, \alpha)$$

$$draft = 68.439\,m$$

$\alpha = -30.86\,deg$

$draft_{real} := draft \cdot \cos(\alpha) = 58.75\,m$

Center of buoyancy in z- and x- direction for calculated draft and trim angle

$z_B(draft, \alpha) = -0.07\,m$

$x_B(draft, \alpha) = 59.872\,m$

Find where the column hits the water

$x := 50\,m$

$d(x) := \tan(\alpha) \cdot x + draft$

Given

$d(x) = 4.295\,m$

$x_1 := Find(x)$

$x_1 = 107.348\,m$

Find out if and where the top leaves the water

$d(L_V) = -32.843\,m$

Given

$d(x) = -4.295\,m$

$x_3 := Find(x)$

$x_3 = 121.724\,m$

Length of waterline

$a_3 := (x_3 - x_1)\cos(\alpha) = 12.341\,m$

$$a_4 := \frac{a_3}{2} = 6.17\,m$$

Distance the center of waterline changes if the top do not leave the water

$$L_e := \begin{cases} \dfrac{(L_V - x_1)}{\cos(\alpha)} - a_4 & \text{if } d(L_V) > -4.295\,m \\ a_4 & \text{otherwise} \end{cases}$$

$$LCF := \frac{2\left[\int_{-a_4}^{L_e} t\left[\frac{4.295\,m}{a_4}\sqrt{(a_4)^2 - t^2}\right] dt\right]}{\left[2 \cdot \int_{-a_4}^{L_e} \frac{4.295\,m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right] dt\right]}$$

$LCF = 0\,m$

The center of floatation of the buoy

$$CF := \frac{x_1 \cdot \cos(\alpha) + a_4 + LCF}{\cos(\alpha)} = 114.536\,m$$

Waterline area of the buoy

$$A_W := 2 \cdot \left[\int_{-a_4}^{L_e} \frac{4.295\,m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right] dt\right] = 83.257\,m^2$$

Calculation of longitudinal moment of intertia of the waterplane

$$I_L := 2\left[\int_{-a_4}^{L_e} t^2\left[\frac{4.295m}{a_4}\sqrt{(a_4)^2 - t^2}\right]dt\right] - 2\cdot\int_{-a_4}^{L_e} \frac{4.295m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right]dt\cdot LCF^2$$

Calculation of the longitudinal metacentric radius

$$BM_L := \frac{I_L}{W_v} = 0.124m$$

Calculation of the metacentric height

$$GM_L := z_B(draft, \alpha) + BM_L - z_G = 0.203m$$

Calculation of transverse moment of intertia of the waterplane

$$I_T := \frac{2}{3}\cdot\int_{-a_4}^{L_e} \left[\frac{4.295m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right]\right]^3 dt$$

Calculation of the transvese metacentric radius

$$BM_T := \frac{I_T}{W_v} = 0.06m$$

Calculation of the tranverse metacentric height

$$GM_T := z_B(draft, \alpha) + BM_T - z_G = 0.139m$$

Case HS4

Data

$F_{sb} := 2540tonne$

$d_{sb} := 13.71m$

$F_{col} := 3110.19tonne$

$d_{col} := 82.18m$

$F_{wb} := 2251.94tonne$

$d_{wb} := 124.739m$

$F_{top} := -220tonne$

$d_{top} := 8m$

$F_b := F_{sb} + F_{col} + F_{wb} + F_{top}$

$\rho := 1025kg\cdot m^{-3}$

Set the diameter over the column

$$D(x) := \begin{cases} (0m) & \text{if } x \ge 156.2m \\ (2.95m) & \text{if } 0m \le x < 6m \\ (8.75m) & \text{if } 6m \le x < 11m \\ (8.62m) & \text{if } 11m \le x < 23.5m \\ (8.63m) & \text{if } 23.5m \le x < 26m \\ (8.62m) & \text{if } 26m \le x < 33.5m \\ (8.63m) & \text{if } 33.5m \le x < 43.5m \\ (8.62m) & \text{if } 43.5m \le x < 55.25m \\ (8.63m) & \text{if } 55.25m \le x < 67m \\ (8.62m) & \text{if } 67m \le x < 113.85m \\ (8.58m) & \text{if } 113.85m \le x < 119.1m \\ (8.6m) & \text{if } 119.1m \le x < 124.3m \\ (8.58m) & \text{if } 124.3m \le x < 129.35m \\ (8.57m) & \text{if } 129.35m \le x < 134.2m \\ (8.83m) & \text{if } 134.2m \le x < 138.85m \\ (8.72m) & \text{if } 138.85m \le x < 147.75m \\ (2.8m) & \text{if } 147.75m \le x < 156.2m \end{cases}$$

Set the radius due to the diameter

$$R(x) := \frac{D(x)}{2}$$

Guess values for α and draft

$\alpha := -10deg$

$L_v := 156.5m$

$draft := 5m$

Define the displaced volume, center of gravity in x- and z- direction

$$W_v := \frac{F_b}{\rho} = 7494.761m^3$$

$$x_G := \frac{\left(F_{sb} \cdot d_{sb} + F_{wb} \cdot d_{wb} + F_{col} \cdot d_{col} + F_{top} \cdot d_{top}\right)}{\left(F_{sb} + F_{wb} + F_{col} + F_{top}\right)} = 74.141m$$

$z_G := -0.04m$

$y_G := 0.02m$

$$zy_G := \sqrt{\left(0 - y_G\right)^2 + \left(0 - z_G\right)^2}$$

Set the tolerance of convertion and the iteration step

$TOL := 0.0$

$CTOL := 0.0$

Put up equations that define the draft and the submerged area

Given

$d(x, draft, \alpha) := \tan(\alpha) \cdot x + draft$

$$a(x, draft, \alpha) := \begin{cases} R(x) & \text{if } d(x, draft, \alpha) \ge R(x) \\ d(x, draft, \alpha) & \text{if } \left|d(x, draft, \alpha)\right| \le R(x) \\ (-R(x)) & \text{if } d(x, draft, \alpha) < -R(x) \end{cases}$$

Area of submerged part

$$A_{cyl}(x, draft, \alpha) := 2\cdot\int_{-a(x,draft,\alpha)}^{R(x)} \sqrt{R(x)^2 - t^2}\,dt$$

Displaced volume should be equal to the earlier defined buoyancy

$$V_v(draft, \alpha) := \int_{0m}^{L_v} A_{cyl}(x, draft, \alpha)\,dx$$

$$V_v(draft, \alpha) = W_v$$

$$A_z(z, draft, \alpha) := 2\left(\frac{z}{\tan(\alpha)} + draft\right)\sqrt{(4.295m)^2 - z^2}$$

Center of buoyancy in z- and x- direction

$$z_B(draft, \alpha) := \frac{\int_{-4.295m}^{4.295m} z\cdot A_z(z, draft, \alpha)\,dz}{V_v(draft, \alpha)}$$

$$x_B(draft, \alpha) := \frac{\int_{0m}^{L_v} x\cdot A_{cyl}(x, draft, \alpha)\,dx}{V_v(draft, \alpha)}$$

Equilibrium when center of garvity and buoyancy is in the same longitudinal position

$$\begin{pmatrix} x_G - x_B(draft, \alpha) \\ -zy_G - z_B(draft, \alpha) \end{pmatrix}\cdot\begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix} = 0$$

Find the draft and trim angle that fulfills the requirements

$$\begin{pmatrix} draft \\ \alpha \end{pmatrix} := Find(draft, \alpha)$$

$draft = 4.116m$

$\alpha = -0.862deg$

$draft_{real} := draft\cdot\cos(\alpha) = 4.116m$

Center of buoyancy in z- and x- direction for calculated draft and trim angle

$x_B(draft, \alpha) = 74.106m$

$z_B(draft, \alpha) = -2.369m$

Find where the column hits the water

$x := 50m$

$d(x) := \tan(\alpha)\cdot x + draft$

Given

$V_v(draft, \alpha) = 7494.763m^3$

$d(x) = 4.295m$

$x_1 := Find(x)$

$x_1 = -11.87m$

Find out if and where the top leaves the water

$d(L_v) = 1.761m$

Given

$d(x) = -4.295m$

$x_3 := Find(x)$

$x_3 = 558.866m$

$a_3 := (x_3 - x_1)\cos(\alpha) = 570.671\text{m}$

$a_4 := \dfrac{a_3}{2} = 285.336\text{m}$

Distance the center of waterline changes if the top do not leave the water

$$L_e := \begin{cases} \dfrac{(L_v - x_1)}{\cos(\alpha)} - a_4 & \text{if } d(L_v) > -4.295\text{m} \\ a_4 & \text{otherwise} \end{cases}$$

$$LCF := \frac{\left[2\int_{-a_4}^{L_e} t\left[\frac{4.295\text{m}}{a_4}\sqrt{(a_4)^2 - t^2}\right]dt\right]}{\left[2\cdot\int_{-a_4}^{L_e} \frac{4.295\text{m}}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right]dt\right]}$$

$LCF = -186.362\text{m}$

The center of flotation of the buoy

$CF := \dfrac{(x_1\cdot\cos(\alpha) + a_4 + LCF)}{\cos(\alpha)} = 87.115\text{m}$

Waterline area of the buoy

$$A_W := 2\cdot\left[\int_{-a_4}^{L_e} \frac{4.295\text{m}}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right]dt\right] = 949.352\text{m}^2$$

Calculation of longitudinal moment of intertia of the waterplane

$$I_L := 2\left[\int_{-a_4}^{L_e} t^2\left[\frac{4.295\text{m}}{a_4}\sqrt{(a_4)^2 - t^2}\right]dt\right] - 2\cdot\int_{-a_4}^{L_e} \frac{4.295\text{m}}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right]dt\cdot LCF^2$$

Calculation of the longitudinal metacentric radius

$BM_L := \dfrac{I_L}{W_v} = 249.412\text{m}$

Calculation of the metacentric height

$GM_L := z_B(\text{draft}, \alpha) + BM_L - z_G = 247.082\text{m}$

Calculation of transverse moment of intertia of the waterplane

$$I_T := \frac{2}{3}\cdot\int_{-a_4}^{L_e} \left[\frac{4.295\text{m}}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right]\right]^3 dt$$

Calculation of the transvese metacentric radius

$BM_T := \dfrac{I_T}{W_v} = 0.428\text{m}$

Calculation of the tranverse metacentric height

$GM_T := z_B(\text{draft}, \alpha) + BM_T - z_G = -1.902\text{m}$

Case HS5

Data

$F_{sb} := 2540\,tonne$

$d_{sb} := 13.71\,m$

$F_{col} := 3110.19\,tonne$

$d_{col} := 82.18\,m$

$F_{wb} := 490.62\,tonne$

$d_{wb} := 28.468\,m$

$F_{top} := 519.02\,tonne$

$d_{top} := 174.86\,m$

$F_b := F_{sb} + F_{col} + F_{wb} + F_{top}$

$L_v := 156.5\,m$

$\rho := 1025\,kg \cdot m^{-3}$

Set the diameter over the column

$$D(x) := \begin{array}{|l} (0m) \ \text{if } x \geq 156.2m \\ (2.95m) \ \text{if } 0m \leq x < 6m \\ (8.75m) \ \text{if } 6m \leq x < 11m \\ (8.62m) \ \text{if } 11m \leq x < 23.5m \\ (8.63m) \ \text{if } 23.5m \leq x < 26m \\ (8.62m) \ \text{if } 26m \leq x < 33.5m \\ (8.63m) \ \text{if } 33.5m \leq x < 43.5m \\ (8.62m) \ \text{if } 43.5m \leq x < 55.25m \\ (8.63m) \ \text{if } 55.25m \leq x < 67m \\ (8.62m) \ \text{if } 67m \leq x < 113.85m \\ (8.58m) \ \text{if } 113.85m \leq x < 119.1m \\ (8.6m) \ \text{if } 119.1m \leq x < 124.3m \\ (8.58m) \ \text{if } 124.3m \leq x < 129.35m \\ (8.57m) \ \text{if } 129.35m \leq x < 134.2m \\ (8.83m) \ \text{if } 134.2m \leq x < 138.85m \\ (8.72m) \ \text{if } 138.85m \leq x < 147.75m \\ (2.8m) \ \text{if } 147.75m \leq x < 156.2m \end{array}$$

Define the radius due to the diameter

$$R(x) := \frac{D(x)}{2}$$

Guess values for α and draft

$\alpha := -3.6\,deg$

$draft := 99\,m$

Define the volume that are submerged, the center of gravity in x- and z- direction

$$W_v := \frac{F_b}{\rho} = 6.497 \times 10^3 \cdot m^3$$

$$x_G := \frac{\left(F_{sb} \cdot d_{sb} + F_{wb} \cdot d_{wb} + F_{col} \cdot d_{col} + F_{top} \cdot d_{top}\right)}{\left(F_{sb} + F_{wb} + F_{col} + F_{top}\right)} = 59.332\,m$$

$z_G := 0.113\,m$

$CTOL := 0.0$

$TOL := 0.0$

Put up equations that define the draft and the submerged area

Given

$$d(x, draft, \alpha) := -\tan(90deg - \alpha)\cdot x - \sin(\alpha)\cdot draft$$

$$a(x, draft, \alpha) := \begin{cases} R(x) & \text{if } d(x, draft, \alpha) \geq R(x) \\ d(x, draft, \alpha) & \text{if } |d(x, draft, \alpha)| \leq R(x) \\ (-R(x)) & \text{if } d(x, draft, \alpha) < -R(x) \end{cases}$$

Submerged length

$$L_{cyl\_sub}(draft, \alpha) := \frac{draft}{\cos(\alpha)}$$

Volume of submerged area

$$V_v(draft, \alpha) := \int_{0m}^{L_{cyl\_sub}(draft, \alpha)} \frac{\pi\, D(x)^2}{4}\, dx$$

Volume of submerged area should be equal to the earlier defined by buoyancy

$$V_v(draft, \alpha) = W_v$$

Area of submerged part defined in z- direction

$$A_z(z, draft, \alpha) := 2\left(\frac{z + \sin(\alpha)\cdot draft}{-\tan(90deg - \alpha)}\right)\sqrt{(4.295m)^2 - z^2}$$

Center of buoyancy and gravity in z- direction

$$z_B(draft, \alpha) := \frac{\int_{-4.295m}^{4.295m} z\cdot A_z(z, draft, \alpha)\, dz}{V_v(draft, \alpha)}$$

$$x_B(draft, \alpha) := \frac{\int_{0m}^{L_{cyl\_sub}(draft, \alpha)} x\cdot\pi\,\frac{D(x)^2}{4}\, dx}{V_v(draft, \alpha)}$$

Equilibrium when center of gravity and buoyancy is in the same longitudinal position

$$\begin{pmatrix} x_G - x_B(draft, \alpha) \\ z_G - z_B(draft, \alpha) \end{pmatrix}\cdot\begin{pmatrix} \sin(\alpha) \\ -\cos(\alpha) \end{pmatrix} = 0$$

Find the draft and α that fulfills the requirements

$$\begin{pmatrix} draft \\ \alpha \end{pmatrix} := \mathrm{Find}(draft, \alpha)$$

$$draft = 116.161m$$

$$\alpha = -4.265deg$$

$$draft_{real} := draft\cdot\cos(\alpha) = 115.839m$$

Center of flotation of the buoy. In this model, always on the buoy axis:

$$CF := 0m$$

Center of buoyancy in z- and x- direction

$$z_B(draft, \alpha) = 3.067\times 10^{-3}\, m$$

$$x_B(draft, \alpha) = 60.806m$$

Length of water line

$$b := \frac{4.295m}{\cos(\alpha)}$$

Moment of inertia

$$I := \pi \cdot 4.295m \cdot \frac{b^3}{4}$$

Metacentric radius

$$BM := \frac{I}{W_v}$$

Metacentric height

$$GM := x_B(draft, \alpha) + BM - x_G = 1.516m$$

Appendix E

Case HS1

Data

$F_{sb} := 2540\,tonne$

$d_{sb} := 13.71m$

$F_{col} := 3110.19\,tonne$

$d_{col} := 82.18m$

$F_{wb} := 2251.94\,tonne$

$d_{wb} := 124.739m$

$F_b := F_{sb} + F_{col} + F_{wb} = 7902.13\,tonne$

$L_v := 169.5m$

$\rho := 1025\,kg \cdot m^{-3}$

Set the diameter over the column

$$D(x) := \begin{cases} 8.59m & \text{if } 0m \le x \le 169.5m \\ (0m) & \text{otherwise} \end{cases}$$

Define the radius due to the diameter

$$R(x) := \frac{D(x)}{2}$$

Guess values for α and draft

$\alpha := -10\,deg$

$draft := 10m$

Define the volume that are submerged, the center of gravity in x- and z- direction

$$W_v := \frac{F_b}{\rho} = 7709.395m^3$$

$$x_G := \frac{\left(F_{sb} \cdot d_{sb} + F_{wb} \cdot d_{wb} + F_{col} \cdot d_{col}\right)}{\left(F_{sb} + F_{wb} + F_{col}\right)} = 72.3m$$

$z_G := 0m$

$CTOL := 0.1$

$TOL := 0.1$

Put up equations that define the draft and the submerged area

Given

$d(x, draft, \alpha) := \tan(\alpha) \cdot x + draft$

$$a(x, draft, \alpha) := \begin{cases} R(x) & \text{if } d(x, draft, \alpha) \ge R(x) \\ d(x, draft, \alpha) & \text{if } \left|d(x, draft, \alpha)\right| \le R(x) \\ (-R(x)) & \text{if } d(x, draft, \alpha) < -R(x) \end{cases}$$

Area of submerged part

$$A_{cyl}(x, draft, \alpha) := 2 \cdot \int_{-a(x, draft, \alpha)}^{R(x)} \sqrt{R(x)^2 - t^2}\, dt$$

Volume of submerged area

$$V_v(draft, \alpha) := \int_{0m}^{L_v} A_{cyl}(x, draft, \alpha)\, dx$$

Volume of submerged area should be equal to the earlier defined by buoyancy

$V_v(\text{draft}, \alpha) = W_v$

Area of submerged part defined in z- direction

$$A_z(z, \text{draft}, \alpha) := 2\left(\frac{z}{\tan(\alpha)} + \text{draft}\right)\sqrt{(4.295m)^2 - z^2}$$

Center of buoyancy and gravity in z- direction

$$z_B(\text{draft}, \alpha) := \frac{\int_{-4.295m}^{4.295m} z \cdot A_z(z, \text{draft}, \alpha)\, dz}{V_v(\text{draft}, \alpha)}$$

$$x_B(\text{draft}, \alpha) := \frac{\int_{0m}^{L_v} x \cdot A_{cyl}(x, \text{draft}, \alpha)\, dx}{V_v(\text{draft}, \alpha)}$$

Equilibrium when center of gravity and buoyancy is in the same longitudinal position

$$\begin{pmatrix} x_G - x_B(\text{draft}, \alpha) \\ z_G - z_B(\text{draft}, \alpha) \end{pmatrix} \cdot \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix} = 0$$

Find the draft and α that fulfills the requirements

$$\begin{pmatrix} \text{draft} \\ \alpha \end{pmatrix} := \text{Find}(\text{draft}, \alpha)$$

$\text{draft} = 5.785m$

$\alpha = -2.272deg$

Find where the column hits the water

$x := 50m$

$x_B(\text{draft}, \alpha) = 72.265m$

$d(x) := \tan(\alpha) \cdot x + \text{draft}$

$z_B(\text{draft}, \alpha) = -0.874m$

Given

$d(x) = 4.295m$

$x_1 := \text{Find}(x)$

$x_1 = 37.546m$

Find out if and where the top leaves the water

$d(L_v) = -0.941m$

Given

$d(x) = -4.295m$

$x_3 := \text{Find}(x)$

$x_3 = 254.016m$

Length of waterline

$a_3 := (x_3 - x_1)\cos(\alpha) = 216.3m$

$a_4 := \frac{a_3}{2} = 108.15m$

Distance the center of waterline changes if the top dont leave the water

$$L_e := \left| \begin{array}{l} \frac{(L_v - x_1)}{\cos(\alpha)} - a_4 \;\; \text{if } d(L_v) > -4.295m \\ a_4 \;\; \text{otherwise} \end{array} \right.$$

$$LCF := \frac{\left[2\left[\int_{-a_4}^{L_e} t\left[\frac{4.295m}{a_4}\sqrt{(a_4)^2 - t^2}\right] dt\right]\right]}{\left[2\cdot\int_{-a_4}^{L_e} \frac{4.295m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right] dt\right]}$$

$L_e = 23.908m$

$LCF = -33.286m$

The center of floatation for the buoy

$$CF := \frac{x_1\cdot\cos(\alpha) + a_4 + LCF}{\cos(\alpha)} = 112.469m$$

Length of each compartment

$$L_c := \begin{pmatrix} 169.5m - 162.5m \\ 162.5m - 156.2m \\ 156.2m - 147.75m \\ 147.75m - 138.85m \\ 138.85m - 134.2m \\ 134.2m - 129.35m \\ 129.35m - 124.3m \\ 124.3m - 119.1m \\ 119.1m - 113.85m \\ 113.85m - 102.15m \\ 102.15m - 90.45m \\ 90.45m - 78.75m \\ 78.75m - 67.0m \\ 67.0m - 55.25m \\ 55.25m - 43.5m \\ 43.5m - 33.5m \\ 33.5m - 23.5m \\ 23.5m - 6m \\ 6m - 0m \end{pmatrix}$$

Steel weight/m of each compartment

$$F_{stm} := \begin{pmatrix} 29.36 \\ 0 \\ 19.97 \\ 37.03 \\ 1.77 \\ 0 \\ 48.02 \\ 2.76 \\ 0 \\ 24.94 \\ 0 \\ 19.09 \\ 26.41 \\ 23.53 \\ 21.79 \\ 0 \\ 26.73 \\ 14.33 \\ 44.24 \end{pmatrix} \frac{\text{tonne}}{\text{m}}$$

$$F_{st} := \overrightarrow{\left(F_{stm} \cdot L_c\right)}$$

$$\sum F_{st} = 3110.41\text{tonne}$$

Weight/m of solid ballast

$$F_{sbm} := \frac{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{sb} \\ 0 \end{pmatrix}}{L_c}$$

Weight/m of water ballast of each compartment

$$F_{wbm} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 48.95 \\ 48.94 \\ 48.82 \\ 46.89 \\ 49.08 \\ 50.76 \\ 50.81 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \frac{tonne}{m}$$

Buoyancy/compartment

$$F_{bm} := \begin{pmatrix} \int_{162.5m}^{169.5m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\ \int_{156.2m}^{162.5m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\ \int_{147.75m}^{156.2m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\ \int_{138.85m}^{147.75m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\ \int_{134.2m}^{138.85m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\ \int_{129.35m}^{134.2m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\ \int_{124.3m}^{129.35m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\ \int_{119.1m}^{124.3m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\ \int_{113.85m}^{119.1m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\ \int_{102.15m}^{113.85m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\ \int_{90.45m}^{102.15m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\ \int_{78.75m}^{90.45m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\ \int_{67m}^{78.75m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\ \int_{55.25m}^{67m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\ \int_{43.5m}^{55.25m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\ \int_{33.5m}^{43.5m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\ \int_{23.5m}^{33.5m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \end{pmatrix} =$$

	0
0	158.756
1	157.332
2	232.703
3	272.053
4	153.129
5	167.716
6	183.263
7	197.83
8	209.018
9	498.673
10	542.382
11	583.415
12	623.599
13	656.527
14	682.662
15	...

$\cdot$tonne

E6

$$\sum F_{bm} = 7902.008\,\text{tonne}$$

$$F_{dn} := \left(F_{sbm} + F_{wbm} + F_{stm}\right)\cdot\cos(\alpha) = \cdot\frac{\text{tonne}}{\text{m}}$$

	0
0	29.337
1	0
2	19.954
3	85.912
4	50.67
5	48.782
6	94.835
7	51.799
8	50.72
9	75.69
10	0
11	19.075
12	26.389
13	23.511
14	21.773
15	...

$$F_{up} := \frac{\overrightarrow{F_{bm}\cdot\cos(\alpha)}}{L_c} = \cdot\frac{\text{tonne}}{\text{m}}$$

	0
0	22.662
1	24.954
2	27.517
3	30.544
4	32.905
5	34.554
6	36.261
7	38.014
8	39.782
9	42.588
10	46.321
11	49.825
12	53.03
13	55.831
14	58.053
15	...

$F_{load} := F_{up} - F_{dn} =$

	0
0	-6.675
1	24.954
2	7.563
3	-55.369
4	-17.765
5	-14.228
6	-58.574
7	-13.785
8	-10.938
9	-33.102
10	46.321
11	30.75
12	26.641
13	32.319
14	36.28
15	...

$\cdot\dfrac{\text{tonne}}{\text{m}}$

$$
\text{vector} := \left|\begin{array}{l} U \leftarrow 0 \\ \text{for } \; i \in 0, 1 .. \left(\text{rows}\left(F_{load}\right) - 1\right) \\ \quad U_i \leftarrow \left(F_{load_i}\right) \cdot L_{c_i} \\ U \end{array}\right.
$$

vector =

	0
0	-46.728
1	157.209
2	63.906
3	-492.781
4	-82.607
5	-69.006
6	-295.8
7	-71.682
8	-57.427
9	-387.297
10	541.956
11	359.779
12	313.035
13	379.751
14	426.294
15	...

$\cdot$tonne

$$V_c := \left| \begin{array}{l} A \leftarrow 0 \\ A_1 \leftarrow vector_0 \\ \text{for } j \in 2,3..(rows(vector) - 1) \\ \quad A_j \leftarrow A_{j-1} + vector_{j-1} \\ A_{19} \leftarrow A_{18} + vector_{18} \\ A \end{array} \right.$$

$V_c =$

	0
0	0
1	-46.728
2	110.481
3	174.387
4	-318.395
5	-401.002
6	-470.008
7	-765.808
8	-837.49
9	-894.916
10	-1282.214
11	-740.258
12	-380.479
13	-67.444
14	312.307
15	...

$\cdot tonne$

$$M_c := \left| \begin{array}{l} C \leftarrow 0 \\ C_1 \leftarrow \dfrac{V_{c_1} \cdot L_{c_0}}{2} \\ \text{for } k \in 2,3..rows\left(V_c\right) - 1 \\ \quad C_k \leftarrow C_{k-1} + \dfrac{\left(V_{c_{k-1}} + V_{c_k}\right) \cdot L_{c_{k-1}}}{2} \\ C \end{array} \right.$$

$M_c =$ (·m·tonne)

	0
0	0
1	-163.547
2	37.276
3	1240.842
4	600.008
5	-1072.589
6	-3184.788
7	-6305.223
8	-10473.797
9	-15021.363
10	-27757.573
11	-39589.031
12	-46145.339
13	-48776.883
14	-47338.313
15	...

$\max(M_c) = 1240.842\,m \cdot tonne$

$\min(M_c) = -48776.883\,m \cdot tonne$

$t := 0.045m$

$I := \pi \cdot R(x)^3 \cdot t$

$\sigma_{top} := \frac{M_c \cdot R(x) \cdot g}{I} =$ (·MPa)

	0
0	0
1	-0.615
2	0.14
3	4.666
4	2.256
5	-4.033
6	-11.976
7	-23.71
8	-39.385
9	-56.486
10	-104.379
11	-148.87
12	-173.524
13	-183.42
14	-178.01
15	...

$$\sigma_{bot} := \frac{M_c \cdot (-R(x)) \cdot g}{I} =$$

	0
0	0
1	0.615
2	-0.14
3	-4.666
4	-2.256
5	4.033
6	11.976
7	23.71
8	39.385
9	56.486
10	104.379
11	148.87
12	173.524
13	183.42
14	178.01
15	...

$\cdot MPa$

$\min(\sigma_{top}) = -183.42 MPa$

Calculations according to DNV rules

$t_c := 0.04m$

$I_c := \pi \cdot R(x)^3 \cdot t_c$

$SF := 1.2$

$$\sigma_{topc} := \frac{M_c \cdot R(x) \cdot g \cdot SF}{I_c} =$$

	0
0	0
1	-0.83
2	0.189
3	6.299
4	3.046
5	-5.445
6	-16.168
7	-32.009
8	-53.17
9	-76.256
10	-140.912
11	-200.974
12	-234.258
13	-247.617
14	-240.314
15	...

$\cdot MPa$

$\min(\sigma_{topc}) = -247.617 MPa$

$\gamma_m := 1.15$

$R_k := 335 MPa$

$$R_d := \frac{R_k}{\gamma_m} = 291.304\,\mathrm{MPa}$$

Case HS2

Data

$F_{sb} := 2540\,\mathrm{tonne}$

$d_{sb} := 13.71\,\mathrm{m}$

$F_{col} := 3110.19\,\mathrm{tonne}$

$d_{col} := 82.18\,\mathrm{m}$

$F_{wb} := 1657.4\,\mathrm{tonne}$

$d_{wb} := 130.743\,\mathrm{m}$

$F_b := F_{sb} + F_{col} + F_{wb}$

$L_v := 156.5\,\mathrm{m}$

$\rho := 1025\,\mathrm{kg\cdot m^{-3}}$

Set the diameter over the column

$$D(x) := \left|\begin{array}{l}
(0\mathrm{m}) \ \text{if } x \ge 156.2\mathrm{m} \\
(2.95\mathrm{m}) \ \text{if } 0\mathrm{m} \le x < 6\mathrm{m} \\
(8.75\mathrm{m}) \ \text{if } 6\mathrm{m} \le x < 11\mathrm{m} \\
(8.62\mathrm{m}) \ \text{if } 11\mathrm{m} \le x < 23.5\mathrm{m} \\
(8.63\mathrm{m}) \ \text{if } 23.5\mathrm{m} \le x < 26\mathrm{m} \\
(8.62\mathrm{m}) \ \text{if } 26\mathrm{m} \le x < 33.5\mathrm{m} \\
(8.63\mathrm{m}) \ \text{if } 33.5\mathrm{m} \le x < 43.5\mathrm{m} \\
(8.62\mathrm{m}) \ \text{if } 43.5\mathrm{m} \le x < 55.25\mathrm{m} \\
(8.63\mathrm{m}) \ \text{if } 55.25\mathrm{m} \le x < 67\mathrm{m} \\
(8.62\mathrm{m}) \ \text{if } 67\mathrm{m} \le x < 113.85\mathrm{m} \\
(8.58\mathrm{m}) \ \text{if } 113.85\mathrm{m} \le x < 119.1\mathrm{m} \\
(8.6\mathrm{m}) \ \text{if } 119.1\mathrm{m} \le x < 124.3\mathrm{m} \\
(8.58\mathrm{m}) \ \text{if } 124.3\mathrm{m} \le x < 129.35\mathrm{m} \\
(8.57\mathrm{m}) \ \text{if } 129.35\mathrm{m} \le x < 134.2\mathrm{m} \\
(8.83\mathrm{m}) \ \text{if } 134.2\mathrm{m} \le x < 138.85\mathrm{m} \\
(8.72\mathrm{m}) \ \text{if } 138.85\mathrm{m} \le x < 147.75\mathrm{m} \\
(2.8\mathrm{m}) \ \text{if } 147.75\mathrm{m} \le x < 156.2\mathrm{m}
\end{array}\right.$$

Define the radius due to the diameter

$$R(x) := \frac{D(x)}{2}$$

Guess values for α and draft

$\alpha := -4\,\mathrm{deg}$

$draft := 10\,\mathrm{m}$

Define the volume that are submerged, the center of gravity in x- and z- direction

$$W_v := \frac{F_b}{\rho} = 7129.356\,\mathrm{m^3}$$

$$x_G := \frac{\left(F_{sb}\cdot d_{sb} + F_{wb}\cdot d_{wb} + F_{col}\cdot d_{col}\right)}{\left(F_{sb} + F_{wb} + F_{col}\right)} = 69.395\,\mathrm{m}$$

$z_G := -0.043\,\mathrm{m}$

$CTOL := 0.1$
$TOL := 0.0$
Put up equations that define the draft and the submerged area
Given
$$d(x, draft, \alpha) := \tan(\alpha)\cdot x + draft$$
$$a(x, draft, \alpha) := \begin{cases} R(x) & \text{if } d(x, draft, \alpha) \geq R(x) \\ d(x, draft, \alpha) & \text{if } \left|d(x, draft, \alpha)\right| \leq R(x) \\ (-R(x)) & \text{if } d(x, draft, \alpha) < -R(x) \end{cases}$$
Area of submerged part
$$A_{cyl}(x, draft, \alpha) := 2\cdot\int_{-a(x,draft,\alpha)}^{R(x)} \sqrt{R(x)^2 - t^2}\, dt$$
Volume of submerged area
$$V_v(draft, \alpha) := \int_{0m}^{L_v} A_{cyl}(x, draft, \alpha)\, dx$$
Volume of submerged area should be equal to the earlier defined by buoyancy
$$V_v(draft, \alpha) = W_v$$
Area of submerged part defined in z- direction
$$A_z(z, draft, \alpha) := 2\left(\frac{z}{\tan(\alpha)} + draft\right)\sqrt{(4.295m)^2 - z^2}$$
Center of buoyancy and gravity in z- direction
$$z_B(draft, \alpha) := \frac{\displaystyle\int_{-4.295m}^{4.295m} z\cdot A_z(z, draft, \alpha)\, dz}{V_v(draft, \alpha)}$$
$$x_B(draft, \alpha) := \frac{\displaystyle\int_{0m}^{L_v} x\cdot A_{cyl}(x, draft, \alpha)\, dx}{V_v(draft, \alpha)}$$
Equilibrium when center of gravity and buoyancy is in the same longitudinal position
$$\begin{pmatrix} x_G - x_B(draft, \alpha) \\ z_G - z_B(draft, \alpha) \end{pmatrix}\cdot\begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix} = 0$$
Find the draft and α that fulfills the requirements
$$\begin{pmatrix} draft \\ \alpha \end{pmatrix} := Find(draft, \alpha)$$
$draft = 6.739m$
$\alpha = -2.648deg$
$z_B(draft, \alpha) = -0.811m$
Find where the column hits the water
$x_B(draft, \alpha) = 69.36m$

$x := 10m$
$z_G - z_B(draft, \alpha)\cdot\sin(\alpha) = -0.08m$

$d(x) := \tan(\alpha)\cdot x + draft$
Given
$d(x) = 4.295m$

$x_1 := \mathrm{Find}(x)$

$x_1 = 52.842\mathrm{m}$

Find out if and where the top leaves the water

$d(L_v) = -0.499\mathrm{m}$

Given

$d(x) = -4.295\mathrm{m}$

$x_3 := \mathrm{Find}(x)$

$x_3 = 238.566\mathrm{m}$

Length of waterline

$a_3 := (x_3 - x_1)\cos(\alpha) = 185.526\mathrm{m}$

$a_4 := \frac{a_3}{2} = 92.763\mathrm{m}$

Distance the center of waterline changes if the top dont leave the water

$$L_e := \begin{cases} \frac{(L_v - x_1)}{\cos(\alpha)} - a_4 & \text{if } d(L_v) > -4.295\mathrm{m} \\ a_4 & \text{otherwise} \end{cases}$$

$$LCF := \frac{\left[2\int_{-a_4}^{L_e} t\left[\frac{4.295\mathrm{m}}{a_4}\sqrt{(a_4)^2 - t^2}\right]dt\right]}{\left[2\cdot\int_{-a_4}^{L_e} \frac{4.295\mathrm{m}}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right]dt\right]}$$

$L_e = 11.006\mathrm{m}$

$LCF = -33.494\mathrm{m}$

The center of floatation for the buoy

$CF := \frac{x_1\cdot\cos(\alpha) + a_4 + LCF}{\cos(\alpha)} = 112.175\mathrm{m}$

Length of each compartment

$$L_c := \begin{pmatrix} 169.5m - 162.5m \\ 162.5m - 156.2m \\ 156.2m - 147.75m \\ 147.75m - 138.85m \\ 138.85m - 134.2m \\ 134.2m - 129.35m \\ 129.35m - 124.3m \\ 124.3m - 119.1m \\ 119.1m - 113.85m \\ 113.85m - 102.15m \\ 102.15m - 90.45m \\ 90.45m - 78.75m \\ 78.75m - 67.0m \\ 67.0m - 55.25m \\ 55.25m - 43.5m \\ 43.5m - 33.5m \\ 33.5m - 23.5m \\ 23.5m - 6m \\ 6m - 0m \end{pmatrix}$$

Steel weight/m of each compartment

$$F_{stm} := \begin{pmatrix} 29.36 \\ 0 \\ 19.97 \\ 37.03 \\ 1.77 \\ 0 \\ 48.02 \\ 2.76 \\ 0 \\ 24.94 \\ 0 \\ 19.09 \\ 26.41 \\ 23.53 \\ 21.79 \\ 0 \\ 26.73 \\ 14.33 \\ 44.24 \end{pmatrix} \frac{tonne}{m}$$

$$F_{st} := \overrightarrow{\left(F_{stm} \cdot L_c\right)}$$

$$\sum F_{st} = 3110.41 tonne$$

Weight/m of solid ballast

$$F_{sbm} := \frac{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{sb} \\ 0 \end{pmatrix}}{L_c}$$

Weight/m of water ballast of each compartment

$$F_{wbm} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 48.95 \\ 48.94 \\ 48.82 \\ 46.89 \\ 49.08 \\ 50.76 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \frac{tonne}{m}$$

Buoyancy/compartment

$$
F_{bm} := \begin{pmatrix}
\int_{162.5m}^{169.5m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{156.2m}^{162.5m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{147.75m}^{156.2m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{138.85m}^{147.75m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{134.2m}^{138.85m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{129.35m}^{134.2m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{124.3m}^{129.35m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{119.1m}^{124.3m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{113.85m}^{119.1m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{102.15m}^{113.85m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{90.45m}^{102.15m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{78.75m}^{90.45m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{67m}^{78.75m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{55.25m}^{67m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{43.5m}^{55.25m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{33.5m}^{43.5m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{23.5m}^{33.5m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\vdots
\end{pmatrix} =
$$

	0
0	0
1	0
2	19.706
3	281.241
4	163.773
5	170.72
6	188.147
7	205.117
8	216.928
9	525.03
10	574.378
11	619.282
12	660.851
13	691.937
14	702.663
15	...

$\cdot tonne$

E17

$\sum F_{bm} = 7315.235\,\text{tonne}$

$F_{dn} := (F_{sbm} + F_{wbm} + F_{stm}) \cdot \cos(\alpha) =$ (table) $\cdot \frac{\text{tonne}}{\text{m}}$

	0
0	29.329
1	0
2	19.949
3	85.888
4	50.656
5	48.768
6	94.809
7	51.785
8	50.706
9	24.913
10	0
11	19.07
12	26.382
13	23.505
14	21.767
15	...

$F_{up} := \overrightarrow{\frac{F_{bm} \cdot \cos(\alpha)}{L_c}} =$ (table) $\cdot \frac{\text{tonne}}{\text{m}}$

	0
0	0
1	0
2	2.33
3	31.566
4	35.182
5	35.162
6	37.217
7	39.404
8	41.275
9	44.826
10	49.04
11	52.874
12	56.183
13	58.825
14	59.737
15	...

$$F_{load} := F_{up} - F_{dn} = \begin{array}{|c|r|} \hline & 0 \\ \hline 0 & -29.329 \\ 1 & 0 \\ 2 & -17.619 \\ 3 & -54.322 \\ 4 & -15.473 \\ 5 & -13.605 \\ 6 & -57.592 \\ 7 & -12.381 \\ 8 & -9.43 \\ 9 & 19.913 \\ 10 & 49.04 \\ 11 & 33.804 \\ 12 & 29.801 \\ 13 & 35.321 \\ 14 & 37.971 \\ 15 & \ldots \\ \hline \end{array} \cdot \frac{tonne}{m}$$

$$\text{vector} := \left| \begin{array}{l} U \leftarrow 0 \\ \text{for } i \in 0, 1 .. \left(\text{rows}\left(F_{load}\right) - 1\right) \\ \quad U_i \leftarrow \left(F_{load_i}\right) \cdot L_{c_i} \\ U \end{array} \right.$$

$$\text{vector} = \begin{array}{|c|r|} \hline & 0 \\ \hline 0 & -205.301 \\ 1 & 0 \\ 2 & -148.881 \\ 3 & -483.464 \\ 4 & -71.952 \\ 5 & -65.987 \\ 6 & -290.838 \\ 7 & -64.382 \\ 8 & -49.509 \\ 9 & 232.983 \\ 10 & 573.764 \\ 11 & 395.506 \\ 12 & 350.159 \\ 13 & 415.016 \\ 14 & 446.154 \\ 15 & \ldots \\ \hline \end{array} \cdot tonne$$

$$V_c := \begin{array}{|l} A \leftarrow 0 \\ A_1 \leftarrow vector_0 \\ \text{for } j \in 2, 3 .. (rows(vector) - 1) \\ \quad A_j \leftarrow A_{j-1} + vector_{j-1} \\ A_{19} \leftarrow A_{18} + vector_{18} \\ A \end{array}$$

$V_c =$ ·tonne

	0
0	0
1	-205.301
2	-205.301
3	-354.182
4	-837.646
5	-909.597
6	-975.584
7	-1266.422
8	-1330.803
9	-1380.313
10	-1147.33
11	-573.565
12	-178.059
13	172.1
14	587.116
15	...

$\max(V_c) = 1963.043 \text{tonne}$

$rows(V_c) = 20$

$$M_c := \begin{array}{|l} C \leftarrow 0 \\ C_1 \leftarrow \dfrac{V_{c_1} \cdot L_{c_0}}{2} \\ \text{for } k \in 2, 3 .. rows(V_c) - 1 \\ \quad C_k \leftarrow C_{k-1} + \dfrac{\left(V_{c_{k-1}} + V_{c_k}\right) \cdot L_{c_{k-1}}}{2} \\ C \end{array}$$

$L_{c_0} = 7\,m$

$M_c =$

	0
0	0
1	-718.552
2	-2011.945
3	-4375.758
4	-9679.391
5	-13741.731
6	-18313.296
7	-23974.36
8	-30727.146
9	-37843.826
10	-52630.534
11	-62697.771
12	-67094.774
13	-67129.784
14	-62669.388
15	...

$\cdot m \cdot tonne$

$\max(M_c) = 0 \cdot m \cdot tonne$

$\min(M_c) = -67129.784 m \cdot tonne$

$t := 0.045m$

$r := 4.295m$

$I := \pi \cdot r^3 \cdot t$

$\sigma_{top} := \frac{M_c \cdot r \cdot g}{I} =$

	0
0	0
1	-2.702
2	-7.566
3	-16.455
4	-36.398
5	-51.674
6	-68.865
7	-90.153
8	-115.546
9	-142.307
10	-197.911
11	-235.768
12	-252.302
13	-252.434
14	-235.661
15	...

$\cdot MPa$

$$\sigma_{bot} := \frac{M_c \cdot (-r) \cdot g}{I} = \begin{array}{|c|r|}\hline & 0 \\ \hline 0 & 0 \\ 1 & 2.702 \\ 2 & 7.566 \\ 3 & 16.455 \\ 4 & 36.398 \\ 5 & 51.674 \\ 6 & 68.865 \\ 7 & 90.153 \\ 8 & 115.546 \\ 9 & 142.307 \\ 10 & 197.911 \\ 11 & 235.768 \\ 12 & 252.302 \\ 13 & 252.434 \\ 14 & 235.661 \\ 15 & ... \\ \hline \end{array} \cdot MPa$$

$\max(\sigma_{top}) = 0 \cdot MPa$

$\min(\sigma_{top}) = -252.434 MPa$

Calculations according to DNV rules

$t_c := 0.04m$

$I_c := \pi \cdot r^3 \cdot t_c$

$SF := 1.2$

$$\sigma_{topc} := \frac{M_c \cdot r \cdot g \cdot SF}{I_c} = \begin{array}{|c|r|}\hline & 0 \\ \hline 0 & 0 \\ 1 & -3.648 \\ 2 & -10.214 \\ 3 & -22.214 \\ 4 & -49.138 \\ 5 & -69.76 \\ 6 & -92.968 \\ 7 & -121.706 \\ 8 & -155.987 \\ 9 & -192.115 \\ 10 & -267.18 \\ 11 & -318.286 \\ 12 & -340.608 \\ 13 & -340.785 \\ 14 & -318.142 \\ 15 & ... \\ \hline \end{array} \cdot MPa$$

$\min(\sigma_{topc}) = -340.785 MPa$

$\gamma_m := 1.15$

$R_k := 335 MPa$

$$R_d := \frac{R_k}{\gamma_m} = 291.304\,MPa$$

Case HS3

Data

$F_{sb} := 25400\,tonne$

$d_{sb} := 13.71\,m$

$F_{col} := 3110.19\,tonne$

$d_{col} := 82.18\,m$

$F_{wb} := 376.22\,tonne$

$d_{wb} := 27.712\,m$

$F_{top} := 519.02\,tonne$

$d_{top} := 174.86\,m$

$F_b := F_{sb} + F_{col} + F_{wb} + F_{top}$

$\rho := 1025\,kg \cdot m^{-3}$

$$D(x) := \begin{cases} (0m) & \text{if } x \ge 156.2m \\ (2.95m) & \text{if } 0m \le x < 6m \\ (8.75m) & \text{if } 6m \le x < 11m \\ (8.62m) & \text{if } 11m \le x < 23.5m \\ (8.63m) & \text{if } 23.5m \le x < 26m \\ (8.62m) & \text{if } 26m \le x < 33.5m \\ (8.63m) & \text{if } 33.5m \le x < 43.5m \\ (8.62m) & \text{if } 43.5m \le x < 55.25m \\ (8.63m) & \text{if } 55.25m \le x < 67m \\ (8.62m) & \text{if } 67m \le x < 113.85m \\ (8.58m) & \text{if } 113.85m \le x < 119.1m \\ (8.6m) & \text{if } 119.1m \le x < 124.3m \\ (8.58m) & \text{if } 124.3m \le x < 129.35m \\ (8.57m) & \text{if } 129.35m \le x < 134.2m \\ (8.83m) & \text{if } 134.2m \le x < 138.85m \\ (8.72m) & \text{if } 138.85m \le x < 147.75m \\ (2.8m) & \text{if } 147.75m \le x < 156.2m \end{cases}$$

$$R(x) := \frac{D(x)}{2}$$

$\alpha := -40\,deg$

$L_v := 169.5\,m$

$draft := 70\,m$

$$W_v := \frac{F_b}{\rho} = 6385.785\,m^3$$

$$x_G := \frac{\left(F_{sb} \cdot d_{sb} + F_{wb} \cdot d_{wb} + F_{col} \cdot d_{col} + F_{top} \cdot d_{top}\right)}{\left(F_{sb} + F_{wb} + F_{col} + F_{top}\right)} = 59.828\,m$$

$z_G := -0.149\,m$

$y_G := 0.072\,m$

$zy_G := \sqrt{(0 - y_G)^2 + (0 - z_G)^2}$

$TOL := 0.0$

$CTOL := 0.1$

Given

$d(x, draft, \alpha) := \tan(\alpha)\cdot x + draft$

$$a(x, draft, \alpha) := \left| \begin{array}{l} R(x) \text{ if } d(x, draft, \alpha) \geq R(x) \\ d(x, draft, \alpha) \text{ if } \left|d(x, draft, \alpha)\right| \leq R(x) \\ (-R(x)) \text{ if } d(x, draft, \alpha) < -R(x) \end{array} \right.$$

$$A_{cyl}(x, draft, \alpha) := 2\cdot\int_{-a(x, draft, \alpha)}^{R(x)} \sqrt{R(x)^2 - t^2}\, dt$$

$$V_v(draft, \alpha) := \int_{0m}^{L_v} A_{cyl}(x, draft, \alpha)\, dx$$

$V_v(draft, \alpha) = W_v$

$$A_z(z, draft, \alpha) := 2\left(\frac{z}{\tan(\alpha)} + draft\right)\sqrt{(4.295m)^2 - z^2}$$

$$z_B(draft, \alpha) := \frac{\int_{-4.295m}^{4.295m} z\cdot A_z(z, draft, \alpha)\, dz}{V_v(draft, \alpha)}$$

$$x_B(draft, \alpha) := \frac{\int_{0m}^{L_v} x\cdot A_{cyl}(x, draft, \alpha)\, dx}{V_v(draft, \alpha)}$$

$$\begin{pmatrix} x_G - x_B(draft, \alpha) \\ -zy_G - z_B(draft, \alpha) \end{pmatrix}\cdot\begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix} = 0$$

$$\begin{pmatrix} draft \\ \alpha \end{pmatrix} := Find(draft, \alpha)$$

$draft = 68.439m$

$\alpha = -30.86deg$

$x_B(draft, \alpha) = 59.872m$

$x := 50m$

$d(x) := \tan(\alpha)\cdot x + draft$

$z_B(draft, \alpha) = -0.07m$

Given

$V_v(draft, \alpha) = 6385.785m^3$

$d(x) = 4.295m$

$x_1 := Find(x)$

$x_1 = 107.348m$

$d(L_v) = -32.843m$

Given

$d(x) = -4.295m$

$x_3 := Find(x)$

$x_3 = 121.724\text{m}$

$a_3 := (x_3 - x_1)\cos(\alpha) = 12.341\text{m}$

$a_4 := \frac{a_3}{2} = 6.17\text{m}$

$$L_e := \begin{cases} \frac{(L_v - x_1)}{\cos(\alpha)} - a_4 & \text{if } d(L_v) > -4.295\text{m} \\ a_4 & \text{otherwise} \end{cases}$$

$$LCF := \frac{2\left[\int_{-a_4}^{L_e} t\left[\frac{4.295\text{m}}{a_4}\sqrt{(a_4)^2 - t^2}\right]dt\right]}{\left[2\cdot\int_{-a_4}^{L_e} \frac{4.295\text{m}}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right]dt\right]}$$

$LCF = 0\text{m}$

$CF := \frac{(x_1 \cdot \cos(\alpha) + a_4 + LCF)}{\cos(\alpha)} = 114.536\text{m}$

Length of each compartment

$$L_c := \begin{pmatrix} 169.5\text{m} - 162.5\text{m} \\ 162.5\text{m} - 156.2\text{m} \\ 156.2\text{m} - 147.75\text{m} \\ 147.75\text{m} - 138.85\text{m} \\ 138.85\text{m} - 134.2\text{m} \\ 134.2\text{m} - 129.35\text{m} \\ 129.35\text{m} - 124.3\text{m} \\ 124.3\text{m} - 119.1\text{m} \\ 119.1\text{m} - 113.85\text{m} \\ 113.85\text{m} - 102.15\text{m} \\ 102.15\text{m} - 90.45\text{m} \\ 90.45\text{m} - 78.75\text{m} \\ 78.75\text{m} - 67.0\text{m} \\ 67.0\text{m} - 55.25\text{m} \\ 55.25\text{m} - 43.5\text{m} \\ 43.5\text{m} - 33.5\text{m} \\ 33.5\text{m} - 23.5\text{m} \\ 23.5\text{m} - 6\text{m} \\ 6\text{m} - 0\text{m} \end{pmatrix}$$

Steel weight/m of each compartment

$$F_{stm} := \begin{pmatrix} 29.36 \\ 0 \\ 19.97 \\ 37.03 \\ 1.77 \\ 0 \\ 48.02 \\ 2.76 \\ 0 \\ 24.94 \\ 0 \\ 19.09 \\ 26.41 \\ 23.53 \\ 21.79 \\ 0 \\ 26.73 \\ 14.33 \\ 44.24 \end{pmatrix} \frac{tonne}{m}$$

$$F_{st} := \overrightarrow{\left(F_{stm} \cdot L_c\right)}$$

$$\sum F_{st} = 3110.41 tonne$$

Weight/m of solid ballast

$$F_{sbm} := \frac{\begin{pmatrix} F_{top} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{sb} \\ 0 \end{pmatrix}}{L_c}$$

Weight/m of water ballast of each compartment

$$F_{wbm} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 37.622 \\ 0 \\ 0 \end{pmatrix} \frac{tonne}{m}$$

Buoyancy/compartment

$$
F_{bm} := \begin{pmatrix}
\int_{162.5m}^{169.5m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{156.2m}^{162.5m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{147.75m}^{156.2m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{138.85m}^{147.75m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{134.2m}^{138.85m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{129.35m}^{134.2m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{124.3m}^{129.35m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{119.1m}^{124.3m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{113.85m}^{119.1m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{102.15m}^{113.85m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{90.45m}^{102.15m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\int_{78.75m}^{90.45m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{67m}^{78.75m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{55.25m}^{67m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{43.5m}^{55.25m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{33.5m}^{43.5m} \rho \cdot A_{cy1}(x, draft, \alpha)\, dx \\
\int_{23.5m}^{33.5m} \rho\, A_{cy1}(x, draft, \alpha)\, dx \\
\vdots
\end{pmatrix} =
$$

	0
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	7.99
8	104.004
9	627.564
10	699.869
11	699.869
12	702.86
13	704.491
14	702.86
15	...

$\cdot tonne$

E28

$$\sum F_{bm} = 6544.969 tonne$$

$$F_{dn} := \left(F_{sbm} + F_{wbm} + F_{stm}\right)\cdot\cos(\alpha) = \begin{array}{|c|r|} \hline & 0 \\ \hline 0 & 88.852 \\ \hline 1 & 0 \\ \hline 2 & 17.143 \\ \hline 3 & 31.788 \\ \hline 4 & 1.519 \\ \hline 5 & 0 \\ \hline 6 & 41.222 \\ \hline 7 & 2.369 \\ \hline 8 & 0 \\ \hline 9 & 21.409 \\ \hline 10 & 0 \\ \hline 11 & 16.387 \\ \hline 12 & 22.671 \\ \hline 13 & 20.199 \\ \hline 14 & 18.705 \\ \hline 15 & ... \\ \hline \end{array} \cdot\frac{tonne}{m}$$

$$F_{up} := \overrightarrow{\frac{F_{bm}\cdot\cos(\alpha)}{L_c}} = \begin{array}{|c|r|} \hline & 0 \\ \hline 0 & 0 \\ \hline 1 & 0 \\ \hline 2 & 0 \\ \hline 3 & 0 \\ \hline 4 & 0 \\ \hline 5 & 0 \\ \hline 6 & 0 \\ \hline 7 & 1.319 \\ \hline 8 & 17.006 \\ \hline 9 & 46.044 \\ \hline 10 & 51.349 \\ \hline 11 & 51.349 \\ \hline 12 & 51.349 \\ \hline 13 & 51.468 \\ \hline 14 & 51.349 \\ \hline 15 & ... \\ \hline \end{array} \cdot\frac{tonne}{m}$$

$$F_{load} := F_{up} - F_{dn} = \begin{array}{|c|r|}\hline & 0 \\ \hline 0 & -88.852 \\ 1 & 0 \\ 2 & -17.143 \\ 3 & -31.788 \\ 4 & -1.519 \\ 5 & 0 \\ 6 & -41.222 \\ 7 & -1.05 \\ 8 & 17.006 \\ 9 & 24.635 \\ 10 & 51.349 \\ 11 & 34.962 \\ 12 & 28.678 \\ 13 & 31.27 \\ 14 & 32.644 \\ 15 & ... \\ \hline \end{array} \cdot \frac{tonne}{m}$$

$$vector := \left| \begin{array}{l} U \leftarrow 0 \\ \text{for } i \in 0, 1 .. \left(rows\left(F_{load}\right) - 1\right) \\ \quad U_i \leftarrow \left(F_{load_i}\right) \cdot L_{c_i} \\ U \end{array} \right.$$

$$vector = \begin{array}{|c|r|}\hline & 0 \\ \hline 0 & -621.964 \\ 1 & 0 \\ 2 & -144.856 \\ 3 & -282.909 \\ 4 & -7.065 \\ 5 & 0 \\ 6 & -208.169 \\ 7 & -5.461 \\ 8 & 89.28 \\ 9 & 288.231 \\ 10 & 600.786 \\ 11 & 409.054 \\ 12 & 336.969 \\ 13 & 367.418 \\ 14 & 383.568 \\ 15 & ... \\ \hline \end{array} \cdot tonne$$

$$V_c := \left| \begin{array}{l} A \leftarrow 0 \\ A_1 \leftarrow vector_0 \\ \text{for } j \in 2,3..(\text{rows}(vector) - 1) \\ \quad A_j \leftarrow A_{j-1} + vector_{j-1} \\ A_{19} \leftarrow A_{18} + vector_{18} \\ A \end{array} \right.$$

$V_c =$ (·tonne)

	0
0	0
1	-621.964
2	-621.964
3	-766.821
4	-1049.73
5	-1056.795
6	-1056.795
7	-1264.964
8	-1270.425
9	-1181.145
10	-892.915
11	-292.129
12	116.925
13	453.894
14	821.312
15	...

$$M_c := \left| \begin{array}{l} C \leftarrow 0 \\ C_1 \leftarrow \dfrac{V_{c_1} \cdot L_{c_0}}{2} \\ \text{for } k \in 2,3..\text{rows}(V_c) - 1 \\ \quad C_k \leftarrow C_{k-1} + \dfrac{\left(V_{c_{k-1}} + V_{c_k}\right) \cdot L_{c_{k-1}}}{2} \\ C \end{array} \right.$$

	0
0	0
1	-2176.875
2	-6095.249
3	-11962.866
4	-20046.515
5	-24944.185
6	-30069.64
7	-35932.082
8	-42524.094
9	-48959.466
10	-61092.718
11	-68025.222
12	-69050.165
13	-65696.606
14	-58204.773
15	...

$M_c =$ (table above) $\cdot m \cdot tonne$

$\max(M_c) = 2276.529\,m \cdot tonne$

$\min(M_c) = -69050.165\,m \cdot tonne$

$t := 0.045m$

$r := 4.295m$

$I := \pi \cdot r^3 \cdot t$

$\sigma_{top} := \frac{M_c \cdot r \cdot g}{I} =$ (table below) $\cdot MPa$

	0
0	0
1	-8.186
2	-22.92
3	-44.985
4	-75.383
5	-93.8
6	-113.073
7	-135.118
8	-159.907
9	-184.106
10	-229.732
11	-255.801
12	-259.655
13	-247.044
14	-218.872
15	...

$$\sigma_{bot} := \frac{M_c \cdot (-r) \cdot g}{I} =$$

	0
0	0
1	8.186
2	22.92
3	44.985
4	75.383
5	93.8
6	113.073
7	135.118
8	159.907
9	184.106
10	229.732
11	255.801
12	259.655
13	247.044
14	218.872
15	...

$\cdot MPa$

$\max(\sigma_{top}) = 8.561 \cdot MPa$

$\min(\sigma_{top}) = -259.655 MPa$

Calculations according to DNV rules

$t_c := 0.04m$

$I_c := \pi \cdot r^3 \cdot t_c$

$SF := 1.2$

$$\sigma_{topc} := \frac{M_c \cdot r \cdot g \cdot SF}{I_c} =$$

	0
0	0
1	-11.051
2	-30.943
3	-60.73
4	-101.766
5	-126.63
6	-152.649
7	-182.41
8	-215.874
9	-248.543
10	-310.138
11	-345.331
12	-350.534
13	-333.51
14	-295.477
15	...

$\cdot MPa$

$\min(\sigma_{topc}) = -350.534 MPa$

$\gamma_m := 1.15$

$R_k := 335 MPa$

$$R_d := \frac{R_k}{\gamma_m} = 291.304\,MPa$$

Case HS4

Data

$F_{sb} := 2540\,tonne$

$d_{sb} := 13.71\,m$

$F_{col} := 3110.19\,tonne$

$d_{col} := 82.18\,m$

$F_{wb} := 2251.94\,tonne$

$d_{wb} := 124.739\,m$

$F_{ext} := -220\,tonne$

$d_{ext} := 8\,m$

$F_b := F_{sb} + F_{col} + F_{wb} + F_{ext}$

$\rho := 1025\,kg \cdot m^{-3}$

$$D(x) := \begin{cases} (0m) & \text{if } x \ge 156.2m \\ (2.95m) & \text{if } 0m \le x < 6m \\ (8.75m) & \text{if } 6m \le x < 11m \\ (8.62m) & \text{if } 11m \le x < 23.5m \\ (8.63m) & \text{if } 23.5m \le x < 26m \\ (8.62m) & \text{if } 26m \le x < 33.5m \\ (8.63m) & \text{if } 33.5m \le x < 43.5m \\ (8.62m) & \text{if } 43.5m \le x < 55.25m \\ (8.63m) & \text{if } 55.25m \le x < 67m \\ (8.62m) & \text{if } 67m \le x < 113.85m \\ (8.58m) & \text{if } 113.85m \le x < 119.1m \\ (8.6m) & \text{if } 119.1m \le x < 124.3m \\ (8.58m) & \text{if } 124.3m \le x < 129.35m \\ (8.57m) & \text{if } 129.35m \le x < 134.2m \\ (8.83m) & \text{if } 134.2m \le x < 138.85m \\ (8.72m) & \text{if } 138.85m \le x < 147.75m \\ (2.8m) & \text{if } 147.75m \le x < 156.2m \end{cases}$$

$$R(x) := \frac{D(x)}{2}$$

$\alpha := -10\,deg$

$L_v := 156.5\,m$

$draft := 5\,m$

$$W_v := \frac{F_b}{\rho} = 7494.761\,m^3$$

$$x_G := \frac{\left(F_{sb} \cdot d_{sb} + F_{wb} \cdot d_{wb} + F_{col} \cdot d_{col} + F_{ext} \cdot d_{ext}\right)}{\left(F_{sb} + F_{wb} + F_{col} + F_{ext}\right)} = 74.141\,m$$

$z_G := -0.04\,m$

$y_G := 0.02\,m$

$$zy_G := \sqrt{(0 - y_G)^2 + (0 - z_G)^2}$$

$TOL := 0.0$

$CTOL := 0.0$

Given

$$d(x, draft, \alpha) := \tan(\alpha)\cdot x + draft$$

$$a(x, draft, \alpha) := \begin{cases} R(x) & \text{if } d(x, draft, \alpha) \geq R(x) \\ d(x, draft, \alpha) & \text{if } |d(x, draft, \alpha)| \leq R(x) \\ (-R(x)) & \text{if } d(x, draft, \alpha) < -R(x) \end{cases}$$

$$A_{cyl1}(x, draft, \alpha) := 2\cdot\int_{-a(x, draft, \alpha)}^{R(x)} \sqrt{R(x)^2 - t^2}\, dt$$

$$V_v(draft, \alpha) := \int_{0m}^{L_v} A_{cyl1}(x, draft, \alpha)\, dx$$

$$V_v(draft, \alpha) = W_v$$

$$A_z(z, draft, \alpha) := 2\left(\frac{z}{\tan(\alpha)} + draft\right)\sqrt{(4.295m)^2 - z^2}$$

$$z_B(draft, \alpha) := \frac{\int_{-4.295m}^{4.295m} z\cdot A_z(z, draft, \alpha)\, dz}{V_v(draft, \alpha)}$$

$$x_B(draft, \alpha) := \frac{\int_{0m}^{L_v} x\cdot A_{cyl1}(x, draft, \alpha)\, dx}{V_v(draft, \alpha)}$$

$$\begin{pmatrix} x_G - x_B(draft, \alpha) \\ -zy_G - z_B(draft, \alpha) \end{pmatrix}\cdot\begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix} = 0$$

$$\begin{pmatrix} draft \\ \alpha \end{pmatrix} := Find(draft, \alpha)$$

$draft = 4.116m$

$\alpha = -0.862deg$

$x_B(draft, \alpha) = 74.106m$

$x := 50m$

$d(x) := \tan(\alpha)\cdot x + draft$

$z_B(draft, \alpha) = -2.369m$

Given

$V_v(draft, \alpha) = 7494.763m^3$

$d(x) = 4.295m$

$x_1 := Find(x)$

$x_1 = -11.87m$

$d(L_v) = 1.761m$

Given

$d(x) = -4.295m$

$x_3 := Find(x)$

$x_3 = 558.866m$

$$a_3 := (x_3 - x_1)\cos(\alpha) = 570.671\,m$$

$$a_4 := \frac{a_3}{2} = 285.336\,m$$

$$L_e := \left|\begin{array}{l} \dfrac{(L_v - x_1)}{\cos(\alpha)} - a_4 \ \text{ if } d(L_v) > -4.295\,m \\ a_4 \ \text{ otherwise} \end{array}\right.$$

$$LCF := \frac{2\left[\displaystyle\int_{-a_4}^{L_e} t\left[\frac{4.295\,m}{a_4}\sqrt{(a_4)^2 - t^2}\right]dt\right]}{\left[2\cdot\displaystyle\int_{-a_4}^{L_e} \frac{4.295\,m}{a_4}\left[\sqrt{(a_4)^2 - t^2}\right]dt\right]}$$

$$LCF = -186.362\,m$$

$$CF := \frac{(x_1\cdot\cos(\alpha) + a_4 + LCF)}{\cos(\alpha)} = 87.115\,m$$

Length of each compartment

$$L_c := \begin{pmatrix} 169.5m - 162.5m \\ 162.5m - 156.2m \\ 156.2m - 147.75m \\ 147.75m - 138.85m \\ 138.85m - 134.2m \\ 134.2m - 129.35m \\ 129.35m - 124.3m \\ 124.3m - 119.1m \\ 119.1m - 113.85m \\ 113.85m - 102.15m \\ 102.15m - 90.45m \\ 90.45m - 78.75m \\ 78.75m - 67.0m \\ 67.0m - 55.25m \\ 55.25m - 43.5m \\ 43.5m - 33.5m \\ 33.5m - 23.5m \\ 23.5m - 6m \\ 6m - 0m \end{pmatrix}$$

Steel weight/m of each compartment

$$F_{stm} := \begin{pmatrix} 29.36 \\ 0 \\ 19.97 \\ 37.03 \\ 1.77 \\ 0 \\ 48.02 \\ 2.76 \\ 0 \\ 24.94 \\ 0 \\ 19.09 \\ 26.41 \\ 23.53 \\ 21.79 \\ 0 \\ 26.73 \\ 14.33 \\ 44.24 \end{pmatrix} \frac{\text{tonne}}{\text{m}}$$

$$F_{st} := \overrightarrow{\left(F_{stm} \cdot L_c\right)}$$

$$\sum F_{st} = 3110.41 \text{tonne}$$

Weight/m of solid ballast

$$F_{sbm} := \frac{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{sb} + F_{ext} \\ 0 \end{pmatrix}}{L_c}$$

Weight/m of water ballast of each compartment

$$F_{wbm} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 48.9 \\ 48.94 \\ 48.82 \\ 46.89 \\ 49.08 \\ 50.76 \\ 50.81 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \frac{tonne}{m}$$

Buoyancy/compartment

$$F_{bm} := \begin{pmatrix} \int_{162.5m}^{169.5m} \rho\, A_{cyl}(x, draft, \alpha)\, dx \\ \int_{156.2m}^{162.5m} \rho\, A_{cyl}(x, draft, \alpha)\, dx \\ \int_{147.75m}^{156.2m} \rho\, A_{cyl}(x, draft, \alpha)\, dx \\ \int_{138.85m}^{147.75m} \rho\, A_{cyl}(x, draft, \alpha)\, dx \\ \int_{134.2m}^{138.85m} \rho \cdot A_{cyl}(x, draft, \alpha)\, dx \\ \int_{129.35m}^{134.2m} \rho\, A_{cyl}(x, draft, \alpha)\, dx \\ \int_{124.3m}^{129.35m} \rho \cdot A_{cyl}(x, draft, \alpha)\, dx \\ \int_{119.1m}^{124.3m} \rho \cdot A_{cyl}(x, draft, \alpha)\, dx \\ \int_{113.85m}^{119.1m} \rho \cdot A_{cyl}(x, draft, \alpha)\, dx \\ \int_{102.15m}^{113.85m} \rho\, A_{cyl}(x, draft, \alpha)\, dx \\ \int_{90.45m}^{102.15m} \rho\, A_{cyl}(x, draft, \alpha)\, dx \\ \int_{78.75m}^{90.45m} \rho \cdot A_{cyl}(x, draft, \alpha)\, dx \\ \int_{67m}^{78.75m} \rho \cdot A_{cyl}(x, draft, \alpha)\, dx \\ \int_{55.25m}^{67m} \rho \cdot A_{cyl}(x, draft, \alpha)\, dx \\ \int_{43.5m}^{55.25m} \rho \cdot A_{cyl}(x, draft, \alpha)\, dx \\ \int_{33.5m}^{43.5m} \rho \cdot A_{cyl}(x, draft, \alpha)\, dx \\ \int_{23.5m}^{33.5m} \rho\, A_{cyl}(x, draft, \alpha)\, dx \\ \int^{23.5m} \cdots \end{pmatrix} =$$

	0
0	0
1	0
2	53.332
3	422.856
4	229.432
5	230.348
6	243.164
7	254.368
8	258.881
9	592.252
10	606.833
11	620.826
12	636.904
13	650.866
14	661.493
15	...

$\cdot tonne$

$\sum F_{bm} = 7690.958\,tonne$

$$F_{dn} := (F_{sbm} + F_{wbm} + F_{stm}) \cdot \cos(\alpha) = \cdot \frac{tonne}{m}$$

	0
0	29.357
1	0
2	19.968
3	85.92
4	50.704
5	48.814
6	94.899
7	51.834
8	50.754
9	75.741
10	0
11	19.088
12	26.407
13	23.527
14	21.788
15	...

$$F_{up} := \frac{\overrightarrow{F_{bm} \cdot \cos(\alpha)}}{L_c} = \cdot \frac{tonne}{m}$$

	0
0	0
1	0
2	6.311
3	47.506
4	49.335
5	47.489
6	48.146
7	48.911
8	49.305
9	50.614
10	51.86
11	53.056
12	54.198
13	55.387
14	56.291
15	...

$$F_{load} := F_{up} - F_{dn} = \cdot \frac{tonne}{m}$$

	0
0	-29.357
1	0
2	-13.657
3	-38.414
4	-1.37
5	-1.326
6	-46.753
7	-2.923
8	-1.449
9	-25.127
10	51.86
11	33.968
12	27.791
13	31.859
14	34.503
15	...

$$\text{vector} := \left| \begin{array}{l} U \leftarrow 0 \\ \text{for } \; i \in 0, 1 .. \left(\text{rows}\left(F_{load}\right) - 1\right) \\ \quad U_i \leftarrow \left(F_{load_i}\right) \cdot L_{c_i} \\ U \end{array} \right.$$

$$\text{vector} = \cdot tonne$$

	0
0	-205.497
1	0
2	-115.401
3	-341.883
4	-6.369
5	-6.429
6	-236.104
7	-15.198
8	-7.609
9	-293.99
10	606.764
11	397.428
12	326.549
13	374.346
14	405.415
15	...

$$V_c := \left| \begin{array}{l} A \leftarrow 0 \\ A_1 \leftarrow vector_0 \\ \text{for } j \in 2,3..(rows(vector) - 1) \\ \quad A_j \leftarrow A_{j-1} + vector_{j-1} \\ A_{19} \leftarrow A_{18} + vector_{18} \\ A \end{array} \right.$$

$V_c =$ (table) $\cdot tonne$

	0
0	0
1	-205.497
2	-205.497
3	-320.898
4	-662.781
5	-669.15
6	-675.579
7	-911.683
8	-926.881
9	-934.489
10	-1228.479
11	-621.715
12	-224.287
13	102.263
14	476.609
15	...

$$M_c := \left| \begin{array}{l} C \leftarrow 0 \\ C_1 \leftarrow \dfrac{V_{c_1} \cdot L_{c_0}}{2} \\ \text{for } k \in 2,3..rows(V_c) - 1 \\ \quad C_k \leftarrow C_{k-1} + \dfrac{\left(V_{c_{k-1}} + V_{c_k}\right) \cdot L_{c_{k-1}}}{2} \\ C \end{array} \right.$$

$M_c =$

	0
0	0
1	-719.239
2	-2013.868
3	-4237.886
4	-8615.256
5	-11711.995
6	-14972.961
7	-18980.797
8	-23761.063
9	-28647.16
10	-41300.525
11	-52124.159
12	-57073.268
13	-57790.158
14	-54389.289
15	...

$\cdot m \cdot tonne$

$\max(M_c) = 0 \cdot m \cdot tonne$

$\min(M_c) = -57790.158 \, m \cdot tonne$

$t := 0.045m$

$r := 4.295m$

$I := \pi \cdot r^3 \cdot t$

$\sigma_{top} := \dfrac{M_c \cdot r \cdot g}{I} =$

	0
0	0
1	-2.705
2	-7.573
3	-15.936
4	-32.397
5	-44.042
6	-56.304
7	-71.375
8	-89.351
9	-107.724
10	-155.306
11	-196.007
12	-214.617
13	-217.313
14	-204.524
15	...

$\cdot MPa$

$$\sigma_{bot} := \frac{M_c \cdot (-r) \cdot g}{I} =$$

	0
0	0
1	2.705
2	7.573
3	15.936
4	32.397
5	44.042
6	56.304
7	71.375
8	89.351
9	107.724
10	155.306
11	196.007
12	214.617
13	217.313
14	204.524
15	...

$\cdot MPa$

$\min(\sigma_{top}) = -217.313 MPa$

Calculations according to DNV rules

$t_c := 0.04m$

$I_c := \pi \cdot r^3 \cdot t_c$

$SF := 1.2$

$$\sigma_{topc} := \frac{M_c \cdot r \cdot g \cdot SF}{I_c} =$$

	0
0	0
1	-3.651
2	-10.223
3	-21.514
4	-43.735
5	-59.456
6	-76.01
7	-96.356
8	-120.623
9	-145.428
10	-209.663
11	-264.609
12	-289.733
13	-293.373
14	-276.108
15	...

$\cdot MPa$

$\min(\sigma_{topc}) = -293.373 MPa$

$\gamma_m := 1.15$

$R_k := 335 MPa$

$$R_d := \frac{R_k}{\gamma_m} = 291.304\,\mathrm{MPa}$$

Appendix F

$\rho := 1025$
$g := 9.807$
$V := 6147$
$L := 1$

$$A := \begin{pmatrix} 1.0489 & 6.8475\cdot10^{-6} & -7.3564\cdot10^{-6} & 6.9863\cdot10^{-4} & -56.566 & 5.2129\cdot10^{-10} \\ -2.0082\cdot10^{-5} & 1.0489 & -7.7453\cdot10^{-6} & 56.566 & 2.1114\cdot10^{-3} & 5.2650\cdot10^{-10} \\ 7.9175\cdot10^{-7} & -2.1718\cdot10^{-7} & 2.2788\cdot10^{-2} & -5.1308\cdot10^{-5} & -5.9159\cdot10^{-5} & -3.1147\cdot10^{-11} \\ -2.2077\cdot10^{-3} & 56.567 & -8.6664\cdot10^{-4} & 4.1010\cdot10^{3} & 0.23216 & -8.4920\cdot10^{-8} \\ -56.567 & -7.6945\cdot10^{-4} & 8.2230\cdot10^{-4} & -7.8705\cdot10^{-2} & 4.1010\cdot10^{3} & -4.6669\cdot10^{-9} \\ 5.1537\cdot10^{-10} & 5.4270\cdot10^{-10} & -3.1885\cdot10^{-11} & -8.4737\cdot10^{-8} & -4.0495\cdot10^{-9} & 5.4292\cdot10^{-13} \end{pmatrix} \cdot\rho\cdot V\cdot L^2$$

$$M := \begin{pmatrix} 0.99986 & 0 & 0 & 0 & -57.109 & 2.6783\cdot10^{-7} \\ 0 & 0.99986 & 0 & 57.109 & 0 & -4.4101\cdot10^{-7} \\ 0 & 0 & 0.99986 & -2.6783\cdot10^{-7} & 4.4101\cdot10^{-7} & 0 \\ 0 & 57.109 & -2.6783\cdot10^{-7} & 7676.2 & 1.0806\cdot10^{-6} & -2.5189\cdot10^{-5} \\ -57.109 & 0 & 4.4101\cdot10^{-7} & 1.0806\cdot10^{-6} & 7676.2 & -1.5298\cdot10^{-5} \\ 2.6783\cdot10^{-7} & -4.4101\cdot10^{-7} & 0 & -2.5189\cdot10^{-5} & -1.5298\cdot10^{-5} & 12.45 \end{pmatrix} \cdot\rho\cdot V\cdot L^2$$

$$C := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8.9854\cdot10^{-3} & -1.9159\cdot10^{-11} & 1.0353\cdot10^{-10} & 0 \\ 0 & 0 & -1.9159\cdot10^{-11} & 1.4831 & 3.1027\cdot10^{-10} & -3.8401\cdot10^{-7} \\ 0 & 0 & 1.0353\cdot10^{-10} & 3.1027\cdot10^{-10} & 1.4831 & -2.4739\cdot10^{-7} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot\rho\cdot g\cdot V\cdot L$$

$$\phi := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -0.9752 & 0 & 0.0176 & 0.018 & 0 \\ 0.9752 & 1 & 0 & -0.018 & 0.0176 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$v := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$i := 0..5$
$\lambda_i := 0.00001$
$CTOL := 0.0001$
$TOL := 0.0001$

Symbolical evaluation of the determinant

$$|-\lambda\cdot(M + A) + C| \rightarrow \frac{1\cdot\left(282.38\times10^{69}\cdot\lambda^7 - 50.17\times10^{69}\cdot\lambda^6 + 2.36\times10^{69}\cdot\lambda^5 - 11.49\times10^{66}\cdot\lambda^4 + 14.82\times10^{63}\cdot\lambda^3\right)}{2.82\times10^{21}\cdot\lambda - 243.23\times10^{18}}$$

We take only the nominator (top part) because the roots of this function are the roots of the top part

$$f(\lambda) := 1.0\cdot\left(6.9468439e71\lambda^7 - 1.2345497e71\lambda^6 + 5.80373073e69\lambda^5 - 2.832928565e67\lambda^4 + 3.657980310e64\lambda^3\right)$$

Coefficients of the polynom constituted by f

$$C := f(\lambda) \text{ coeffs}, \lambda \rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ 36.58\times10^{63} \\ -28.33\times10^{66} \\ 5.8\times10^{69} \\ -123.45\times10^{69} \\ 694.68\times10^{69} \end{pmatrix}$$

Roots of the polynom

$$G := \text{polyroots}(C) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 2.66\times10^{-3} \\ 2.66\times10^{-3} \\ 86.18\times10^{-3} \\ 86.2\times10^{-3} \end{pmatrix}$$

Function to calculate the period

$$H(\lambda) := \frac{2\cdot\pi}{\sqrt{\lambda}}$$

The zero eigenvalues have to be removed they lead to infinite periods

$$\Lambda := \begin{pmatrix} G_3 \\ G_4 \\ G_5 \end{pmatrix} = \begin{pmatrix} 2.66\times10^{-3} \\ 2.66\times10^{-3} \\ 86.18\times10^{-3} \end{pmatrix}$$

Eigenperiods

$$\overrightarrow{H(\Lambda)} = \begin{pmatrix} 121.83 \\ 121.72 \\ 21.4 \end{pmatrix}$$

Appendix G

2.7 SUMMARY OF MODEL PROPERTIES

ALL COORDINATES ARE GIVEN IN THE INPUT COORDINATE SYSTEM

THE RADII OF GYRATION AND CENTRIFUGAL MOMENTS OF THE MASS MATRIX

AND THE RESTORING COEFFICIENTS ARE GIVEN RELATIV TO THE MOTION REFERENCE POINT

(ORIGIN OF THE GLOBAL COORDINATE SYSTEM).

UNITS DATA:

```
ACCELERATION OF GRAVITY            G       = 9.80665E+00   [L/T**2]
WATER DENSITY                      RHO     = 1.02500E+03   [M/L**3]
```

GEOMETRY DATA:

```
CHARACTERISTIC LENGTH                   L        = 1.00000E+00   [L]
VERTICAL COORDINATE OF STILL WATER LINE   -ZLOC    = 0.00000E+00   [L]
NUMBER OF NODES IN THE MORISON MODEL      NMNOD    =  682
NUMBER OF MORISON ELEMENTS                NMELM    =  680
           OF WHICH                       20 2-D ELEMENTS
                                         660 POINT MASSES
NUMBER OF MORISON SUBELEMENTS             NMSEL    =  681

NUMBER OF BASIC PANELS                         =  561
NUMBER OF SYMMETRY PLANES IN
THE PANEL MODEL                                =    0
TOTAL NUMBER OF PANELS                         =  561
DISPLACED VOLUMES OF THE PANEL MODEL      VOL 1    = 6.14738E+03   [L**3]
                          VOL 2    = 6.14738E+03
                          VOL 3    = 6.14738E+03
```

MASS PROPERTIES AND STRUCTURAL DATA:

```
MASS OF THE STRUCTURE                 M        = 6.30020E+06   [M]
WEIGHT OF THE STRUCTURE               M*G      = 6.17838E+07   [M*L/T**2]
CENTRE OF GRAVITY                     XG       =-4.41074E-07   [L]
                                      YG       =-2.67869E-07   [L]
                                      ZG       =-5.71171E+01   [L]
ROLL  RADIUS OF GYRATION              XRAD     = 8.76201E+01   [L]
PITCH RADIUS OF GYRATION              YRAD     = 8.76201E+01   [L]
YAW   RADIUS OF GYRATION              ZRAD     = 3.52870E+00   [L]
ROLL-PITCH CENTRIFUGAL MOMENT         XYRAD    =-1.08073E-06   [L**2]
ROLL-YAW   CENTRIFUGAL MOMENT         XZRAD    = 2.51929E-05   [L**2]
PITCH-YAW  CENTRIFUGAL MOMENT         YZRAD    = 1.52999E-05   [L**2]
```

HYDROSTATIC DATA:

DISPLACED VOLUME VOL = 6.14739E+03 [L**3]
MASS OF DISPLACED VOLUME RHO*VOL = 6.30108E+06 [M]
WATER PLANE AREA WPLA = 5.52366E+01 [L**2]
CENTRE OF BUOYANCY XCB =-5.70078E-08 [L]
YCB =-2.04375E-08 [L]
ZCB =-5.56656E+01 [L]
TRANSVERSE METACENTRIC HEIGHT GM4 = 1.48313E+00 [L]
LONGITUDINAL METACENTRIC HEIGHT GM5 = 1.48313E+00 [L]
HEAVE-HEAVE RESTORING COEFFICIENT C33 = 5.55228E+05 [M/T**2]
HEAVE-ROLL RESTORING COEFFICIENT C34 =-1.18387E-03 [M*L/T**2]
HEAVE-PITCH RESTORING COEFFICIENT C35 = 6.39722E-03 [M*L/T**2]
ROLL-ROLL RESTORING COEFFICIENT C44 = 9.16465E+07 [M*L**2/T**2]
PITCH-PITCH RESTORING COEFFICIENT C55 = 9.16465E+07 [M*L**2/T**2]
ROLL-PITCH RESTORING COEFFICIENT C45 = 1.91723E-02 [M*L**2/T**2]

EQUILIBRIUM OF STATIC FORCES AND MOMENTS:

SUM OF TOTAL BUOYANCY AND GRAVITY FORCES F3 = 8.60400E+03 [M*L/T**2]
STATIC MOMENT ABOUT THE X-AXIS M1 = 1.52871E+01 [M*L**2/T**2]
STATIC MOMENT ABOUT THE Y-AXIS M2 =-2.37286E+01 [M*L**2/T**2]
CORRESPONDING VERTICAL TRANSLATION = 1.54963E-02 [L]
TRIM ANGLE IN ROLL ALFAX = 9.55722E-06 [DEG]
TRIM ANGLE IN PITCH ALFAY =-1.48347E-05 [DEG]

4.2 STATIC RESULTS

MASS INERTIA COEFFICIENT MATRIX

	1	2	3	4	5	6
1	9.9986E-01	0.0000E+00	0.0000E+00	0.0000E+00	-5.7109E+01	2.6783E-07
2	0.0000E+00	9.9986E-01	0.0000E+00	5.7109E+01	0.0000E+00	-4.4101E-07
3	0.0000E+00	0.0000E+00	9.9986E-01	-2.6783E-07	4.4101E-07	0.0000E+00
4	0.0000E+00	5.7109E+01	-2.6783E-07	7.6762E+03	1.0806E-06	-2.5189E-05
5	-5.7109E+01	0.0000E+00	4.4101E-07	1.0806E-06	7.6762E+03	-1.5298E-05
6	2.6783E-07	-4.4101E-07	0.0000E+00	-2.5189E-05	-1.5298E-05	1.2450E+01

HYDROSTATIC RESTORING COEFFICIENT MATRIX

	1	2	3	4	5	6
1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

3	0.0000E+00	0.0000E+00	8.9854E-03	-1.9159E-11	1.0353E-10	0.0000E+00
4	0.0000E+00	0.0000E+00	-1.9159E-11	1.4831E+00	3.1027E-10	-3.8401E-07
5	0.0000E+00	0.0000E+00	1.0353E-10	3.1027E-10	1.4831E+00	-2.4739E-07
6	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

TOTAL ADDED MASS MATRIX

	1	2	3	4	5	6
1	1.0489E+00	6.8475E-06	-7.3564E-06	6.9863E-04	-5.6566E+01	5.2129E-10
2	-2.0082E-05	1.0489E+00	-7.7453E-06	5.6566E+01	2.1114E-03	5.2650E-10
3	7.9175E-07	-2.1718E-07	2.2788E-02	-5.1308E-05	-5.9159E-05	-3.1147E-11
4	-2.2077E-03	5.6567E+01	-8.6664E-04	4.1010E+03	2.3216E-01	-8.4920E-08
5	-5.6567E+01	-7.6945E-04	8.2230E-04	-7.8705E-02	4.1010E+03	-4.6669E-09
6	5.1537E-10	5.4270E-10	-3.1885E-11	-8.4737E-08	-4.0495E-09	5.4292E-13

EIGEN VALUES:			EIGEN VECTORS:					
NO	PERIOD [T]	ANG. FREQ.	1	2	3	4	5	6
1	INFINITE							
2	INFINITE							
3	INFINITE							
4	1.2185E+02	5.1564E-02	1.0000	-0.9752	0.0000	0.0176	0.0180	0.0000
5	1.2185E+02	5.1566E-02	0.9752	1.0000	0.0000	-0.0180	0.0176	0.0000
6	2.1405E+01	2.9354E-01	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000

Appendix H

Calculation of period needed for small volume assuption

$D := 8.59m$

$\lambda_{min} := 5 \cdot D = 42.95m$

$$T_{min} := \sqrt{\frac{\lambda_{min}}{1.56\frac{m}{s^2}}} = 5.247s$$