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Comparison of operational limit estimations for FRP laminates – effects of material modelling and stochastic behaviour

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Abstract

Transport companies need vehicles with higher efficiency to remain competitive and comply with environmental regulations. Lightweight construction is one way to increase the efficiency of a vehicle. In some vehicles, lightweight construction can reduce the fuel consumption, increase the cargo capacity and reduce the environmental footprint. Fibre-Reinforced Plastics (FRPs) are convenient materials for lightweight construction due to their high stiffness and strength to weight ratio; however, FRPs are expensive materials compared to steel and sometimes to aluminium. Therefore, a FRP structure should have the least amount of material as possible so as to increase the benefits of lightweight construction and to be as inexpensive as possible; FRP structures should also be beneficial enough to motivate the added acquisition cost.

Operational limits determine the magnitude of the load that a material can carry safely, thus, in a sense, operational limits determine how much material it is required for a certain component or part. FRPs are heterogeneous anisotropic materials that exhibit several modes of material degradation and failure. The calculation of their operational limits is generally simplified, which may lead to under predicted operational limits and consequently, to unnecessarily heavy and expensive FRP structures.

The work presented in this thesis intends to contribute to the body of knowledge regarding the estimation of operational limits for FRPs, and to determine if more accurate methods for estimating the operational limits of FRPs can motivate higher material utilization compared to current design rules. Both objectives aim at rendering lighter, cheaper and safer FRP structures. This investigation compares the operational limits of tension-loaded FRP laminates calculated through several methods. The methods are intended to be generally applicable to FRP structures; however, in this work, the methods are applied to structures that follow marine structure design rules and guidelines. The comparison evaluates the effects that material modelling and stochastic behaviour have on the estimation of operational limits.

Keywords: FRP, probability, reliability, safety.

Preface

This thesis presents work carried out during the years 2009-2012 at the Department of Shipping and Marine Technology, Division of Marine Design at Chalmers University of Technology. This work was carried out as a part of the European Union project Breakthrough in European Ship and Shipbuilding Technologies (BESST) [Grant agreement no.: 233980, Call id: FP7-SST-2008-RTD-1, www.besst.it].

I would like to express my sincere gratitude to my supervisor Professor Jonas Ringsberg and my co-supervisor Adjunct Professor Erland Johnson for letting me loose, but nudging me in the right direction. Grateful acknowledgements to my colleagues Måns Håkansson and Sven-Erik Hellbratt at Kockums for providing me with insights and a myriad of test coupons for future investigations.

Special thanks to the friendly crew on the Department of Shipping and Marine Technology, these three years have been a delight.

This thesis is dedicated to my niece Maria Emilia. I hope it serves as a very educating bedtime story.

Gothenburg, August 2012.

Luis Felipe

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List of Appended Papers

Paper I Sánchez L., Ringsberg J.W., Johnson E. (2011). *Optimization of composite maritime structures – effects of uncertainties on design criteria and limits.* In: Advances in Marine Structures – Proceedings of the Third International Conference on Marine Structures (MARSTRUCT 2011), Hamburg, Germany, 28-30 March, 2011, p. 707-714.

The author of this thesis contributed to the ideas presented, planned the paper with the co-authors, carried out the numerical simulations and wrote most of the manuscript.

Paper II Sánchez-Heres L.F., Ringsberg J.W., Johnson E. (2012). *Study on the possibility of increasing the maximum allowable stresses in Fibre-Reinforced Plastics*. Journal of Composite Materials (available online, issue number pending).

The author of this thesis contributed to the ideas presented, planned the paper with the co-authors, performed the numerical simulations and wrote the manuscript.

Paper III Sánchez-Heres L.F., Ringsberg J.W., Johnson E. (2012) *Effects of matrix cracking on the estimation of operational limits of FRP laminates*. In: Proceedings of the Fifteenth European Conference in Composite Materials (ECCM15), Venice, Italy, 24-28 June, 2012.

The author of this thesis contributed to the ideas presented, planned the paper with the co-authors, performed the numerical simulations and wrote the manuscript.

“The future is already here – it’s just not very evenly distributed”

~ William Gibson (1993)

1 Introduction and motivation

Like any other, transport companies strive to reduce costs in order to remain competitive and earn money, as well as to comply with present and upcoming regulations so as to be allowed to do business.

Regulations are in place to protect people, property and the environment. Climate change, environmental degradation and health concerns have led to the instalment of regulations that limit the amount of contaminants (e.g. NO_X, SO_X, noise) that a vehicle can release. It is clear that these limits will become more stringent in the future, with the further addition of limits for greenhouse gas emissions.

Transport companies rely on vehicles, which have two main costs associated with them: acquisition cost and operational cost. Depending on the approach of the transport company one might be more important than the other. A company that plans to acquire a vehicle and to use it for a short period of time might not consider convenient to pay a premium for an advanced vehicle with low operational costs. However, most transport companies do employ their vehicles for periods of time sufficiently long so as to benefit from reductions on the operational costs.

Present and future environmental regulations regarding energy efficiency, pollution and greenhouse gas emissions (Dings 2012 and European Commission 2011), as well as cost requirements can only be met by making transport more efficient. Efficiency denoting the amount of resources needed to transport an amount of cargo or people a particular distance at a certain speed. In this context resources can be understood as a variety of things, such as: energy, personnel, maintenance, number of vehicles, and even space. Nowadays, energy efficiency is especially important due to its direct link to fuel consumption and emission of contaminants and greenhouse gases.

There are several ways of improving the energy efficiency of transport. Effective supply chain management and logistics can reduce the energy usage of the whole transport system by shortening the distance vehicles must travel as well as lowering their speed. Both measures are highly effective as the energy consumption of a vehicle is proportional to the travelled distance and to the square of the travelling speed. The energy efficiency of the vehicles themselves can also be improved. Aerodynamic and hydrodynamic analyses can render vehicle designs with decreased drag, while better engines, machinery, and propulsion systems can cutback a vehicle's thermal and mechanical energy losses. An additional solution for reducing the energy consumption or improving the energy efficiency of a vehicle is to reduce its weight. Lighter vehicles require less energy or are able to transport more cargo using the same amount of energy. Lightweight construction can be achieved through a variety of approaches; one of them is the utilization of Fibre-Reinforced Plastics (FRPs) as construction materials.

1.1 Fibre-Reinforced Plastics (FRPs) past and present use

FRPs have been around for more than 70 years, their high strength and stiffness to weight ratio makes them very convenient materials for lightweight structures. Glass Fibre-Reinforced Plastics (GFRPs) revolutionized the boat building industry in the 40s by allowing the inexpensive construction of whole series of identical boats that did not rot or corrode and were easy to repair, lowering considerably the costs of buying and owning a boat. Throughout the years GFRPs and Carbon Fibre-Reinforced Plastics (CFRPs) have been used extensively to improve the operational performance of a wide range of vehicles. Naval ships and submarines incorporate these materials to reduce fuel consumption and maintenance costs, as well as to increase the cargo capacity and stability of the vessel (with the additional benefit of decreasing its radar and magnetic signature) (Chalmers 1991 and Mouritz et al. 2001). Due to similar benefits, CFRPs are used extensively in the aircraft and sport car industry. One of the most recent examples is the jet airliner Boeing 787 Dreamliner, an aircraft that is 50% composite by weight and consumes 20% less fuel than similar non-composite aircrafts (Boeing 2012).

Even though FRPs have been used for such a long time, and that their high strength and stiffness to weight ratio makes them very suitable materials for vehicle structures, their use has never become widespread in the transport industry (with the exception of the boat and yacht industry). The reasons for this reluctance can be narrowed down to two aspects: price and motivation. In general, FRPs with mechanical properties similar to aluminium or steel are expensive, and therefore, their use has been limited to applications where performance is paramount and price is generally not an issue, such as military applications and sport cars. Cheap fuel prices and little environmental awareness have not helped the adoption of FPRs either. For 50 years there has been little motivation for making cars, trucks, ships and airplanes lighter, so as to reduce fuel consumption. Furthermore, design and manufacturing of FRP structures is regarded as a complex process compared to steel or aluminium structures. This added complexity has additionally dissuaded vehicle manufacturers from using FRPs.

Nowadays the reasons for not using FRPs are disappearing. The steady rise in fuel prices and increasing environmental concerns motivate the use of lightweight construction in vehicle structures. The price of carbon fibre will most likely decrease in the following decade as a result of an increase of demand and competition, as well as novel manufacturing technologies product of current research (US Department of Energy 2012). New applications for FRPs in vehicles are arising, and with them new technical and economical challenges.

1.2 FRPs in marine structures

In some ways FRPs seem to be the perfect material for the construction of marine structures. FRPs are lightweight, buoyant, stiff and strong, they do not rot or corrode, and they have a long fatigue life. Even though they have so many desirable properties their use has been limited mostly to boats, yachts, high-speed light craft, naval ships and submarines (Chalmers 1991).

One of the main reasons why FRPs have not been used in commercial ships (such as container ships and cruise ships) is because they are combustible. In general FRPs burn, release black smoke, and loose their structural strength at temperatures significantly lower than those of aluminium or steel. The international prescriptive rules Safety Of Life At Sea (SOLAS) used to limit the materials out of which a ship could be constructed to "steel or equivalent" with respect to fire resistance. Since FRPs are combustible they were not allowed. FRPs have been used in boats, yachts, high-speed light craft, naval ships and submarines because they do not have to comply with the SOLAS rules. In 2002, the SOLAS rules changed, and today, they allow the use of construction materials other than steel or equivalent, as long as the material meets the same safety objectives and functional requirements. This means that FRP structures can be used in SOLAS vessels as long as the fire safety of the structure is guaranteed.

At present, the construction of a whole commercial ship out of FRPs in unlikely. The largest full FRP vessel today is the Mirabella V yacht with a displacement of 740 metric tons and a length of 75.2 meters, a dwarf compared to the Allure of the Seas, a 362 meters long cruise ship with a ~100,000 metric tons displacement. However, commercial ships can still benefit from FRPs construction. A successful technique utilized in naval vessels is to build the superstructure out of FRPs (Mouritz et al. 2001). This technique provides two principal benefits: first, it reduces the maintenance costs of the vessel since FRPs do not corrode and have a long fatigue life; and second, it improves the ship's stability by reducing the structural topside weight. The latter also increases the ship's efficiency by allowing the transport of more cargo or by reducing the fuel consumption (Mouritz et al. 2001 and Chalmers 1994).

Several industry projects have evaluated the benefits, drawbacks, challenges and potential solutions of adopting FRPs in commercial ships. The LÄSS project (Lightweight Construction Applications at Sea, 2005 – 2009) investigated GFRP and CFRP sandwich superstructures, among other lightweight designs, for high-speed light craft and RoPax ships. For both ships FRPs were predicted to give reductions in structural weight; however, the payback time of the FRP structures differed greatly, from years to more than a decade (Hertzberg 2009). Similarly, the LÄSS-C project (Lightweight Construction of a Cruise Vessel; 2008 – 2011) investigated the use of FRPs for the upper decks of a cruise ship. The investigation concluded that the weight reduction due to the use of FRPs instead of steel, would allow the construction of about 100 more passenger cabins. Additionally, the investigation determined that the payback time of the price difference between the FRP structure and the steel one would be of 2.5 years (Evegren 2011). Currently, one of the work packages of the EU-project BESST (Breakthrough in European Ship and Shipbuilding Technologies; 2009 – on going) continues to investigate the use of FRPs as innovative materials for lightweight construction in passenger ships, ferries and mega-yachts. The results will be applicable to a large extent to other types of ships.

The use of FRPs for marine structures presents several challenges. One of the most problematic is economical. In general, the acquisition cost of a FRP structure is higher than the one of a similar structure in aluminium or steel (Hertzberg 2009). Even though the benefits of using FRPs are also higher, the acquisition cost can make FRP structures economically unattractive. Most ship owners are willing to pay a premium for a more efficient ship, as long as the increased efficiency pays for the premium in a reasonable amount of time, typically less than five years. A FRP structure needs to have an acquisition cost in line with its efficiency benefits to be economically attractive. One of the largest parts of the acquisition cost of a FRP structure is the material itself; therefore, it is of great importance to guarantee that the FRPs are used effectively.

Due to the nature of the marine industry, ships are designed conservatively. The design time is quite limited, as the ship owners will not wait more than five years from the moment they order the ship to the moment it is delivered. Furthermore, ships are usually task specific, and therefore, only a limited amount of sister ships, if any, are constructed. Since the design will not be used hundreds or thousands of times, like in the car or aircraft industries, there is a significant economical limitation on how many man-hours can be spent in the design (Shenoi and Dodkins 2000). Limited time and limited resources force the designers to rely on fast and trustworthy tools and methods, usually the ones stated in design rules.

In the maritime industry, design rules are issued by classification societies, as they are entrusted with the task of guaranteeing the safety of new ship designs. Due to the diversity of possible ship designs and the variation of engineering competence and engineering resources worldwide, the design rules are simple to use and understand. This simplicity, unfortunately, can come with a lack of accuracy, and consequently large safety factors that can lead to very conservative designs.

To summarize, marine FRP structures are commonly designed with simple and fast tools and methods, as well as large safety factors due to time and cost constraints. The design methodology can render too conservative designs that can be too expensive to be considered economically attractive. Marine FRP structures could therefore benefit from more accurate design methodologies that would lead to the usage of less material through its better utilization.

1.3 Operational limits of FRPs

One of the most important tasks of designing a structure is to guarantee that it has the right safety level. In structural engineering, safety level is understood as the combination of the consequences of structural failure and the probability that such failure would occur. For example, the failure of a cup holder in a car would probably only lead to dirty interiors and an angry customer; therefore, a cup holder design with a probability of failure of $1/10^3$ has an acceptable safety level. In contrast, a car suspension support with a probability of failure of $1/10^3$ would have an unacceptable safety level since its failure would lead to a very serious and possibly fatal accident, such a critical component should have a probability of failure of at least $1/10^6$ to have an acceptable safety level (DNV 2010a). Operational limits guarantee the desired safety level in structures, they mark the load threshold over which the safety level is not met, or in other words, for a given consequence of failure, the limit load after which the probability of failure of a structure is too high.

The accurate prediction of safety levels and operational limits of vehicle structures is extremely important. The unexpected failure of a structure due to the over estimation of an operational limit could lead to: interruption of service, expensive repairs, increased maintenance, bad reputation, complete loss of the vehicle, damage to the environment and most importantly loss of lives. On the other hand, a structure with under estimated operational limits would not fail; however, the structure will be unnecessarily heavy and strong. The extra weight and strength increases the manufacturing and operational costs of the vehicle and decreases its efficiency. This issue is exacerbated in FRP structures since FRPs are expensive, and the main reasons for using them is to increase the vehicle's efficiency.

Safety concerns, and therefore, operational limits, determine the type and quantity of material that is needed in a structure. The accurate estimation of operational limits is a crucial task on the design of an efficient lightweight structure, as they determine if the material is utilized to its fullest.

2 Objective

The objective of the work presented in this thesis is twofold: first, to contribute to the body of knowledge regarding the estimation of operational limits for FRP laminates, and second, to investigate the possibility of improving the utilization of FRP laminates intended for marine structures through a more accurate estimation of its operational limits.

2.1 Limitations

There is a wide variety of FRPs laminates available in the market. Their mechanical properties are not only dependent on the type of fibre and plastic matrix, but also on the architecture of the material itself. The failure event of a chopped strand mat FRP laminate (medium sized fibres randomly oriented in a plane) is significantly different from the one of a unidirectional FRP laminate (long fibres aligned in a single direction). To study the operational limits of all the different types of FRPs would be a monumental task; therefore, the work presented in this licentiate thesis was limited to FRPs made out of long unwoven 'straight' carbon and glass fibres with discrete orientations. There are two types of FRPs that match this description: prepregs and Non-Crimp Fabrics (NCFs). Out of all the different types of FRPs, prepregs have the best mechanical properties, and therefore they have been heavily studied. NCF laminates are more commonly used in maritime structures; however, prepregs are fundamentally similar to NCF laminates. All the analyses performed in this work use prepreg data since at the beginning of the research the author had no access to NCFs material data. The prepreg data used in the analyses is of non-aged specimens (no environmental effects: humidity, temperature, UV, etc.).

Even though they are commonly regarded as materials, FRP laminates are structures. Most of the uncertainty concerning the failure of FRP laminates arises from this convenient oversimplification. The work in this thesis analyses only FRP laminates in order to focus on the causes of failure uncertainty and on the effects it has on the operational limits of laminates with the intention, as mentioned before, of estimating these limits more accurately. For this reason, the operational limits of marine FRP structures such as stiffeners and joints are not analysed in this thesis. Additionally, to limit the scope of the failure uncertainty, the laminates are analysed only under monotonically increasing tension loads with low loading rates. The design rules of the classification society Det Norske Veritas (DNV 2010a and 2010b) are used to provide a baseline of the current utilization of FRP laminates.

3 Material modelling and stochastic behaviour of FRPs

This section contains a brief summary of some basic knowledge needed to estimate the operational limits of FRPs. The goal of this summary is not to cover all the subjects and details presented in the appended papers I to III, but to elucidate the reasoning behind the work. Consequently, this summary contains only two main subjects: material modelling and stochastic behaviour of FRPs.

3.1 Material modelling

3.1.1 The definition of failure and degradation

Failure is the state or action of not functioning. For the sake of the discussion, the function of a FRP is to safely carry a load for an extended period of time; therefore, the inability to do so constitutes FRP failure. Degradation is the deterioration or reduction of a property. In this work, degradation of FRPs is the reduction of the mechanical properties that allow the material to perform its function, in other words, degradation *leads* to failure.

3.1.2 Failure of FRP plies

Plies are the building block of FRP laminates. For practical purposes plies are considered to be a thin arrangement of long unidirectional fibres embedded in a matrix, somewhere between 0.1-1.0 mm thick. Figure 1 presents the archetype of a ply.

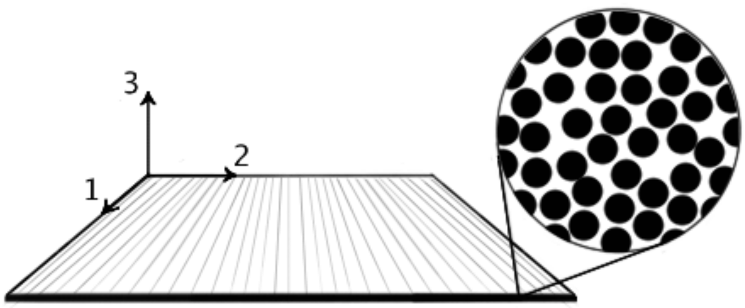

Figure 1. FRP ply and its local coordinate system
(1: longitudinal direction; 2 and 3: transverse direction).

The fibres are aligned in the length direction and randomly distributed throughout the width and thickness. Due to this particular fibre arrangement, FRP plies are heterogeneous transversely isotropic materials, in other words, the mechanical properties of the ply (stiffness moduli, Poisson's ratios, thermal expansion coefficients and failure strengths) are different in the longitudinal and transverse directions. A consequence of this mismatch is that depending on the loading condition, plies might present different failure modes and extension-shear couplings, in addition to different deformations with respect to the load direction (Mallick

2008). Figure 2 presents some of the different failure modes that can be observed in a FRP ply.

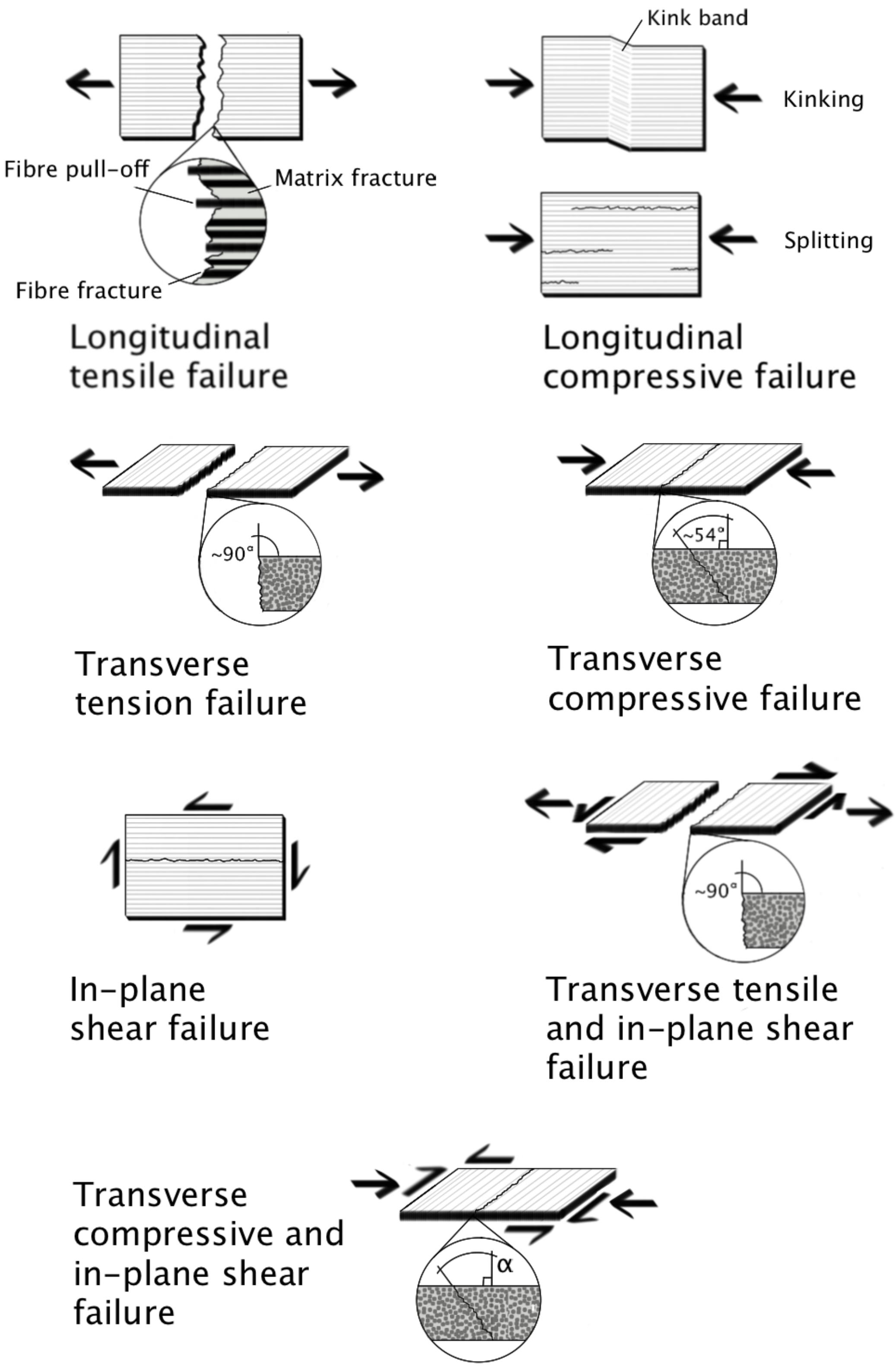

Figure 2. Modes of failure of a FRP ply.

When a ply is subjected to a critical tension load in the longitudinal direction, the fibres start to fracture at different locations along their length. Furthermore, in some occasions the bonding between the matrix and the fibres starts to fail as well. Fibre fracture is a progressive process (Alava et al. 2006 and Sutherland and Soares 1997). Some fibres fracture earlier because the load is not evenly distributed among the fibres and because the strength of all the fibres is not the same. The fracture of a fibre leads to a redistribution of its load to the adjacent fibres and matrix, increasing locally the stresses, and therefore, promoting more fracture. The ply does not fail when the first fibre fractures, it fails when a critical amount of fibres break, rendering the ply unable to carry the load. Typically, the matrix does not fracture before fibres fracture because the fracture strain of the matrix is larger than the one of the fibres, matrix fracture occurs eventually but only as a result of the fibre fractures.

Several failure modes can be observed in plies subjected to a compressive critical load in the longitudinal direction. The most common failure mode is kinking, the formation of a kink band bounded by fibre breaks (Budiansky and Fleck 1993). The initiation of a kink band is considered to be a microbuckling problem governed at a large extent by the misalignment of the fibres, as well as the nonlinear deformation of the matrix. Instability of the fibres leads to their rotation and consequently the formation of a kink band. Other failure modes are fibre fracture in compression and splitting (the formation of matrix cracks parallel to the fibres).

Matrix fracture (aka. Intra Fibre Fracture) occurs in a ply when it is subjected to a transverse tensile or compressive load, as well as an in-plane shear load. Matrix cracks initiate where a ply has its largest imperfections, for example voids of inclusions. Once the cracks are formed they propagate throughout the ply, forming a fracture surface that splits the ply. Experimental evidence suggests that a ply stressed in any of the transverse directions (2 and 3 in Figure 1) fractures in one of two ways: tensile fracture or shear fracture in the planes with the highest respective stresses (Knops 2008). Additionally, it has been observed that compressive stresses hinder shear fracture at some extent. The fracture surface can be oriented at different angles because of the ways that the transverse and in-plane shear stresses can interact.

3.1.3 Degradation and failure of FRP laminates

Laminates are thin multi-layered structures built by stacking plies on top of each other at particular orientations with respect to its global coordinate system, as shown in Figure 3. This building technique has its benefits and drawbacks. By stacking the plies at different orientations one is able to obtain a laminate with a strength and stiffness similar to the one of a ply in the longitudinal direction in more than one direction, making the FRPs actually useful as structural materials.

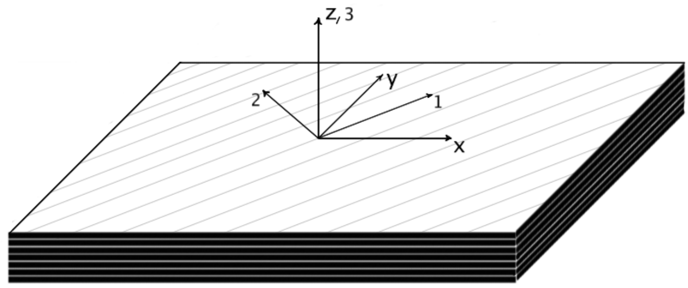

Figure 3. FRP laminate and the global (x,y,z) and local (1,2,3) coordinate systems.

Unfortunately, complications arise due to the transversely isotropic mechanical properties of the plies and the layered construction of the laminate. Chief among these complications is delamination. The mismatch between the Poisson's ratios and/or the extension-shear couplings of adjacent plies lead to interlaminar stresses. These stresses can relatively easily cause delamination between the plies since the bond is not reinforced; consequently the strength of the bond depends mostly on the fracture toughness of the plastic matrix. Delamination is not only dependent on fracture toughness the plastic matrix. It is also influenced by the stacking sequence of the laminate and the loading condition. The interlaminar stresses depend on the mismatch between the Poisson's ratios and/or extension-shear couplings of adjacent plies.

An additional complication in laminates is progressive matrix cracking. In a laminate, a number of matrix cracks may appear due to transverse thermal or mechanical stresses, in one or several of the plies. The presence of matrix cracks may not significantly affect the thermal and mechanical stiffness of the laminate or cause its immediate fracture (Nairn 2000). Plies with matrix cracks can still carry loads in the transverse direction since the load is transferred to the adjacent plies through interlaminar shear stresses before and after the matrix cracks. Matrix cracks do not cause immediate laminate fracture because the plies are bounded by other plies with different orientations, effectively arresting the crack propagation. The presence of matrix cracks, however, may lead to the initiation of other types of damage, such as delamination, or cause a substantial increase on the susceptibility of the laminate to moisture absorption and chemical degradation. If the load is increased, more matrix cracks will appear, and at certain crack density the thermal and mechanical stiffness of the laminate will then start to exhibit significant reductions.

Progressive matrix cracking is dependent on the material properties, the laminate stacking sequence and the thickness of the plies. If a ply, bounded only on one side, is subjected to an increasing transverse stress, matrix cracking will initiate earlier than if the ply was bounded in both sides. Furthermore, the stiffness of the bounding plies, in the direction normal to the crack surface, has also an effect on the initiation and development of matrix cracking.

Matrix cracks also develop when the laminate is subjected to an increasing shear stress. In the beginning micro-cracks in the matrix start to develop, and as the shear stress increases, the micro-cracks grow into full matrix cracks (Greve and Pickett 2006). A common test to measure the shear stiffness and strength of a ply is to apply a unidirectional load to a laminate with plies in the ±45° directions. The resulting stress-strain curve is non-linear due to the initiation and development of matrix cracks and the subsequent delamination. Both damage mechanisms are dependent on the stacking sequence and the plies thicknesses.

All these complications make laminate fracture a mixture of different types of damage. As mentioned before, the presence, initiation and development of these types of damage depends on the FRP material and the characteristics of the laminate. For example, a quasi-isotropic laminate has a balanced number of plies oriented in the 0°, ±45° and 90° directions, giving it plane quasi-isotropic mechanical and thermal stiffness. As shown in Figure 4, the failure of such a laminate under uniaxial tension is a mixture of matrix cracking, delamination and fibre fracture (Masters and Reifsnider 1982).

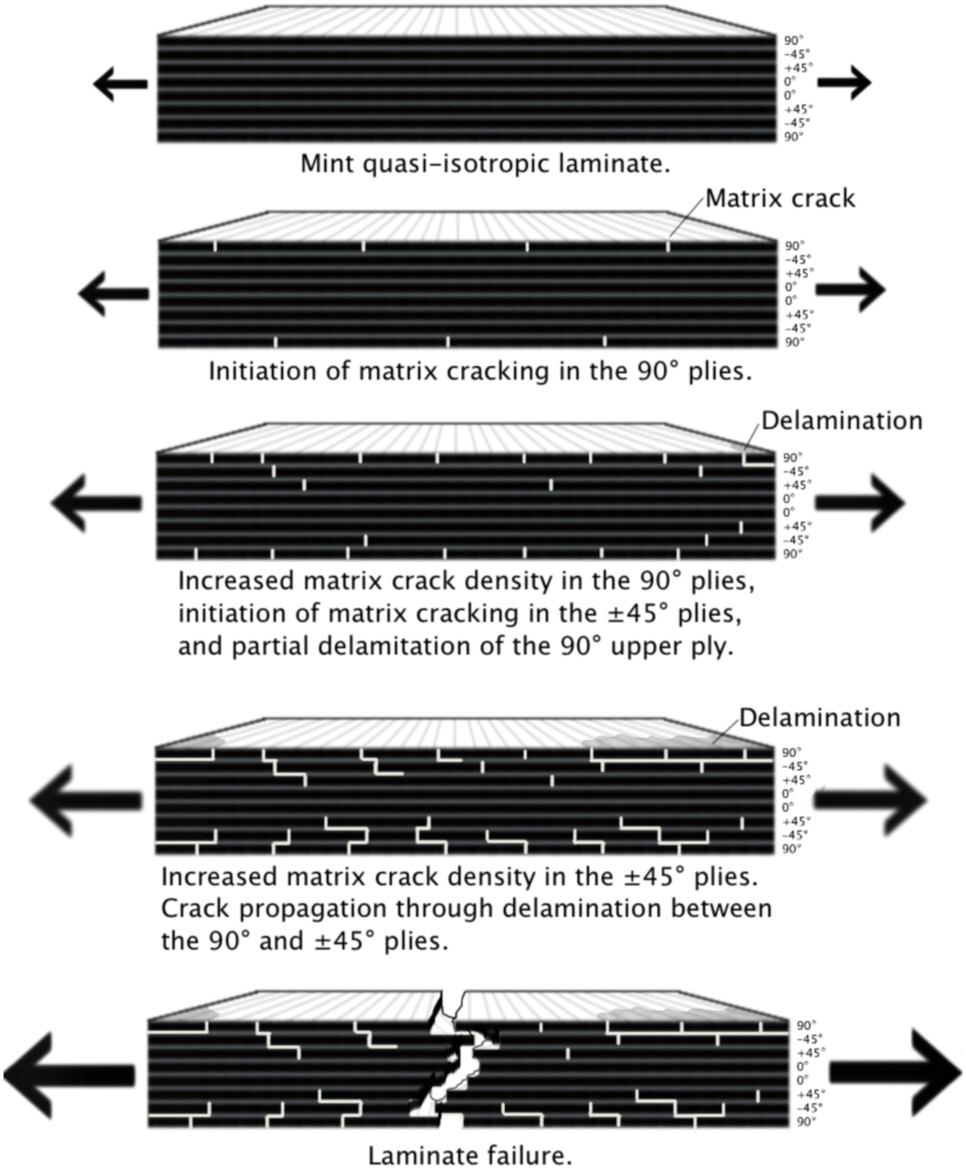

Figure 4. Degradation and failure of a FRP quasi-isotropic laminate under uniaxial tension.

Most commonly damage begins in the 90° plies with the initiation of matrix cracking, followed by matrix cracking in the ±45° plies and delamination. As the load is increased the number of matrix cracks and delaminated areas extend until the 0° plies fracture in the longitudinal direction, leading to overall laminate fracture. The fracture of the 0° plies is promoted by the progressive loss of carrying capacity of the ±45° and 90° plies due to matrix cracking and delamination, as well as the stress concentrations that these types of damage can generate.

It is not easy to define what constitutes 'laminate failure' considering the complex accumulation of damage that FRP laminates present when loaded. FRP failure was defined previously as the inability of the material to safely carry a load for an extended period of time. For a FRP ply, fibre fracture and matrix fracture constitute clearly ply failure, as the ply is split by any of those two failure modes. In contrast, some FRP laminates can sustain a large amount of matrix fractures in the form of matrix cracks, in addition to delamination, before overall laminate fracture. Consequently, it could be said that matrix cracking and delamination are not failure modes of FRP laminates, but material degradation. This statement is true if the only purpose of the FRP laminate is to carry a load, and if the presence of matrix cracks and delamination does not affect the capacity of the laminate of carrying a load safely for an extended period of time. Because of this last restriction it is tempting to consider a laminate as failed when it exhibits matrix cracks or delamination; however, this simplification could lead to a quite restricted use of the material since matrix cracking and delamination can initiate at stresses much lower than the ones where laminate fracture is observed.

3.1.4 Modelling

A very convenient and fairly accurate assumption is to consider that FRP plies present a linear elastic response to loading up to ply failure. This linear elastic assumption can be extended to laminates by using a lamination theory to estimate how each one of the plies contributes to the laminate stiffness and behaviour (Mallick 2008). The accuracy of a laminate linear elastic response model is questionable after the initiation of matrix cracking, since matrix cracks can reduce the mechanical and thermal stiffness of the laminate. To be on the conservative side, a laminate linear elastic response model with reduced matrix-dominated ply properties can be used (DNV 2010a). The reduction has the purpose of approximating the response of the model to the one of a degraded laminate. The ply matrix-dominated properties can be reduced in a variety of ways. The most conservative approach is to set the matrix-dominated properties of all the plies to 1% of their original value from the beginning. This approach considers that the laminate presents full degradation and that its plies are only capable of carrying a load in the fibre direction.

An approach closer to reality is to start the analysis with the non-degraded laminate linear elastic model and to progressively reduce the matrix-dominated properties of the plies as they start to exhibit matrix cracking. The difficulty of this approach is to determine when and how much should the properties be reduced. In the literature, laminate matrix cracking is most commonly considered to be a discrete fully developed event that occurs at certain stress states in the plies, fully or partially degrading their matrix properties (Garnich and Akula 2009). This consideration is somewhat in line with the observed behaviour of stand-alone plies, but not with the behaviour of plies embedded in a laminate. Most of the matrix failure criteria, used to predict matrix cracking, are not capable to account for the influence that the ply thickness and the adjacent plies have on the initiation of matrix cracking; therefore, their estimates of matrix cracking initiation are too conservative. For all these reasons, laminate response models with progressive degradation must be able to predict accurately the initiation and continuous development of matrix cracking, as well as its effects on the laminate properties.

Several analyses capable of such predictions (at least to some extent) have been proposed during the last 20 years. One of these analyses is the fracture mechanics variational analysis, capable of predicting the initiation and development of matrix cracking in cross-ply laminates by means of a fracture energy-based criterion (Nairn 2000).

4 Stochastic behaviour

The operational limits of a material cannot be determined without estimating its probability of failure under different loading conditions. The probability of failure of a material depends on the stochastic properties and the behaviour of the material itself, as well as the characteristics of the load it is subjected to, whether it is deterministic or stochastic. For example, the probability of failure of a bridge would depend on the stochastic properties of the material (stiffness and strength) and its construction (dimensions and tolerances), in addition to the maximum loading condition the bridge is subjected to, in this case a mixture of deterministic (maximum capacity of cars and trucks) and stochastic loads (wind and earthquakes). Such a bridge would be considered to be 'reliable' if its probability of failure was inline with the consequences of its failure. In this case the most prominent failure consequences would be the loss of human lives and infrastructure, therefore, a widely agreed acceptable failure probability for such consequences would be in the order of 10^{-6}. This work considers all the loads to be deterministic in order to simplify the estimation of the probabilities of failure.

4.1.1 Stochastic material properties

A stochastic material property (e.g. stiffness or strength) can be represented mathematically through a probability distribution (DNV 1992). Probability distributions assign a probability to each one of the possible values of the material property. Theoretically, the true probability distribution of a material property can be found through the testing of a large enough number of "identical" specimens; however, this method is never used due to the very large number of tests that would be required to confidently estimate the shape of the probability distribution, making the approach time and cost prohibitive. In practice, material properties are assumed to have a particular probability distribution that theoretically agrees with the property itself. For example, a chain is only as strong as its weakest link, therefore, following weakest-link theory, chain strength could be assumed to have a Weibull probability distribution. In many cases, however, it is not obvious, or clear, which probability distribution agrees with a property, leading to an uncertainty on the choice of probability distribution.

Depending on the type of probability distribution, one or more parameters might be needed to characterize it, for example, the Weibull distribution requires two parameters, the scale and the shape parameter. These parameters are usually determined by fitting the assumed probability distribution to a set of experimental observations. One problem with this methodology is that the limited number of experimental observations gives no information about the shape of the tails (the least frequent occurring values) of the true probability distribution; therefore, the fit of the assumed distribution might be correct in the mode of the distribution (the most frequently occurring values) but incorrect in the tails, leading to an uncertainty on the fit of the assumed probability distribution tails. This uncertainty is extremely important on the estimation of probabilities of failure. Very low probabilities of failure, like the ones used to define operational limits, depend on the tail values of the probability distributions of the material properties.

4.1.2 Theoretical probability distributions for FRPs

FRPs are usually considered to present a failure scatter considerably larger than other materials, like for example steel or aluminium. A large failure scatter in FRPs seems reasonable if one considers the large number of damage mechanisms that these materials can exhibit. The initiation and development of these damage mechanisms depend on a large number of factors, some of which are innate to the fibres and matrix, and some of which are created during the manufacturing of the laminate. Micro damage in the fibres, voids and inclusions in the matrix, residual stresses, local high or low concentrations of fibres in the plies, uneven curing of the matrix and many more imperfections and characteristics can cause stochastic behaviour. One possible approach for estimating the stochastic behaviour of a FRP would be to estimate the probability distribution of these imperfections and their effects; however, a much more practical one is to estimate the probability distribution of more measurable FRP material properties, such as ply stiffness and strength.

The stiffness of a FRP ply can be considered to be the average stiffness of a large number of microelements conforming the material itself; therefore, following the central limit theorem, the material stiffness can be considered to have a normal probability distribution. In a similar fashion, the Poisson's ratios and thermal expansion coefficients can also be considered to follow a normal distribution. The probability distribution of the strength of a FRP is most likely dependent on the loading direction. The transverse strength of a ply can be considered to follow a Weibull distribution. Matrix fracture is a brittle event, governed by the size of the largest imperfection (in a sense, the weakest link). On the other hand, the probability distribution of the ply strength in the longitudinal direction is a very uncertain issue (Alava et al. 2006 and Sutherland and Soares 1997). The strength of an individual fibre follows a Weibull probability distribution, while the strength of a bundle of fibres does not. For a bundle of fibres, where each fibre carries an equal fraction of the load, the fracture of one fibre does not lead to bundle fracture, but to an equal redistribution of the load carried by the fractured fibre. Statistical models predict that the strength of such a fibre bundle system will have a normal distribution; however, in a FRP ply the fibres do not carry the same load fraction and upon fracture of a fibre the load is not equally distributed among all the other fibres. As pointed out in subsection 3.1.2, ply fracture in the fibre direction under tension is a complex process and to the author's knowledge, there are no satisfactory statistical models that can predict its probability distribution.

4.1.3 Estimation of the stochastic behaviour of FRP laminates

To be able to estimate the stochastic behaviour of a FRP laminate it is necessary to account for the stochastic properties of the FRP plies that constitute the laminate, as well as the stochastic deviations from the ideal FRP laminate construction, mainly variations on the thicknesses and orientation angles of the plies. Due to the large number of stochastic properties that influence the behaviour of the laminate, and the complexity of some of the laminate response models, it is difficult to estimate the stochastic behaviour of a laminate through analytical methods. The Monte Carlo method is a numerical algorithm capable of estimating the stochastic behaviour of complex mathematical models that depend on large numbers of stochastic input variables.

The method utilizes simulations to characterise the stochastic output of the model. In each simulation a set of randomly sampled stochastic input variables is evaluated, giving one of the possible stochastic output values.

Figure 5 shows an example of the Monte Carlo simulation used to estimate the stochastic matrix cracking behaviour of the plies of a laminate. The red dots in the input variables would be randomly sampled values, while the ones in the output would be matrix cracking strains predicted by the deterministic model with those input values. With a large number of simulations the probability distribution of the output can be characterized. The main drawback of the Monte Carlo method is that it requires very large number of simulations to accurately characterize the tails of the output probability distribution of the model. A rule of thumb states that to estimate accurately the probability P that an output variable will be smaller or equal to a certain value, the minimum number of simulations should be $N = 100/P$. For example, to have accurate estimations at a probability $P = 1/10^6$, the number of Monte Carlo simulations needed would be $N = 10^8$. This requirement limits the use of the Monte Carlo method to models that can be evaluated in a sufficiently small amount of time.

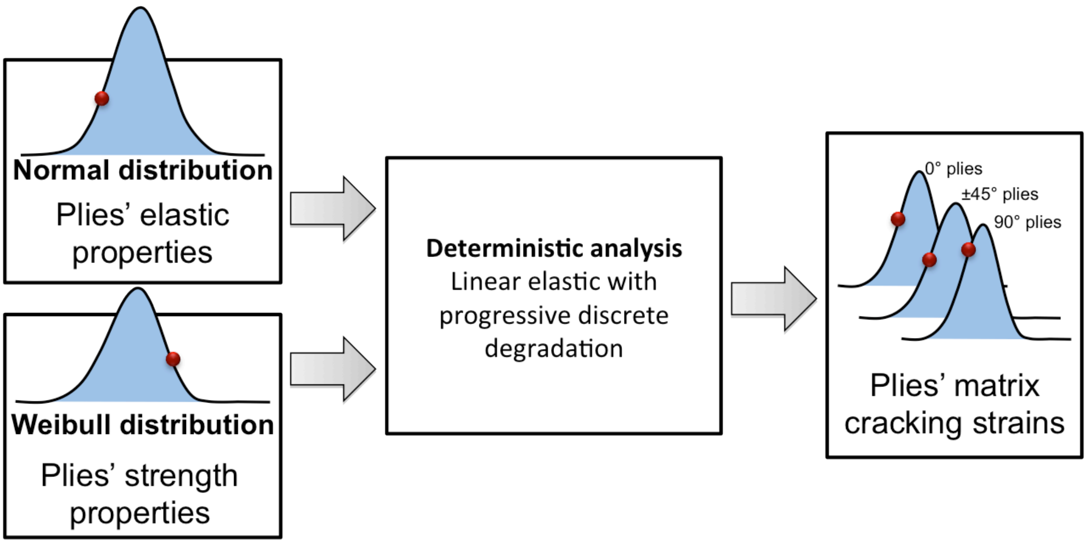

Figure 5. Example of a Monte Carlo simulation. Randomly sampled stochastic input for a deterministic model gives a particular output, the repetition of this process allows the estimation of the stochastic response of the model.

5 Methodology

This section contains a brief overview of the methodology used in this work. The purpose of this overview is to emphasize and clarify the comparisons made in the appended papers I to III so as to fulfil the established objective.

The work in the appended papers utilizes four laminate response models for estimating the operational limits of FRP laminates. Different assumptions and complexity levels are used in the laminate response models so as to determine the importance of such material modelling considerations. The four laminate response models are:

- **Linear elastic model (Papers I, II and III)**
  - The laminate is considered to have a linear elastic response up to failure.
  - The degradation effects of matrix cracking and delamination are neglected.
  - First ply failure on the longitudinal direction is considered as laminate failure.
- **Linear elastic with full degradation (Paper III)**
  - The laminate is considered to have a linear elastic response up to failure.
  - The laminate is fully degraded due to matrix cracking. The plies are only capable of carrying loads in the fibre direction; therefore, the matrix-dominated thermal and mechanical properties are set to 1% of their original value.
  - First ply failure on the longitudinal direction is considered as laminate failure.
- **Linear elastic with progressive discrete degradation (Paper I)**
  - The laminate is considered to have a linear elastic response up to matrix cracking in one of the plies.
  - Matrix cracking is considered to be a sudden and fully developed event, it occurs only once in each ply.
  - Ply matrix cracking is predicted with a stress-formulated criterion, the embedded ply is assumed to crack at the same stress as a stand-alone ply.
  - Matrix cracking in a ply degrades its matrix-dominated thermal and mechanical properties to an amount determined by the reduction factors.
  - The reduction factors are calibrated against experimental laminate stress-strain curves.
  - The degradation effects of delamination are neglected.
  - First ply failure on the longitudinal direction is considered as laminate failure.

- **Linear elastic with progressive continuous degradation (Papers II and III)**
  - The laminate is considered to have a linear elastic response up to matrix cracking in one of the plies.
  - Matrix cracking in a ply is considered to be a continuous and accumulative event.
  - Ply matrix cracking initiation and development is predicted with a fracture energy formulated criterion, capable of accounting for the effects that ply thickness and adjacent plies have on the initiation and development of matrix cracking.
  - Matrix cracking degrades continuously the thermal and mechanical properties of the laminate.
  - The degradation effects of delamination are neglected.

First ply failure on the longitudinal direction is considered as laminate failure, and therefore, as the Ultimate Limit State (ULS) of all the laminates in all the analyses. Due to the lack of own experimental measurements, all material properties, stress-strain and average crack density vs. laminate stress curves were obtained from a literature survey.

Several comparisons, between operational limits calculated with different probability distributions for the FRP ply properties, are used to determine the effect that such assumptions, regarding probability distributions, have on the estimation of the operational limits.

In Paper I the analyses were carried out with the finite element software ANSYS (ANSYS Inc. 2009). The laminate response models and failure criteria were incorporated to the software by means of macros written in ANSYS Parametric Design Language. For the Monte Carlo simulations the probabilistic design module incorporated in ANSYS was used. In Papers II and III all the analyses were carried out in MATLAB (MathWorks Inc. 2011) in order to shorten the calculation time.

6 Summary of the work in the appended papers

The work presented in the appended papers deals with current maratime industry practice and modern theoretical developments, in a sense it explores the possible benefits of bringing the industry up to date. On the most basic level the appended papers attempt to answer two main questions:

Papers I and II:

Can a more accurate estimation of the operational limits motivate a higher utilization of FRP laminates compared to current practice?

Paper III:

How much does the matrix cracking influence the estimation of operational limits for FRP laminates?

In the following, a brief summary and results of each paper are presented. The summaries focus on the description and the conclusions of each paper, as an attempt of presenting as clear as possible the main scientific contributions of this thesis.

6.1 Paper I: Optimization of composite maritime structures – effects of uncertainties on design criteria and limits

This paper assesses through deterministic and probabilistic analyses the benefits and drawbacks of the linear elastic and the linear elastic with progressive discrete degradation models. Both the deterministic and the probabilistic analyses utilize a quasi-isotropic carbon epoxy laminate, subjected to a tensile biaxial loading as a study case. The deterministic analysis compares the laminate response predicted with both models against experimental measurements and maximum allowable strains determined by design rules. The probabilistic analysis compares the stochastic matrix cracking response, predicted with the linear elastic with stochastic degradation model, against the maximum allowable strains.

It is clear from the deterministic analysis, as shown in Figure 6, that between the two models, the linear elastic with progressive discrete degradation model is capable of predicting a stress-strain response closer to the experimental measurements, however, this model has several flaws and problems. First, matrix cracking seems to be under predicted in all the plies, causing a mismatch between the experimental and the predicted stress-strain curves. Second, this mismatch occurs close to the maximum allowable strains, exactly where the model should be accurate. The probabilistic analysis reaffirms this problem by showing that the model predicts that matrix cracking will most definitely occur in several of the plies before the maximum allowable strains are reached.

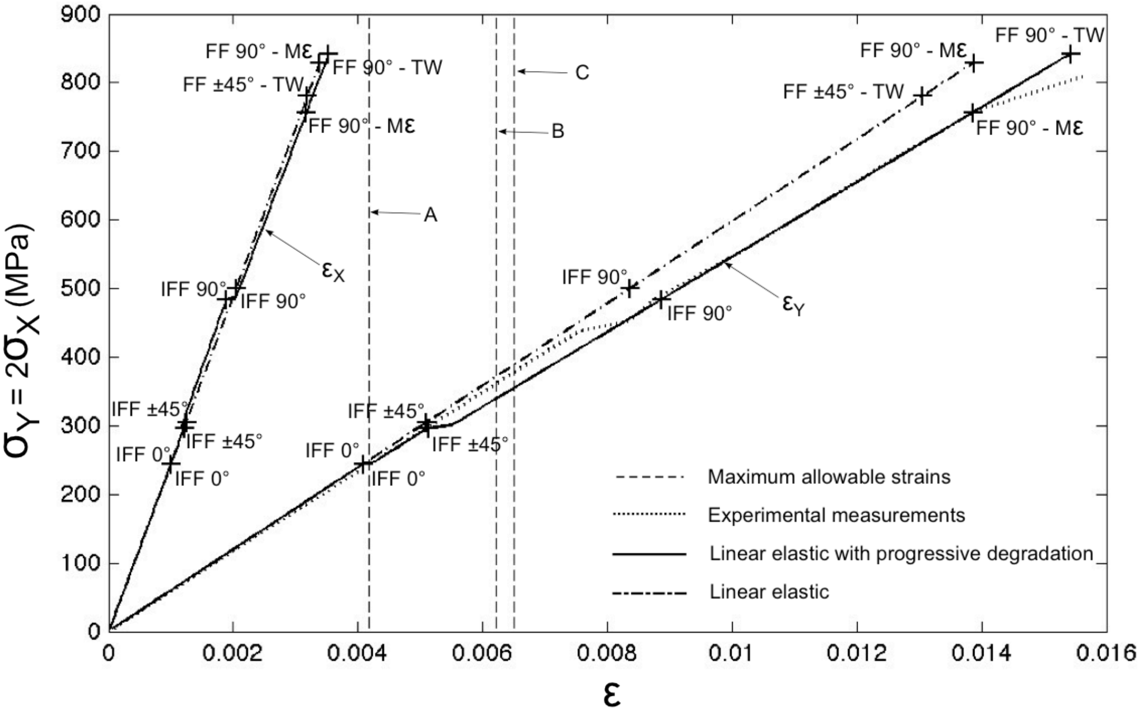

Figure 6. Stress-strain response of a quasi-isotropic carbon/epoxy laminate to monotonic biaxial tensile loading. (IFF: Inter Fibre Fracture (matrix cracking), FF: Fibre Fracture, Mε: maximum strain failure criterion, TW: modified Tsai-Wu failure criterion, A: maximum allowed strains as stated in DNV 2010b, B and C: maximum allowed strains as stated in DNV 2010a; for more details, see Paper I).

The main conclusion of the assessment is that the linear elastic with progressive discrete degradation model provides no benefits when compared to the linear elastic model. The progressive discrete degradation model just ads complexity and uncertainty to the structural analysis, and cannot be used to motivate an increase of the maximum allowed loads.

6.2 Paper II: Study on the possibility of increasing the maximum allowable stresses in fibre-reinforced plastics

In this paper, the operational limits of a group of cross-ply laminates are estimated with two methodologies: through probabilistic analyses using the linear elastic with progressive *continuous* degradation model, and via deterministic analyses using the linear elastic model with safety and model factors as stated by design rules. Both sets of operational limits are compared to determine if the operational limits estimated with the design rules methodology are too conservative, and therefore, if higher operational limits can be motivated. Additionally, the operational limits are compared against probabilistic measurements of matrix cracking initiation and development (see Figure 7), so as to qualitatively evaluate the suitability of such operational limits. An uncertainty analysis is performed to determine how does the type of probabilistic distributions of the stochastic properties, used in the probabilistic analyses, affect the estimation of the operational limits.

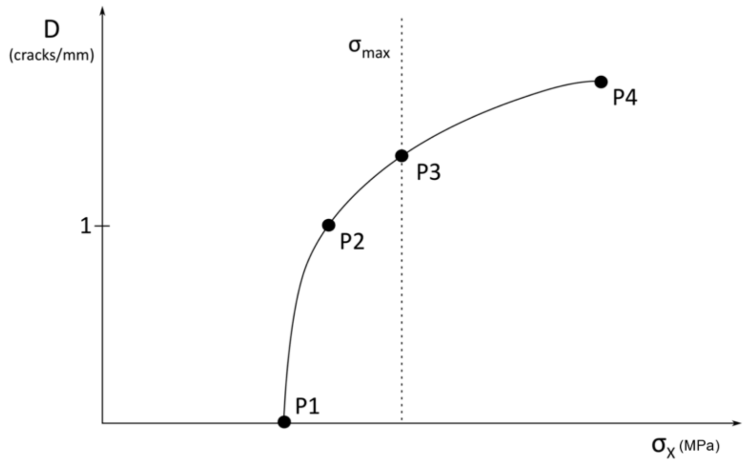

Figure 7. Measurements of interest in the average crack density vs. longitudinal stress curve of cross-ply laminate. (P1: stress at matrix cracking onset, P2: stress at matrix cracking density equal to 1 crack/mm, P3: crack density at the maximum allowed longitudinal laminate stress, P4: stress and crack density at the ULS.)

The investigation concludes that the operational limits estimated with the design rules methodology are unlikely to provide the desired safety level. This conclusion is based on two main observations. First, depending on the type of distribution chosen for the ultimate longitudinal tensile strength, the operational limits calculated by means of probabilistic analyses can be higher or lower than the ones calculated through the design rules methodology as shown by the two types of boxplot in Figure 9 (for a description of the boxplots, see Figure 8). Since none of the distribution types can be considered right or wrong, the operational limits estimated with the design rules methodology are in an uncertainty region. Second, the linear elastic with progressive continuous degradation analyses show that the cross-ply laminates are very likely to present considerably high matrix crack densities at the operational limits. The effects of such high crack densities, beyond the reduction of the thermal and mechanical stiffness of the laminates, are not considered, and therefore, the estimated probabilities of failure at the operational limits are not conservative.

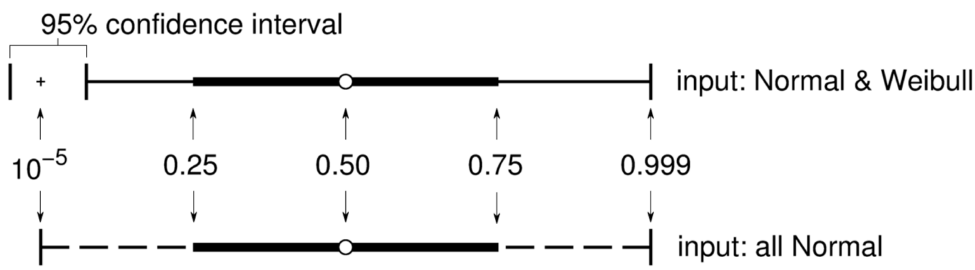

Figure 8. Boxplots are used to describe the shape of the distributions of the stochastic model responses. The left and right whiskers indicate the quantiles where the state has a probability of occurrence equal or less than 10^{-5} and 0.999, respectively. The sides of the box indicate the 0.25 and 0.75 quantiles while the white circle marks the location of the median.

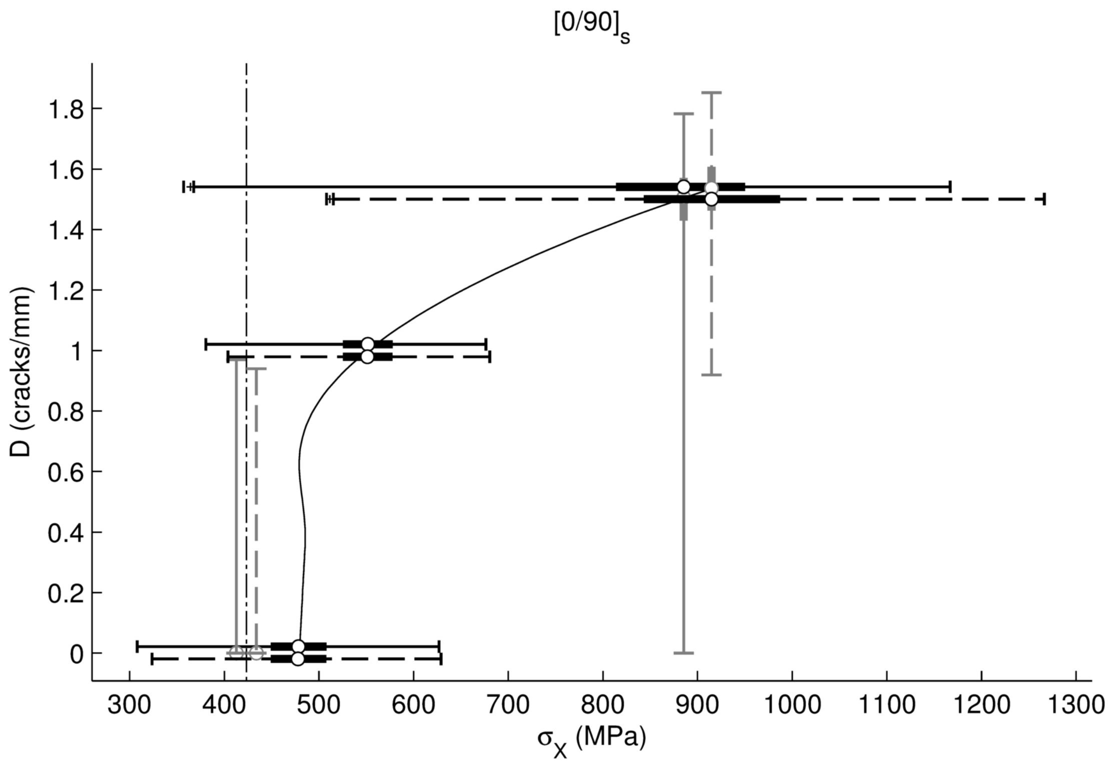

Figure 9. Comparison between the deterministic average crack density vs. laminate stress curve of a $[0/90]_S$ carbon/epoxy cross-ply laminate, and the stochastic response of the measurements of interests indicated in Figure 8.

6.3 Paper III: Effects of matrix cracking on the estimation of operational limits of FRP laminates

In this paper, the operational limits of a group of carbon epoxy and glass epoxy cross-ply laminates are estimated through probabilistic analyses using different laminate response models. The aim is to estimate the effects that matrix cracking and its modelling technique have on the estimation of the operational limits. Three laminate response models are used: linear elastic, linear elastic with progressive continuous degradation, and linear elastic with full degradation. The investigation includes an uncertainty analysis to determine the effects that the types of distribution used for the stochastic properties have on the estimation of the operational limits.

The investigation concludes that the importance of the matrix cracking and its modelling technique depends on the FRP material. For the carbon epoxy laminates the different models gave similar operational limits, as seen in Figure 10, since matrix cracking does not reduce significantly the laminate stiffness. Therefore, the linear elastic model with full degradation is considered to provide a conservative estimation of the operational limits, without significantly underestimating the material's usability. The glass fibre epoxy laminates, on the other hand, present a far more significant reduction of laminate stiffness due to matrix cracking. The linear elastic model with full degradation will most likely lead to a very conservative design with poor material utilization.

For all the operational limits, the laminates are predicted to present high crack densities, and therefore, the laminate's tolerance to them must be investigated. In addition to these conclusions the investigation points out that the uncertainty on the true shape of the distribution of the longitudinal tensile strength has a larger effect on the estimation of the operational limits than the reduction of laminate stiffness due to matrix cracking.

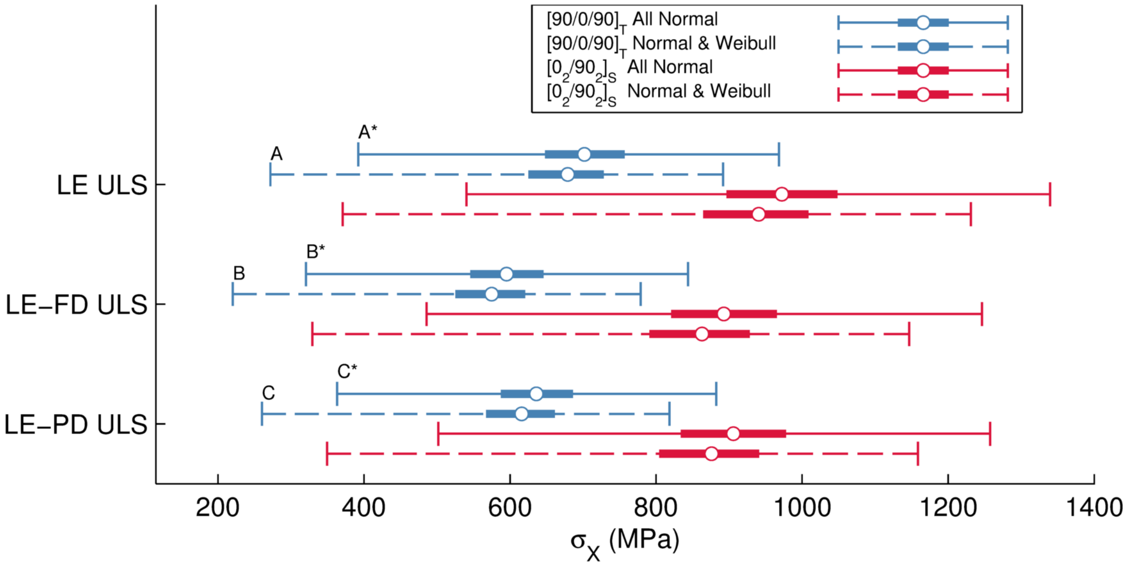

Figure 10. Comparison of ULS stochastic predictions for the $[90/0/90]_T$ & $[0_2/90_2]_S$ carbon/epoxy laminates with different laminate response models (LE: linear elastic, LE-FD: linear elastic full degradation, LE-PD: linear elastic with progressive continuous degradation, the boxplots follow the description presented in Figure 8 without the confidence interval; for more details, see Paper III).

7 Conclusions

The objectives of this work were:

- To contribute to the body of knowledge regarding the estimation of operational limits for FRP laminates.

  How much does the matrix cracking influence the estimation of operational limits for FRP laminates?

- To investigate the possibility of improving the utilization of FRP laminates intended for marine structures through a more accurate estimation of its operational limits.

  Can a more accurate estimation of the operational limits motivate a higher utilization of FRP laminates compared to current practice?

Overall, the work presented in three appended papers lead to three main conclusions:

1. The uncertainty on the true shape of the probabilistic distribution of the ultimate longitudinal tensile strength is the most important parameter on the estimation of operational limits for FRP laminates through probabilistic analyses.

2. FRP laminates made out of carbon epoxy or glass epoxy are likely to present high crack densities at their operational limits. Better understanding of the short and long term effects of high crack densities is needed in order to define a 'maximum allowable crack density' and guarantee a reliable design.

3. The importance of laminate response modelling depends on the FRP material. Carbon epoxy laminates can be evaluated with a linear elastic model with full degradation without obtaining too conservative estimations. Glass epoxy laminates are much more sensitive to the laminate response modelling and could benefit of a progressive continuous matrix cracking model.

The investigations presented in the appended papers do answer at some extent the questions posed in the objectives. The effects of matrix cracking on the estimation of operational limits due to the redistribution of loads were assessed. Also, the investigations showed that laminate response models with increased accuracy through the incorporation of the effects of matrix cracking cannot motivate higher utilization of FRP laminates compared to current practice; however, they do point out the most important uncertainties that influence the estimation of operational limits. Future analyses dealing with these uncertainties appropriately might motivate a higher utilization of FRP laminates.

8 Future work

Non-Crimp Fabrics

Non-Crimp Fabrics (NCFs) are increasingly becoming the most widely used type of FRP for vehicles. Even though prepregs are fundamentally similar to NCFs, their differences might be large enough to partially change the conclusions presented in this thesis. Future investigations will characterize NCFs and study their behaviour so as to determine if the methodology here presented requires adjustments and improvements. The uncertainties influencing the most the estimation of the operational limits will be investigated thoroughly with the intent of maximizing the utilization of the material.

Guidelines for optimizing FRP structures to goal-based standards

One ambition of the author is to create guidelines for optimizing FRP structures with respect to safety, weight, cost and environmental impact, following goal-based standards (DNV 2010a). The work presented in this thesis contributes mostly to the optimization with respect to safety and weight. Future work will aim at simplifying as much as possible the estimation of operational limits through probabilistic analysis.

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