ICE FORCES
ON CYLINDRICAL AND CONICAL COLUMNS

by

Stefan Karlsson and Peter Strindö
PREFACE

This diploma work (MS dissertation) was performed in the Department of Hydraulics at Chalmers University of Technology, in cooperation with Götaverken Arendal AB, Gothenburg.

We wish to thank all the staff of technical advisers in the Section of Hydrodynamics of Götaverken Arendal for spending a lot of energy on time consuming discussions with us, the undersigned.

Furthermore, we would like to give special thanks to

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- Lars Bergdahl (CTH) who was our tutor at Chalmers.

Gothenburg in March 1985

Stefan Karlsson  Peter Strindö
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SUMMARY

The purpose with the present work is to develop a computer program for calculation of ice forces on offshore structures.

Our work is based upon test reports from Wärtsilä Arctic Research Center. Proceeding from these reports and various theoretical models we have developed a program which corresponds to the tests carried out by the WARC.

The work consists of two parts:

- Literature studies. (Theoretical studies.)
- Computer program with a manual.

All the calculation methods applied in this context are based on the condition that the structure is fixed and rigid. Three different main types of ice - level ice, pack ice and rubble fields. Furthermore different calculation methods are required for vertical and conical structures respectively. The results deriving from the various calculation methods show fairly large divergencies.

On account of this the reliability of these methods is not to be overestimated.
1. Introduction

To facilitate the extension of offshore activities to colder latitudes with the predominance of ice it is of the greatest importance to estimate the forces on the structures.

The difficulty in estimating these forces are obvious since the properties of ice are very varying and the calculation methods differ.

In this work we have applied some of the best known theories.

On the basis of the most valid theories described in our literature we have compiled a computer program.

Our literature studies mainly refer to a report by P. Trägårdh and B. Forsman /37/. The chapter on ice properties is from "Mechanical properties of sea ice" by F.U. Häusler /38/.
2 Different Methods to Calculate Ice Forces

The methods reviewed apply to the calculation of forces exerted by drifting ice on single vertical fixed cylinders. The methods are divided into three groups with respect to different types of ice. Arctic ice conditions comprise more than these three types and various combinations are also frequent. The ultimate ice load on a cylindrical leg is, however, assumed to be exerted by one of these three types. For real offshore structures the ice usually interacts with more than one cylindrical element and some particular problems concerning multi-columned structures are briefly dealt with.

2.1 Ice Forces on Cylinders

Although level ice is not very frequent in the Arctic, most ice model tests are made in such ice. Level ice is easy to define in theoretical analyses and to simulate in experiments and therefore provides a good basis for further studies. The force from an ice sheet on a cylinder is limited by the pressure that breaks the sheet. Two different failure modes are possible - crushing or buckling.

Crushing

The crushing force, \( F \), is determined by the size of the contact area and the crushing strength

\[
F = D \cdot H \cdot \sigma_{cr} \quad (1)
\]

where

\[
D = \text{cylinder diameter}
\]

\[
H = \text{ice thickness}
\]

\[
\sigma_{cr} = \text{crushing strength of the ice}
\]

The crushing strength is not related to a well-defined stress state and different investigators suggest different expressions for \( \sigma_{cr} \) as a function of the uniaxial compressive strength, ice thickness, cylinder diameter and relative velocity. The following general expression is useful for comparisons of the proposed methods:

\[
\sigma_{cr} = C \cdot m \cdot K \cdot \sigma_{cx} \quad (2)
\]

where \( \sigma_{cx} \) is the maximum uniaxial compressive strength of the ice with respect to the strain rate.
C-Indentation factor

C depends on the multiaxial stress state in the ice when it is crushed. The coefficient indicates the ratio of the crushing strength at indentation to the uniaxial compressive strength for the corresponding ice:

\[ C = \frac{\sigma_{Cr}}{\sigma_c} \]  

(index i - indentation) \hspace{1cm} (3)

In seekoo C is fixed to 3 corresponding to the test results, or C could be calculated by the formula

\[ C = \left( \frac{5 \cdot H}{D} + 1 \right)^{0.5} \hspace{1cm} /31/ \] \hspace{1cm} (4)

Based on von Mises yield criterion Michel has derived

\[ C = 2.97 \text{ for } \dot{\varepsilon} < \dot{\varepsilon}_0 \] \hspace{1cm} (5)

Other investigators have shown test results where C depends on D and H. Korzhavin was the first to present empirical indentation data and he obtained C equal to 2.5 for D/H = 1. For wider structures (higher D/H) C tends to 1.0. Hirayama et al /14/ have presented empirical formulas where

\[ C = \text{const.} \cdot D^{-0.32} H^{0.1} \] \hspace{1cm} (6)

Saeki et al /30/ have made tests in natural sea ice and derived an expression where

\[ C = \text{const.} \cdot D^{-0.5} \] \hspace{1cm} (7)

Michel & Toussaint /21/ claim that the D-dependencies are caused by changed strain rate (defined by /18/) when D was varied at the tests. Hirayama et al's test results have also been used to derive the expression

\[ C = 2 \left( 1 + \left( \frac{D}{H} \right)^{-0.6} \right) \hspace{1cm} /15/ \] \hspace{1cm} (8)
Natural ice is anisotropic and the strength properties vary with the hydrostatic pressure and differ in tensile and compression. This implies that the von Mise's yield criterion cannot describe the plastification of ice correctly. Based on triaxial compression tests Reinecke & Remer /28/ have derived yield criterion for isotropic ice. When an ice edge is crushed by a flat indentor $D/H\rightarrow\infty$ corresponds to a state of plane stress and $D/H\rightarrow0$ corresponds to plane strain. According to Reinecke & Remer an upper limit for the crushing strength at plane stress is given by

$$C = 1.4 + 0.97(D/H)^{-1}$$  \hspace{1cm} (9)

and at plane strain by

$$C = 5.13$$  \hspace{1cm} (10)

Ralston /27/ has used strength data from anisotropic fresh water ice in a generalized von Mise's yield criterion and obtained the following C-values:

<table>
<thead>
<tr>
<th></th>
<th>plane strain</th>
<th>plane stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>upper limit</td>
<td>4.47</td>
<td>3.28</td>
</tr>
</tbody>
</table>
| lower limit    | 3.77         | 2.98         |  \hspace{1cm} (11)

Croasdale /7/ has presented another theoretical plastification analysis for indentation. It is based on a simple Tresca yield criterion and in a simplified form an upper limit for C can be expressed by

$$C = 1 + 0.304(D/H)^{-1}$$  \hspace{1cm} (12)
A comparison of the various values and expressions of C is shown in Fig. 1.

Fig. 1. Stress state coefficients versus aspect ratio
m -shape coefficient

The value of $m \leq 1$ and depends on the shape of the body that interacts with the ice.

When an ice sheet is continuously crushed against a cylinder, the pressure varies over the contact zone and the ice is not crushed simultaneously over the whole surface. Therefore it is usual to base empirical force prediction methods on indentation tests, where a punch indents a flat ice edge.

In this case the pressure is distributed equally over the contact area when crushing is initiated. The indentation force depends on the shape of the indentor and with $m = 1$ for a flat indentor the following values have been obtained:

<table>
<thead>
<tr>
<th>Indentor shape and corresponding $m$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Indentor shape" /></td>
<td><img src="image2" alt="Indentor shape" /></td>
</tr>
<tr>
<td><img src="image3" alt="Indentor shape" /></td>
<td><img src="image4" alt="Indentor shape" /></td>
</tr>
</tbody>
</table>

At continuous crushing - or penetration - which is more relevant to application on arctic offshore structures, the shape-dependence is harder to identify and it is probably less than at indentation /9/. Thus $m = 1$ will be used in the following calculation of penetrating cylinders. Therefore Seekoo is used with $m = 1$. 
K - contact coefficient

Due to incomplete contact the ice force at continuous crushing or penetration is less than at indentation. \( K \) indicates the portion of the surface \( D \cdot H \) where ice and structure are in contact during crushing.

In Seekoo the value for the contact coefficient consists of two components:

\( K_1 \) depends on strain velocity

\( K_2 \) depends on width of the structure if the strain rate is high enough.

For \( K_1 \) following strain rate dependence has been used: /16/

\[
K_1 = 1 \text{ when } \dot{\varepsilon} < 5 \cdot 10^{-4} \text{s}^{-1} \\
K_1 = 0.3 \text{ when } \dot{\varepsilon} \geq 5 \cdot 10^{-4} \text{s}^{-1}
\]
The component $K_2$ is also dependent on strain rate as well as on the width of the structure: /16/

$K_2 = 1$ when $\dot{\epsilon} < 5 \cdot 10^{-4}\text{s}^{-1}$ and is given by the function in Fig.2 for $\dot{\epsilon} > 5 \cdot 10^{-4}\text{s}^{-1}$.

Fig.2 $K_2$ as function of D/H ratio when $\dot{\epsilon} > 5 \cdot 10^{-4}$.
According to Michel & Toussaint /21/ K depends on the strain rate. In the ductile range \( (\varepsilon < \varepsilon_0) \) \( K \sim 0.6 \) and for brittle failure \( K \sim 0.2 - 0.3 \). Bruen et al /5/ have studied the local contact conditions for ice-structure interaction and note that the contact coefficient increases with increasing temperature. This temperature dependence may be stronger than the weakening of the ice with increasing temperature and hence the global ice load may be greater in warm ice than in cold.

Hirayama et al's test results have been used to show that K decreases with increasing size of the surface \( D \cdot H \).

\[
K = \left( \frac{D \cdot H}{A_0} \right)^{-0.165}
\]  

(13)

where \( A_0 = 0.35 \text{ cm}^2 \) is a reference area determined by the ice crystal size.

The model tests referred to have all been made with a ratio of structure diameter to ice crystal diameter >10 and thus the crushing strength has not been affected by scale effects due to lack of similitude of crystal size in model and full scale. Kry /18/ has performed two similar penetration tests in two model scales, which differed by a factor ten. The results indicate that the ice failure pressure decreases as the scaling factor tends to one.

Model tests made in different scales and in fresh water or saline ice all show the same tendencies and agree reasonably well with theoretical analyses. Only a few field measurements of forces exerted by arctic ice are available for comparison but they indicate that the basic formula (1) and (2) are useful for the prediction of full scale crushing forces.

The last factor in the pressure formula is the compression strength of ice can be stated as follows:

\[
\sigma_{cx} = \kappa \cdot \sigma_{cx}^{	ext{max}}
\]  

(14)

where \( \kappa \) is a strain-rate-dependent strength factor. 

\( \sigma_{cx}^{	ext{max}} \) is the maximum compression strength of one year old arctic sea ice. The standard value of \( \sigma_{cx} \) in Seekoo is 3.5 MPa.
\( \alpha \) - strain rate coefficient

The ice strength depends on the strain rate \( \dot{\varepsilon} \). \( \alpha \) is the ratio of the compressive strength at a certain \( \varepsilon \) to the maximum compressive strength with respect to \( \dot{\varepsilon} \).

\[
\alpha = \frac{\gamma_c}{\gamma_{cX}}
\]  
(15)

In Seekoo \( \alpha \) is calculated according to Michel /20/

\[
\alpha = \left[ \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right]^{0.32} \quad \text{for} \quad \dot{\varepsilon} \leq \dot{\varepsilon}_0
\]  
(16)

which is also described in Fig. 3.

Fig. 3. \( \alpha \) as function of \( D \) and \( V \).
$\dot{\varepsilon}_0$ is the strain rate that causes the highest compressive strength, $\sigma_{cx}$. Fig. 4.

![Graph showing uniaxial compressive strength of S2 ice (lake ice with horizontal c-axes) at -10°C](image)

For $\dot{\varepsilon} > \dot{\varepsilon}_0$ the fracture becomes brittle and $\sigma_{c} < \sigma_{cx}$ and independent of $\varepsilon$. In most cases of structure - ice interaction it is supposed that $\dot{\varepsilon} > \dot{\varepsilon}_0$. If, however, the ice sheet is stopped, $\dot{\varepsilon}$ will be equal to $\dot{\varepsilon}_0$ at a certain moment and when design is concerned $\varepsilon$ is to be taken equal to 1.

In uniaxial compression tests the strain rate is defined by

$$\dot{\varepsilon} = \frac{V}{a} \quad (17)$$

where

$V$ = relative velocity and

$a$ = height of the specimen
When tests are made on ice sheets it is more difficult to identify the size of the deformed area and various definitions of the strain rate have been proposed. For example, Michel /20/ used

\[ \dot{\varepsilon} = \frac{V}{4D} \]  

(18)

and in a study by Exxon

\[ \dot{\varepsilon} = \frac{V}{2D} \]  

(19)

was used. Hirayama et al /14/ define \( \dot{\varepsilon} \) from

\[ \dot{\varepsilon} = 3.05 \cdot V^{1.45} \cdot D^{-0.25} \cdot H^{-0.60} \]  

(20)

The various definitions complicate comparisons between the methods.
Buckling

As regards vertical cylinders we also have to take into consideration that ice in some cases buckles, especially, when the ice thickness is small compared to the width of the structure. The possible failure mode of buckling in an ice field depends also on the type of contact between the ice field and the structure. Buckling is very sensitive to all kinds of irregularities in the ice sheet, and the buckling force has been observed to be less than theoretical calculations would indicate.

The pressure corresponding to the buckling of an infinitely wide ice sheet is:

\[ \frac{p}{\rho_0} = \frac{\sqrt{S \cdot g \cdot D_f}}{H} \]  \hspace{1cm} (21)

according to Wang /33/

where \( D_f \) is the flexural rigidity

\[ D_f = \frac{E \cdot H^3}{12(1 - \nu^2)} \]  \hspace{1cm} (22)

For a structure the buckling load depends on the boundary condition which is somewhere between frictionless and clamped condition. For frictionless boundary the nominal buckling pressure can be estimated with the following equation:

\[ \frac{p}{\rho_0} = 1 + \frac{3.32}{\frac{D}{1} + \frac{1}{4} \left( \frac{D}{1} \right)^2} \]  \hspace{1cm} (23)

according to Sodhi & Hamza /32/

where \( l \) is the characteristic length of the ice plate.

\[ l = 4 \sqrt[4]{\frac{D_f}{S_g}} \]  \hspace{1cm} (24)

The relation between \( p/\rho_0 \) and \( D/l \) is shown in Fig. 5.
The buckling force will be

\[
F = \left[ 1 + \frac{3.32}{D} \left( \frac{D}{1 + \frac{1}{4}D^2} \right) \right] \cdot p_0 \cdot H \cdot D
\]  

(25)
2.2 Theory for Conical Collars

For many reasons the ice forces might reach unacceptably high levels and reduce the operability of the installation. In order to avoid this or even increase the operability there are different methods of reducing ice forces. The most efficient method is to fit a cone or make the leg conical instead of cylindrical.

In the following pages there is a brief description of different methods for calculating ice forces against conical structures.

Simple Theory (two Dimensions):

Consider the initial interaction between ice and the sloping face.

\[ H = N \cdot \sin \alpha + \mu \cdot N \cdot \cos \alpha \] (26)

\[ V = N \cdot \cos \alpha - \mu \cdot N \cdot \sin \alpha \] (27)

\[ H = V \left( \frac{\sin \alpha + \mu \cdot \cos \alpha}{\cos \alpha - \mu \cdot \sin \alpha} \right) \] (28)

Fig. 6. Initial interaction between ice and sloping structure.
The maximum value of $V$ will be limited by the bending strength of the ice.

In this case, assume that the ice sheet can be represented by a beam on an elastic foundation,

$$ V_f' = \frac{6 \cdot Mo}{bt^2} $$  \hspace{1cm} (29)

where $b$ is the thickness of the beam and $t$ is the ice thickness.

For a semi-infinite beam on an elastic foundation the maximum bending moment ($Mo$) due to $V$ is given by Hetenyi [13].

$$ Mo = \frac{V}{\beta \cdot \frac{\pi}{4}} \cdot \sin\left(\frac{\pi}{4}\right) $$  \hspace{1cm} (30)

there $\beta$ is a characteristic length

$$ \beta = \left(\frac{g \cdot g \cdot b^{0.25}}{4EI}\right) $$  \hspace{1cm} (31)

Combining equations (29), (30) and (31)

$$ V = 0.68 \cdot V_f' \cdot b \left(\frac{g \cdot g \cdot t^5}{E}\right)^{0.25} $$  \hspace{1cm} (32)

This is the force required to break the advancing ice. For subsequent interactions a force is also required to push the ice up the slope.
Fig. 7. General interaction between ice and sloping structure.

\[ P = \frac{Z}{\sin \alpha} \cdot t \cdot b \cdot g \cdot \sin (\sin \alpha + \mu \cdot \cos \alpha) \]  

(33)

There \( Z \) is the height reached by the ice on the slope and \( g \) is the density of the ice.

The total horizontal force given on the structure per unit is

\[
\frac{H}{b} = 0.68 \cdot \frac{g}{f} \cdot \left( \frac{g \cdot w \cdot t^5}{E} \right)^{0.25} \cdot \frac{\sin \alpha + \mu \cdot \cos \alpha}{\cos \alpha - \mu \cdot \sin \alpha} \\
+ Z \cdot t \cdot g \cdot \left( \frac{(\sin \alpha + \mu \cdot \cos \alpha)^2}{\cos \alpha - \mu \cdot \sin \alpha} + \frac{\sin \alpha + \mu \cdot \cos \alpha}{\tan \alpha} \right)
\]

(34)
Three-dimensional Theory

In the three-dimensional case the same mechanisms apply but the zone of ice failure extends wider than the structure. Also for structures of circular section the effective angle for ice ride-up is reduced and ice pieces can slide around the structure without fully riding-up. These effects are illustrated conceptually in Figure 13. Intuitively it will be appreciated that 3-D effects cause divergence from the simple theory more for narrow structures than for wide structures.

It is generally assumed that the essence of the ice force problem on a conical structure reduces to the prediction of the forces necessary to fail a series of ice wedges formed by radial cracking of the ice as it advances against the cone, see Figure 8.

Figure 8 Ice action on sloping structures (3-D effects).
Bercha and Danys /3/ have also analysed the effect of in-plane compressive stresses on the flexural failure of the ice sheet. For steep, rough structures the effect can be significant and increases the horizontal force.

An approach for ice forces on a conical structure using plastic limit analysis has been proposed by Ralston /26/. His results can be expressed in the form

\[ H = A_4 \left( A_1 \frac{q_f t^2}{v} + A_2 \frac{g \rho w t D^2}{v_f t^2} + A_3 \frac{\rho w t (D^2 - D_T^2)}{v_f t^2} \right) \]  \hspace{1cm} (35)

\[ V = B_1 H + B_2 \frac{\rho w t (D^2 - D_T^2)}{v_f t^2} \]  \hspace{1cm} (36)

where \( D_T \) is the top diameter, \( D \) is the water line diameter, \( q_f \) is the flexural strength, \( t \) is the ice thickness and \( \rho w \) the density of water, \( A_1 \) and \( A_2 \) are coefficients dependent on \( \frac{S_w}{g} \cdot \frac{D^2}{v_f t^2} \), and \( A_3, A_4, B_1 \) and \( B_2 \) are coefficients dependent on the cone angle \( \alpha \) and friction \( \mu \). Values for these coefficients are reproduced in this report in Figure 9.

It should be noted that Ralston's analysis includes both the forces due to ice ride-up and ice breaking. In equation (35) the first two terms are due to ice breaking and the third term results from the ice pieces sliding over the surface of the cone.

It is of interest to note that for narrow structures, Ralston's theory predicts the ride-up component to be small compared with ice breaking, see Figure 10. For wide structures the ice ride-up component becomes a larger part of the total force, see Figure 11.
FIG. 9 ICE FORCE COEFFICIENTS FOR PLASTIC ANALYSIS.
Figure 10. Horizontal force vs ice thickness - narrow structure (Ralston's theory).

Figure 11. Horizontal force vs ice thickness - wide structure (Ralston's theory).
Experimental Data

At this time there are only two series of laboratory experiments for which data are available. In 1970, tests were conducted in the Arctic model basin on 45° angle cones up to 100 cm in diameter with ice up to 7 cm thick. Results from these tests have been reported by Edwards and Croasdale (1976). In 1971, tests were conducted with cones up to 28 cm in diameter with ice up to 3.5 cm thick by Afanasev, Dolgopolov and Shvaishtein (1971).

An empirical relationship derived by Edwards and Croasdale /9/ from their tests is given as:

$$H = 1.6 \theta_f t^2 + 6.0 \frac{\gamma_w g d t^2}{1.93 \cdot 1}$$

(for a 45° angle cone with an ice to cone friction coefficient of 0.05).

The investigators proposed that the first term represents the ice breaking portion of the ice force and the second term represents the ice clearing component.

From observations of their tests Afanasev et al /1/ proposed the following formula based on elastic plate theory

$$H = \frac{\theta_f t^2 \cdot S_x \cdot \tan \alpha}{1.93 \cdot 1}$$

(38)

where $S_x$ is the length of the circumferential crack given as

$$S_x = 1.76(r + \frac{\pi}{4} \cdot l)$$

(39)

where $r$ is the cone radius at ice level, and $l$ is the characteristic length given by

$$l = \left(\frac{E t^3}{12 \cdot \frac{\gamma_w g (1 - \nu^2)}}\right)^{0.25}$$

(40)

where $E$ is Young's modulus and $\nu$ is Poisson's ratio.
Bercha's Example - 18.3 m Diameter Cone

To examine results for a wider structure consider the example of an 18.3 m diameter cone, with the other variables as defined in Table 1. This is the largest diameter structure considered by Bercha and Danys /3/ in their paper.

Again the Ralston formula predicts the highest force, but the ice breaking force is in reasonably close agreement with Bercha and Danys.

The most significant point about the data presented in Table 1 is that the ice ride-up or clearing forces are now quite large. The simple 2-D theory probably over predicts the ride-up forces but both the correlations of Ralston and Edwards and Croasdale suggest ride-up forces of around 1000 KN. Clearly for a structure of this width the ride-up forces cannot be ignored.
TABLE 1. HORIZONTAL FORCE ON AN 18.3 m DIAMETER CONICAL TOWER (BERCHA'S EXAMPLE)

ASSUMPTIONS: D=60ft. (18.3m), t=3ft. (0.91m), $\sigma$=100 psi (700kPa)
$v=0.33$, $E=1\cdot10^6$psi ($700\cdot10^6$kPa), $\alpha=45^\circ$
$\mu=0.15$, freeboard = 20 ft (6.1m)

<table>
<thead>
<tr>
<th></th>
<th>Breaking Force (kN)</th>
<th>Ride-up Force (kN)</th>
<th>Total Force (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bercha and Dany's /13/</td>
<td>1558</td>
<td>-</td>
<td>1558</td>
</tr>
<tr>
<td>Ralston /26/</td>
<td>1964</td>
<td>1196</td>
<td>3160</td>
</tr>
<tr>
<td>Simple 2-D Theory</td>
<td>355</td>
<td>1896</td>
<td>2251</td>
</tr>
<tr>
<td>Simple 2-D Theory (Adjusted)</td>
<td>845</td>
<td>1896</td>
<td>2741</td>
</tr>
<tr>
<td>Afanasev et. al. /1/</td>
<td>711</td>
<td>-</td>
<td>711</td>
</tr>
<tr>
<td>Edwards and Croasdale /9/</td>
<td>922</td>
<td>900</td>
<td>1822</td>
</tr>
</tbody>
</table>
2.3 Broken Ice

Broken ice includes various conditions, which have to be analysed by different methods to obtain reliable force predictions. The size and the concentration of the broken floes or blocks govern the most appropriate calculation method.

In open pack-ice, where the floes are separated and large compared with the structure diameter, the ultimate impact force of a single floe can be calculated from energy dissipation. In closer pack-ice forces are transmitted between the floes and if their size is of the same order or smaller than the structure, inertia forces dominate the interaction. The inertia forces from the broken ice are included in some of the empirical and theoretical calculation methods for the resistance of icebreaking ships. This resistance component can be separated, and in some cases applied to structure interaction with broken ice, but unfortunately not to cylindrical structures.

Seekoo is programmed with a very appropriate calculating method for pack-ice, which reduces the force from sheet ice on cylinders and cones due to the degree of ice concentration. Fig. 12. /23/.

![Dimensionless plot of ice load.](image)

Fig. 12
In ice conditions where the surface concentration is more than 100%, that is when floes and blocks are also distributed vertically the ice can be considered a granular material. A granular material according to Mohr-Coulomb is characterized by a friction angle, $\phi$, and a cohesion, $C_1$, and the shear strength is given by

$$\tau_s = C_1 + \mu_n \cdot \tan \phi$$  \hspace{1cm} (41)

where

$$\mu_n = \text{stress normal to the plane of failure.}$$

The ratio of the structure diameter to the size of the ice blocks governs the validity of this approach. For example interaction between a large cassion platform and an arctic rubble field, formed by floes in a size range of 2-8 ice thicknesses may be satisfactorily described by the Mohr-Coulomb theory. For narrow structures and piles the theory may apply to brash and mush ice.

Due to great uncertainty in determining cohesion and internal friction angle of ice (in broken ice and rubble fields) the calculations in Seekoo have been simplified.

2.4 Rubble Fields

Prodanovic /25/ has carried out model tests with a cylinder penetrating a rubble field. The cohesion and the friction angle were determined by shear box tests and the ratio of structure diameter to rubble floe size ranged from 1/2 to 2. The results were consistent with a theoretical solution according to a plastic limit analysis of a linear Mohr-Coulomb material. An upper limit of the penetration force for a cylinder is given by

$$F = (1 + aH/D(1 + bH/D)) D \cdot H \cdot \sigma_c$$  \hspace{1cm} (42)

where

$$\sigma_c = 2 \cdot C_1 \tan (\pi/4 + \phi/2)$$ is the compressive strength of the ice mass in plain strain

$C_1$ = cohesion

$H$ = thickness of the rubble field

$a$ and $b$ are constants depending on the friction angle $\phi$. Fig. 13.
Fig. 13 Values of $a$ and $b$ in (42) versus friction angle (25)
Another method to calculate the forces in rubble fields is an empirical formula according to Joensuu /16/ which is used in Seekoo. The forces on cylinders will be

\[
F = 10 \cdot \rho_w \cdot g \cdot H_s^2 \cdot D \quad (43)
\]

and for cones breaking upwards

\[
F = 10 \cdot \rho_w \cdot g \cdot H_s^2 \cdot D + 45 \cdot \rho_w \cdot g \cdot \frac{H_s^3}{\tan \alpha} \quad (44)
\]

where

\[
\rho_w = \text{density of water}
\]

\[
H_s = \text{sail height}
\]

\[
D = \text{breadth of the structure}
\]

\[
\alpha = \text{cone angle}
\]

The rubble field is considered to be "loose" (the ice blocks are not frozen together). Thus no crushing takes place.

It must be stressed that the lack of data on ridge and rubble field properties make these calculation methods inadequate for design considerations.

First-year ridges can be partly consolidated, for example a re-frozen zone often occurs near the waterline. In such cases a level ice-breaking force could be added to the ridge-breaking force.
3. Multicolumned Structures

The calculation methods hitherto reviewed only apply to single cylinders. In real offshore structures a number of cylindrical legs usually interact at icebreaking. Some effects of this are briefly described below.

It is not likely that all the legs encounter the most critical ice condition simultaneously. In sheet ice, for example, some legs will probably only encounter broken ice. This may also suggest that a multicolumned structure should be orientated in a way that minimizes the number of legs encountering critical ice conditions. Another problem affected by the orientation of the structure is the breaking of consolidated ridges. The failure mode and the exerted forces are expected to differ considerably, if the ridge breaks against one cylinder in a corner of the structure or against two separate cylinders.

When the dynamics of the breaking forces is considered it is obvious that the load peaks are unlikely to occur simultaneously on all legs. This implies a reduction of the relative amplitude of the total force compared with the loads on the single cylinders, Figure 14. This is also consistent with Kry's theory /17/ of force reduction due to icebreaking in independent zones on wide structures.
Characteristics of dynamic loads on four cylinders and their sum /15/
The effects described above indicate that ice loads on multicolumned structures may be lower than predicted by the methods for single cylinders. There are, however, other effects that will magnify the ice loads on multicolumned structures. It is well-known that broken ice often accumulates between and in front of the legs of a multicolumned structure. This can cause increased forces but is not always undesirable, because the accumulation can prevent ice from hitting drill or riser equipments. The problem is hard to treat theoretically and model tests are recommended for more thorough investigations.

In a model test described by Noble & Singh /23/ the effect of ice accumulation is demonstrated. Three different half models of semi-submersibles with two, three and four cylindrical legs were tested in broken ice. Each test run started at a surface concentration of 50% and progressed until it reached 100%. A plot of the ice loads obtained versus the concentration is given in Figure 15. From the tests it is evident that the ice accumulation can extend wider than the structure. It is also noteworthy that the four-legged structure gives considerably lower loads than the others.
According to Joensuu /16/ the total force on the structure can be described as the force on one column multiplied with the factor $K_2$ and the number of legs, which is done in Seekoo. On page 8 the factor $K_2$ is described, where width of the complete structure is equal to the number of columns multiplied by the diameter.

Fig. 15 Results of modeltest with multicolunmed structures in broken ice /23/
4. Mechanical Properties of Sea Ice

Uniaxial compressive strength

The dependencies of the uniaxial compressive strength of sea ice on the most important parameters are qualitatively as follows:

- the ice gets stronger with decreasing temperature (brine volume decreases)
- the ice gets weaker with increasing salinity (brine volume increases)
- the ice strength increases with increasing strain rate in the ductile failure range and remains roughly constant in the brittle failure range (limit $\varepsilon \approx 10^{-4}$ to $10^{-3}$ s$^{-1}$).
- the ice strength parallel to the growth direction (vertical) usually is much greater than in plane with the ice cover (horizontal).

For sea ice, usually, the strength dependencies on temperature and salinity are combined by using the brine volume as a parameter. The brine volume $v_b$ can be evaluated by means of the empirical formula

$$v_b = S_i (0.532 - \frac{49.185 \circ C}{T_i}) /10/ \quad (45)$$

where $S_i$ is the absolute ice salinity and $T_i$ is the ice temperature in $\circ C$.

This formula is valid only between -0.5 to $-22.9\circ C$.

In a most comprehensive study Peyton /24/ analysed among other things the dependency of the compressive strength at sea ice on brine volume variations (Figure 16).
Peyton's equation is

\[ \sigma_R = 1.08 \text{ MPa}(1 - \sqrt[0.672]{\nu_b}) \]  \hspace{1cm} (46)

Weeks and Assur /35/ suggested an equation

\[ \sigma_R = 1.65 \text{ MPa}(1 - \sqrt[0.275]{\nu_b}) \]  \hspace{1cm} (47)

which also describes the low brine volume data points of Peyton's study but is limited to values of \( \nu_b \leq 0.25 \). At higher \( \nu_b \)-values \( \sigma_R \) is assumed to remain constant.

---

**Fig. 16** Stress rate corrected compressive strength versus square root of brine volume (Peyton, 1966).

---

---
Frederking /11/ shows a relation between uniaxial compressive strength and temperature (Figure 17) which is used in Seekoo's menu.

![Figure 17](image)

Joensuu /16/ shows on a standard value at 3.5 MPa.
Flexural Strength

Many investigators have determined values for the flexural strength of sea ice. The relation between brine content and flexural strength as determined from in-situ cantilever beam tests is particularly consistent, as shown in the investigations of Weeks and Andersson /2/, Brown /4/ and Butkovich /6/ (Figure 18).

From Figure 18, the equation obtained for \( f_f \) as a function of \( \nu_b \) is

\[
\sigma_f = 0.75(1 - \sqrt[3]{\nu_b}) \text{(MN/m}^2\text{)} \text{ for } \sqrt[3]{\nu_b} < 0.33 \quad (48)
\]

These results suggest that the flexural strength remains roughly constant for brine volumes for which \( \sqrt[3]{\nu_b} > 0.33 \).

This equation have been changed by Weeks and Assur /36/ to the formulas

\[
\sigma_f = 0.69(1 - \sqrt[3]{\nu_b}) \text{(MN/m}^2\text{)} \text{ for } \sqrt[3]{\nu_b} \leq 0.35 \quad (49)
\]

\[
\sigma_f = 0.20 \text{ MN/m}^2 \text{ for } \sqrt[3]{\nu_b} > 0.35.
\]

From large scale in-situ cantilever beam and simple beam tests together with small scale simple beam tests Dykins /8/ developed a relation which can be expressed as

\[
\sigma_f = 1.08(1 - \sqrt[3]{\nu_b}) \text{(MN/m}^2\text{)} \quad (50)
\]
Fig. 18. Flexural strengths as measured from in situ cantilever beam tests versus square root of brine volume.

A comparison of the results shows that in both cases the curves have similar slopes as well as similar intercepts on the flexural-strength axis.

Joensuu /16/ means that the flexural strength is quite independent of temperature variations and gives the value 0.4 MPa.

Elasticity Modulus

With increasing brine volume the elastic modulus decreases. The results of Langleben and Pounder /19/ from small scale acoustic test on young sea ice can be described by the linear equation

\[ E = 10.0(1 - 0.00351 \sqrt{v_b}) \]  

(51)
Anderson's /2/ results (Figure 19) show a steeper decrease with an elastic modulus of about 4.5 GPa at $V_b = 40 \%$ and about 1.0 GPa at $V_b = 100 \%$ brine volume.

![Graph of elastic modulus vs. brine volume](image1.png)

**Figure 19**: Elastic modulus of sea ice as determined by seismic techniques versus brine volume (Anderson, 1958[8]). The three triangular points are the results of static tests performed by Dykins (1971).

Gold /12/ presents the relationship elastic modulus v.s temperature (Figure 20) which is used in Seekoo's menu.

![Graph of elastic modulus vs. temperature](image2.png)

**Figure 20**.

Joensuu /16/ presents a standard value of 5.5 GPa.
Poissons Ratio

Gold /12/ presents the relationship poisson's ratio v.s. temperature shown in (Figure 20) which is used in Seekoo's menu.

Joensuu /16/ gives the value of ν to 0.69.

Friction

The uncertainty on the physical nature of friction may be the reason for the paucity of information on this subject. Schwarz and Weeks /31/ have collected friction data published in literature. They found static friction coefficients between ice and steel of different surface conditions to be varying between $C_f=0.03-0.04$ for sea ice and wet smooth steel and up to $C_f=0.40-0.70$ also for sea ice and steel, all being rather independent of surface pressure.
5. Conclusions

In regard to differences between published theories for calculating ice loads and uncertainties to determine the mechanical properties of sea ice, there is still a great requirement to continue with the research in this domain.

The computer program, which has been developed, compares very well with the results obtained from the ice model tests carried out.
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7. Appendix

User's manual for Seekoo.
user's manual
for
SEEKOO

SEPTEMBER 1984

Götaverken Arendal AB
INTRODUCTION

To design an offshore installation is a very complex task. Additional design conditions related to the arctic environment make the design process of arctic offshore installations even more complex. The most obvious arctic environmental condition is, of course, the low temperature. One consequence of this is ice, and it is a well-known fact to anybody involved with arctic activities that the variety of ice features is very great. It is quite simple to define a homogeneous ice sheet and its properties have been investigated by many scientists. To define other ice features as broken ice (ice channels, floe ice, rubble fields, etc) ice ridges (first and multiyear) is far more complicated. However, many theoretical and experimental investigations have been performed and also published.

Considering ice forces the most natural shape of a structural member on an arctic offshore installation is cylindrical or conical.

Many methods for calculating ice forces on cylindrical and conical members have been published during the last decade. There is quite a span of divergence in the presented data.

Seekoo is Inuit (Canadian Eskimo language) and means ice, glacier.
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**LIST OF SYMBOLS**

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<td>A</td>
<td>Radius of the circular crack</td>
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<td>D</td>
<td>Diameter</td>
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<td>DT</td>
<td>Top diameter</td>
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<td>E</td>
<td>Elasticity modulus</td>
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<tr>
<td>F</td>
<td>Force</td>
</tr>
<tr>
<td>g</td>
<td>Gravity constant</td>
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<td>H</td>
<td>Ice thickness</td>
</tr>
<tr>
<td>l</td>
<td>Characteristic length</td>
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<tr>
<td>p</td>
<td>Pressure</td>
</tr>
<tr>
<td>po</td>
<td>Two-dimensional buckling pressure</td>
</tr>
<tr>
<td>R</td>
<td>Radius of the water plane</td>
</tr>
<tr>
<td>V</td>
<td>Velocity</td>
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<tr>
<td>Z</td>
<td>Cone depth</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>Water density</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poissos ratio</td>
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<tr>
<td>$\alpha$</td>
<td>Cone angle</td>
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<tr>
<td>$\sigma_F$</td>
<td>Flexural strength</td>
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<tr>
<td>$\sigma_{CX}$</td>
<td>Uniaxial compressive strength</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Friction coefficient</td>
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**ABBREVIATIONS USED**

WARC Wärtsilä Arctic Research Centre
1.0 - GENERAL INFORMATION

The primary purpose of SEEKOO is to calculate the horizontal ice forces on an offshore structure in different types of ice. The different types of ice, the program considers, are level ice, pack ice and rubble fields. The program also considers the number of columns and the shape: cylindrical or conical.

For cones it is assumed that they are downward breaking. For rubble fields, however, the program only assumes an upward breaking cone.

SEEKOO is written in BASIC for HP 9845 and HP 9836.

1.1 - Basic Assumptions

The structure is assumed to be fixed. The fixed structure can be defined as having a negligible deformation from ice action. Any deformations that do occur, are heavily damped, i.e. the dynamic response of the structure is not significant. In this program, the interaction between the ice failure process and the response of the structure has not been considered.
2.0 - THEORY

To evaluate the ice capability of a certain structure the level ice performance is a good starting point because level ice is not so complex as pack ice or rubble fields.

There are several different methods for theoretical calculations of ice forces for level ice on structures. In SEEKOO there are three options for calculating ice forces on columns:

1. Vertical cylinders in level ice, according to Korzhavin, with corrections for ice concentration in pack ice conditions.

2. Conically shaped collars in level ice, according to Ralston with corrections for ice concentration in pack ice conditions.

3. Vertical cylinders and conically shaped collars in rubble fields, according to Wärtsilä.

2.1 - Level Ice Failure on Vertical Cylinder

The possible failure modes of a uniform ice field moving against the structure are crushing and buckling. The actual governing failure mode is the one which has the lowest value. The nominal ice pressure acting a vertical cylinder is:

\[ P = \frac{F}{D \times H} \]

where \( F \) is the horizontal force, \( D \) is the diameter and \( H \) is the ice thickness.

2.1.1 - Crushing

The following theory has been developed by Korzhavin. The used theory is a modified one. According to Korzhavin, the nominal pressure is dependent on the uniaxial crushing strength of the ice.

This can be stated as follows:

\[ P = C \times m \times k \times \sigma_{cx} \]

where

- \( C \) = Indentation factor
- \( m \) = Shape factor
- \( k \) = Contact coefficient
- \( \sigma_{cx} \) = Crushing strength
The last factor in the pressure formula is the compression strength of ice $\sigma_{cx}$ which can be stated as follows:

$$\sigma_{cx} = \alpha \times \sigma_{cx}^\wedge$$

where $\alpha$ is a strain-rate-dependent strength factor

$\sigma_{cx}^\wedge$ is the maximum compression strength of one year old arctic sea ice. The value of $\sigma_{cx}^\wedge$ is 3.5 MPa.

The value of $\alpha$ is shown in the figure No. 2.1.

Fig. 2.1 Strength factor $\alpha$ with different velocities of ice and widths of structure.
In Korzhavin's original equation (4) a velocity term was included, but it is generally accepted that it can be omitted if the ice strength is specified for the appropriate velocity or strain rate.

According to Michel's (1977) test results (5), he defines strain rate as:

\[ \mathcal{E} = \frac{V}{4 \times D}, \text{ where } V \text{ is the ice velocity.} \]

The indentation factor \( C \) is highly dependent on the test conditions in which it has been defined. In these calculations \( C \) is fixed to 3 to correspond with the test result (6). But research has been aimed at determining the effect of what is called the ratio factor \( D/H \) viz the value of \( C \).

Croasdale and Korzhavin et al. propose that \( C \) tends to be 1.0 for a wide structure and is close to 2.5 for narrow structures (\( b/h = 1.0 \))

\[ C = \left( \frac{5 \times H}{D} + 1 \right)^{0.5} \] (7)

The shape factor \( m \) for a cylinder is \( \approx 1.0 \).

The contact coefficient consist of two components:

- \( K_1 \) dependent on strain velocity
- \( K_2 \) dependent on width of the structure

For \( K_1 \) following strain rate dependence has been used: (8)

\[ K_1 = 1 \quad \text{when } \mathcal{E} < 5 \times 10^{-4} \]
\[ K_1 = 0.3 \quad \text{when } \mathcal{E} \geq 5 \times 10^{-4} \]
The component $K_2$ is also dependent on strain rate as well as on the width of the structure: (8)

$$K_2 = 1, \quad \text{when } \dot{\varepsilon} < 5 \times 10^{-4}$$

Fig. 2.3 The dependence of component $K_2$ of the aspect ratio $D/H$ when $\dot{\varepsilon} > 5 \times 10^{-4} \ 1/s$
2.1.2 - Buckling

Regarding vertical cylinders one should also take into consideration that ice in some cases buckles, especially when the ice thickness is small compared to the width of the structure. The possible failure mode of buckling in an ice field is also dependent on how the ice field and the structure are joined together. Buckling is very sensitive to all kinds of irregularities in ice properties and the buckling force has been observed to be less than theoretical calculations would indicate.

The pressure corresponding to the buckling of an infinitely wide ice sheet is:

\[ P_0 = \frac{1}{g} \frac{3 \times E \times H^3}{12 (1 - \nu^2)} \]

where \( D_f \) is the flexural rigidity

\[ D_f = \frac{E \times H^3}{12 (1 - \nu^2)} \]

For a structure the buckling load depends on the boundary condition which is somewhere between frictionless and clamped condition. For frictionless boundary the buckling nominal pressure can be estimated with the following equation:

\[ \frac{P}{P_0} = 1 + \frac{3.32}{\frac{D}{1} + \frac{1}{4} \left( \frac{D}{1} \right)^2} \]

according to Sodhi & Hamza 1977

where \( l \) is the characteristic length of the ice plate.

\[ l = \sqrt[4]{\frac{D_f}{8 \cdot g}} \]

The buckling force

\[ F = \left( 1 + \frac{3.32}{\frac{D}{1} + \frac{1}{4} \left( \frac{D}{1} \right)^2} \right) \times P_0 \times H \times D \]
2.2 - Level Ice Failure on Cone

The following theory has been proposed by Ralston (12). It is based on the plastic limit analysis and leads to an upper bound estimate of the horizontal force. Level ice failure addresses the situation shown in figure below.

![Figure 2.4 Ice sheet failure against a conical structure.](image)

A conical structure with inclination $\alpha$ from the horizontal, waterline diameter $D$ and top diameter $D_T$ is subject to the force imposed by an advancing ice sheet of thickness $H$.

The leading side of the exposed surface of the cone is assumed to be covered by a single thickness layer of broken ice pieces. The effective friction coefficient between the ice and the structure is denoted by $\mu$. The strength of ice sheet is characterized by its flexural strength $Q_f$.

A pure bending failure criterion is used for the ice sheet in this analysis.

The result of the analysis can be expressed in the form.
\[ R_H = \left\{ \frac{1}{3} + \frac{2.711}{3} \frac{\ln \rho}{\rho} \frac{\sigma_f}{t} t^2 + 0.075 (\rho^2 + \rho - 2) \rho_w g t \frac{D^2}{t} \right. \]
\[ + \frac{0.9 \rho_w g t}{4 \cos \alpha} \left( 1 + \frac{\mu E(\sin \alpha)}{\tan \alpha} \right) \]
\[ - \frac{0.9 \rho_w g t}{4 \tan \alpha} \left( D^2 - D^2 \right) f(\alpha, \mu) g(\alpha, \mu) \}
\] \[ \frac{\tan \alpha}{1 - \mu g(\sigma, \mu)} \]

Where the functions \( f(\alpha, \mu) \) and \( g(\alpha, \mu) \) are defined by:

\[ f(\alpha, \mu) = \sin \alpha + \mu \cos \alpha F(\sin \alpha) \]
\[ g(\alpha, \mu) = \left[ \frac{1}{2} + \frac{\alpha}{\sin 2 \alpha} \right] \sqrt{\frac{\pi \sin \alpha}{4} + \frac{\mu \alpha}{\tan \alpha}} \]

and the functions \( F \) and \( E \) are complete elliptic integrals of the first and second kind.

The parameter \( \varrho = A/R \) that optimizes this, is the solution of the equation.

\[ \rho - \ln \rho + 0.0830 (2\rho + 1) (\rho - 1)^2 \left( \frac{\rho_w g D^2}{\sigma_f t} \right) = 1.369 \]

For downward breaking cones the value of \( \varrho \) must be replaced by \( \varrho_w \). \text{(13)}
2.3 - Pack Ice

Due to great uncertainties in determining cohesion and the internal friction angle of ice, the calculations for pack ice have to be considerably simplified.

When calculating the ice force from pack ice, the force is estimated as level ice and then corrected for the degree of ice concentration (fig. 2.5). (1)

IMPORTANT!

The theory and model test results, on which the pack ice forces are based, are very questionable. However, we accept it for the time being, but future work to improve the calculations is most desirable.

Fig. 2.5 Dimensionless plot of ice load.
2.4 - Forces in Rubble Field

The forces on cylinders in a rubble field can be calculated by using the empirical formula:

\[ F = 10 \times \rho \times g \times H_s^2 \times D \]

For upward breaking cones the formula is

\[ F = 10 \times \rho \times g \times H_s^2 \times D + 45 \times \rho \times g \times H_s^3 \text{tan} \alpha \]

Observe! This formula is only valid for upward breaking cones.

\[ \rho = \text{density of water} \]
\[ H_s = \text{sail height} \]
\[ D = \text{breadth of the structure} \]
\[ \alpha = \text{cone angle} \]

The rubble field is considered to be "loose" (the ice blocks are not frozen together). Thus no crushing takes place. (8)

*) The ratio sail height/keel depth is 4.5.
2.5 - Multicolumn Structures

The calculation methods hitherto reviewed only apply to single cylinders. In real offshore structures a number of cylindrical legs usually interact at icebreaking. Some effects of this are briefly described below.

It is not likely that all the legs encounter the most critical ice condition simultaneously. In level ice, for example, some legs will probably only encounter broken ice. This may also suggest that a multicolumn structure should be oriented in a way that minimizes the number of legs encountering critical ice conditions.

The effects described above indicate that ice loads on multicolumn structures may be lower than predicted by the methods for single cylinders. There are, however, other effects that will magnify the ice loads on multicolumn structures. It is well known that broken ice often accumulates between and in front of the legs of a multicolumn structure.

According to Joensuu (8) the total force on the structure can be described as the force on one column multiplied with the factor $K_2$ and the number of legs.

On page 9 the factor $K_2$ is described, where width of the complete structure is equal to the number of columns multiplied by the diameter.

For rubble field the total force on a multicolumn structure is corrected according to experience. (For rubble fields, SEEKOO is only checked for 4 columns.)
3.0 - PROGRAM STRUCTURE

The SEEKOO program is written in BASIC. It is built around a main program from which a number of subroutines are called. This is illustrated by the flow chart, which also on the next page contains a brief description of the various subroutines.

SEEKOO is divided into two sections, one for single point calculation and one for plotting and tabulation. First section must always be processed.

SEEKOO incorporates a standard menu with fixed values for ice properties. They are stored in a subroutine and can be changed interactively.

The standard parameters are:

- water density 1025 kg/m³
- elasticity modulus 5.5 GPa
- uniaxial compressive strength 3.5 GPa
- flexural strength 0.4 MPa
- poisson's ratio 0.69
- friction coefficient 0.03
3.1 - Flow Chart for SEEKOO

START

Input

MENU

Select option

INP

CAL

Select option

Input

Select option

Input

Select option

CAL

Select option

PLOT

Select option

END

Input: type of structure, type of ice and number of columns

Menu of standard parameters

To change the values of the standard parameters

Input of geometry

Calculation of single forces

New single calculation or plotting and tabulation

Functions options

Varying options

Calculation

More varying options

Plotting or tabulation

Further calculation or stop
3.2 - Subroutines

**MAIN PROGRAM**

**MENU**
- Sub MENU Rubble fields
- Sub MENU Cone/Level ice
- Sub MENU Cylinder/Level ice

**INP**
- Sub INP Rubble fields
- Sub INP Cone/Level ice
- Sub INP Cylinder/Level ice

**CAL**
- Sub CAL Cross cylinder/level ice
- Sub CAL Cone Cone/level ice
- Sub CAL Pack ice
- Sub CAL Rubble fields

**PLOT**
- Sub PLOT Plotting and tabulation Cylinder
- Sub PLOT Plotting and tabulation Cone
3.3 - Restrictions

The program has the following restrictions:

**CYLINDER:**
The maximum permissible cylinder diameter is 25 m. The ratio diameter/ice thickness has to be greater than 1. (D/H > 1).

**CONE:**
The maximum permissible cone depth is 10 m. The permissible angles are between 30 and 60 degrees.

**LEVEL ICE:**
The maximum permissible ice thickness is 3 m.

**RUBBLE FIELDS:**
The maximum permissible sail height is 3 m.

**PACK ICE:**
The minimum permissible ice concentration is 70%.

**PLOTTING AND TABULATING:**
The maximum permissible number of curves in one diagram is 5.
4.0 - USE OF SEEKOO

4.1 - How to Start

The tape cartridge is equipped with an auto start routine.

Before you insert the tape cartridge make sure that the machine is switched off. Insert the tape cartridge in the right hand slot and the "AUTOST" key is latched. Turn the machine on. Answer the questions, which appear on the terminal screen. For activating SEEKOO, enter the program number, then press "Cont" and the auto start routine will load SEEKOO.

4.2 - Program Running

SEEKOO is run interactively to give the user full control over the calculations. The program is executed by selecting the appropriate options from the menus appearing on the terminal screen.

Numbers (0-3) are generally used as options when moving from one block to another in the program. The program proceeding is controlled with the key "Cont".

4.3 - Input Data

SEEKOO expects two basic types of user input:

- An integer entry
- A numerical entry, where the user has to give a numerical value

Input data for SEEKOO is organized in the following groups:

1. Heading
2. Type of structure
3. Type of ice
4. Number of columns
5. Ice properties
6. Geometry
7. Plotting and tabulating
4.3.1 - Heading

Two lines are reserved for the project name or identification in the plot routine. Each line has 35 available characters. If plotting and tabulating are not to be used, project name can be omitted.

The date must be given with 6 characters. (Example 840820). Enter the characters and press "Cont".

4.3.2 - Type of Structure

Two types of structures are possible: cylindrical or conical.

![Cylindrical and Conical Structures](image)

Each type is specified by a number:

- Cylinder (1)
- Cone (2)

Enter applicable number and press "Cont".

4.3.3 - Type of Ice

Three different types of ice can be treated by the program. The actual types are specified by a number:

- Level ice (0)
- Pack ice (1)
- Rubble fields (2)

Enter applicable number and press "Cont".
4.3.4 - Number of Columns

The program calculates, if desired, the ice force for a multi-column structure with maximum 8 legs.

Enter applicable number of columns and press "Cont".

4.3.5 - Ice Properties

Due to great uncertainties in determining ice properties, SIKOU has the option to change the values, and this has to be done in the menu. If no alterations of the properties are done press 0 and "Cont", then the program continues with geometry. If one wants to change any of the properties press 1 and "Cont", then the properties appear one after one, with some explanations on how they are varied. The values of the properties in the tables correlates to (8) with some corrections for the standard values fitted. If one doesn't want to change a particular value, just press "Cont" and the standard value is still the same. If the properties are to be altered, type the new value and then press "Cont".

4.3.6 - Geometry

Input data for a cylindrical structure in level ice is:

- Ice thickness
- Diameter
- Ice velocity

http://example.com
Input data for a conical structure in level ice is:

- Ice thickness
- Cone depth
- Column diameter
- Cone angle
- Ice velocity

[Diagram of a conical structure with labels for cone depth, ice velocity, ice thickness, cone angle, and column diameter]
For pack ice and rubble fields following has to be added:

**Pack ice**

Ice concentration (in percent)

**Rubble fields**

Sail height

Keel height = 4.5 x sail height.

In this section data has to be given with numerical entry. A question appears then on the terminal screen. Enter the numerical value and press "Cont".
4.3.7 - Plotting and Tabulating

When the user enters the "Plot" options in order to perform graphing, following input is required:

(These questions appear on the terminal screen.)

- Define which variables are to be plotted.
- Define which variables are to be varied.

Give the values of these variables:

Example: Force vs ice thickness

![Graph of Force vs Ice Thickness](image)

In this example we have: Diameter is varied and force vs. ice thickness is to be plotted. The plot routine can store five independent curves in the same run.

Before the graph is displayed on the terminal screen, the coordinate axes have to be specified.

A temporary table will appear on the screen after every calculation.

This table will help the user to specify the coordinate axes.

1. Enter permissible length of the X-axes.
2. Enter the corresponding force on the Y-axes.
When the graph is displayed the result can be received in three different ways:

1. Plotted on paper
2. Tabulated on printer
3. Graphics (to see the graph again)

In this section date has to be given with numerical entry and integer entry. Enter the option and press "Cont".
5.0 - EXAMPLE OF PROGRAM RUNNING:

When the program is started:

**WELCOME**
**TO**
**SEEKOO**

appears on the screen (and stays for five seconds).

*THIS IS A COMPUTER PROGRAM FOR CALCULATION OF ICE FORCES ON CYLINDRICAL AND CONICAL COLUMNS IN LEVEL ICE, PACK ICE AND RUBBLE.

BY

STEFAN KARLSSON AND PETER STRINDS

READY? THEN PRESS CONT.
SEEKOO IS DIVIDED INTO TWO SECTIONS. ONE FOR SINGLE POINT CALCULATIONS AND ONE FOR PLOTTING AND TABULATION. THE SECTION FOR PLOTTING AND TABULATION APPEARS AFTER THE SINGLE POINT CALCULATION.

READY? THEN PRESS CONT

CONT

TWO LINES ARE RESERVED FOR THE PROJECT NAME. EACH LINE CAN TAKE 0-35 CHARACTERS.

NAME OF THE PROJECT? (LINE 1)
SEEKOO CONT

NAME OF THE PROJECT? (LINE 2)
CONT

(If any of the lines has more than 35 characters, the machine beeps and the text appears, you may try again.)

MORE THAN 35 CHARACTERS

DATE (6 CHARACTERS)?
84 08 17 Cont
CYLINDERS OR CONES? CYLINDERS (1) CONES (2)
1 CONT

TYPE OF ICE? LEVEL ICE (0), PACK ICE (1), RUBBLE FIELD (2)
0 CONT

NUMBER OF COLUMNS?
1 CONT

USED VALUES FOR THE ICE PROPERTIES
---------------------------------------------
WATER DENSITY 1016.00 (Kg/m³)
ELASTICITY MODULUS 5.50 (GPa)
UNIAXIAL COMPRESSION STRENGTH 3.50 (MPa)
POISSON'S RATIO .69
INDENTATION FACTOR 3.00
CONTACT COEFFICIENT 0.3 OR 1.0

If you want to change the values on the properties type (1) else (0)
0 CONT

ICE THICKNESS IN METER
.5 CONT
CYLINDER DIAMETER IN METER
15 CONT
ICE VELOCITY IN METER/SECOND
.5 CONT
THE CALCULATIONS IS MADE FOR 1 CYLINDRICAL COLUMN IN LEVEL ICE.

USED PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICE THICKNESS</td>
<td>0.50 (m)</td>
</tr>
<tr>
<td>CYLINDER DIAMETER</td>
<td>15.00 (m)</td>
</tr>
<tr>
<td>ICE VELOCITY</td>
<td>0.50 (m/s)</td>
</tr>
<tr>
<td>ELASTICITY MODULUS</td>
<td>5.50 (GPa)</td>
</tr>
<tr>
<td>(UNIAXIAL COMPRESSION STRENGTH)</td>
<td>3.50 (MPa)</td>
</tr>
</tbody>
</table>

FORCE = ___________ (KN)

GO ON WITH PLOTTING (0), NEW CALCULATION (1), STOP (2)?

1 CONT

CYLINDERS OR CONES? CYLINDERS (1) CONES (2)

2 CONT

TYPE OF ICE? LEVEL ICE (0), PACK ICE (1), RUBBLE FIELDS (2)

0 CONT

NUMBER OF COLUMNS?

4 CONT
USED VALUES FOR THE ICE PROPERTIES

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of Water</td>
<td>1016.00 (Kg/m^3)</td>
</tr>
<tr>
<td>Flexural Strength</td>
<td>0.40 (MPa)</td>
</tr>
<tr>
<td>Coefficient of Friction</td>
<td>0.03</td>
</tr>
</tbody>
</table>

IF YOU WANT TO CHANGE THE VALUE ON THE PROPERTIES TYPE (1) ELSE (0)
1 CONT

(We just want to check the properties without changing them.)

*** DENSITY OF WATER ***

THE DENSITY OF WATER DEPENDS MAINLY ON WATER TEMPERATURE AND SALINITY. WHEN THE WATER TEMPERATURE IS ABOUT 0 DEGREES CELSIUS AND THE SALINITY FLUCTUATES, IS FOLLOWING VALUES OF DENSITY REQUIRED.
(THE STANDARD VALUE IS 1016 Kg/m^3)

<table>
<thead>
<tr>
<th>Salinity (0/00)</th>
<th>Density (Kg/m^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000</td>
</tr>
<tr>
<td>10</td>
<td>1008</td>
</tr>
<tr>
<td>20</td>
<td>1016</td>
</tr>
<tr>
<td>30</td>
<td>1024</td>
</tr>
<tr>
<td>40</td>
<td>1032</td>
</tr>
</tbody>
</table>

WRITE THE WATER DENSITY (Kg/m^3)
CONT

(When one doesn't want to change the value it is enough to press CONT.)
*** FLEXURAL STRENGTH ***

The flexural strength of ice is quite independent of temperature variations. The standard value is 0.4 MPa.

Write the flexural strength (MPa)

CONT

*** COEFFICIENT OF FRICTION ***

The coefficient of friction depends on type of color, material on the legs and temperature. The standard value is 0.03.

Write coefficient of friction

CONT

Ice thickness in meter

0.5 CONT

Cone depth in meter

6 CONT

Column diameter in meter

15 CONT

Cone angle in degrees

45 CONT

Ice velocity in meter/second

0.1 CONT
THE CALCULATIONS IS MADE FOR 4 CONICAL COLUMNS IN LEVEL ICE.

**USED PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Unit 1</th>
<th>Value 2</th>
<th>Unit 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICE THICKNESS</td>
<td>0.50</td>
<td>m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CONE DEPTH</td>
<td>6.00</td>
<td>m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>COLUMN DIAMETER</td>
<td>15.00</td>
<td>m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CONE ANGLE</td>
<td>45.00</td>
<td>Deg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ICE VELOCITY</td>
<td>0.10</td>
<td>m/s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WATER DENSITY</td>
<td>1016.00</td>
<td>Kg/m^3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FLEXURAL STRENGTH</td>
<td>0.40</td>
<td>MPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRICTION COEFFICIENT</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**FORCE =** \((\text{KN})\)

GO ON WITH PLOTTING (0), NEW CALCULATION (1), STOP (2) ?

0  CONT

FORCE vs. CONE DEPTH  (0)
FORCE vs. ICE THICKNESS  (1)
FORCE vs. CONE ANGLE  (2)

? 1  CONT

IF YOU WISH TO PLOT OR TABULATE MORE THAN ONE CURVE IN THE SAME DIAGRAM, YOU MUST SELECT WHICH PARAMETER TO BE VARIED BETWEEN THE CURVES.

VARYING NUMBER OF COLUMNS (1) OR CONE DEPTH (0)

1  CONT
NUMBER OF COLUMNS?

4  CONT

(Now the text

**** CALCULATION IN PROCEEDING 88.0 ****

appears blinking on the screen).

ICE THICKNESS

<table>
<thead>
<tr>
<th>(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

FORCE

<table>
<thead>
<tr>
<th>(MN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>

CONE DEPTH = 6.0 (m)

NUMBER OF COLUMNS 4

IF YOU WANT TO PLOT OR TABULATE THIS CALCULATION YOU HAVE TO SAVE IT.

SAVE (1) ELSE (0)

1  CONT

TOTALLY YOU CAN PLOT 4 MORE CALCULATIONS IN THE SAME DIAGRAM

FURTHER CALCULATIONS (1) ELSE (0)

1  CONT
NUMBER OF COLUMNS?

6 CONT

**** CALCULATION IN PROCEEDING 89.0 ****

<table>
<thead>
<tr>
<th>ICE THICKNESS (m)</th>
<th>FORCE (MN)</th>
<th>CONE DEPTH = 6.0 (m)</th>
<th>NUMBER OF COLUMNS 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

IF YOU WANT TO PLOT OR TABULATE THIS CALCULATION YOU HAVE TO SAVE IT

SAVE (1) ELSE (0)

1 CONT

TOTALY YOU CAN PLOT 3 MORE CALCULATIONS IN THE SAME DIAGRAM

FURTHER CALCULATIONS (1) ELSE (0)

0 CONT
SIZE OF ICE THICKNESS AXES IN METER

1 CONT

SIZE OF FORCE AXES IN MN

4 CONT

(The graphics curves appear on the screen, and stay there for five seconds.)

PLOTTING ON PAPER (0)
TABULATED (1)
GRAPHICS (2)
NEW CALCULATION (3)

0 CONT
SEEKOO

CONE IN LEVEL ICE

FORCE vs. ICE THICKNESS

CONE DEPTH: 6.0 (m)
COLUMN DIAMETER: 15.0 (m)
CONE ANGLE: 45.0 (Deg)
FLEXURAL STRENGTH: 0.4 (MPa)

FORCE (MN)

4 COLUMNS
6 COLUMNS
PLOTTING ON PAPER (0)
TABULATED (1)
GRAPHICLES (2)
NEW CALCULATION (3)

? 1 CONT
CONF IN LEVEL ICE
FORCE vs. ICE THICKNESS
INPUT DATA

<table>
<thead>
<tr>
<th>ICE THICKNESS (m)</th>
<th>FORCE 4 LEGS (MN)</th>
<th>FORCE 6 LEGS (MN)</th>
<th>FORCE 0 LEGS (MN)</th>
<th>FORCE 0 LEGS (MN)</th>
<th>FORCE 0 LEGS (MN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
PLOTTING ON PAPER (0)
TABULATED (1)
GRAPhICS (2)
NEW CALCULATION (3)

Now we stop!
6.0 - REFERENCES

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"Engineering Properties of Sea Ice"
Journal of Glaciology, Vol 19, No. 81, 1977

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(10) Wang, Y.S.,


(13) WARC Test Report No. A 106

(14) WARC Test Report No A 94
"Model Tests in Ice with a Cylindrical Leg of an Oil Drilling Jack-up Platform", for GVA, October 1983