The Limits of Digital Backpropagation in Nonlinear Coherent Fiber-Optical Links

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Abstract *The performance of a single-channel fiber-optical link is evaluated with linear (dispersive) and nonlinear equalization. The results show a quadratic growth of the noise variance with input power for a system with nonlinear equalization and also justify the known cubic growth for linear equalization.*

Introduction

In the design of advanced coding and modulation techniques for fiber-optical channels [1,2], an accurate channel model is essential. In fiber-optical channels, the nonlinear Schrödinger equation describes the propagation of light. These channels are nonlinear, and due to the lack of analytical solutions and the complexity of numerical approaches, deriving the statistics of such channels is in general cumbersome. Hence, many efforts have been devoted to computing the statistics for simplified models, e.g., memory-less nonlinear channels with single- [3 4] and dual-polarization (DP) [5] signals. The performance of uncoded systems after mitigating the deterministic impairments can also be computed using an accurate channel model.

Recently, an analytical model was proposed for a fiber-optical link using wavelength-division multiplexing with electronic dispersion compensation (EDC) [6,7]. Moreover, a discrete-time model [8] is proposed for an uncompensated single-channel fiber-optical link with EDC. In this discrete-time model (Fig. 1), closed-form expressions are derived for the noise distribution and channel attenuation (the nonlinearly-induced loss of the transmitted signal power to noise) as a function of the transmitted power and the channel parameters.

In this paper, the performance of a polarization-multiplexed uncompensated single-channel fiber-optical link with linear and nonlinear equalization is evaluated with analytical and numerical methods. In the analysis, we take into account the crosstalk between the signal in both polarizations. We also include the inline interaction between the transmitted signal and the amplified spontaneous emission (ASE) noise due to the Kerr-effect in different spans. Moreover, we introduce a new analytical model for a fiber-optical link with a nonlinear equalization [9], which shows a nearly quadratic growth of noise variance with input power for moderate transmit powers. The existing analytical result [8] for linear dispersion equalization is also used in the performance analysis.

Continuous-Time Model

A fiber-optical channel imposes linear and nonlinear effects on the transmitted signal. These impairments are described by the Manakov equation (with attenuation included) as [10]

$$j\frac{\partial \mathbf{u}}{\partial z}(t,z)-\frac{\beta_2}{2}\frac{\partial^2 \mathbf{u}}{\partial t^2}(t,z)+\gamma\|\mathbf{u}(t,z)\|^2\mathbf{u}(t,z)+j\frac{\alpha}{2}\mathbf{u}(t,z)=\mathbf{0},$$

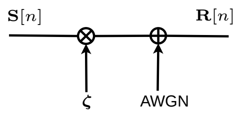

Fig. 1: The equivalent nonlinear discrete-time model for an uncompensated fiber-optical link with linear or nonlinear equalization.

where $\mathbf{u}$ is the DP electric field with complex components, γ is the fiber nonlinear coefficient, α is the attenuation coefficient, β_2 is the group velocity dispersion, t is the time coordinate in a co-moving reference frame and z is the propagation distance. A fiber-optical link with N spans of length L is considered according to Fig. 2. Each span consists of a standard single-mode fiber (SMF) followed by an erbium-doped fiber amplifier (EDFA).

In this paper, we use the split-step Fourier method (SSFM) both to construct the analytical discrete-time model as well as to simulate a fiber-optical channel numerically. In this method, each SMF span is modeled by a concatenation of M segments with linear and nonlinear effects as shown in Fig. 2. The length of each segment, L/M, should be chosen small enough to ensure that the linear and nonlinear effects act independently. The nonlinear effect of each segment is given by $\tilde{\mathbf{u}}(t,\ell_{m,i}) = \mathbf{u}(t,\ell_{m,i})\exp(j\gamma L_{\mathsf{eff}}\|\mathbf{u}(t,\ell_{m,i})\|^2)$, for $m=0,\ldots,M-1$, where $L_{\mathsf{eff}}=[1-\exp(-\alpha L/M)]/\alpha$ and $\ell_{m,i} = m(L/M)+iL$. The linear propagation is described in the time domain by $\mathbf{u}(t,z) = \exp(-\alpha z/2)\mathbf{u}(t,0)*h(t,z)$, where $*$ denotes convolution and $h(t,z) = \exp[jt^2/(2\beta_2 z)]/\sqrt{j2\pi\beta_2 z}$ is the dispersive impulse response. As shown in Fig. 2, the linear effect in each segment is described by $\mathbf{u}(t,\ell_{m+1,i}) = A\mathbf{u}(t,\ell_{m,i})*h(t,L/M)$, where $A \triangleq \exp[-\alpha L/(2M)]$ is the signal attenuation for each segment. The symbols $\mathbf{S}[n] = (S_{\mathsf{x}}[n],S_{\mathsf{y}}[n])$, e.g., DP-QPSK, are transmitted every T seconds with a pulse shaping filter $g(t)$. It is assumed that $\mathbb{E}\{|S_{\mathsf{x}}[n]|^2\} = P_{\mathsf{x}}T$, where P_{x} is the average transmitted power in polarization x. We assume that each EDFA compensates for the attenuation in one fiber span and adds a circular white complex Gaussian ASE noise in each span with variance σ^2 in each polarization. Finally, we define $\phi_{\mathsf{x}} \triangleq \gamma\alpha^{-1}P_{\mathsf{x}}$, $\phi_{\mathsf{n}} \triangleq \gamma\alpha^{-1}\sigma^2$, and the dispersion length [10] $L_{\mathsf{D}}=T^2/|\beta_2|$.

Discrete-Time Model

The discrete-time model for segment m from span i of the continuous-time model shown in Fig. 2 is depicted

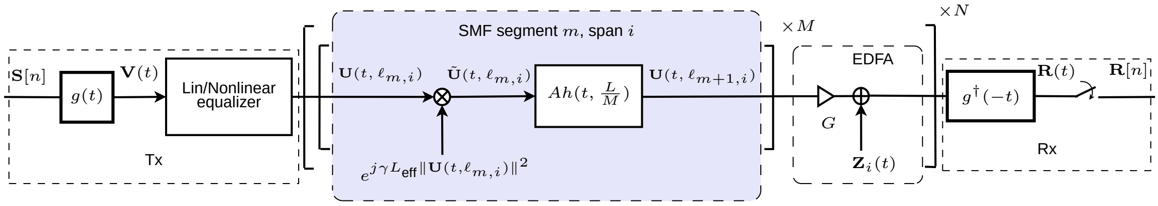

Fig. 2: A baseband continuous-time model based on SSFM for a fiber-optical link with N spans of SMF fiber $i = 0, \ldots, N-1$, each consisting of M segments $m = 0, \ldots, M-1$, and electronic equalization ($G = \exp(\alpha L/2)$ is the gain of each EDFA to compensate for the signal attenuation of corresponding span, $A = \exp[-\alpha L/(2M)]$ is the signal attenuation for one segment, and $g(t)$ is the pulse shaping filter).

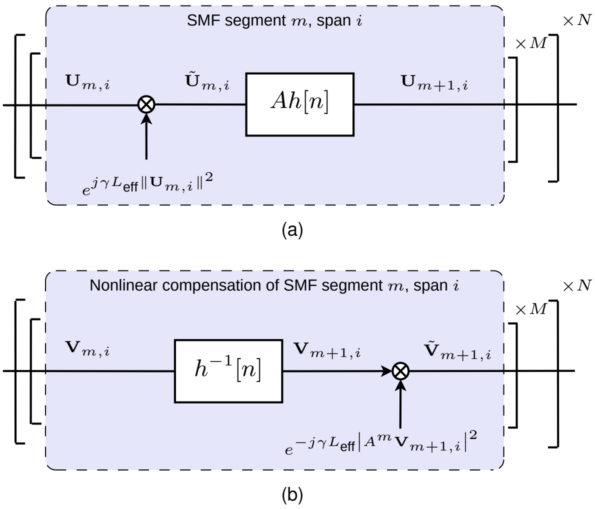

Fig. 3: A baseband discrete-time model for a fiber-optical link with (a) N spans, $i = 0, \ldots, N-1$, of SMF fiber, each consisting of M segments $m = 0, \ldots, M-1$, and (b) electronic nonlinear equalization based on digital backprogation [9] ($h^{-1}[n]$ is the inverse of the filter $h[n]$).

in Fig. 3(a). The Lin/Nonlinear equalizer unit can be a linear equalizer which solely compensates for CD or a nonlinear equalizer based on backpropagation [9]. In this paper, we assume a pseudo-linear fiber-optical data transmission. The band-limited CD filter for each segment is given by $h[n] = h\,(t, L/M) * \operatorname{sinc}(t/T)\,\big|_{t=nT}$.

Linear Dispersion Equalization

The channel linear dispersive effect can be compensated by using the inverse of the channel chromatic dispersion $h\,(t, NL)$. It has been shown [8] that the link with this linear equalizer, i.e., electronic dispersion compensation, can be modeled as a channel shown in Fig. 1, with an AWGN and a complex constant attenuation ζ_{x} in polarization x. The AWGN variance in polarization x is given by $N\sigma^2 + \sigma^2_{\text{NL}}$, where the nonlinear noise variance σ^2_{NL} shows a cubic growth with ϕ^3_{x}. Moreover, the complex constant attenuation ζ_{x} describes the nonlinearly-induced loss of the transmitted signal power to AWGN noise as well as the phase rotation. As shown in the numerical results, the Gaussian distribution yields an accurate channel model for linear dispersion equalization.

Nonlinear Equalization

The channel nonlinear Kerr effects can be categorized as signal-signal, signal-noise, and noise-noise interactions. The first term is deterministic, while the other two are random. Therefore, one may exploit nonlinear equalization based on backpropagation to mitigate the effect of the first term. Since the system performance is significantly improved in this case, the Gaussian assumption is not sufficiently accurate in the tails of the noise probability density function. Thus, in contrast to linear equalization, the analytical result derived based on the Gaussian assumption is solely used as a lower bound for the system performance.

In short, the system performance with backpropagation is bounded by the performance of an AWGN channel shown in Fig. 1, with a complex constant attenuation ζ_{x} in polarization x. The nonlinear noise variance, introduced in this AWGN channel, is proportional to $\phi^2_{\text{x}}\phi_n$. Thus, for moderate transmit powers the noise variance is a quadratic function of the transmit power rather than cubic as it is for linear equalization. The behavior of the nonlinear noise variance for two systems has been depicted in Fig. 6 for linear and nonlinear equalization in the next section.

Numerical Results

In this section, we evaluate the accuracy of the derived model for two fiber-optical links with symbol rates of 32 and 38 Gbaud. The symbol error rate (SER) is computed with both analytical and numerical methods. For the numerical SSFM, the Manakov equation (1), is used to model the nonlinear propagation with two polarizations with a segment length of $0.03\ L_{\text{D}}$. In the simulations, the receiver is assumed to have perfect knowledge of the polarization state. Moreover, ASE noise with the same bandwidth as the EDFA filters is added in each span. The EDFA filters are assumed to be unity gain with double-sided bandwidth equal to the used sampling frequency, which is twice as large as the signal bandwidth.

The input bits to the DP-QPSK modulator are generated as independent, uniform random numbers. The following channel parameters are used for the numerical simulations: the dispersion coefficient $D = 17$ ps/(nm km), the nonlinear coefficient $\gamma_{\text{SMF}} = 1.4$ W^{-1}km^{-1}, the optical wavelength $\lambda = 1.55\ \mu$m, the attenuation coefficient $\alpha_{\text{SMF}} = 0.2$ dB/km, $N = 25$ spans with the span length $L = 150$ km, and the ASE noise factor $n_{\text{sp}} = 1.5$. We also consider a root raised

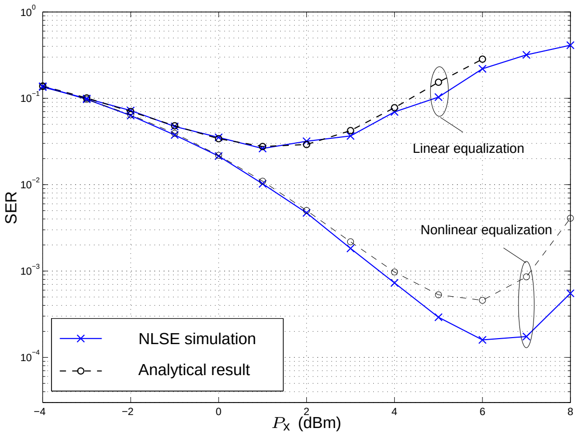

Fig. 4: The SER of a system with 38 Gbaud DP-QPSK versus transmitted power per polarization P_x.

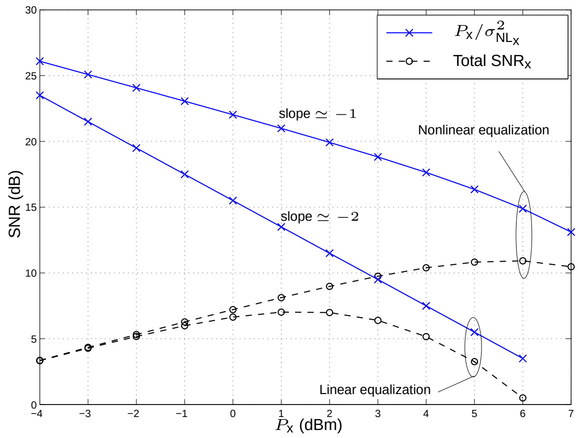

Fig. 6: The total SNR$_x$ and the normalized variance of the nonlinear noise for a system with a symbol rate of 38 Gbaud and a dispersion coefficient of $D = 17$ ps/(nm km).

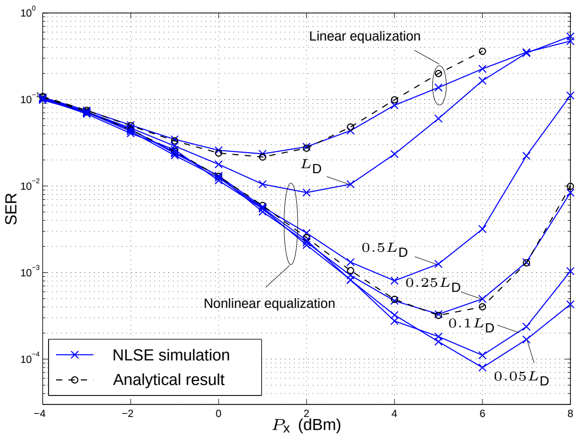

Fig. 5: The SER of a system with 32 Gbaud DP-QPSK versus transmitted power per polarization P_x. The nonlinear equalizer has been performed with segment lengths of L_D to $0.05\ L_D$.

cosine (RRC) pulse shape with an excess bandwidth of 0.25 and a truncation length of 16 symbols.

As seen in Figs. 4 and 5, the analytical results for the system performance show a good agreement with the simulation results for linear equalization. For nonlinear equalization, the analytical upper bounds of SER are applied for segment lengths smaller than $0.25L_D$. However, one may exploit an analogous approach to find the upper bound for larger segment lengths.

The performance of nonlinear equalization is very sensitive to the chosen segment length. As shown in Fig. 5, by decreasing the segment length from L_D to $0.05\ L_D$, the system SER decreases significantly. However, the improvement due to decreasing the segment length gets negligible for segment lengths smaller than $0.05\ L_D$.

Finally, the total SNR$_x$, $|\zeta_x|^2 P_x/(N\sigma^2 + \sigma^2_{NL_x})$ and the inverse of the normalized nonlinear noise variance $\sigma^2_{NL_x}/P_x = 1 - |\zeta_x|^2$ for the system with 38 Gbaud are plotted versus the transmitted power P_x in Fig. 6. This figure illustrates the cubic growth of the nonlinear noise variance with the input power for linear equalization and the quadratic growth for nonlinear equalization.

Conclusions

The results show that a discrete-time AWGN channel model with a complex attenuation provides an accurate channel model for uncompensated fiber-optical links with linear and nonlinear equalization. The complex attenuation as well as the AWGN variance are functions of input power and the channel parameters. For linear dispersion equalization, i.e. EDC, the noise variance shows a cubic growth with input power, while by applying nonlinear equalization, the growth is mitigated to nearly quadratic with input power. The SSFM numerical results justify the accuracy of this model for a symbol rate of 28 Gbaud and above.

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