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Department of Signals and Systems
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Project report

Master's Thesis

An MPC Approach To Low Speed Lane Guidance

Author:
Andreas Nilsson

Abstract

This thesis focuses on a low speed lane keeping problem. In particular, the objective is to constrain the vehicle motion inside its own lane on a highway, by controlling its lateral dynamics. The problem is approached with an output feedback control, as well as a Model Predictive Control. The output feedback control exceeded the constraints in some cases. The MPC achieved the control objectives.

The MPC presents a simple way to handle the problem constraints. The MPC is a constrained optimal control technique that minimises a cost function subject to the system dynamics and a set of state and input constraints. The main component in MPC is a model of the system that is used to predict its future behaviour. A bicycle model is used to model the vehicle dynamics.

The results presented in this thesis are obtained from simulations, in a Matlab/SIMULINK environment with the Model Predictive Control Toolbox.

An output feedback controller is enough to achieve the control objectives at different velocities, without a delay.

A Model predictive control achieves the control objective when a 0.3 s delay is present.

Acknowledgements

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Least but not last I would like to thank Volvo Technology for an incredible opportunity to see and practice how the automotive industry works.

Contents

1 Introduction

Active safety and autonomous driving technologies are intensively studied at the moment in the automotive industry. There are different systems being developed that not only assist the driver and help increase safety, but also improve fuel efficiency. A big part of this research is making the vehicles more autonomous. Some vehicle functionalities that are looked at are: lane keeping, platooning.

This thesis will focus on the development of a low speed ($0 - 50\,\mathrm{km/h}$) lateral lane keeping system for trucks. The control purpose is to keep the vehicle in its lane, i.e. to control the vehicle derivation from the road center-line The vehicle's position w.r.t. the road center-line. The position can be measured with a lane camera, for example. The primary objective is to keep the vehicle in its own lane. Further objectives are limiting the torque on the steering actuator and making sure that the ride comfort is not compromised by limiting the lateral acceleration.

Volvo technology is taking part in the HAVEit project. A so called Automated Queue Assistance function (AQuA) is being developed. The purpose is to be able to control a vehicle at low speeds; with focus on driving in traffic queues. This give rise to a lot of so called, stop and go behaviour. The traffic queues also presents a dynamic and changing environment that an autonomous vehicle safely needs to operate in. The function being developed in the HAVEit project should control the vehicle in the lateral as well as the longitudinal direction.

This thesis is going to approach the lateral lane keeping problem from a different angle. The thesis will use similar sensory information but will focus on guaranteeing that no constraints put on the system are broken. As such, a big part of this thesis will be to find a good way of dealing with potential constraints. A classical controller used for lane keeping usually corrects an error as it appears rather than trying to predict errors. When a real person uses a vehicle, the person will look ahead and try to stay on the road with the help of current as well as future information. A person can estimate where the vehicle will be positioned some time in the future, as well as the curves of the road. By trying to mimic this behaviour, a better control system could potentially be developed.

A reference vehicle is used; a Volvo truck that was used in the HAVEit project[1], a Volvo truck[2]. A lane camera added to the front of the vehicle. The purpose of the sensor is to detect where the lane markings are with respect to the vehicle.

[1] An EU project Volvo Technology took part in. http://www.haveit-eu.org/
[2] http://datasheets.volvotrucks.com/getfile.aspx?id=1024338

1.1 Literature Survey

This section overview the state of the art on lane keeping technologies. Four different papers will be presented, with a description of their contribution and their approach to similar problems.

Kosecka et al. (1997) focuses on a lateral vehicle control problem. A vision system is used as sensor to track the lane. The paper presents a linear state space model of the vehicle with the road curvature treated as a disturbance.

In the paper a lead-lag controller is used to control the vehicle's lateral movement. Data from experiments performed in a real vehicle, are also presented. A lot of the focus of the paper is placed on the performance of the vision system, how the look ahead distance affects the problem. An observer is used to estimate the states in the vehicle model.

Mouri and Furusho (1997) describes the vehicle with a linear state space model. The paper covers the use of both a PD (Proportional-Derivative) controller and an LQG (Linear Quadratic Gaussian) controller to solve a lane keeping problem.

For this thesis this paper is interesting because it covers a similar problem. The paper presents a solution that utilises an optimal control — LQG.

Keviczky et al. (2006) present an approach to autonomous lateral and yaw vehicle control via front steering in a double lane change scenario.

The vehicle model used in this paper is a bicycle model with non-linear tire characteristics.

This paper is relevant for this thesis as it presents a scenario where a bicycle model is used in conjunction with an MPC, even if the way of generating the reference path is different from this thesis.

Shimakage et al. (2002) describes another lane keeping problem. This paper presents a detailed model of the steering wheel actuator and describes a lane-keeping method where the input is limited by the turning torque of the steering wheel. The proposed controller is a H_2 optimal control.

In the paper a non-linear model is used to model the steering actuator and a bicycle model is used to model the vehicle from the steering input to the wheel angle.

The paper tests the controller in both simulations and in a real testing vehicle.

1.2 Purpose

The list below presents the purpose of this thesis:

- Identify a suitable vehicle model that can be used with an output feedback control and with an MPC.
- Analyse if a classical feedback controller is sufficient to solve a lane keeping problem, or if an MPC should be used instead.
- Implement an MPC controller and analyse its performance if feedback control seems insufficient.

1.3 Goals

Next are the goals of the thesis,

- Find a suitable vehicle model that can be used for low speed lane keeping.
- Model the steering actuator with included delays.
- Solve the lateral control problem with a classical controller.
- Implement, and solve the problem with a controller.
- Implement a lane keeping MPC controller in a test vehicle.

1.4 Limitations

This thesis work will assume that sensor data is provided by other systems and that the data is accurate and measurement errors are negligible.

This thesis focuses on a low speed driving scenario. In low speeds it is assumed that tires can be described by linear models.

The steering actuator in a truck is a complex system, often with combined electrical and hydraulic components. This thesis will assume that the steering actuator can be described as a simple delay.

Even if the thesis aims to deal with low-speed behaviour it will not assume that the vehicle stops. The thesis will not deal with stop-and-go behaviour.

2 Nomenclature

m	Mass of vehicle.
l_r, l_f	Distance from centre of gravity to the wheel axles.
$C_{\alpha r}(t), C_{\alpha f}(t)$	Cornering stiffness constant front and rear.
$F_{yr}(t), F_{yf}(t)$	Lateral tyre forces.
$\theta_v r(t), \theta_v f(t)$	Slip angles present.
$\dot{f}(t)$	should be interpreted as the first time derivative of f with regards to time t, $\frac{df(t)}{dt}$
$R(s)$	Radius of the road at the point s of the road.
$c_0(s)$	The road curvature w.r.t. time, $c_0(s) = \frac{1}{R(s)}$.
$\psi_t(s)$	The tangential angle of the road.
v_x	The longitudinal speed of the vehicle.
$v_y(t)$	The lateral speed of the vehicle.
$y(t)$	is the vehicle's movement in the lateral direction, $\dot{y} = v_y$ is the lateral speed at the time t.
$\psi(t)$	the yaw angle at the time t.
$e_y(t)$	the offset from the centre-line of the lane.
$e_\psi(t)$	the error of the heading angle.
$u(k+\tau)$	The real control signal at the time $k+\tau$.
$\hat{u}(k+\tau\|k)$	The predicted control signal at the time $k+\tau$, based on the available data at the time k.
H_p	The length of the prediction horizon.
H_u	The control signal horizon.
$\|\|x\|\|_Q^2$	The squared 2-norm function: $x^T Q x$
f_{max}	The maximum frequency with which the road changes curvature.
ω_{max}	The angular frequency of f_{max}

3 Vehicle Modelling

This section presents a physical model describing the motion of the vehicle in its lane.

3.1 Coordinate Systems

This section will describe the different coordinate systems adopted in the vehicle model. Consider a global coordinate frame $X(t)$ and $Y(t)$ as in Fig. 1 . Assume the road can be described as a function of the path length coordinate s. Every part of the road will have some radius $R(s)$ and a curvature $c_0(s) = \frac{1}{R(s)}$. For every point of the road s it is then possible to determine the tangent angle as, (Velenis and Tsiotras, 2005; Casanova et al., 2000)

$$\psi_t(s) = \int_0^s c_0(s) \text{ ds}, \tag{3.1}$$

every point of the road can then be described as a point with coordinates $x(s)$ and $y(s)$, where

$$x(s) = \int_0^s cos(\psi_t(s)) \text{ ds}, \tag{3.2}$$
$$y(s) = \int_0^s sin(\psi_t(s)) \text{ ds}. \tag{3.3}$$

Assuming that the vehicle follows the road, the longitudinal velocity can be expressed as

$$v_x(t) = \frac{ds}{dt}. \tag{3.4}$$

3.2 Bicycle Model

The vehicle is modelled at each axis through a bicycle model. The speed v_x in the longitudinal direction can be considered as a constant every time instant.

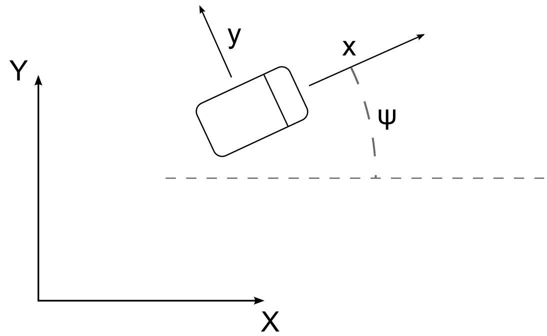

Figure 1: Coordinate Systems

In a bicycle model the two wheels are lumped into a single wheel. It will be assumed that a simple linear tire model can be used due to the considered low speeds. It is also assumed that the small angles approximation can be used.

The equations describing the yaw and lateral motion are

$$
\begin{aligned}
m\dot{v}_y(t) &= F_{yf}(t) + F_{yr}(t), \qquad (3.5)\\
I_z\ddot{\psi}(t) &= l_f F_{yf}(t) - l_r F_{yr}(t),
\end{aligned}
$$

where $F_{yf}(t)$ and $F_{yr}(t)$ are the tire forces at the front and rear axles of the vehicle respectively, which are a function of the cornering stiffness of the tires $C_{\alpha f}$, $C_{\alpha r}$, respectively, the wheel angle $\delta(t)$, and the vehicle states.

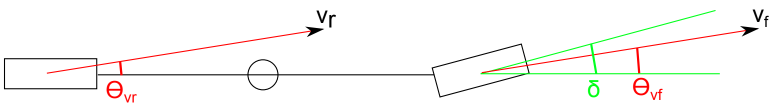

Figure 2: The vectors v_r and v_f are the velocity vectors for respective tire.

The vehicle travels with a longitudinal velocity v_x and a lateral velocity v_y. The two velocities, v_x and v_y can be used to form a vector describing the resultant velocity and its direction. It is then possible to find the slip angles, the difference between the actual travelling direction of the wheel and where it is pointed,

$$
\begin{aligned}
\alpha_f(t) &= \delta(t) - \theta_{vf}(t), \qquad (3.6)\\
\alpha_r(t) &= -\theta_{vr}(t), \qquad (3.7)
\end{aligned}
$$

where,

$$\theta_{vf}(t) = atan\left(\frac{v_y + l_f\dot{\psi}(t)}{v_x}\right), \tag{3.8}$$

$$\theta_{vr}(t) = atan\left(\frac{v_y - l_r\dot{\psi}(t)}{v_x}\right). \tag{3.9}$$

The two angles, $\theta_{vf}(t)$ and $\theta_{vr}(t)$ are the angles of the velocity vectors for the rear and front tires, respectively, as shown in Fig. 2 . (Rajamani, 2006)

The lateral tire forces acting on the wheels are $F_{yf}(t)$ and $F_{yr}(t)$, Rajamani (2006) state that they are proportional to the slip-angle, for small angles. The lateral tire forces can be written as

$$\begin{aligned} F_{yf}(t) &= 2C_{\alpha f}(\delta(t) - \theta_{vf}(t)), \\ F_{yr}(t) &= 2C_{\alpha r}(-\theta_{vr}(t)). \end{aligned} \tag{3.10}$$

Combining Eq. 3.5 and Eq. 3.10 gives the following system of ordinary differential equations,

$$\begin{bmatrix} \dot{v}_y(t) \\ \ddot{\psi}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{2C_{\alpha f}+2C_{\alpha r}}{mv_x} & -v_x - \frac{2C_{\alpha f}l_f - 2C_{\alpha r}l_r}{mv_x} \\ -\frac{2l_fC_{\alpha f}-2l_rC_{\alpha r}}{I_zv_x} & -\frac{2l_f^2C_{\alpha f}+2l_r^2C_{\alpha r}}{I_zv_x} \end{bmatrix}}_{A_c} \begin{bmatrix} v_y(t) \\ \dot{\psi}(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{2C_{\alpha f}}{m} \\ \frac{2l_fC_{\alpha f}}{Iz} \end{bmatrix}}_{B_c} \delta(t). \tag{3.11}$$

The model can can be written as

$$\dot{x}(t) = A_c(v_x)x(t) + B_c(v_x)u(t).$$

This is a Linear Parameter Varying Model, shown in Section 3.4 . The wheel angle $u(t) = \delta(t)$ is the input for the model.

3.3 Road Curvature

Denote by $e_y(t)$ the offset to the center line and $e_\psi(t)$, the relative angle to the center line. $c_0(t)$ describes the road curvature at time t, which are assumed to be measured. (Cerone and Regruto, 2003)

Fig. 3 shows the relationship between road curvature and a road with continuous radius.

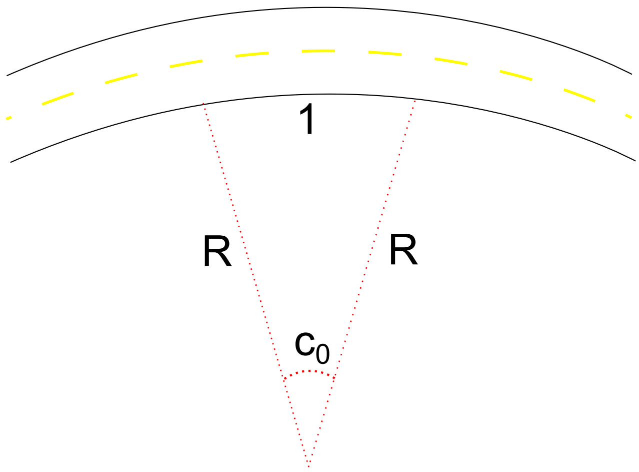

Figure 3: The relationship between the curvature and the road. Where $c_0 = \frac{1}{R}$, R represents the turning radius of the road at some particular time.

Based on small angles approximation the differential equations, Eq. 3.13 and Eq. 3.12 are derived to describe vehicle position and orientation w.r.t. the lane orientation based on the vehicle states. (Cerone et al., 2002)

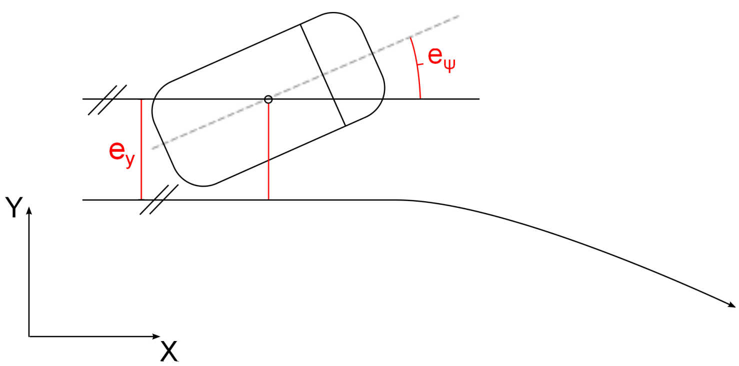

Figure 4: The vehicle's position relative to the center-line. In a perfect scenario the vehicle would be lined up without any angle against the center-line, or without any offset. The picture illustrates what $e_y(t)$ and $e_\psi(t)$ represents.

$$\dot{e}_y(t) = v_y(t) + v_x e_\psi(t) - v_x c_0(t) L, \tag{3.12}$$
$$\dot{e}_p(t) = \dot{\psi}(t) - v_x c_0(t) L, \tag{3.13}$$

where L is called look ahead distance. Fig. 4 shows how $e_y(t)$ and $e_\psi(t)$ are geometrically defined w.r.t. the road center-line. The state space model in Eq. 3.11 is augmented with Eq. 3.12 and Eq. 3.13 , forming a new state vector $x(t)$. The road curvature is included in the model as a disturbance.

$$x(t) = \begin{bmatrix} v_y(t) & \dot{\psi}(t) & e_y(t) & e_\psi(t) \end{bmatrix}^T,$$

$$\dot{x}(t) = \begin{bmatrix} A_{c11} & A_{c12} & 0 & 0 \\ A_{c21} & A_{c22} & 0 & 0 \\ 1 & 0 & 0 & v_x \\ 0 & 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} B_{c1} \\ B_{c2} \\ 0 \\ 0 \\ 0 \end{bmatrix} \delta(t) + \begin{bmatrix} 0 \\ 0 \\ -v_x L \\ -v_x L \end{bmatrix} c_0(t). \tag{3.14}$$

If c_0 can be measured then it can be represented as a state, or as a measured disturbance (see above). When there is an input delay present the state space model can be written as

$$\dot{x}(t) = A_c x(t) + B_c u(t-\tau) + D_c c_o(t), \tag{3.15}$$

where τ is a delay, in this case $\tau = 0.3\,\mathrm{s}$.

3.4 Linear Parameter Varying Model

From Eq. 3.11 can be seen that the longitudinal velocity v_x is treated as a constant value. However, in reality a vehicle have a varying longitudinal velocity. It is of interest to somehow linearise the system around the current v_x.

As such, the model need to be able to update the state matrices to take v_x into account. This is done by rewriting the state space model as

$$\dot{x}(t) = A(v_x)x(t) + B(v_x)u(t),$$
$$y(t) = C(v_x)x(t),$$

where A, B, C are updated based on v_x. By using a linear parameter varying model it is possible to treat the state space model as a linear system.

4 Constraints

This section will present the constraints that acts on the states, i.e. comfort, vehicle position and orientation, and the rate $\dot{u}(t)$, i.e., the rate of change in control signals.

The vehicle can not pass the lane boundaries during normal operation. A lane for a highway has a width of 3.25 m according to (Vägverket, 2004). The truck considered in this project is about 2.5 m wide including mirrors. That leaves 0.375 m on each side of the vehicle. For comfort reasons a constraint is added to the lateral acceleration. The steering command and its rate of change should also be constrained to operate within the capability of the steering actuator.

The road itself is assumed to have a curvature c_0 between 0 and 0.002. It is also assumed that the road curvature changes with, at most, a frequency of $f_{max} = 1/2\,\mathrm{Hz}$, which translates into a frequency range of,

$$f_{max} = 1/2\,\mathrm{Hz}, \tag{4.1}$$

$$\omega_{max} = f_{max} 2\pi = \pi\,\mathrm{rad/s}, \tag{4.2}$$

$$\omega = [0, \pi]\,\mathrm{rad/s}. \tag{4.3}$$

The following is a list of the boundaries.

- Lane boundary. $\pm 0.15\,\mathrm{m}$
- Comfort, lateral acceleration. $\pm 0.2\,\mathrm{m/s^2}$
- Actuator limitations. $\pm 0.1\,\mathrm{rad}$ and $\pm 0.1\,\mathrm{rad/s}$
- Road Curvature. ± 0.002 with a frequency range of $\omega = [0, \pi]\,\mathrm{rad/s}$

These physical constraints will later be used as specification for the designed controllers.

5 Control Design

This section describes how a lane keeping problem can be solved with an output feedback controller designed based on the model in Eq. 3.14 .

5.1 Control Objective

The objective is to design a lane keeping algorithm. The objective is to keep the vehicle inside its lane. It is desirable to have no steady state position and orientation error. The controller should not violate the constraints in Section 4 . Since the system is subject to a disturbance, in the form of the road curvature, the problem can be formulated as a disturbance rejection problem.

The bode plots of the open loop transfer functions from the disturbance to $e_y(t)$ and $e_\psi(t)$ can be seen in Fig. 5 . They shows how the vehicle would behave without a feedback controller when a disturbance acts on the system.

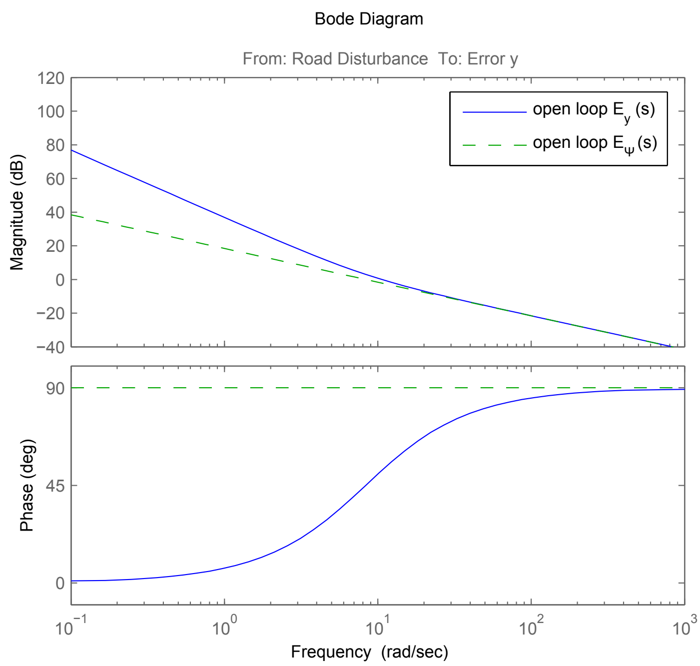

Figure 5: The Open loop frequency response from the disturbance to the errors $e_y(t)$ and $e_\psi(t)$. As can be seen from both the variables they are amplified at low frequencies. If there is a continious disturbance the error will continue to grow. This implies that for the open loop system, if the road turns the vehicle will not follow it.

5.1.1 The Vehicle Model

For control design purposes, the vehicle is represented as the state space model. Note that the road curvature enters the system as the disturbance $d(t) = c_0(t)$.

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t),$$
$$y(t) = Cx(t).$$

The disturbance $d(t)$ need to be rejected as it is the primary factor that affects $e_y(t)$ and $e_\psi(t)$.

5.1.2 Steady State Error

With the vehicle model used in Section 3.2 , there is an inherent shortcoming. During steady state cornering, both $e_y(t)$ and $e_\psi(t)$ can not be zero at the same time. On the other hand, during steady state cornering the state vector $\dot{x}(t) = 0$. For every new time instant t the errors $e_y(t)$ and $e_\psi(t)$ will be updated.

Assume that the variables will be constant during steady state cornering: $\dot{x}(t) = 0$, $v_x = \bar{v}_x$ and $c_0(t) = \bar{c}_0$. This gives us the following relations

$$\dot{e}_y(t) = \dot{y}(t) + v_x e_\psi(t) - v_x c_0(t)L = 0, \tag{5.1}$$
$$\dot{e}_\psi(t) = \dot{\psi}(t) - v_x c_0(t)L. \tag{5.2}$$

During steady state cornering $\dot{y}(t) = 0$, and $\dot{e}_y(t) = 0$. This gives:

$$v_x e_\psi(t) - v_x \bar{c}_0 L = 0, \tag{5.3}$$
$$e_\psi(t) = \bar{c}_0 L. \tag{5.4}$$

Hence, the $e_\psi(t)$ can never become zero. As such, there is a orientation steady state error.

5.2 Output Feedback Control

The vehicle model have two output variables $e_y(t)$ and $e_\psi(t)$ which should be regulated toward zero. Recall that

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}^T = Cx(t) = \begin{bmatrix} e_y(t) \\ e_\psi(t) \end{bmatrix}^T, \tag{5.5}$$

this means that both are feed back with some gain $K(s)$ as shown in Fig. 6 .

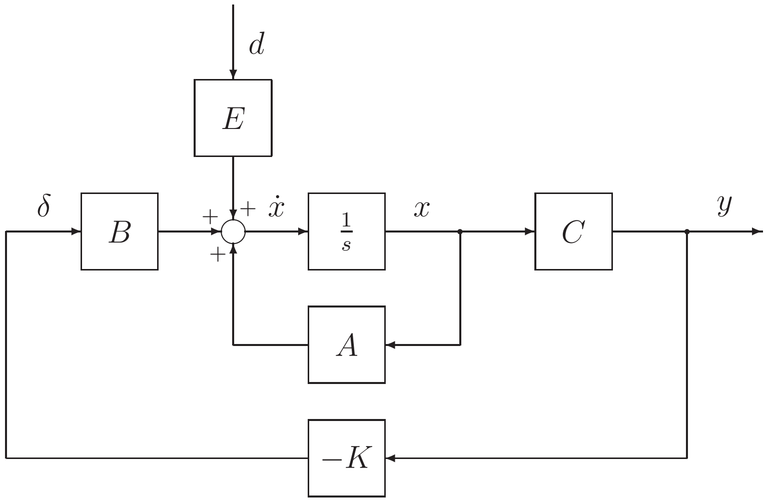

Figure 6: Block diagram of the State space model used to design the Classical Controls.

5.2.1 Loop-Shaping

The gain $K(s)$ is a vector containing $k_1(s)$ and $k_2(s)$. By choosing $k_1(s)$ and $k_2(s)$ it is possible to move the zeros and poles of the system. This is called loop shaping. This technique will be used to create a set of controllers that can be used to regulate the system and change the frequency response, so the system does no violate any constraints. Examples of loop-shaping can be seen in Fig. 7 and Fig. 8 .

5.2.2 Derivation of Transfer functions

To design the controller it is necessary to know how the disturbance will affect different variables such as $e_y(t)$, $e_\psi(t)$; or how it will affect the lateral acceleration and the control signal. Given an output feedback control law, the control input signal can be expressed as

$$u(t) = -K(t)y(t). \tag{5.6}$$

By combining Eq. 5.5 with Eq. 5.6 and the state space model, the Laplace transforms of $y_1(t)$ and $y_2(t)$ are

$$Y_1(s) = G_1^u(s)U(s) + G_1^d(s)D(s), \tag{5.7}$$
$$Y_2(s) = G_2^u(s)U(s) + G_2^d(s)D(s). \tag{5.8}$$

Combine Eq. 5.7 , Eq. 5.8 with Eq. 5.6

$$\begin{aligned}
&U(s) = -K(s)Y(s),\\
&U(s) = -\begin{bmatrix} k_1(s) & k_2(s) \end{bmatrix}\begin{bmatrix} G_1^u(s)U(s) + G_1^d(s)D(s) \\ G_2^u(s)U(s) + G_2^d(s)D(s) \end{bmatrix},\\
&U(s)\left(1 + k_1(s)G_1^u(s) + k_2(s)G_2^u(s)\right) = -D(s)\left(k_1(s)G_1^d(s) + k_2(s)G_2^d(s)\right),
\end{aligned}$$

$$G_u^d(s) = \frac{U(s)}{D(s)} = -\frac{k_1(s)G_1^d(s) + k_2(s)G_2^d(s)}{1 + k_1(s)G_1^u(s) + k_2(s)G_2^u(s)}. \tag{5.9}$$

From this transfer functions it is possible to derive the four transfer functions that are of interest:

$$Y_1(s) = [G_1^u(s)G_u^d(s) + G_1^d(s)]D(s) \Rightarrow G_{y1}^d(s) = G_1^u(s)G_u^d(s) + G_1^d(s), \tag{5.10}$$
$$Y_2(s) = [G_2^u(s)G_u^d(s) + G_2^d(s)]D(s) \Rightarrow G_{y2}^d(s) = G_2^u(s)G_u^d(s) + G_2^d(s). \tag{5.11}$$

The lateral acceleration $A(s)$ is obtained by taking the derivative of the lateral velocity $V_y(s)$ (that is: $A(s) = sV_y(s)$). Assume that there is a transfer function from the control signal $U(s)$ to the lateral velocity $V_y(s)$ in Eq. 3.14 called $G_{v_y}^u(s)$, then

$$G_a^d(s) = G_{v_y}^u(s)G_u^d(s)s. \tag{5.12}$$

This gives four transfer functions: $G_{y1}^d(s), G_{y2}^d(s), G_u^d(s), G_a^d(s)$.

5.2.3 Constraint Limitations

As can be seen in Section 4 there are a lot of different constraints the system is subject to. Some of these constraints work against each other. E.g., better

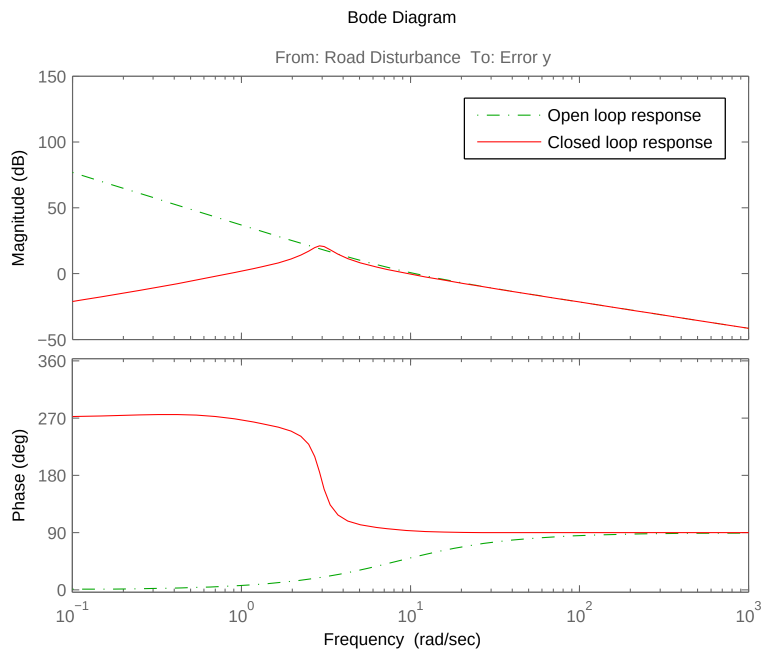

Figure 7: The bode plot frequency response of $E_y(s)$. The plot shows the open-loop frequency response and an example of a closed-loop frequency response. It is desirable to have low frequency damping; this attenuates disturbances such as the one present. The peak of the frequency response, where it crosses zero, will determine the overshoot of the system when there is a change in the curvature.

disturbance rejection might lead to larger control signals.

$e_y(t)$ and $e_\psi(t)$ are the two main variables that are keeping the vehicle at the center-line. Minimising both should be the main priority, it implies that the vehicle will stay on its path. However, as shown in Section 5.1.2 both can not be zero during steady state cornering. By changing the feedback law it is possible to change the steady state error of $e_y(t)$ and $e_\psi(t)$. There are different possible scenarios: either let $e_\psi(t)$ go to zero or make a compromise and let both have some kind of steady state error. Finding what is desirable and then acting on it will be a factor that decides how the feedback law will look.

It is assumed that the road curvature changes with a frequency of $\omega = [0, \pi]$ rad/s as described in Section 4 . This measurement will create the first bound for the constraints.

The next bound is on the amplitude of the bode plot. By looking at the constraints acting on the system, a limit for the different transfer functions can be found. The interesting constraints are the maximum constraints on lateral acceleration a_{max},

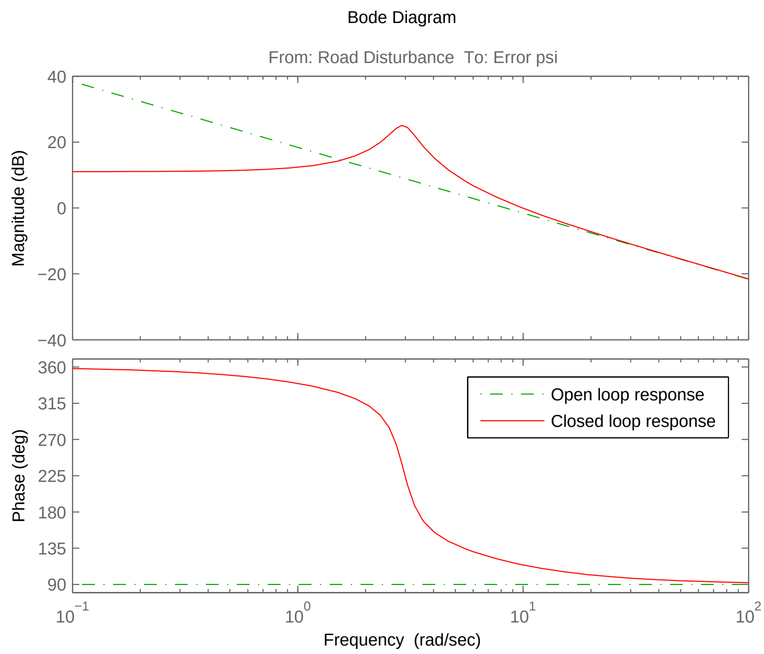

Figure 8: The bode plot of the frequency response of $E_\psi(s)$. Note how the plot does not have the same amount of low-frequency damping as the other figure. This is because of the coupling and the nature of the bicycle model used.

the road curvature d_{max} and on the control signal u_{max}. The relation between the road curvature and the lateral acceleration can be found in the transfer function between the two, $G_a^d(s)$. This gives the following bound

$$\lim_{s\to 0} G_a^d(s) < \frac{a_{max}}{d_{max}} = \frac{0.2}{0.002} = 100, \tag{5.13}$$

$$20log_{10}(100) = 40\,\text{dB}. \tag{5.14}$$

Likewise, for the control signal a bound is calculated as

$$\lim_{s\to 0} G_u^d(s) < \frac{u_{max}}{d_{max}} = \frac{0.1}{0.002} = 50, \tag{5.15}$$

$$20log_{10}(100) = 34\,\text{dB}. \tag{5.16}$$

The constraints can then be added in the respective bode plots of the transfer functions.

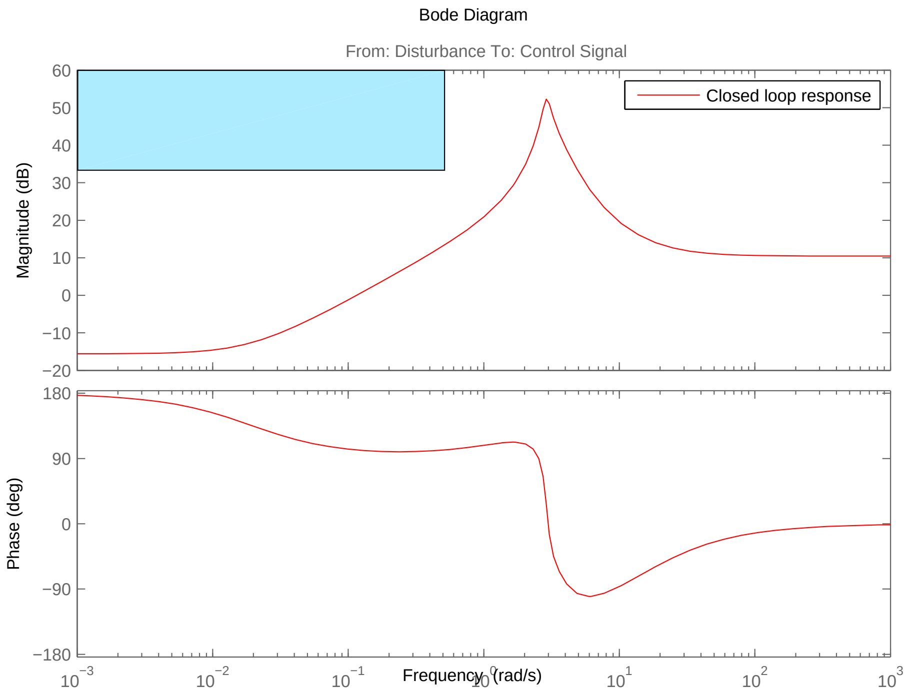

Figure 9: The bode plot of the transfer function from the disturbance to the control signal. The box represents the constraints on the control signal, the maximal road curvature and frequency of the road curvature.

From the plots Fig. 9 , Fig. 10 , Fig. 11 it can be seen that it is possible to stays within the constraints. This will be tested with simulations.

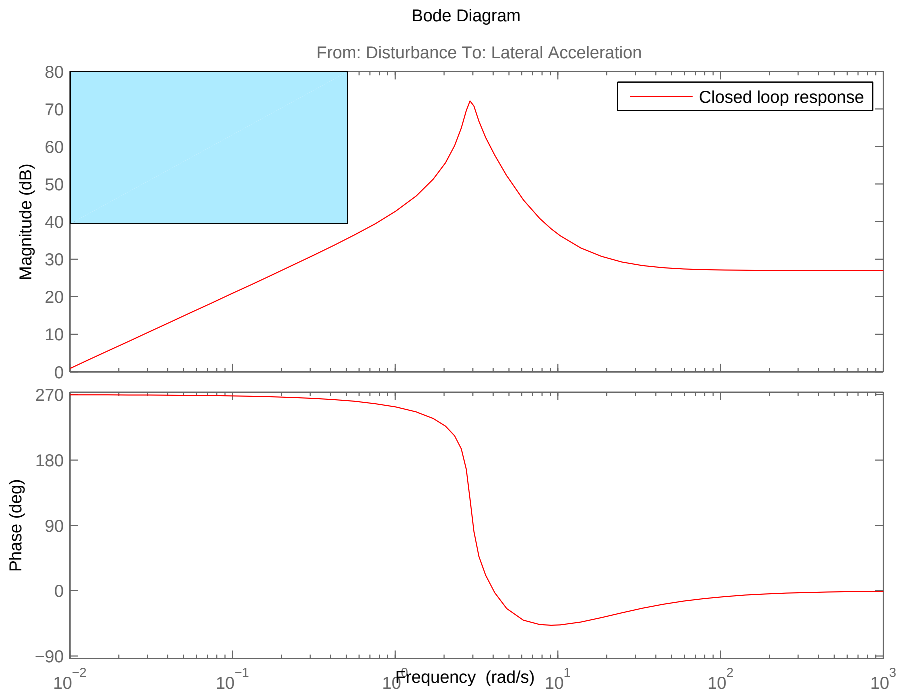

Figure 10: The bode plot of the transfer function from the disturbance to the lateral acceleration. The box represents the constraints on the lateral acceleration and the maximal road curvature as well as its frequency. The constraints are very close to the actual curve.

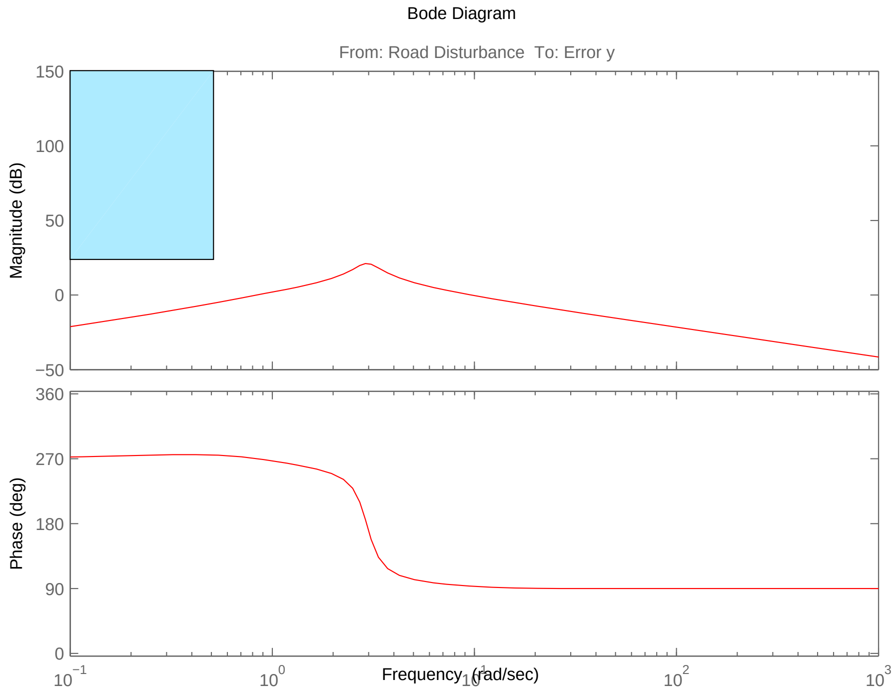

Figure 11: The bode plot of the transfer function from the disturbance to $e_y(t)$. The box is the constraints on $e_y(t)$.

6 Model Predictive Control Design

This section will describe what a Model Predictive Control (referred to as MPC) is, and describe the MPC formulation used in this thesis.

MPC is a control technique primarily found and used in the process- and petrochemical industry where the methodology where proposed and developed. Some of the earliest mentions of predictive control can be found in articles from the 1970's (Richalet et al., 1978; Garcia et al., 1989).

6.1 Introduction

Model Predictive Control is a control type designed to control constrained, multivariable, systems. As the name indicates MPC is a model based control scheme, where the differences between some plant model and the actual plant affects the control performance.

An MPC is a constrained optimal control technique. MPC minimises a cost function subject to the system dynamics and a set of state and input constraints. Essentially this means that a dynamic system model is fed previous data, state and input constraints and some input trajectory. A cost function $V(k)$ is minimised to find whatever input trajectory presents the best results, and helps the system follow some specific reference path. The result is the optimal control trajectory (Bemporad and Morari, 1999).

6.2 The Plant Model

Considering a linear state space model,

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + \Gamma d(k), \\ y(k) &= Cx(k) + Du(k), \end{aligned} \tag{6.1}$$

where $x(k)$ is the state vector, $u(k)$ is the control signal, $y(k)$ is the output, $d(k)$ an input disturbance and A, B, C, D, Γ are matrices of appropriate dimensions. Assume that the system is augmented with

$$\Delta u(k) = u(k) - u(k-1), \tag{6.2}$$

now $\Delta u(k)$ describes the difference between the control signals, the increment, instead of the absolute value (Åström and Wittenmark, 1997; Camacho and Bordons, 2003). To account for this, the State Space model is redefined to include an additional state describing the relation between the current and previous control signal, i.e.,

$$\tilde{x}(k) = \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix}, \tag{6.3}$$

which results in the following model:

$$\begin{aligned} \tilde{x}(k+1) &= \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} B \\ I \end{bmatrix} \Delta u(k) + \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} d(k), \\ \tilde{y}(k) &= \begin{bmatrix} C & 0 \end{bmatrix} \tilde{x}(k) + D\Delta u(k), \end{aligned} \tag{6.4}$$

where I is the identity matrix. This can be written as

$$\begin{aligned} \tilde{x}(k+1) &= \tilde{A}\tilde{x}(k) + \tilde{B}\Delta u(k) + \tilde{\Gamma} d(k), \\ \tilde{y}(k) &= \tilde{C}\tilde{x}(k) + \tilde{D}\Delta u(k), \end{aligned} \tag{6.5}$$

where

$$\tilde{A} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}, \tilde{B} = \begin{bmatrix} B \\ I \end{bmatrix}, \tilde{C} = \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} . \tilde{D} = D.$$

6.3 Problem Formulation

At the time k, some control sequence u^*, from $k, ..., k+H_p-1$ minimize the cost function $V(k)$ subject to the plant model and the constraints acting on the system. Where H_p is the control horizon. The first control sequence, $u^*(0)$ is used as a control input for the plant model to gain

$$x(k+1) = Ax(k) + Bu^*(0), \tag{6.6}$$

where $x(k+1)$ updates the optimization problem for the next time step. Using only the first control signal in u^* is called Receding Horizon Control (Glad and Ljung, 2000; Morari and Lee, 1998).

6.4 Constraints and Cost Formulation

For this specific project the constraints used are the same ones that were presented in Section 4 . They are used to form the following input and output constraints,

$$\begin{aligned} x(k) &\in X, \\ u(k) &\in U. \end{aligned} \tag{6.7}$$

The considered cost function for the MPC problem formulated is reported in Eq. 6.7 . The cost function takes the two errors $e_y(t)$ and $e_\psi(t)$, and the steering angle.

$$V(k) = \sum_{i=0}^{H_u} e_y(k)^2 Q_y + e_\psi(k)^2 Q_\psi + \delta^2 R \tag{6.8}$$

$$\min V(k):$$

$$\begin{aligned} x(k+1) &= f(x, \delta, c_0) \\ c_0(k) &\in [-0.002, 0.002] \\ a_y(k) &\in [-0.2, 0.2] \\ e_y(k) &\in [-0.15, 0.15] \\ \delta(k) &\in [-0.1, 0.1] \\ \dot{\delta}(t) &\in [-0.1, 0.1] \\ \delta(k) &= \delta(t) \\ x(k) &= x(t) \end{aligned}$$

7 Simulations

This section will present the simulations performed, it will compare the output feedback control (Section 5.2) with the MPC (Section 6.4) by different test cases.

7.1 Test Cases

The vehicle considered in the simulations has the following parameters.

Test Vechile

$C_{\alpha f}$	$75700\,\mathrm{rad}^{-1}$
$C_{\alpha r}$	$75700\,\mathrm{rad}^{-1}$
I_z	$90000\,\mathrm{kgm}^2$
m	$15000\,\mathrm{kg}$
l_f	$3.045\,\mathrm{m}$
l_r	$1.755\,\mathrm{m}$

The road first goes straight then after for some time, then the road left. After 20 s the vehicle reverse its turning direction. The curvature of the road is $c_0 = \pm 0.002$. The path is plotted in Fig. 12 .

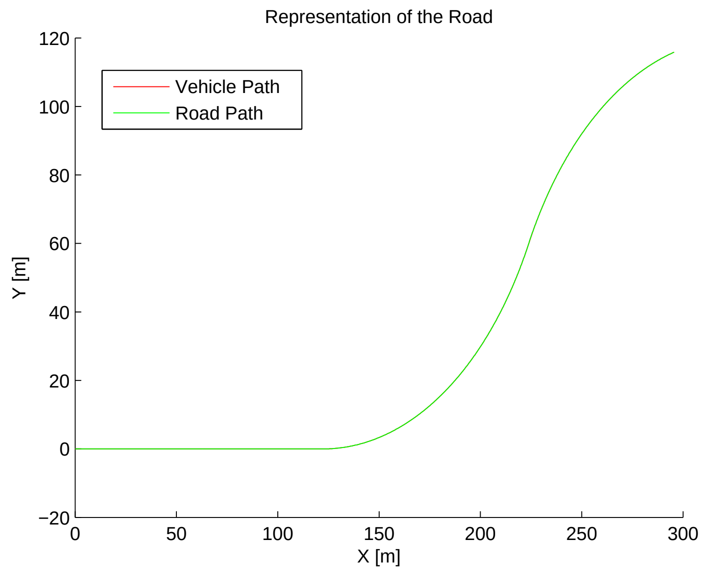

Figure 12: A comparison of the vehicle's path and the road. This should give an idea of how the experiment is performed. This example comes from the experiments in Section 7.2.1 at 30 km/h.

All the simulations are done in MATLAB and Simulink. MATLAB is used for loop-shaping. Simulink is used as a testing environment. Moreover, the MATLAB Model Predictive Toolbox is used. The controller will be tested in the same Simulink environment as the classical controller.

7.2 Output Feedback Controller Simulations

These experiments shows how the classical controller behaves and are used to determine whether the constraints are exceeded. This simulation serves as a baseline to show how a classical controller would behave during such a situation.

7.2.1 Output Feedback Controller

This controller demonstrates how the vehicle behaves when regulated by an output feedback controller. The vehicle is travelling on the road described in Section 7.1 . The control law used for the controller are:

$$k_1 = 0.9 + 0.5s + \frac{3.9 + 3s}{s} \tag{7.1}$$

$$k_2 = 0.01 + \frac{1s + 0.1}{1 + 0.9s} \tag{7.2}$$

The result of the simulation are presented in the following figures. In this simulation the vehicle is travelling at 5 km/h, 30 km/h and 30 km/h.

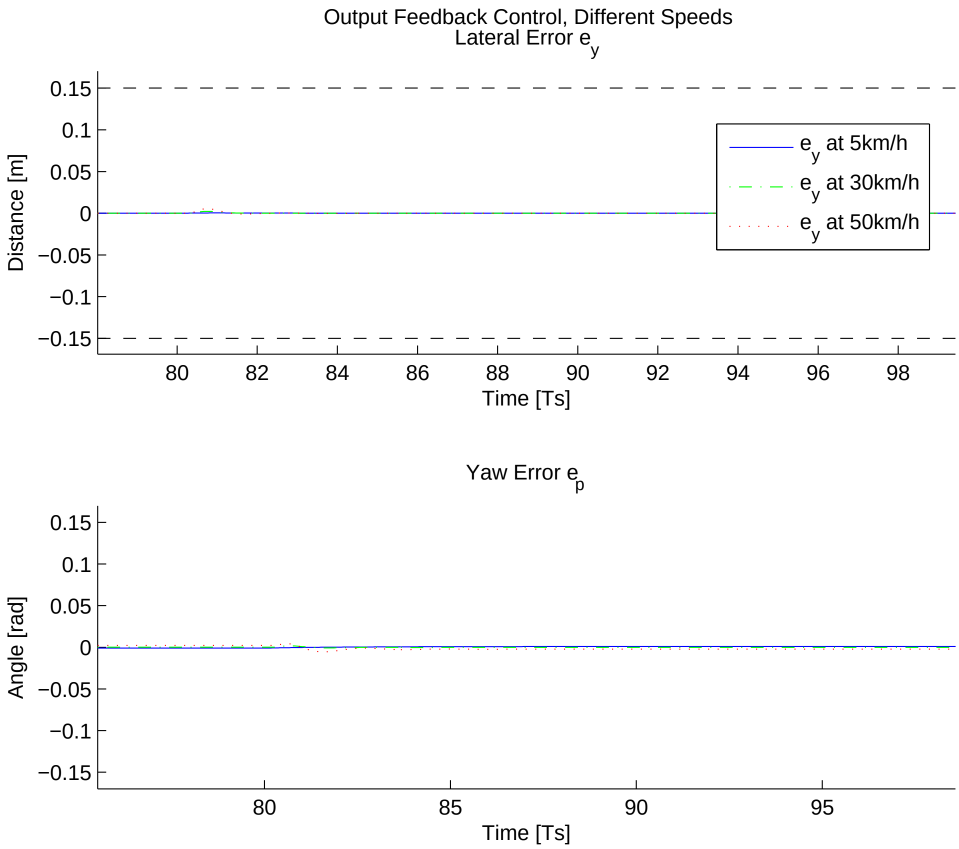

Figure 13: The vehicle's position and orientation offset w.r.t. the centre-line. The vehicle is not allowed to have an error greater than 0.2 m. *Notice that the vehicle stays in the lane for all three speeds.*

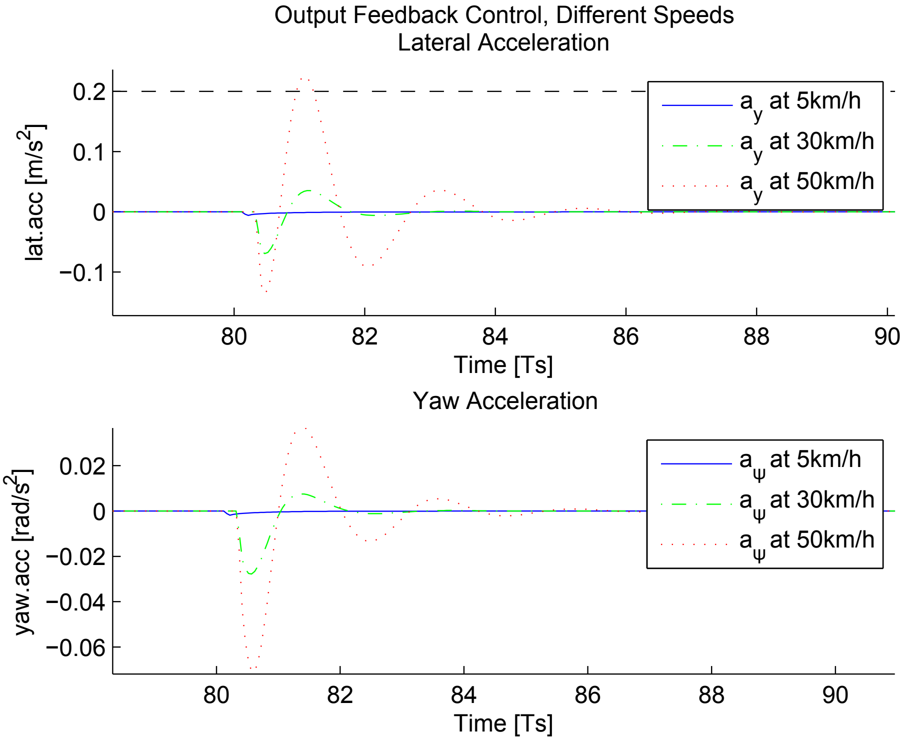

Figure 14: The acceleration spikes that comes with the change in curvature. Notice that the acceleration is constant after the initial change in curvature. In the case of 50 km/h *the vehicle exceeds the acceleration constraints.*

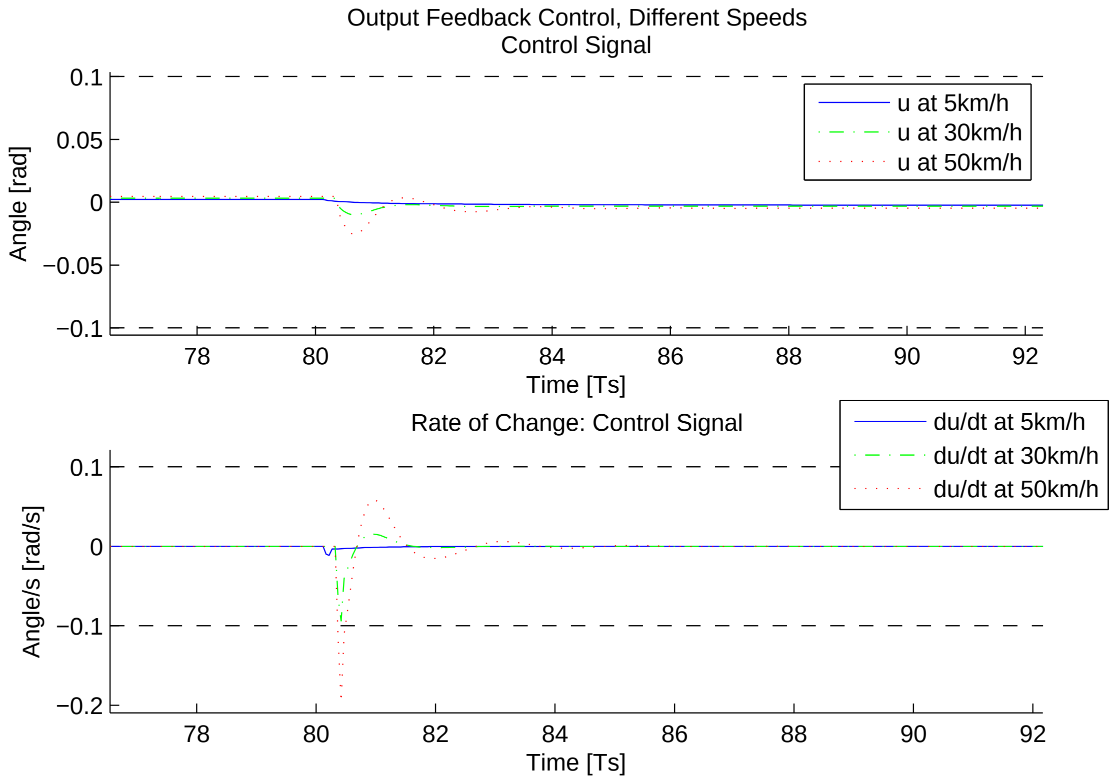

Figure 15: The control signals and the rate of change in the control signal. It can be seen that it grows outside of the set constraints. This is an example of the problems with tuning a feedback controller.

The controller keeps the vehicle inside its lane. But at 50 km/h the vehicle exceeds the constraints for rate of change in the control signal and the lateral acceleration.

7.3 Model Predictive Control Experiments

The simulations are done to compare the performance of different predictive controllers.

7.3.1 LQ Controller

This section will present how an LQ controller performs, an LQ controller is the same thing as a MPC without constraints and with an infinite prediction horizon

H_p. The unconstrained simulation will only be presented from the 30 km/h case. The road will be the same as in the other cases.

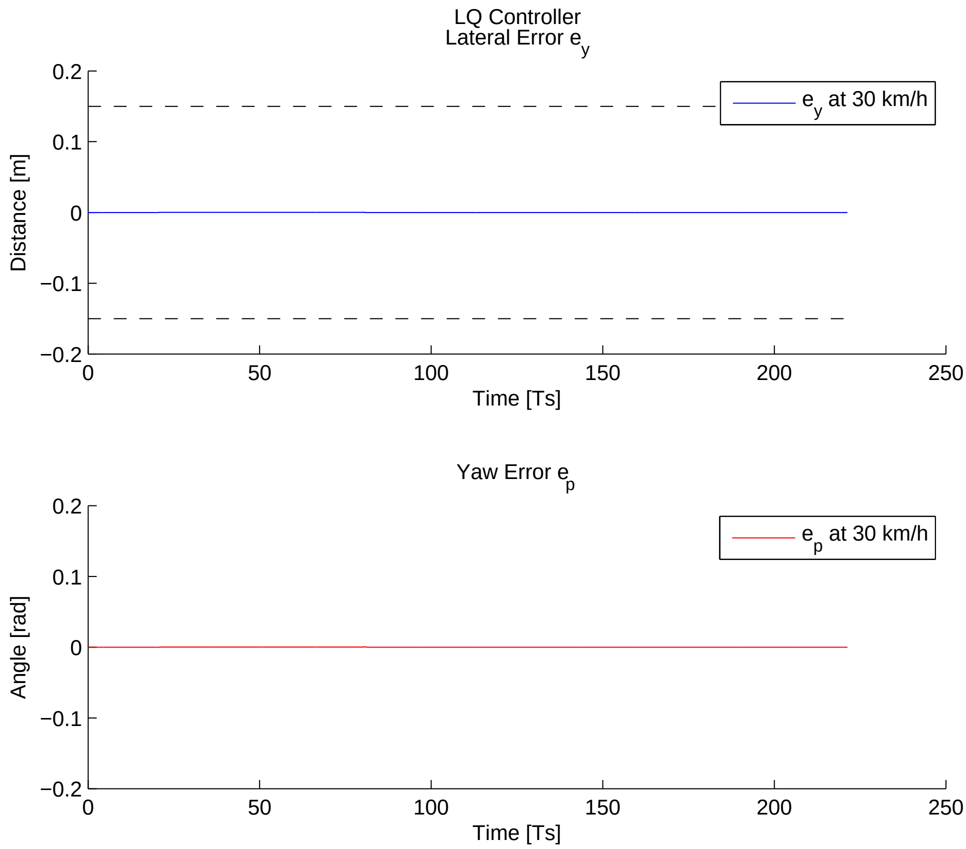

Figure 16: The errors $e_y(t)$ and $e_\psi(t)$ against the road. Notice how small the error are. $e_y(t)$ is well within the constraints at 30 km/h.

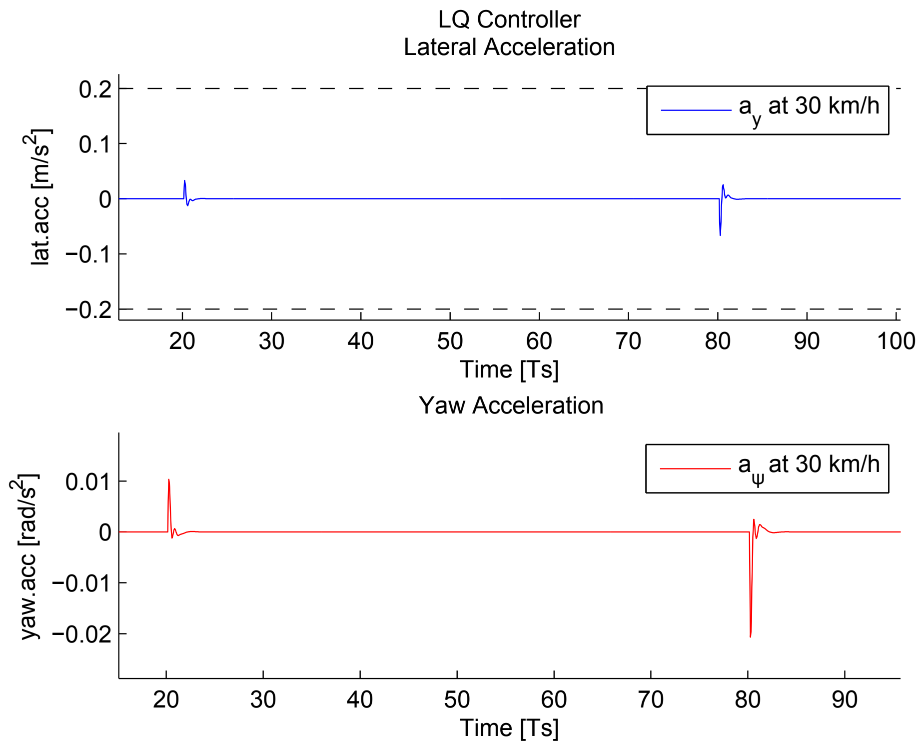

Figure 17: The acceleration corresponding to the road change, notice how it keeps within the limits of the controller.

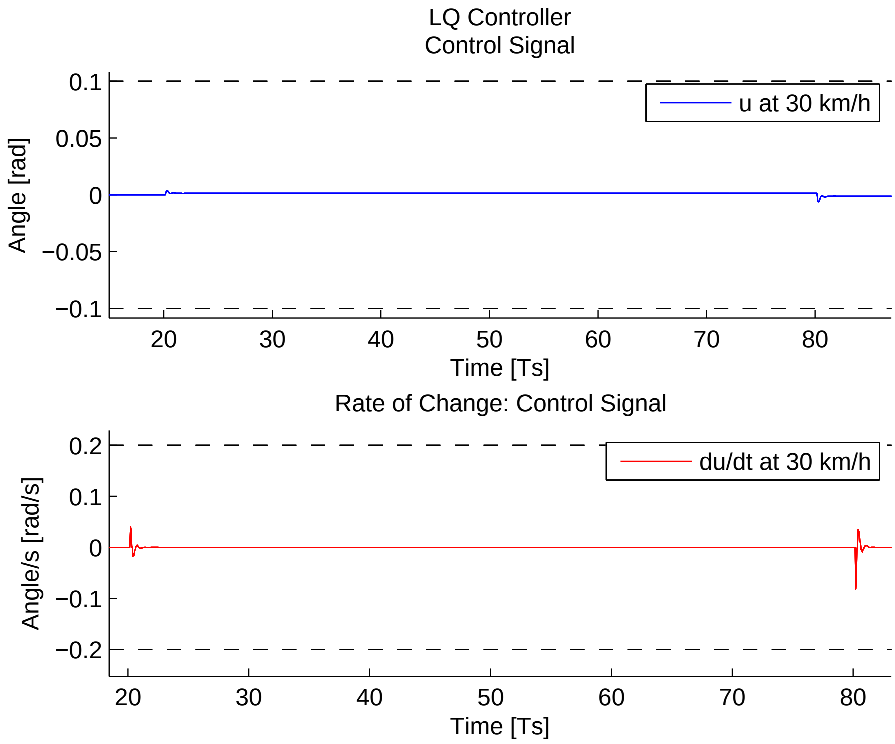

Figure 18: In yet another scenario the predictive controller does not actually seem to need any constrains to be "under" the set constrains.

From Fig. 16 , Fig. 17 and Fig. 18 it can be seen that the boundaries in this scenario are not exceeded. Even without any kind of soft or hard constraints.

7.3.2 MPC with speed comparison

This simulation will be used to measure how an MPC will handle the same scenario as the previous controller. To investigate how hard constraints affect the problem. The simulation is done on the same road as previous simulations, where $c_0 = 0.002$ and the road turns first in one direction, then switches to the opposing side after some time. The velocities used in these simulations are 5 km/h, 30 km/h and 50 km/h. It is desirable to see how the speed affect the controller, if it has a significant impact on the errors and the passenger comfort.

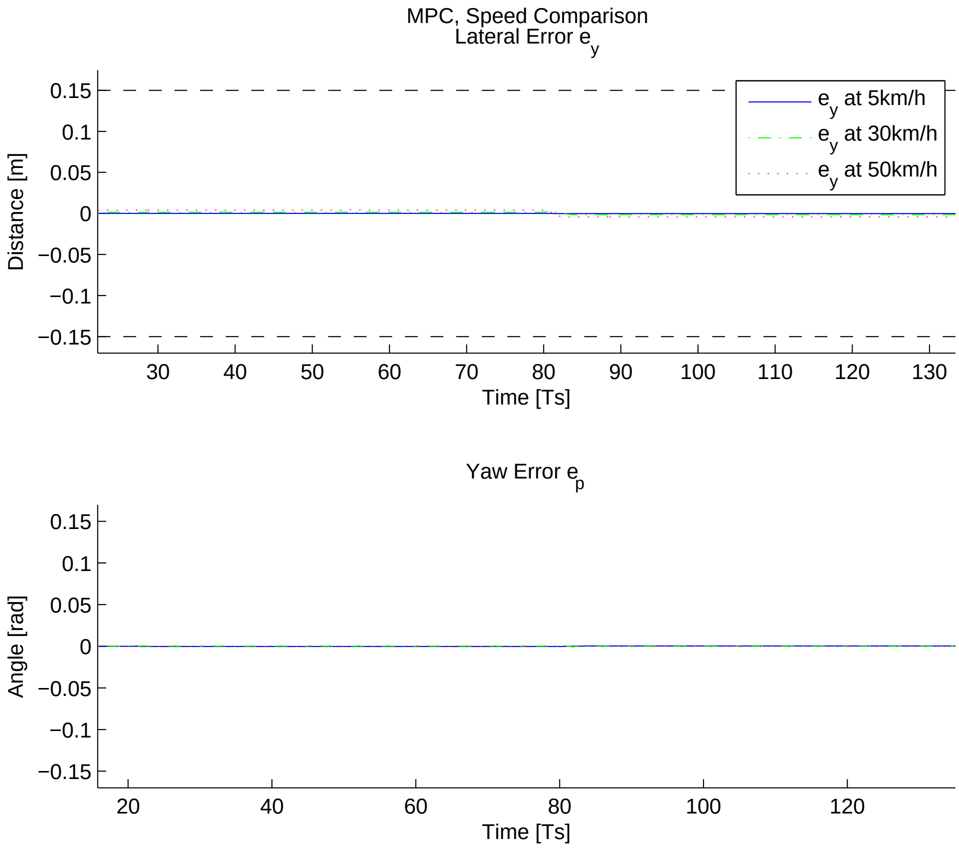

Figure 19: The road side error $e_y(t)$ and the angle error $e_\psi(t)$ at the different speeds: 5 km/h, 30 km/h and 50 km/h. It is important to remember that MPC is an optimal control and as such, you can not expect a linear behaviour because of different speeds. However none of the different tests went out of bounds, it still shows that there will be differences with varying conditions.

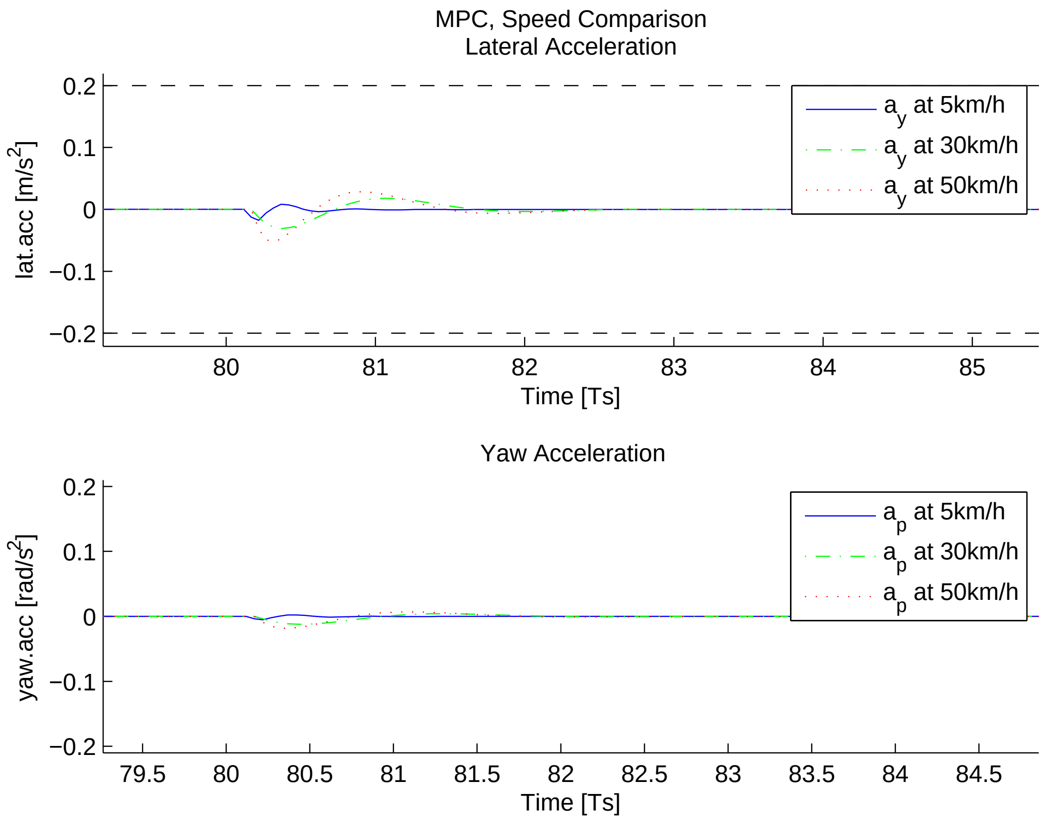

Figure 20: Comparison of the acceleration at some different speeds: 5 km/h, 30 km/h *and* 50 km/h. *Note: this figure only shows the data from the second turn in the road.*

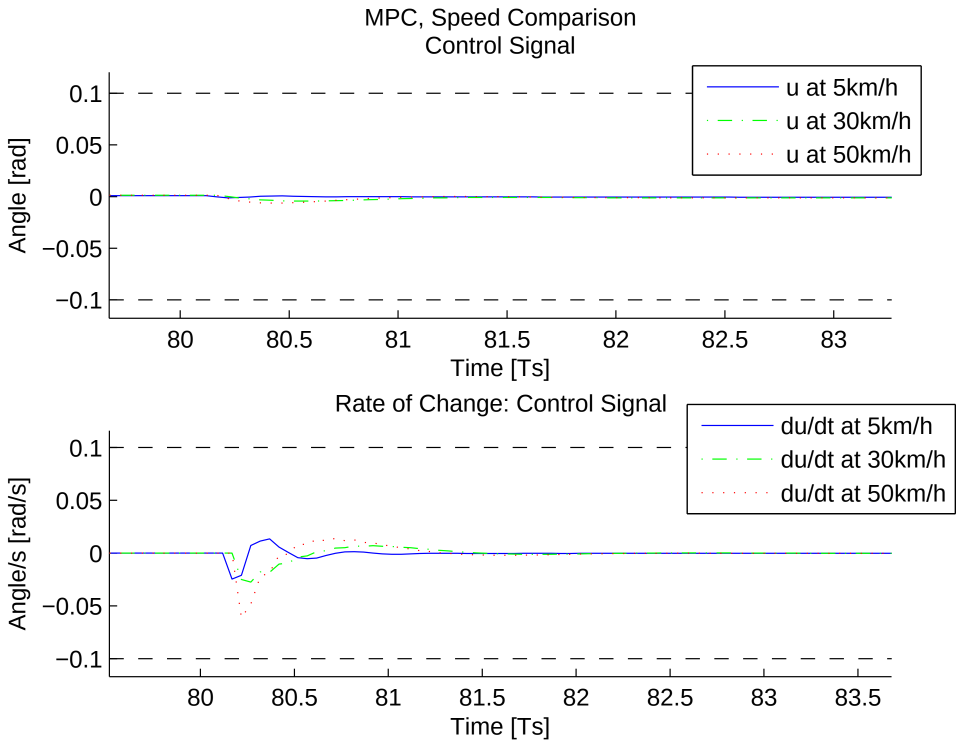

Figure 21: How the control signals differs in the different speed scenarios.

In this case the MPC handled all the cases and they stayed within the bounds. With each increase in speed the acceleration and change of control signal became slightly larger.

7.3.3 LQ with delay

To further see if constraints are necessary or not the unconstrained MPC was tested in a scenario with an added input delay. This was done in a simulation with a model with an input delay of 0.3 s. The simulation is conducted on the same road as before with a road curvature of $c_0 = 0.002$. The vehicle is travelling at a speed of 30 km/h. The delay was included in the controller, so it is tuned with a delay in mind.

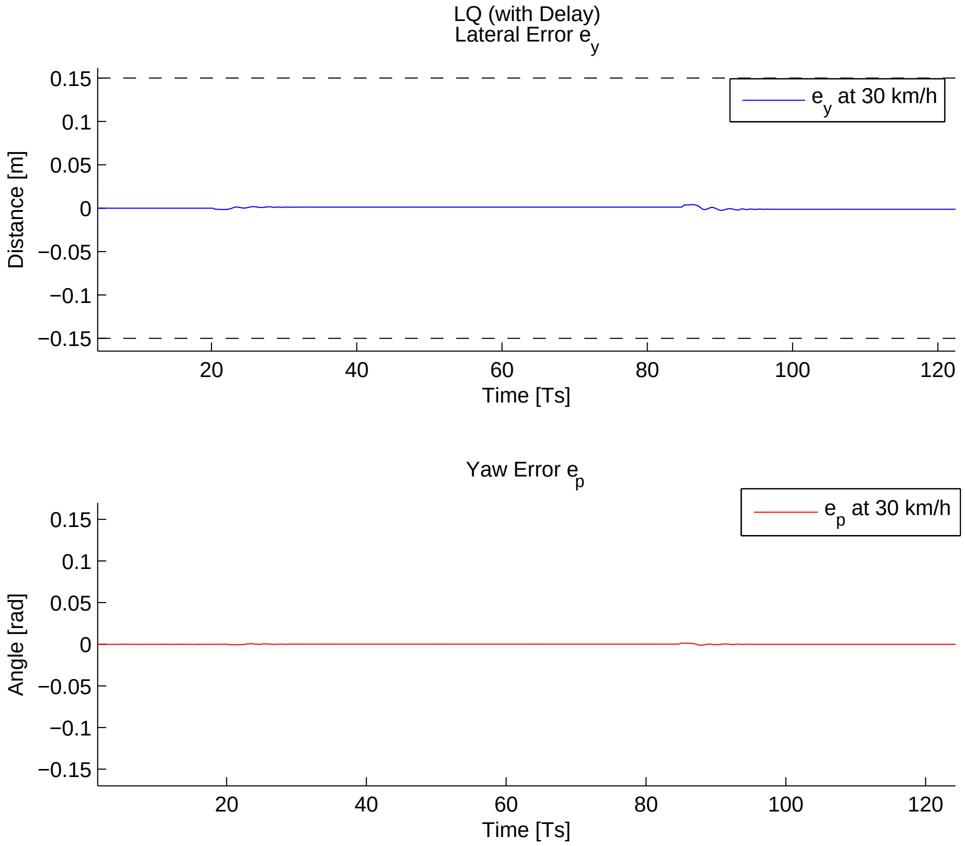

Figure 22: How an MPC without constraints handles the road when a delay of 0.3 s *is present. The vehicle behaves within constraints.*

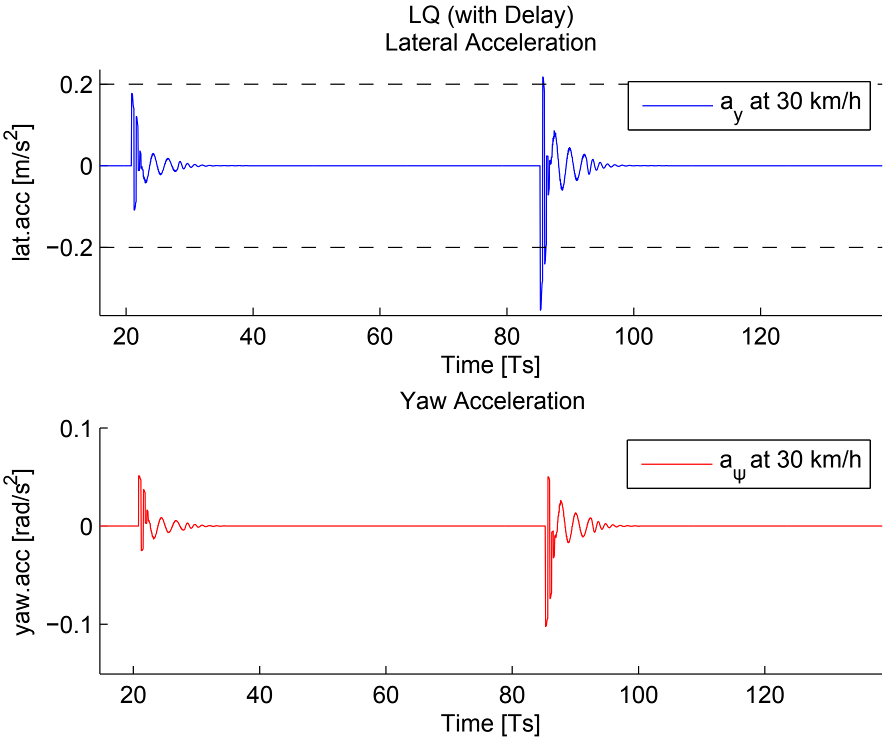

Figure 23: The acceleration does not stay within its bounds. Notice the oscillations.

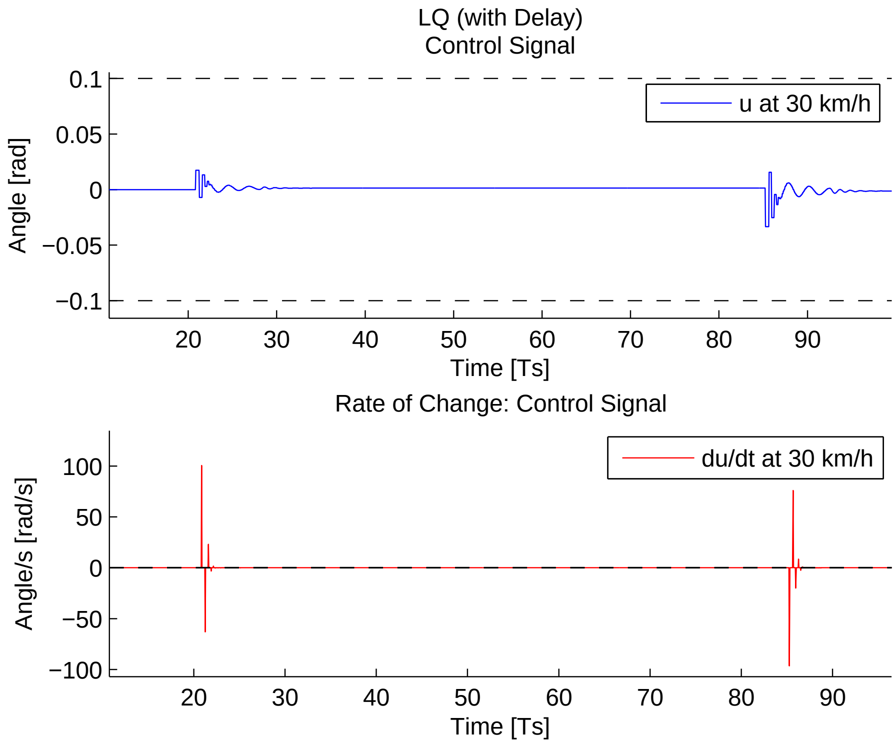

Figure 24: The change in the control signal is out of bounds by a large margin.

The biggest problem with this simulation was the oscillatory behaviour that can be seen. The controller exceeds the constraints on the rate of change of the control signal, by a large margin. But also the acceleration increased.

7.3.4 Constrained MPC with delay

This simulation is done to show how a MPC controller with constraints handles an added input delay. The input delay is added to represent the steering actuator that was originally planned to be a part of this thesis. This simulation is conducted on a road that first turns in one direction and then is turned in the other direction with $c_0 = 0.002$. The vehicle is travelling at a speed of 30 km/h. Compared to the previous case, this one is constrained. The most interesting constraint ought to be the rate of change of the control signal. The controller is detuned with the delay in mind.

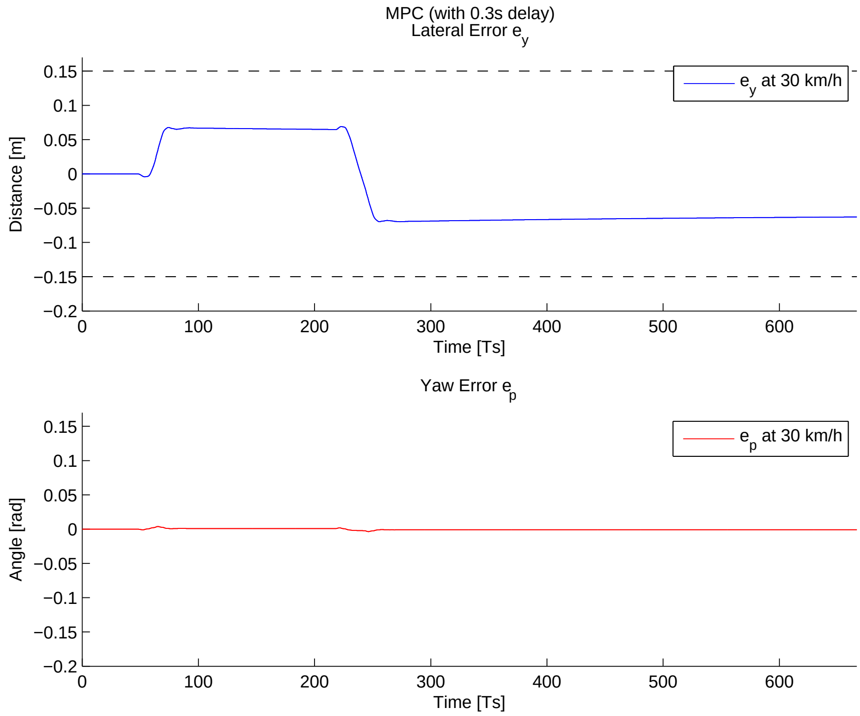

Figure 25: The Figure shows how the delayed MPC relates to the road. It does not go out of its constraint bounds. The delay added was 0.3 s*.*

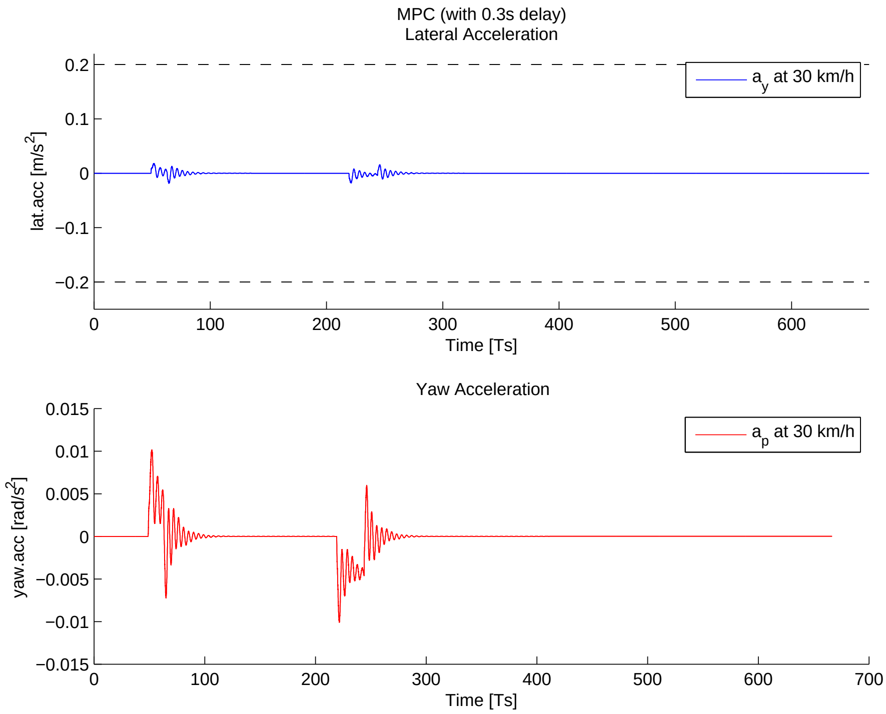

Figure 26: The acceleration in this case is also kept at an acceptable range. Note the difference in oscillation compared to the other case.

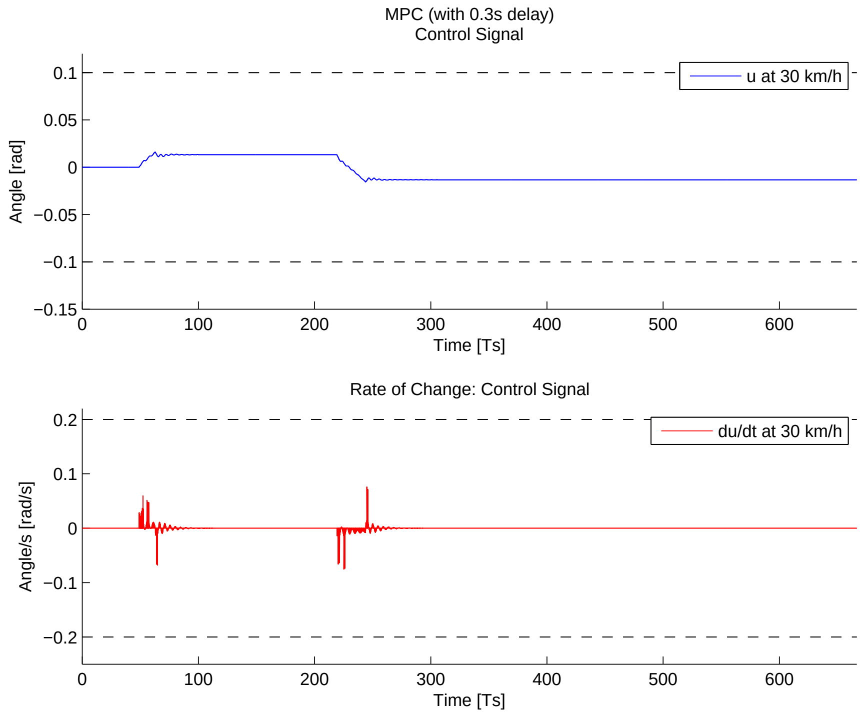

Figure 27: This simulation shows the biggest difference between the two cases. The change in the control signal is much smaller in this case.

There is more oscillations present in this case, however all the variables are kept within bounds. As such, the MPC handles the delay without problem.

8 Discussion

The model primarily chosen due to the fact that it presented the lateral acceleration and yaw acceleration. This as opposed to an simpler model that was considered with a different set of states. The model that was used is quite complicated compared the complexity of the problem itself. This presented some limitations, which might or might not have been present with another choice of model. For example, the classical controller would be significantly easier to tune, when a simpler model was used.

The work structure of this thesis was in the beginning of the project geared toward doing some testing with a real vehicle. On a real vehicle it would be easier to capture the real dynamics of the system. However, due to time constraints on the project, the controllers were not tested on a real vehicle. Any kind of insight into the dynamics of this systems come from intuition and simulations alone. Because of that, an even better simulation environment could perhaps have benefited the project.

The steering actuator also provided difficulties. Initially the project was created with the steering actuator as a part of the model. The idea was to simply represent it with a filter and a delay. Due to time constraints the actuator was further simplified and represented by the delay. The problem with modelling a steering actuator was that a lot of existing models are non-linear and quite advanced due to the combination of hydraulics and electronics. However with lack of testing data it was not possible to properly model the actuator in the vehicle.

The classical control approach that was used in this thesis was to use an output feedback controller. The controller could not handle large changes, E.g. if the vehicle would have taken more load (a fully loaded vehicle versus an empty vehicle for example), or a vehicle at different speeds, affecting its performance. This could have been solved by creating some kind of law for the creation of the control parameters. This in itself would mean a more advanced controller.

Another potential solution for this problem (that was later used) was the MPC. The MPC controller was not chosen to deal with an environment with changing speed, but with the use of MPC the problem was solved. Because MPC always re-calculates trajectories and goes for an optimal approach.

The MPC, was primarily chosen as a means to handle all the constraints. With a classical controller the addition of more constraints added further complications when it came to tune the controller. The MPC is specifically designed to deal with such cases and with constraints. However as it was later shown in the simulations, the constrained MPC was not directly beneficial to use in most scenario. However it could be argued that the constraints added would be needed in a real scenario, but this is only an assumption. It is also worth noting that the constraints themselves might have to be adjusted to better represent the reality. In this study they were more used as a proof of concept.

The classical control and the optimal control was never really compared. However the most interesting results came from the comparison with different velocities for the MPC and the addition of a delay. Whilst the added delay showed that there could be even more tuning done to further improve on the predictive controller's results.

The output feedback controller could probably be improved to better handle delays and re-calculate its control law each instance, with that being the case it is probably a better solution to the problem than an MPC. The best motivation for using an MPC is still to easily deal with constraints.

8.1 Future Works

As stated earlier, there was a few things left out that could have benefited this project. Especially real life testing on a vehicle platform. More testing would provide important information, and would introduce another range of problem: how do you properly implement the MPC on the vehicle computer. Also it would raise the question of what MPC software should be used.

Another area that need further work would be the model of the steering actuator, it was originally a goal of this thesis. That objective was not fulfilled. A model need to be created and it needs to be validated against the real vehicle. There is a high probability that this model will be non-linear and as such this will introduce new problems, when used together with a predictive controller. Then it would be possible to properly introduce torque as input to the system, and as such the ability to limit it.

9 Conclusions

The following are some of the conclusions that was drawn from this project.

- A bicycle model was sufficient to model the vehicle, but had limitations.
- Loop-shaping is a viable solution, but it was hard to use due to constraints; more constraints would create more problems.
- An Output-Feedback controller worked, but would need additional structure to work better at different speeds and settings.
- The lateral control problem could be solved using an MPC. However MPC does not seem necessary to solve a lane keeping problem at low speeds.
- The Predictive Controllers could handle the constraints , although a non-constrained controller also worked well when no delays were present.
- With the addition of a delay, the MPC was suitable.
- The MPC was not implemented on a truck as stated in the goals.

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