Numerical Modelling of Coastal Defence Structures

Overtopping as a Design Input Parameter

Master’s Thesis in the International Master’s Programme Geo and Water Engineering

MATTHIEU GUÉRINEL

Department of Civil and Environmental Engineering
Division of Water Environment Technology
Engineering Hydraulics
CHALMERS UNIVERSITY OF TECHNOLOGY
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SUPERVISORS:
Matthew Batman, Ramböll Sverige AB
Lars Bergdahl, Chalmers Tekniska Högskola

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Department of Civil and Environmental Engineering
Division of Water Environment Technology
Engineering Hydraulics
Chalmers University of Technology
SE-412 96 Göteborg
Sweden
Telephone: + 46 (0)31-772 1000

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Wave overtopping of a seawall at Hartlepool, United Kingdom.

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ABSTRACT
Breakwaters are coastal defence structures which are widely spread around the world and are used to absorb or dissipate the energy of the incoming waves in order to insure the water’s tranquillity behind the breakwater. There are many different types of breakwaters, but two main families of breakwaters dominate: sloping and vertical structures. Different phenomena are taken into account in the breakwaters’ design process, which are normally guided by the incoming waves’ action. This project focuses principally on the overtopping, as well as on the run-up, but also on the structure’s stability.

After acquiring in-depth knowledge regarding the breakwaters’ design, a numerical model is developed to calculate the mean overtopping discharge as an input parameter in order to get the structure’s optimum height. The stability of the structure is then calculated, thus completing the design process.

The initial work involved setting up models, for all possible types of structures, of the overtopping discharge that occurs on a structure, and has been validated by comparing the results with existing models and structures. Parallel to this, structure stability models have also been developed and validated the same way.

Once the models were validated, they were used as a basis to perform the calculations in reverse order and use limit overtopping discharge values as input parameters. It is thus possible to see how important the breakwater’s functionality definition is regarding the final dimensions of the structure. Besides, it also allows the check of the different structures’ responses to overtopping, depending on their type and on the sea parameters.

Finally, an economical dimension was added to the models in order to compare the different structures’ prices. Furthermore, an extension of the comparisons to other parameters than overtopping is made using the same models. Having the prices of the different structures for a project can thus be used as a support for the decisions makers.

Key words: Coastal defence, breakwater, overtopping, wave run-up, stability, sloping structure, vertical structure, numerical modelling
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Preface

This project has been carried out in cooperation with the Water Environment Technology Division at Chalmers University of Technology, and the Bridges and Harbours division of Ramböll Sverige AB in Göteborg, from January to June 2009. It has been initiated by Mr Matthew Batman and Mr Sten Munthe from Ramböll Sverige AB, and Professor Lars Bergdahl from Chalmers University of Technology.

The study focuses on the overtopping of coastal defence structures under waves attack, but also on the stability of these structures. Models have been developed in order to design and estimate the price of different types of breakwaters, which can be used as a decision support for the choice of the type of structure to realise.

Göteborg May 2009
Matthieu Guérinel
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Last, but not least, I am particularly indebted to Mr René-Paul Angot for transmitting me his passion of civil engineering and hydraulics, and without whom these fields might have stayed a mystery to me.
Notations

Roman upper case letters

$A_0$ Coefficient for the wave momentum flux parameter
$A_1$ Coefficient for the wave momentum flux parameter
$A_c$ Armour crest size
$B$ Sloping structure’s berm width
$B_p$ Width of the upright section
$B_M$ Rubble-mound foundation’s berm width
$C_r$ Armoured crest berm reduction factor
$D_n$ Nominal stone diameter
$D_{50}$ Median rocks nominal diameter
$F_G$ Weight force of the vertical structure
$F_{wave}$ Wave force
$G_c$ Armoured structures’ crest berm width
$H$ Wave height
$H_{m0}$ Spectral significant wave height
$H_{max}$ Maximum significant wave height
$H_s$ Significant wave height
$K_D$ Stability coefficient
$L$ Wave length at the depth $h_b$
$L'$ Wave length at the depth $h'$
$L_{berm}$ Horizontal length between $-1.0 \cdot H_s$ and $1.0 \cdot H_s$
$L_{m-1,0}$ Spectral wave length
$M_{50}$ Medium mass of rocks
$M_D$ Depth-integrated wave momentum flux
$M_G$ Vertical structure’s weight moment
$M_{Hz}$ Wave’s moment at the bottom of the vertical structure
$M_{Hzfinal}$ Final wave’s moment at the bottom of the vertical structure
$M_{up}$ Moment of the uplift force
$M_{upfinal}$ Final moment of the uplift force
$N$ Stone shape coefficient factor
$N_m$ Melby’s stability parameter
$N_{od}$ Toe’s damage level
$N_{ow}$ Number of overtopping waves during a storm
$N_w$ Number of waves during a storm
$N_{z}$ Number of waves during a storm
$N_s$ Stability coefficient
$P$ Structure’s permeability coefficient
$P_{Hz}$ Total wave pressure on the front of a vertical structure
$P_{Hzfinal}$ Final total wave pressure on the front of a vertical structure
$P_{MF}$ Wave momentum flux parameter
$P_{ov}$ Probability of overtopping per wave
$P_{upfinal}$ Final vertical structure’s uplift pressure
$P_{uplift}$ Vertical structure’s uplift pressure
$P_o$ Overtopping volume’s probability of exceedence
$Q$ Dimensionless overtopping discharge
$R_{u2%}$ 2% exceedence run-up level
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<td>$U_{up}$</td>
<td>Reduction factor for the uplift pressure</td>
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<td>$U_{Mtup}$</td>
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<td>$W$</td>
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**Roman lower case letters**

- $a$: Empirical coefficient for wave overtopping
- $a$: Probability of exceedence function factor
- $b$: Empirical coefficient for wave overtopping
- $b$: Probability of exceedence function factor
- $c$: Overtopping normally distributed function mean
- $c_1$: Run-up empirical coefficient
- $c_2$: Run-up empirical coefficient
- $c_3$: Run-up empirical coefficient
- $d$: Water depth at a rubble-mound foundation
- $d_n$: Coefficient for impulsive conditions on composite structure
- $d_{15\text{core}}$: Grain size diameter exceeded by 15% of the core material
- $d_{25\text{filter}}$: Grain size diameter exceeded by 85% of the filter material
- $d_{85\text{core}}$: Grain size diameter exceeded by 85% of the core material
- $d_h$: Water depth above a sloping structure’s berm
- $g$: Acceleration of gravity
- $h$: Water depth
- $h'$: Water depth at the vertical structure’s bottom
- $h_a$: Coefficient for impulsive conditions
- $h_b$: Water depth at the top of a toe berm
- $h_b$: Water depth at a distance of 5·$H_s$ of the vertical structure
- $h_c$: Vertical structure elevation above SWL
- $h_c$: Height above SWL to which waves pressure is acting
- $h_s$: Water depth at the toe of the structure
- $k$: Reduction coefficient for a parapet
- $p_1$: Wave’s pressure at the SWL
- $p_{1\text{imp}}$: Wave’s pressure at the SWL under impulsive conditions
- $p_2$: Wave’s pressure at the top of a vertical structure
- $p_3$: Wave’s pressure at the bottom of a vertical structure
- $p_e$: Vertical structure’s heel pressure
- $p_u$: Vertical structure’s uplift pressure
- $q$: Mean overtopping discharge
- $q_{parapet}$: Mean overtopping discharge with a parapet
$q_v$  Mean overtopping discharge for a vertical structure
$s_m$  Mean wave steepness
$s_{m-1,0}$  Spectral wave steepness
$s_{mc}$  Critical wave steepness
$t$  Storm duration
$t_e$  Distance on which the heel pressure is exerted

**Greek upper case letter**

Δ  Density of stones in water

**Greek lower case letters**

α  Slope angle
α  Parapet inclination angle
α*  Wave’s pressure for impulsive conditions parameter
α₁  Wave’s pressure parameter
α₂  Wave’s pressure parameter
α₃  Wave’s pressure parameter
α₄  Wave’s pressure for impulsive conditions parameter
α₅  Wave’s pressure for impulsive conditions parameter
α₆  Wave’s pressure for impulsive conditions parameter
α₇  Wave’s pressure for impulsive conditions parameter
α₈  Smooth structures’ top wall inclination angle
β  Wave incidence angle
γₚ  Reduction factor for the presence of a berm
γᵣ  Surface roughness reduction factor
γ_{surfing}  Surface roughness reduction factor for surging waves
γᵥ  Reduction factor for the presence of a vertical wall
γᵦ  Wave incidence angle reduction factor
δ₃  Wave’s pressure for impulsive conditions parameter
δ₁₁  Wave’s pressure for impulsive conditions parameter
δ₂  Wave’s pressure for impulsive conditions parameter
δ₂₂  Wave’s pressure for impulsive conditions parameter
η*  Maximum wave’s pressure elevation above SWL
κ  Vertical structure’s rubble-mound foundation parameter
μ  Friction coefficient
ξ_{m-1,0}  Wave breaker parameter (or Iribarren number)
ξ_{m-1,0c}  Transition breaker parameter
ρ  Fluid mass density
ρₕ  Stone mass density
ρ_{str}  Vertical structure mass density
ρₜ  Water mass density
1 Introduction

Coastal defence structures are often needed to protect maritime structures such as harbours and are thus widely spread all over the world. As they are used as shelters against the waves’ actions, their function is to dissipate or reflect the waves’ energy in order to have still water on their landward side. This ensures for instance the tranquillity of a ship channel or an inlet, or the diminishing of the bed load transport. Despite the fact that they are present in almost all projects of coastal protection, there are still some uncertainties regarding some of their parameters and phenomena to which they are submitted. The reason for this is that the main available data regarding those structures is empirical.

Indeed, during a long time there was no real scientific foundation on the rules the designers were using, and they were thus only relying on their past experiences and observations. However, many experiments have been carried out over the past century in order to get more and more accurate design methods, allowing thus to reduce costs and material consumption, but also risks of structures’ failure.

The attention has been first focused on the stability of the structures, for them to be resistant enough to the waves’ action, but people realised that the structure’s stability should not be the only parameter to look after. Phenomena like waves’ run-up and overtopping also appeared to be considered due to their impact on the structure’s integrity and functionality.

1.1. Background and scope

Wave run-up and overtopping are phenomena that always occurred on coastal defence structures, but they only started to be studied in-depth in the beginning of the eighties, and even more extensively in the past ten years. The wave overtopping action appeared indeed to be decisive for the geometrical structure design.

The purpose of this thesis work is thus to develop detailed knowledge regarding the overtopping process on different types of coastal defence structures, as well as the wave run-up process, since these two phenomenon are antagonistic. Besides, in order to have a complete overview of the coastal defence structures’ design, the issue is also to study the stability of sloping and vertical structures.

As many phenomena are taken into account, the main objective of this thesis is to develop models that permit to realise a complete breakwater’s section design for sloping and vertical structures, depending on the limit mean overtopping discharge to which the structure will be submitted to. This thus gives the optimum design of a section for a given overtopping rate. Furthermore, the utilisation of the models can be extended to other parameters than overtopping, with the idea of using them in order to compare financially the two different types of structures mentioned before. The decisions makers would thus have a better view on the different prices, and might use these results to get to a decision.
1.2. Outline

The basic waves’ parameters needed for the calculation of the waves’ run-up, overtopping and structure stability are presented in part 2.

Then, part 3 describes the wave run-up phenomenon. It presents the empirical theory existing behind the wave run-up on smooth and armoured structures and explains the difference that exist between those two kinds of sloping structures. Nothing appears there concerning the vertical structures since it does not really occur on them. After the waves’ run-up theory follows the wave overtopping definition in part 4. It shows the different theories existing for both sloping and vertical structures and their derivates, the notion of impulsive and non-impulsive breaking for vertical structures, as well as the overtopping volume per wave. Besides, the existing overtopping limit thresholds used for the later design are also presented.

The stability of both sloping armoured- and vertical structures is exposed in part 5. It presents how waves’ parameters are related to the armour’s stones and underlayers’ nominal diameter in the case of sloping structure, but also the pressures’ repartition of the waves on vertical structures and their stability safety factors.

The results of the models in which all the previous theoretical parts were implemented are finally shown in part 6. The method followed to develop the models is described, with some examples of the results obtained. Furthermore the economical aspect is also shown with some comparisons between armoured structures and vertical structures.
2 Wave parameters

As they are the ones which will influence the structure design, waves’ parameters importance is far from being negligible. A wave is usually defined by three basic parameters: its height, its period and its length, and other parameters can thereafter be determined from those.

![Wave parameters diagram](image)

Figure 2.1: Presentation of the wave parameters

2.1.1 Wave height

The wave height $H$ corresponds to the difference in metres between the crest and the trough of a wave, as showed in Figure 2.1. For an irregular wind wave it can be presented in different manners, such as $H_{1/3}$, which corresponds to the average value of the highest third of a group of waves, but also as $H_{m0}$, which corresponds to the spectral significant wave height. In this report, only $H_{m0}$ is used. These values are almost the same in deep water, but in shallow water differences of 10 to 15% can occur, and $H_{m0}$ is more representative. There are some methods that allow calculating a wave height distribution and the significant wave height $H_{1/3}$ based on the spectral significant wave height (Battjes & Groenendijk, 2000).

2.1.2 Wave period

As for the wave height, there are different types of period that might be considered. Indeed, the conventional wave periods are the peak period $T_p$ (period giving the peak of the spectrum), the average period $T_m$ (from a wave spectrum or a wave record) and the average period of the highest third of a group of waves $T_{1/3}$. $T_{1/3}$ and $T_p$ are pretty much the same. Besides, the relation between $T_m$ and $T_p$ usually lies between $0.79 – 0.91$ (Bergdahl, 2008). The period used for the wave run-up and overtopping is the spectral period $T_{m-1.0}$. The advantage of this period is that it gives more weight to the longer periods in the spectrum, being thus more representative of the whole spectrum, and makes the calculations easier when the wave spectra are double-peaked or
flattened. In the case of single-peaked spectrum (which are the most common, for instance Pierson-Moskovitz or JONSWAP), the relationship between \( T_p \) and \( T_{m-1,0} \) is close to the one between \( T_p \) and \( T_m \). Indeed, \( T_p \approx 1.1 \cdot T_{m-1,0} \).

### 2.1.3 Wave length

The wave length corresponds to the distance between two waves. It depends only on the wave period and thus takes the same mark as the period used to calculate it (spectral, peak, average...). The length is approximated as follow:

\[
L_{m-1,0} = \frac{g \cdot T_{m-1,0}^2}{2 \cdot \pi}
\]

Once that these parameters have been determined, it is possible to calculate two other parameters (see section 2.1.4 below) which are crucial for the calculation of the wave run-up and overtopping, as well as for the structure’s stability determination.

### 2.1.4 Wave steepness \( s \) and breaker parameter \( \xi \)

The wave steepness is the ratio of the wave height to the wave length defined as:

\[
s_{m-1,0} = \frac{H_{m0}}{L_{m-1,0}}
\]

The steepness usually varies from 0.01 to 0.06. Values located close to 0.01 are representative of swell seas, with long periods, whereas high steepnesses around 0.06 represent wind generated sea. Depending on the foreshore slope, wind sea wave might break due to the shoaling effect which conducts to a lower steepness, since the wave period does not vary that much when waves break.

The wave steepness is useful in the determination of the breaker parameter, or Iribarren number \( \xi \). This parameter combines the structure slope and the wave steepness. It has been defined by Iribarren as following (d'Angremond & van Roode, Breakwaters and Closure Dams, 2004):

\[
\xi_{m-1,0} = \frac{\tan \alpha}{\sqrt{s_{m-1,0}}}
\]

where \( \alpha \) is the structure’s slope angle [deg].

As for the wave period and height, its subscript depends on the type of period and height used before. In the case of run-up and overtopping, as \( H_{m0} \) and \( T_{m-1,0} \) are commonly used, this parameter will be presented as \( \xi_{m-1,0} \).

The value of \( \xi_{m-1,0} \) informs us on the kind of breaking the structure is going to be exposed to. In figure 2, the different kinds of breaking are presented. The breaking is changing while \( \xi_{m-1,0} \) increases. There are four different types of breaking: spilling, plunging, collapsing and surging. Depending on which kind of breaking occurs on the structure, the run-up and overtopping values will vary, as well as the stability parameters.
After realising all these preliminary questions, it is possible to start the calculation of the wave run-up and the overtopping on the structure, and to complete the design by realising the structure’s stability calculation.
3 Structure wave run-up

Shore protection structures, which have a relatively mild slopes, around 1:1.5 or milder, are always submitted to a first phenomenon called run-up before being overtopped. It is mainly considered in the realisation of dikes and seawalls, but can also be studied for rough slopes, armoured with for example rocks or concrete blocks.

Literally, the run-up corresponds to the point that an incoming wave impacts on a structure, and it is always represented as the vertical difference (in metres) between this highest point and the still water level (also called SWL). In order to be more precise, as all kinds of incoming waves have a stochastic nature, the run-up is presented as $R_{uX\%}$, where $X$ corresponds to the percentage of waves that will actually exceed the wave run-up height (Van der Meer & Stam, Wave run-up on smooth and rock slopes of coastal structures, 1992). It is quite usual to have the run-up height calculated for a 2% level of exceedence.

![Wave run-up level Ru2%](image)

Figure 3.1: Wave run-up level Ru2%

3.1 Run-up on smooth structures

One can see on Figure 3.1 the physical representation of the wave run-up level. $R_{u2\%}$ is the run-up height, $L$ is the wave length, $H_s$ the significant wave height, $h_s$ the water depth at the toe of the structure, and $\alpha$ the slope angle of the structure. As stated before, the 2% run-up value means that there will only be 2% of all the incoming waves that will run over it, even if there is still, and will probably always be, a little bit of uncertainty due to the waves’ stochastic nature. As for the origin of the 2% run-up value, it has usually given good indications regarding the structure design, ever since it has been employed during the 1930’s by Delft Hydraulics. Indeed, they used to consider this value, because it appeared that if only 2% of the waves reach the top of the structure crest, especially in the case of dikes, the crest itself and the structure inner slope would not be affected, and thus would not need any specific protection (Van der Meer & Janssen, Wave run-up and wave overtopping at dikes, 1994).

The wave run-up depends mainly on the wave parameters, as well as on the structure properties. It is usually first calculated as the “relative” run-up, that is to say the 2% run-up $R_{u2\%}$ over the significant wave height $H_s$. Its formulation varied several times with time, depending on the different experiments that have been done.

Its latest expression is written as follows (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007):
\[ \frac{R_{u2\%}}{H_{m0}} = c_1 \cdot \gamma_f \cdot \gamma_\beta \cdot \gamma_b \cdot \xi_{m-1,0} \quad \text{with a maximum of} \] 

\[ \frac{R_{u2\%}}{H_{m0}} = \gamma_f \cdot \gamma_\beta \cdot \left( c_2 - \frac{c_3}{\sqrt{\xi_{m-1,0}}} \right) \]  

\[ (3.1) \]

Where:

- \( R_{u2\%} \): Run-up height exceeded by 2\% of the incoming waves [m]
- \( H_{m0} \): Significant wave height [m]
- \( c_1, c_2, c_3 \): Empirical coefficients [-]
- \( \gamma_f, \gamma_\beta, \gamma_b \): Influence factors for the slope surface roughness \( (f) \), the wave incidence angle \( (\beta) \), and the presence of a berm \( (b) \) [-]
- \( \xi_{m-1,0} \): Breaker parameter (or Iribarren number) [-]

The breaker parameter is here an important parameter because the run-up evolution can change considerably if the waves are breaking or not (see Figure 3.2).

Equation (3.1) is the basic equation for determining the run-up, but the three empirical coefficients can vary. Indeed, there are two different formulae, derived from equation (3.1), which can be used. One can choose to define the wave run-up according to a probabilistic way or a deterministic way:

- For the deterministic design:
  
  \[ \frac{R_{u2\%}}{H_{m0}} = 1.75 \cdot \gamma_f \cdot \gamma_\beta \cdot \gamma_b \cdot \xi_{m-1,0} \quad \text{with a maximum of} \] 
  
  \[ \frac{R_{u2\%}}{H_{m0}} = 1.00 \cdot \gamma_f \cdot \gamma_\beta \cdot \left( 4.3 - \frac{1.6}{\sqrt{\xi_{m-1,0}}} \right) \]  
  
  \[ (3.2) \]

- For the probabilistic design:
  
  \[ \frac{R_{u2\%}}{H_{m0}} = 1.65 \cdot \gamma_f \cdot \gamma_\beta \cdot \gamma_b \cdot \xi_{m-1,0} \quad \text{with a maximum of} \] 
  
  \[ \frac{R_{u2\%}}{H_{m0}} = 1.00 \cdot \gamma_f \cdot \gamma_\beta \cdot \left( 4.0 - \frac{1.5}{\sqrt{\xi_{m-1,0}}} \right) \]  
  
  \[ (3.3) \]

One can notice that the difference between the two factors is not so big, but it can still have an impact on the wave run-up level and thus the structure crest height \( R_c \), especially since this is the relative wave run-up which is calculated in these equations. That is why it is advised to use the deterministic design formula when it is about assessing real-case structure wave run-ups, since it includes a safety margin from the probabilistic design (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007). The probabilistic formula is preferred when it is question of comparing some experiments measurements with the theory, with some lower and upper exceedance lines for example.
Figure 3.2 shows the draw of equations (3.2) and (3.3), and it appears clearly that using the deterministic design formula (upper curve) will lead to slightly higher values and thus to a safer design. Besides, one can notice the difference in steepness in both of the curves around $\xi_{m-1,0} = 1.75$, which corresponds to the transition between breaking waves and non-breaking waves (sometimes noted $\xi_{tr}$). The first part of the curves corresponds to the breaking waves, such as spilling or plunging waves, for which the relative run-up rises quickly. The second part of the curves, more smooth, corresponds to the surging waves, which are not considered to be breaking, and whose behaviour when arriving on the structure does not evolve considerably regarding the increase of $\xi_{m-1,0}$. The difference between the two curves can seem small, but as it is a ratio which is represented, the final difference between two different run-up levels can be quite important.

### 3.2 Run-up on armoured structures

For both smooth sloped structures and armoured sloped structures the relative run-up formulae have the same form. There is only a change in the surface roughness reduction factor, as follow:
\[ \frac{R_{u2\%}}{H_{m0}} = 1.75 \cdot \gamma_f \cdot \gamma_{\beta} \cdot \gamma_b \cdot \xi_{m-1,0} \quad \text{with a maximum of} \]

\[ \frac{R_{u2\%}}{H_{m0}} = 1.00 \cdot \gamma_{fsurging} \cdot \gamma_{\beta} \cdot \left( 4.3 - \frac{1.6}{\sqrt{\xi_{m-1,0}}} \right) \]  

(3.4)

\[ \gamma_{fsurging} = \gamma_f + (\xi_{m-1,0} - 1.8) \cdot \left( 1 - \frac{\gamma_f}{8.2} \right) \]

\[ \gamma_{fsurging} = 1.0 \quad \text{for} \quad \xi_{m-1,0} > 10 \]

Equation (3.4) corresponds to the deterministic design formula. As one can see in it, in the case of armoured structure, one has to take into account the waves surging property. If the structure is permeable then the relative run-up will reach a maximum value of 2.11, because the water can penetrate the structure core. But if the structure is impermeable, then the waves surging effect makes them only going up and down on the structure’s slope, and thus induces that there is always water in the armour layer (Van der Meer & Janssen, Wave run-up and wave overtopping at dikes, 1994). This way, the roughness has almost no more effect, and the structure behaves more and more like an impermeable smooth slope as \( \xi_{m-1,0} \) increases.

The impact of the reduction factors can be more or less important on the relative run-up value, which is presented in the following part.

### 3.3 Determination of the reduction factors

The run-up value can vary a lot depending on the wave properties as well as on the structure itself. Some of those properties are defined by the use of the reduction factors. Therefore it is a matter of great importance to correctly define these factors.

#### 3.3.1 Wave incidence angle reduction factor \( \gamma_{\beta} \)

As stated in its name, \( \gamma_{\beta} \) influences the run-up depending on the wave angle approach. The 0° angle corresponds to a normal wave attack to the structure. The higher the wave is inclined, the lower the run-up is. The angle can be illustrated as follows in Figure 3.3:

![Wave approach angle β](image)

Figure 3.3: Wave approach angle \( \beta \)

Delft hydraulics was a forerunner in studying the wave angle effect. One used to differentiate the long-crested waves (swell) from the short-crested and there were thus
two different formulas to determine $\gamma_\beta$ (De Waal & Van der Meer, 2002). Experiments have been continuously performed over the years, and as a result, it now appears more accurate to consider mainly the short-crested waves. The factor $\gamma_\beta$ can be determined using the simple relation (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007):

$$\gamma_\beta = 1 - 0.022 \cdot |\beta| \quad \text{for} \quad 0^\circ \leq |\beta| \leq 80^\circ$$

$$\gamma_\beta = 0.824 \quad \text{for} \quad |\beta| > 80^\circ$$

(3.5)

Some studies however showed that in the case of small angles, the resulting run-up value can be underestimated a little, and one can thus choose to use $\beta = 0^\circ$ if the wave angle approach is less than 20° (Schüttrumpf, Barthel, Ohle, Möller, & Daemrich, 2003).

### 3.3.2 Berm reduction factor $\gamma_b$

There are many dikes and other breakwaters which are constructed with more than one slope. Indeed, in some cases, one can opt for a presence of a berm in the middle of a slope. It is considered as a berm when it is a part of the structure slope profile whose slope is not steeper than 1:15 (Van der Meer, Wave run-up and overtopping, 1998).

![Figure 3.4: Structure berm representation](image)

In Figure 3.4, one can see the different parameters used to determine $\gamma_b$. $B$ corresponds to the width of the berm, which cannot have a higher value of $0.25 \cdot L_0$. The parameter $d_h$ corresponds to the location of the berm regarding the still water level. It is important to note that the berm can be located under or above the still water level, and the value is positive when the berm is located under the SWL. $L_{berm}$ is the horizontal length between the 2 points lying at $1.0 \cdot H_s$ below and above the berm level.

The reduction value is calculated as follow:
The conditions in equation (3.6) show that the run-up reduction due to the presence of a berm in the structure cannot be more than 40%. That is why it can be of interest to define the berm width in order to have such a reduction. This can only happen when the berm is located at the still water level ($d_h = 0$), with an optimum width $B = 0.4 \cdot L_{berm}$. However, if the berm lies above or below the still water level, the value $\gamma_b = 0.6$ cannot be reached, but the lower value still appears at the optimum berm width.

### 3.3.3 Surface roughness reduction factor $\gamma_f$

As one can guess, there are many different kinds of surface layers for both smooth structures and armoured structures. The choice of these layers can depend on many different factors, such as cost and availability, but also their efficiency regarding the run-up reduction and the surrounding environment of the future structure. That is the reason why it can be necessary to conduct a cost efficiency analysis when several solutions are possible for a project.

Here again, the factors are different when considering a smooth slope or an armoured slope.

#### 3.3.3.1 Smooth structures surface roughness reduction factor

The reduction factors of the smooth structures usually have a rather low influence on the run-up, compared to the armoured structures. Indeed, the surface layer of dikes and embankments is often made of grass, asphalt, or blocks revetments.

In the following table, the different values of $\gamma_f$ are displayed for the most common elements used to realize the surface layer of the structure (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007).
Table 3.1: Values of $\gamma_f$ for some common surface element

<table>
<thead>
<tr>
<th>Surface element type</th>
<th>$\gamma_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>1.0</td>
</tr>
<tr>
<td>Asphalt</td>
<td>1.0</td>
</tr>
<tr>
<td>Closed concrete blocks</td>
<td>1.0</td>
</tr>
<tr>
<td>Grass</td>
<td>1.0</td>
</tr>
<tr>
<td>Basalt</td>
<td>0.90</td>
</tr>
<tr>
<td>Small blocks over 1/25(^{th}) of surface</td>
<td>0.85</td>
</tr>
<tr>
<td>Small blocks over 1/9(^{th}) of surface</td>
<td>0.80</td>
</tr>
<tr>
<td>1/4 of stone setting 10cm higher</td>
<td>0.90</td>
</tr>
<tr>
<td>Ribs (optimum dimensions)</td>
<td>0.75</td>
</tr>
</tbody>
</table>

As one can see in Table 3.1, the maximum reduction possible due to the surface roughness on the wave run-up is about 20% when using small blocks on the slope surface. It is however still more common to see structures with surface elements inducing almost no reduction at all as stated before.

3.3.3.2 Armoured structures surface roughness reduction factor

Contrary to the smooth structures, the surface roughness reduction factor of the armoured structures has a major influence on the wave run-up. Indeed, the armoured structures are usually made of big blocks of rocks, concrete, or special shaped concrete elements. This can confer to the structure permeability since the water can penetrate the spaces in-between these blocks. Such structures are thus more likely to dissipate much more of the wave energy, and thus reduce considerably the run-up compared to smooth structures, whose permeability is null. Table 3.2 on the following page shows the latest values of $\gamma_f$ for the most common armours. The values of $\gamma_f$ are the results of many laboratory tests, except for the three last elements (Dolosse, Berm Breakwater, Icelandic Bermbreakwater), for which the $\gamma_f$ values have been extrapolated (Bruce, Van der Meer, Franco, & Pearson, 2006).

One can see that in the case of armoured structures, the run-up reduction due to the surface roughness can reach 65%, which is much more than the one observed for the smooth structures. Besides, it is also interesting to notice in the case of the rocks, one can choose between having a permeable core or not. For both cases (1 or 2 layers), the reduction due to permeability is only 5% as shown in Table 3.2, but regarding the relative run-up, having permeability in the core will increase the value to a maximum of 2.11 (See section 3.2).
Table 3.2: Values of $\gamma_f$ for the armoured structures surface layer elements

<table>
<thead>
<tr>
<th>Armour type</th>
<th>Nb of layers</th>
<th>$\gamma_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>-</td>
<td>1.00</td>
</tr>
<tr>
<td>Rocks with a permeable core</td>
<td>2</td>
<td>0.40</td>
</tr>
<tr>
<td>Rocks with an impermeable core</td>
<td>2</td>
<td>0.55</td>
</tr>
<tr>
<td>Rocks with a permeable core</td>
<td>1</td>
<td>0.45</td>
</tr>
<tr>
<td>Rocks with an impermeable core</td>
<td>1</td>
<td>0.60</td>
</tr>
<tr>
<td>Concrete cubes</td>
<td>2</td>
<td>0.47</td>
</tr>
<tr>
<td>Concrete cubes</td>
<td>1</td>
<td>0.50</td>
</tr>
<tr>
<td>Antifer</td>
<td>2</td>
<td>0.47</td>
</tr>
<tr>
<td>Accropode</td>
<td>1</td>
<td>0.46</td>
</tr>
<tr>
<td>Tetrapod</td>
<td>2</td>
<td>0.38</td>
</tr>
<tr>
<td>Core-Loc™¹</td>
<td>1</td>
<td>0.44</td>
</tr>
<tr>
<td>Xbloc™²</td>
<td>1</td>
<td>0.45</td>
</tr>
<tr>
<td>Haro</td>
<td>2</td>
<td>0.47</td>
</tr>
<tr>
<td>Dolosse</td>
<td>2</td>
<td>0.43</td>
</tr>
<tr>
<td>Berm Breakwater</td>
<td>2</td>
<td>0.40</td>
</tr>
<tr>
<td>Icelandic bermbreakwater</td>
<td>2</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Rocks are more likely to be chosen as armour elements as they offer a significant reduction of the wave run-up, fit in rather well in the surrounding environment and are almost present everywhere. However, if it appears that their availability is poor in some areas and thus makes them expensive, one should compare them to some concrete structures, which could sometimes be much more suitable than the rocks.

1 Concrete armour unit patented by the U.S. Army Corps of Engineers’
2 Concrete armour unit patented by Delta Marine Consultants
Figure 3.5: Influence of $\gamma_f$ on the relative run-up

Figure 3.5 above shows the relative run-up for three different kinds of structures. The first curve corresponds to a smooth impermeable structure ($\gamma_f = 1$), which is the same as the one drawn for the deterministic design in Figure 3.2. The second and the third curves correspond to an armoured breakwater with 2 layers of rocks, respectively impermeable ($\gamma_f = 0.55$) and permeable ($\gamma_f = 0.40$). One can clearly see what impact the choice of the surface element can have on the relative run-up, and then at a later stage, on the design of the structure itself. As in Figure 3.2, the first part of each curve evolves drastically, which corresponds to the breaking waves, and the gradient then reduces once the waves stop plunging but surge on the structure instead.

Besides, one can also notice that the last curve remains constant after a certain point, which corresponds to the relative run-up maximum value of 2.11 for the structures with a permeable core.

The formulae presented above are the most known and used nowadays. However, there is another method of calculation which also gives good results. Even if one cannot find it in manuals such as the Coastal Engineering Manual (USACE, 2002), it can still be used for comparison to check if the results seem to be in a good range of values or not.
3.4 Wave momentum flux based run-up

Another approach in the wave run-up calculation, more physical, has been considered by Hughes. It is based on the maximum depth-integrated wave momentum flux (Hughes, Wave momentum flux for coastal structures design, 2003). It focuses on the surface momentum whose direction is parallel to the wave propagation. As a wave encounters a solid object on its trajectory, its momentum reverses, and thus the acting force on this object is equivalent to the change of rate of this momentum, also called wave momentum flux.

3.4.1 Wave momentum flux parameter \( P_{MF} \)

The wave momentum flux is defined as a dimensionless parameter such as:

\[
P_{MF} = \left( \frac{M_F}{\rho \cdot g \cdot h^2} \right)_{\text{max}}
\]

(3.7)

Where:

- \( M_F \): Depth-integrated wave momentum flux
- \( \rho \): Fluid mass density [kg/m\(^3\)]
- \( g \): Gravitational acceleration [m/s\(^2\)]
- \( h \): Water depth [m]

Using Fourier’s transformation on the nonlinear waves of his experiments, Hughes’ work lead to the following empirical expression of the wave momentum flux (Hughes, Wave momentum flux for coastal structures design, 2003):

\[
P_{MF} = A_0 \cdot \left( \frac{H_s}{h} \right)^{-A_1}
\]

with

\[
A_0 = 0.639 \cdot \left( \frac{H_s}{h} \right)^{2.026}
\]

and

\[
A_1 = 0.180 \cdot \left( \frac{H_s}{h} \right)^{-0.391}
\]

(3.8)

It is interesting to note here that the water depth is taken into account, whereas it is not in the equations resulting from Van der Meer’s work (Van der Meer & Janssen, Wave run-up and wave overtopping at dikes, 1994) (see section 3.1 to 3.3).

In order to apply this to the run-up, Hughes’ theory resides in the assumption that the weight of water in the upper area of an impacting wave evolves proportionally to \( P_{MF} \) (see Figure 3.6).
3.4.2 Run-up on smooth impermeable slopes

Depending on the slope angle $\alpha$, the area A (see Figure 3.6) changes. Besides, in order to simplify the formulae, Hughes chose to consider this area as a triangle so that the wave’s curvature is not taken into account.

For smooth impermeable slopes, the wave run-up based on the wave momentum flux has two formulations. Indeed the difference is also made for non-breaking and breaking waves (Hughes, Estimating irregular wave runup on rough, impermeable slopes, 2005):

- For the non-breaking waves:
  \[
  \frac{R_{u2\%}}{h_s} = 1.75 \cdot \left[1 - \exp\left(-1.3 \cdot \cot \alpha \right)\right] \cdot \sqrt{\frac{P_{MF}}{h_s}}
  \]
  for \(1 \leq \cot \alpha \leq 4\)  \(3.9\)

- For the breaking waves:
  \[
  \frac{R_{u2\%}}{h_s} = 4.4 \cdot \left(\tan \alpha \right)^{0.47} \cdot \sqrt{\frac{P_{MF}}{h_s}}
  \]
  for \(1.5 \leq \cot \alpha \leq 30\)  \(3.10\)

3.4.3 Run-up on armoured impermeable slopes

The way to determine the wave run-up on armoured (or rough) impermeable slopes is the same as for the smooth slopes. A reduction factor is however used due to the roughness of the slope. Besides, the waves are here assumed to be breaking in most of the cases. Based on the wave run-up formulae for smooth structures, one has the following:
\[
\frac{R_{u2\%}}{h_s} = 4.4 \cdot (\tan \alpha)^{0.47} \cdot \sqrt{P_{MF}} \cdot 0.505
\]

(3.11)

for \( 2 \leq \cot \alpha \leq 4 \)

where 0.505 represents the surface roughness reduction factor [-]

The limitation in Hughes formulae is that it is available for only two kinds of structures. It is however interesting to notice that another method of wave run-up calculation exists and that it gives close results to the general ones (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007). Still, as equation (3.1) and its derivations applies for many structures and is more widely spread it is recommended to use it, but Hughes’ formulae can still be used as “backup” calculations when some doubts on the results are present.

One sees in this part that run-up is an important parameter to consider in the construction of protection water structures. Despite that, it is quite common to build protection water structures which crest height \( R_c \) is lower than the run-up value \( R_{u2\%} \), and for which overtopping will occur.
4 Wave overtopping

As soon as the crest height $R_c$ of a structure is exceeded by the highest wave run-up, wave overtopping occurs. It represents the amount of water passing over the structure and is usually determined as an average discharge per linear meter of width $[(m^3/s)/m]$.

Contrary to the wave run-up phenomenon, overtopping can occur over both sloping and vertical structures, and the way to define it is obviously proper to both kind of structure.

Figure 4.1: Average overtopping rate $q$

It is a matter of great importance to well determine the overtopping rate, because consequences can be dramatic on the surrounding environment and on the structure itself, especially for structures whose purpose is to avoid flooding. A first classification in the allowable rates is available as follow (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007):

$q < 0.1 (l/s)/m$: Insignificant with respect to structure’s crest- and rear strength.
$q = 1 (l/s)/m$: In the case of dikes, the inner slopes made of grass or clay may erode.
$q = 10 (l/s)/m$: The overtopping start to have a significant impact for smooth structures and moderate for armoured structures.
$q = 100 l/s/m$: Smooth structures’ crest and inner slope must be protected, and wave transmission may occur for the armoured structures.

Moreover, there are two different kinds of overtopping, which mainly depend on the wave breaker parameter $\xi_{m-1.0}$. Indeed, when a structure is overtopped by an homogeneous sheet of water, which corresponds to a value of $\xi_{m-1.0}$ greater than two, one usually calls it “green water”, whereas when the wave breaker parameter is lower than two, which means that waves break on the structure seaward slope, the overtopping consists in a mix of water and air, commonly called “white water” or overtopping spray. One must note that in the case of overtopping spray, the water can be carried much further as the “green water” if there is a strong wind, and the water’s salinity can thus have an effect on structures or vehicles located behind the defence, as well as on the vegetation.
As for the wave run-up, there are different models to predict the mean overtopping discharge of a structure, which are also empirical, and there is usually only one model related to one type of structure (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007).

There is however one main formula to calculate the mean overtopping discharge, the terms of which are changed in order to be more representative of one kind of structure.

\[
q = a \cdot \exp \left( -b \cdot \frac{R_c}{H_{m0}} \right)
\]

(4.1)

Where:

- \( q \) : Mean overtopping discharge \([(m^3/s)/m]\)
- \( g \): acceleration of gravity (taken as \( g = 9.81 \text{ m/s}^2 \) \[m/s^2]\)
- \( H_{m0} \): Significant wave height \[m]\)
- \( a \) and \( b \): Empirical coefficients [-]

In equation (4.1), \( a \) and \( b \) are the empirical coefficients which will vary according to the kind of structure considered. One has to note that these coefficients are not always single values, but can also be a product of different subsidiary parameters.

There are three main types of structure to be considered for overtopping. The two first are the sloping structures, which are differentiated depending on whether an armour is present or not, and the last one is the vertical structures (USACE, 2002).

4.1 Wave overtopping of smooth structures

As written before, the wave overtopping occurs when the crest height \( R_c \) is exceeded by the maximum run-up level. In the case of the smooth structures, one could tend to think that, as for the wave run-up, the overtopping is higher for a smooth structure than for an armoured structure. And he would be right. Indeed, for the same reason as the run-up, which is that there is no armour to dissipate the waves energy, the smooth structure’s mean overtopping discharge will be much dangerous than the armoured structure’s one for a same geometry.

Here also, one can compute the mean overtopping discharge using either a probabilistic design or a deterministic design, and it is obviously advised to use the deterministic design in the case of real projects. Moreover, one has to pay attention to the Iribarren number \( \xi_{m,-1,0} \), because the formulae change whether \( \xi_{m,-1,0} < 5 \) or \( \xi_{m,-1,0} > 7 \). The formulae are the following (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007):
For the deterministic design:

\[ \zeta_{m-1,0} < 5: \]

\[ \frac{q}{\sqrt{g \cdot H_{m0}^3}} = \frac{0.067}{\sqrt{\tan \alpha}} \cdot \gamma_b \cdot \xi_{m-1,0} \cdot \exp \left( -4.3 \cdot \frac{R_c}{\xi_{m-1,0} \cdot H_{m0} \cdot \gamma_f \cdot \gamma_b \cdot \gamma_v} \right) \]  

with a maximum of \[ \frac{q}{\sqrt{g \cdot H_{m0}^3}} = 0.2 \cdot \exp \left( -2.3 \cdot \frac{R_c}{H_{m0} \cdot \gamma_f \cdot \gamma_b} \right) \]  

\[ \zeta_{m-1,0} > 7: \]

\[ \frac{q}{\sqrt{g \cdot H_{m0}^3}} = 0.21 \cdot \exp \left( -\frac{R_c}{H_{m0} \cdot \gamma_f \cdot \gamma_b \cdot (0.33 + 0.022 \cdot \xi_{m-1,0})} \right) \]  

For the probabilistic design:

\[ \zeta_{m-1,0} < 5: \]

\[ \frac{q}{\sqrt{g \cdot H_{m0}^3}} = \frac{0.067}{\sqrt{\tan \alpha}} \cdot \gamma_b \cdot \xi_{m-1,0} \cdot \exp \left( -4.75 \cdot \frac{R_c}{\xi_{m-1,0} \cdot H_{m0} \cdot \gamma_f \cdot \gamma_b \cdot \gamma_v} \right) \]  

with a maximum of \[ \frac{q}{\sqrt{g \cdot H_{m0}^3}} = 0.2 \cdot \exp \left( -2.6 \cdot \frac{R_c}{H_{m0} \cdot \gamma_f \cdot \gamma_b} \right) \]  

\[ \zeta_{m-1,0} > 7: \]

\[ \frac{q}{\sqrt{g \cdot H_{m0}^3}} = 10^c \cdot \exp \left( -\frac{R_c}{H_{m0} \cdot \gamma_f \cdot \gamma_b \cdot (0.33 + 0.022 \cdot \xi_{m-1,0})} \right) \]  

\[ (4.5) \]

Where:

- \( c \): Mean of a normally distributed function with a standard deviation of 0.24, whose value is 0.92 (thus \( 10^{0.92} = 0.12 \) (Van der Meer, Technical Report Wave run-up and overtopping at dikes, 2002) [-].

Although there is no equation available for \( 5 \leq \zeta_{m-1,0} \leq 7 \), one can interpolate linearly the overtopping discharge after computing it for \( \zeta_{m-1,0} = 5 \) and \( \zeta_{m-1,0} = 7 \).

One can see in equations (4.1) to (4.5) that the mean overtopping discharge \( q \) is not directly calculated. Indeed, it is more common to first calculate it as a dimensionless discharge \( Q \).

\[ Q = \frac{q}{\sqrt{g \cdot H_{m0}^3}} \]  

\[ (4.6) \]
This gives log-linear curve which are more appropriate to make comparisons. Besides, one can easily determine $q$ using equation (4.6) afterwards.

Figure 4.2: Probabilistic and deterministic design relative overtopping discharge $Q$

The difference between deterministic and probabilistic design is presented in Figure 4.2. The relative overtopping discharge is plotted against the relative freeboard height $R_c / H_{m0}$ in order to have the evolution of the general trend. The plain curve ($Q_{\text{det}}(R_c)$) represents the deterministic design formula and the dashed one the probabilistic design formula. When the allowable mean overtopping discharge is high, there is not such a large difference between the two curves. However, the allowed value is most of the time a lot less than 0.1 (m$^3$/s)/m, and as in Figure 4.2 the graph y-scale is logarithmic, it appears clearly that the choice in the design formula is decisive as to the achievement of a realistic result.

4.2 Wave overtopping of armoured structure

There had been many different way of estimating the wave overtopping on the armoured structures, and it was not always easy to choose which one to use (USACE, 2002). Researchers and engineers however noticed that the seaward slope steepness is often the same for armoured structures (around 1:1.5), which leads to a single equation, equal to the maximum of equation (3.2) (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007):
\[
\frac{q}{\sqrt{g \cdot H_{m0}^3}} = 0.2 \cdot \exp \left( -2.3 \cdot \frac{R_c}{H_{m0} \cdot \gamma_f \cdot \gamma_f} \right)
\] (4.7)

Albeit a single formula is available, one can choose to use equation (4.2) if the slope angle appears to be lower or larger than 1:1.5.

### 4.3 Wave overtopping of vertical structures

The determination of the mean overtopping discharge for the vertical structures is different than the one of the sloping structures. Indeed, as the structure is vertical, one first has to check if it is submitted to impulsive conditions or not. Impulsive conditions occur when the waves are breaking “on” the vertical wall, such as the waves smash almost vertically into the structure. These conditions can lead to forces which are 10 to 40 times greater than under normal conditions.

On Figure 4.3, one can see a vertical structure submitted to impulsive breaking waves. It shows clearly that the whole energy of the wave is transmitted to the wall in a single short hit. These conditions usually appear when the wave height is large compared to the water depth when approaching the structure. The shoaling effect then makes the waves becoming bigger and steeper and some waves might thus break directly at the structure. With such conditions, the overtopping discharge consists of a high jet of water that can reach high heights up the structure.

![Figure 4.3: Illustration of an impulsive condition](image)

In the case of composite vertical structure, those conditions can be even more likely to occur, since the mounds located in front of the structure might intensify the shoaling effect.
One can clearly see in Figure 4.4 that the presence of rubble mound at the base of the structure induces a reduction in water depth and thus increases the shoaling effect. In order to determine these conditions, the following formulae can be used (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007):

\[
\begin{align*}
    h_* &= 1.35 \cdot \frac{h_s}{H_{m0}} \cdot \frac{2\pi \cdot h_s}{g \cdot T_{m-1.0}^2} \\
    d_* &= 1.35 \cdot \frac{d}{H_{m0}} \cdot \frac{2\pi \cdot h_s}{g \cdot T_{m-1.0}^2}
\end{align*}
\]

Equation (4.8) is valuable for plain vertical structures, and equation (4.9) for composite vertical structures. The limits values are the same for both equations. Non-impulsive conditions prevail for values greater than 0.3, and impulsive conditions occur when the values are lower than 0.2. Values located between 0.2 and 0.3 correspond to the transition between breaking and non breaking waves. If this happens, one should calculate the overtopping for both non-impulsive and impulsive conditions, and then choose the largest value.

It is also possible to have a faster estimation of such conditions to occur or not. Indeed, in his book, Goda defined a questionnaire for judging the danger of impulsive breaking wave pressure on a vertical structure. Besides, if a rubble mound foundation is present, one can also have an estimation of the influence of the berm width on the generation of impulsive breaking (Goda, 2000).

### 4.3.1 Mean overtopping discharge under non impulsive conditions

Goda is one of the benchmark researchers regarding the design of the maritime vertical structures, and he defined a complete set of diagrams that allows one to check easily what would be the overtopping rate for a given structure (Goda, 2000). However, some experiments have still been performed in order to be more and more precise. People such as Franco (Franco & Franco, 1999), (Franco, de Gerloni, & Van der Meer, Wave overtopping on vertical and composite breakwaters, 1994) tried to...
develop or improve existing formulae. The most recent formula regarding wave overtopping of vertical structures is the following:

\[
\frac{q}{g \cdot H_{m0}^3} = 0.04 \cdot \exp \left( -1.8 \cdot \frac{R_c}{H_{m0}} \right)
\]  

(4.10)

Equation (4.10) can be applied for both plain vertical structures and composite structures. Indeed, in the case of non impulsive conditions, the variation of wave overtopping between these two structures is almost negligible. Once again, one can see that the equation is based on the model of equation (4.1).

### 4.3.2 Mean overtopping discharge under impulsive conditions

If the structures are submitted to an impulsive breaking, then the difference between plain vertical structures and composite vertical structures can be important.

- **Plain vertical structures:**
  
  For breaking waves:

  \[
  \frac{q}{h^2_\ast \sqrt{g} \cdot h_3^3} = 2.8 \cdot 10^{-4} \cdot \left( R_c \cdot \frac{h_\ast}{H_{m0}} \right)^{-3.1} \quad \text{for} \quad 0.03 < R_c \cdot \frac{h_\ast}{H_{m0}} < 1.0
  \]  

  (4.11)

  For broken waves:

  \[
  \frac{q}{h^2_\ast \sqrt{g} \cdot h_3^3} = 3.8 \cdot 10^{-4} \cdot \left( R_c \cdot \frac{h_\ast}{H_{m0}} \right)^{-2.7} \quad \text{for} \quad h_\ast \cdot \frac{R_c}{H_{m0}} < 0.02
  \]  

  (4.12)

Equation (3.12) is normally used in the case of very shallow water. One can note that there is no equation for \( 0.02 < h_\ast \cdot \frac{R_c}{H_{m0}} < 0.03 \). This corresponds to the transition between breaking wave’s impulsive conditions and broken wave’s impulsive conditions. There is nowadays no sufficient data that allow having a precise estimation in this range. However, it is advised to use equation (4.11) until the value of 0.02.

- **Composite vertical structures:**
  
  In the case of composite vertical structures, the toe, usually made of rubble mound, can have different effects on the waves depending on its dimensions. In the case of a small toe which is only used as foundation of the structure the calculations are the same as for a plain vertical wall (the toe is simply ignored). However, if the toe is moderate, with non negligible dimensions regarding the rest of the structure, one has to follow the approach presented hereinafter. The last configuration possible for a composite vertical structure is to have rubble-mounds that emerge above the still water level. In this case, one can calculate the overtopping discharge using equation (4.7).
\[ \frac{q}{d_t^2 \sqrt{g \cdot h_*^2}} = 7.8 \cdot 10^{-4} \left( \frac{d_* \cdot R_c}{H_{m0}} \right)^{-2.6} \quad \text{for} \quad 0.03 < h_* \cdot \frac{R_c}{H_{m0}} < 1.0 \]

and \( h_* < 0.3 \)

\( d_* \) and \( h_* \) are calculated as presented in equations (4.8) and (4.9). If the conditions of equation (4.13) are not fulfilled, then one has to refer either to the first case (small toe) or to the third one (emerging mounds).

The way to determine overtopping for different kinds of structure have been exposed here, but it might change depending on the intrinsic properties of the structure. Indeed, in the case of the sloping structures, one can apply some reduction factors, which sometimes are the same as for the wave run-up, and in the case of the vertical wall, one can sometimes have a battered wall or a bullnose at the top of the structure.

### 4.4 Reduction factors for sloping structures

As for the wave run-up, there are some reduction factors that one can apply depending on the waves’ and structures’ properties. Some of these factors are the same for both run-up and overtopping, but others differ. Still, subparts are done for reduction factors which are the same for run-up and overtopping, so as to mark the fact that they have the same importance as the others.

#### 4.4.1 Surface roughness reduction factor \( \gamma_f \)

The surface roughness reduction factor \( \gamma_f \) is the same for wave run-up and overtopping for both smooth and armoured structures.

##### 4.4.1.1 Smooth structures surface roughness reduction factor

See Table 3.1.

##### 4.4.1.2 Armoured structures surface roughness reduction factor

See Table 3.2.
Figure 4.5: Surface roughness reduction factor influence on overtopping

Figure 4.5 is an example of the importance that represents the choice of the surface layer. The upper dashed curve ($q_{\text{smooth}}$) represents the mean overtopping discharge for a smooth impermeable structure and the plain curve ($q_{\text{rub}}$) represents the mean overtopping discharge for a two layers rock permeable structure ($\gamma_f = 0.4$). One can clearly see how the surface layer properties influence severely the overtopping, and thus how interesting it is to have such armours when control of the overtopping is needed.

### 4.4.2 Berm reduction factor $\gamma_b$

As for the surface roughness reduction factor, the berm reduction factor $\gamma_b$, which just applies for smooth structures, is the same as the one used for the run-up. See section 3.3.2.

### 4.4.3 Wave incidence angle reduction factor $\gamma_\beta$

#### 4.4.3.1 $\gamma_\beta$ for smooth structures

The effect of the wave incidence angle on the mean overtopping discharge is the same as explained in section 3.3.1. However, its reductive effect on overtopping is slightly bigger than on run-up, as one can see in the following equation (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007):

$$
\gamma_\beta = 1 - 0.033 \cdot |\beta| \quad \text{for} \quad 0^\circ \leq |\beta| \leq 80^\circ
$$

$$
\gamma_\beta = 0.736 \quad \text{for} \quad |\beta| > 80^\circ
$$

(4.14)
Here again, as for equation (3.5), one can choose not to consider any reduction until $\beta = 20^\circ$, since it appears that these formulae overestimate a bit the reduction of overtopping. Choosing to have no reduction until $\beta = 20^\circ$ thus gives a safety margin in the mean overtopping rate prediction.

### 4.4.3.2 $\gamma_\beta$ for armoured structures

The wave incidence angle reduction factor formula for armoured structures has the same base as those presented before (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007):

$$
\gamma_\beta = 1 - 0.063 \cdot |\beta| \quad \text{for} \quad 0^\circ \leq |\beta| \leq 80^\circ
$$

$$
\gamma_\beta = 0.496 \quad \text{for} \quad |\beta| > 80^\circ
$$

(4.15)

One can see in equation (4.15) that the limit value for $\gamma_\beta$ is much lower than for the smooth structures. This is mainly due to the armour properties of the structures, which start to dissipate the waves’ energy as soon as they reach them.

### 4.4.4 Wave wall reduction factor $\gamma_v$

In the case of smooth structures such as a dike, one can sometimes find a small wall disposed on top of the structure in order to reduce the overtopping rate. There is however no precise knowledge regarding the effect of the wall. Still a formula is available, but it does not take into account the wall height. When the wall is vertical, one can assume $\gamma_v = 0.65$. Otherwise $\gamma_v$ can be determined as:

$$
\gamma_v = 1.35 - 0.078 \cdot \alpha_{\text{wall}}
$$

(4.16)

where:

$\alpha_{\text{wall}}$: wall angle in degrees (for a vertical wall, $\alpha_{\text{wall}} = 90^\circ$)

### 4.4.5 Armoured crest berm reduction factor $C_r$

It is quite common for the armoured structures to have a crest berm width $G_c$ equal to three stones’ nominal diameters. In this case, the overtopping is equal to the one defined by equation (4.7). This being the case, it is still possible to reduce the amount of overtopping by creating a wider crest berm. The factor $C_r$ is determined this way:

$$
C_r = 3.06 \cdot \exp \left( -1.5 \cdot \frac{G_c}{H_{m0}} \right)
$$

(4.17)

To apply this factor, one should first calculate the mean overtopping discharge using equation (4.7), and then multiply it by $C_r$ (Note that the maximum value of $C_r$ is 1).
4.5 Parapet on vertical wall

It is quite common to see some parapets (or bullnoses) at the top of the vertical structures, which allow a significant reduction of the overtopping, and also of the crest height of the structure.

![Diagram of a parapet and its main parameters](image)

Figure 4.6: Illustration of a parapet and its main parameters

The reduction in overtopping brought by the presence of a parapet is usually defined as the ratio \( k \) between the mean overtopping rate of a structure with a parapet and the mean overtopping rate of the same structure without parapet, such as (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007):

\[
k = \frac{q_{\text{parapet}}}{q_v}
\]  

(4.18)

The parapet can be inclined seaward (\( \alpha < 90^\circ \)) or landward (\( \alpha > 90^\circ \)). It appears logically that a landward inclination would give even more overtopping. That is why it is not treated here, as the aim is to reduce the overtopping discharge.
One can see in Figure 4.7 the different steps to follow in order to determine the ratio $k$ and thus the mean overtopping discharge of a vertical wall with parapet (Pearson, Bruce, Allsop, Kortenhaus, & Van der Meer). It is however not advised to consider values of $k < 0.05$, because the accuracy of the results after this limit cannot be guaranteed. The parameters present in Figure 4.7, if not defined before, correspond to the one presented in Figure 4.6.

As the limit value is 0.05 for the parapet reduction, it is obvious that the parapet can be really efficient ($k = 0.05$ represents an overtopping reduction to about one twentieth of a pure vertical structure). One should thus always consider them, since the price of a structure can considerably be decreased thanks to them.

### 4.6 Wave overtopping volume

The previous subparts with regard to overtopping are focused on the main overtopping discharge. As it is a “mean” value, it means that for single waves, the overtopping volume can be higher or lower than this value. It is thus interesting to have a look at the probabilities for waves to overtop a structure or not. The basic relationship is the same for all kinds of structures. Indeed, the probability distribution function for the wave overtopping volume follows a Weibull distribution. The probability of exceedance $P_v$ of having an overtopping volume greater than a volume $V$ is given by (USACE, 2002):
\[ P_v = P(V \geq Y) = \exp \left[ -\left( \frac{V}{a} \right)^b \right] \]  

(4.19)

The terms \( a \) and \( b \) change for sloping or vertical structures (see section 4.6.4 after).

Figure 4.8: Probability of exceedance of an overtopping volume per wave \( V \)

When looking at Figure 4.8, one can note the typical behaviour of an exceedance probability function. Indeed, the bigger the overtopping volume is, the less is the probability for it to happen.

From equation (4.19), one can modify it in order to check the volume per wave for a given probability of exceedance (USACE, 2002):

\[ V = a \cdot (\ln(P_v))^{\frac{1}{b}} \]  

(4.20)

Finally, the maximum overtopping volume likely to happen depends on the number of overtopping waves during a storm period \( t \), and can be calculated using:

\[ V_{\text{max}} = a \cdot (\ln(N_{aw}))^{\frac{1}{b}} \]  

(4.21)

These formulae are applicable for each kind of structures. However, the way to determine the number of overtopping waves which is used in these previous formulae depends essentially on the kind of structure considered.
4.6.1 Probability of overtopping per wave for smooth structures
The probability of overtopping per wave is defined as the ratio:

\[ P_{ov} = \frac{N_{ow}}{N_w} \]  \hspace{1cm} (4.22)

If a Rayleigh distribution is assumed for the wave run-up, one can have, based on the run-up value \( R_{u2\%} \) (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007):

\[ P_{ov} = \exp \left[ -\left( \sqrt{-\ln(0.02)} \cdot \frac{R_c}{R_{u2\%}} \right)^2 \right] \]  \hspace{1cm} (4.23)

Using the result of equation (4.23), one can easily find the number of waves that will overtop for a given number of incoming waves.

4.6.2 Probability of overtopping per wave for armoured structures
In the case of an armoured structure, one can use equation (4.23), but there is another formula available, which follows a Weibull distribution rather than a Rayleigh distribution:

\[ P_{ov} = \exp \left[ -\left( \frac{A_c \cdot D_n}{0.19 \cdot H_{m0}^2} \right)^{1.4} \right] \]  \hspace{1cm} (4.24)

This equation takes into account the armour unit size as well as the armour crest width \( A_c \) (in most of the cases \( A_c = R_c \)). Its result is normally lower than equation (4.23) and that is the reason why it is advised to calculate it for both cases to check the difference, letting then the appreciation to the designer.

4.6.3 Probability of overtopping per wave for vertical structures
The calculation of the number of overtopping waves and thus of the overtopping probability differs slightly in the case of vertical structures. Indeed, there is no run-up on the vertical structures, and once again, the formulae are not the same whether the waves’ conditions are impulsive or not.

Franco found a good expression for normal (non-impulsive) conditions (Franco, de Gerloni, & Van der Meer, Wave overtopping on vertical and composite breakwaters):

\[ P_{ov} = \frac{N_{ow}}{N_w} = \exp \left[ -\frac{1}{k} \cdot \frac{R_c}{H_{m0}} \right]^2 \] \quad with \quad k = 0.91 \hspace{1cm} (4.25)
As before, it changes for the impulsive conditions. One thus has:

\[ P_{ov} = 0.031 \cdot \frac{H_{m0}}{h_* \cdot R_c} \]  

(4.26)

Where \( h_* \) corresponds to the parameter calculated in equation (4.8)

### 4.6.4 Determination of the parameters a and b

#### 4.6.4.1 For sloping structures

For both smooth and armoured structures \( a \) and \( b \) are the same. \( a \) may vary whereas \( b \) remains constant:

\[ a = 0.84 \cdot T_m \cdot \frac{q}{P_{ov}} = 0.84 \cdot T_m \cdot q \cdot \frac{N_w}{N_{ow}} = 0.84 \cdot q \cdot \frac{t}{N_{ow}} \]  

(4.27)

\[ b = 0.75 \]

#### 4.6.4.2 For vertical structures

The coefficients \( a \) and \( b \) vary depending on whether the conditions are impulsive or not, which implies the calculation of \( h_* \) as presented in section 4.3. Moreover, it is first necessary to calculate the average volume per overtopping wave \( V_{bar} \) as:

\[ V_{bar} = \frac{q \cdot T_{m-1,0} \cdot N_w}{N_{ow}} \]  

(4.28)

Then, one has:

- For non-impulsive conditions (\( h_* > 0.3 \)):
  \[ a = 0.74 \cdot V_{bar} \quad \text{and} \quad b = 0.66 \quad \text{for} \quad s_{m-1,0} = 0.02 \]  

(4.29)

- For impulsive conditions (\( h_* < 0.3 \))
  \[ a = 0.90 \cdot V_{bar} \quad \text{and} \quad b = 0.82 \quad \text{for} \quad s_{m-1,0} = 0.04 \]  

(4.30)

Once that all these values have been calculated, it is interesting to look after the risk that they might represent. Indeed, they would not be so relevant if there were not any values representing a type of danger.
4.7 Limits values for overtopping

Many experiments on overtopping realized over the past decades drove to many different results about the allowable overtopping discharges. From all these data, a relevant table has been set up as one can see on Figure 4.9 (USACE, 2002).

![Figure 4.9: Limits values of mean overtopping discharge q](image)

Figure 4.9 gives a good overview of the risks encountered for a given mean overtopping value, in function of a considered entity. One can see that the range of allowable overtopping can here go from 0.001 (l/s)/m to 200 (l/s)/m, which mainly vary due to the structure’s functionality.

The latest observations made have been realised within the CLASH project (Crest Level Assessment of Coastal Structures by full-scale monitoring, neural network prediction and Hazard analysis on permissible wave overtopping), a European project that take inventory of many different overtopping observations in order to use it as a database (Pullen, Allsop, Bruce, Kortenhaus, Schüttrumpf, & Van der Meer, 2007). The overtopping discharges fixed by this project are close to the one present in Figure 4.9, but are in a way more apprehensible.
Furthermore, it also shows the limits in maximum overtopping volume per wave on a structure. One still has to note that the volumes limits are more indicative than exhaustive, and should thus rather focus on the mean overtopping discharge $q$. This is mainly due to the fact that only few data on individual overtopping are available.

Table 4.1: Mean overtopping discharge and maximum individual volume limits

<table>
<thead>
<tr>
<th>Hazard type</th>
<th>Limit mean discharge $q$ [l/m/s]</th>
<th>Maximum volume $V_{\text{max}}$ [l/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driving at moderate / high speed, impulsive overtopping inducing high velocity water jets$^1$</td>
<td>$0.01 – 0.05$</td>
<td>$5 – 50$</td>
</tr>
<tr>
<td>Aware pedestrian not surprised by the coming waves. Structure crest and rear face not damaged even if not protected</td>
<td>$0.1$</td>
<td>$20 – 50$$^2$</td>
</tr>
<tr>
<td>Damage to equipment set $5 – 10$ m behind the defence$^1$</td>
<td>$0.4$</td>
<td>-</td>
</tr>
<tr>
<td>Structure elements of a building$^3$</td>
<td>$1$</td>
<td>-</td>
</tr>
<tr>
<td>Trained staff ready to get wet, no falling jets from overtopping / Grass covered structure crest and rear face not damaged</td>
<td>$1 - 10$</td>
<td>$1000 – 2000$</td>
</tr>
<tr>
<td>Sinking small boats set $5 – 10$ m behind the defence, larger yacht damaged$^1$</td>
<td>$10$</td>
<td>$1000 - 10000$</td>
</tr>
<tr>
<td>Driving at low speed, no falling jets from overtopping</td>
<td>$10 - 50$</td>
<td>$100 – 1000$</td>
</tr>
<tr>
<td>Significant damage or sinking of larger yachts / Grass covered structure damaged</td>
<td>$50$</td>
<td>-</td>
</tr>
<tr>
<td>Well protected structure crest and rear slopes not damaged</td>
<td>$50 – 200$</td>
<td>-</td>
</tr>
<tr>
<td>Armoured structure promenade damaged</td>
<td>$200$</td>
<td>-</td>
</tr>
</tbody>
</table>

$^1$ For an overtopping discharge estimated right at the defence
$^2$ Only for the pedestrians, no volume defined for the structure
$^3$ For an overtopping rate calculated at the building
5 Structure Stability

The way to determine the stability of a structure is obviously not the same for a sloping structure than for a vertical structure. Indeed, in the case of sloping structures, the attention is mainly focused on the dimensions of a single median armour unit, whereas in the case of vertical structures, one has to look after the whole structure unit.

This report presents only the stability for rock armoured structures and vertical structures. Indeed, they are the structures used for the hereinafter study, and it is thus not necessary to present all stability’s formulae for each different kind of armour. One can refer to the Coastal Engineering Manual to have them (USACE, 2002).

5.1 Rock armoured structures stability

Since decades people look after a way to get a good structure’s stability, in order to have the right armour stone size, not too small for not being taken away by the waves, and not too big for not having a too expensive structure.

5.1.1 Iribarren formula

Iribarren (Hedar, 1960) is one of the first instigators in that field. He assumed a simple relation between the wave force, the stone nominal diameter, the wave height, the volumic mass of stones and water and the acceleration of gravity. It leads to the following equation:

\[ F_{wave} = \rho_w \cdot g \cdot D_n^2 \cdot H_s \]  

(5.1)

Where:

- \( F_{wave} \): Wave force [N]
- \( \rho_w \): Water mass density [kg/m³]
- \( g \): Gravitational acceleration [m/s²]
- \( D_n \): Nominal stone size [m]
- \( H_s \): Significant wave height [m]

It does not take into account the block shape or the wave period, but it was still a good march for that time. By doing the forces equilibrium for the downrush and the uprush, Iribarren found (d’Angremond & van Roode, Breakwaters and Closure Dams, 2004):

- For the downrush:

\[ W \geq \frac{N \cdot \rho_s \cdot g \cdot H_s^3}{\Delta^3 \cdot (\mu \cdot \cos \alpha - \sin \alpha)^3} \]  

with \( \Delta = \frac{\rho_s}{\rho_w} - 1 \)  

(5.2)

- For the uprush:

\[ W \geq \frac{N \cdot \rho_s \cdot g \cdot H_s^3}{\Delta^3 \cdot (\mu \cdot \cos \alpha + \sin \alpha)^3} \]  

with \( \Delta = \frac{\rho_s}{\rho_w} - 1 \)  

(5.3)
Where:

\( \mu \): Friction coefficient [-]
\( \rho_s \): Stone mass density [kg/m³]
\( N \): Stone shape coefficient factor [-]

For rough angular quarry stone, which is the most common, \( N \) and \( \mu \) can be taken as (d'Angremond & van Roode, Breakwaters and Closure Dams, 2004):

- For the downrush: \( N = 0.43 \); \( \mu = 2.38 \)
- For the uprush: \( N = 0.849 \); \( \mu = 2.38 \)

### 5.1.2 Hudson Formula

Approximately at the same time, experiments have been realised by the United States Army Corps of Engineers. Hudson proposed a formula which is still in use nowadays (USACE, 2002):

\[
\frac{H_s}{\Delta \cdot D_{n50}} = (K_D \cdot \cot \alpha)^{\frac{1}{3}} \quad \text{or} \quad \frac{M_{50}}{K_D \cdot \Delta^3 \cdot \cot \alpha} = \rho_s \cdot \frac{H_s^3}{\Delta^3 \cdot \cot \alpha}
\]  

(5.4)

Where:

\( D_{n50} \): Nominal diameter of the median rocks [m]
\( M_{50} \): Medium mass of rocks [kg]
\( K_D \): Stability coefficient [-]

The determination of \( K_D \) is based on the notion of damage. The damage of a structure is here ranked in 4 categories:

- No damage: This means that no armour units are displaced under wave action.
- Initial damage: Only a few units are displaced. It is common to say that it is initial damage when 0 to 5% of the stones are moved.
- Intermediate damage: 5 to 10% of the units are displaced, but neither the underlayer nor the filter is directly exposed to direct wave attack.
- Failure: More than 20% of the stones are being displaced, and the filter layer or the underlayer is directly exposed to wave attack.

After considering one kind of damage, one can pick the corresponding value of \( K_D \) using Table 5.1 or Table 5.2. As we want the structure to hold on for a certain period of time -the design is usually made with a fifty years return period wave- it is rather common to choose to have an initial damage only.
Table 5.1: $K_D$ values according to Shore Protection Manual 1977

<table>
<thead>
<tr>
<th>Stone shape</th>
<th>Placement</th>
<th>Damage D 0 – 5%</th>
<th>Damage D 5 – 10%</th>
<th>Damage D 10 – 15%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Breaking waves</td>
<td>Nonbreaking waves</td>
<td>Nonbreaking waves</td>
</tr>
<tr>
<td>Smooth, rounded</td>
<td>Random</td>
<td>2.1</td>
<td>2.4</td>
<td>3.0</td>
</tr>
<tr>
<td>Rough, angular</td>
<td>Random</td>
<td>3.5</td>
<td>4.0</td>
<td>4.9</td>
</tr>
<tr>
<td>Rough, angular</td>
<td>Special¹</td>
<td>4.8</td>
<td>5.5</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1 comes from the Shore Protection Manual edited in 1977. However, since some problems occurred using these values a new table was published in the 1984 edition, as presented in Table 5.2.

Table 5.2: $K_D$ values according to Shore Protection Manual 1984

<table>
<thead>
<tr>
<th>Stone shape</th>
<th>Placement</th>
<th>Damage D = 0 to 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Breaking waves</td>
</tr>
<tr>
<td>Smooth, rounded</td>
<td>Random</td>
<td>1.2</td>
</tr>
<tr>
<td>Rough, angular</td>
<td>Random</td>
<td>2.0</td>
</tr>
<tr>
<td>Rough, angular</td>
<td>Special¹</td>
<td>5.8</td>
</tr>
</tbody>
</table>

When comparing Table 5.1 and Table 5.2, one can see that in Table 5.2, the only damage considered is the initial one, and that the values vary most for the breaking waves. However, one of the most important changes in the edition of 1984 is that the wave height considered is $H_{1/10}$. Usually, $H_{1/10} = 1.27 \cdot H_s$. Thus it appears obvious that the stone size values will be much larger in this case. In many design, these values appeared to be way too conservative.

In order to have a good design, one can decide to choose a value located in-between those obtained from the two editions, or to use another formula, as the one defined by Van der Meer (Van der Meer, Rock Slopes and Gravel Beaches under Wave Attack, 1988).

5.1.3 Van der Meer formula

The stability formula of Van der Meer arrived after the one of Hudson. It is interesting and somehow more accurate for the reason that he decided to include other properties of the waves than the significant wave height $H_s$, as well as structure properties (Van der Meer, Rock Slopes and Gravel Beaches under Wave Attack, 1988). Indeed, after

¹ Special placement means that the long axis of the stones is placed perpendicularly to the slope face.
different series of experiments, he found out that apart from the parameters used in Hudson’s formula, the structure’s stability also varies in function of the wave steepness $s$, the structure slope angle $\alpha$, the structure permeability $P$ and finally the storm duration $t$. Two different formulae are established. One concerns the plunging waves and the other one the surging waves. The first step is thus to calculate the transition breaker parameter value $\xi_{m-1,0c}$:

$$\xi_{m-1,0c} = \left(6.2 \cdot P^{0.31} \cdot \sqrt{\tan \alpha}\right)^{\frac{1}{(P+0.5)}}$$

(5.5)

Then, one has:

- For plunging waves, $\xi_{m-1,0} < \xi_{m-1,0c}$:

$$\frac{H_s}{\Delta \cdot D_{n50}} = 6.2 \cdot S^{0.2} \cdot P^{0.18} \cdot N_z^{-0.1} \cdot \xi_{m-1,0}^{-0.5}$$

(5.6)

- For surging waves, $\xi_{m-1,0} > \xi_{m-1,0c}$:

$$\frac{H_s}{\Delta \cdot D_{n50}} = 1.0 \cdot S^{0.2} \cdot P^{-0.13} \cdot N_z^{-0.1} \cdot \sqrt{\cot \alpha \cdot \xi_{m-1,0}}^P$$

(5.7)

Where:

- $S$: Damage level [-]
- $P$: Structure permeability [-]
- $N_z$: Number of waves [-]

However, in order to be valid, equations (5.6) and (5.7) have to respect the following conditions:

- If $\cot \alpha \geq 4$ one should only use equation (5.6)
- The number of waves $N_z$ should not exceed 7500
- The permeability $P$ should be comprised between 0.1 and 0.6

The wave steepness should be comprised between 0.005 and 0.06.

### 5.1.3.1 Choice of the permeability $P$

The permeability depends of the way the armoured structure is set up. It has been defined as one can see in Figure 5.1 (Van der Meer, Rock Slopes and Gravel Beaches under Wave Attack, 1988):
Figure 5.1: Permeability coefficients as defined by Van der Meer

It is quite usual for a rock armoured sloping structure to have the same set up as b) in Figure 5.1, leading to a permeability value $P = 0.4$.

5.1.3.2 Damage level $S$

The damage level depends on the structure erosion with time. This erosion is relatively small, and is mainly due to the settlement of the structure, as well as to the removal of a few rocks that lost stability. Based on the different common slopes a structure can have, one has (USACE, 2002):

Table 5.3: Definition of the damage level $S$

<table>
<thead>
<tr>
<th>Slope</th>
<th>Initial damage</th>
<th>Intermediate damage</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 : 1.5</td>
<td>2</td>
<td>3 – 5</td>
<td>8</td>
</tr>
<tr>
<td>1 : 2</td>
<td>2</td>
<td>4 – 6</td>
<td>8</td>
</tr>
<tr>
<td>1 : 3</td>
<td>2</td>
<td>6 – 9</td>
<td>12</td>
</tr>
<tr>
<td>1 : 4</td>
<td>3</td>
<td>8 – 12</td>
<td>17</td>
</tr>
<tr>
<td>1 : 6</td>
<td>3</td>
<td>8 – 12</td>
<td>17</td>
</tr>
</tbody>
</table>

As in the Hudson’s formulae, one rather uses the values of $S$ for an initial damage, as we want the structure to fulfil its function for all its design life time.
After choosing those values, one can easily determine the nominal diameter $D_{h50}$ corresponding to a sea state, and thus the corresponding median weight $W_{50}$. The evolution of stability and weight evolves as follow:

![Figure 5.2: Stability and breaker parameter evolution](image)

It is interesting to have a look at the behaviour of the structure’s stability. It is drawn against the wave steepness in Figure 5.2, since the steepness is the only variable present in both stability and breaker parameter formulae (note that the y-scale differs for the stability parameter and the breaker parameter). Here, it is really easy to see the difference between surging and plunging waves as defined in equations (5.6) and (5.7). Indeed, the stability curve’s gradient changes abruptly when $\xi_{m-1,0}$ reaches the value $\xi_{m-1,0c}$ (for this case $\xi_{m-1,0c} = 3.77$), for which the stability is the lowest. As the nominal diameter is determined from the stability parameter, one can calculate the corresponding median stone weight considering the stone as a cube. Thus, the median stone weight can be plotted against the wave steepness as in Figure 5.3. As it could be expected, the lower the stability is, the higher is the stone weight. The weight is here presented in kilograms, with the same input parameters as in Figure 5.2.
Figure 5.3: Median stone weight evolution

The highest median stone weight (and consequently lowest structure’s stability) corresponds to the worst breaking conditions for the structure, as calculated in equation (5.5). That is the reason why it might be interesting to make the structure geometry vary in order to find the best compromise between cost and efficiency.

Once the nominal diameter has been found, it is easy to determine the diameters of the filter and core layer using Figure 5.1. However, it is important to have a look at the retention and permeability criteria. The first one prevents leeching of the core material through the filter layer and the second one prevents the hydraulic gradient being too high across the layer. They are respectively defined as:

\[
\frac{d_{15\text{filter}}}{d_{95\text{core}}} < 4 \text{ to } 5 \quad (5.8)
\]

\[
\frac{d_{15\text{filter}}}{d_{15\text{core}}} > 4 \text{ to } 5 \quad (5.9)
\]

Where:

- \(d_{15\text{filter}}\): grain size diameter exceeded by 85% of the filter material [m]
- \(d_{85\text{core}}\): grain size diameter exceeded by 15% of the core material [m]
- \(d_{15\text{core}}\): grain size diameter exceeded by 85% of the core material [m]
Nowadays, Van der Meer’s equations for structures’ stability are the most spread and used for the design of armoured structures. However, as for the wave run-up equations, the U.S. Army Corps of Engineers developed a formula for stability based on the wave momentum flux (Hughes, Wave momentum flux for coastal structures design, 2003).

5.1.4 Melby’s formula

As for the wave run-up, Melby (Melby, 2005) tried to define a more physical relation to define the stability of an armoured structure, using the wave momentum flux as a basis. The reason of this research is that he argued that Hudson’s and Van der Meer’s formulae have a too simplistic approach on the fluid force, and that the water depth and wave period at the toe of the structure should be considered as well.

The formulae have been developed using the laboratory results of Van der Meer, which thus allowed a later comparison. One of the most important changes is that it includes the water depth and the number of waves attacking the structure. As for Van de Meer’s formulae, the formula is not the same depending on whether the waves are plunging or surging (Melby, 2005):

- **Plunging waves:**

  \[ N_m = 5.0 \cdot \left( \frac{S}{\sqrt{N_z}} \right)^{0.2} \cdot P^{0.18} \cdot \sqrt{\cot \alpha} \]  \[ (5.10) \]

- **Surging waves:**

  \[ N_m = 5.0 \cdot \left( \frac{S}{\sqrt{N_z}} \right)^{0.2} \cdot P^{0.18} \cdot \cot \alpha^{0.5-p} \cdot s_m^{\frac{-p}{3}} \]  \[ (5.11) \]

\( s_{mc} \) corresponds to the critical wave steepness on the structure (inducing the lowest stability number), such as:

\[ s_{mc} = -0.0035 \cdot \cot \alpha + 0.028 \]  \[ (5.12) \]

\( N_m \) is the stability parameter. It is defined as (Melby, 2005):

\[ N_m = \sqrt{\frac{K_a \cdot P_{MF}}{\Delta}} \cdot \frac{h_s}{D_{n50}} \]  \[ (5.13) \]

In these equations:

- \( K_a \): 1 [-]
- \( P \): Structure permeability (See Figure 5.1) [-]
- \( N_z \): Number of waves during a storm, using the mean period \( T_m \) [-]
- \( s_m \): Mean wave steepness using the mean wave length \( L_m \) [-]

For the value of \( P_{MF} \), refer to section 3.4.1.
The results obtained thanks to these equations are usually a bit lower than the ones obtained with equations (5.6) and (5.7) from Van der Meer. That is why one should rather uses both of the design formulae in order to compare them. Besides, as it is usually made for a preliminary design, the stability should be verified when applicable by using a small scale model.

As the water depth at the toe of many rubble-mound structures is usually not so important, it is quite common to have a structure toe, which is here to support the main armour layer, but also to prevent any damages that could result from the scour effect.

### 5.1.5 Toe stability

In order to support the armour weight and protect the water to infiltrate the structure core, a toe is usually needed for shallow and intermediate water. During a long time, a rule of thumb defined by the Shore Protection Manual was saying that the indicative stone weight of the toe should be ten times lower than the armour median stone weight. However no precise rules were defined regarding it.

Therefore some researches have been performed (d’Angremond, Van der Meer, & Gerding, Toe structure stability of rubble mound breakwaters, 1995), and a formula has been set up for the determination of a nominal toe diameter:

\[
\frac{H_s}{\Delta \cdot D_{n50}} = \left( \frac{0.24 \cdot h_b}{D_{n50}} + 1.6 \right) \cdot N_{od}^{0.15}
\] (5.14)

Figure 5.4: Representation of a structure toe

In equation (5.14), \(h_b\) is the water depth at the top of the toe berm as one can see on Figure 5.4.

The parameter \(N_{od}\) is similar to the parameter \(S\) in the formulae of Van der Meer, and varies in function of the damage level one wishes to have. More precisely, it corresponds to the number of units displaced out of the armour layer within a strip width of \(D_{n50}\). The different values are (USACE, 2002):

- 0.5 for assuring no damage at all
- 2 for an acceptable damage
- 4 for a severe damage

In the case of an acceptable damage, it is possible to choose \(N_{od} = 1\) to have a bigger safety margin (d’Angremond & van Roode, Breakwaters and Closure Dams, 2004).
This equation is valid for irregular waves, regardless of their breaking kind. However, it applies for some given ranges, which are representative of a large panel of structures:

- \( 0.4 < \frac{h_b}{h_s} < 0.9 \)
- \( 0.28 < \frac{H_s}{h_s} < 0.8 \)
- \( 3 < \frac{h_b}{D_{n50}} < 25 \)

The dimensions of the toe usually follow a rule-of-thumb saying that the height should be around 2 to 3 times \( D_{n50} \) and the length around 3 to 5 times \( D_{n50} \).

Once that the nominal diameter of the toe is found, one can almost consider that the design of the rubble mound breakwater is done. Despite this, it is still important to have a look at the breakwater head, where the wave action is not exactly the same as on the breakwater body. This is however not treated in this report.

The rubble-mound breakwaters’ design is basically focused on getting the corresponding armour stone nominal diameter to a sea state, from which stem the other dimensions. In comparison to that, the stability of a vertical structure is totally different. Indeed, one has to consider the whole structure as an entity in order to obtain the best design for the structure to be stable.

### 5.2 Stability of vertical structures

Even if most protection water structures are sloping structures, vertical structures are another well spread kind of structures, especially for large water depths. Their use is pretty common in Japan, and Japanese engineers and researchers such as Goda (Goda, 2000) developed practical formulae for vertical structures’ stability.

![Cross section of a typical vertical breakwater](image)

Figure 5.5: Cross section of a typical vertical breakwater

There are three major risks that can cause the failure of a vertical breakwater: the sliding, the overturning, and the foundation failure (inducing for instance a circular slip of the whole structure). As the foundation failure concerns the geotechnical aspect of the foundation, only the stability of the vertical structure itself is treated in this report, that is to say the sliding and overturning stability. One can see in Figure 5.6 the representation of the stability failure due to sliding and overturning. In order to well design the structure, one has to look after the different pressures that are being applied on the structure. To calculate these pressures, one has first to define the design wave
height $H_{\text{max}}$ which is not the same the one used for the sloping structures’ design, as well as the elevation up to which the wave will exert pressure.

According to Goda (Goda, 2000), the design wave height for vertical structure is the highest wave of the sea state considered. Seaward of the surf zone, it is taken as $H_{\text{max}} = 1.8 \cdot H_{m0}$, whereas within the surf zone, $H_{\text{max}}$ is the highest of random breaking waves at a distance located $5 \cdot H_{m0}$ seaward of the structure, where the water depth is $h_b$. Goda realized graphs that allows to determine easily these wave heights (See appendixes I to IV). It is important to note that the period of this maximum design wave height is the same as the one of the significant wave height $H_{m0}, T_{m\cdot1.0}$.

Once the maximum wave height determined, one can calculate the elevation up to which the wave pressure is exerted (Goda, 2000):

$$\eta^* = 0.75 \cdot (1 + \cos \beta) \cdot H_{\text{max}}$$  \hspace{1cm} (5.15)

As for the overtopping discharge calculations, $\beta$ [deg] corresponds to the wave angle attack.

### 5.2.1 Determination of waves’ pressures on vertical structures

Waves’ pressures do not exert uniformly on the whole structure, but varies regarding depth. The repartition of all the pressures can be represented as in the following figure.
Figure 5.7: Wave pressure distribution on a vertical structure

As one can see on Figure 5.7, there are four main pressures exerting on the structure. The pressures on the front of the vertical wall are defined according to the following formulae (Goda, 2000):

\[
p_1 = \frac{1}{2} \cdot (1 + \cos \beta) \cdot (\alpha_1 + \alpha_2 \cdot \cos^2 \beta) \cdot \rho \cdot g \cdot H_{\text{max}}
\]  \hspace{1cm} (5.16)

\[
p_2 = \begin{cases} 
  p_1 \cdot \left(1 - \frac{h_c}{\eta^*}\right), & \eta^* > h_c \\
  0, & \eta^* \leq h_c 
\end{cases}
\]

\[
p_3 = \alpha_3 \cdot p_1
\]  \hspace{1cm} (5.17)

in which:

\[
\alpha_1 = 0.6 + \frac{1}{2} \cdot \left[ \frac{4 \cdot \pi \cdot h_s/L}{\sinh(4 \cdot \pi \cdot h_s/L)} \right]^2
\]  \hspace{1cm} (5.19)

\[
\alpha_2 = \min\left\{ \frac{h_b - d}{3 \cdot h_b} \cdot \frac{H_{\text{max}}}{d} \cdot \left( \frac{H_{\text{max}}}{H_{\text{max}}} \right)^2, \frac{2 \cdot d}{H_{\text{max}}} \right\}
\]  \hspace{1cm} (5.20)

\[
\alpha_3 = 1 - \frac{h'}{h_s} \cdot \left[ 1 - \frac{1}{\cosh(2 \cdot \pi \cdot h_s/L)} \right]
\]  \hspace{1cm} (5.21)
Moreover, there is an uplift pressure exerted on the bottom of the structure as showed in Figure 5.7. This pressure is due to the buoyancy but also the waves’ action, and is calculated as follow:

\[
p_u = \frac{1}{2} \cdot (1 + \cos \beta) \cdot \alpha_1 \cdot \alpha_3 \cdot \rho_w \cdot g \cdot H_{max} \tag{5.22}
\]

As \( H_{\text{max}} \) is usually pretty important regarding \( H_{\text{m0}} \), the pressures might seem to be really high. However, the choice of this wave height for the design resides in the principle that the breakwater should be able to counter, during a storm, the single wave exerting the largest pressure among all storm waves.

The coefficient \( \alpha_1 \) increases with the wave period. This shows that the pressure of a wave gets bigger for longer periods. The coefficient \( \alpha_2 \) corresponds to the increase in pressure in function of the water depth.

It is quite usual to have a rubble-mound foundation to spread the structure weight on the sea bottom. Sometimes these rubble-mounds have a significant height, and the same impulsive conditions can occur as for the overtopping (see section 4.3). It is thus important to check if such conditions are likely to occur or not, since it can have a non-negligible impact at a later stage regarding the section design. This can be made by rewriting equation (5.16) in the following way (Goda, 2000):

\[
p_{1\text{imp}} = \frac{1}{2} \cdot (1 + \cos \beta) \cdot (\alpha_1 + \alpha^* \cdot \cos^2 \beta) \cdot \rho \cdot g \cdot H_{\text{max}} \tag{5.23}
\]

In which:

\[
\alpha^* = \text{max}\{\alpha_2, \alpha_I\} \tag{5.24}
\]

\[
\alpha_I = \alpha_{IH} \cdot \alpha_{IB} \tag{5.25}
\]

And:

\[
\alpha_{IH} = \text{min}\{H_{\text{max}}/d, 2.0\} \tag{5.26}
\]

\[
\alpha_{IB} = \begin{cases} 
\cos \delta_2 / \cosh \delta_1, & \delta_2 \leq 0 \\
1/(\cosh \delta_1 \cdot \sqrt{\cosh \delta_2}), & \delta_2 > 0 
\end{cases} \tag{5.27}
\]

\( \alpha_{IH} \) and \( \alpha_{IB} \) result from the following conditions:

\[
\delta_1 = \begin{cases} 
20 \cdot \delta_{11}, & \delta_{11} \leq 0 \\
15 \cdot \delta_{11}, & \delta_{11} > 0 
\end{cases} \tag{5.28}
\]

\[
\delta_2 = \begin{cases} 
4.9 \cdot \delta_{22}, & \delta_{22} \leq 0 \\
3.0 \cdot \delta_{22}, & \delta_{22} > 0 
\end{cases} \tag{5.29}
\]
\[
\delta_{11} = 0.93 \cdot \left( \frac{B_M}{L} - 0.12 \right) + 0.36 \cdot \left( 0.4 - \frac{d}{h_s} \right) \tag{5.30}
\]

\[
\delta_{22} = -0.36 \cdot \left( \frac{B_M}{L} - 0.12 \right) + 0.93 \cdot \left( 0.4 - \frac{d}{h_s} \right) \tag{5.31}
\]

In equations (5.30) and (5.31), \(B_M\) corresponds to the width of the seaward rubble mound. If \(p_{\text{imp}}\) appears to be lower than the pressure \(p_1\) calculated before, then one can consider that there is no impulsive breaking on the structure. In the contrary case, one should use \(p_{\text{imp}}\) instead of \(p_1\) for the calculations.

Once all these pressures have been calculated, it is possible to calculate the total wave pressure as well as its moment on the front of the structure and on its bottom. Thus, one has for the front of the structure (Goda, 2000):

\[
P_{hz} = \frac{1}{2} \cdot (p_1 + p_3) \cdot h' + \frac{1}{2} \cdot (p_1 + p_2) \cdot h_c \tag{5.32}
\]

\[
h^*_c = \min(\eta^*, h_c)
\]

\[
M_{hz} = \frac{1}{6} \cdot (2 \cdot p_1 + p_3) \cdot h'^2 + \frac{1}{2} \cdot (p_1 + p_2) \cdot h' \cdot h_c^* + \frac{1}{6} \cdot (p_1 + 2 \cdot p_2) \cdot h_c^2 \tag{5.33}
\]

And for the uplift pressure:

\[
P_{\text{uptift}} = \frac{1}{2} \cdot p_u \cdot B_v \tag{5.34}
\]

\[
M_{up} = \frac{1}{3} \cdot p_u \cdot B_v^2 \tag{5.35}
\]

\(B_v\) corresponds here to the width of the upright section. It is usually first estimated and its optimum value can be iterated afterwards using the stability safety factors (see section 5.2.2 after). The step coming right after the determination of the wave’s pressure is the design of the upright section.

### 5.2.2 Design of the vertical structure upright section

The value of the structure’s crest height is determined at an early stage by the wave parameters and the overtopping discharge that one allows. Here, the focus is made on the way to get the optimal section width \(B_v\) for the structure to be resistant enough to counter the waves’ attack without moving. In order to do that, one has to look after the safety factors against sliding and overturning, but the first step is to determine the weight of the structure itself.

For that, \(B_v\) is also estimated here. One thus has (USACE, 2002):
\[ F_G = \rho_{\text{str}} \cdot B_v \cdot (h' + h_c) \cdot g - \rho_w \cdot g \cdot B_v \cdot h' \] (5.36)

where:
\( \rho_{\text{str}} \): Structure mass density [kg/m³]

The resulting moment is thus:
\[ M_G = \frac{1}{2} \cdot B_v^2 \cdot g \cdot [\rho_{\text{str}} \cdot (h' + h_c) - \rho_w \cdot h'] \] (5.37)

Further studies carried out later by Van der Meer, Juhl and van Driel showed that the equations about waves’ pressure were a bit conservative. That is why some reduction factors have been elaborated as follow (USACE, 2002):

\[ P_{hz\text{final}} = U_{hz} \cdot P_{hz}, \quad U_{hz} = 0.90 \] (5.38)
\[ P_{up\text{final}} = U_{up} \cdot P_{up\text{lift}}, \quad U_{up} = 0.77 \] (5.39)
\[ M_{hz\text{final}} = U_{Mthz} \cdot M_{hz}, \quad U_{Mthz} = 0.81 \] (5.40)
\[ M_{up\text{final}} = U_{Mtup} \cdot M_{up}, \quad U_{Mtup} = 0.72 \] (5.41)

Thanks to those equations, one can finally assess the structure’s stability against sliding and overturning.

\[ SF_S = \frac{\mu_f \cdot (F_G - P_{up\text{final}})}{P_{hz\text{final}}} \] (5.42)
\[ SF_{Ov} = \frac{M_G}{M_{up\text{final}} + M_{hz\text{final}}} \] (5.43)

where:
\( \mu_f \): friction coefficient between the upright section and the foundation [-].
\( \mu_f = 0.6 \) for friction between concrete and rubble stones. For different materials, other values can be found in appendix VIII (USACE, 2002).

According to the Japanese design, which is one of the most advanced in this field, the values of these safety factors should not be less than 1.2. Since \( B_v \) is present in some of the parameters used in these equations, it is thus possible to determine the optimum width using iterative calculations.

It is necessary to note that the attention should also be given to the rubble-mound and the seabed, in order to check if they have enough resistance to carry the structure. There is however a simplified technique to examine the heel pressure in the case of seabeds made of a dense sand layer or good bearing capacity soil. In those two cases, the distribution beneath the bottom of the section is assumed to be trapezoidal or
triangular. The largest contact pressure at the heel can thus be calculated as (Goda, 2000):

\[
p_e = \begin{cases} 
\frac{2 \cdot W_e}{3 \cdot t_e}, & t_e \leq \frac{1}{3} \cdot B_v \\
\frac{2 \cdot W_e}{B_v} \cdot \left(2 - 3 \cdot \frac{t_e}{B_v}\right), & t_e > \frac{1}{3} \cdot B_v 
\end{cases}
\]

(5.44)

In which:

\[
t_e = \frac{M_e}{W_e}, \quad M_e = M_G - M_{up\text{final}} - M_{hz\text{final}}
\]

(5.45)

\[
W_e = F_G - P_{up\text{final}}
\]

The value of \( p_e \) should normally be kept under 400 to 600 kPa, in order to insure safety regarding the carrying capacity.

Once that all these calculations have been performed the design of the vertical structure is almost done. If the structure is a concrete caisson, which is quite common, it is usually prefabricated, and made of several intervals with partition walls. Normally, depending on the maximum length that can be realized, the length of a caisson is equal to 0.5 to 2 times its width. Furthermore, the length between each wall interval should not exceed 5 m. The outer wall thickness should be around 0.4 to 0.5 m, and the inner walls around 0.2 m. As for the bottom slab, it is common to set it around 0.5 to 0.7 m (Goda, 2000). The step following the vertical structure design is the design of the rubble mound foundation on which it lies, as well as the foot protection blocks.

### 5.2.3 Vertical structure’s rubble-mound foundation design

The rubble-mounds foundation’s function is to spread the load of the vertical structure over the seabed. They normally have a minimum height of 1.5 m (Goda, 2000) and their depth should not be too deep either, in order to facilitate underwater operations, for instance levelling evenly the mound surface. The value of the berm width \( B_M \) of the rubble mound foundation (in front of the vertical structure) should be around 5 (normal conditions) to 10 m (large storm waves). The rear-side foundation berm just has the function of spreading the load on the seabed. Therefore the choice of its width is left to the designer. Besides, it is common to set the rubble-mound slope at 1:2 to 1:3 for the seaward side and 1:1.5 to 1:2 for the harbour side.

As for the armoured breakwaters, even if it is here submerged, the foundation must have armour units with a sufficient weight in order to withstand the wave action. Based on the Hudson’s formula, Tanimoto found a formula to determine the minimum stone median weight (Goda, 2000):

\[
W_{50} = \frac{\rho_s}{N_s^3 \cdot \Delta^3} \cdot H_{m0}^3
\]

(5.46)
Contrary to Hudson’s equation (equation (5.4)), one can notice that the slope angle does not appear in Tanimoto’s formula. Moreover, the stability coefficient $N_s$ is not chosen among values in a table as $K_D$ in equation (5.4), but defined as (Goda, 2000):

$$N_s = \max \left\{ 1.8, \left( 1.3 \cdot \frac{1 - \kappa}{\kappa \pi} \cdot \frac{h'}{H_{m0}} + 1.8 \cdot \exp \left( -1.5 \cdot \frac{(1 - \kappa)^2}{\kappa \pi} \cdot \frac{h'}{H_{m0}} \right) \right) \right\} \quad (5.47)$$

with:

$$\kappa = \frac{4 \cdot \pi \cdot h'/L'}{\sinh(4 \cdot \pi \cdot h'/L')} \cdot \sin^2 \left( \frac{2 \cdot \pi \cdot B_M}{L'} \right) \quad (5.48)$$

where:

$L'$: Wave length at the depth $h'$
$B_M$: Rubble-mound’s berm width

Figure 5.8: Evolution of the foundation stone size and weight with the water depth $h$

Figure 5.8 presents the evolution of the stone’s size and diameter in function of an increasing water depth. This has been realized with a constant rubble-mound foundation of 3 m and a single significant wave height. It is logical to have a decreasing stone size since the waves’ effect reduces with the increasing depth. One can however notice a peak on the curve, which corresponds to a growth in the wave height due to the presence of the foundation close to the Still Water Level. Besides, further studies on the same case showed that impulsive conditions occur at that depth. It is also important to say that even if they are not needed as Figure 5.8 points it out,
one can choose to have big blocks in deep water as well, in order to facilitate the setting-up of the foundation. The foundation’s surface can still be made of small grains to guarantee the levelling.

The value of $\kappa$ changes when waves are oblique and at the breakwater head, but this is not treated here.

As well as having two layers of rock on the foundation, it is quite common to set foot-protection concrete blocks at the front and the rear of the upright section, on the top of the embankment (two rows at the front and one at the rear).

![Disposition of the foot protection blocks](image)

Figure 5.9: Disposition of the foot protection blocks

These foots are normally rectangular, with weights from 10 to 40 tons, depending on the waves’ parameters and water depth. An example of a typical foot is presented in Figure 5.10 (USACE, 2002):

![Japanese foot protection block](image)

Figure 5.10: Japanese foot protection block

As they are rectangular, they allow chocking properly the upright section. There is no formula to design them, but one can use the graph of appendix IX.

Once this last design step has been performed, the design of the vertical structure is fulfilled. This way, if both designs of the armoured and vertical structures are done, it is possible to compare them in function of different parameters such as the allowable overtopping discharge or the water depth, regarding the needed materials amount as well as their price. By setting up different models, it has been possible to establish such comparison, as presented in the next part.
6 Development of the numerical models

All the equations that have been presented so far have been used for the realisation of models for the different kinds of structures, with the use of the software MathCAD. The main objective was to get a comparison of the price evolution in function of the mean limit overtopping discharge for both armoured and vertical structures, as presented in section 4.7.

As the models were set up, the comparison has thereafter been extended to the water depth. Besides, it is still possible for one to make new comparisons with these models, since it is really easy to change the parameters we want to investigate. Another advantage of these models is that they allow one to design the section of an armoured breakwater or vertical breakwater in a short amount of time.

Four main steps represent the work done for getting the final models and results:

- Determination of the mean overtopping discharge and volumes
- Determination of the crest height $R_c$ for a limit mean overtopping discharge
- Addition of the structures’ stability calculation
- Introduction of the price dimensions for comparing purposes

It has been decided to present only one example for sloping structures and one for vertical structures, since the design logic is the same for any structure, and describing examples for all the different kinds of structures would be tedious and also demand too much space.

6.1 Run-up, mean overtopping discharge and volumes

As mentioned before, the first step is to create a file for each kind of structure, that calculates the mean overtopping discharge which would occur for a given sea state. In the case of sloping structures, the run-up has also been determined and then entered as the value of the crest height $R_c$.

The input parameters are almost the same for each kind of structures. Besides, the definition of the units is already made in MathCAD, which is much more convenient. For each file, the general parameters input looks as follow:

<table>
<thead>
<tr>
<th>General parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep water significant wave height : $H_0 := 2m$ One can directly use $H_{m0}$ if available</td>
</tr>
</tbody>
</table>

The input period used (peak, mean, spectral) depends on the available data. The other types of periods can still be estimated afterwards.

| Wave period : $T_p := 8sec$ or $T_m - 1.0$ if available. |

Slope Angle: $\alpha := \text{deg}$

Given $\tan(\alpha) = 0.4 \quad \alpha := \text{Find}(\alpha) \quad \alpha = 21.801 \text{deg}$

Wave attack angle : $\beta := 0$

(0 = perpendicular waves)

Depth at the toe of the structure : $h_s := 10m$

Figure 6.1: Model general input parameters
Thanks to this, it is then easy to calculate parameters used for the run-up and overtopping determination such as the wave length and the breaker parameter. As for the reduction factors, they are usually calculated according to certain conditions. That is the reason why their determination has been made thanks to the programming tools of MathCAD, or with the use of control listboxes. An example is displayed hereunder:

**Reduction factors:**

\[
\gamma_f :=
\begin{align*}
\text{Smooth impermeable surface} & \quad \gamma_f = 0.4 \\
\text{Rocks (1 layer, impermeable core)} \\
\text{Rocks (1 layer, permeable core)} \\
\text{Rocks (2 layers, impermeable core)} \\
\text{Rocks (2 layers, permeable core)} \\
\text{Cubes (1 layer, random positioning)} \\
\text{Cubes (2 layers, random positioning)} \\
\text{Antifers} \\
\text{Haro's} \\
\end{align*}
\]

**Oblique waves reduction factor :**

\[
\beta = 0 \\
\gamma_\beta := \begin{cases} 
1 - 0.0063 \cdot \beta & \text{if } 0 \leq \beta \leq 80 \\
1 - 0.0063 \cdot \beta & \text{otherwise} 
\end{cases}
\]

\[
\gamma_\beta = 1
\]

Figure 6.2: Definition of the reduction factors by the use of programming tool

Once that all the factors have been determined, it is then possible to calculate the wave run-up and mean overtopping discharge to which the structure will be submitted. Figure 6.3 shows an example of it, for an armoured structure. Note that the value of \( q \) appears in \( \text{m}^2/\text{s} \), but this is due to the fact that the software automatically simplified the units (as the normal unit would be \( \text{(m}^3/\text{s})/\text{m} \)).

Furthermore, these first files calculating the mean overtopping discharge (using equations of section 4) are also interesting in the calculation of the required crest height for a given overtopping discharge.
6.2 From the discharge \( q \) to the crest height \( R_c \)

The first files created for the calculation of the mean overtopping discharge have been a great help for the creation of the reverse course. Indeed, they have been used as a basis, since MathCAD disposes of a Boolean calculus tool which thus spares one to re-write all the equations presented before to get the crest height corresponding to a given discharge.

The limit values used for the mean overtopping discharge are the one displayed in Table 4.1. They have been set in a one column matrix in an increasing order. An example has been displayed for the calculation of the different crest heights in the case of a vertical structure, with \( h_s = 10 \) m, \( H_{m0} = 1.84 \) m and \( T_{m-1,0} = 10.4 \) sec (Note: \( d_t \) corresponds to \( d_* \); it is not possible to enter it this way in MathCAD, as * is an operator).

The value of \( Q \) given at the top of Figure 6.4 is needed for the iterative calculus to start at this value. By entering the right function, the corresponding values of \( Q \) are then directly computed. The same procedure is followed to get the crest height value \( R_c \), and the results can finally be summarised as in Figure 6.5. One can notice here how large the difference in crest height can be for two different mean overtopping discharges.
Figure 6.4: Iterative calculation of $Q$

$$Q_{cv} := 1 \quad j := 0..9$$

$$q_{cv} := \begin{pmatrix}
0.000001 \\
0.00001 \\
0.00003 \\
0.0001 \\
0.0004 \\
0.001 \\
0.01 \\
0.02 \\
0.05 \\
0.2
\end{pmatrix} \mathrm{m}^3 \mathrm{m}^{-1} \mathrm{sec}^{-1}$$

$$\text{Given } q_{cv} = \begin{cases}
Q_{cv} \sqrt{g \cdot H_{m0}}^3 & \text{if } d_t > 0.3 \\
Q_{cv} \cdot d_t^2 \cdot \sqrt{g \cdot d^3} & \text{if } d_t < 0.3
\end{cases}$$

$$\text{Find } Q_{cv}(j) = F(q_{cv}(j))$$

<table>
<thead>
<tr>
<th>$q_{cv}$</th>
<th>$Q_{cv}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.279⋅10^{-6}</td>
</tr>
<tr>
<td>1</td>
<td>1.279⋅10^{-5}</td>
</tr>
<tr>
<td>2</td>
<td>3.838⋅10^{-5}</td>
</tr>
<tr>
<td>3</td>
<td>1.279⋅10^{-4}</td>
</tr>
<tr>
<td>4</td>
<td>4.10^{-4}</td>
</tr>
<tr>
<td>5</td>
<td>1.279⋅10^{-3}</td>
</tr>
<tr>
<td>6</td>
<td>0.01</td>
</tr>
<tr>
<td>7</td>
<td>0.02</td>
</tr>
<tr>
<td>8</td>
<td>0.05</td>
</tr>
<tr>
<td>9</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Figure 6.5: Determination of the crest height for a given overtopping rate

The stability is defined after this part which allows a complete section design. It is especially needed for the materials volume and price calculations.
### 6.3 Structure’s stability addition

The stability of the structure is not directly linked to the overtopping discharge and can be determined separately. However, it is interesting to have those two different things together in the same model regarding the convenience of having a whole design in one file. In the case of armoured structures, the median stone weight has been calculated using the formulae of Van der Meer (see section 5.1.3), which gives (same parameters as in Figure 6.1):

\[
L_0m := \frac{g \cdot T_m^2}{2 \pi} \quad L_0m = 82.553m
\]

\[
s_{om} := \frac{Hm_0}{L_0m} \quad s_{om} = 0.022 \quad \xi_m := \tan(\alpha) \sqrt{s_{om}} \quad \xi_m = 3.349
\]

\[
\xi_{mc} := (6.2P^{0.31}\sqrt{\tan(\alpha)}) \left( \frac{1}{P+0.5} \right) \quad \xi_{mc} = 3.768
\]

Number of waves \( N_z \) : \( N_z := \frac{1}{T_m} \) \( N_z = 4.95 \times 10^3 \)

\[
\text{Stab} := \frac{6.2 S^{0.2} \cdot P^{0.18} \cdot N_z^{0.1}}{\xi_m} \quad \text{Stab} = 1.409
\]

\[
\left( 1.0 S^{0.2} \cdot P^{0.13} \cdot N_z^{0.1} \cdot \sqrt{\cot(\alpha)} \cdot \xi_m^P \right) \quad \text{if} \quad \xi_m \geq \xi_{mc}
\]

\[
\text{Stab} = \frac{Hm_0}{\Delta \cdot D_{n50}} \quad D_{n50} := \frac{Hm_0}{\Delta \cdot \text{Stab}} \quad D_{n50} = 0.83m
\]

\[
V_{Dn50} := D_{n50}^3 \quad V_{Dn50} = 0.572m^3
\]

The average required weight is then:

\[
W_{50} := V_{Dn50} \rho_s \quad W_{50} = 1.515 \times 10^3 \text{ kg}
\]

\[
W_{50} = 1.515 \text{ tonne}
\]

Figure 6.6: Median Stone weight calculation

However, the obtained value corresponds just to the mean weight of the stones. As they are quite big, their grading is assumed to be rather narrow. Moreover, the filter and core size are determined following the rule of Van der Meer as one can see in Figure 5.1 (usually the model b) is followed, giving a permeability \( P = 0.4 \).
Stone size repartition:

It is advised to take a narrow grading in the case of rubble mounds, such as $\frac{D_{50}}{D_{15}} < 1.5$

$D_{n50} = 0.83\text{m}$

$\frac{D_{85}}{D_{15}} = 1.25$  

The best repartition would consist in having $(D_{85}+D_{15}) / 2 = D_{50}$

$D_{15} := D_{n50} / 1.125$  

$D_85 := 1.25 D_{15}$

$D_{15} = 0.738\text{m}$

$V_{D_{15}} := D_{15}^3$  

$V_{D_{85}} := D_{85}^3$

$W_{15} := V_{D_{15}} \rho_s$  

$W_{85} := V_{D_{85}} \rho_s$

$D_{85} = 0.402\text{m}^3$  

$V_{D_{85}} = 0.784\text{m}^3$

$W_{15} = 1.064\text{tonne}$  

$W_{85} = 2.079\text{tonne}$

Filter & Core layers:

Usually, $W_{\text{filter}} = (W_{50})/15$ to $(W_{50})/10$, or $D_{n\text{filter}} = D_{n50}/2$ to $D_{n50}/3$

It is common to see $D_{n\text{filter}} = D_{n50}/2$ for $P = 0.4$ (V.d. Meer 88a)

Following the Van de Meer values (V. der Meer, 1988):

$D_{n50\text{filter}} = 0.5 D_{n50}$

Thus, $W_{50\text{filter}} := D_{n50\text{filter}}^3 \rho_s$

And: $D_{n50\text{core}} := 0.25 D_{n50\text{filter}}$

So that: $W_{50\text{core}} := D_{n50\text{core}}^3 \rho_s$

Assume a quarry run with a wide grading (common): $\frac{D_{85\text{core}}}{D_{15\text{core}}} = 2.5$

$D_{15\text{core}} := D_{n50\text{core}} / 1.75$

$W_{15\text{core}} := D_{15\text{core}}^3 \rho_s$

$D_{85\text{core}} := 2.5 D_{15\text{core}}$

$W_{85\text{core}} := D_{85\text{core}}^3 \rho_s$

And, Permeability criterion: $\frac{D_{15\text{filter}}}{D_{15\text{core}}} > 4$ to $5$ => $D_{15\text{filter}} = 5 D_{15\text{core}}$

Retention criterion: $\frac{D_{15\text{filter}}}{D_{85\text{core}}} < 4$ to $5$

$D_{15\text{filter}} = 0.296\text{m}$

$W_{15\text{filter}} := D_{15\text{filter}}^3 \rho_s$

$W_{85\text{filter}} := D_{85\text{filter}}^3 \rho_s$

Finally: $D_{85\text{filter}} := 2 D_{n50\text{filter}} - D_{15\text{filter}}$

$W_{15\text{filter}} = 69.034\text{kg}$

$W_{85\text{filter}} = 402.605\text{kg}$

$D_{85\text{filter}} = 0.534\text{m}$  

$W_{15\text{filter}} = 69.034\text{kg}$  

$W_{85\text{filter}} = 402.605\text{kg}$

$D_{n50\text{filter}} = 0.415\text{m}$  

$W_{50\text{filter}} = 189.428\text{kg}$  

$D_{n50\text{core}} = 0.104\text{m}$  

$W_{50\text{core}} = 2.96\text{kg}$

$D_{15\text{core}} = 0.059\text{m}$  

$W_{15\text{core}} = 0.552\text{kg}$  

$D_{85\text{core}} = 0.148\text{m}$  

$W_{85\text{core}} = 8.629\text{kg}$

$D_{85\text{filter}} = 0.534\text{m}$  

$W_{85\text{filter}} = 402.605\text{kg}$

Figure 6.7: Stone size repartition - Armour, Filter and Core
When a toe is needed, the toe stability is calculated using equation (4.14). In the case presented in Figure 6.7, no toe is needed since the water depth is deep enough (the value of \( D_{n50toe} \) thus appears as undefined in the model).

In the case of a sloping structure, once that the different layer diameters have been determined, it is easy to define the thickness of the different layer as well as the volume per metre width of a section. This volume thus allows determining the price per metre of the structure, which is presented thereafter.

In the case of the vertical structure, the same way is followed for the rubble-mound foundation, but the upright section has to be divided in each of its component, that is, the submerged walls made in a special concrete, the sand used to fill the intervals of the caisson, and the top of the structure made in a normal quality concrete. The volume of each of these materials is determined thanks to the rules defined in section 5.2.2 in order to be able to establish price estimation per linear metre.

The main goal desired by adding the structure’s stability to the overtopping in the model is to have a tool that permits to design a whole structure by just entering the sea- and waves’ parameters as input. The economic dimension is then easy to add to get a first cost estimation of the structure.

6.4 Structures’ costs comparison related to overtopping

Once that the models have been set for both armoured and vertical structures, including overtopping, stability and materials’ cubing, it is relatively easy to define prices per metre for each structure. Indeed, the prices of material are usually displayed in per cubic metre. The volume of material just has to be determined for one metre width and be multiplied by its corresponding price.

As the dimensions of the crest height evolve whether the overtopping discharge increases or decreases, so are the volumes and thus the prices. An example is shown in Figure 6.9. This breakwater is designed for a wave height \( H_{m0} = 1.84 \) m with a water depth at the toe of the structure of \( h_s = 10 \) m. The price has here been set after looking at some existing structure prices. Indeed, the prices of the stones can for instance vary in function of the stone quality or its geographical location compared to the structure’s site. Moreover, the prices can also vary depending on the stone sizes. If it occurs, it can easily be modified in the model to obtain a price per layer and then a final price per linear metre. Here, a unique price per m3 has been considered for the rubble-mound. In order to define the cost per linear metre of structure, the rubble mound is divided in 3 areas as follows (see Figure 6.9 for the calculations):

![Figure 6.8: Rubble-mound sub-areas](image-url)
As a result, the price evolution can be traced against the mean overtopping discharge, and the MathCAD regression tools can give us the regression curve corresponding to this structure. If the same sea and wave parameters are used in the vertical structure model, the two regression curves can be represented on the same graph in order to perform the comparison. Figure 6.10 represents the evolution of price for the both types of structure.
Figure 6.9: Price estimation of an armoured breakwater

A1 corresponds to the first slope surface:

\[ A_1 = \left( \frac{R_c + h}{\tan(\alpha)} \right)^2 \]

A2 corresponds to the surface under the breakwater crest:

\[ A_2 = B \left( R_c + h \right) \]

A3 corresponds to the surface of the landward slope:

\[ A_3 = \left( \frac{R_c + h}{\tan(\alpha)} \right)^2 \]

<table>
<thead>
<tr>
<th>( R_c ) m</th>
<th>( h ) m</th>
<th>( A_1 ) m²</th>
<th>( A_2 ) m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>212.108</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>191.189</td>
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</tr>
<tr>
<td>2</td>
<td>2</td>
<td>181.59</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>171.355</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>159.938</td>
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</tr>
<tr>
<td>5</td>
<td>5</td>
<td>152.607</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>134.946</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>129.842</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>123.245</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>113.592</td>
<td>9</td>
</tr>
</tbody>
</table>

Landward slope angle: \( \alpha_1 := \cot(\alpha_1) = 1.5 \)

Thus, total area \( A_{\text{tot}} := A_1 + A_2 + A_3 \)

<table>
<thead>
<tr>
<th>( A_{\text{tot}} ) m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
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<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
</tbody>
</table>

Sek := \( 1 \)

The price / m³ for stones of any size is defined as:

\[ V_{\text{linear}} = \frac{A_{\text{tot}}}{1000 \text{Sek m}^3} \]

Thus, the price per meter for the structure is:

\[ P_m = P_{\text{unitary}} \cdot V_{\text{linear}} \]

Finally

<table>
<thead>
<tr>
<th>( q ) m</th>
<th>( R_c ) m</th>
<th>( h ) m</th>
<th>( V_{\text{linear}} ) m³</th>
<th>( P_m ) Sek m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>4.564</td>
<td>0</td>
<td>4.075 \times 10^5</td>
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<tr>
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<td>1</td>
<td>3.827</td>
<td>1</td>
<td>3.69 \times 10^5</td>
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<td>2</td>
<td>2</td>
<td>3.476</td>
<td>2</td>
<td>3.513 \times 10^5</td>
</tr>
<tr>
<td>3</td>
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<tr>
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<td>7</td>
<td>1.395</td>
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<td>2.556 \times 10^5</td>
</tr>
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<td>1.102</td>
<td>8</td>
<td>2.433 \times 10^5</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>0.658</td>
<td>9</td>
<td>2.253 \times 10^5</td>
</tr>
</tbody>
</table>
Figure 6.10: Price evolution for a vertical structure (P(x)) and an armoured structure (Pr(x)) against overtopping (x)
When having a look on the graph on Figure 6.10, one can see how the price is evolving exponentially when decreasing the allowable overtopping discharge, which in a way can be expected from the beginning by looking at equations (3.7) to (3.13).

By superposing the two different curves, it is then possible to see in which case one structure is more economically interesting than the other at a given overtopping rate and for the same waves- and sea’s properties, as shown in the following figure:

![Figure 6.11: Price comparison between an armoured structure -Pr(x)- and a vertical structure -P(x)-](image)

In Figure 6.11, the price is presented in Swedish kronor. One can clearly see the point at which one a structure (here the armoured one) becomes more expensive than the other one. Results such as this one can thus be important concerning the choice of structure type.

Another interesting and non-negligible point of having established models which allow designing and compare structures is that the comparison is not restricted to overtopping only. Indeed, thanks to the MathCAD’s interface, it is possible to change the parameter that one wants to use as basis for a comparison.

### 6.5 Extension to other parameters

The study and use of overtopping as a design input parameter was one of the main aims when starting this study. The fact is that once MathCAD’s sheets have been set up, the author realised that it might also be interesting to use them to perform comparisons between structures with other input parameters than the overtopping discharge, but for instance water depth or wave height and period.

As it appears that most of the coastal defence structures located in deep water are often vertical structures, it has been decided to have a look at the price evolution of the two kinds of structure stated before in function of the water depth, for a fixed
overtopping discharge and wave parameters. Thanks to the regression functions, it is possible to realise a comparison between the prices as follow (the maximum water depth considered here is 30 m):

Figure 6.12: Structure price evolution comparison depending on the water depth

As one could expect it, one notices in Figure 6.12 that there is one point at which the armoured structure (plain line) becomes more expensive than the vertical one (dashed line). By using solving tools it is easy to determine the point where structures have the same price. It seems logical that the armoured structure becomes more expensive after a certain depth, since the presence of slopes induces an important increase in material volume.

Although the variation of other input parameters is not presented here, it can easily be made as mentioned before.

Besides, representing the price according to one variable input parameter is not the only thing possible thanks to these models. It is indeed possible to represent it in function of two variables by the use of matrices, despite the fact the fact that a comparison cannot be made directly. Here it has been realised for a vertical structure depending on the allowable overtopping discharge and the water depth. It is thus
possible for instance to see the evolution of the price at a precise water depth according to the variation in the overtopping discharge, and vice-versa.

Figure 6.13: Vertical breakwater's price evolution in function of \( q \) and \( h_s \)

Figure 6.13 gives us an overview of how the price would vary depending on the location of the structure in the sea and the overtopping discharge that could handle the structure. As expected, the highest cost occur for a minimum allowable overtopping discharge of \( q = 0.1 \) l/m/s, at the maximum water depth considered (here \( h_s = 30 \) m).

Numerically, it can be presented as in Figure 6.14. As nine values are used for the overtopping discharge limits and fourteen values for the water depth, the prices are displayed in a 14x9 matrix. This can be changed by adding or removing values in one of these two parameters. It is however much more pleasant and easy to understand to look at a bar plot as displayed in Figure 6.13.
Figure 6.14: Price evolution's matrix
7 Conclusion

The models realisation for designing a whole breakwater section according to its wave overtopping and stability gives anticipated results.

Indeed, about the overtopping, it shows clearly that the structure dimensions increase considerably when decreasing the mean allowable overtopping discharge, and so does the price. Thus it is a matter of great importance to well define at the early stage of a project the desired functionality of the breakwater, in order to make good first price estimation and in the same way avoid ulterior unexpected costs (it has been seen that a small difference in the crest height can have huge consequences on the overtopping discharge). This is even more true when considering a vertical structure that might be inclined to waves’ breaking impulsive conditions. If for instance the budget is limited, the designer might thus have to look after other dispositions of the breakwater that might allow raising the overtopping discharge and thus reduce the crest height.

The other conjecture when starting the work was that one structure might become more interesting than the other on an economical point of view depending on parameters such as the water depth. This has been verified for the water depth when it appears clearly that one structure becomes more expensive than another type after a certain depth. However, heed should be paid on the fact that the prices of materials can change depending on the geographical location of the structure to build. In the case presented before, prices were set for a given place, and depending on the abundance of certain materials the unitary prices can vary a lot.

The use of these models gives a good first estimation of a structure dimension and prices. However, it is advised not to rely just on it since some phenomena are still missing. Indeed, further studies could be made and implemented in these models regarding for instance the wave reflection on the structures as well as the wave transmission through the structures. Those parameters are relevant regarding the wave height in front of and behind the structure, which might be important for the tranquillity of the harbours and the ship movements for example.

Besides, these models have been realised according to the last updated design formulae and methods. That is why it is a major point to keep an eye out for new formulae, which seem likely to be modified each 5 years.

Finally, in the case of large projects, it is advised not to trust only the theoretical methods, but also to realise several small-scale tests to ensure that the designed structure has the required stability and height to counter the waves action, and carry out modifications if necessary.
8 References


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Pearson, J., Bruce, T., Allsop, W., Kortenhaus, A., & Van der Meer, J. W. Effectiveness of recurve walls in reducing wave overtopping on seawalls and breakwaters. 29th International Conference on Coastal Engineering (pp. 4404-4416). Lisbon: American Society of Civil Engineers.


Appendixes

Appendix I – Wave height estimation – bottom slope 1/10
Appendix II – Wave height estimation – bottom slope 1/20
Appendix III – Wave height estimation – bottom slope 1/30
Appendix IV – Wave height estimation – bottom slope 1/100
Appendix V – Maximum Wave height in the surf zone
Appendix VI – Relation maximum wave height/water depth
Appendix VII – Shoaling of non-linear waves
Appendix VIII – Friction coefficient $\mu$ values
Appendix IX – Foot protection block dimensions

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III
IV
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VII
VIII
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X
Appendix I – Wave height estimation – bottom slope 1/10

Wave Height Ratio, $\frac{H_{\text{max}}}{H_0}$

Wave Height Ratio, $\frac{H_{L/3}}{H_0}$

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Appendix II – Wave height estimation – bottom slope 1/20

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Appendix III – Wave height estimation – bottom slope 1/30

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Appendix IV – Wave height estimation – bottom slope 1/100

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Appendix V – Maximum Wave height in the surf zone

Note: \((H_{1/3})_{\text{peak}}\) refers to the maximum value of \(H_{1/3}\) in the surf zone.

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Appendix VI – Relation maximum wave height/water depth

Note: $(h_{\text{peak}})_{1/3}$ refers to the water depth at which $H_{1/3}$ takes the maximum value in the surf zone.
Appendix VII – Shoaling of non-linear waves

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Appendix VIII – Friction coefficient $\mu$ values

Levelling method for rubble mound

![Rough leveling](image)

![Fine-rubble leveling](image)

Bottom pattern of caissons

![Flat](image) ![Clog-shaped](image) ![Spike](image)

Table of Dynamic friction coefficients

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<th>Bottom pattern</th>
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<th>$\bar{\mu}_{max}$</th>
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$\mu_{max}$ dynamic friction coefficient corresponding to maximum tensile load
$\bar{\mu}_{max}$ dynamic friction coefficient corresponding to the average of the peak tensile loads
$\mu_{const}$ dynamic friction coefficient corresponding to constant tensile load
$\mu$ dynamic friction coefficient corresponding to caisson displacement $S$
$S$ caisson displacement
Appendix IX – Foot protection block dimensions

![Diagram showing foot protection block dimensions](image)

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Specification of outer block geometries in meters. Hole volume should be approx. 10% of total volume.